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Computational Methods in Structural Dynamics and Earthquake Engineering
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PREFACE

This volume contains the full-length papers presented in the 7th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2019) that was held on June 24-26, 2019 in Crete, Greece.

COMPDYN 2019 is one of the 32 Thematic Conferences of the European Community on Computational Methods in Applied Sciences (ECCOMAS) to be held in 2019 and is also a Special Interest Conference of the International Association for Computational Mechanics (IACM). The purpose of this Conference series is to bring together the scientific communities of Computational Mechanics, Structural Dynamics and Earthquake Engineering, to act as the forum for exchanging ideas in topics of mutual interests and to enhance the links between research groups with complementary activities. We believe that the communities of Structural Dynamics and Earthquake Engineering will benefit from their exposure to advanced computational methods and software tools which can highly assist in tackling complex problems in dynamic and seismic analysis and design, while also giving the opportunity to the Computational Mechanics community to be exposed to very important engineering problems of great social interest. The COMPDYN 2019 Conference is supported by the National Technical University of Athens (NTUA), the European Association for Structural Dynamics (EASD), the European Association for Earthquake Engineering (EAEE), the Greek Association for Computational Mechanics (GRACM).

The editors of this volume would like to thank all authors for their contributions. Special thanks go to the colleagues who contributed to the organization of the Minisymposia and to the reviewers who, with their work, contributed to the scientific quality of this e-book.

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- European Community on Computational Methods in Applied Sciences (ECCOMAS)
- European Association for Structural Dynamics (EASD)
- European Association for Earthquake Engineering (EAEE)
- Greek Association for Computational Mechanics (GRACM)
- Hellenic Society for Earthquake Engineering (HSEE)
- School of Civil Engineering, National University of Athens (NTUA)
- Hellenic Republic-Region of Crete
- Municipality of Heraklion

Plenary Speakers and Invited Session Organizers

We would also like to thank the Plenary and Semi-Plenary Speakers and the Minisymposia Organizers for their help in the setting up of a high standard Scientific Programme.

Plenary Speakers: Michel Bruneau, Álvaro Cunha, Michael Fardis, Charbel Farhat, Christian Soize, Alexander Vakakis

Semi-Plenary Speakers: Christoph Adam, Gian Paolo Cimellaro, Eleni Chatzi, Geert Degrande, Boris Jeremic, Shinobu Yoshimura

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EXPERIMENTAL ASSESSMENT OF STRENGTHENING STRATEGIES AGAINST THE OUT-OF-PLANE COLLAPSE OF MASONRY INFILLS IN EXISTING RC STRUCTURES

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Abstract

Past and more recent seismic events worldwide clearly showed that a crucial issue for life-safety and loss reduction due to earthquakes for existing reinforced concrete (RC) buildings is related to the out-of-plane (OOP) collapse of infill masonry walls. In literature, few studies addressed this paramount topic, above all about the proposal of strengthening strategies to prevent the infills’ collapse. This paper presents an experimental work about the assessment of possible strengthening solutions designed to mitigate or prevent the out-of-plane collapse of masonry infills in existing RC buildings. Three nominally identical full-scale one-bay-one-story RC frames were built and infilled with a thin masonry wall made up of horizontal hollow clay bricks. The first specimen was representative of the enclosure of a typical existing RC building in the Mediterranean region in its “as-built” condition. The remaining two specimens were strengthened against the out-of-plane collapse by means of two different strengthening techniques based on the application of innovative systems made up of high-ductility mortar plaster and fibre-reinforced polymer nets. All the tests consisted in the application of a semi-cyclic (loading-unloading-reloading) history of imposed displacements in the OOP direction by means of small pneumatic jacks through a uniform distributed load. Experimental results are shown in terms of OOP force-displacement responses, deformed shapes and damage evolution. In the end, the results of the tests are compared to assess the effectiveness of the selected strengthening techniques and to provide a support towards the choice of the best strategies for future further investigations and applications.

Keywords: RC buildings, masonry infills, out-of-plane collapse, strengthening strategies, experimental tests.
1 INTRODUCTION

Seismic events worldwide clearly showed that a crucial issue for life-safety and loss reduction due to earthquakes for existing reinforced concrete (RC) buildings is related to the out-of-plane (OOP) collapse of infill masonry walls. In the last thirty years, a quite limited number of experimental tests was presented in the literature on unreinforced masonry infills in RC frames under OOP loading (e.g., [1]-[4]). Even less studies addressed the paramount topic of the strengthening strategies to prevent the infills’ OOP collapse. The latter point is still a frontier issue for the most recent research works and it represents the focus of the present paper.

The integration of the infill panels on the substructure and respective improvement of the OOP collapse vulnerability can be achieved by using different techniques, such as: fiber reinforced polymers (FRP), engineered cementitious composites, textile reinforced mortars (TRM) or bed joints reinforcement. Experimental studies can be found in the literature concerning this topic by means of the realization of mechanical characterization tests on small panels or OOP tests in infill panels embedded in RC frames.

Regarding the TRM technique, Guidi et al. [5] carried out combined IP-OOP tests with the aim of characterizing the OOP behaviour of infill walls made with different types of masonry units, with and without reinforcement. The authors tested four specimens (two as-built and two strengthened) and the strengthening strategy consisted on the application of a special quadriaxial net made with hybrid glass fibers, that was casted in an extra fiber-reinforced plaster layer. From the test results, the authors concluded that strengthened specimens reached 30% higher OOP strength, due to the development of an arch mechanism, even for higher values of previous IP drift. Later, Koutas et al. [6] studied the development and performance of new textile-based anchors. Two different textiles were used as externally bonded reinforcement of the specimens: (i) a commercial textile with equal quantity of epoxy-coated glass fibers in two orthogonal directions; and (ii) lighter commercial textile (with a larger mesh size) made of elastomeric polymer-coated E-glass with equal quantity of fibers in two orthogonal directions. From the tests, the authors pointed that the increment of the fibers quantity in the anchors resulted in a non-proportional increase of the forces carried by them. The anchors placed between two layers of textiles were more effective, about 50%, than the ones placed on top of a single layer. Da Porto et al. [7] assessed the effectiveness of different plasters and TRM to retrofit infill panels made with hollow clay horizontal bricks subjected to combined IP-OOP tests. Different solutions were tested, namely: i) special lime-based plaster with geo-polymer binder; ii) bidirectional composite meshes with inorganic matrices (TRM); iii) TRM improved by anchorage of the mesh to the RC frame. From the results, the authors pointed out that the OOP behaviour of ‘light’ clay damaged panels after IP test clearly improved with plasters made of natural hydraulic lime, with or without geopolymer binder, reaching around 2.7 times higher OOP strength. The anchorages embedded in the TRM fixed to the upper beam did not change significantly the panel OOP capacity. The authors also pointed out the reduction of the displacement at the maximum strength and allowed reaching a controlled failure mode. Martins et al. [8] tested an innovative strengthening technique based on TRM technique with meshes based on composite rod composed by an external polyester protection. The idea of the authors was that the core provides the reinforcement and the external polyester braided rod contribute with the residual strength and deformation capacity which controls the damage and avoids brittle failure of the masonry units. Flexural strength tests perpendicular to the horizontal bed joints were carried out and the results indicated that the flexural cracking load increased between 7% to 34% for manufactured mesh with glass and carbon fibers, respectively. The authors also found that cracked stiffness was about 50% and 10% of the initial stiffness in the case of meshes made
with carbon fiber and glass fiber, respectively. The bending strength was higher in the specimens strengthened with the manufactured meshes comparatively with the equivalent commercial meshes.

All the studies herein presented revealed interesting results in terms of increase of the deformation capacity of the panels and prevention of brittle failures. From a technical point of view all of the techniques are effective if the retrofit material is well anchored to the surrounding frame and bonded to the panel. Different types of anchors were found in the literature, with different materials, with different application procedures, etc., but without proper design and application of the anchors, the strengthening strategy may not be effective.

This paper presents an experimental work performed in the Laboratory of Earthquake and Structural Engineering of the Civil Engineering Department of the University of Porto in cooperation with the Department of Structures for Engineering and Architecture of the University of Naples Federico II. The experimental tests presented herein are about the assessment of possible strengthening solutions designed to mitigate and prevent the OOP collapse of masonry infills in existing RC buildings.

Three nominally identical full-scale one-bay-one-story RC frames were built and infilled with a thin masonry wall made up of horizontal hollow clay bricks. The first specimen was representative of the enclosure of a typical existing RC building in the Mediterranean region in its “as-built” condition. The remaining two specimens were strengthened to reduce the out-of-plane collapse vulnerability by means of two different strengthening techniques based on the application of innovative systems made up of high-ductility mortar plaster and fibre-reinforced polymer nets. All the tests consisted in the application of a semi-cyclic (loading-unloading-reloading) history of imposed displacements in the OOP direction by means of small pneumatic jacks through a uniform distributed load. The mechanical properties of the adopted materials have been characterized and presented. The paper presents the experimental results analysed in detail in terms of OOP force-displacement responses, deformed shapes and damage evolution.

In the end, the results of the tests are compared to assess the effectiveness of the selected strengthening techniques and to provide a support towards the choice of the best strategies for future further investigations and applications.

2 EXPERIMENTAL TEST PROGRAM

Three nominally identical full-scale one-bay-one-story RC frames were built and infilled with a thin masonry wall made up of horizontal hollow clay bricks. The first specimen was representative of the enclosure of a typical existing RC building in its “as-built” condition. On the other two identical specimens, two different strengthening techniques were applied to prevent the out-of-plane collapse, both based on the application of high-ductility mortar plaster and fibre-reinforced polymer nets, as described in Section 2.1. Materials properties, instrumentation and setup adopted for these tests are briefly described in Sections 2.1 and 2.2.

2.1 Specimens descriptions

The testing campaign presented herein comprised three nominally identical full-scale, one-bay-one-story RC frames infilled with a thin masonry wall made up of horizontal hollow clay units. The infill panels’ geometric dimensions were defined as 4.20x2.30m (length and width respectively). The columns’ and beams’ cross sections were 0.30x0.30m and 0.30x0.50m, respectively. Figure 2 shows the schematic layout of the specimen geometry. All the infill panels have equal geometry with the above-mentioned dimensions, made of (300x200x100) mm hollow clay horizontal bricks (with 110mm thickness), as shown in Figure 1.
No reinforcement was used to connect the infill panel and the surrounding RC frame, and no gaps were adopted between the panel and the frame. A traditional mortar M5 class was considered a suitable choice for the construction of the panels. The testing campaign is composed by the as-built specimen (herein designated “AB-OOP”) and two retrofitted specimens (R1-OOP and R2-OOP respectively).

Concerning to the RC frame material properties, a C20/25 concrete was assumed and steel reinforcement A500 class were used (with a nominal mean yielding stress equal to 500MPa).

In the next sub-sections, the strengthening solutions adopted for each strengthened specimen (panels R1-OOP and R2-OOP) will be described. Table 1 reports a summary of the three specimens tested and analysed in the following sections.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>G-FRP net on the infill</th>
<th>Connection to the frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB-OOP</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>R1-OOP</td>
<td>Yes</td>
<td>No (Plastic connectors)</td>
</tr>
<tr>
<td>R2-OOP</td>
<td>Yes</td>
<td>Yes (L-shape bars connectors)</td>
</tr>
</tbody>
</table>

Table 1: Experimental tests.

2.1.1 Specimen R1-OOP
The strengthening solution adopted for the specimen R1-OOP was a textile reinforced mortar composed by a glass-fibre net designated “FASSANET ARG 40” commercialized by FASSA BORTOLO, with a matrix 4x4cm, a tensile strength equal to 56.25kN/m and a maximum ultimate strain equal to 3%.
The mortar used for the plaster was a ductile one, designated “SISMA” and commercialized also by FASSA BORTOLO. The mean compression and tensile strengths of the plaster mortar at the day of the test were around 24.4MPa and 6.7MPa, respectively. The net was positioned and fixed to the RC frame and to panel by means of plastic connectors. Thus, the application procedure of this strengthening strategy started by the application of 1cm plaster. Then the net was positioned and fixed with the plastic connectors. The roll of net was provided with 1 meter width and 50 meters length. Five vertical strips were used to strengthen the wall, as can be observed in Figure 3. The overlap length used between each vertical strip were assumed to be 10cm, and for the transition RC frame-infill panel it was assumed a duplicate net with an overlap equal to 30cm (15cm for the RC frame and 15cm for the infill panel). The disposition and distribution of the connectors is shown in Figure 3a, and the general view of the specimen R1-OOP is shown in Figure 3b. At the end, an additional 1cm layer of ductile mortar was applied, so that the final thickness of the retrofitting plaster was equal to 2cm.

![Figure 3: Specimen R1-OOP: (a) strengthening schematic layout and (b) general view.](image)

2.1.2. Specimen R2-OOP

The strengthening solution selected for specimen R2-OOP was similar to the one adopted for specimen R1-OOP. The difference among them was related to the anchorage of the net to the frame. In this case, L-shape connectors were used to fix the net to the RC frame (Figure 4).

The application procedure adopted to apply this connectors was: 1) application of the first layer of plaster with thickness equal to 1cm; 2) application of the net; 3) drilling a hole with \(\phi 6\)mm diameter and 10cm length for each connector; 4) full filling of the hole with epoxy resin
(provided by the manufacturer); 5) application of the L-shape connector; and 6) application of the second layer of 1cm plaster. The net and the plaster were the same used in the specimen R1-OOP.

Figure 4: Specimen R2-OOP: Detail of the L-shape connector.

2.2 Test Setup, instrumentation and loading protocol

The OOP test setup consists in the application of a distributed OOP loading through 28 pneumatic actuators that mobilized the entire infill panel surface resorting to wood plates with dimensions 0.5x0.5m² placed between the actuators and the panel. The pneumatic actuators were linked to four horizontal alignments performed by HEB140 steel shapes which reacted against five vertical alignments performed by HEB200 steel shapes. The horizontal alignments were coupled with hinged devices that allow lateral sliding. This steel reaction structure is a self-equilibrated structure designed with a concept similar to the previous experimental campaigns carried out by Furtado et al. ([1]-[2]). The steel structure is attached to the RC frame in twelve points (5 in the bottom and 5 in the top beam and 2 in middle-height columns) with steel bars that are coupled with load cells that allow monitoring the OOP loadings. Figure 5a and Figure 5b shows the schematic layout and the general view of the test setup.

Concerning to the instrumentation assumed for all the tests (Figure 5c), 34 displacement transducers were used to measure the OOP displacements of the panel, OOP displacements of the frame, relative displacements between the panel and the frame and vertical displacement of the top beam.

Lastly, the loading protocol consisted on the application half-cyclic OOP displacements (loading-unloading) that were imposed with steadily increasing displacement levels, targeting the following nominal peak displacements: 0.5, 1, 1.5, 2, 2.5, 3.5; 5; 7.5; 10mm; and so on 5 by 5 mm until a maximum OOP displacement of 120 mm. Two half-cycles were repeated for each lateral deformation demand level at the control node to evaluate the strength degradation.

3 EXPERIMENTAL RESULTS

In this Section, the results of the experimental testing procedure described above are presented and analysed test by test. OOP load (F_{OOP}) versus OOP displacement in the centre of the infill panel (d_{OOP, center}) responses are shown, together with the deformed shapes along given vertical alignments and damage evolution from the first observed cracks on the panel to the end of the test.
3.1 Specimen AB-OOP

Figure 6 shows the semi-cyclic OOP force-displacement response for the as-built specimen (AB-OOP) and the deformed shape of the panel along three vertical alignments (as shown in Figure 5c) at the peak load condition. First, note that the OOP displacement used in Figure 6a (and in the similar ones in the following analyses) is the displacement monitored by the displacement transducer located in the geometrical centre of the panel (W13 in Figure 5c).

The initial (secant) stiffness of this response – calculated as the ratio between $F_{\text{OOP}}$ and $d_{\text{OOP,centre}}$ at the first peak related to the first applied displacement level – is equal to $k_{\text{OOP,sec,in}}=8.89$ kN/mm. By increasing the applied OOP displacement, a first visible (macro-) cracking was observed on the panel for an applied OOP displacement in the centre equal to 2.5mm, at $F_{\text{OOP,cr}}=21.81$ kN (see Figure 6a). At this stage, a horizontal crack along a mortar bed joint occurred in the middle of the panel, as shown in Figure 7a. The secant stiffness related to this first cracking is thus slightly lower than the initial one, and in particular equal to 8.72 kN/mm. Secant stiffness progressively reduced during the test, and progressively wider cracks appeared in the panel, drawing on it a quite clear “pavilion” shape until the peak load is reached (see Figure 6b and Figure 7b). The pavilion-deformed shape highlights the existence of a double-arch (horizontal and vertical) resisting mechanism, as expected for an infill panel connected with the surrounding frame along four-edges [3]. The maximum OOP load corresponding to this stage was equal to $F_{\text{OOP,max}}=52.68$ kN at $d_{\text{OOP,centre,max}}=39.55$ mm. The corresponding secant stiffness thus reduced to 1.33 kN/mm. At about 45 mm of applied OOP
displacement, the infill panel totally collapsed out of its plane, after its detachment from the top beam, and the crushing of the hollow clay bricks in the compressed portions of the panel (see Figure 7c).

![Figure 6: Test AB-OOP - F_{OOP}-d_{OOP} response (a) and deformed shape along vertical alignments at F_{OOP,max} (b).](image)

![Figure 7: Test AB-OOP cracking pattern– First cracking (a); peak load (b); end of the test (c).](image)

### 3.2 Specimen R1-OOP

Figure 8 shows the OOP force-displacement response for the first retrofitted specimen (R1-OOP) and the deformed shape of the panel along five vertical alignments (see Figure 5c) at the peak load.

For this test, the initial (secant) stiffness of the response – calculated as explained before – is equal to $k_{OOP, sec, in}=29.15$ kN/mm, namely significantly higher (+228%) than the $k_{OOP, sec, in}$ related to the specimen AB-OOP. Such a difference is mainly ascribable to the presence of the plaster for the specimen R1-OOP. By increasing the applied OOP displacement, first visible (macro-) cracks were observed on the panel for an applied OOP displacement in the centre equal to 3.6 mm, at $F_{OOP,cr}=70.47$ kN (see Figure 8a). At this stage, hairline horizontal and vertical cracks appeared in the middle of the panel, as shown in Figure 9a. The secant stiffness related to this first cracking thus reduced to $19.58$ kN/mm. Secant stiffness progressively reduced during the test, and progressively wider cracks appeared in the panel, with additional diagonal cracks in the bottom portion of the panel, until the peak load was reached (Figure 9b). The maximum OOP load corresponding to this stage was equal to $F_{OOP,max}=95.95$ kN at $d_{OOP,centre,max}=15.00$ mm. At peak load, a significant detachment from the top beam was measured, as
shown in Figure 8b. From the achievement of the peak load to the end of the test, there were
the progressive widening of the central cracks, the detachment of the reinforcing plaster for the
top part of the frame, and a pronounced slippage of the plastic connectors from the top beam
and from the lateral columns (see Figure 9c).

Figure 8: Test R1-OOP - $F_{OOP}$-$d_{OOP}$ response (a) and deformed shape along vertical alignments at $F_{OOP,\text{max}}$ (b).

Figure 9: Test R1-OOP cracking pattern – First cracking (a); peak load (b); end of the test (c).

3.3 Specimen R2-OOP

Figure 10 shows the OOP force-displacement response for the retrofitted specimen R2-OOP
and the corresponding deformed shape at the peak load, as for the previous tests. For this test,
the initial secant stiffness – calculated as explained before – is equal to $k_{OOP,\text{sec.in}}=34.85$ kN/mm,
namely slightly higher than the $k_{OOP,\text{sec.in}}$ related to specimen R1-OOP, likely due to the stronger
degree of connection between the retrofitting plaster on the panel and the frame. For increasing
applied OOP displacement, a first visible (macro)-cracking was observed on the panel, at
$F_{OOP,\text{cr}}=89.73$kN and $d_{OOP,\text{center}}$ equal to about 3 mm (see Figure 10a). At this stage, a hairline
horizontal crack appeared in the middle of the panel together with some smaller vertical cracks
on the bottom, as shown in Figure 11a.

Secant stiffness progressively reduced, and progressively wider cracks appeared in the panel,
with additional diagonal cracks in the bottom portion of the panel, vertical central cracks and
horizontal cracks at the infill-top beam interface, until the peak load was reached (Figure 11b).
The maximum OOP load corresponding to this stage was equal to $F_{OOP,\text{max}}=116.70$kN at
d_{OOP,\text{center,max}}=15.34$mm. The above-mentioned horizontal cracks at the infill-top beam interface
highlighted the increasing OOP sliding of central bricks on the top of the panel (visible on the backside of the wall and measured by the top LVDTs, as shown in Figure 10b) involving “monolithically” bricks and retrofitting plaster.

From the achievement of the peak load to the end of the test, there were the progressive widening of the central cracks, the crushing of some clay bricks in the bottom and a slight OOP sliding along the infill-bottom beam interface. The damage state at the end of this test, at $d_{OOP, centre}$ practically equal to the infill wall thickness, is shown in Figure 11c. It is worth noting that, at the end of the test, the system “infill panel + retrofitting plaster” detached from the upper part, but it still remained connected along the columns and to the bottom part of the frame. In the top of the panel where the sliding was observed, at the end of the test, the connectors were still in-situ, but the glass fibre net was locally cut around the connectors.

![Figure 10](image1.png)

Figure 10: Test R2-OOP - $F_{OOP}$-$d_{OOP}$ response (a) and deformed shape along vertical alignments at $F_{OOP,max}$ (b).

![Figure 11](image2.png)

Figure 11: Test AB-OOP cracking pattern – First cracking (a); peak load (b); end of the test (c).

4 COMPARISONS OF THE RESULTS

Figure 12 shows a comparison among the test results presented in the previous section, in terms of $F_{OOP}$-$d_{OOP,center}$ envelope (Figure 12a) and of secant stiffness ($k_{sec}$) evolution (Figure 12b). Please, note that envelopes in Figure 12a are shown until the last first-cycle peak. Additionally, Table 2 provides a summary of the results commented above.

It can be noted that the maximum $F_{OOP}$ for the retrofitted specimens are 1.82 and 2.22 times the $F_{OOP,max}$ related to the AB-OOP specimens, for tests R1-OOP and R2-OOP, respectively. This aspect can be particularly important for typical code-based safety checks for the out-of-
plane collapse of masonry infills, since they are generally carried out in terms of strength (e.g. [9], [10]).

Significant force increment are observed at the first (macro-) cracking condition: $F_{OOP,cr}$ is 3.23 and 4.11 times the related value for the AB-OOP specimen, for tests R1-OOP and R2-OOP, respectively. Secant stiffness is also significantly affected by the presence of the retrofitting plaster, by increasing of at least of $+228\%$ with respect to AB-OOP specimen.

On the contrary, the OOP displacement at the peak OOP load ($d_{OOP,center,max}$) is about the 40% of the related displacement of AB-OOP specimen for both the retrofitted tests. The displacements corresponding to 20% of strength reduction (namely, corresponding to the 80% of the maximum load) on the envelopes ($d_{OOP,center,u,80\%}$) are also reported in Table 2. The corresponding deformation capacity, calculated as the ratio between $d_{OOP,center,u,80\%}$ and $d_{OOP,center,max}$, are 53% and 43% higher than the reference specimens AB-OOP, for specimens R1-OOP and R2-OOP, respectively.

![Figure 12: Comparison of the results - $F_{OOP}$-$d_{OOP}$ envelopes (a) and secant stiffness evolutions (b).](image)

<table>
<thead>
<tr>
<th></th>
<th>AB-OOP</th>
<th>R1-OOP</th>
<th>R2-OOP</th>
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<th>R2/AB</th>
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<tr>
<td>$F_{OOP,cr}$ (kN)</td>
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<td>$d_{OOP,center,max}$ (mm)</td>
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<td>$d_{OOP,center,u,80%}$ (mm)</td>
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<td>$\mu_{OOP,center,u,80%}$ (-)</td>
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<td>1.76</td>
<td>1.65</td>
<td>1.53</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Table 2: Comparisons of the results.

An additional interesting comparison among the presented test results can be carried out in terms of observed “failure mode”, described in the previous sub-sections. Certainly, the most critical point of this kind of retrofitting strategy is the connection between the system “ductile mortar plaster + fibre-net” and the surrounding frame. An effective connection is necessary to prevent a premature physical collapse of the panel out of its plane. Actually, for the retrofitted specimen with an effective plaster-frame connection (R2-OOP), the system “infill panel + retrofitting plaster” did not collapse out of its plane for an OOP displacement equal to the infill thickness. Nevertheless, to improve the ductility of this retrofitting system, particular care should be still addressed to the proper definition of the typology of the connectors and their spacing. To this aim, future desirable experimental tests should provide additional useful data.
5 CONCLUSIONS

This paper presented an experimental work performed in the Laboratory of Earthquake and Structural Engineering of the Civil Engineering Department of the University of Porto in cooperation with the Department of Structures for Engineering and Architecture of the University of Naples Federico II, about the assessment of possible strengthening solutions designed to mitigate or avoid the out-of-plane collapse of hollow clay infills in existing RC buildings.

Three nominally identical full-scale one-bay-one-story RC frames were built and infilled with a thin masonry wall. The first specimen was representative of the “as-built” condition. The remaining two specimens were strengthened to improve the out-of-plane collapse vulnerability by means of two different strengthening techniques based on the application of high-ductility mortar plaster and fibre-reinforced polymer nets. All the tests consisted in the application of a semi-cyclic (loading-unloading-reloading) history of imposed displacements in the OOP direction by means of small pneumatic jacks through a uniform distributed load.

The experimental results have been showed in terms of OOP force-displacement responses, and damage evolution, and compared to each other. It was observed that the OOP strength capacity at OOP load at first cracking significantly increases (more than +200%) for the retrofitted specimens with respect to the as-built reference test, mainly due to the significant tensile strength of the adopted fibre-reinforced mortar. Similarly, the OOP secant stiffness significantly increases, as expected. On the contrary, the infill OOP displacement at peak load reduces in retrofitted infills by about 60%. Nevertheless, note that, for the retrofitted specimen with an effective plaster-frame connection, the system “infill panel + retrofitting plaster” did not collapse out of its plane for an OOP displacement equal to the infill thickness.

In conclusion, certainly the presented data can be useful to provide a support towards the choice of the best strategies for future further investigations and applications. Additional experimental data will be certainly important to improve the OOP retrofitting system for masonry infills, with particular care to the plaster-frame connection system.

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REFERENCES


INFLUENCE OF INFILL PANELS AND FLOOR SYSTEM IN THE FRAGILITY ANALYSIS OF EXISTING RC BUILDINGS: A CASE STUDY

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Abstract

In the last few years, the study of the seismic fragility of existing Reinforced Concrete (RC) buildings has been the object of a wide interest within the scientific community, especially after the experiences provided by recent hazardous events, which have shown the vulnerability of the existing RC building stock. The recent studies about this topic have shown that the probabilistic approach of Performance Based Earthquake Engineering (PBEE) is the most suitable methodology for taking into account the different sources of uncertainty in fragility analysis, considering both the nature of the seismic demand and the problems related to the incomplete knowledge. For an accurate probabilistic seismic fragility estimation, a fundamental aspect is to consider the contribution of all the elements that constitute the building system, such as the secondary structural (floor system) and non-structural elements (infill panels).

In this paper, the focus is aimed on the investigation of the role of secondary and non-structural elements in the evaluation of fragility curves of existing RC buildings. In particular, a real case study, representative of the typical existing school buildings of the Southern Italy, is analysed and discussed.

Keywords: PBEE; seismic fragility; existing RC buildings; infill panels; floor system
1 INTRODUCTION

The seismic vulnerability of existing RC building is one of the main topics of interest for the scientific community, considering the large economic losses on the existing building stock due to the past seismic events. In particular, the buildings designed according to old technical codes are not built for counteracting seismic actions, which are unpredictable and in continuous evolution, as occurs in the Mediterranean zone. Consequently, the necessity of an effective assessment of existing RC buildings is primary for avoiding the large economic losses due to the damage of buildings and for providing reliable retrofit solutions [1-3].

In this view, one of the most suitable tools is provided from the approach of Performance Based Earthquake Engineering (PBEE) [4], which allows to estimate the damages and the losses resulting from possible earthquakes in a probabilistic way. In particular, through the application of a mathematical formulation, the methodology provides a probabilistic distribution in terms of output metrics and describes the performance of the building analyzed in terms of needs and objectives of owners.

Generally, the approach is developed in four steps schematized as follows:
- a probabilistic definition of the seismic demand to which the structure is subjected;
- a probabilistic definition of the structural capacity, accounting for several and increasing intensity of the seismic demand;
- a probabilistic definition of the damage of the structure, as function of an Engineering Demand Parameter (EDP) and of an Intensity Measure (IM), previously defined;
- an estimation of the possible economic losses due to the damages that the building can suffered.

For a correct estimation of the losses and damages, one of the most important phases is the structural modelling of the building, since the initial hypotheses play a fundamental role in the results of the structural behaviour. In particular, the structural model, which is typically a Finite Element (FE) one, can incorporate the presence of the elements that really constitute the building, such as the infill panels and the floor system, according to different modelling hypotheses [5-7], and account for the uncertainty of the mechanical parameters for which there is typically an uncompleted knowledge [8-10]. In addition, in this phase, it is possible to account for all failure mechanisms, such as bending and shear for the structural elements and beam-column joints [11-12]. Clearly, for a detailed numerical model, the analyst has to consider these latter elements, in order to make sure for the result. Nevertheless, on the other side, this means an increment of the time and computational efforts of the analysis, which leads to an ordinary simplification of the numerical modelling phase.

The aim of this paper is to investigate the role of the secondary structural element (floor system) and non-structural elements in the structural analysis phase of existing RC buildings and, subsequently, its effect in the damage response, as well as fragility curves. To this scope, a real case study has been analysed, which is an existing RC school building in the Province of Foggia, Southern Italy, representative of the similar buildings in the entire geographic zone.

2 ROLE OF FLOOR SYSTEM AND INFILL PANELS IN THE SEISMIC BEHAVIOR OF EXISTING RC BUILDINGS

The observations about the damages on existing RC buildings after the seismic events have shown the fragility of the building stock, with reference to both structural, secondary structural and non-structural elements [13-14]. In particular, also the earthquakes with low intensity damage the infill panels and the floor system, with related economic losses that can be a greater part of the future retrofit cost for the building.
The typical phenomena that occur in these elements, with reference to typical existing RC buildings in Mediterranean areas, can be displayed in figure 1, such as the in-plane failures of the infill panels (and the out of plane failures) and the hallow bricks crumbling for the floor system, considering a constructive systems constitute by ribbed slab with RC joists.

![Figure 1: Post-earthquake damages on infill panels and floor system](image)

Hence, in order to identify how to account for the contribute of secondary structural and non-structural elements in numerical models, the next paragraphs show the behavior under seismic actions and the numerical modelling, in a context of global analysis, for the abovementioned elements.

### 2.1 Floor system: behavior under seismic actions and numerical modelling

In the evaluation of the performance of existing RC buildings, the floor system plays a fundamental role for several reasons. First of all, the slab is a system, which is orthogonally loaded from gravity and vertical actions. This means that the primary role of the floor system is to transmit the gravity actions to the structural skeleton and then, it is usually designed accounting for the maximum effects of all vertical loads.

From the horizontal actions point of view, the role of the floor system is to transmit the loads among the vertical elements, proportionally to them stiffness. To this scope, in new designs, the slab must be intended as infinitely rigid. However, in the existing buildings, this latter condition is not always ensured, due to design mistakes and decay of in-place materials. In addition, as shown in [15], the real in-plane stiffness of a RC ribbed slab depends from other parameters, such as the number of storeys, the in-plane shape ratio, the dimension of vertical elements, the orientation of the joists and so on. The main effect of this condition lead to a different distribution of the stresses among vertical elements, which are loaded in low predictable way.

Generally, in FE models, the usual hypothesis to assume is the rigid floor, through the application of an internal constraint among each node of the floor. For accounting the real stiffness of the floor system, the scientific literature and some international technical codes purposes some modelling methodologies, to apply on 3D FE models, able to simulate the deformability of the floor under horizontal actions and then, the real distribution of the stresses among the structural vertical elements. The more simplified procedure is provided from the “strut model” [16], which consists in the simulation of the fields of the slab with two cross equivalent struts, with dimensions computed through the equivalence between the slab stiffness \( K_{\text{slab}} \) and the equivalent strut stiffness \( K_{\text{strut}} \). \( K_{\text{slab}} \) is defined in equation 1, \( K_{\text{strut}} \) is defined in equation 2 and from their equivalence, it is possible to define the unique unknown term, which is the strut section \( A_{\text{sec}} \). The other terms indicate the slab dimension orthogonal to seismic action \( L' \), the inertia moment of slab section \( J \), the shear area of section slab \( A_s \), the elastic modulus of
slab material ($E_s$), the shear modulus of slab material ($G_c$), the elastic modulus of strut material ($E_s$) and the length of the strut ($L_s$).

\[
K_{\text{slab}} = \frac{1}{12}\frac{L^3}{E_s} + \frac{L}{A_s G_c}
\]  

\[
K_{\text{strut}} = \frac{E_s A_s}{L_s}
\]

The use of this modelling methodology allows of avoiding of a-priori hypotheses about floor stiffness in both linear and non-linear analyses [10, 17].

For an accurate estimation of the slab, the literature suggests the use of macro-models (shell elements), which can be calibrated on experimental tests [18] and sensitivity analyses performed on detailed micro-modelling [5-7]. In the case of RC ribbed slab, the macro-model is going to be orthotropic, because it behaves differently in the two in-plane direction.

The main advantage of these kind of models, despite the increment of computational efforts, consists in the possibility to compute the in-plane stresses due to seismic actions, in order to assess the performance of the elements that constitute the floor system (always avoiding a-priori hypotheses about floor stiffness).

2.2 Infill panels: behavior under seismic actions and numerical modelling

The non-structural elements as the infill panels can assume a primary role in the study of RC buildings behavior, especially for the existing ones. In fact, in this latter case, the masonry panels are conceived as directly linked with the surrounding frame, which changes the structural behavior (strongly influenced under seismic actions) and makes the infill panels contribute important and not negligible in the structural analysis.

Generally, the masonry infills in existing RC buildings can induce from one hand a benefit effect, with an increment of energy dissipation, stiffness and strength [19] and subsequent reduction of the horizontal displacement due to seismic actions. On the other hand, an increment of the stiffness, with a reduction of fundamental periods, can cause an increment of the seismic demand, which the structure is not able to counteract. This effect can lead premature local collapses, induced in the structural elements, with a change of global equilibria of the stress states. In addition, as highlighted in [20], infills panels can provide a change in terms of global structural behavior, due to the addition of torsional effects and development of soft-storey mechanisms, as in the cases of pilotis.

Leaving the out of plane behavior of infill panels, an accurate structural modelling has to predict the in-plane mechanisms, with the subsequent effects on the elements that constitute the surrounding frame. In particular, as highlighted in [21], the failure mechanisms to consider can be summarized as the diagonal compression failure, the diagonal cracking failure, the sliding shear failure, the corner crushing failure and the frame failure crushing.

It is clear that, a FE model can predict the abovementioned typologies of failures, based on the accurateness of the masonry panel modelling. With this regard, the nonlinear behavior of infill panels can be simulated through models to different scales, with different computational efforts in the analysis.

The scientific literature provides several modeling methodologies and, in a practice-oriented view, the macro-models to equivalent struts are the more used and developed. In particular, as shown in [22], the calibration of the linear and nonlinear behavior equivalent strut depends from the setting of three parameters, which are the width of the strut ($b_w$), the constitutive law of the panel and the number of the struts used. In [23], a lot of formulations by scientific literature for
these three parameters are provided. In this work, the infill panels are going to be modelled to single strut for each panel and direction (2 cross braces), by computing the slenderness of the strut ($\lambda$), useful for the elastic stiffness, according to equation 3 [24] and by considering as nonlinear constitutive law, the model proposed in figure 2 [25].

$$\lambda = \sqrt{\frac{E_w t_w \sin 2\theta}{4 E_c I_p H_w}}$$  

(3)

The terms of equation 3 indicate the elastic modulus of the masonry ($E_w$), the thickness of the panel ($t_w$), the slope angle of the panel’s diagonal ($\theta$), the elastic modulus of concrete ($E_c$), the moment of inertia of column adjacent to panel ($I_p$) and the height of panel ($H_w$).

Figure 2: Constitutive law proposed in [25]

3 SEISMIC FRAGILITY OF EXISTING RC BUILDINGS

A fundamental phase of vulnerability analysis of existing RC buildings, by using the paradigm of the PBEE, is the study of the seismic fragility, which provide a complete information about the possible damages due to increasing seismic action, accounting for the existing uncertainty sources present in the problem [26]. The fragility of buildings can be formally defined as a lognormal function, which defines the structural behaviour, conditioned by an established IM. In other words, this function provides as result some fragility curves, which can be obtained from the probability of violating some limit states, to which some given values of IM are associated. As reported in [27], fragility curves are ruled by mathematical relationships, like the one in equation 4:

$$F_{LS}(IM) = P(\text{Limit State violated}|IM) = P(IM > IM_c|IM)$$  

(4)

where $IM_c$ is the capacitive IM, which correspond to the EDP defined for the limit state to investigate.

Hence, a fragility curve depends from the IM associated to a limit state (and the related EDP) to investigate and this, in turn, depends from the FE model of the case study. As abovementioned, the modelling or not of the secondary structural and non-structural elements, strongly modify the structural behaviour and, consequently, the building fragility.

If for numerical models that account for the seismic behaviour of infill panels, the scientific literature provides some studies [28-30], it does not present cases in which the floor system is considered. In particular, based on the results obtained in [5, 23], the presence of infill panels provides an increment of the stiffness of vertical elements, which can induce a greater deformability of floor system, especially when infill are defined as ‘strong’ (with mechanical features with high stiffness – strength values). This means that the modelling or not of the floor system can induce some differences in the definition of fragility curves.
3.1 Case study

The case study analysed is an existing RC school building located in the city of Cerignola, in the Province of Foggia, Apulia Region, Southern Italy. The building is a part of a scholastic complex built in 70’s and it was designed accounting for only gravity loads and in absence of specific details for seismic actions. The knowledge of building is provided for an investigation carried out within an Agreement between “AdB Puglia” and Polytechnic University of Bari [31] and all geometrical features of the structural elements and mechanical parameters of in-situ materials were extensively investigated and available.

![Figure 3: Structural information about case study](image)

The structural configuration of the building with indication about structural elements, the data about the mechanical features of in-situ materials provided from destructive and non-destructive tests and the loads acting on the building are summarized in figure 3. This provides the values of $f'_{cm}$ and $f'_{ym}$, which are respectively the in-situ compressive strength of concrete and steel rebar and the loads $G_1$, $G_2$ and $Q$, which are respectively the gravity permanent and the live loads.

The building has a regular in-plane shape, 2 floors above ground and a staircase in one corner. The slab typology is a RC ribbed slab, with constant joists dimensions (height 20 cm, width 10 cm, spaced 50 cm) and thickness of top concrete slab of about 4 cm. The orientation of the slab is in Y direction without resistant frames, according to reference system in figure 3. Plinths connected by beams constitute the foundations. Concerning to the masonry infills, through some investigation tests, they were characterized, according to the parameters reported in table 1.
Table 1: Mechanical parameters of infill panels

<table>
<thead>
<tr>
<th>Mechanical parameters</th>
<th>Value (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_w$</td>
<td>3080</td>
</tr>
<tr>
<td>$E_w\theta$</td>
<td>1495</td>
</tr>
<tr>
<td>$G_w$</td>
<td>1233</td>
</tr>
<tr>
<td>$f_{tp}$</td>
<td>0.36</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>2.5</td>
</tr>
</tbody>
</table>

3.2 Numerical models and eigenvalue analyses

Based on the data collected, the structural FE model of the building has been modelled by using the software OpenSees [32]. In particular, the behavior of beams and columns have been simulated through a lumped plasticity approach by using `beamWithHinges` model, in which the constitutive law of plastic hinges is simulated through the Pinching4 material. Each hinge is able to account for the concepts developed by Ibarra [33] for properly weighting the hinges versus the elastic beam. The laws assumed, in terms nonlinear moment-curvature of the sections, have been computed according to Eurocode 8 [34]. For both beams and columns, the unique mechanism accounted is the bending, considering for the columns a constant axial stress, derived by seismic combination of loads. All columns are fixed to the ground, through external restraints; the present staircase is modeled considering its influence, in terms of masses, on competence beams. Based on the topic of the work, the brittle shear mechanisms and possible nodes’ failures have been neglected, avoiding add other variables in the problem to investigate.

In order to investigate the fragility of the case study, accounting for the secondary structural and non-structural elements, four numerical models have been developed, as shown in figure 4, based on the structural skeleton previously defined, in the order and with the assigned tag of the list below:

- B1 – Rigid floor – Bare frame;
- B2 – Flexible floor – Bare frame;
- B3 – Rigid floor – Infilled frame;
- B4 – Flexible floor – Infilled frame.

The flexible floor has been modelled by using the strut model and the modelling parameter of each strut has been calibrated according to eqs. 1 and 2, while the rigid floor has been modelled through internal constraints. The infill panels have been simulated as declared in the section 2.2, according to eq. 3 and fig. 2. Each strut has been modelled with `corotationalTruss` element. Firstly, for each model, an eigenvalue analysis has been performed and table 2 reports the values obtained, in terms of periods. The results show that models B1 and B2 are similar (as expected), while models B3 and B4 present some differences, considering higher values in B4, due to the flexibility of the floor.

<table>
<thead>
<tr>
<th></th>
<th>$T_1$ (s)</th>
<th>$T_2$ (s)</th>
<th>$T_3$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.903</td>
<td>0.6758</td>
<td>0.6265</td>
</tr>
<tr>
<td>B2</td>
<td>0.906</td>
<td>0.681</td>
<td>0.6295</td>
</tr>
<tr>
<td>B3</td>
<td>0.2131</td>
<td>0.1297</td>
<td>0.1231</td>
</tr>
<tr>
<td>B4</td>
<td>0.2544</td>
<td>0.1682</td>
<td>0.1413</td>
</tr>
</tbody>
</table>

Table 2: Periods of the models investigated
3.3 Seismic behavior through nonlinear analyses

On each model previously defined, pushover analyses have been performed in both direction, by applying only the uniform load pattern and neglecting the eccentricity. The scopes of pushover analysis are manifold. In fact, firstly, it is going to be analysed the FE models response in the inelastic field and after, the definition of the EDP parameters for the definition of the limit states thresholds, according to [35]. The results, depicted in figure 5 as base shear ($V_b$) versus roof displacement ($\delta_R$), show that the model B4 is more flexible then model B3, with a shift of the pushover curve peak, due to the greater deformability of the floor. In addition, considering the properties of in-situ masonry, the infills contribute goes to higher displacement values. This effect leads to condition of the EDP values for identifying the limit states. In fact, according to the common practice, the achievement of the limit states is defined with a local criterion. This means that, by considering a flexible floor, the serviceability limit state (IO – Immediate Occupancy) is achieved later than the equivalent model with rigid floor and vice versa for the ultimate limit states (LS – life safety, NC – near collapse).

Table 3 shows the values of the EDP chosen (max $\theta_i$ – maximum interstorey drift), for all models and all limit states accounted. In particular, the criteria selected for the achievement of the limit states are summarized in the list below:

- IO – achievement of the failure for the first panel in a storey or 0.5% for bare models;
- LS – achievement of the ¾ of ultimate rotation for a certain percentage of structural elements (about 50%);
- NC – achievement of the ultimate rotation for the first column.
Figure 5: Pushover analysis in Y direction for numerical models

Table 3: EDP values for the models investigated

<table>
<thead>
<tr>
<th></th>
<th>B1 - max θi [%]</th>
<th>B2 - max θi [%]</th>
<th>B3 - max θi</th>
<th>B4 - max θi</th>
</tr>
</thead>
<tbody>
<tr>
<td>IO</td>
<td>0,5</td>
<td>0,5</td>
<td>0,168</td>
<td>0,522</td>
</tr>
<tr>
<td>LS</td>
<td>3,55</td>
<td>3,55</td>
<td>3,44</td>
<td>3,21</td>
</tr>
<tr>
<td>NC</td>
<td>4,41</td>
<td>4,41</td>
<td>4,32</td>
<td>4,15</td>
</tr>
</tbody>
</table>

In order to study the structural capacity in probabilistic terms, Incremental Dynamic Analyses (IDAs) have been performed on each numerical model [36]. The seismic input is represented by 30 accelerograms, selected according to intensity level related to PGA of the building site, which is equal to 0.212g (medium intensity). In particular, all records have been taken from the INNOSEIS database [37], which collects some records of high and medium intensity ones.

All IDAs are plotted in figure 6, where the selected IM is the $S_{agm}(T_{1-3})$, which is the geometric mean of the spectral accelerations related to the first three periods of the building, computed as in equation 5 and according to the methodology proposed in [38].

$$S_{agm}(T_{1-3}) = \left[ \prod_{i=1}^{3} S_a(T_i) \right]^{1/3} \quad (5)$$

In figure 6, IDAs on rigid floor models (B1 and B3) are reported in light grey, while the IDAs on flexible floor models are reported in dark grey. As expected, the IDAs trend of B1 and B2 is similar and different from the one of B3 and B4. The presence of strong infill increases the seismic action to which building can be subjected.

3.4 Seismic fragility curves under different scenarios

The structural analysis previously performed is propedeutic to the identification of the structural damages. In particular, adopting an IM-basis criterion, it is possible to compute the lognormal distribution of EDP, for each IM value and then, to compute each median and dispersion.

Through the conditioning of the structural capacity with the EDP value for each limit state, fragility curves have been computed. With this regard, figure 7 shows the results in terms of damage states. In particular, green curves indicate IO fragility ones, blue curves indicate LS fragility ones and red curves indicate NC fragility ones. The continuous lines indicate the
fragility curves on the rigid floor models (B1 and B3), while dotted lines indicate the fragility curves on the flexible floor models (B2 and B4).

The results show that for bare frame models, the fragility curves for rigid floor and flexible floor models are similar, for all limit states, which means that, as expected, the modelling of the floor deformability is not necessary.

Different result is shown for the infilled models. In particular, for all limit states the fragility curves are shifted to right. This result suggests that at the same IM value, the probability of exceeding a certain limit state, for a given EDP, is higher in the model with flexible floor, evidence due to a different distribution of the seismic action among structural elements, which lead to the anticipation of the failure mechanisms in some ones.

Figure 6: IDAs on numerical models

Figure 7: Fragility curves for the building, to serviceability and ultimate limit states
4 CONCLUSIONS AND FURTHER DEVELOPMENTS

The work presented the effects of the floor deformability of existing RC buildings under seismic actions. This aspect, strongly conditioned by some geometrical parameters of the buildings, as the number of storeys, the in-plane shape ratio, the dimension of vertical elements, the orientation of the joists and so on. The presence of the infill panels, as shown from the scientific literature, increases the stiffness of the vertical system of the building and, if the masonry can be defined as “strong”, the failure of the struts that simulate the infills behavior can occur to high displacements.

The situation previously defined happens in the case study analyzed, an existing RC school building in the Province of Foggia, Southern Italy. In particular, the structural behavior of the building has been investigated through 4 FE models, accounting and not for the influence of floor system (modelled by using a strut method) and infill panels (modelled through 2 cross braced).

For each model, eigenvalue and pushover analyses have been carried out, showing the different behavior obtained through different modellings. Furthermore, in a PBEE view, IDAs have been performed, in order to provide fragility curves for the building at each limit state. Local criteria have been fixed for determining the achievement of the limit states, according to a practice-oriented use.

The results in terms of damage states, suggest that the modelling of the building with the rigid floor, can induce to have results not conservative. In fact, the floor deformability can lead to a different distribution of the stresses due to seismic actions, with a premature failure of structural elements and a greater probability of violating a certain limit state.

The future development of the work is the analysis of the same problem here shown, by considering possible retrofit solutions that increase the stiffness of the vertical elements, such as reinforced infills or the addition of RC walls on the building side.

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EXPERIMENTAL ASSESSMENT OF STRENGTHENING STRATEGY TO IMPROVE THE MASONRY INFILLS OUT-OF-PLANE BEHAVIOR THROUGH TEXTILE REINFORCED MORTAR

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Abstract

Throughout the last years, the study of the masonry infill walls out-of-plane behavior is being observed by the scientific community with special attention, mainly due to damages observed during post-earthquake scenarios, where several number of damages and out-of-plane collapses of infill panels were found. Different causes are pointed to justify the seismic vulnerability of these type of elements, most of all related to deficient construction practices, which are nowadays still applicable. Based on this motivation, two experimental quasi-static, full-scale, out-of-plane tests were carried out on RC frames that were built and infilled with a thin masonry wall made up of horizontal hollow clay bricks. The first specimen is representative of the enclosure of a typical existing RC building in the Southern countries in its “as-built” condition. The second specimen was strengthened with textile-reinforced mortar using glass fiber mesh. Both specimens were subjected to semi-cyclic (loading-unloading-reloading) history of imposed displacements by means of small pneumatic jacks through a uniform distributed load. Experimental results will be presented and detailed in terms of out-of-plane force-displacement responses and damage evolution. In the end, the results of the tests are compared to assess the effectiveness of the strengthening technique.

Keywords: masonry infill walls, out-of-plane testing, strengthening strategies, textile-reinforced mortar.
1 INTRODUCTION

One of the major challenges concerning the earthquake risk mitigation is the vulnerability assessment of existing buildings that were not designed according to modern codes and the development of effective retrofit techniques. Over the last few years, it is visible an increase of interest regarding the study of the masonry infill walls and their influence in the response of reinforced concrete buildings when subjected to earthquakes. Recent post-earthquake damages survey reports recognized that the masonry infill walls played an important role in the seismic response of the reinforced concrete buildings. Similarly, the infill panels’ seismic behavior is being characterized by extensive damages and collapses due to combined in-plane and out-of-plane (OOP) loadings. The infill panels’ OOP collapse is being responsible by innumerous fatalities, material and economical losses [1-3].

Different authors reported that the masonry infill walls’ OOP behavior is strongly affected by the following issues [4, 5]: existence or not of connection between the panel and the reinforced concrete frame elements; existence of not of connection between leafs (in case of double-leaf infill walls); inadequate panel’ width support (very common constructive procedure adopted for thermal bridges’ prevention), boundary conditions, panel slenderness, inadequate construction execution of the last horizontal bed joint and lastly, the existence of previous damage. The infill panels’ collapse can result in plan and/or vertical irregularities, which can trigger global failure mechanisms.

Considering the small number of experimental and numerical studies in this field and based on the well common masonry infill walls’ presence in the reinforced concrete buildings in Portugal, it is fundamental to carry out studies to characterize the seismic behavior of these panels and to develop efficient retrofit strategies that will improve their performance and prevent their collapse when subjected to earthquakes.

The present work focusses in the assessment of the efficiency of a strengthening technique based on textile-reinforced mortar (TRM) to improve the OOP behavior of a full-scale infill panel. For this, two specimens tested to OOP loadings applied by pneumatic actuators. The first specimen is representative of the enclosure of a typical existing RC building in the Southern countries in its “as-built” condition. The second specimen was strengthened with textile-reinforced mortar using glass fiber mesh. Experimental results will be presented and detailed in terms of out-of-plane force-displacement responses and damage evolution. In the end, the results of the tests are compared to assess the effectiveness of the strengthening technique.

2 EXPERIMENTAL CAMPAIGN

2.1 Specimens’ description

The experimental campaign was composed by two quasi-static OOP tests of full-scale infill panels by using pneumatic actuators that applied a uniform load. The geometric dimensions of the tested panels are 4.20x2.30m (length and height, respectively), representative of building stock existing in the Southern European Countries [6] (Figure 1). The infill panel is surrounded by an envelope reinforced concrete frame, which is composed by columns and beams with sections 30x30cm² and 30x50cm², respectively.

Both panels were built with horizontal hollow clay bricks which geometric dimensions are 150x200x300mm (thickness, height and length respectively), without any mechanical connection and any gap between the panel and the envelope frame. Both panels were built aligned with the external side of the RC beam as well as using a typical workmanship. Concerning the remaining materials, it was selected a traditional mortar type M5, concrete class C20/25 and reinforcement steel class A500.
The as-built panel, herein designated “INF_09” was built and tested until the collapse. After the panel removal, it was built a new panel, which was strengthened 10 days after. This panel, herein designated “INF_10” was strengthened using TRM solution with a glass fiber net. Fassa Bortolo provided the net selected, which was the FASSANET ARG 40, with a matrix 4x4cm and a tensile strength equal to 56.25kN/m. Metallic connectors (ϕ6mm and 8cm length) ensured the connection of the net to the panel and that two of the brick walls were crossed by (Figure 2a and 2b). In order to ensure a better fixation of the net to the wall, plastic discs with a diameter of 6 cm were used in the top of the metallic connector. Regarding the anchorage of the net to the RC frame it was designed a solution made with M8 metallic connectors with a plastic disk with 6cm diameter in the top, as can be observed in Figure 2c and 2d. Thus, the application of the strengthening followed the following steps:

1) Application of a first layer of plaster (thickness around 0.5cm);
2) Placement and positioning of the net;
3) Fixation of the net with the connectors;
4) Application of a second layer of plaster 2cm thick.

The mortar used for the application of the plaster was a current one, M5 class. It was applied five vertical strips with 1 meter width each, being the transition among them with 10cm width. The transition between the panel and the frame elements was reinforced with two layers of net as recommended by the suppliers with a total width equal to 30cm (15cm in the panel plus 15cm in the frame elements). The schematic layout of the strengthening strategy adopted is plotted in Figure 3.
Figure 2: Retrofit application process: a) detail of the metallic connectors and plastic disk used in the panel; b) general view of the net fixed to the panel; c) detail of the M8 metallic connectors and plastic disk used to fix the net to the frame; and d) detail view of the application.

Figure 3: Schematic layout of the retrofit strategy (units in meters).
2.2 Test Setup

The experimental test consisted on the application of a uniform OOP load applied by 28 pneumatic actuators, which are linked to a self-equilibrated reaction steel structure composed by four horizontal alignments made with HEB140 steel profiles and five vertical alignments made with HEB220 steel profiles (Figure 4). The vertical alignments are hinged, allowing their rotation during the tests. The steel reaction structure is attached to the envelope frame in twelve points (five in each top and bottom beam and one in each column). In each of these connections, it was placed a load cell that allowed to monitor the loads during the tests.

![Figure 4: Test setup a) schematic layout; b) lateral view; and c) front view.](image)

2.3 Test Setup

The instrumentation was composed by 21 displacement transducers, thirteen of them related to the monitoring of the panel OOP displacements and the remaining eight to the rotation between the panel and the envelope frame (Figure 5). Apart of that, and as explained in the previous subsection, twelve load cells were used to monitor the loadings developed during the test. The pressure level inside the pneumatic actuators was set by two pressure valves which were
controlled according to the target and measured OOP displacement of the central point of the infill panel (the control node and variable) continuously acquired during the tests using a data acquisition and control system developed in the National Instruments LabVIEW software platform [7].

Figure 5: Test instrumentation - scheme layout.

Two half-cyclic (loading-unloading) OOP displacements were imposed with steadily increasing displacement levels, targeting the following nominal peak displacements: 0.5; 1; 2.5; 5; 7.5; 10; 15; 20; 25; 30; 35; 40; 45; 50; 55; 60; 65 and 70mm. The central geometric point of the panel was selected as the control point since it was expected that is the region where it will occur the largest deformation of the panel.

3 EXPERIMENTAL RESULTS

The results of the two tests are here presented and discussed in terms of damages observed, cracking pattern and force-displacement results. In the first part of this section, the individual results will be detailed and later will be presented the comparison among the specimens to assess the efficiency of the strengthening solution.

3.1 Specimen Inf_09

During the test, it was not detected any damage until an OOP displacement equal to 5mm. At this level of OOP displacement occurred the plaster detachment in some parts of the panel. After that, at the OOP displacement equal to 7.5mm it was observed the beginning of a horizontal cracking at 1/3 of the panel height. When the panel reached the OOP displacement equal to 15mm, the horizontal crack becomes more pronounced and at the same time appeared a vertical crack at the middle of the panel, from the top until the horizontal crack. Quite often, when the OOP displacement reached 25mm diagonal cracks were visible, which started in the same alignment of the horizontal crack until the bottom of the panel. At the end, at the OOP displace-
At a displacement equal to 30mm it occurred the panel collapse. The cracking pattern was essentially trilinear as evidenced in Figure 6a. Figure 6b and 6c presents the panel immediately before and after the collapse.

![Figure 6a: Specimen Inf_09: a) Cracking pattern; b) Final damage before the panel collapse; c) Beginning of the panel collapse; d) Final damage after the panel collapse.](image-url)
Figure 7 presents the force-displacement response, from which it is possible to observe that for the OOP displacement equal to 2mm occurred the first decrease of strength, which was quickly recover and followed by a progressive increase until the 6mm (instant where it was visible the beginning of plaster detachment). After that, it can be verified a progressive increase of the OOP strength until reach a maximum peak load equal to 61.2kN which occurred for an OOP displacement equal to 29mm. Thereafter, at the OOP displacement equal to 29.8mm occurred suddenly the panel collapse without any visible previous decrease of the OOP strength.

![Figure 7: Specimen Inf_09: Force-displacement response.](image)

### 3.2 Specimen Inf_10

During the test of the panel Inf_10, it was not visible any crack until the OOP displacement equal to 2mm, similarly to the panel Inf_09. At this moment it was noticed some plaster detachment. When the panel reached an OOP displacement equal to 10mm it was visible a horizontal cracking at the same height as the one observed in the specimen Inf_09 (1/3 of the panel height). After that, at the OOP displacement equal to 20mm two additional horizontal cracks parallel to the first one appeared the middle of the panel height and at 2/3 of the panel height. Following that, small diagonal cracks occurred until the four corners of the panel. When the panel OOP displacement reached 30mm, the two horizontal cracks were more and more visible (placed at 1/3 and 2/3 of the panel height) and the corresponding diagonals developed from both to the panel corners. Finally, the panel reached the OOP displacement equal to 70mm when it was visible a horizontal crack at the transition between the panel and the bottom RC beam (corresponding to the detachment of the panel from the frame). The panel collapse was prevented which allow to conclude that the strengthening solution was effective. Figure 8 presents the cracking pattern observed.
From the force-displacement curve (plotted in Figure 9), it is possible to observe that the load increased progressively until reach the maximum peak load equal to 77.5kN (OOP displacement equal to 30.6mm, which basically means 1/5 of the panel thickness). After this point, it was visible the beginning of the strength degradation. The displacement transducers were not capable to capture the maximum value of that displacement (which based on the remaining transducers allows to estimate that were around 50mm), because they exceeded their measuring range, however it was possible to measure the value of the residual displacement of the wall (about 41.9mm) and to restart the test from that point. After that moment, the stiffness degradation was enhanced, but still ten more cycles were possible to perform until reach the ultimate displacement equal to 71.7mm without any strength degradation. The ultimate strength was 39kN.
3.3 Global comparison

Through the comparison between both specimens, plotted in Figure 10, it is possible to observe that both reached similar initial stiffness, as well as the force corresponding to the appearance of the first cracking and the respective OOP displacement. Regarding to the maximum peak load, the strengthened specimen achieved an increment equal to 30% for an OOP displacement 5% higher. The major contribution of the reinforcement was the prevention of the OOP collapse and the possibility to achieve OOP displacements 2.5 times higher.

![Figure 10: Global comparison: Force-displacement response of the Specimens Inf_09 and Inf_10.](image)

4 CONCLUSIONS

In the assessment of existing buildings and in the design of new buildings, the consideration of the masonry infill walls presence should be mandatory as well as particular attention must be given to the connection of the panel to the envelope frame. Recent earthquakes evidenced that the infill panels are vulnerable to OOP loadings, which could result in serious human and economic consequences. Thus, the present work was carried out with the aim of assess the efficiency of a specific retrofit technique to improve the OOP behavior and prevent the panel collapse. The technique was textile-reinforced mortar by using a glass fiber net. Two specimens (one as-built and one strengthened) were tested under uniform OOP loadings applied by pneumatic actuators. The test of the as-built panel allow confirming the vulnerability of the panel, since it occurred the collapse without any prior decrease of the OOP strength. A trilinear cracking was observed without any detachment of the panel from the envelope frame. The retrofit technique revealed to be very efficient since it prevented the OOP collapse, improved the maximum peak load about 30% and the deformation capacity 2.5 times. The technique was very easy to apply and using traditional workmanship without spending long time during the application. Future tests will be carry out to assess the efficiency of similar solutions to combined in-plane and OOP loadings.

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SEISMIC LOSS ANALYSIS OF A MODERN RC BUILDING
ACCOUNTING FOR UNCERTAINTY OF INFILL STRUT MODELING
PARAMETERS

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Abstract

Probabilistic seismic risk analysis is affected by several sources of uncertainty. Investigating the influence of uncertainties on loss analysis results is important to understand their impact on computed risk. Even though the topic has been the focus of many previous studies, however, only a few studies focused on the effects of modeling uncertainty of often-neglected non-structural elements, such as masonry infill walls in reinforced concrete (RC) moment frame buildings. This paper explores the effect of uncertainty related to modeling parameters of masonry infill walls on the seismic loss estimates for a modern code designed infilled RC frame building. The unreinforced masonry (URM) infill walls are modeled using equivalent strut models and probability density functions (PDFs) are assigned to selected infill strut parameters, based on existing literature on analytical and experimental studies. Using a latin hypercube sampling (LHS) method to sample the PDFs, a set of two-dimensional models are generated and are subjected to nonlinear response history analyses (NRHAs) at increasing seismic intensities. The building seismic performance in terms of damage and direct losses is evaluated for two performance models, the first one considering only the median values of infill strut parameters, the second one considering the uncertainties of such parameters. For both performance models, element fragility characterization is completed by state-of-the-art fragility and consequence functions for clay brick infill walls. Results are presented for an intensity-based risk analysis as well as for complete life-cycle analysis, which outputs expected annualized losses.

Keywords: Modern RC building, URM infill, parameter uncertainty, seismic loss analysis, FEMA P-58, EAL.
INTRODUCTION

Probabilistic seismic loss analysis is affected by a great number of aleatory and epistemic uncertainties. The large number of steps sequentially included in the seismic risk and loss assessment process entail the propagation of such uncertainties [1, 2].

The effects of record-to-record variability and modeling uncertainties on the seismic response of structures have been extensively studied in the literature [3–5]. Recently, Alam and Barbosa [6] proposed a probabilistic formulation to incorporate model class uncertainties in probabilistic seismic demand assessment (PSDA) and used several infill-strut model classes for the probabilistic assessment of drift hazard demand for an unreinforced masonry (URM) infilled reinforced concrete (RC) frame buildings. Seismic performance of this building typology is highly uncertain due to uncertainties in infill properties as well as the complex interaction of infills with surrounding frame, which contribute to increased lateral strength and initial stiffness of infilled frames [7, 8]. In addition to the above-mentioned sources of uncertainty, a seismic loss analysis requires including several other sources of uncertainties, such as the definition of the building collapse behavior or the element damage and loss characterization, which can greatly affect the seismic loss results [9].

The main objective of this study is to incorporate the effect of uncertainty in URM infill wall parameters on the estimates of seismic financial losses for a modern infilled RC frame building. To this end, URM infill panels are modeled through an equivalent strut macro modeling approach. A large number of two-dimensional (2D) finite element models (FEMs) of the infilled RC frames are developed using median infill single-strut force-displacement backbone curves and 200 additional random realizations of the strut backbone curves obtained using the latin hypercube sampling (LHS) method. For each structural model, nonlinear response history analyses (NRHAs) are performed at eight seismic intensities of shaking, in order to obtain the structural response parameters for seismic loss analysis. Two different performance models are implemented and compared in terms of dispersions and median repair costs and expected annual loss (EAL). Lastly, the uncertainty propagation from the response analysis results to the seismic losses is presented.

MODERN INFILLED RC FRAME BUILDING

A modern ductile RC building is selected to evaluate the effect of infill strut parameter uncertainty on seismic loss estimation. The earthquake lateral force resisting system consists of infilled RC Special Moment Resisting Frames (SMRFs) in both the horizontal directions, designed following ACI 318-08 [10] and ASCE/SEI 7-05 [11] standards for a medium seismicity site. The building is an irregular six-story structure, with three bays in Y-direction and five bays in X-direction, as shown in Figure 1. The concrete compressive strength is $f'_c = 24$ MPa, and the reinforcing steel used is Grade 420 with a nominal yield strength of $f_y = 414$ MPa. As can be seen in Figure 1(a), infill partition walls are used only at second and third stories in the Y-direction, while the X-direction exterior frames are fully infilled, as shown in Figure 1(b). Solid clay bricks are used as infill wall panels.

INFILLED RC FRAME MODELING AND UNCERTAINTIES

The selected building is modeled in OpenSees [12] through a two-dimensional FEM, implementing the infilled RC frame in the Y-direction shown in Figure 1(a). Figure 2(a) shows the FEM scheme adopted to model the infilled RC frames. Columns and beams are modeled using distributed plasticity force-based beam-column elements with fiber-section. The concrete uniaxial stress-strain response is simulated using the Concrete02 model [13], while the stress-strain
response of the reinforcing steel fibers is modeled through the bilinear Steel01 model with kinematic hardening ratio of 0.01. The effect of concrete confinement is also considered in the definition of the concrete stress-strain response of the core of the cross-sections; P – Δ effects are considered. Beam-column joints are modeled using partially rigid-end offsets. The possible column shear failure due to infill-frame interaction is modeled through a zero-length nonlinear shear spring, which is an adapted version of the spring in [14] per [6], located at both ends of columns.

Figure 1 Modern infilled RC building: (a) elevation views along Y-direction; (b) elevation views along X-direction (dimensions in cm).

Following the approach in Sattar and Liel [15], URM infill walls are modeled with equivalent diagonal single struts, concentric in the beam-column joint nodes, as presented in Figure 2(a). The strut width is computed following the formulation presented in Mainstone [16]. Figure 2(b) illustrates the backbone and main parameters required to define the backbone curve. The infill strut parameters are treated as random variables and the relevant statistics of these random variables are listed in Table 1. Statistics on model bias are determined based on the approach and results presented in Burton and Deierlein [17], except for the parameter \( \frac{V_c}{V_{cap}} \), which is based on the experimental observations reported in FEMA 307 [18]. Using these statistics in Table 1, LHS is used to generate 200 random realizations of the infill strut backbone curves, which are shown in Figure 3.

Together with modeling parameter uncertainty, record-to-record variability is taken into account using 30 natural ground motion (GM) records for eight hazard levels (HLs) from 50% in 50 years to 1% in 200 years. A hybrid stripe analysis (HSA;[6]) approach is adopted to perform NRHAs. Further details on the infilled RC building modeling and GM selection are reported in Alam and Barbosa [6].
Figure 2 Infilled RC frame FEM: (a) infill diagonal concentric strut; (b) infill parameters that define the four-branch backbone.

<table>
<thead>
<tr>
<th>Distribution parameters</th>
<th>Random variable</th>
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<tbody>
<tr>
<td>$\lambda_{V_{\text{cap}}}$</td>
<td>$\lambda_{K_e}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$K_{e,\text{model}}$</td>
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<td>8.35</td>
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<tr>
<td>0.48</td>
<td>0.65</td>
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</table>

Legend: COV = coefficient of variation.

Table 1 Parameters of lognormal random variables: mean value, $\mu$, and coefficient of variation, COV.

Figure 3 Original infill strut backbone and 200 simulated realizations of the strut backbone curves.

4 PERFORMANCE MODELS

In this work, the monetary seismic building performance is carried out via the FEMA P-58 methodology [19]. The response analysis results, expressed as Engineering Demand Parameters (EDP) median profiles, are used as input for the element damage analysis. These are reported here for the computational model with the original infill strut formulation proposed in Sattar and Liel (SL10_O), and the 200 additional models designated as SL10_B. It is worth noting that the response of the 200 SL10_B models is reported as a single median result, which takes
into account the effect of the infill strut model bias. A collapse fragility function, which expresses the probability of incurring the building collapse “C” with increasing seismic intensity measure (IM) level, $P(C|im)$, is also developed for SL10_O and for SL10_B, to obtain seismic losses resulting from collapse.

The peak interstory drift ratio (IDR) and the peak floor acceleration (PFA) median profiles are reported in Figure 4 for five out of the eight seismic IMs employed in the seismic loss analysis, both for IDR (a – c) and PFA (f – l). The results of the median IDR profiles reveal a marked gap between the original model and the model with bias at the stories where infill panels are located, i.e. at the second and third stories. The increment in IDR value at these stories reveals the great influence on response analysis results of the infill strut parameters uncertainty previously defined in Table 1. The differences in IDR values at second and third stories reduce in percentage with increasing seismic intensity, since the contribution of the infill walls decreases due to progressive damage in infills, which results in frame-only response at higher HLs. In particular, the IDR ratio of SL10_B to SL10_O model decreases from 360% at 30% in 50 years HL to 153% at 1% in 200 years HL.

Figure 4 EDP median profiles at 30%, 10% 5%, 1% in 50 years HLs and 1% in 200 years hazard levels: (a) - (e) peak interstory drift ratios (IDR); (f) – (l) peak floor accelerations (PFA).

With the EDP results and the building geometric characteristics, two performance models are implemented through the Seismic Performance Prediction Program (SP3) software [20]. The first one is related to the SL10_O computational model, while the second is related to the response analysis results of the 200 realizations generated considering the effect of infill strut parameter uncertainties (SL10_B). To implement a performance model for seismic loss analysis, a set of fragility and consequence functions are selected to describe damage and loss features of RC elements, URM infill walls, and nonstructural elements. The set of fragility and consequence functions is extracted from the FEMA P-58 database, except for the masonry infill walls, which follow the proposal by Cardone and Perrone [21]. The repair costs proposed for the Italian case by Cardone and Perrone are converted to equivalent USA costs, in case the
repair or the replacement of openings is required; in particular, the equivalence is based on an average ratio of USA to Italian repair costs for infill walls at the different damage states.

5 EFFECT OF INFILL PARAMETER UNCERTAINTY ON SEISMIC LOSS

A comparison between the two performance models, SL10_O and SL10_B, is performed to evaluate the effect of the infill strut model bias on seismic losses; the comparison is developed in terms of intensity-based loss analysis results and EAL. The uncertainty propagation from NRHA results to seismic losses is also reported. In this regard, the uncertainty of response analysis results and seismic losses is expressed in terms of logarithmic standard deviation, for the maximum IDR (MIDR), $\beta_{MIDR}$, and for repair costs, $\beta_{RC}$.

Figure 5 reveals that both the MIDR and repair cost dispersions are higher for SL10_B at lower return periods. The difference between the SL10_O and SL10_B dispersions diminish for MIDR dispersion (Figure 5a) with increasing seismic intensity, while repair cost dispersions for SL10_O performance model even exceed ever so slightly those of SL10_B (Figure 5b) at the four highest HLs. Moreover, Figure 5 reveals a considerable amplification of $\beta$ from the MIDR structural response to the seismic repair costs. This increment decreases increasing the seismic intensity, varying from 335% at the lowest HL to 15% at the highest HL. These outcomes are probably due to the progressive failure of infill panels with increasing seismic intensity. As a result, the effect of infill strut backbone variation on seismic loss analysis results diminishes as HLs increase (Figure 5b).

![Figure 5](image-url) Propagation of infill wall parameters uncertainty, expressed as logarithmic standard deviation, $\beta$: (a) MIDR dispersion; (b) normalized repair costs dispersion.

Figure 6 shows the median repair cost with increasing HLs, normalized to the total replacement cost of the building. Figure 6 reveals that performance model with infill parameter uncertainty (SL10_B) has larger median repair costs than the performance model with the original infill strut formulation (SL10_O). Differences increase up to the 2475 return period HL, then reduce at higher HLs.

The effect of infill wall parameter uncertainty also impacts the life-cycle loss analysis results. The EAL parameter is equal to 0.073% for SL10_B performance model, whereas it is equal to 0.055% for SL10_O, thus an increase of 34% occurs when model uncertainty of the infill wall parameters is considered.

The most relevant HLs in EAL estimate are the most frequent HLs, hence such significant increment in EAL value is mainly due to the increase in repair costs uncertainty at lower HLs due to the effect of infill strut model bias, previously shown in Figure 5. The increment in the
median values of repair costs, presented in Figure 6, is low at more frequent HLs, therefore its effect on EAL increment is less significant.

Figure 6 Effect of infill wall parameters uncertainty on the median value of normalized repair costs.

6 CONCLUSIONS

In this paper, the effect of uncertainties in URM infill walls parameters on seismic losses was evaluated for a modern infilled RC building. Uncertainty in URM infill walls is accounted for by developing 200 FEMs considering model uncertainty of the equivalent infill strut backbone parameters. Nonlinear response history analyses were performed for the resulting set of FEMs at eight seismic intensities. Two different performance models were implemented: the first one included the FEM that considered only the median values of infill strut parameters (SL10_O), the second one included the 200 FEMs that considered the infill strut parameters uncertainty (SL10_B). To evaluates the effect of the infill strut model uncertainty on seismic losses, a comparison between the two implemented performance models was carried out in terms of repair costs and EAL. Results indicate that repair cost dispersions are higher for SL10_B at lower HLs, whereas at highest HLs the dispersions for SL10_O performance model even exceed those of SL10_B. Moreover, results indicate that a large amplification of dispersions from MIDR parameter to seismic repair costs, although this amplification is less notable at larger seismic intensities.

With respect to median losses, the SL10_B performance model results in larger median repair costs than SL10_O performance model. Differences increase up to the 2475 return period HL but decrease at higher seismic intensities. Life-cycle analysis reveals that the EAL parameter increases by 34% when uncertainty of the infill wall model parameters was considered, confirming the significant effect of infill strut parameter uncertainty especially at more frequent seismic intensities.

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NONLINEAR DYNAMIC ASSESSMENT OF THE OUT-OF-PLANE RESPONSE AND BEHAVIOUR FACTOR OF UNREINFORCED MASONRY INFILLS IN REINFORCED CONCRETE BUILDINGS

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Abstract

Current building codes address the issue of the seismic safety check of non-structural elements, such as unreinforced masonry (URM) infill walls. The force-based safety assessment of these elements includes the verification with respect to out-of-plane (OOP) seismic accelerations. To this aim, on one hand different force-demand models, based on simplified floor spectra, are proposed by codes; on the other hand, OOP strength-capacity models are not always provided; in addition, the capacity models proposed by codes do not always account for the OOP strength reduction due to in-plane (IP) damage, i.e., for the so-called IP/OOP interaction. Moreover, within such a force-based framework, current codes also provide behaviour/response modification factors with different values ranging from the unit to more than two. However, the source and reliability of these values have never been checked.

In this study, non-linear time-history incremental dynamic analyses are performed on reinforced concrete buildings designed according to Eurocodes and different for design seismic acceleration and number of storeys. The buildings are infilled by three different infill layouts. For infill walls, a recently-proposed modelling strategy accounting for their OOP response and collapse, respectively affected and promoted by the IP/OOP interaction, is implemented. The results of more than thirty thousand dynamic analyses allow the evaluation of the seismic acceleration capacity of infills with respect to the OOP collapse and of the parameters influencing it. In addition, a robust and reliable evaluation of the OOP behaviour factor is performed. Such a value is compared with current code provisions and proposed for a potential application in the seismic safety check of URM infills within a code-based linear elastic approach.

Keywords: Infill wall, out-of-plane, in-plane/out-of-plane interaction, nonlinear time-history analysis, seismic assessment, safety assessment.
1 INTRODUCTION

In recent years, a growing interest is arising on the experimental, theoretical and numerical assessment of unreinforced masonry (URM) infills’ out-of-plane (OOP) response under seismic actions. The main issues investigated are i) the pure OOP response of URM infills and ii) the effects on the OOP response of the damage due to in-plane (IP) seismic action (IP/OOP interaction). For which concerns the first topic, experimental and theoretical studies were presented, especially in past years [1-4]. For which concerns the second topic, experimental, analytical and numerical studies were presented, especially in recent years [5-15].

It is well-known that the OOP collapse of URM infills can occur also with a very ruinous and abrupt overturning phenomenon that, above all, can harm human life safety. Hence, it seems appropriate considering the attainment of the OOP collapse of URM infills the same as the attainment of Life Safety Limit State (LS). That being said, it is worth to remember that Eurocode 8 [16] requires the verification of nonstructural components against the seismic action within the construction assessment at LS (section 2.2.2(6)P).

Presently, Eurocode 8 proposes a simplified floor spectrum for the calculation of the seismic acceleration/force demand acting on acceleration-sensitive nonstructural components in section 4.3.5. A specific formulation for the calculation of the OOP strength of URM infill walls is not provided. However, the formulation proposed in Eurocode 6 [17] for the lateral resistance of one-way spanning masonry walls accounting for one-way arching action may be used for this aim. Similarly, also American [18] and New Zealand [19] codes propose simplified response spectra and also OOP strength models specifically dedicated, in this case, to URM infills. Therefore, it is clear that current codes propose a force-based safety check of URM infills with respect to OOP seismic actions. Within this approach, the assumption of a behaviour/response modification factor is allowed to reduce the elastic seismic force demand calculated by means of the proposed floor response spectra. More specifically, the behaviour factor proposed by ASCE/SEI 7-10 is equal to 2.5, the one proposed by Eurocode 8 is equal to 2, that proposed by NZSEE is equal to one. The source of these values has never been justified or investigated. It is also worth to mention that IP damage affects the OOP response of URM infills [5, 7, 10, 13, 14, 20, 21]. This phenomenon is known as “IP/OOP interaction” and it is not clear whether the above behaviour factors account it.

In this study, 16 Reinforced Concrete (RC) buildings designed to Eurocodes are infilled by three infill layouts different for geometric and mechanical properties. A recently-proposed infill wall macro-model is applied to model URM infills by accounting for their OOP response and for the IP/OOP interaction effects. Nonlinear incremental dynamic analyses are performed on the case-study buildings.

The results of the analyses are used to present and discuss the global response of the case-study buildings and to investigate the influence of the IP/OOP interaction effects and of the OOP collapse of infills on such a response. Mean values of the PGA demand at which the first OOP collapse of infills occurs and the parameters having a potential influence on it, namely the number of storeys and the design PGA of the construction, are presented and discussed.

The results of the analyses are used to evaluate and propose a safe-sided and practice-oriented value of the OOP behaviour factor of URM infills accounting for the IP/OOP interaction effects that can be used for their seismic safety assessment in a linear elastic framework.

2 CASE-STUDY BUILDINGS AND ANALYSIS PROCEDURE

The case-study buildings are provided of 5 and 3 bays in the X and Z direction, respectively. All bays spans are 4.5 m long while the inter-storey height is always equal to 3 m. Each building has been designed for gravity and seismic loads by applying the Response Spectrum Analysis
method according to Eurocode 2 [22] and Eurocode 8. The case-study buildings are distinguished for different values of the design Peak Ground Acceleration (PGA) at LS (PGAd) (0.05, 0.15, 0.25, and 0.35 g) and for different number of storeys (2, 4, 6, and 8). The materials used for the building design are class C28/35 concrete and reinforcing steel with characteristic yielding stress equal to 450 N/mm$^2$. The buildings were designed on a stiff and horizontal type A soil. Eurocode 8 Type 1 elastic spectrum, which is recommended for high-seismicity zones, was used to evaluate horizontal seismic actions. A behaviour factor equal to 4.68 was applied in the design process. All buildings resulted regular in plan while not regular elevation due to a non-gradual stiffness reduction along their height. P-δ effects resulted negligible. A lateral deformability verification at Damage Limitation Limit State (DL) was performed under a seismic action defined by applying a scaling factor equal to 0.4 times the response spectrum at LS [23]. The longitudinal and transverse reinforcements for beams and columns were determined according to force demands assessed through RSA and by applying capacity design rules.

The RC elements’ non-linearity was modelled in OpenSees [24] by using ModIMKPeakOriented Material with response parameters determined according to Haselton et al. [25] and with the introduction of the cracking point.

Three infill layouts are considered. The first is constituted by a two-leaf (thickness: 80+120 mm) URM ‘weak’ infill wall (weak layout, WL), the second is constituted by a two-leaf (thickness: 120+200 mm) URM ‘intermediate’ infill wall (intermediate layout, ML), the third is constituted by a one-leaf (thickness: 300 mm) URM ‘strong’ infill wall (strong layout, SL). The mechanical properties of these infills are those calculated for the masonry wallets tested by Calvi and Bolognini [7] for the WL and the ML and those by Guidi et al. [20] for the SL (Table 1). Note that the value of masonry shear strength of Guidi et al.’s specimens is not provided, so it is set to 0.30 N/mm$^2$ according to Table 3.4 of Eurocode 6.

Each infill wall is introduced in the structural model by using a couple of equivalent struts whose non-linear behaviour is modelled based on Panagiotakos and Fardis proposal [26]. According to this modelling approach, the slope of the softening branch of the force-displacement IP behaviour relationship is a fraction $\alpha$ of the infill initial elastic stiffness, while the infill residual strength is herein set to zero. In [27] it is suggested to set $\alpha$ to a value between -1.5% and -5%. For the 80-, 120- and 200-mm thick leaves $\alpha$ is set to -1.6% while for the 300-mm thick leaf it is set to -3.6%. These values yield to predictions of the softening stiffness and ultimate IP displacement in good accordance with the experimental evidences shown by Calvi and Bolognini (specimen 2) for $\alpha$=-1.6% and by Guidi et al. (specimen URM-U) for $\alpha$=-3.6%. The IP behaviour characteristic points are reported in Table 2.
The OOP behaviour of IP-undamaged infills is modelled by using the lumped-plasticity empirical-based modelling strategy proposed by Ricci et al. [12] in the updated version described in [28]. This modelling strategy consists in defining for IP-undamaged infills a trilinear elastic-cracked-plastic OOP backbone. In addition, the IP damage effects on infills’ OOP behaviour and vice-versa modelled by means of supplementary IP and OOP backbones that mutually-neutralize themselves or activate based on the IP and OOP displacement demands calculated step-by-step during the nonlinear time-history analysis. For the definition of the IP and OOP degraded backbones relationships are proposed to calculate the coordinates of the characteristic points of the IP-undamaged OOP backbone. A peculiarity of this modelling strategy is the flexibility with respect to the definition of the IP and OOP damaged and undamaged backbones, for which whichever material model, hysteretic rule and degradation rule can be used. The OOP behaviour characteristic points of the IP-undamaged case-study infill leaves are reported in Table 4. Based on the expected first-mode deformed shape, the mass participating to the first OOP vibration mode of each leaf was set to 66% of the infill total mass.
Table 3: IP-undamaged infills’ OOP behaviour characteristic points.

Ten ground motions were selected among the records of seven different European earthquakes collected in the Engineering Strong-Motion (ESM) Database [29] in order to perform Incremental Dynamic Analyses (IDAs) [30]. Significant characteristics of the selected ground motions are reported in Table 3.

<table>
<thead>
<tr>
<th>#</th>
<th>ESM ID</th>
<th>Country</th>
<th>Date</th>
<th>Mw</th>
<th>Repi [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ME-1979-0003</td>
<td>Montenegro</td>
<td>15/04/1979</td>
<td>6.9</td>
<td>19.7</td>
</tr>
<tr>
<td>3</td>
<td>IT-1984-0004</td>
<td>Italy</td>
<td>07/05/1984</td>
<td>5.9</td>
<td>10.1</td>
</tr>
<tr>
<td>4</td>
<td>IT-1997-0006</td>
<td>Italy</td>
<td>26/09/1997</td>
<td>6.0</td>
<td>26.5</td>
</tr>
<tr>
<td>5</td>
<td>IT-1997-0137</td>
<td>Italy</td>
<td>14/10/1997</td>
<td>5.6</td>
<td>8.70</td>
</tr>
<tr>
<td>6</td>
<td>SI-1998-0010</td>
<td>Italy</td>
<td>12/04/1998</td>
<td>5.7</td>
<td>23.5</td>
</tr>
<tr>
<td>7</td>
<td>IT-1998-0103</td>
<td>Italy</td>
<td>09/09/1998</td>
<td>5.6</td>
<td>18.0</td>
</tr>
<tr>
<td>8</td>
<td>GR-1999-0001</td>
<td>Greece</td>
<td>07/09/1999</td>
<td>5.9</td>
<td>19.7</td>
</tr>
<tr>
<td>9</td>
<td>TK-1999-0294</td>
<td>Turkey</td>
<td>13/09/1999</td>
<td>5.8</td>
<td>13.8</td>
</tr>
<tr>
<td>10</td>
<td>IT-2009-0009</td>
<td>Italy</td>
<td>06/04/2009</td>
<td>6.1</td>
<td>26.2</td>
</tr>
</tbody>
</table>

Table 3: Selected Ground Motions for Incremental Dynamic Analyses.

The selection of records was performed searching among the bidirectional registration of stations based on Eurocode 8 type A soils, consistently with the design soil type. Consistently with the choice of using Eurocode 8 Type I design spectrum, only earthquakes with magnitude between 5.5 and 7 and only registration of stations with epicentral distance between 10 and 30 km were considered. Both horizontal components of the selected records were simultaneously matched to the 5%-damped Eurocode 8 design spectrum at Life Safety Limit State by using wavelets through the RspMatchBi software [31].

IDAs were performed by scaling each selected and matched record for a set of pre-determined scale factors in order to obtain for each horizontal direction an incremental PGA (selected as Intensity Measure) vs maximum IDR (selected as Engineering Demand Parameter) curve. A total of 32 scale factors ranging from 0.067 to 10 were considered. This allowed performing the IDAs for 32 values of PGA roughly equal in both directions and ranging from 0.010 g to 1.50 g.

The analyses were carried out by applying mass- and tangent stiffness-proportional Rayleigh damping rules for two control vibration modes. A “global” and a “local” mode were selected as control modes. For instance, the first control mode corresponds the first natural frequency of
the infilled structure, while the second control mode corresponds to the mode associated to the frequency closer to the infill natural frequency in the OOP direction. The assigned damping ratio is equal to 5% both for the first global and for the second local control mode: the last choice is due to the lack of exhaustive studies on this topic, which is worth to be investigated in the future.

3 SEISMIC RESPONSE OF THE CASE-STUDY BUILDINGS

In this section, the global response of the case-study buildings represented by IDA curves is discussed. The mean PGA at the attainment of the first OOP collapse, PGA$_{c}$, is assessed for all case-study buildings. Note that the results of the analyses are summarized, for the sake of brevity. More detailed results are reported in [28].

The global seismic response of the case-study buildings is discussed by comparing their IDA curves. Remember that two different models (W/O and W/) of 48 buildings were analysed (16 different RC frames x 3 different infill layouts) under the action of 10 bidirectional records scaled per 32 different scale factors. A total of more than 30 thousand time-history analyses were performed. For the sake of simplicity, the IDA curves are not reported for all case-study buildings but only for some of them, in order to show in a simple and direct way how the IP/OOP interaction effects influence the seismic response of RC buildings.

In Figure 1, the IDA curves for building 8P35_WL are shown for both the W/O and the W/ models.

![Figure 1: IDA curves for building 8P35_WL in the X (a-c) and Z (b-d) directions for the W/ (a-b) and the W/O (c-d) models.](image)

The OOP collapses of infills (and their removal from the structural model) is visible from the sudden increase in the lateral displacement demand. Of course, this jerk is visible also in the IDA curves of the W/O model (Figure 1d), but for a higher PGA demand. It should be noted that, as expected, wide OOP collapses are registered in the more deformable direction of the building, i.e., along Z global direction.

In general, if the IP/OOP interaction effects are neglected, the lateral displacement demand acting on buildings is underestimated. For example, at PGA equal to 0.35 g, the median displacement demand in Z direction for 8P35_WL building is roughly equal to 10 mm according to the results of the analyses on the W/O model, while it is roughly equal to 35 mm for the W/ model.

Note also that, especially for the W/ model, the IDA curves are characterized by a jagged trend, with many “resurrections”. This is quite expected, as the entity of the IP/OOP interaction effects as well as the number and location of infills collapsed and removed from the structural model can vary significantly (and not necessarily “monotonically”) at increasing intensity level.

The mean and standard deviation of the PGA$_{c}$ values, the least accounting for the record-to-record variability and for the (small) variability due to the different design PGA of the case-study buildings, are reported in Tables 4-5. The mean values of the PGA$_{c}$ are shown in Figure.
2 for each case-study building. Note that, for what concerns the W/O models of SL-infilled buildings, the OOP collapse of infills was not registered, even at the higher PGA, for some records. Hence, the value reported in Table 4 for the W/O models of SL-infilled buildings is not the mean but the lower PGA_c value registered among the few ones obtained from the analyses. This circumstance is pointed out also in Figure 2, in which the PGA_c values for the W/O models of SL-infilled case-study buildings are reported with a dashed line.

In W/O models the OOP collapse of infills always occurs at the building last storey. It is observed that PGA_c decreases at increasing number of storeys, which is expected, as in higher buildings the OOP seismic demand at the last storey is higher with respect to lower buildings. In addition, PGA_c does not show a clear tendency with the design PGA, most likely due to the influence of the record-to-record variability on the OOP seismic demand, which is also dependent on the relationship existing between the OOP period of the infill and the vibration period of the building. Note that all trends for SL layouts are not very clear, most likely because the values reported and compared are not, as already stated, an average value but a minimum value obtained from a reduced number of analyses performed up to the OOP collapse of such a type of infills. Clearly, as expected, due to the higher OOP undamaged strength, SL buildings are provided with a significantly higher PGA_c with respect to WL and ML infills. If the IP/OOP interaction is neglected, all buildings result safe with respect to the OOP collapse of infills, given that for all infill layouts PGA_c is always higher than the design PGA at LS.

In W/ models, PGA_c is quite independent on the design PGA and on the total number of storeys of the building for WL and ML infills. However, some trends are slightly visible, with PGA_c decreasing at increasing number of storeys, which is expected, as higher buildings – being more deformable – are more prone to the IP/OOP interaction, and increasing at increasing design PGA, which is also expected, as buildings designed for higher seismic acceleration – being more stiff – are less prone to the IP/OOP interaction. Clearly, as expected, due to the higher OOP undamaged strength and to the lower impact of the IP/OOP interaction effect, SL buildings are provided with a significantly higher PGA_c with respect to WL and ML infills. It is observed that mid- and high-rise buildings in mid- and high-seismicity zones are not safe with respect to the OOP failure of infills if WL and ML infills are used. Only SL infills are always safe with respect to the IP/OOP interaction.

<table>
<thead>
<tr>
<th>W/O models</th>
<th>2P mean</th>
<th>st.dev.</th>
<th>4P mean</th>
<th>st.dev.</th>
<th>6P mean</th>
<th>st.dev.</th>
<th>8P mean</th>
<th>st.dev.</th>
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<tbody>
<tr>
<td>WL</td>
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<td>0.08</td>
<td>0.58</td>
<td>0.12</td>
<td>0.51</td>
<td>0.12</td>
<td>0.48</td>
<td>0.13</td>
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<tr>
<td>ML</td>
<td>0.91</td>
<td>0.25</td>
<td>0.88</td>
<td>0.19</td>
<td>0.73</td>
<td>0.18</td>
<td>0.71</td>
<td>0.18</td>
</tr>
<tr>
<td>SL</td>
<td>1.20*</td>
<td>-</td>
<td>1.31*</td>
<td>-</td>
<td>1.30*</td>
<td>-</td>
<td>1.24*</td>
<td>-</td>
</tr>
</tbody>
</table>

*: lower bound

Table 4: Mean and standard deviation of the PGA_c [g] values obtained for the W/O models.

<table>
<thead>
<tr>
<th>W/ models</th>
<th>2P mean</th>
<th>st.dev.</th>
<th>4P mean</th>
<th>st.dev.</th>
<th>6P mean</th>
<th>st.dev.</th>
<th>8P mean</th>
<th>st.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WL</td>
<td>0.25</td>
<td>0.06</td>
<td>0.23</td>
<td>0.05</td>
<td>0.20</td>
<td>0.04</td>
<td>0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>ML</td>
<td>0.33</td>
<td>0.07</td>
<td>0.32</td>
<td>0.06</td>
<td>0.27</td>
<td>0.05</td>
<td>0.27</td>
<td>0.07</td>
</tr>
<tr>
<td>SL</td>
<td>0.73</td>
<td>0.18</td>
<td>0.73</td>
<td>0.14</td>
<td>0.60</td>
<td>0.13</td>
<td>0.58</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 5: Mean and standard deviation of the PGA_c [g] values obtained for the W/ models.
It is clear from the comparison of the results obtained on the W/O and on the W/ models that accounting for the IP/OOP interaction is necessary to not perform an unsafe assessment of the seismic performance of buildings at LS.

4 OUT-OF-PLANE BEHAVIOUR FACTOR

The main aim of this section is the definition of the behaviour factor associated with the OOP seismic response of URM infills.

Each IP-undamaged infill of the case-study buildings can be considered as a Single Degree Of Freedom (SDOF) dynamic system provided of a mass $m^*$ equal to the mass participating to the first OOP vibration mode, of an initial stiffness, $K_{el,und}$, of a certain strength, $F_{max,und}$, and of a certain displacement capacity $d_{u,und}$. As already stated, due to the OOP modelling strategy adopted for infill walls in this study, the SDOF that represents them has an evolutionary behaviour during non-linear time-history analyses as its displacement capacity, strength and initial stiffness varies during the analyses due to IP/OOP interaction.

Now, consider a specific record used for the non-linear time-history analysis of a certain case-study building. As during IDAs the record is scaled, it is possible to define the lowest scale factor that multiplied for the unscaled record produces the first OOP infill collapse. Such collapse occurs at a certain storey of the case-study building. So, it is possible to associate to the specific record considered and to the specific case-study building:

i. the storey at which the first OOP infill collapse occurs and the acceleration and displacement floor response spectra of the scaled record at the storey at which the first OOP collapse occurs. This floor spectrum can be considered a “capacity floor spectrum” for the specific record and for the specific “capacity condition” considered, i.e., the first OOP infill collapse;

ii. the “residual” OOP initial stiffness, $K_{el,dam}$, reduced due to the IP action effects, that the infill/SDOF had when it collapsed;

iii. the “residual” OOP strength, $F_{max,dam}$, reduced due to the IP action effects, that the infill/SDOF had when it collapsed;
iv. the “residual” OOP displacement capacity, $d_{u,\text{dam}}$, reduced due to the IP action effects, that the infill/SDOF had when it collapsed.

As it is well-known, the behaviour factor of an SDOF is given by the ratio between the seismic force that the SDOF would have known if it was elastic, $F_{\text{el}}$, over its strength, $F_{\text{max}}$.

Now, consider that in general, for the sake of simplicity, the IP/OOP interaction is not explicitly modelled. So, usually, $K_{\text{el,dam}}$ and $F_{\text{max,dam}}$ are not known by the practitioner. For this reason, the q-factor can be calculated by considering the OOP seismic demand evaluated for the W/ model but entering the capacity acceleration floor spectrum with the vibration period associated with $K_{\text{el,und}}$ and dividing the elastic force by $F_{\text{max,und}}$, as reported in Figure 3.

![Figure 3. Determination of the behaviour factor for the W/ model with reference to the OOP elastic stiffness of the IP-undamaged infill.](image)

In this case, it should not be surprising that q-factors even lower than the unit are obtained, given that the OOP seismic demand considered is consistent with the one associated with an IP-damaged infill while the OOP strength considered is that associated with the IP-undamaged infill. This occurs as if the OOP seismic demand should be indeed increased to account for the detrimental effects of the IP/OOP interaction if such a demand is compared with the OOP strength of the IP-undamaged infill.

The values obtained for the q-factor are shown in Figure 4 for the W/ models. It is observed that the average q-factor is quite independent on the design PGA of the building, while it increases at increasing total number of storeys of the building.

The 16th, 50th and 84th percentile values of the q-factors obtained for the considered infill layouts and accounting for the record-to-record variability and for the variability associated with the different number of storeys and with the different design PGA of the buildings are reported in Table 6. In other words, one can use such percentiles of the q-factor only by knowing the infill typology and independently on the building number of storeys and design PGA.

Based on the above results, the most appropriate and practical way to calculate in a linear elastic framework the OOP seismic demand on URM infills and to verify them against the seismic action by accounting for the IP/OOP interaction (but without explicitly modelling it) is:

i. calculating the seismic demand as a function of the initial elastic stiffness (and of the corresponding period) of the IP-undamaged infill;
ii. calculating the OOP capacity as a function of the OOP strength of the IP-undamaged infill;
iii. assuming a behaviour factor equal to 1.
The proposed value is consistent with the 16\textsuperscript{th} percentile of the values obtained from the analyses’ results and it seems, to the Authors’ judgment, sufficiently conservative for all infill layouts.

Such value of the behaviour factor can be considered as representative of a specific mechanical phenomenon. If the IP/OOP interaction is not explicitly modelled and the OOP seismic demand and capacity of infills is calculated as if the infill were not IP-damaged, the beneficial effects of the potential OOP ductility capacity of the infill should not be considered to perform a safety-sided simplified safety check, as if the beneficial effects of the ductility capacity were neutralized by the detrimental effects of the IP/OOP interaction. In addition, it is worth to mention that the value herein suggested for the behaviour factor is equal to that assumed by NZSEE 2017 [19] for URM infills.

![Figure 4. Mean values of the q-factor for all case-study buildings.](image)

<table>
<thead>
<tr>
<th>percentiles</th>
<th>WL</th>
<th>ML</th>
<th>SL</th>
</tr>
</thead>
<tbody>
<tr>
<td>16\textsuperscript{th}</td>
<td>1.12</td>
<td>0.98</td>
<td>0.88</td>
</tr>
<tr>
<td>50\textsuperscript{th}</td>
<td>1.27</td>
<td>1.18</td>
<td>1.06</td>
</tr>
<tr>
<td>84\textsuperscript{th}</td>
<td>1.41</td>
<td>1.35</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table 6: Values of the q-factor for all infill layouts.

5 CONCLUSIONS

In this study, the seismic response of 16 RC buildings designed to Eurocode 8 different for the number of storeys and for the design PGA at LS and infilled by three different URM infill layouts is investigated by means of nonlinear time-history dynamic analyses. A recently-proposed modelling strategy is adopted for introducing infill walls in the nonlinear model of the case-study buildings. For each case-study building, two different models (W/O and W/) are analyzed. In W/O models, the IP and OOP seismic response of infills is considered up to the IP or OOP collapse of infills, at which they are removed from the structural model. In addition, in the W/ models of the case-study buildings, the IP damage effects on the OOP response and the OOP damage effects on the IP response of infills are considered, too: this phenomenon is named IP/OOP interaction. The following conclusions can be drawn:

- The seismic displacement demand acting on the buildings may be highly underestimated if the IP/OOP interaction effects are neglected. For example, at PGA equal to 0.35 g, the median displacement demand in Z direction for 8P35_WL building is roughly equal to 10 mm according to the results of the analyses on the W/O model, while it is roughly equal to 35 mm for the W/ model.
• The collapse PGA of infills is significantly overestimated if the IP/OOP interaction is neglected. For mid- and high-rise buildings with two-leaf infills (WL and ML layouts) in mid- and high-seismicity zones are not safe with respect to the OOP collapse of infills at LS, i.e., such buildings are safe with respect to the attainment of LS on the “structural” side but not on the “nonstructural” side, as the capacity PGA at the first OOP collapse is lower than the design PGA at LS.

• Based on the analyses results, for the seismic safety assessment of URM infills in a simplified code-based force-based framework, the use of a behaviour factor equal to the unit seems to be the more correct, simple and safe choice.

• In future studies, simplified floor spectra accounting for the IP/OOP interaction effects in infill walls can be proposed. In addition, the numerical analyses herein carried out will be performed on RC buildings with strengthened and reinforced infill walls.

ACKNOWLEDGMENTS

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REFERENCES


SEISMIC PERFORMANCE OF PORTUGUESE MASONRY INFILL WALLS: FROM TRADITIONAL SYSTEMS TO NEW SOLUTIONS

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Abstract

The use of masonry infill walls in RC structures is a common solution in Portugal for more than 50 years. These walls are used to build the envelope of the buildings and as interior partitions. Since these walls are built only after the hardening of the RC elements, they are assumed as non-structural elements, and not considered in the structural design. However, when buildings are subjected to seismic action, these walls assume a structural behavior. This leads in many cases to the existence of an unsatisfactory behavior of infill walls, resulting in damage of the walls, which put human lives in danger and cause extensive economic losses. This seismic vulnerability of masonry infill walls is well recognized by the scientific community and has been observed in the recent earthquakes in southern Europe. In the case of a seismic event of high intensity in Portugal, it is expected that similar problems will occur in the Portuguese masonry infill walls, given the similarity of the construction typologies, in southern Europe. It is therefore important to study the masonry infill walls used in Portugal over the years, to fully understand their seismic behavior, and thus be able to propose and study new masonry infill systems for new construction.

Thus, in this paper is made a characterization of Portuguese masonry infill walls in terms of evolution over the years. After being presented the problems that arise from the seismic vulnerability of these walls. In an attempt to solve some of these problems is presented in this paper a new system of masonry infill walls, to be used in new construction, with the objective of present a better seismic behavior. Experimental validation of the system is performed, using the in-plane and out-of-plane tests performed at the University of Minho. At the end of the paper, a comparison is made between the new system and the traditional cavity walls, to assess the improvement of the seismic behavior in relation to the traditional solutions.

Keywords: Masonry infill walls, In-Plane behavior, Out-of-Plane behavior, Airbag, Experimental characterization, Seismic behavior, Cyclic tests.
1 INTRODUCTION

Masonry infill walls are the most common solution to build enclosures walls in Portugal, mainly in reinforced concrete structures, leading to the need of production of thousands of masonry units annually [1]. In the last 60 years, several authors have carried out studies [2]–[5] to assess the influence of masonry infill walls on the reinforced concrete structure, trying to understand what is their contribution to the lateral strength and stiffness of buildings. According to these authors the masonry infill walls contribute significantly to the performance of buildings and may have a positive influence on the lateral strength and stiffness of buildings, as well as in energy dissipation. However, there are still problems in the behavior of these walls especially when they are subjected to seismic action [6], because masonry infill walls assumes a structural behavior for which they have not been designed. Since masonry infill walls are considered non-structural elements in the current regulations [7], they are not considered in the current structural design practice. Usually there is no verification of their safety against seismic actions. This leads to a high level of damage to moderate earthquakes, resulting in high economic losses and endangering human lives [8]–[10].

The recent earthquakes in Lefkada in 2003 [11], L’Aquila in 2009 [12], Van in 2011 and Emilia Romagna in 2012 [13], among others, have clearly revealed the vulnerability of non-structural elements in contemporary architecture buildings with reinforced concrete structures. These earthquakes allowed to observe that contemporary structures in reinforced concrete have a reasonable ability to withstand seismic loads, given that were designed for this purpose according to the current design codes [7]. But in the case of masonry infill walls these earthquakes confirmed their vulnerability to seismic actions. The most common pathologies are the separation between masonry panels and structural elements, diagonal cracking, and out-of-plane partial or total collapse. Sometimes it is possible to observe some kind of damage like soft-story or short column associated with the improper use of masonry infill walls.

In the case of a seismic event of high intensity in Portugal, it is expected that similar problems will occur in the Portuguese masonry infill walls, given the similarity of the construction typologies. Despite masonry infills walls being a widely used construction solution in Portugal, the construction systems for this type of walls remains the same for many years, apart from some minor changes. In Portugal the masonry infill walls, had its great advent in the 60’s, with the massification of reinforced concrete structures. Since then until our days the masonry infills walls do not change, apart from some changes in thickness and introduction of thermal isolation between the leaves. The construction system remains based in cavity walls without any connection between leaves, constructed with horizontal perforated bricks and poor mortars. Masonry units have always been ceramic clay bricks, with horizontal perforation and high percent of voids with weak mechanical properties. In Figure 1, it is possible to see the standardized masonry units used in Portugal. However, their vulnerability under seismic actions is recognized by scientific community and proven past earthquakes. Nevertheless, they continue to be used, because until now does not exist any seismic resistant system developed to be used in new construction, with clear design procedures and construction guidelines.

Eurocode 8 [7], is silent in this case and only present a simplified procedure for the calculation of the out-of-plane action, but does not provide design recommendations. On the other hand, this code considers that verification of the safety of non-structural elements is guaranteed if the relative displacements between floors are limited. However, states that appropriate measures should be taken to avoid brittle failure and premature disintegration of infill walls.

To address the existing problems with masonry infill walls, it is important continue the study the masonry infill walls used in Portugal over the years, to fully understand their seismic behavior, and thus be able to propose and study reinforcement systems for existing walls.
and new masonry infill systems for new construction, that can withstand seismic action and propose design guidelines that can be used by structural designers.

![Horizontal perforated masonry units](image)

Figure 1: Horizontal perforated masonry units used in Portugal.

Thus, in this paper is made a characterization of Portuguese masonry infill walls in terms of evolution over the years, and in terms of geometrical characterization. After are presented the problems that arise from the seismic vulnerability of these walls. In an attempt to solve some of these problems is presented in this paper a new system of masonry infill walls, to be used in new construction, with the objective of present a better seismic behavior. An experimental validation of the system is performed, using the in-plane and out-of-plane tests performed at University of Minho.

At the end of the paper, a comparison is made between the new system and the traditional cavity walls, to assess the improvement of the seismic behavior in relation to the traditional solution applied in Portugal.

2 MASONRY INFILL WALLS IN PORTUGAL

2.1 Evolution of Portuguese masonry infill walls

Masonry infill walls are a very common constructive solution to build the envelope of buildings in Southern and Central European countries, particularly in Portugal, Spain, Italy, Greece among others. This type of walls continues to be widely used, especially in reinforced concrete structures, because they remain an economical and durable solution. In addition, they are relatively easy to construct and provide great architectural freedom and are still capable of meeting a number of requirements such as watertightness, acoustic comfort, thermal comfort, fire safety and good air quality [14].

However, there are several possible solutions to build masonry infill walls. The wide variety of existing mortars and masonry units, as well as the use of connectors and reinforcements make possible the existence of thousands of different combinations that can be translated in different solutions to this kind of walls. Different solutions are often associated with different geographies.

In Portugal the evolution of masonry enclosures walls is linked to the evolution of Portuguese building stock. The most significant moment in the evolution of masonry walls is the transition between the old masonry buildings and the buildings built with in reinforced concrete structure. The massification of reinforced concrete structures, in the late 1940s, first on slabs, replacing the wooden floors, and then on the vertical structural elements, caused masonry enclosures walls to lose their structural function, turning into simple infill elements. This trend occurred in parallel with the declining use of stone, that start being replaced by clay bricks. Clay brick units were developed in different sizes and shapes, progressing from traditional solid small units to large horizontally perforated elements with a high percentage of voids.
The widespread use of reinforced concrete associated with the importance of maintaining the watertightness of the enclosures led to the generalization of enclosure cavity walls, made of clay bricks, in the 1960s (see Figure 2). The use of cavity walls, in addition to ensuring watertightness, reduce the load on the structure, increase the productivity in construction, reduce costs, simplify the development of works in height.

Since its generalization in the 1960s, the masonry enclosures walls have undergone some evolutions, as shown in Figure 2. The typical 1960s solution is a double wall solution, where the external wall is thicker (usually 0.15m) than the inner wall (usually 0.11m), the wall panels are not connected at all, and the cavity between the walls is filled with air. The masonry units used are horizontal drilling and the mortar was produced on-site.

In the 70's a reduction of the thickness (usually 0.11m) of the external leaf was observed, which would increase in thickness in the 80's, in order to meet the thermal requirements, in addition the cavity between the wall panels, was now filled with thermal insulation. Since the 1990s and nowadays, the cavity wall solution has been replaced by the use of the single wall, using thicker walls, with thermal insulation in external side of the wall [5].

Although horizontal drilling masonry units are still widely used, in recent years the use of so-called thermal and acoustic bricks has been introduced and they have vertical drilling and improved thermal and acoustic properties. In terms of materials throughout all these years the most used masonry units have always been ceramic clay bricks, much like other European countries [16].

2.2 Geometrical characterization of Portuguese masonry infill walls

For the geometric characterization of masonry enclosures, were used the data collected in two previous studies [5], [17], which analyzed design plants of reinforced concrete buildings built in Portugal.

Furtado et al. [17] analyzed 80 architectural and structural design drawings to collect geometric data about reinforced concrete elements, and masonry enclosures. For each building were collected the following parameters: the story height with emphasis on ground floor, that usually is different from the others, ground floor area, dimensions of columns and beams with information about the reinforcement, and slabs thickness. Furthermore, 1400 masonry infill walls, from the analyzed buildings, were studied to define the most common typologies of openings and make a geometric characterization of masonry infill walls. The main goal was to characterize these types of non-structural walls used in Portuguese construction practice.

The analyzed buildings are mainly located in the districts of Lisbon, Aveiro and Porto, and there are also buildings in the districts of Braga, Viseu, Coimbra and Leiria. These buildings...
were constructed between 1950 and 2010. The period between 1950 and 1960 represents 11.77% of the analyzed buildings, and represents the buildings constructed prior to the entry into force of any seismic regulation. 52.93% of the analyzed buildings were constructed between 1960 and 1985, having been constructed with the first Portuguese seismic regulation [18]. The buildings constructed between 1985 and 2010, were designed with the most recent seismic regulation in Portugal [19], [20] before the Eurocode 8 [7]. The analyzed buildings constructed in this period from 1995 to 2010 represent 35.30%. These percentages are very similar to those obtained from the 2011 census, for reinforced concrete buildings.

In the case of the number of floors, the analyzed buildings have a large amplitude from 2 to 10 floors, and most buildings have between 3 and 6 floors representing 70.6% of the analyzed buildings.

Turning now to the geometric features of buildings. From the analysis of the buildings, the authors concluded that normally the height of the ground floor, is different from the height of the remaining floors. It was possible to conclude that ground floor of buildings presents heights between 2.8m and 3.5m. The most common height is 3.0m representing 39.70% followed by 2.8m with 25% of the buildings. The weighted average of the ground floor considering all the buildings is 3.19m. In terms of upper floors, the heights vary between 2.5m and 3.0m. The most used heights are 2.7m and 2.8m, representing almost 50% of the buildings. The weighted average for upper floors is 2.77m.

In terms of reinforced concrete structural elements, the authors collect information about beams and columns. For beams were collected the length, the height, the width and the reinforcement ratio of longitudinal reinforcement. For columns were collected information about the dimensions, width and depth, and reinforcement, longitudinal reinforcement ratio, and transversal reinforcement.

Beams length is usually conditioned by architectural options. For this study the authors, recorded beams with a length of 2.5m to 6.5m. The most usual length is 5m to be used in more than 26% of the elements analyzed. The great majority of buildings (69.42%) have beams with lengths between 4m and 5.5m. The weighted average for the length of the beams is 4.42m. Also, Pereira [5], in a similar study carried out on 24 reinforced concrete buildings, reported an average beam length of about 4.5m. In the case of beams height, the weighted average is 0.43m. Were observed beams height between 0.20m and 0.55m, but the most used heights are 0.40m and 0.45m, representing 44.59% of the analyzed beams. In terms of beams width, the most used width is 0.25m, the weighted average of all analyzed elements is 0.29m, having been observed widths between 0.15m and 0.4m. The last parameter analyzed in beams was the reinforcement ratio. The reinforcement ratio from 0.50% to 0.75% is clearly the most used, with 20% more elements than the second most used 0.25 to 0.50%. The weighted average value presents a value of 0.63%.

For the columns, the authors analyzed the width, depth, longitudinal reinforcement ratio and transversal reinforcement. In terms of width there is a clear tendency to use the range between 0.25 and 0.30 m, with values ranging from 0.15 to 0.40m. The weighted average width of the columns is 0.26m. In the case of depth, there is a greater dispersion of the values used in the analyzed columns. Values between 0.20m and 0.55m were observed. The weighted average has a value of 0.38m. In terms of longitudinal reinforcement ratios, the distribution is similar to beams, the interval 0.50 to 0.75, is the one that presents greater number of elements analyzed in a clear distance for the other intervals. The weighted average has a value of 0.61%. The distribution of the transversal reinforcement shows that all analyzed elements had transversal reinforcement made of stirrups with 6mm rods, deferring only in the space between stirrups. The most common spacing is 0.15m, and was recorded in 40% of the columns, followed by 0.25m spacing recorded in 24% of the analyzed columns.
After characterizing the reinforced concrete elements, the authors worked on the characterization of the masonry infill walls of analyzed buildings. It was studied the presence of openings in this type of walls, identifying the most common typologies of walls, in more than 1400 studied walls. It was also studied the dimensions of the openings in the masonry panel. The authors identified thirteen different types of walls identified in Figure 3, which differ in the number of openings and their position in the masonry panel.

![Figure 3: Masonry infill wall types](image)

From the 1400 masonry infill panels of the analyzed buildings, it was concluded that panel type 1 and type 2 are the most used, each representing 17% of the total of infill panels. From remaining typologies, types 6, 8, 4 and 12 are highlighted, which stand out from the other typologies with 15%, 11%, 10, and 9% respectively.

The distribution of openings height and width was measured in the infill panels. For openings height was registered openings ranging from 0.5m to 2.5m. It was possible to identify the interval of 1.0m to 1.5m as the most common value of height for openings. The weighted average for the height of the openings in the panels is 1.43m. In the case of the width of the openings in the panels, there were openings with values from 0.5m to 4m. But clearly the
range of 1.5m to 2.0m, and the range of 1.0m to 1.5m, are the most common, absorbing over 60% of the analysed openings. The weighted average width of the openings is 1.80m.

In conclusion, the buildings constructed in the recent past in Portugal, are characterized by having a reinforced concrete structure, with an envelope made using masonry infill walls. In terms of characteristics, based on the studies presented previously, it can be said that a typical frame presents beams of 4.42m with a section of 0.29m width by 0.43m of height with a reinforcement percentage of 0.63%. In the case of columns, they have a height of 2.77m, with a section of 0.26m wide by 0.38 deep. The longitudinal reinforcement has a value of 0.61% of the section area, and the transversal reinforcement is composed of 6mm stirrups spaced of 0.15m. The openings have an average width of 1.8m with a height of 1.43m, and 17% of the walls do not have any opening.

3 SEISMIC VULNERABILITY OF MASONRY INFILL WALLS

Despite all the advantages attributed to masonry infill walls constructed with ceramic units, it is often the case that these infill walls present several pathologies, namely excessive cracking and problems related with moisture, which are associated with the poor quality of the workmanship and the lack of detail in terms of design [14]. The occurrence of earthquakes has also demonstrated its inadequacy in terms of performance to the seismic action. Masonry infill walls are not considered as structural elements since they do not support vertical loads and therefore there are no specific rules or guidelines for their design and detail, including in Eurocode 6 [21]. However, when an earthquakes occur, these walls play an important role in the overall behavior of the building, generally having a positive influence on the overall behavior of the building increasing the lateral strength and stiffness, contributing also to energy dissipation [4]. If the damage of the walls is controlled, they can promote mechanisms of energy dissipation and control the relative displacements between floors. On the other hand, an irregular distribution of the walls in height, can lead to mechanisms of global collapse (soft story) or local collapse, at the level of the columns due to shear forces induced by infill walls (short pillar effect)[4].

Although masonry infill walls are not structural elements, individually they have to withstand seismic actions that are induced in the in-plane direction and in the out-of-plane direction [22]. The recent earthquakes in Lefkada in 2003 and Parnitha in 1999 in Greece, the earthquakes of Láquila 2009, Emilia in 2011 in Italy, and Christchurch in 2010 in New Zealand, showed an high level of damage for non-structural elements, with the masonry infill walls to present an inadequate seismic behavior.

Often the infill walls detach from the reinforced concrete elements and present diagonal cracking as a result of the relative displacement between floors incompatible with their deformation capacity and the development of stresses higher than the tensile strength of the masonry (see Figure 4). Sliding is also possible at the level of the mortar joints, as a result of the shear failure, leading to the detachment of the plaster. This cracking may or may not be repairable, depending on the density and thickness of the cracks. The behavior of the masonry infill walls presents a particular vulnerability to out-of-plane actions. For the seismic action is frequent the occurrence of a global collapse of masonry panel, as result of its rotation around the edges (beams and columns), which is associated with the poor connection between them and the infill wall. In the absence of adequate connection between the wall and the frame, it is impossible to develop the arch mechanism, typical of walls connected in two, three or four edges (unidirectional or bidirectional arch mechanism), which allows the wall to resist to out-of-plane actions with considerable deformations without loss of stability [23]. The seismic vulnerability of infill walls is also associated with the lack of connection between the panels in the case of cavity walls. In this case, the two panels work independently and the high slen-
derness results in fragile collapse of the panels, in particular of the external panel, (see Figure 4). Another aspect to be considered in the vulnerability of cavity walls consists of the poor support of the external leaf, when it is intended to use this for the correction of thermal bridges [10]. From the seismic point of view, inadequate support from the external leaf can contribute to accelerate the collapse of the wall in the out-of-plane direction. This type of collapse should be avoided because produce severe damages and loss of human lives.

Figure 4: Masonry infill damage, (a) separation between infill and frame, (b) in-plane damage, (c) out-of-plane collapse, (d) out-of-plane failure.

4 NEW SOLUTIONS FOR MASONRY INFILL WALLS

To address these issues of seismic vulnerability in masonry infill walls, the INSYSME project (606229) has as the main objective the development of innovative systems for masonry enclosures walls that resist better to seismic action. These solutions are designed to be used in new construction, using traditional materials. This section focuses on the presentation of a new constructive system for masonry infill walls in Portugal in order to improve the seismic performance of masonry infill walls.

During the development of the new system, three main types of solutions have been identified for the development of innovative enclosure masonry systems. The main goal is to solve the above-mentioned problems arising under the point of view of seismic behaviour. Notwithstanding, problems and aspects also related to the service behaviour and to non-mechanical behaviour of the infill walls, are tackled.

The three main concepts on which the systems rely are: (1) keeping the enclosure wall rigidly attached (adherent) to the frame, but using either or both robust units and internal (mainly steel rebars, as in reinforced load bearing masonry) or external (mainly reinforced plasters)
reinforcements; (2) keeping the enclosure wall rigidly attached (adherent) to the frame, or slightly disconnected, but allowing the internal deformation of the wall to occur, by means of special devices, special units, or special sliding or deformable vertical or horizontal joints; or (3) disconnecting the enclosure system from the top beam and/or from the columns, in order to allow relative displacements between the wall and the frame to occur without interactions. It is also possible to create hybrid systems using more than one of the solutions described above.

At the University of Minho was created one system called UMSystem. The UMSystem proposed at the University of Minho, also called Térmico system (see Figure 5), uses the concept of maintaining the infill rigidly attached to the frame, using internal reinforcement and connectors between the infill and frame. This system is a single-leaf clay masonry wall made with a commercial vertical perforated masonry unit produced in Portugal. The proposed system uses a M10 pre-mixed commercial mortar in the bed joints, and dry head joint with interlocking. To improve the in-plane and out-of-plane performance of masonry infill walls, truss reinforcements were used in the bed joints. Additionally, the walls are connected to the columns by metallic connectors at each two rows where bed joint reinforcement is applied. The masonry infill panel was built with 294x187x140mm bricks with vertical perforation, using murfor RND 0.5 100 reinforcement and in each two rows, and murfor L +100 anchors to connect the infill and RC frame at the same levels of reinforcements.

![Figure 5: Masonry infill system proposed at University of Minho.](image)

The idea of UMSystem is making the infill and the frame one system, increasing the initial stiffness by using connectors and reinforcement, which not only helps to increase the maximum load, as to control cracking and the out-of-plane collapse.

5 EXPERIMENTAL PROGRAM

5.1 Description of the specimen

The reinforced concrete frame considered in the present study is representative of the actual building practice in Portugal. The definition of the typical RC frame was based on an extensive work of geometrical characterization of Portuguese masonry infill presented in point
2.2. Due to the laboratory limitations, it was decided to test reduced scale specimens (2/3). For this, Cauchy’s Similitude Law was considered. Therefore, the geometry of the frame was reduced to 1.5 times and the reinforcing scheme was updated so that the relation between resisting bending moments and shear resisting forces could be well correlated between full and 1:1.5 scale frames. The geometry and reinforcement scheme adopted for the 2/3 scale RC frame are shown in Figure 6.

![Figure 6: RC frame used in experimental tests, (a) geometric scheme, (b) reinforcement scheme.](image)

The frame had 2575 mm and 1770 in length and height, respectively. The dimensions of beam and columns sections were 270×160 mm and 160×160 mm, respectively. The masonry infill panel was built with 294x187x140mm bricks with vertical perforation, using murfor RND 0.5 100 reinforcement and in each two rows, and murfor L +100 anchors to connect the infill and RC frame at the same levels of reinforcements. The steel used for the construction of RC frame and for reinforcement of masonry infill panel was a A400NR, and A500NR. In case of concrete, a C55/67 class was used for the construction of the RC frame. A M10 mortar was adopted for the laying of the masonry units. The thickness of the horizontal joints was assumed to be 0.5cm. The mechanical proprieties of materials and masonry specimens was obtained at university of Minho by Silva et al. [24].

5.2 Test setup and instrumentation

The test setup for the in-plane loading of the infilled frames is shown in Figure 7(a). The infilled frame was placed on two separated steel beams of HEA300 that were firmly attached to the strong floor to avoid their sliding on the floor. The sliding of the infilled frame was prevented by bolting an L-shape steel profile to each side of the steel beam and its uplifting was also prevented by bolting two rectangular-shape steel profiles to the steel beams. The rectangular shape steel profile was made by welding two UNP140. The out-of-plane movement of the enclosure frame was restrained by putting the L-shaped steel frame on each side of the upper beam. Those profiles were bolted to the upper steel beams. Three rollers were placed on upper L-shaped profiles to minimize or even completely eliminate the friction between them and the upper reinforced concrete beam during in-plane loading.

Two vertical jacks were mounted on the top of the columns to apply the vertical load of 200 KN, corresponding to 30% of the column’s axial force capacity. Those jacks are pinned to the lower steel beams by means of four vertical rods of Φ16mm. A hydraulic actuator with capacity of 250kN was attached to the reaction wall to apply the in-plane cyclic loading to the specimen. A steel plate of 400x300x30mm was connected to the hydraulic actuator that ap-
plies the load in positive direction from right to left direction. This steel plate was connected to other one with the same dimensions by 2Ф50mm steel rods to enable to pull the specimen in the negative direction. These steel plates enable also to have a uniform distribution of the horizontal load in the cross-section of the upper beam.

An instrumentation scheme to measure the in-plane most relevant displacements during the in-plane testing is shown in Figure 7(b). Eighteen linear variable differential transformer (LVDT) devices were used to record the displacement in selected points. From them, two LVDTs were mounted on the masonry infill to measure the deformation of the infill (L1 and L2), and eight LVDTs were used to measure the relative displacement of the infill with respect to its surrounding frame (L3, L4, L5, L6, L7, L8, L9 and L10). The LVDTs L11 and L12 were placed to measure the sliding and uplifting of the infilled frame with respect to the steel profile. Four LVDTs L13, L14, L15 and L16 measure the sliding and uplifting of the steel profiles with respect to the strong floor. LVDTs L17 and L18 measure the horizontal displacement of the upper reinforced concrete frame.

The test setup for out-of-plane loading is shown in Figure 8(a). The infilled frame was supported on the same steel profiles used for the in-plane testing setup. The out-of-plane restriction at the top and bottom RC beams was strengthened so that out-of-plane displacements at the boundaries could be prevented. For this, four steel rods connected to a steel device, connected in turn to the horizontal steel profiles were added at each side of the top RC beam, see Figure 8. The out-of-plane loading is applied by means of an airbag that is connected to an external supporting frame. Four rollers were mounted in the bottom part of the supporting frame enabling its moving along the direction of applied load without friction. The supporting frame was also kept in touch with four load cells to measure the load that is applied to the infill walls through the airbag, see Figure 8(a), where a detail about the system of the four load cells is shown (section A-A). The supporting frame, to which the load cells are attached, was firmly connected to the strong floor and to the lateral reaction wall, which prevented completely any uplifting and sliding of the out-of-plane reaction structure.

The instrumentation plan of the out-of-plane testing is shown in Figure 8(b). A total number of fifteen LVDTs were placed on the specimen to monitor its deformation while the out-of-plane load is applied. From them, nine LVDTs record the displacement history of the infill panel during loading (LVDT L1 to L9). Four LVDTs measure the relative displacement between infill and its surrounding frame (L10 to L13) and two LVDTs measure the out-of-plane movement of the upper and bottom reinforced concrete beam (L14 and L15).
5.3 Loading pattern for in-plane and out-of-plane tests

The in-plane testing was performed under displacement control by imposing different predefined levels of displacement by the hydraulic actuator, (see Table 1). The loading pattern was computed in accordance with FEMA 461 [25]. It is composed of sixteen different sinusoidal steps that starts from displacement of 0.5mm (0.026% drift) up to the lateral displacement of 75mm, corresponding to a lateral drift of 3.94%. Each step was repeated two times except for the first step that repeated six times.

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Drift (%)</th>
<th>Displacement (mm)</th>
<th>Number of repetitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.026</td>
<td>0.50</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>0.037</td>
<td>0.70</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.051</td>
<td>0.98</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0.072</td>
<td>1.37</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
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<td>2</td>
</tr>
<tr>
<td>6</td>
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</tr>
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</tr>
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<tr>
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</tr>
</tbody>
</table>

Table 1: In-Plane loading pattern

The amplitude $a_{i+1}$ of step $i+1$ is 1.4 times of the amplitude $a_i$ of step $i$. Table 1 shows the drifts, the displacements and the number of repetitions for each cycle adopted for in-plane testing.

In the case of loading pattern for out-of-plane, a quasi-static cyclic testing (one cyclic direction), the same approach was used, being the test conducted also in displacement control [26]. The first amplitude was repeated for six times and the others repeated two times to investigate the strength degradation of the specimen at each displacement increment. The point selected to control the test was the midpoint of the masonry infill wall (mid height and at mid length). The loading was performed in one direction to monitor the deformation of the infill, propagation of the cracks and performance of the interfaces between infill and reinforced con-
crete frame. In out-of-plane load pattern for the cycle sixteen the displacement used was 77.78mm, corresponding to a lateral drift of 4.08%.

The experimental campaign is composed of two specimens to be tested under in-plane loading (a bare frame, and a RC frame with the UMSystem), and, one specimen to be tested under the out-of-plane loading.

6 TEST RESULTS

6.1 In-Plane behavior

The in-plane lateral force-displacement diagram obtained for UMSystem and Bare Frame during in-plane loading is shown in Figure 9. The forces recorded correspond to the load cell of the actuator placed at the middle height of the upper beam. In the case of the displacements, these correspond to the LVDT 18, also placed in the middle height of the upper beam.

As mentioned before, positive direction is the direction where the hydraulic actuator pushes the specimen, whereas the negative direction is the direction that the actuator pulls the specimen by two plates that connected with two thick rods.

This diagram presents some differences in the positive and negative directions. The specimen reached maximum load of 166.65kN (+147% than bare frame) at displacement of 19.83mm corresponding to lateral drift of 1.04%. After the peak load the lateral force decreased until a residual strength around 95kN for 55.56mm displacement (2.92 % drift). On the other hand, for the negative direction, the specimen reached its peak load of -155.25kN at displacement of 19.60mm (lateral drift of 1.03%). Lateral force was gradually decreased and reached a force around -84kN at the displacement of 55.56mm (2.92% drift).

The test stopped because of the non-repairable damage of infill, depicted also by a reduction of force that can be observed in Figure 9(a). The force response during the second cycle is almost the same of first cycle, for firsts steps of loading in elastic range of the wall. After these firsts’ steps, it is possible to see the reduction for second cycle. This reduction is approx. 10.75% for positive direction and 10.63% for negative direction.

Figure 9: In-Plane force-displacement diagram, (a) UMSystem, (b) Bare Frame.
To assess the influence of masonry infill wall in the global cyclic behavior, was carried out one in-plane test only in the bare frame. The force-displacement diagram obtained for Bare Frame is shown in Figure 9(b). By comparing the force-displacement diagrams for RC bare frame and RC infilled frame, it is observed that an increase on the lateral strength was observed in case of the RC infilled frame. The bare frame achieved a maximum load of 67.60kN at displacement of 53mm corresponding to a lateral drift of 2.78%. After the peak load was reached, a reduction of lateral load of about 17% was recorded at a displacement of about 75mm (3.93% drift). For the negative direction, the specimen presents a similar behavior. The maximum load for this direction was -54.38kN at displacement of 46.98mm (2.47% drift). For the last cycle of 75mm the lateral force was decreased for a value around -52kN.

Comparing these two in-plane tests, it is possible to conclude that the use of masonry infill wall means the increase on the lateral resistance of approximately 147%. However, and as expected, the stiffness of infilled RC frame is higher than stiffness exhibited by the bare frame. The drift for maximum load is lower in the infilled frame, being of 1.04%, while in bare frame the drift corresponding to the maximum lateral resistance was 2.78%.

The damage patterns at different levels of displacement are presented in Figure 10. The damage pattern of infilled wall starts to be represented in Figure 10(a) at cycle 8, for a displacement of 7.32mm corresponding to a lateral drift of 0.34%. At this level of deformation, the cracking initiate at the interface between top beam and masonry infill, and in the interfaces between the columns and the infill wall, occurring the separation between the infill and the bare frame. After the damage begins by appearing in the central part of the wall and progress through the corners. In next cycles, the cracks increase their dimensions, and the damage in the interface between masonry infill and bare frame increase. The damage involves also some cracking at the brick units. At the end of the test in Figure 10(c) same parts of masonry units fall from the wall, this is more clearly in top of infill, and along the diagonal of the wall.

(a)  (b)  (c)

Figure 10: Damage pattern of UMSystem, (a) at 7.32mm, (b) at 14.35mm, (c) at 55.56mm.

For the bare frame the cracks start at cycle 8, for a drift of 0.34% (7.32 mm displacement), at bottom part of columns. For increasing displacements, the cracks concentrate in the top intersections between the columns and beams.

6.2 Out-of-plane behavior

The force-displacement behavior for the out-of-plane test of UMSystem is shown in Figure 11. After an initial linear behavior, the response become nonlinear before the maximum load is reached. The system reached maximum load of 117.05kN at a displacement of 53.65mm corresponding to lateral drift of 3.28%. After the peak load, the behavior presents same softening, and the lateral force decreased until a resistance around 108kN for 64mm displacement (3.92 % drift), before stopping the test. The maximum displacement applied was around 64mm, near to wall collapse. The test stopped because the imminent collapse of infill, depict-
ed also by a reduction of force that can be observed in Figure 11. The force response during the second cycle is almost the same of first cycle, for firsts steps of loading in elastic range of the wall. After these firsts’ steps, it is possible to see the reduction for second cycle. This reduction for maximum force has a value from approx. 6.59%.

The damage patterns at different levels of displacement for the out-of-plane test are presented in Figure 12. The first cracks start to appear in central part of wall in cycle 9, 0.45% drift (7.38mm). After this in next cycles until the final of test the cracks open from the central part to the corners of the wall. It is possible to observe the development of an arch mechanism in vertical direction. At the final stage of test, it’s clearly stair step crack, progressing from the central part of the infill to the corners, as result of the development of the arching mechanism in the vertical and horizontal direction. Additionally, it is seen same cracks in the middle of the columns, that can result from que transmission of stress the occur due to the use of metal connectors between the infill and the columns.

6.3 Comparison with traditional cavity walls

After briefly presenting the results obtained for UMSystem, to the In-Plane, and Out-of-Plane direction, it is important to compare the performance of the system, with the traditional solutions used in Portugal, in order to evaluate the performance of the UMSystem, compared to the current situation in Portugal.
For this, were used the results of two tests performed by Akhoundi [27], in cavity walls that are representative of traditional solution applied in Portugal. This comparison is possible because the tests performed by Akhoundi uses a bare frame with the same dimensions of the bare frame used in UMSystem, and the tests were performed in the same laboratory of University of Minho following the same test procedure. The masonry panel tested by Akhoundi is a cavity wall, without any connection between the leafs, made with an external leaf of 80mm and an inner leaf of 60mm, which gives the same cross section of UMSystem.

In the Figure 13, are presented the monotonic envelopes of the obtained from the experimental hysteretic loops, of the in-plane and out-of-plane tests.

Comparing the monotonic envelopes obtained for in-plane test of UMSystem, with the envelopes obtained for the traditional wall and bare frame (see Figure 13(a)), it is possible to conclude that the presence of an infill wall panel highly increases the stiffness and the maximum lateral force that the system can withstand. The UMSystem present the high value of lateral resistance, increase the value in 147% in relation to the bare frame, and 24% in relation to traditional cavity wall. The maximum lateral force on the traditional cavity wall is reached for a lower drift value (0.54%), while for the UMSystem the maximum is reached for a drift of 1.04%. The stiffness of the two systems is quite similar until the traditional wall approaches its maximum capacity. After reaching the maximum the tendency of the systems is to lose lateral resistance, approaching the behavior of the curve of the reinforced concrete frame, for great values of displacements.

The monotonic envelopes obtained for out-of-plane test of UMSystem, and traditional wall are presented in Figure 13(b). From the comparison of the curves, it is possible to conclude that the UMSystem present a much larger value of lateral resistance, increase the value in 194% in relation to traditional cavity wall. This large difference can in part be explained by the operation of the traditional wall, where only the exterior leaf works, when the wall is subjected to out-of-plane loads. The maximum lateral force on the traditional cavity wall is reached for a lower drift value (1.48%), while for the UMSystem the maximum is reached for a drift of 3.28%, which shows the that UMSystem can accommodate large displacements before reach its maximum capacity. The stiffness of the two systems is similar in the initial
phase for displacements lower than 3mm. After that the traditional wall approaches its maximum capacity, and the UMSystem continues the loading process. After reaching the maximum the tendency of the systems is to lose lateral resistance, the traditional cavity wall, have a great capacity of deformation reaching almost 80mm, of maximum displacement. In the case of the wall of the UMSystem, the loss of strength after the maximum is more pronounced, but the system can continue to accommodate deformation, to a value close to 65mm, when the test was stopped due to the eminence of collapse.

7 CONCLUSIONS

This paper presents and discuss same results obtained for in-plane and out-of-plane cyclic testing carried out on a new solution defined to improve masonry infilled RC frames. This system was developed by University of Minho in the scope of a European project INSYSME.

In the in-plane direction the masonry infill achieves higher resistance in about 24% than the traditional solution of cavity walls. The global behavior of hysteric curves was similar in the case of two systems, both presenting softening after peak load, but in case of strength degradation the traditional cavity wall presents more degradation. In case of stiffness, both systems presented similar values until the traditional wall approaches its maximum capacity.

For the out-of-plane direction, it was possible to apply a uniform load using an airbag, which is considered to be more representative of the out-of-plane lateral induced load by earthquakes. The resisting mechanism in both tests was an arching mechanism more evident in vertical direction. In both specimens it was possible to catch the post-peak behavior showing same softening. The UMSystem present a must larger value of lateral resistance, increase the value in 194% in relation to traditional cavity wall, which can be explained by the fact of only the external leaf of traditional wall be loaded, when the wall is subjected to out-of-plane test.

From the presented results, it can be concluded that this UMSystem, presents good indicators with respect to the seismic behavior, representing an improvement in the behavior, compared to the traditional cavity walls used in Portugal.

8 ACKNOWLEDGMENTS

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SEISMIC ANALYSIS AND RETROFITTING WITH FRP OF AN OLD MASONRY CLOCK TOWER

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Abstract

The seismic analysis of old masonry structures represents an important issue nowadays. Many researches have been developed to highlight on this important subject and how to apply retrofitting strategies appropriate for this kind of problems. This paper deals with the numerical seismic analysis and the benefit of the FRP retrofitting method on the seismic behavior of an old masonry clock tower located in Tripoli, Lebanon is a country with a high seismicity and is well known by its historical monuments and sites indeed. The chosen structure, called Al Tall clock tower, is one of the most significant monuments of Tripoli and was built by the Otto-mans in 1902. Therefore, a three dimensional finite element model is used to assess the seismic vulnerability of the structure by using the non-linear time history analysis first. Then, a pushover analysis was carried out and both results were compared. The obtained deformed shape and stress maps were used to highlight on the failure modes and the weak zones within the tower. The results exhibit the vulnerable behavior of the structure and the developed damaged zones along its height. Furthermore, a numerical model of the structure strengthened using fiber reinforced polymer (FRP) sheets wrapped around these critical zones is created. The analyses showed an improvement of the tower lateral strength and are promising in terms of reducing the seismic vulnerability of the old masonry monuments.

Keywords: Seismic Analysis, Finite Element Method, FRP, Retrofitting.


1 INTRODUCTION

Earthquakes represent one of the natural threats that, for a long time, have been an important issue studied by researchers and structural engineers. In this regard, the seismic vulnerability has been defined as the inability of an existing structure to withstand the effects of seismic forces. Many researches have been carried out in order to investigate the response of structures, made of different materials, when subjected to a predicted seismic loading. Different types of structures can be subjected to seismic risks and require a particular attention, such as buildings, towers, bridges, dams and nuclear power plants. The purpose of these studies is to extract the essential information and conclusions that are needed to establish new guidelines for seismic design and retrofitting strategies.

Recent earthquakes have underlined the urgent need for wide monitoring and safety assessment of historical and architectural heritage. Therefore, the seismic vulnerability assessment of existing old masonry buildings and monuments have become nowadays an important and complex design issue. The structural design of a new masonry building represents, relatively, a simple job compared to the investigation of seismic behavior of old masonry buildings due to various uncertainties at the level of the geometry, the mechanical properties of the materials and elements, the foundations systems, the soil properties, the past loading history...etc. Thus, handling all these data is mandatory before carrying out any structural investigation or proposing a retrofitting intervention.

The prediction of seismic behavior of existing masonry structures has attracted some attention in the last decades. Several codes such as the American code (ACI 530-02/ASCE 5-02/TMS 402-02) and the Eurocode 6 (DD ENV 1996-1-1: 1996) have set guidelines for strengthening and rehabilitation of existing structures.

The investigation of the seismic behavior of structures is mainly possible through three methods that allow checking their seismic performance and vulnerability. The first one is the experimental assessment that is being extensively used until today i.e. the shaking table experiment that reproduces the motion of the soil on which the physical model has been laid. These experiments provide crucial information and provide data to calibrate computer models in numerical simulation. This latter represents the second investigation method and has been continuously developing in the last 40 years. The third one was recently introduced and developed as an economic alternative experimental method that combines numerical simulation and physical testing. Furthermore, this new method on the hand involves online calculations and control and on the other hand the continuous measurement of the properties of a the studied structure, allowing researchers to model simulate the seismic response of a structure in a realistic way. Several common testing methods, including quasi-static testing (QST), shake table testing (STT), effective force testing (EFT), pseudo-dynamic (PSD) testing and real-time dynamic hybrid simulation (RTDHS) can be performed using the developed hybrid testing system.

The main objective of this paper is to present the numerical seismic analysis of the old masonry clock tower located in Tripoli, Lebanon. Firstly, the description of the structure and its geometry is presented with a brief report of its history. Then, the obtained finite element modeling results related to the non-linear time history and pushover analysis and the will be shown, compared and discussed. In addition, two strengthening methods, which are the fiber reinforced polymer (FRP) sheets wrapping and the repointing method, were modeled and the obtained results will be compared in terms of strength and stiffness. And finally, conclusions and perspectives will be discussed.
2 DESCRIPTION OF AL TALL CLOCK TOWER

Lebanon is on the countries with high seismic hazard, as it is located on the boundary between the Arabian and African tectonic plates. Although the high seismic risk that threatens Lebanon, there is no official building code that takes it into account and draws guidelines and recommendations to ensure the building of new earthquake-proof structures or to help preserving the existing old structures and in particular the old masonry monuments. In this regard, the clock tower of Al Tall located in Tripoli, Lebanon was chosen as a case study of this work.

This famous tower is one of the most recognizable sites in Tripoli (Figure 1.a). Built in 1902, it was a gift to the locals from the Ottoman Sultan Abdulhamid II at his 10th anniversary as a sultan. Standing at a height of 30 meters with five floors (Figure 1.b), the structure looms over the Tal district and is a staple of what it means to be from Tripoli.

The walls of the tower at the made of 60 cm thick sandstone blocks along the height of the structure. The square section of the tower decreases along its height starting by a 5 m × 5 m at the ground floor level (Figure 1.c) with four openings (2.1 m × 2.6 m) and reaching a section of 4 m × 4 m at the top with smaller openings (1.2 m × 2.6 m). The clock is in the fifth floor withstanding the upper load of a small stone pyramid hat. The tower footing is made of very thick stone blocks and is connected with the exterior walls. Therefore, the tower can be identified as a slender cantilever structure. The interior of the structure involves a stair ensuring the circulation from the bottom of the tower up to the fifth floor. A balcony exists at the 3rd floor level and can be considered as a mass concentration along the tower’s height. This mass has a negligible effect on the dynamic behavior of the structure due to its small value indeed.

The assessment of the mechanical properties of the masonry material was not possible due to several constraints imposed by the municipality of Tripoli. Hence, the mechanical properties of materials were evaluated considering some existing previous technical reports and recommendations. Moreover, the results of in-situ and laboratory tests on similar masonry materials, obtained from the debris of the old historical structures existing in the old city of Tripoli, were useful to estimate the mechanical properties of the tower’s materials. Limestone was used for bonding between the stone blocks in the walls.

Figure 1: Al Tall clock tower as it is today with its elevation and the ground floor plan layout
3 NUMERICAL MODELING

3.1 Materials properties

A finite element analysis was carried out in order to understand properly the dynamic behavior of the clock tower, predict the possible damaged zone when subjected to seismic loading and then study the different possible strengthening/retrofitting strategies of this monument. SAP2000 (CSI) software has been used for modelling the considered structure. The used material mechanical properties are given in Table 1. These values were given by previous technical reports related to laboratory tests on specimens taken from residues of the old historical structures in the old souks of Tripoli.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\gamma$ (kN/m$^3$)</td>
<td>20</td>
</tr>
<tr>
<td>Masonry compressive strength $f'_m$ (MPa)</td>
<td>40</td>
</tr>
<tr>
<td>Mortar compressive strength $f_m$ (MPa)</td>
<td>5</td>
</tr>
<tr>
<td>Tensile strength $f_t$ (MPa)</td>
<td>1.0</td>
</tr>
<tr>
<td>Shear strength $\tau_0$ (MPa)</td>
<td>0.3</td>
</tr>
<tr>
<td>Modulus of elasticity $E$ (GPa)</td>
<td>12</td>
</tr>
<tr>
<td>Shear modulus $G$ (GPa)</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Table 1: Mechanical properties of limestone.

3.2 Numerical model

Unreinforced masonry represents a composite heterogeneous material composed of masonry blocks and mortar. This latter ensures the bond between the stone blocks and then the overall strength is governed by the physical and mechanical properties of the components. Therefore, the material is considered as anisotropic and should be modeled by mean of a numerical model able to represent this anisotropy. This task has been a challenge for a while and recently two different numerical approaches have been presented and have become more and more used: micro-modeling and macro-modeling. The first one uses the finite element method to model the discretised masonry with the adoption of suitable constitutive non-linear behavior laws. The second approach requires the modeling of the structure by mean of piers and spandrels elements where both are connected by mean of rigid offsets. In this regards, several studies have have been carried out related to these approaches [1, 3, 4, 6, 8, 11, 12].

In this research, a three-dimensional finite element model representative of the structure was created through SAP2000 in order to perform seismic analysis. The elements are modeled by
mean of 3D non-linear layered shell elements (Figure 2.a). Moreover, two different behavior laws were used to represent to the vertical stresses $S_{11}$, the horizontal stresses $S_{22}$ and the shear stresses $S_{12}$ (Figure 2.b). Figure 3 shows the stress-strain curve of $S_{11}$ and $S_{22}$ based on the work of Kaushik et al. where (2007) [5] where $f_m$ is the compressive stress in masonry, $f'_{m}$ is the masonry compressive strength, $\varepsilon_m$ is the compressive strain in masonry and $\varepsilon'_{m}$ is the peak strain in masonry i.e. the compressive strain corresponding to $f'_{m}$.

On the other hand, the shear strength criteria is defined in terms of cohesion and the friction between block and mortar. It is represented by mean of Coulomb friction criterion:

$$\tau = c + \sigma \tan \varphi$$

(1)

Where $\sigma$ is the vertical stress and $\varphi$ represents the friction angle between elements. The 3D finite element model used to investigate the seismic behavior of the clock tower is shown in Figure 4.
3.3 Seismic load and dynamic characteristics

Three seismic accelerograms have been artificially generated in three direction NS (x-direction), EW (y-direction) and V (z-direction) in order to be used in the dynamic analysis of the tower. These three seismic signals represent the seismic risk level in Lebanon. Moreover, the response spectrum functions were derived from the given signals. It is necessary to note that the soil-structure interaction and the base rotations are not taken into account in this work. Figure 5 shows the three seismic signals and the corresponding response spectrum.

![Seismic Signals and Response Spectra](image)

Figure 5: synthetic acceleration-time graphs of the seismic risk in Lebanon and their response spectrums.

3.4 Modal analysis

Table 2 shows the details about the first five dynamic modes. The periods of the dynamic modes and their participation factors were computed.

<table>
<thead>
<tr>
<th>Modes</th>
<th>1\textsuperscript{st}</th>
<th>2\textsuperscript{nd}</th>
<th>3\textsuperscript{rd}</th>
<th>4\textsuperscript{th}</th>
<th>5\textsuperscript{th}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periods (s)</td>
<td>1.12</td>
<td>1.12</td>
<td>0.32</td>
<td>0.32</td>
<td>0.16</td>
</tr>
<tr>
<td>Participation factor (%)</td>
<td>39</td>
<td>39</td>
<td>8</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2: The first five modes and their participation factors

The contribution of the first four modes is very significant about 94% where the first mode participation is 39%. The 5\textsuperscript{th} mode, which represents a torsional mode, has almost no effect on the dynamic behavior of the tower. In-situ measurements of the modal frequencies using the ambient vibrations technique should have been done in order to validate the numerical values. However, it didn’t happen due to some technical problems.
3.5 Non-linear temporal dynamic analysis

In Figure 6, the time history behavior in x-direction (NS) of the tower is given. The seismic behavior of the tower exhibits a maximum top displacement of 5.5 cm, a maximum base bending moment of 15000 kN.m and a maximum shear force at base of 1210 kN.

![Displacement x](image)

![Base Moment](image)

![Base Shear X](image)

Figure 6: Time history evolution of the top x-displacement, the base bending moment and the base shear force of the tower

The distribution of the maximum axial and shear stresses are shown in Figure 7. The results show that the maximum stress values given by the FEM analysis are 4.2 MPa in compression, 0.9 MPa in tension and 1 MPa in shear. Based on the obtained stress mapping, the tower is damaged in tension in different zones and in particular close to the openings as the tensile capacity of the masonry material is 1 MPa.

![Stress Distribution](image)

Figure 7: Distribution of axial stress (left) and shear stress (right) along the tower height
3.6 Pushover analysis

A non-linear static analysis, so called pushover analysis, was also performed on the tower. This conventional method represents an alternative simple way to investigate the seismic behavior of an existing structure with a small computational cost when compared to non-linear time history analysis. Therefore, the finite element model is subjected to constant gravity loads with the application of lateral loads that increase monotonically [7, 10]. These forces are proportional to the product of the mass and the first modal shape amplitude at each level. The response of the structure is represented by a capacity curve characterizing the relationship between the top displacement and the base shear force. Then the capacity curve is transformed to a capacity spectrum curve used to find its intersection point with the inelastic acceleration response spectrum curve: this point is called the performance point (Figure 8) of the structure and subsequently can be used to find the design base shear force, top displacement and damage level. In this matter, the performance spectral displacement can be used to evaluate the damage limit state according to Lagomarsino and Penna (2003) [9] as shown in Table 3.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>Spectral displacment S_d</th>
</tr>
</thead>
<tbody>
<tr>
<td>No damage</td>
<td>S_d &lt; 0.7 D_y</td>
</tr>
<tr>
<td>Slight</td>
<td>0.7 D_y &lt; S_d ≤ D_y</td>
</tr>
<tr>
<td>Moderate</td>
<td>D_y &lt; S_d ≤ D_y + 0.25 (D_u - D_y)</td>
</tr>
<tr>
<td>Extensive</td>
<td>D_y + 0.25 (D_u - D_y) &lt; S_d ≤ D_u</td>
</tr>
<tr>
<td>Complete</td>
<td>S_d &gt; D_u</td>
</tr>
</tbody>
</table>

Table 3: Performance levels and criteria according to Lagomarsino and Penna (2003).

The obtained top displacement was 4.3 cm where the base shear force was 530 kN. These values were far from those obtained during the non-linear time history analysis. Therefore, a multimodal pushover analysis was a must in order to take into account the contribution of higher modes in the dynamic behavior of the tower. Table 4 shows that by combining the effects of the first two modes the results become quite close to those obtained by the non-linear time history analysis. The combination is based on the SRSS method (Square Root of the Sum of the Square):

\[ u_i^{SRSS} = \sqrt{\sum_{j=1}^{n} u_{i,j, max}^2} \]  

Figure 8: Finite element model of the clock tower with the window openings.
By looking to the spectral displacement (4.3 cm) obtained during the pushover analysis, the damage state of the tower is considered as extensive. Consequently, the tower requires a strengthening procedure must be carried out in order to prevent the total collapse of the tower if an earthquake hits the region. Two main methods were studied in this work: the fiber reinforced polymer (FRP) sheets wrapping and the repointing method.

### 3.7 Structural strengthening of the tower

The two investigated strengthening methods were discussed in [2]. On the first hand, FRP materials that greatly meet the concept of ‘engineered material’, are being more and more applied for the restoration of vertical and horizontal structural elements as well as isolated monuments (statues, etc…). This reinforcement is made of different kinds of fibers (carbon, glass, polyvinylalcohol, etc...) impregnated in a polymeric matrix. Moreover, it allows the increase of the flexural strength of the vertical structural elements. On the other hand, deep repointing is a widely applied technique in all types of masonry. However, deep repointing is expected to be efficient in enhancing the mechanical properties of masonry only in some cases and under several conditions. This operation involves the partial replacement of the mortar joints with better quality mortar, in order to improve the masonry mechanical characteristics, and it should be applied in the case the deterioration is localized only in the mortar. The described operation can increase the masonry resistance of both vertical and horizontal loads, but the best results are obtained especially in term of deformation, which are also greatly diminished due to the confinement effect of the joints.

The numerical model was updated to model the two strengthening techniques. Therefore, at a first stage by adding a new layer, representative of the FRP material, on the highly damaged tensile zones. The added FRP material has a young modulus of 100 GPa, 1mm thickness and an ultimate tensile strength of 3500 MPa. A linear elastic behavior law for the FRP material was adopted. In a second stage, the repointing method was modeled by increasing the mortar compressive strength to 38 MPa.

![Figure 9: Distribution of the maximum axial stresses (a- repointing method/ c - FRP reinforcement) and the maximum shear stresses (b- repointing method/ d - FRP reinforcement) along the tower](image)

<table>
<thead>
<tr>
<th></th>
<th>Time history</th>
<th>Pushover (2 modes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top displacement (cm)</td>
<td>5.5</td>
<td>4.9</td>
</tr>
<tr>
<td>Base shear force (kN)</td>
<td>1210</td>
<td>1125</td>
</tr>
</tbody>
</table>

Table 4: Comparison between the results of time history analysis and multimodal pushover analysis of the tower.
Figure 9 shows that after adding the FRP sheets the maximum compressive stress is reduced to 2.2 MPa and the maximum tensile stress became 0.5 MPa. Moreover, the maximum shear stress is reduced to 0.8 MPa. On the other side, the tower reinforced using the repointing method exhibits a maximum compressive stress of 4 MPa, a maximum tensile stress of 0.8 MPa and a maximum shear stress of 1.1 MPa.

A pushover analysis was also carried out using the two reinforced models and the new capacity curves are redrawn as shown in Figure 10. As shown, the ultimate strength of both models, in terms of base shear force, increased from 850 kN to 1200 kN. However, the FRP reinforced model behavior is more ductile than the model strengthened with the repointing method.

![Figure 10: Capacity curves of the unreinforced and reinforced models.](image)

4 CONCLUSIONS

This work presented the seismic investigation of an old masonry clock tower in Tripoli, Lebanon. A 3D finite element model was created for this purpose and used in the time history and pushover analysis. The results showed the necessity to adopt a multimodal pushover analysis in order to get congruent results for both analyses. Moreover, the distribution of damage along the numerical model of the tower lead to propose strengthening strategy. Two reinforcement methods were investigated: the repointing method and the FRP sheets wrapping. Both methods helped improving the lateral capacity of the tower with a main difference in the lateral behavior: the first reinforced model exhibited a rigid behavior less ductile than the behavior of the FRP reinforced FE model. However, precautions should be taken regarding the low fire resistance of FRP material i.e. the use of minimum coating/mortar, which can play an aesthetic role as well. Finally, a further research on the seismic behavior of masonry structures using a distinct element method is interesting as this method can represent easily the cracks/discontinuities formation and propagation through the structure.

REFERENCES


COMPUTATIONAL PREDICTION OF THE STABILITY OF TENSEGRITY STRUCTURES

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Abstract

The lightweight nature of tensegrity structures calls for the formulation of computational tools that are able to analyze the stability problem of such structures, both in statics and dynamics. The present work analyzes the concepts of prestress-stability and superstability with reference to a special class of tensegrity structures recently appeared in the literature, named class theta tensegrity prisms. Such structures are formed by two triangular prisms superimposed one over the other, and two different sets of strings connecting the two prisms. The given results can be easily generalized to general tensegrity structures and can be usefully incorporated in form-finding and topology optimization codes.

Keywords: class θ tensegrity systems, form-finding, prestress-stability, superstability.
1 INTRODUCTION

Tensegrity structures are three dimensional mechanical systems formed by a discontinuous set of compressive elements (bars or struts) connected each other through tensile elements (cables or strings). Such systems differ from conventional truss structure due to the fact that the equilibrium of the structure is guaranteed by the action of self-equilibrated sets of axial forces in bars and cables (in absence of external loads), which stabilize infinitesimal mechanisms from the freestanding configuration. Structural problems of relevant importance to tensegrity systems are related to the existence of freestanding configurations, and the stability of the static and dynamic equilibrium problems under the action of external forces [1]-[7]. It is worth remarking that tensegrity structures offer lightweight solutions to structural design problems under a large variety of loading conditions [8]-[15].

A new class of tensegrity structures has been recently introduced in the literature, with the name of class theta tensegrity prisms [16]. Such structures are formed by the superimposition of two T3 prisms (i.e., two triangular prisms formed by 3 bars each [Skelton and de Oliveira]), which are connected each other through two different set of cables. A first set of cables connects the external nodes of the structure, while the second set connects the inner nodes and provides a connection between the two superimposed prisms. The Greek letter has been introduces by Bieniek to label such systems [16], since it mimics the geometry of a structure formed by external and internal cables.

Due to the presence of self-stress and infinitesimal mechanisms, different notions of stability can be introduced with reference to tensegrity systems [17], [18], [19], with special emphasis on prestress-stability and superstability (Sect. 2). The present work specializes such definitions with reference to class theta tensegrity prisms (Sect. 3). A benchmark example referred to this structural typology is analyzed through a numerical approach (Sect. 4), which can be easily generalized to study the stability of arbitrarily shaped tensegrity systems.

2 STABILITY NOTIONS FOR TENSEGRITY STRUCTURES

Let us consider the current configuration of an elastic tensegrity structure formed by a total number \( M \) of bars and strings that obey the following constitutive law

\[
t_m = k_m (\ell_m - \bar{\ell}_m), \quad m = 1, ..., M
\]

where \( \bar{\ell}_m \) denotes the length of the generic member in the unstressed configuration of the lattice (rest-length) and \( k_m = E_m A_m / \bar{\ell}_m \) denotes the axial stiffness coefficient. Here, the quantity \( E_m \) denotes the Young modulus of the \( m \)-th member, while \( A_m \) denotes the cross-section area.

The equilibrium problem of the lattice is written as follows [19]:

\[
A \, t = f
\]

where \( A \) denotes the instantaneous static matrix of the structure; \( t \in \mathbb{R}^M \) denotes the vector of the axial forces carried by bars and strings; and \( f \in \mathbb{R}^Q \) denotes the external force vector. \( Q \) being the total number of degrees of freedom of the structure. The transpose \( B \) of the static matrix rules the kinematic problem of the structures, and its null space groups the infinitesimal mechanisms of the structure [19]. The equilibrium problem of the structure is ruled by the equations:
where \( \frac{\partial \ell_m}{\partial q_r} \) is the \((m,r)\) entry of \( A \). By differentiating both sides of Eqn. (3) one time with respect to time, and denoting time-derivatives by superimposed dots, we obtain the incremental form of the equilibrium problem as follows

\[
K_T \ddot{q} = \dot{f}.
\]

Here, \( K_T \) is the tangent stiffness matrix of the structure, which is computed through the summation of the material stiffness matrix \( K_M \) and the geometric stiffness matrix \( K_G \).

In particular, \( K_M \) depends on the material stiffness coefficients \((k_1, ..., k_m)\) and on the cosine directors of the member axes \((\frac{\partial \ell_m}{\partial q_r})\). It can be written as follows:

\[
K_{M_{rs}} = k_m \frac{\partial \ell_m}{\partial q_r} \frac{\partial \ell_m}{\partial q_r}, \quad (r, s = 1, ..., Q),
\]

Moreover, \( K_G \) depends on the tensions acting in the current configurations of the members \((t_1, t_2, ..., t_M)\) and on the variations of the cosine directors of the member axes (i.e., \( \frac{\partial^2 \ell_m}{\partial q_r \partial q_s} \)):

\[
K_{G_{rs}} = t_m \frac{\partial^2 \ell_m}{\partial q_r \partial q_s}, \quad (r, s = 1, ..., Q).
\]

From Eqn. (6), it results that the geometric term of \( K_T \) is null when the members of the structure are not in tension or in case of infinitesimally-small deformations.

By analyzing Eqn. (5), we can deduce that \( K_M \) can be also written in the following form:

\[
K_M = A \text{ diag}(k_1, k_2, ..., k_M) A^T.
\]

This means that it is reasonable to write that \( B = A^T \) ([18][20][21]).

Then, the expression \( K_M u_M = 0 \) representing a zero-material-stiffness mode, has the same meaning of writing \( \ddot{q}_M \) multiplied by an infinitesimal time interval (that represents the mechanism of the structure), where the quantity \( u_M \) is the virtual displacement.

If \( K_T \) is positive-definite in correspondence with a generic configuration, the latter is defined as stable and it results

\[
K_T u \cdot u > 0
\]

where \( u \) is a generic virtual displacement from the current configuration [17][18][21][22].

Moreover, we can define a prestress-stable configuration if it results

\[
K_G u_M \cdot u_M > 0
\]

in correspondence with each non-trivial mechanism \((u_M \neq 0)\) [17][18][21][22].

In the prestress-stable configuration, the tension applied to the members stabilizes the structure by avoiding the formation of zero-stiffness modes.

Finally, it is sayd a superstable configuration the one that corresponds to a prestressable tensegrity structure and to a nonnegative value of \( K_G \).
This means that a superstable structure is stable along all the possible virtual displacements from the analyzed configuration and it does not depend on prestress and material properties [17].

3 FORM-FINDING OF CLASS THETA TENSEGRITY PRISMS

This Section illustrates a mathematical approach employed to optimize the shape of a Class $\theta=1$ tensegrity tetrahedron [24][25]. We first consider the tetrahedron $T_2$ illustrated in Fig. 1. The analyzed structure is formed by four bars of length $b$ and two sets of four strings of length $c$ (internal cables) and $l$ (external cables). On introducing the Cartesian frame $\{O,x,y,z\}$ depicted in red in Fig. 1, we can obtain the nodal coordinate vectors depending on $c$, $l$ and $\alpha$, that denotes the arbitrary angle between internal strings and the y-axis.

The length of the bars can be easily computed by using the following expression:

$$b = \frac{1}{4}c^2 + \frac{1}{2}l^2 + \frac{1}{4}(l + c\cos \alpha)^2 + \frac{1}{4}l^2 \cos \left(\frac{\pi}{4} - \alpha\right)^2$$

The geometrically feasible configurations of the tensegrity tetrahedron are obtained by letting $\alpha$ vary between $\alpha=0^\circ$ (Figure 1c) and $\alpha=45^\circ$ (Figure 1d).

We now consider the class $\theta=1$ triangular tensegrity prism centered at the origin of a Cartesian frame $\{O,x,y,z\}$ [26] (Figure 3).
This structure is formed by six bars of equal length $b$ and two sets of \textit{internal} and \textit{external} cables. Inner cables are formed by 6 elements of equal length $c$, connecting the nodes TB1-BT1-TB2-BT2-TB3-BT3 (Figure 3).

The nine external cables include:

- two set of three horizontal cables with length $l$, connecting the nodes T1,T2,T3 and B1,B2,B3 (Fig. 3);
- three horizontal cables T1-B1, T2-B2 and T3-B3 with equal length $v$ (Fig. 3).

The kinematics of such a structure can be described by varying two aspect angles $\alpha$ and $\beta$ describing the slope of the internal strings with respect to the xy-axes and the twisting angle between the two external bases of the structure, respectively.

By varying the values of $\alpha$ and $\beta$ and the lengths of the connectivities, we could design different configurations. Fig. 3 shows the extreme non-equilibrium configurations of the analysed structure for $\beta = 0^\circ$ and $\beta = 60^\circ$.

Few examples of equilibrium configurations are illustrated in Figure 4. In particular, Figure 4a corresponds to $l = v = 2c$ (regular configuration) and Figure 4b corresponds to $l = v = c$ (expanded configuration).
Making use of the previous results, we finally obtain [26]:

\[
b = \left(\frac{1}{3} \cdot l \cdot \cos \frac{\beta}{2}\right)^2 + \frac{1}{4} c^2 (1 + 3 \cos^2 \alpha)^2 + cl \cdot \cos \alpha \left(\cos \frac{\beta}{2} + \frac{1}{3^{1/2}} \cdot \sin \frac{\beta}{2}\right) + \cdots \\
\cdots + \frac{1}{4} v^2 + \frac{1}{2} c \cdot v \left(1 - \frac{4}{3} \left(\frac{l}{v}\right)^2 \cdot \sin^2 \left(\frac{\beta}{2}\right)\right)^{1/2} \cdot \sin \alpha^{1/2}
\]

(11)

An approach to the form-finding of the illustrated tensegrity structures is to look for the zero this approach we refer to [16].

4 PRESTRESS STABILITY AND SUPERSTABILITY

Let us now study the stability problem of a tensegrity theta prism with reference to the model structure shown in Fig. 1. The freestanding configuration of the examined structure shows \( l = 120 \text{mm}, v = 125 \text{mm} \) and \( c = 55 \text{ mm} \) [16]. The compressive members are made of grade 8.8, M4 threaded steel bars plated with white zinc, while the strings consist of Spectra ™ cables with 0.76 mm diameter [23].

Figure 5: Freestanding configuration of a class theta tensegrity prism corresponding to \( l = 120 \text{mm}, v = 125 \text{mm} \) and \( c = 55 \text{ mm} \).
We begin by studying the kinematic problem of the structure in Fig. 1 through the approach presented in [16], which leads us to recognize that such a structure exhibit the ten infinitesimal mechanisms shown in Fig. 2 form the freestanding configuration (upon constraining rigid body motions).

Next, we pass to compute the geometrical tangent stiffness matrix \( K_{TG} \) of the structure under consideration employing the approach outlined in Sect. 2 and diffusely described in [16]. The self-states of self-stress in the freestanding configuration is characterized by the following force densities \( x_{i-j} \) (positive in tension in the strings, and in compression in the bars), which are parametrized in terms of the force density \( x \) acting in the base strings (free parameter). The force densities acting in the vertical strings are equal to 1.01\( x \), while those acting in the internal strings are computed as follows: \( x_{10-7} = x_{11-8} = x_{12-9} = 3.61x \), \( x_{7-11} = x_{8-12} = x_{9-10} = 4.44x \). Finally, the force densities acting in the bars are equal to 1.79\( x \) in the self-stressed state. The geometric stiffness matrix of the structure, \( K_{TG} \), can be cast into the following block form

\[
K_{TG} = x \begin{bmatrix}
G_1 & G_2 & 0 & G_3 \\
G_2 & G_4 & G_5 & 0 \\
0 & G_5 & G_6 & G_7 \\
G_3 & 0 & G_8 & G_9 \\
\end{bmatrix}
\]

where the block-matrices \( G_i \) \((i = 1, \ldots, 9)\) are given by:

\[
G_1 = \begin{bmatrix}
0.96 & 0.23 & 0.72 & -0.45 & 0.5 & 0 & -0.093 & -0.29 & 0 \\
0.23 & 0.77 & 0.032 & 0.5 & -0.55 & 0 & -0.29 & -0.91 & 0 \\
0.72 & 0.032 & 0.7 & 0 & 0 & -1 & 0 & 0 & -1 \\
-0.45 & 0.5 & 0 & 0.62 & -0.033 & -0.33 & -0.96 & -0.21 & 0 \\
0.5 & -0.55 & 0 & -0.033 & 1.1 & -0.64 & -0.21 & -0.045 & 0 \\
0 & 0 & -1 & -0.33 & -0.64 & 0.7 & 0 & 0 & -1 \\
-0.093 & -0.29 & 0 & -0.96 & -0.21 & 0 & 1 & -0.19 & -0.39 \\
-0.29 & -0.91 & 0 & -0.21 & -0.045 & 0 & -0.19 & 0.72 & 0.61 \\
0 & 0 & -1 & 0 & 0 & -1 & -0.39 & 0.61 & 0.7
\end{bmatrix}
\]
\[
G_2 = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.95 & 0.23 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.23 & -0.055 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.97 & -0.024 & 0.2 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.024 & -0.99 & -0.11 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.2 & -0.11 & -0.055 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.97 & 0.024 & -0.2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.024 & -0.99 & -0.11 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 & -0.11 & -0.055
\end{bmatrix},
\]

\[
G_3 = \begin{bmatrix}
0.59 & -0.43 & -0.72 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.43 & 1.6 & -0.26 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.72 & -0.26 & 1.4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.17 & -0.23 & 0.13 \\
0 & 0 & 0 & -0.23 & 0.48 & 0.76 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.13 & 0.76 & 1.4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.67 & 0.59 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.67 & 1.2 & -0.49 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.59 & -0.49 & 1.4
\end{bmatrix},
\]

\[
G_4 = \begin{bmatrix}
0.96 & -0.23 & -0.72 & -0.093 & 0.29 & 0 & -0.45 & -0.5 & 0 \\
-0.23 & 0.77 & 0.32 & 0.29 & -0.91 & 0 & -0.5 & -0.55 & 0 \\
-0.72 & 0.032 & 0.7 & 0 & 0 & -1 & 0 & 0 & -1 \\
-0.093 & 0.29 & 0 & 1 & 0.19 & 0.39 & -0.96 & 0.21 & 0 \\
0.29 & -0.91 & 0.19 & 0.72 & 0.61 & 0.21 & -0.045 & 0 & 0 \\
0 & 0 & -1 & 0.39 & 0.61 & 0.7 & 0 & 0 & -1 \\
-0.45 & -0.5 & 0 & -0.96 & 0.21 & 0 & 0.62 & 0.033 & 0.33 \\
-0.5 & -0.55 & 0 & 0.21 & -0.045 & 0 & 0.033 & 1.1 & -0.64 \\
0 & 0 & -1 & 0 & 0 & -1 & 0.33 & -0.64 & 0.7 \\
0.59 & 0.43 & 0.72 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.43 & 1.6 & -0.26 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.72 & -0.26 & 1.4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0.67 & -0.59 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.67 & 1.2 & -0.49 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.59 & -0.49 & 1.4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.7 & 0.23 & -0.13 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.23 & 0.48 & 0.76 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.13 & 0.76 & 1.4
\end{bmatrix},
\]

\[
G_5 = \begin{bmatrix}
4.4 & -2.2 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.2 & 2.1 & -0.11 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.3 & -0.11 & 6.1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4.6 & 2.1 & -0.24 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2.1 & 1.9 & -0.2 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.24 & -0.2 & 6.1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.72 & 0.097 & -0.056 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.097 & 5.7 & 0.31 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.056 & 0.31 & 6.1
\end{bmatrix},
\]

\[
G_6 = \begin{bmatrix}
-3.6 & 0 & 0 & -1.4 & 1.8 & -1.0 & 0 & 0 & 0 \\
0 & -0.28 & 0.96 & 1.8 & -3.4 & -0.59 & 0 & 0 & 0 \\
0 & 0.96 & -3.3 & -1.0 & -0.59 & -4.1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1.1 & -1.4 & 0.83 & -4.4 & 0 & 0 \\
0 & 0 & 0 & -1.4 & -2.8 & -0.48 & 0 & -0.34 & 1.2 \\
0 & 0 & 0 & 0.83 & -0.48 & -3.3 & 0 & 1.2 & -4.1 \\
-1.4 & -1.8 & 1.0 & 0 & 0 & -1.1 & 1.4 & -0.83 & 0.0 \\
-1.8 & -3.4 & -0.59 & 0 & 0 & 1.4 & -2.8 & -0.48 & 0.0 \\
1.0 & -0.59 & -4.1 & 0 & 0 & 0 & -0.83 & -0.48 & -3.3
\end{bmatrix},
\]

\[
G_7 = \begin{bmatrix}
\end{bmatrix}
\]
By prescribing a prestrain \( p = 0.1\% \) in the base strings, we get \( x = 113.41 \text{ N/m} \). Let us refer to this value of the free parameter to numerically characterize the self-stress state of the structure under examination. It is easily verified that such a structure is prestress-stable since the work done by KG for each of the mechanisms in Figure 6 is positive. In fact, it results:

\[
\begin{align*}
K_G \cdot \ddot{u}_1 &= 324.56, \\
K_G \cdot \ddot{u}_2 &= 224.581, \\
K_G \cdot \ddot{u}_3 &= 596.017, \\
K_G \cdot \ddot{u}_4 &= 711.805, \\
K_G \cdot \ddot{u}_5 &= 345.87, \\
K_G \cdot \ddot{u}_6 &= 400.564, \\
K_G \cdot \ddot{u}_7 &= 739.289, \\
K_G \cdot \ddot{u}_8 &= 506.349, \\
K_G \cdot \ddot{u}_9 &= 711.595, \\
K_G \cdot \ddot{u}_{10} &= 449.881
\end{align*}
\]

Making use of the Eigenvalues and Eigenvectors functions of Mathematica®, we easily recognize that the KG matrix given by Eqn. (13) shows seven negative eigenvalues, which are associated with the eigenmodes graphically illustrated in Figure 7. We therefore conclude that such a structure is not superstable [17].

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K_G \cdot \ddot{u}_4 &= 711.805, \\
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K_G \cdot \ddot{u}_6 &= 400.564, \\
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5 CONCLUDING REMARKS

We have reviewed the form-finding problem of class theta tensegrity prisms and have studied the stability problem of such structures with reference to an illustrative example. The given results have shown that the examined structure is prestress-stable but not superstable. This implies that it is not stable independently of the selfstress level and material properties [17]. The large number of infinitesimal mechanisms exhibited by tensegrity theta prisms justify the technical interest toward such structures, which are able to exhibit both soft and stiff modes [16]-[26]. Their use for the design of novel seismic metamaterials [27]-[28] and bandgap structures [29]-[31] is addressed to future work. We also plan to generalize the constitutive response of cables and bars to account for hyperelasticity, nonlinear, nonlocal and unilateral response of the material [32]-[45], through future research. Finally, we plan to design shells and membranes with tensegrity architecture [46]-[48] and to fabricate physical models of tensegrity structures through advanced additive manufacturing techniques [49]-[51].

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Computational Modeling of the Dynamics of Active Sunscreens with Tensegrity Architecture

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Abstract

Recent studies have investigated the use of tensegrity structures for the construction of active solar façades of Energy Efficient Buildings. In this paper we simulate the dynamics of shading screens with tensegrity architecture through an in-house developed code that handles rigidity constraints on the deformation of the bars. We present numerical results illustrating the dynamic response of a tensegrity solar façade and its morphing capabilities, which require minimal storage of internal energy.

Keywords: Tensegrity Structures; Dynamic Solar Façades; Energy Efficient Buildings.
1 INTRODUCTION

An efficient management of building technology systems translates to energy savings, reduction of CO2 emissions and improvement of the environment in full compliance with the most common national and international directives. A solar shading, according to international regulations [1], is a system that "applied to the outside of a transparent glazed surface allows a variable and controlled modulation of the energetic and optical luminous parametrics in response to solar stresses". The basic principle of their operation consists in avoiding the indoor overheating caused by direct solar radiation only during the summer, without penalizing the entry of natural light during the winter season. The building envelope (made up of opaque and transparent parts) is the skin of the building through which the flows of energy are exchanged with the surrounding environment. Today, thanks to technological and energy advances, it is possible to make the building envelope a dynamic surface able to guarantee adequate thermal comfort by reducing energy consumption.

Recent studies have shown that tensegrity lattices can be adapted for the construction of active solar screen for energy-efficient smart buildings [2]-[4]. Tensegrity lattices consists of networks of prestressable truss structures, obtained by connecting compressive members (bars or struts) through pre-stretched tensile elements (cables or strings). Motivated by nature, where tensegrity concepts appear often and at different length-scales, as for instance in cells, in the microstructure of the spider silk, and in the arrangement of bones and tendons for control of locomotion in animals and humans, engineers have only recently developed efficient analytical methods for exploiting tensegrity concepts in engineering design [5]-[8]. The present work aims to design, model and control active solar façade based on tensegrity units that support solar panels and/or sound-proof panels. Such units are controlled by cables tension, and are used to protect the building from intense solar radiation.

We propose a tensegrity solution that is inspired by the shading envelope of the well-known Al Bahar Towers in Abu Dhabi, designed in 2012 by Aedas Architects [9]-[10], using a different technology, with the aim to recall the ancient Islamic architecture characterized by the famous “mashrabiya.” The elements forming the “origami” panels of the tensegrity shading screens analyzed in the present work are in the fully opened configuration at night, and pass progressively in the fully closed configuration during daylight hours, by controlling the tension in selected cables. Their aim is to improve the thermos-hygrometric indoor conditions by reducing the direct solar exposure.

We simulate the dynamics of the tensegrity shading screens through an in-house developed code that handles a rigidity constraint on the deformation of the bars. We present schematic results illustrating the mechanics of the proposed tensegrity solar façade and its morphing capabilities, which require minimal storage of internal energy, and reduced operation costs (due to small friction between parts), as compared, for instance, to the piston-actuated technology adopted by Aedas Architects.

2 DYNAMICS OF A TENSEGRITY SUNSCREEN MODULE

A key goal of the present work is to analyze the dynamic of a tensegrity sunscreen model through an algorithm that handles a rigidity constraint on the deformation of the bars (makes use of a Runge-Kutta scheme for the time-integration of the equations of motion [11]). By describing the bars as rigid members and the strings as elastically deformable elements, we can develop the dynamics of an arbitrary tensegrity network in vector form. The latter formulation can be conveniently associate with standard FE models, because such models usually make consider a large number of degree of freedom and in nonlinear dynamic analysis are not easy to implement [11]-[35].

By analyzing a general tensegrity network made up of $n_n$ nodes (or joints), $n_b$ bars and $n_s$ strings or cables, we assume the bars (i.e., the compressed members) behave as straight rigid
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bodies with uniform mass density (for the numerical values see the table 1), constant cross-section, and negligible rotational inertia about the longitudinal axis; the cables are instead modeled as straight elastic springs that can carry only tensile forces.

Figure 1: Schematic view of the tensegrity sunscreen model. The red colored members are cables; the black thick lines are bars. For graphic reasons, only one third of the basic module was highlighted, the remaining part is grayscale.

We can describe the geometry of the tensegrity module by defining in the three-dimensional Euclidean space, the generic node \( i \), with \( i \in [1, \ldots, n_n] \) located by the vector \( \mathbf{n}_i \in \mathbb{R}^3 \), and the external force vector \( \mathbf{p}_i \in \mathbb{R}^3 \). Through these vectors \( \mathbf{n}_i \) and \( \mathbf{p}_i \), we introduce the following nodal and force matrices:

\[
\mathbf{N} = [\mathbf{n}_1 \quad \mathbf{n}_2 \quad \ldots \quad \mathbf{n}_i \quad \ldots \quad \mathbf{n}_{n_n}] \in \mathbb{R}^{3 \times n_n} \\
\mathbf{P} = [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \ldots \quad \mathbf{p}_i \quad \ldots \quad \mathbf{p}_{n_n}] \in \mathbb{R}^{3 \times n_n}
\]

By collecting all the \( \mathbf{b}_k, \mathbf{s}_k, \mathbf{r}_k \) vectors, we get the following bars matrices \( \mathbf{B} \), strings matrices \( \mathbf{S} \) and center of mass matrix \( \mathbf{R} \) necessary to define the position of the structural elements:

\[
\mathbf{B} = [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \ldots \quad \mathbf{b}_k \quad \ldots \quad \mathbf{b}_{n_b}] \in \mathbb{R}^{3 \times n_b} \\
\mathbf{S} = [\mathbf{s}_1 \quad \mathbf{s}_2 \quad \ldots \quad \mathbf{s}_k \quad \ldots \quad \mathbf{s}_{n_s}] \in \mathbb{R}^{3 \times n_s} \\
\mathbf{R} = [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \ldots \quad \mathbf{r}_k \quad \ldots \quad \mathbf{r}_{n_r}] \in \mathbb{R}^{3 \times n_r}
\]

where the vector \( \mathbf{r}_k = (\mathbf{n}_i + \mathbf{n}_j) / 2 \) defines the center of mass of the \( k \)th bar between nodes \( i \) and \( j \).

By introducing the connectivity matrices of bars, cables and center of mass, respectively \( \mathbf{C}_B \in \mathbb{R}^{n_b \times n_b}, \mathbf{C}_S \in \mathbb{R}^{n_s \times n_s} \) and \( \mathbf{C}_R \in \mathbb{R}^{n_r \times n_r} \) we can rewrite the above matrices as follows:
The general element $C_{Bij}$ (or $C_{Sij}$) is equal to: -1 if vector $b_i$ (or $s_i$) is directed away from node $j^{th}$; 1 if vector $b_i$ (or $s_i$) is directed toward node $j^{th}$, and 0 if vector $b_i$ (or $s_i$) does not touch node $j$. In this way we can describe the class number of a tensegrity network and say that a tensegrity system is of class $m$, if the maximum number of bars concurring in each node is equal to $m$ [6],[12]-[13]. We remember that the system we are considering is a tensegrity of class 6.

By defining the force densities parameter (member force divided by length) of the $k^{th}$ cable as:

$$\gamma_k = \max \left[ 0, k_k \left( 1 - \frac{L_k}{s_k} \right) \right] + \gamma_{ck}$$

(with $k_k$ the material stiffness coefficient of the $k^{th}$ cable, $L_k$ the rest length, $s_k$ the cable force vector ($s_k = n_j - n_i$) and $\gamma_{ck}$ the force density due to damping), we can write the equations of motion of a class 1 tensegrity system as follows [5]:

$$\ddot{N}M + NK = P$$

where:

$$M = C_B^T \hat{m} C_B \frac{1}{12} + C_R^T \hat{m} C_R \in \mathbb{R}^{n_x \times n_x}.$$  

$$K = C_S^T \gamma C_S - C_B^T \hat{\lambda} C_B \in \mathbb{R}^{n_x \times n_x}.$$  

and $\hat{\lambda}$ is the force density in the bars per each unit length. In the field of form finding tensegrity methods, the force density method is used to convert the nonlinear equilibrium systems into linear systems through the nonlinear transformation between force density (the control variable that appears linearly in the system) and rest length [14]. Numerical results about the dynamic analysis of class 6 tensegrity sunscreen module that we are considering will be discuss in the section 3.

### 3 NUMERICAL RESULTS

The present Section is devoted to the description of the tensegrity model adopted and to the computation of the dynamic analysis under the actuation motion. A mechanical modeling of tensegrity sunscreen system has been recently presented in [2], with reference to a quasi-static deformation process under wind-induced forces by neglecting dynamical effects.

The sunscreen model is a tensegrity class 6 system constituted by 12 bars and 3 strings (figure 1). The Nodes 2, 3 and 7 are connected through deformable cables (red-colored members), which are stretched in the fully closed configuration and rest with fully opened screen.

The physical model of the tensegrity sunscreen model makes use of Aluminum alloy hollow tubes for the bars and nylon-fiber ropes for the strings. The mechanical data are given in the table 1, while the different frames of the tensegrity system actuation motion is illustrated in figure 2.
Symbols and names | Units | Bars | Strings
---|---|---|---
E (Young’s modulus) | GPa | 72 | 3.9
\( \rho \) (mass density) | kg/m\(^3\) | 2700 | 1140
\( \sigma \) (yield stress) | MPa | 260 | 616
\( k_{str} \) (string stiffness) | N/m\(^2\) | - | 77734
mass \( \text{bar} \) (mass) | kg | 25 | -

Table 1: Mechanical data of analyzed tensegrity system.

4 CONCLUDING REMARKS

The use of shielding as a technical device is now a consolidated trend in the energy efficiency building field. In this paper we have promoted the use of tensegrity architecture for the design of active shading systems able to reduce carbon dioxide emissions and indoor energy consumption. The use of screens with moving panels, as represented by the evolution of the other shielding systems, has found a success that is gradually increasing, especially in the hottest places.

We have studied the use of tensegrity solution to optimizing the façade technology of the Al Bahar Towers through a dynamic analysis. The results obtained have highlighted the peculi-
arities of the tensegrity structures where the modular panels are controlled by stretching or relaxing a limited number of selected cables and offer minimal mass elements.

Future research lines are addressed on the possible applications of 3d/4d printing [15]-[17] for the design of automated movable structures thanks to photovoltaic sensors or photocells on one hand, and the use of horizontal deployable systems to promote natural ventilation in multi-story double-skin facades on the other. We also address the generalization of the constitutive laws employed in the present work to account for more complex hyperelastic behaviors [18]-[20], the bimodular/no tension response of bars [21]-[26], and nonlinear / nonlocal response [27]-[31] to future work. Additional future research will deal with the employment of shell-like structures [32]-[34] and three-dimensional lattices with coupled soft/stiff response [35]-[37] for the design of dynamic solar facades.

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THE SEISMIC PERFORMANCE OF STEEL BUILDINGS RETROFITTED WITH STEEL CABLES AGAINST PROGRESSIVE COLLAPSE

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Abstract

Code-conforming buildings designed against earthquake have an inherent robustness against horizontal loads. However, when they are subjected to member-loss scenarios, typically they develop particularly large deflections. To avoid such an undesirable behaviour, various methods have been employed to increase the buildings’ progressive collapse resistance. Such a method is the installation of steel cables in strategically selected bays of the building. As described in existing works, it is possible to properly define the cable properties so that they do not affect the seismic performance of the structures when assessed using the applicable design codes up to the design earthquake, but they are effective in element loss scenarios. This paper investigates the effect of different cable configurations in the seismic performance and collapse potential of steel buildings retrofitted against progressive collapse. It aims to identify the alteration in the structural behaviour and propose an approach in order to optimize the beneficial effect of the installed components.

Keywords: Progressive collapse assessment, steel cables, steel buildings, seismic performance.
1 INTRODUCTION

Buildings designed against earthquake are able to withstand seismic excitations up to the Design Base Earthquake, while their response is within the limits of the defined performance level. To achieve that, the individual member capacities need to be large enough to receive the loads induced by all considered actions. At the same time, the total energy dissipation capacity of the structural system should be adequate to sustain the earthquake excitation, without extensive damage that would cause undesirable performance, or even collapse of the system. However, when a damage occurs which causes the partial loss of strength or total failure of a load bearing element which transfers the loads vertically, typically a column, systems designed only against earthquake do not necessarily possess the required mechanisms to receive the extra loads from the failed elements safely. Hence, as the internal stresses of the neighbouring elements, typically the beams, increase significantly, they develop large deflections and fail to restrict the disproportionate extent of structural damage. To increase the progressive collapse resistance of such buildings, they need to be retrofitted appropriately.

Design guidelines such as [1] and [2] use the Alternate Load Path method as a means to design buildings against progressive collapse. The aim of this method is to design a structure which is able to redistribute the loads applied on it when local failure occurs, e.g. when one or multiple structural members of the building are lost. In such a case, loads from the damaged elements are received by neighbouring elements and transferred through alternative load paths safely to the ground. In order to apply the alternate load path method, an artificial local failure is modelled, which is simulated with the removal of certain structural members. The structure in its ‘damaged’ state is required to receive the applied loads and its response is assessed against defined performance criteria. The same procedure is used to assess the progressive collapse resistance of an existing structure.

According to previous studies ([3] [4]), that strengthening elements which transfer loads horizontally or in a diagonal direction, taking into account the structural performance against earthquake as well, can significantly enhance a building’s progressive collapse resistance, as they play a significant role in the alternate load paths employed. In a previous work by the authors [5] it was illustrated that it is possible to install steel cables in a structure without affecting its earthquake response. This is achieved by introducing an initial sagging to the cables. Sagging cables do not interact with other structural elements and so they do not alter the load paths used to transfer gravitational loads to the ground while the building is at rest. For the cables to be activated, large displacements between the two ends need to develop. If the length of the cables is large enough, these cables can be disabled during an earthquake and, so the building’s performance is not affected. An adequate cable length for that would be equal to \((1+IDR_{\text{max},i,\text{DBE}})L_0\), where \(L_0\) is the distance between the two ends of the cable at the rest state of the building and \(IDR_{\text{max},i,\text{DBE}}\) is the maximum interstorey drift developed in storey \(i\) where the cables are installed during the Design Base Earthquake. Alternatively, \(IDR_{\text{max},i,\text{DBE}}\) can be substituted by the maximum admissible interstorey drift at the desirable performance level. Even though, up to the DBE (or an earthquake that causes an \(IDR_{\text{max},i}\) equal to the selected level) retrofitted buildings perform as designed initially, if the ground motion is strong enough to exceed this limit, retrofitted buildings can become irregular due to the additional stiffness provided by the cables installed. The effect such an intervention might have to a building’s collapse potential needs to be investigated before the application of any retrofit method.
2 STRUCTURAL MODELLING AND ANALYSIS

In this work, a prototype building was designed against earthquake, using the optimization procedure described in [6]. The procedure was selected so that the designed building will have the desirable performance against earthquake, while at the same time the additional over-strength will be limited, as its structural members will have the minimum feasible sizes. The building is a six-storey residential steel building, comprising by five bays in each direction. The total height of the first storey is 4.2m, while in all other storeys it is 3.5m. The total beam span is 5m in both directions. In x-direction beam-column connections are moment restrained, so that Moment Resisting Frames (MRFs) are formed, while in y-direction, beams are simply supported. In both directions, steel bracings are installed in the middle bay of each side of the building. Standard HEB sections were used for the columns, IPE sections for the beams and L-shaped sections for the bracings. All columns were grouped in (a) corner columns, (b) peripheral in x-direction, (c) peripheral in y-direction and (d) internal columns. Every two storeys a different element group was defined for beams and columns. Bracings were grouped based on the plane they are installed on (x-z or y-z plane). Element grouping is shown in Fig. 1.

![Figure 1: Assessed building - 3D model and section grouping (top slab removed for visualization purposes).](image)

All individual structural members were designed according to EN1993-1 [7]. The building was designed against earthquake using the provisions of EN1998-1 [8], at the Collapse Prevention performance level. A maximum admissible interstorey drift of 4% was assumed, according to [9]. Following, the building was retrofitted against progressive collapse using steel cables. The cables were installed according to the ‘belts’ method described in [4]. The particular cable layout was selected as it concentrates all cables in particular storeys of the building. Hence, when the buildings exceed the selected drift level for the sagging cables, it will be highly irregular in height due to the additional stiffness. The provisions of UFC 4-023-03 [1], to assess the progressive collapse resistance of buildings also described in the GSA guidelines [2], were used. The building was subjected to four damage scenarios and its performance was assessed. The simulated damage scenarios include the loss of (a) a corner column at the base of the building, (b) a peripheral column in y-direction at the base of the building, (c) a corner column at the third storey and (d) the loss of multiple neighbouring elements (i.e. three columns and two beams at the corner in the first storey and a corner column in the second storey). The cable size was selected so that the maximum developed plastic rotation at any beam of the damaged building does not exceed 6° for High Level of Protection against progressive collapse, according to [1].
The assessed buildings were simulated using the OpenSEES software [10]. Three-dimensional models were used in order to take into account the contribution of the elements in the perpendicular direction on the building’s seismic response and progressive collapse resistance. Structural elements were simulated using force-based distributed plasticity ‘fiber’-elements. Bilinear elastoplastic stress–strain behaviour was assumed for beams, columns and bracings. Cables were modelled using the ‘Elastic – Perfectly Plastic’ material, the properties of which were defined so that it will account for the initial sagging of the cables, while it will have no compressive resistance. The modified Ibarra-Krawinkler model [11] was used to capture stiffness and strength deterioration at the end of the beam, near the flange reinforcing plates. Five types of analyses were employed:

a) A force-controlled linear static analysis under gravitational loads, to check against the individual member checks according to EN1993-1-1.

b) An eigenvalue analysis to define the fundamental period of the buildings.

c) A displacement-controlled Static PushOver analysis in each horizontal direction to evaluate the performance of the building under the design earthquake during the initial design procedure.

d) A force-controlled Static PushDown analysis for each simulated damage scenario, allowing the assessment of the progressive collapse resistance of the retrofitted building.

e) Multiple Nonlinear Time-History Analyses to perform the required Incremental Dynamic Analyses for each earthquake considered.

3 APPLICATION

To investigate the effect the applied retrofit approach has on the building’s structural response their collapse potential needs to be assessed. This is achieved through the definition of suitable fragility functions. To construct the fragility functions, 15 strong ground motions were strategically selected so that their characteristics vary particularly. A larger number of earthquakes could provide more accurate results regarding the buildings’ expected performance. However, as the purpose of this work is to perform an initial investigation of the retrofit approach’s effect on the building’s seismic response, the selected set of earthquakes is considered to be adequate. Both buildings were submitted to a series of appropriately scaled earthquakes at multiple intensity levels. Specifically, all earthquakes were scaled from 0.01g up to a maximum intensity of 10g, at a step of 0.01g. The maximum interstorey drift was recorded for each storey, as well as the total roof drift. Collapse is assumed to occur at a maximum interstorey drift of 8% according to [12]. Fig. 2 shows the maximum recorded interstorey drift at each intensity level versus the intensity level for the unretrofitted and the retrofitted building. It can be noticed that, up to a maximum interstorey drift of 4%, the performance of both buildings is the same: for each earthquake, at every intensity level, the same maximum interstorey drift is developed. However, in the retrofitted building, for larger intensity levels where larger maximum interstorey drifts develop (so the installed cables contribute to the structural performance) the curves seem to deviate from those corresponding to the unretrofitted building. In particular, the gradient of the curves seems to reduce, i.e. the same maximum interstorey drift is achieved in the retrofitted building at a lower intensity level than in the retrofitted building due to the same earthquake. Hence, an overall deterioration of the performance is observed for larger earthquake intensities. This is the result of the additional stiffness attributed to particular storeys. While two storeys become quite stiffer than before, developing much smaller drifts, other storeys remain the same and, eventually develop larger displacements. To visualise the overall effect in the building’s collapse potential, the respective fragility curves were defined. As in
both buildings there are cases where buildings do not exceed the selected interstorey drift for collapse, the method described in [13] for results yielded by truncated incremental dynamic analyses was used. Fig. 3 shows the fragility curves for the two buildings. [12]

![Graphs showing IDA curves for retrofitted and unretrofitted buildings](image)

Figure 2: IDA curves for (a) the unretrofitted building and (b) the retrofitted building.

![Graphs showing fragility curves for retrofitted and unretrofitted buildings](image)

Figure 3: Fragility curves for (a) the unretrofited building and (b) the retrofitted building.
Comparison between the two curves shows that, while the building’s performance is altered and, in general, the building tends to collapse in smaller intensity levels for the same earthquake, this effect is not significant enough to alter the fitted fragility function substantially. Hence, the detrimental effect of the irregularity in height being the result of the installation of cables as a retrofit means against progressive collapse, seems to be within acceptable limits. However, to make the decision whether to retrofit or not against progressive collapse using such a method, an engineer needs to weight the benefits gained from the enhanced progressive collapse resistance of the building, against the losses due to deteriorated seismic performance in high intensity levels.

4 CONCLUDING REMARKS

In this work, the issue of possible adverse results in the building’s seismic response when trying to enhance structural performance against an additional hazard, i.e. progressive collapse, was presented. Buildings not designed explicitly against progressive collapse do not possess the required robustness to avoid disproportionate damage due to a relatively small-scale initial cause. Hence, retrofit methods are employed to enhance their progressive collapse resistance. An effective method is the installation of steel cables in strategically defined bays of the building, which can be installed with an initial sagging, so that they do not interfere with its seismic performance. However, for high seismic intensities, these buildings become irregular in height, which results in deterioration of their overall performance. In the assessed building, the level of deterioration was found to be limited enough to allow an engineer to apply the particular method, provided that the long-term benefits are more than the respective losses. As this was an initial investigation of the issue, to provide more accurate results, a more detailed investigation is required, which will take into account (a) additional ground motions, (b) more cable layouts and (c) more levels of initial sagging.

REFERENCES


DAMAGE DETECTION OF MIXED CONCRETE/STEEL FRAME SUBJECTED TO EARTHQUAKE EXCITATION

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Abstract

This paper deals with the application of wavelet analysis on damage detection in mixed concrete/steel frame structures subjected to earthquake excitation. Such buildings are typically the result of a building initially constructed as a reinforced concrete building and, at a later time, more storeys were added as steel moment resisting frames. These structures consist of reinforced concrete frames at the lower storeys and steel frames at the upper storeys. They are characterized by the material inconsistency in height. The proposed method of wavelet analysis of structural response is an output-only damage detection method. Non-linear dynamic analysis has been performed and response data at each story are obtained which are used as simulation data. Damage in the frame is due to hysteretic behaviour of columns and beams. Since the dynamic behaviour of an inelastic structure subjected to an earthquake excitation is a non-stationary process, discrete and continuous wavelet analysis were performed in order to retrieve the simulation response data. The proposed method is based on the assumption that there is a correlation between structural damage, due to non-linear behaviour of structural elements and spikes that can be clearly detected in the wavelet details. This is supported by the fact that at the time when the spikes are recorded, structural damage occurs as well. The numerical results indicate that the discrete wavelet analysis is a promising method for the detection of damage in structures without the need for visual inspection.

Keywords: Mixed concrete/steel frame, Damage detection, Discrete wavelet analysis, Continuous wavelet analysis, Structural dynamics, Earthquake engineering.
1 INTRODUCTION

Damage observed during the service life of a structure may be caused by excessive earthquake excitations. The first step to detect the existence and the location of the damage in structure is the visual inspection. However, in some cases visual inspection may not be feasible; e.g. in hotels or hospitals where the interior of the building is covered by fixed furniture, equipment or plasterboards. Bridges in viaducts are also a case where it is difficult to access and observe the damage of critical structural elements which are located below the bridge.

Damage detection includes the determination of the existence, severity and location of the damage, as well as the prediction of the remaining service life. In order to apply a damage detection method an appropriate tool is structural health monitoring (SHM), which provides the suitable data for making the appropriate decisions. SHM, is an efficient strategy to monitor system performance and make corresponding maintenance decisions. A main group of methods for damage detection is modal analysis methods, which are based on the fact that the change in structural properties causes a variation in the modal parameters, natural frequencies, damping ratios and mode shapes. Many analytical and experimental studies have been conducted to establish analytical correlations between damage severity and modal parameters. Kirmsner, [1], investigated the relationship between natural frequencies and the introduction of a crack in an iron beam. A literature review on methods of damage detection using vibration signals for structural and mechanical systems was provided by Fan and Qiao, [2]. Another work based on changes in modal parameters is that of Humar et al. [3]. Ciambella et al. [4] investigated damage localization and assessment based on eigenfrequencies and eigenvectors curvatures.

Neural network approaches are more innovative methods and can also be used for damage detection. Wu et al. [5] trained a neural network to recognize the behaviour of an undamaged structure as well as the behaviour of a structure with various possible damage states. When the trained network is subjected to the measurements of the structural response, it is able to detect any existing damage. Masri et al. [6] trained a neural network with measurements from a healthy structure and this trained network was fed comparable vibration measurements from the same structure under different episodes of response in order to monitor the health of structure. Vanik and Beck [7] and Chandrashekhar and Ganguli [8] used fuzzy logic to determine the damage location. Friswell and Mottershead [9] used a combination of sensors and an analytical model of the structure for the damage detection. Yun et al. [10] used genetic algorithms for their damage detection approach. Papadimitriou and Ntotsios [11] updated the parameters of the model that is related to damage so that the dynamic characteristics of the model corresponded to the sensor measurements. Sakellariou and Fassois [12] introduced a stochastic output error for damage detection and assessment (location and quantification) in structures under earthquake excitation. Chatzi et al. [13] propose a methodology for the on-line identification of non-linear hysteretic systems where the parameters of the system are unknown and also the nature of the analytical model describing the system is not clearly established. Dertimanis and Chatzi [14] investigate a hybrid optimization algorithm to the state–space parameter estimation problem. The hybrid algorithm was designed in a way that takes advantage of its deterministic and stochastic counterparts, combining fast local convergence and increased reliability in the search of the global optimum.

Wavelet analysis is another tool for damage detection in structures [15], [16]. Taha et al. [17] presented a view of wavelet transformation and its technologies. They discussed specific needs of health monitoring addressed by wavelet transformation. Kim and Melhem [18] provide a review of the research that has been conducted on damage detection by wavelet analysis. Hou et al. [19] proposed a wavelet-based approach for structural damage detection. Their model consisted of multiple breakable springs that may suffer either irreversible damage when the
response exceeds a limit value or the cumulative number of cycles of motion exceeds the fatigue life. In any case, occurrence of damage and the time when it takes place can be clearly determined in the details of the wavelet decomposition of these data. Alonso et al. [20] used orthogonal wavelet decomposition for identifying the stiffness loss in a single degree of freedom spring-mass-damper system. Their work shows that pseudo-alias effects caused by the orthogonal wavelet decomposition (OWD) affect damage detectability. Rucka and Wilde [21] use neuro-wavelet technique to detect damage in beam, plate and shell structures; their results were also validated with experiments. Hera and Hou [22] applied wavelet analysis for the detection and location the damage. They found that structural damage due to sudden breakage of structural brace elements can be detected by spikes in the wavelet details. In the work of Khatam et al. [23], wavelet analysis is used for damage identification in beams subjected to harmonic loading. The damaged region can be determined by the spatial distribution pattern of the observed spikes. Soyoz and Feng [24] worked theoretically and experimentally on damage detection of bridge structures. Noh et al. ([25], [26]) introduced three wavelet-based damage-sensitive features (DSFs) which are defined as functions of wavelet energies at particular frequencies and specific time instances. These DSFs can be used to diagnose structural damage.

Adaptive-scale damage detection strategy for plate structures based on wavelet finite element model was developed by He and Zhu [27]. Law et al. [28] worked on statistical damage classification method based on wavelet packet analysis. Liu et al. [29] developed a structural time-varying damage detection method using synchro-squeezing wavelet transform. Wang et al. [30] used discrete wavelet transform for time-varying physical parameter identification of shear type structures. Finally, Fan et al. [31] proposed a novel transmissibility concept based on wavelet transform for structural damage detection.

A computational procedure for damage identification is developed based on discrete and continuous wavelet analysis. Nagarajaiah and Basu [32] presented a wavelet-based distortion energy approach in the work of Bukkapatnam et al. [33] as a method for quantifying and locating the damage to structural systems. Goggins et al. [34] used a wavelet-based equivalent linearization technique to determine the temporal variations in frame stiffness that occurs due to brace yielding and buckling. Lima et al. [35], use wavelet analysis for damage detection of non-linear structures. Damage detection of frame structures subjected to earthquake excitation using discrete and continuous wavelet analysis is presented in the works of Pnevmatikos et al. ([36], [37]).

Applications of damage detection for buildings with material irregularity in height like concrete-steel frame structures (mixed concrete-steel frame structures) is very limited. These structures consist of reinforced concrete frames at the lower storeys and steel frames at the upper storeys. Such buildings are typically the result of a building initially constructed as a reinforced concrete building and at a later time more storeys were added as steel moment resisting frames. Due to the different time of construction, those buildings are often designed with different design codes and approaches. However, if the lower part of the building was constructed after the 90’s, then both parts are designed according to the Eurocodes, so it can be considered as a mixed concrete/steel building since its original design.

In this paper damage detection in mixed concrete/steel frame structures subjected to earthquake excitation is presented. Damage into the structure is introduced by the non-linear behaviour of the structural members. The data to be analysing were obtained by non-linear analysis of the mixed concrete/steel frame structure. Since the dynamic behaviour of inelastic structures during an earthquake is a non-stationary process, wavelet analysis is the most appropriate tool. A computational procedure for damage identification is developed based on discrete and
continues wavelet analysis. The proposed procedure is based only to the output response of the structure in each story.

2 BACKGROUND OF CONTINUOUS AND DISCRETE WAVELET ANALYSIS

Wavelet analysis provides a powerful tool to characterize local features of a signal. Unlike the Fourier transform, where the function used as the basis of decomposition is always a sinusoidal wave, other basis functions can be selected for the wavelet shape according to the features of the signal. The basis function in wavelet analysis is defined by two parameters: scale and translation. These properties lead to a multi-resolution representation for non-stationary signals.

The continuous wavelet transform of a signal, \( f(t) \), is defined as:

\[
f(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \overline{\Psi\left(\frac{t-b}{a}\right)} dt
\]

where \( a, b \) are the scale and translation parameters respectively and \( \overline{\Psi} \) denotes the complex conjugate of \( \Psi \). The functions, \( \Psi(t,a,b) \), are called wavelets. They are dilated and translated versions of the mother wavelet, \( \Psi(t) \).

By discretizing the parameters, \( a \) and \( b \), a discrete version of the wavelet transform (DWT) is obtained. The procedure becomes more efficient if dyadic values of \( a \) and \( b \), are used, i.e.

\[
a = 2^j \quad b = 2^k \quad j, k \in Z
\]

where \( Z \) is a set of positive integers. The corresponding discretized wavelets \( \psi_{j,k} \) are defined as:

\[
\psi_{j,k}(t) = 2^{-j/2}\psi\left(2^{-j}t - k\right)
\]

where \( \psi_{j,k} \) forms an orthonormal base. In the discrete wavelet analysis, the signal can be represented by its approximations and details. The signal is passed through a series of high pass filters, which relate to details, to analyse the high frequencies, as well as through a series of low-pass filters, which relate to approximations, in order to analyse the low frequencies. The detail at level, \( j \), is defined as:

\[
D_j = \sum_{k \in Z} a_{j,k}\psi_{j,k}(t)
\]

where \( a_{j,k} \) is defined as:

\[
a_{j,k} = \int_{-\infty}^{\infty} f(t) \overline{\psi_{j,k}(t)} dt
\]

and the approximation at level \( J \) is defined as:

\[
A_J = \sum_{j > J} D_j
\]

Finally, the signal, \( f(t) \), can be represented by:

\[
f(t) = A_J + \sum_{j \leq J} D_j
\]

The discrete wavelet transform (DWT) can be very useful for on-line health monitoring of structures, since it can efficiently detect the time of a frequency change caused by stiffness degradation.

3 DAMAGE DETECTION METHODOLOGY OF MIXED CONCRETE/STEEL FRAME STRUCTURE SUBJECTED TO EARTHQUAKE EXCITATION

Damage detection methodology is an extension of the authors’ work ([36], [37]) and is based on the fact that when plastic hinges have developed during excitation, the frequency of the
system changes and this causes a shift of frequency of the response signal. On the other hand, when a signal changes its frequency and analysed by discrete wavelet transform this is imprinted as spikes in details of its signal. In the present work, two more indicators were introduced to distinguish automatically the occurrence of spikes in the detailed signal. Furthermore, the use of continues wavelet transform of the response signal is introduced.

The strategy which is used to detect the damage in the framed structure is as follows:

1. Output-only response signal for each floor is analysed by discrete wavelet analysis and the details of the signal are obtained. Two indicators that show the existence of spikes are calculated.
2. If spikes are observed in the details of the response signal, this indicates that a structural element which is related to the corresponding floor (beams in the floor and columns from above or below the floor), goes beyond the yielding point and damage has occurred. The time instant that the spikes appear in the details, represent the time when damage has occurred to the structural element.
3. If no spikes are observed in the detail of the response signal for the floor, the corresponding elements for this floor would remain elastic and no damage would appear.

The above steps are shown schematically in Figure 1.

A question which arises: “How could one define the existence or not of spikes in the detail of output signal in order to come to a conclusion on whether a damage occurred or not?” To answer this question, two numerical indices were defined. The first is the \( R1 \) value index. This index is the percentage of how many times the picks of the wavelet detail of output signal in each floor exceeds the proportion of maximum absolute value \( p \cdot \max D \). The value of \( p \), is taken as 50\%. If the calculated value of \( R1 \) is small enough (\( R1 \leq 5\% \)), then this is an indicator that spikes exist in the wavelet detail of signal. Otherwise, if \( R1 > 5\% \), then no spikes caused by structural damage appear in the wavelet details and, consequently, no damage has occurred. The second index is the value of \( R2 \) index. This index is the ratio of the maximum absolute
value of the wavelet detail of output signal to the mean value of absolute signal. When the value of $R2$ is high enough ($R2 > 30$) this means that in the detail signal a spike exists so damage occurred in a local region of measurement. When the value of $R2$ is lower than 20 this means that there is not a clear spike in the detail signal, so no damage occurred. In the grey zone between values from 30 to 10 once should combine and look at the $R1$ value in order to conclude of damage in structure. The graphical representation of $R1$ and $R2$ numerical indicators are shown in Figures 2 and 3.

![Graphical representation of $R1$ numerical index and limits of $R1$ value for damaged structure (a) and healthy structure (b).](image-url)
4 NUMERICAL CASE STUDY

The above damage detection strategy has been applied to a five story three bay concrete/steel mixed framed structure shown in Fig. 1. The frame has three bays with 5m span each, while the typical story high is 3m. The dead and live loads are 40kN/m and 15kN/m, respectively, and they are applied on the beams as distributed loads. The materials used for the lower part are C20/25 concrete with B500C reinforcement steel. The upper steel part consists of S275 structural steel. Details about materials properties, sections and modelling of frame are in the work of Pnevmatikos et al. [38].

The frames were subjected to Santa Barbara (1978) earthquake excitation. The non-linear dynamic analysis and the response of the frames has been completed using the software programme SAP2000nl. The wavelet analysis of the response signal has been done using MATLAB software, and specifically using the wavelet toolbox. Fifth order Daubechies wavelets and eight levels of the details have been used for the wavelet analysis of the response signal. Other types of wavelets which are sensitive to damage detection can also be used.
The earthquake excitation was scaled up in order plastic hinges appears in structural members. In figure 4 the response of frame with scale factor equal to 1 and 2.8 is shown. It is shown that in the column (a), is the healthy frame with no damage while in column (b) is the frame with slight damage where the sections reach the yield point. The damage detection procedure is applied when the structure has low level of damage otherwise the damage can be visually inspected. The $R_1$ and $R_2$ values for each case were calculated and shown in figure 4. It is clear that the $R_1$ and $R_2$ values for healthy and damaged structure follow the limits that presented in figure 2 and 3.

The $R_1$ and $R_2$ values were obtained from the response at point B (5th floor) from both frames. It was found that they satisfy the criteria of healthy frame for both cases which are presented in figure 2 and 3. It is worth to note that the $R_1$ and $R_2$ values at point B (5th floor) from damaged structure satisfy the criteria of healthy structure because at 5 floor no damage was observed. In lower floors, for example third floor, of damaged frame the $R_1$ and $R_2$ values satisfy the limits...
of damaged structure presented in Figures 2 and 3. This is because at this area of structure is
damage since plastic hinges occurred.

Continues wavelet analysis was also performed. In Figure 5 the chance of frequencies during
the time of excitation in 3D space and in contour plot as well as the cone of influence is shown
for damage and healthy structure at second floor.

![Figure 4: Continue wavelet transform from response measured at 1st floor for healthy (b) and damaged (a) frame.](image)

5 CONCLUSIONS

A procedure of discrete and continues wavelet analysis for damage detection of a mixed
concrete/steel framed structure subjected to earthquake excitation has been performed. Discrete
wavelet analysis is a good tool to analyze the non-stationary dynamic behaviour of inelastic
structures and capture the damage that occurs in buildings. It was shown that structural damage
of a story level, can be detected by spikes in the wavelet details. Tow numerical indicator \( R1 \)
and \( R2 \), and their limits in order the structure to be healthy or damaged were proposed.

This procedure helps engineers in two ways. Firstly, after an earthquake event, engineers
concentrate their inspection on a specific building, from all the instrumented buildings, which
they have the \( R \) values between the damage limits. Secondly, they need to uncover specific
floors from the chosen building in order to perform a detailed visual inspection of the structural
elements located at this floor. This approach can also be used as an alarm procedure helping
engineers proceed with the visual inspection at the appropriate region of the building. The nu-
merical results of mixed concrete/steel framed shows the effectiveness of the wavelet approach
for damage detection in structures subjected to earthquake excitation.
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SENSOR PLACEMENT SELECTION FOR SHM OF BUILDINGS

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Abstract

The objective of this work is to propose a method to improve the sensor layouts for building structural health monitoring. The selected layouts should give the maximum information on the dynamic behavior of a building due to external forces while taking into account any constraints on accessibility, cost, and sensor number or type available. The proposed method is implemented in two steps, first, the dynamic behavior of the building is simulated by a structural analysis software well-known tools such as SAP2000 or ETABS, and then, the results of its modal analysis are fed into a selection/optimization tool, developed in Excel-VBA and MatLab, that finds the most appropriate positions for the sensors in order to register the maximum mass-displacement/accelerations in the building. The results of the method are promising and in a following step will be compared to installed sensor layouts on existing buildings, in order to investigate the contribution of the proposed method to cost & complexity reduction of the SHM sensor installation.

Keywords: Structure Monitoring, Sensor Placement, Optimization, eigenmodes, modal analysis.
1 INTRODUCTION

The deterioration of civil structures has attracted the public attention in recent years. This is because a number of catastrophic events that have been widely covered by the media. Bridges collapses, building losses their stability etc. Mississippi I-35W Bridge 2007, Moradi Bridge in Genoa 2018, and recently bridges in Kavala 2018 are some examples and shows that the effectiveness of strategic inspection of structure has increased.

In Europe, scientists warn about the state of bridges in the networks of most European countries, while in USA an ASCE report (2017) estimates that about 188,000,000 vehicles cross structurally inadequate bridges every day in the US. In Greece due to the lack of funding for timely retrofitting, average condition of the infrastructure is expected to continue deteriorating in the foreseeable future. In Greece the application of SHM to civil structures is very limited and the gap in the decision-making process for the maintenance of the structures based on monitoring systems is very big.

Except of natural hazards, extreme loadings or aging of the environmental impact which can cause damage to the structure the man-made disasters can cause damage on civil infrastructures, as highlighted by the numbers of terrorist attack which can have catastrophic impact on society. Therefore, real-time monitoring and automatic condition assessment of the civil structures are critical for crisis response, structure safety and infrastructure maintenance.

One way to assess the condition of structures is by visually inspection. Inspection is usually conducted rarely, which may not be timely enough for capturing rapidly growing damage. Furthermore, visual inspections are highly subjective, significantly different condition ratings can be given for the same structure by different inspectors. Visual inspections can only identify damage that is visible on the structural surface, damage located below the surface often remains unrevealed. As a result, there is a need for reliable structural health monitoring systems, SHM, that can automatically and quantitatively assess the real-time condition of civil structures. Damage in buildings may be caused by excessive earthquake excitation, severe environmental conditions, degradation of the material’s properties, fatigue, cumulative crack growth, etc observing during their service life. In civil structures often the existence and the location of the damage can be determined through visual inspection. As mention above visual inspection may not be feasible. To ensure structural safety and low maintenance cost, structural health monitoring, (SHM), is an efficient strategy to monitor system performance and make corresponding maintenance decisions.

A SHM system measures structural performance and operating conditions with various types of sensing devices, and evaluates structural conditions using certain damage diagnosis or prognosis algorithms. In order to make a good decision about the condition of structure the placement of sensors plays an important role to give the appropriate data.

The international concern about the state of infrastructures encourages the adoption of advanced fault diagnosis and fault planning systems to guide the planning of their maintenance actions. SHM replaces routine inspections and on-site data collections with remote monitoring systems, providing real-time information on safety, efficiency, reliability and remaining life of structures.

Structural health monitoring (SHM) provides an innovative tool to ensure the safety of infrastructures. Sensor devices and topology of devices plays an important role in SHM, because mechanical analyses, such as model updating, damage identification and condition evaluation, depend on acquired data from sensors. From the other hand, the number of sensors installed in a structure is constrained by the cost associated with data acquisition systems, on line processing data as well as the initial installation of the sensors. The goal is to gather most information about the state of structures using as few sensors as possible. Therefore, a procedure to
obtained a limited number of sensors is of crucial importance in the design of an effective SHM on a structure.

An appropriate configuration of sensors should satisfy that the measured responses could be used by the damage identification method. Also the recorded time-history data are informative about the condition of the structure, and they are still usable in noise condition. Finally, the cost of instrumentation, communication and maintenance should be minimized.

Structures deteriorate due to aging of the environmental impact, thus the degradation of stiffness occurs and structural dynamic characteristics such as the natural frequencies and the mode shapes change. By instrumenting structures with a sensor topology, their vibration can be continuously measured and the dynamic characteristics extracted from these signals. At the same time, interpreting the sensor data by intelligent software in terms of structural health, damage can be detected at its early stage and the reliability of structures assessed in real-time.

Instrumentation sensor devices for long-term structural health monitoring, includes accelerometers, displacement or velocity sensors, strain gauges, pressure sensors and sensors to detect environmental conditions. Data recorders are installed on the sites with the capacities of auto-triggering and data storage. Power supply systems consisting of AC power, UPS, solar panels and rechargeable batteries are specially designed to provide power backup in case of power disruption during earthquakes or another crisis.

2 MODAL ANALYSIS DISPLACEMENT, MODES

One of the most used analysis in seismic analysis of structures is the response spectrum analysis. This analysis is based on the eigen-modes of the structure. The response is calculated for each mode and the total response is obtained by superposition of the responses of each mode. The eigen modes of a structure are calculated by a modal analysis. Knowing the mass and initial stiffness of the structure in Modal analysis is the eigenmodes $\Phi_i$, eigenperiods $T_i$ or eigenfrequencies $f_i$ and the corresponding damping ratios $\xi_i$ of the structure are obtained from the solution of the following eigenvalue problem, Chopra [1]:

$$[K - \omega^2 M]_{n \times n} \Phi = 0$$

$$\Rightarrow \Phi = 0$$

$$\Rightarrow \{\Phi_1, \Phi_2, \ldots, \Phi_n\} \Rightarrow T_i = \frac{2\pi}{\omega_i}, f_i = \frac{\omega_i}{2\pi}, i = 0, \ldots, n - 1$$

$$C_n = \Phi_n^T C \Phi_n$$

$$M_n = \Phi_n^T M \Phi_n$$

$$\xi_i = 2C_n M_n \omega_n$$

The participation factor $\Gamma_i$, and corresponding eigen-mass ratio for the $i^{th}$ eigenmode are given as:

$$\Gamma_i = \frac{\Phi_i^T M E_i}{\Phi_i^T M \Phi_i}, i = 1, \ldots, n$$

$$M_i = \Gamma_i \{\Phi_i\}^T [M] \{\nu\}$$

where $E$ is the direction matrix for the earthquake and $S_{d,i}(T_i, \xi_i)$ is the spectral acceleration of a structure which has eigenperiod $T_i$.

The seismic forces $F_{q,i}$ for the $i^{th}$ eigenmode are given as:

$$F_{q,i} = M \Phi_i \Gamma_i S_{d,i}(T_i, \xi_i), i = 1, \ldots, n$$

The maximum seismic forces $F_q$ for each degree of freedom are obtained combining with Square Root of Sum Squares method (SRSS) the seismic forces from each eigenmode, thus:

$$F_q = \sqrt{\sum_i^n F_{q,i}^2}$$
The calculation of the eigenmodes-displacements of each mode are given with the follow formula:

\[ \{u_i\} = \Gamma_i \cdot q \cdot \frac{S_d(T)}{\omega_i^2} \{\Phi_i\} \]  

(6)

Based on eigenmodes-displacements the internal forces of each mode are calculated. The total displacements or total internal forces are calculated with the method of superposition, (Sun Root of Sum Square SRSS, or Complete Quadratic Combination, CQC).

From the above formulation it is shown the dynamic characteristics of the structure (eigenmodes, eigen periods) play significant role to calculate the response of structure. Instant to calculate them numerically, by modal analysis, dynamic characteristics can be obtaining from existing structure in more reliable way using a monitoring system, consisting of sensors, installed to the structure. However, in order to capture all mode of vibrations one needs to place the sensors in appropriate places inside the structure.

In this paper the way how to choose the appropriate locations of sensors to be installed to the structure is examined. The choice of the location is based on an optimization procedure. The locations of the sensors were strategically selected by optimization analysis. The procedure takes into account that the location of sensors should be placed in the points where has maximum modal displacement. Furthermore, the weighting factor of this petitions is related to the mass participation ratio of the corresponding mode. The number of modes shape that are take into account in order give the optimum location of sensors are those which their sum of modal mass contributes up to 90% of the mass of structure. This is imprinted to Eurocode 8, [3], for chosen the numbers of modes in order to calculate the total displacement. The general idea for chosen locations of sensors based on eigenmodes is shown in figure...

Figure 1: Appropriate and non-appropriate locations of sensors based on eigenmodes of structure.

3 SENSOR PLACEMENT

In this work we focus on the displacement monitoring of selected nodes of a building. Our aim is to monitor & register the displacement of the nodes, using as few as possible sensors without compromising accuracy. Unlike a naive brute-force method, that would place a sensor on any node of the building and monitor everything, we want to intelligently select specific nodes that will provide us with approximately the same information but in a significantly lower cost and simpler implementation.

Not all nodes’ displacements are equally useful, from the information point of view. Each node has a different behavior, a different mass, and, a different effect on the building’s structural health. In addition, a node’s displacement information may be redundant, as for instance, for two nodes connected by a beam, where it is sufficient to monitor the displacement of only
one of them, along the beam direction. Our aim is to provide the engineer with a tool that will help the decision process for sensor placement, according to his aims and restrictions.

To demonstrate our method, we assume that the monitoring system has a limited number of acceleration/displacement sensors, and that the engineer wants to minimize their number provided that, he will adequately collect all required information regarding the building’s oscillation modes under investigation.

The problem is therefore an optimization problem where we try to select the best positions to place our sensors in order to monitor the required amount of information with the minimum number of sensors. The search for the solution can be done in two ways, either exhaustively by trying all possible solutions, or, by searching heuristically the solution space and converge faster to a very good if not optimal solution. The first can be done in smaller buildings with fewer nodes, while the second is more suitable for large constructions with thousands of nodes.

3.1 Placement selection & optimization

Our problem is an optimization problem where we try to best monitor our building under constrained resources. Constrains comprise the available number or type of sensors, as well as any budget or accessibility restrictions. Our goal is to record the elements displacement as accurately as possible, while minimizing resources without violating any constrains.

Typically in a building there are many candidate positions especially in large buildings (corners, edges, plate centers, edge centers, joints, etc., for all floors). This combined with the different sensor types (e.g., 1, 2, or 3D accelerometers), results in too many combinations of potential positioning setups. For installing \( K \) sensors in a building with \( N \) possible positions, the available combinations to select the best, will be: \( C = N!/(K! (N-K)!) \). Budget and other restrictions drive us to find the smaller, simpler and least expensive layout that satisfies our monitoring requirements.

For the problem under consideration, two tools were developed. One is suitable for simpler and smaller buildings, it is implemented using a programmed spreadsheet, and it considers all possible positions in order to propose the best combination. The other tool is based on Heuristics and GAs to search for the best positions and is suitable for large and complex buildings with thousands of placement combinations.

3.1.1 Simple Tool for Smaller buildings

When a small & simple building is under investigation, searching for the best solution can be done with exhaustive calculation, of all possible positions and their combinations. Due to the small number of nodes, it can be done by a common tool like a spreadsheet, programmed to calculate, sort and find the top positions that meet our criteria and constrains (Fig. 2).

For our implementation we use an Excel-VBA spreadsheet. The spreadsheet reads the data extracted from the simulation program, namely SAP2000, and then, calculates the contribution or importance of each node from its displacement on his degrees of freedom. This is done from the point of view of mass-displacement importance, as explained earlier.

To do that an artificially created index is created, the Displacement Index (DI). DI is calculated from the product (Mass x Displacement), squared summed for all its motion directions and then square rooted. The total index is calculated for each node from the sum of all significant modes (typically covering the 90% of the participating mass). Finally, the nodes are placed in a descending order of importance according to DI. The top nodes are then selected, based on the maximum number of sensors available (Eq. 7).
\[ DI_N = \sum_{m=1}^{M} DI_{Nm}, \ \text{where,} \ \ DI_{Nm} = \sqrt{\sum_{i=1}^{3} (\text{Mass}_N \times \text{Displ}_{Ni})^2} \]  

where, \( N \) is the node, \( M \) the significant modes and \( i \) is the displacement coordinate (x,y,z).

3.1.2 Heuristic search for Larger and complex buildings

When facing a large & complex building with hundreds or thousands of nodes it is difficult to solve it by just testing all possible combinations in short time. In that case heuristic methods [6,7,8,9] can search faster to find the best combination of a large number of candidate positions. Genetic Algorithms are able to search for an near-optimum solution in a fraction of time required by an exhaustive search. A GA tool is therefore implemented to investigate its performance when a large building is under investigation.

GA needs an evaluation function to assign an index to each possible position. For GA we will use the same index, DI presented above that is based on the results of a simulation tool such as SAP2000 or ETABS. The GA fitness function combines the evaluation results with any constrains or restrictions posed and any weighting provided, to calculate a final score for each scenario/position.
Reading SAP files ... MODES 1 & 2
USE CASE 1 - Optimization for a Small Building
Building & Checking 60 initial scenarios...
GA Initialization
GA RUN Started
with 60 population members for 35 generations...
fval =
0.39945
0 1 0 1 0 0 0 1 0 1 0 0 0 1 0 1 0 0 0 1 0 1
0 3 0 6 0 0 0 12 0 15 0 0 0 21 0 24 0 0 0 30 0 33

Figure 3: GA results for a typical two floor building, and the final convergence after just 15 generations for the positions of 8 sensors for monitoring mode 1 & 2

Work Flow of the proposed Tool:
1. Model the building in the Simulation tool.
2. Define the various Seismic Loads to be used
3. Define the maximum number of sensors
4. Define other constrains & restrictions
5. Run the Simulations to produce Displacement & Mass Participation results
6. Submit all user & simulated data to the GA to adjust the fitness function
7. Define GA’s running parameters
8. Generate the initial random population for GA
9. Calculate the overall score of the population based on the fitness function
10. Generate the new population (offsprings)
11. Repeat Step 9 & 10 for a number of Generations until convergence is achieved
12. Report the final population of the suggested sensor positions.

In Figure 3 another small building example is shown to demonstrate the method functionality. It is a typical two floor building, and the final convergence of the GA is achieved after just 15 generations for the positioning of 8 XY sensors and for monitoring the displacements caused by modes 1 & 2.

4 FURTHER TESTING ON MORE COMPLEX BUILDINGS

For the verification and evaluation of the method, four 15th story reinforced concrete buildings were selected and modeled in structural analysis software SAP2000 [4]. One building is symmetrical with central core, the other two is irregular in plane and in elevation and the fourth one is irregular both in plane and in elevation. The building with elevation irregularity is shown in figure 4.

![Figure 4: 3D and elevation view for the 15th reinforced concrete building.](image)

For every building, modal analysis was performed and the dynamic properties such as eigen periods, eigen-modes the mass participation factor were calculated. For the 15th reinforced concrete building with elevation irregularity the first three modes are shown in figure 5. It is observed that the first two modes are translational modes along in x and y direction while the third mode is the rotational mode along the vertical z axis.

The corresponding eigen-periods, the participating factor and the mass participation factor for each mode are collected. It is seen that the 19 first eigen modes needs to considered in opti-
mization algorithm in order to find the optimum locations for the sensors. Similar results are obtained when we examined the other three buildings. The large number of significant modes, and the very large number of nodes and positions drive us to develop further the GA implementation of the tool in order to use it in bigger and more complex buildings.

![First mode](image1.png)  ![Second mode](image2.png)  ![Third mode](image3.png)

Figure 5: The first three modes of vibration of the 15th reinforced concrete building.

5 CONCLUSIONS

A method for detecting the best locations for sensors in a building was proposed. The method is based on the eigenmodes of the monitored building, the mass ratio and the participation factors of each mode. To find the appropriate locations for sensors is very crucial in order to perform health monitoring. The method is based on typical results of structural analysis software and runs on a spreadsheet or a MatLab environment. It is demonstrated using simple building. The results of the method are promising and in a following step will be used in complex buildings and also will be compared to installed sensor layouts on existing buildings, in order to investigate the contribution of the proposed method to cost & complexity reduction of the SHM sensor installation.
REFERENCES


ELASTIC AND INELASTIC NEAR FAULT INPUT ENERGY SPECTRA

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Abstract

An attenuation model for input energy spectra under near fault ground motions is developed in this study. The ultimate objective is developing an energy based seismic performance assessment procedure under near fault ground motions. The presented study presents the first phase of this endeavor. The main purpose of this study is to develop a reliable model for predicting the input energy spectra of near-fault ground motions for linear elastic and inelastic systems, and to evaluate the effect of damping and lateral strength on energy dissipation demands. An attenuation model has been developed through one-stage nonlinear regression analysis. Comparative results revealed that near-fault ground motions have significantly larger energy dissipation demands, which are very sensitive to earthquake magnitude and soil type. The effect of damping on elastic and inelastic near fault input energy spectra is insignificant. Near fault input energy spectra for inelastic systems is dependent on lateral strength ratio R for short period systems, however, there is almost no dependency on lateral strength for intermediate and long period systems, recalling an equal energy rule. This is a significant advantage for an energy-based design approach.

Keywords: Input Energy, Near Fault Ground Motions, Attenuation Model, Regression Analysis, Equal Energy Rule.
1 INTRODUCTION

Damage occurs in structural systems during an earthquake if the input energy imparted by the ground motion cannot be dissipated simultaneously by the inherent linear viscous damping mechanism of the system. The excess energy is in turn dissipated by inelastic energy absorption and dissipation mechanisms for satisfying energy equilibrium. This is the main cause of seismic damage in structural systems. For a correct assessment of plastic energy dissipation and the associated damage in a structural system, three main ingredients are necessary. These are the input energy imposed on an inelastic system during seismic excitation, distribution of plastic energy dissipation in the structural system, and the consequent damage that occurs at these plastic regions in dissipating the plastic energy. The basic objective of the presented study is devoted to accomplish the first task. A prediction model for input energy imposed on the elastic and inelastic systems is developed under the effect of near fault ground motions.

2 ELASTIC AND INELASTIC NEAR FAULT INPUT ENERGY SPECTRA

Input energy is slightly different for linear elastic and inelastic systems. Moreover, strong ground motions from near-fault (NF) earthquakes impose higher energy dissipation demands compared to the ordinary far-fault (FF) ground motions from all distances.

Exact description of elastic and inelastic input energy spectra for recorded earthquake ground motions is obtained theoretically by integrating the equation of motion over time for a class of SDOF systems [1-3]. Design input energy spectra for elastic or inelastic systems, on the other hand, can be estimated by two procedures: Either by employing prediction equations based on the site and source characteristics of ground motions recorded in the past as well as the SDOF response parameters obtained under these ground motions [4-7], or by developing practical scaling rules relating the elastic or inelastic system and energy response parameters with the intensity parameters of recorded ground motions [8-12]. In fact, the most practical approach for obtaining input energy spectra for inelastic systems with different damping values (ζ) and lateral strength ratios (R), is applying scaling factors to a reference elastic input energy spectra derived for 5 percent damping. These approaches are schematized in Figure 1, where \( E_{ie} \) is the input energy for a linear elastic system (elastic input energy), \( E_{iy} \) is the input energy for a yielding system (inelastic input energy), and \( T, \xi, R \) are the period, damping ratio and lateral strength ratio, respectively.

![Figure 1. Different schemes for converting input energy spectra for a linear elastic system to the energy spectra for a yielding system.](image-url)

Operation paths (1) - (2), or (3) - (4) can be followed for converting the reference 5% damped elastic input energy spectrum into the inelastic input energy spectra for different \( \xi \) and \( R \). \( F_R \) and \( F'_R \) are the elastic to inelastic scaling functions for constant damping, and \( F_\xi \) and \( F'_\xi \) are the damping scaling functions for elastic and inelastic systems respectively in Figure 1. The scaling operations summarized in Figure 1 are quite well established for the accel-
eration response spectra or design spectra. However, the effect of $\zeta$ and $R$ on input energy spectra is not as distinct as in the acceleration response spectra [12]. We will investigate whether such scaling relations can be defined for near fault elastic and inelastic input energy spectra in this study.

Ground motions recorded at close distances to the fault may possess special features that significantly affect seismic energy demand on structural systems when compared to ground motions with broad distance characteristics. Housner [13] pointed out earlier that at near source locations, the relation between ground motion intensity and earthquake magnitude is not apparent as has been sometimes supposed, especially for moderate to larger magnitudes. Ground motions close to a fault are significantly affected by the faulting mechanism, direction of rupture propagation relative to the site (forward directivity effect), and the permanent ground displacement at the site (fling step effect). Depending on these effects, ground motions in the near-fault region may exhibit impulsive characteristics [14]. Thus, seismic response of structures in the near fault of rupture has to be evaluated differently from those in the far fault due to possible impulsive characteristics. Nonlinear response of degrading systems have been further investigated under near-fault ground motions with emphasis on inelastic displacement demands for seismic performance evaluation [15, 16].

While there are several studies on the analysis of near-fault ground motion records and their effects on acceleration and displacement spectra, there is not much research on the near-fault input energy spectra. Prediction of input energy spectra has been extended to linear elastic and inelastic systems subjected to near-fault ground motions in this study. A comprehensive evaluation of the effects of damping ratio $\zeta$ and lateral strength ratio $R$ on inelastic input energy spectra is presented.

3 NEAR-FAULT GROUND MOTIONS

A batch of 157 near-fault ground motion (GM) accelerograms with two horizontal components, each representing free field motion, is selected in order to study the near-fault effects on seismic input energy. This batch is a subset of the ground motion database employed in [17]. The database for this study includes GM records which were recorded at closest fault distances ($R_{epi}$) not longer than 30 km. The moment magnitudes ($M_w$) of earthquakes producing these ground motions ranges from 5.69 to 7.62. Figure 2 shows the $M_w$ versus $R_{epi}$ scatter diagram for the ground motions in the database. Additionally, Table 1 presents the distribution of records in the compiled GM batch with respect to shear wave velocity of the upper 30 meters of soil profile (VS30) according to NEHRP site classification. Ground motion sites in the database with VS30 values larger than the limiting value of 360 m/s (NEHRP A, B and C) are designated as stiff soil type, whereas those with lower VS30 values (NEHRP D and E) are specified as soft soil type. In the foregoing analysis, the input energy spectrum of each ground motion in the database is calculated separately for both horizontal component by integrating the equation of motion over time, defined in relative energy terms for the associated SDOF system. Then the input energy spectrum of ground motion is obtained as the geometric mean of the two horizontal ground motion components where spectral ordinates are obtained either in terms of input energy $E_i$ (J) for a unit mass, or in terms of energy equivalent velocity $V_{eq}$ (cm/s) where $V_{eq} = \sqrt{2E_i / m}$. 

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Figure 2. Magnitude - distance distribution of the near-fault ground motions used in this study

<table>
<thead>
<tr>
<th>$V_{S30}$ Range (m/s)</th>
<th>NEHRP Classification</th>
<th># of Records</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;180</td>
<td>E</td>
<td>-</td>
</tr>
<tr>
<td>180-360</td>
<td>D</td>
<td>65</td>
</tr>
<tr>
<td>360-760</td>
<td>C</td>
<td>88</td>
</tr>
<tr>
<td>760-1500</td>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>&gt;1500</td>
<td>A</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. Distribution of ground motions with respect to VS30

4 NEAR-FAULT ATTENUATION MODEL FOR ELASTIC AND INELASTIC INPUT ENERGY

The attenuation model in [18, 19] given in Equation (1) is employed for predicting input energy spectra for elastic and inelastic systems in terms of energy equivalent velocity $V_{eq}$.

$$
\log(V_{eq}) = b_1 + b_2 M + b_3 M^2 + (b_4 + b_5 M) \log \left( R_{jb} \right)^2 + b_6 + b_7 S_s + b_8 S_A + b_9 F_N + b_{10} F_R
$$

In Equation (1), $M$ is the moment magnitude and $R_{jb}$ is the Joyner-Boore distance in kilometers. $S_S$ and $S_A$ are dummy variables representing the influence of site class. $S_S$ is 0 and $S_A$ is 1 for stiff sites, and $S_S$ is 1 and $S_A$ is 0 for soft sites. $F_N$ is zero and $F_R$ is 1 for reverse faulting, and the opposite for normal faulting. They are both 0 for strike slip. The model given in Equation (1) has been modified by simplifying the geometrical decay term, and replacing the Joyner-Boore distance $R_{jb}$ with the distance to epicenter $R_{epi}$. Accordingly the form of geometrical decay is simplified as ($\log (R_{epi})$), without using any additional term. This is due to the fact that seismic waves reach the station from many parts of long rupture in the near-fault regions unlike in the far-fault case where the source is idealized as a point, and hence geometric decay of the earthquake is small [20]. After implementing these modifications, the attenuation model derived for near-fault ground motions becomes,

$$
\log(V_{eq}) = b_1 + b_2 M + b_3 M^2 + (b_4 + b_5 M) \log R_{epi} + b_6 S_s + b_7 S_A + b_8 F_N + b_{10} F_R
$$

The regression coefficients in Equation (2) are determined by a one-stage nonlinear regression analysis at the specified period values for observed (computed) spectral values of linear elastic and inelastic systems separately. The basic reason for this choice is the presence of several single recorded events in the database, generally in the lower earthquake magnitude ranges. Two-stage nonlinear regression analysis technique gives more weight to these less well-recorded earthquakes in the database, which may lead to the violation of magnitude satu-
ration. Accordingly, two-stage analysis overestimates spectral energy at higher magnitudes and underestimates at lower magnitudes [18, 20]. A mass proportional 5% viscous damping ratio is used in the analysis. Elastic, perfectly plastic force-deformation model (bilinear model with zero strain hardening) is employed for inelastic systems which leads to more conservative responses [21]. Three different lateral strength ratios ($R=2, 4$ and $6$) are employed for defining the level of inelasticity. A near-fault ground motion may possess impulsive characteristics, but certainly it is unpredictable at this state of knowledge. Hence, it is preferred in this study that ground motions in the database are not classified into separate groups, such as with and without directivity pulses. It is also worthwhile to note that a pulse-like ground motion may exhibit impulsive characteristics only within a narrow band of orientations. Therefore, including the probability of occurrence of more demanding components exhibiting impulsive features in the design stage may produce overestimation of design energy values. This situation is also valid for the fling effect. Therefore, fault directivity and fling effects are not included explicitly in the prediction model in order to reduce complexity, and to maintain the reliability of results. The regression coefficients computed for elastic ($R=1$) and inelastic SDOF systems ($R=4$) are presented in Tables 2 and 3 respectively, for 5% damping.

<table>
<thead>
<tr>
<th>$T$ (s)</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
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Table 2. Regression coefficients calculated for linear elastic systems ($R=1$) with $\zeta=5\%$
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Table 3. Regression coefficients calculated for yielding systems with $R=4$ and $\zeta=5\%$.

5 NEAR-FAULT ELASTIC INPUT ENERGY PREDICTION

Near-field ground motions may exhibit particular characteristics that affect seismic demand on structures. In order to observe these differences, the variations of 5% damped elastic $V_{eq}$ spectra with distance $R_{eq}$ are obtained from the NF attenuation model derived in this study (Equation (2) and Table 2). They are plotted for a strong earthquake and stiff soil type for the mean and mean ± one sigma at four specified periods of 0.5, 1.0, 2.0 and 4.0 seconds in Figure 3. The computed (observed) $V_{eq}$ spectral ordinates of the ground motions from the selected earthquakes at these periods are also plotted on the corresponding graphics in scatter form. Chi-Chi (1999) earthquake with moment magnitude of 7.62 is selected for comparative evaluation. Fault rupture mechanism of this earthquake is reverse-oblique.

At longer periods, sensitivity of $V_{eq}$ to distance completely vanishes for the model. This is consistent with the NF condition where seismic waves reach the design site from many parts of long rupture synchronously, hence the distance effect is lost. Computed values from the ground motions of selected earthquake (circular dots) also display a gradual variation with distance in these figures, and get closer to each other at longer periods.
Figure 3. Comparison of the computed $V_{eq}$ with the mean and mean ± one sigma of NF attenuation model for $M_{w}=7.62$ Chi-Chi (1999) earthquake, stiff soil type.

Mean elastic input $V_{eq}$ spectra of ground motions selected from Northridge-01 (1994) and Imperial Valley (1979) earthquakes are predicted by the NF attenuation equations developed both in this study and in [20]. The range of epicentral distances for the selected records are 12-16 km for the Northridge-01 (1994) and 17-23 km for the Imperial Valley (1979) earthquakes. The computed elastic input $V_{eq}$ spectra of the selected ground motions from the two earthquakes are shown in Figure 4 for two different soil types, along with their mean spectra and the estimated mean spectra from the two prediction models. Epicentral distance is employed in this study whereas it is Joyner-Boore distance in Ambraseys and Douglas. The middle values of the considered distance bands are used in the predictions. It can be observed from Figure 4 that the results of both studies are in fairly good agreement with each other, and match well with the data obtained from the mean of 7 and 5 ground motions for stiff and soft soils, respectively. The differences between the results of two models are perhaps due to different assumptions in these studies. Ambraseys and Douglas employ the maximum component whereas the geometric mean of two horizontal components are employed in this study. This is the main reason for larger energy predictions by Ambraseys and Douglas in the period range of 0.2-2.0 seconds that they have considered. However, the choices on the distance parameter (epicentral vs. Joyner and Boore) are not expected to play a role on the differences of results because both definitions are consistently accounted for in the associated models.
The sensitivity of mean input $V_{eq}$ spectra to magnitude, distance, fault type and soil type is also evaluated for two magnitudes, three fault distances and three fault types. The mean elastic input energy spectra obtained from the NF model for these parameters are presented in Figures 5, 6 comparatively. It can be inferred from the figures that reverse and strike-slip faults impose 15-40% higher energy demands compared to the normal faults especially at the medium period region. Moreover, spectral values from strike-slip faults fall slower with period compared to the reverse and normal faults. The effect of soil type is more prominent at large magnitudes where ground motions on soft soils impose considerably higher energy demands than those on stiff soils. The soft-to-stiff $V_{eq}$ ratio is about 1.40 for $M_{w}$ 7.0 and 1.25 for $M_{w}$ 6.0 on average along the period axis. Besides, the effect of epicentral distance on $V_{eq}$ completely disappears at longer periods, i.e. $T > 4$ seconds.

Figure 4. The comparison of the computed mean elastic $V_{eq}$ spectra of ground motions selected from $M_{w}$=6.69 Northridge-01 (1994) and $M_{w}$=6.53 Imperial Valley (1979) earthquakes with the estimated mean spectra from the two prediction equations.

Figure 5. Variation of elastic input energy spectra $V_{eq}$ with earthquake magnitude, obtained from NF attenuation model for different soil types and fault mechanisms, $R_{eq}$=15 km.

Figure 6. Variation of elastic input energy spectra $V_{eq}$ with epicentral distance, obtained from NF attenuation model for different soil types and fault mechanisms, $M_{w}$= 7.0.
6 THE EFFECT OF DAMPING ON NEAR FAULT ELASTIC AND INELASTIC INPUT ENERGY SPECTRA

The effect of damping on elastic and inelastic input energy ($E_i$) spectra of NF ground motions is evaluated by computing the $E_i$ spectra for three GM records from the compiled database presented in Table 2. These are GM111 from Cape Mendocino (1992) Earthquake ($M_w$ 7.01, reverse faulting) recorded at Cape Mendocino Station (stiff soil type, $R_{epi}$ = 10.36 km), GM310 from Loma Prieta (1989) Earthquake ($M_w$ 6.93, reverse-oblique faulting) recorded at San Jose-Santa Teresa Hills Station (stiff soil type, $R_{epi}$ = 20.13 km), and GM33 from Kocaeli, Turkey (1999) Earthquake ($M_w$ 7.51, strike-slip faulting) recorded at İzmit station (stiff soil type, $R_{epi}$ = 5.31 km). Geometric mean of the two horizontal components are used in calculating the input energy spectra. Elastic ($R=1$) and inelastic ($R=4$) input energy spectra of these ground motions are compared in Figure 8 and Figure 9 respectively for three different damping ratios of 2%, 5% and 10%.

It is observed from Figure 8 that damping ratio has a slight effect on near fault elastic $E_{ie}$ spectra along the entire period range where higher damping leads to smoother spectral curves. This is also consistent with [22]. Furthermore, damping ratio has almost no influence on the near fault inelastic input energy spectra $E_{iy}$ due to the reduced effect of damping on inelastic behavior, as inferred from Figure 9. The effect of damping on the mean spectra will perhaps be completely diminished, although this is not exercised herein but quite obvious. Therefore, it can be suggested that $F_{ie}=1$ and $F_{iy}=1$ in Figure 1 for all damping ratios and all periods. There is no need for scaling the near fault elastic and inelastic input energy spectra for damping. Accordingly, the regression coefficients given in Tables 2-3 for 5% damping can be employed to predict the near fault input energy spectra of elastic and inelastic systems respectively, for all damping ratios.

![Figure 7](image7.png)
Figure 7. Comparison of elastic input energy spectra ($R=1$) of three near fault ground motions for 2, 5 and 10 percent damping ratios.

![Figure 8](image8.png)
Figure 8. Comparison of inelastic input energy spectra ($R=4$) of three near fault ground motions for 2, 5 and 10 percent damping ratios.
7 THE EFFECT OF LATERAL STRENGTH RATIO ON NEAR FAULT INELASTIC INPUT ENERGY SPECTRA

Lateral strength ratio $R$ has been identified as one of the most important parameters for characterizing nonlinear behavior of structural systems. Thus, its influence on input energy spectra for yielding systems is further examined in this study. Five percent damped reference elastic ($R = 1$) and inelastic ($R = 2, 4, 6$) input energy spectra are computed for the three near fault GM records selected in the previous section, i.e. GM111, GM310 and GM33. Elastic and inelastic input energy spectra of these records for 5% damping are shown in Figure 9. It can be observed that although the variations of $E_i$ with $R$ are somewhat different among the three NF GM’s, there are common trends. First, it is clear that $E_i$ for $R = 1$ and $R = 2$ are very close in the average sense where $R = 2$ smoothens the elastic spectra for $R = 1$, but follows almost the same trend along the entire period axis. Second, $E_i$ for $R = 4$ and $R = 6$ fall below $E_i$ for $R = 1$ and $R = 2$ consistently at the long period region ($T > 1$ second) for the considered GM’s. However the differences between $R = 4$ and $R = 6$ are small, and not very sensitive to $T$.

In order to illustrate the relation between inelastic and elastic input energy spectra, the spectral ratios of inelastic energy spectral ordinates $E_i$ for $R = 2, 4$ and 6 to the elastic spectral ordinates $E_i$ ($R = 1$) are computed for 5% damping and plotted in Figure 10. It can be observed from this figure that the inelastic spectra for $R = 2$ fluctuates almost around 1 however those corresponding to $R = 4$ and $R = 6$ start falling below 1 after $T > 1$ second. Hence, it is difficult to suggest a simple form for the elastic-to-inelastic scaling functions $F_R$ and $F''_R$ introduced in Figure 1, which is reminiscent of an equal energy principle. A healthier conclusion may perhaps be reached on the mean quantities rather than those obtained from individual NF GM’s.

At this stage, we will focus on the predicted mean values of inelastic input energy spectra in order to evaluate the estimation accuracy of the derived prediction equation for inelastic input energy demands, and base our decisions on the sensitivity of mean inelastic $E_i$ to $R$. 

Figure 9. Comparison of input energy spectra of three near fault ground motions for four different $R$ values.

Figure 10. Comparison of the ratios of inelastic energy spectral ordinates ($R = 2, 4$ and 6) to the elastic energy spectral ordinates ($R = 1$) for the three near fault ground motions.
The observed values of 5% damped $V_{eq}$ for $R=4$ for the ground motions from Chi-Chi (1999) and Imperial Valley-06 (1979) earthquakes are plotted on the mean ± 1 sigma variations of $V_{eq}$ with $R_{epi}$, predicted from Equation (2) and Table 3 for the source parameters of these earthquakes in Figures 11 and 12. Both figures reveal the success of the attenuation model in catching the trends of the observed inelastic $V_{eq}$ ordinates. Almost all of the observed (calculated) values fall within the mean ± 1 sigma range.

![Graphs showing comparison of computed inelastic $V_{eq}$ with mean and mean ± 1 sigma of NF attenuation model for $M_w=7.62$ Chi-Chi (1999) earthquake. $R=4$, stiff soil type.](image)

Inelastic $V_{eq}$ spectra of ground motions selected from the Chi-Chi (1999), Northridge-01 (1994) and Imperial Valley (1979) earthquakes recorded at fairly similar distances, and their mean spectra are calculated for $R=4$ and plotted in Figure 13 along with the mean spectra estimated from Equation (2) and Tables 2 and 3 for $R=1$ and $R=4$, respectively. The ranges of distances for the selected GM records are 19-25 km for the Chi-Chi (1999), 11-17 km for the Northridge-01 (1994) and 17-23 km for the Imperial Valley (1979) earthquakes. Their central values are employed in Equation (2) as $R_{epi}$. The estimated inelastic mean spectra from the proposed attenuation model with the regression coefficients for $R=4$ predicts the computed mean spectra with good accuracy. It can also be observed that the estimated mean spectra for $R=4$ and $R=1$ from the developed model display similar spectral variations despite slight differences in short periods.
Figure 12. Comparison of the computed inelastic $V_{eq}$ with the mean and mean ± 1sigma of NF attenuation model for $M_w=6.53$ Imperial Valley (1979) earthquake. $R=4$, soft soil type.

Figure 13. Inelastic $V_{eq}$ spectra ($R=4$) of ground motions selected from Chi-Chi (1999), Northridge-01 (1994), Imperial Valley (1979) earthquakes, and the comparison of their mean spectra with the estimated mean spectra from the developed attenuation model for $R=4$ and $R=1$.

An additional comparison of predicted near fault input energy spectra for four different $R$ values ($R=1, 2, 4, 6$) is presented in Figure 14. Reverse faulting and stiff soil condition are considered for two earthquake magnitudes of $6.0$ and $7.0$, and two epicentral distances of $10$ km and $20$ km. It can be clearly observed from Figure 14 that at intermediate and long periods ($T > 0.5$ s), reducing the lateral strength of the system by increasing $R$ reduces the near fault input energy demand only slightly, regardless of the earthquake magnitude and distance to epicenter. However, for short period systems with $T < 0.5$ seconds, reduced lateral strength increases input energy demand. The ratios of predicted inelastic to elastic spectral energy ordinates are given in Figure 15. It is clear that at intermediate and long periods ($T > 0.5$ s), the strength factor $R$ has no effect on input energy. However input energy demand of inelastic systems increase rapidly with $R$ for short period systems ($T < 0.5$ s).

The effects of earthquake magnitude and distance on the input energy spectra estimated from the NF prediction model derived in this study can be observed from Figures 16 and 17. Magnitude is more prominent on input energy spectral ordinates for elastic and inelastic
(yielding) systems when compared to the distance parameter. Increasing magnitude from 6.0 to 7.0 increases spectral ordinates almost 2.5 times whereas increasing distance from 10 to 20 km reduces spectral ordinates by only 15% on average.

![Graphs showing input energy spectra for different magnitudes and distances.](image)

Figure 14. Predicted input energy spectra from the attenuation model developed in this study for reverse faulting and stiff soil condition cases considering $M_w=6.0$ and $7.0$ and $R_{epi}=10$ km and 20 km.

Sensitivity of the estimated mean inelastic $V_{eq}$ spectra to magnitude, distance, fault type and soil type is further studied for two magnitudes, three fault distances, three fault types and two soil types. The spectral relations obtained from Equation (2) and Table 3 for $R=4$ are shown in Figures 16-17 comparatively. It can be inferred from these figures that, similar to elastic input energy, reverse and strike-slip faults impose 20-30% higher energy demands compared to the normal faults, and energy demands of strike-slip faults exhibit a slower fall with periods. The effect of soil type is more prominent at large magnitudes, and input energy of ground motions on soft soils are considerably higher than those on stiff soils. The soft-to-stiff $V_{eq}$ ratios calculated from the presented results are on average 1.26 for $M_w=7.0$ and 1.20 for $M_w=6.0$. These ratios are lower when compared to the elastic case, which were 1.40 and 1.25, respectively. Inelastic input energy spectra are sensitive to distance for epicentral distances closer than 15 km, but distance sensitivity disappears after 15 km for the near fault ground motions.
Figure 15. Ratios of inelastic energy spectral ordinates \((R = 2, 4\) and \(6)) to the elastic energy spectral ordinates \((R = 1)) predicted from the attenuation model developed in this study for reverse faulting and stiff soil conditions considering \(M_w = 6.0\) and \(7.0\) and \(R_{epi} = 10\) km and \(20\) km.

Figure 16. Variation of inelastic input energy spectra \(V_{eq} (R=4)) with earthquake magnitude obtained from NF attenuation model for different soil types and fault mechanisms, \(R_{epi}=15\) km.

Figure 17. Variation of inelastic input energy spectra \(V_{eq} (R=4)) with epicentral distance obtained from NF attenuation model for different soil types and fault mechanisms, \(M_w=6.0\).
8 CONCLUSIONS

An attenuation model is developed for predicting the input energy spectra of near-fault ground motions for linear elastic and inelastic systems through one-stage nonlinear regression analysis, by using a selected batch of NF ground motions. Ground motions are classified as NF if their closest fault distances are less than 30 km, and shorter or equal to the associated rupture lengths. Near-fault input energy demand is very sensitive to earthquake magnitude, slightly sensitive to source distance less than 15 km, but not sensitive to distance after 15 km.

The effect of damping on input energy spectra for elastic and inelastic systems is negligible. Near fault input energy spectra for inelastic systems has some dependency on lateral strength ratio \( R \) for short period systems; however, there is no dependency on lateral strength for intermediate and long period systems. Accordingly, elastic input energy spectra obtained for a reference 5% damped system can be practically employed to predict the inelastic input energy spectra for intermediate and long period systems for all damping ratios with slight conservatism. This result recalls an equal energy rule, which can be considered as a significant advantage for an energy-based design approach. However, we have not attempted to propose approximate elastic-to-inelastic scaling relations in this study. Instead, the proposed attenuation model described in Equation 2 and Tables 2-3 can be consistently employed for estimating the mean elastic and inelastic input energy spectra from a near-fault earthquake source with a given magnitude, fault type, soil type at the site, and source to site distance.

REFERENCES


ULTIMATE ENERGY DISSIPATION CAPACITY AND COLLAPSE BEHAVIOR OF MULTI-STORY STEEL FRAME WITH SHS COLUMN UNDER BIAXIAL EXCITATION

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Abstract

In seismic design, Moment Resisting Frame (MRF) is generally designed to form the overall sway mechanism. However, because of the strain hardening of beams and panel zones and decreasing of the strength of columns caused by bi-directional horizontal external forces, it is possible to form the weak column mechanism. Once weak column mechanism is formed, frames tends to collapse because of deterioration of restoring force caused by local buckling in column and P-Δ effect. In this study, bi-directional response analysis of weak column type steel MRFs using realistic hysteresis model of columns considering local-buckling are conducted. From analytical results, it is clarified that, once columns start to deteriorate, deformation of the story progressed in one direction. Historic dissipation energy until collapse was not much larger than that of monotonic loading due to damage caused by orthogonal excitation.

Keywords: Energy Dissipation Capacity, Moment Resisting Frame, Square Hollow Section Column, Local Buckling, Bi-Axial Ground Motion, Collapse
1 INTRODUCTION

In seismic design, Moment Resisting Frame (MRF) is generally designed to form the overall sway mechanism under severe earthquake. However, it is possible to form the weak column mechanism. That is because strength of beams and panel zones rises due to strain hardening. Also, strength of columns decreases due to the bi-directional horizontal external forces. These effects are considered in the current seismic design method, however due to the uncertainty of the earthquake ground motion etc., the formation of the weak story mechanism cannot be certainly prevented. In the full-scale shaking table test of 4 story steel moment frame conducted at E-defense in 2007 [1,2], specimen building made sway mechanism under design level earthquake. However, under extremely strong ground motion, mechanism changed to weak story type and specimen building was completely collapsed.

The causes to collapse the weak column type frame are strong bi-axial seismic force, P-Δ effect and deterioration of restoring force caused by local buckling in column. Many studies focused on the collapse behavior of multi-story frames under severe earthquake have been conducted. In the studies focused on the influence of P - Δ effect, generally ideal historical behavior was considered to clarify its influence. Analysis taking into consideration the deterioration behavior of members has also been carried out [3]. These studies have taken into consideration the influence of the deterioration behavior of the members in addition to the P - Δ effect, however they are limited to the in-plane behavior considering single horizontal ground motion. These studies have taken into consideration the influence of the deterioration behavior of the members in addition to the P - Δ effect, however they are limited to the in-plane behavior considering single horizontal ground motion. It is because the current hysteresis models of steel members considering the deteriorating behavior [4, 5] are limited to the in-plane behavior.

In this study, at first, by comparing with experimental results it is verified that bi-directional behaviors of Square Hollow Section (SHS) columns including deteriorating range subjected to horizontal bi-directional external force can be reproduced by applying the in-plane hysteresis model of SHS columns to Multiple Shear Springs (MSS). Then, multi-story weak column type steel MRF with SHS columns is modified to the multi-story lumped mass shear model and conducted a series of response analyses considering bi-axial horizontal ground motion. From analytical results, collapse behavior and ultimate energy dissipation capacity of weak column type steel MRFs with SHS columns under bi-axial excitation is clarified.

2 ANALYTICAL MODEL

Lumped mass model with Multiple Shear Spring (MSS) [6] shown in Figure.1, is used to analyze the responses of steel MRF subjected to bi-directional ground motion. MSS consists of the equal shear springs which placed at an equal angle express the behaviors of each stories. The strength \( q_s \) and the stiffness \( k_s \) of the springs constituting the MSS are given by the equations (1) and (2) from the shear strength \( Q_s \) and stiffness \( K_s \) of the story.

\[
q_s = \sum_{i} Q_s |\cos \theta_i| 
\]

(1)
Where, $N_s$ is the number of the springs in MSS, $\theta_t$ is the direction of each spring. Considering accuracy of calculation, number of shear springs in a story is set 16.

The analytical model assumes a weak column type moment resisting frame consist with cold-formed SHS column and H-shaped beam. Since it is a weak column type MRF, its ultimate behavior is dominated by the ultimate behavior of the column. The SHS column has small directionality of the structural performance, it is suitable for applying MSS.

For the shear strength of the first story, it is set as 0.25 times of the total weight of MRF. It corresponds with strength of columns of the 1st story. Furthermore, to form weak 1st story mechanism that is most severe case in earthquake damage, other stories are kept elastic. In addition, P-Δ effect is considered in the analysis.

![Figure 1 Lumped Mass Model Connected by MSS](image)

3 PARAMETERS OF RESPONSE ANALYSIS

Parameters of the analytical model are number of the story $N$ (=4, 8, 12), width-to-thickness ratio of RHS column $D/t$ (=20, 29). The number of the story was set to examine the influence of the difference of fundamental natural period. Regarding the width-to-thickness ratio, the model with $D/t=20$ represents a building with moderate ductility, and the model with $D/t=29$ represents the building without enough ductility. As for the hysteresis model, polyliner hysteresis model of cold-formed SHS column considering deteriorating behavior governed by local buckling [5] is adopted for columns.

In all models, weight of each story $W_i$ is 1,000(kN), height of each story is 3.5(m). In order to make weak 1st story mechanism, all stories except 1st story are kept in elastic range. Fundamental natural period $T_1$, yield shear strength of the first story $Q/y_1$, and the stiffness distribution $\{K_i\}$ of the analytical models are shown in Table 1.
As input waves, NS and EW components of El-Centro record (1940 Imperial Valley Earthquake), Taft record (1952 Kern County Earthquake), Hachinohe record (1968 Tokachi-oki Earthquake), JMA-Kobe record (1995 Kobe earthquake) and JMA-Sendai record (2011 Tohoku earthquake) were used. In the analysis, amplification factor is multiplied to the acceleration data to change intensity of excitation up to collapse.

Table 1 Characteristics of analysis models

<table>
<thead>
<tr>
<th>Model</th>
<th>$T_1$ (sec)</th>
<th>$W_i$ (Const) (kN)</th>
<th>$h_i$ (Const) (m)</th>
<th>$Q_{y1}$ (kN)</th>
<th>${ K_i }$ (×10³ kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Stories</td>
<td>0.775</td>
<td>1,000</td>
<td>3.5</td>
<td>1,200</td>
<td>{ 68.6, 57.6, 47.4, 31.3 }</td>
</tr>
<tr>
<td>8 Stories</td>
<td>1.046</td>
<td>1,000</td>
<td>3.5</td>
<td>2,400</td>
<td>{ 137.1, 130.0, 115.2, 104.9, 94.7, 80.3, 62.7, 42.0 }</td>
</tr>
<tr>
<td>12 Stories</td>
<td>1.262</td>
<td>1,000</td>
<td>3.5</td>
<td>3,600</td>
<td>{ 205.7, 202.2, 187.1, 172.8, 162.0, 152.7, 142.1, 128.4, 112.0, 94.0, 74.5, 48.4 }</td>
</tr>
</tbody>
</table>

4 COLLAPSE BEHAVIOR OF WEAK STORY TYPE MULTI STORY FRAMES UNDER BI-AXIAL GROUND MOTION

Orbit of the 1st story and hysteresis of the collapsed and its orthogonal directions are shown in Figure 2 (a) – (d). Case A (12 stories, $D/t=20$, Hachinohe) and Case B (4 stories, $D/t=29$, Kobe) represent the cases which the dissipated energy in the collapsed direction, discussed in the next section, were relatively large. Case C (8 stories, $D/t=20$, Taft) and Case D (8 stories, $D/t=29$, Sendai) represent the cases which the dissipated energy in the collapsed direction were relatively small. In the figure, analytical results of collapse level and 75% of collapse level are shown. For the hysteresis, the analysis result of monotonic loading is indicated by a broken line.

Under the relatively low input level that model does not reach its maximum strength, such as 75% of collapsed level excitations of Cases C and D, deformation of the story did not progress in one direction. However, under the relatively high input level that stiffness of the story becomes negative according to the deterioration caused by local buckling, such as collapsed level excitations of all cases, displacement progresses to one direction after deterioration of the stiffness is observed.

Comparing the hysteresis of each cases in collapsed direction, in the cases of A and B, stable behavior is appeared until story reaches its maximum strength. On the other hand, in the cases of C and D, displacement progresses to one direction without obvious cyclic behavior under collapse level excitation.
(a) Case A Model; 12 stories, D/t=20  Input Wave; Hachinohe

(b) Case B Model; 4 stories, D/t=29  Input Wave; Kobe
(c) Case C Model; 8Stories, D/t=20 Input Wave; Taft

(d) Case D Model; 8stories, D/t=29 Input Wave; Sendai
Figure 2 Orbit of the 1st story and hysteresis of the 1st story in collapsed and its orthogonal directions
5 DISSIPATED ENERGY BY WEAK STORY TYPE MULTI STORY FRAMES UNTIL COLLAPSE UNDER BI-AXIAL GROUND MOTION

In this chapter, seismic performance of model is discussed with hysteretic dissipated energy $E_p$, calculated by equation (3). Here, $Q_i$ is restoring force of the story, $W_i$ is weight of the story, $h_i$ is the story height, $\Delta \delta_i$ is increment of story drift, $N$ is the total number of story and $t_0$ is the duration of input wave. In $E_p$, loss of the energy dissipation capacity due to $P-\delta$ effect is considered.

$$E_p = \int_0^{t_0} \sum_{i=1}^{N} \left[ \left( Q_i - (\sum_{j=1}^{N} W_j) / h_i \right) \cdot \Delta \delta_i \right] dt$$  \hspace{1cm} (3)

Hysteretic dissipated energy in the collapse direction under collapsed level excitations $E_{pc}$ are divided by the dissipated energy under monotonic loading $E_{gm}$ and shown in Figure 3. In most of the cases, the ratio of $E_{pc}$ to $E_{gm}$ is around 1.0 to 1.5 regardless of the deformation capacity (width-to-thickness ratio $D/t$) of the column. In some cases, i.e. cases C and D etc., the ratio falls below 1.0. In those cases, the energy dissipation capacity was reduced due to the influence of damage caused by the response in the orthogonal direction.

![Figure 3: Hysteretic dissipated energy in the collapsed direction under collapsed level excitations](image)

6 CONCLUSIONS

A series of response analyses of weak column type multi-story steel frames under bi-axial ground motion are carried out. Hysteresis model used in analysis is based on the realistic behavior of column including the deterioration range governed by local buckling. From analytical results, following conclusions are obtained.

1. Focusing on the orbit of the 1st story, under the relatively low input level that model does not reach its maximum strength, deformation of the story did not progressed in one direction. However, under the relatively high input level that stiffness of the story becomes
negative according to the deterioration caused by local buckling, displacement progresses to one direction after deterioration is observed.

2. In many cases, hysteretic dissipated energy in the collapse direction under collapsed level excitation $E_{pd}$ became 1.0 to 1.5 times of the hysteretic dissipated energy under monotonic loading $E_{pm}$ regardless of the deformation capacity (width-to-thickness ratio $D / t$) of the column. However, in some cases, $E_{pd}$ became slightly lower than $E_{pm}$ due to the influence of damage caused by response in the orthogonal direction.

REFERENCES


A VARIATIONAL APPROACH FOR ENERGY-BASED ANALYSIS OF NEAR-FAULT PULSE-LIKE SEISMIC RECORDS

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Keywords: Variational Mode Decomposition, Pulse-like ground motion, Pulse period.

Abstract. Near-fault earthquakes often exhibit a pulse in the velocity time-history that mainly occurs in the strike-normal direction at locations towards which the earthquake rupture has propagated. The extraction of the dominant impulsive component embedded into such kind of seismic records and the proper consideration of the other signal modes are critical tasks for a reliable prediction of the behavior of structural systems subjected to earthquakes close to active faults. Within this framework, the present work is concerned with the application of the Variational Mode Decomposition technique for processing near-fault pulse-like seismic signals. Specifically, this numerical technique is herein applied to some fault-normal horizontal pulse-like seismic velocity time-histories in order to evaluate its suitability in detecting the mode with the dominant pulse. Final results demonstrate that such variational approach is able to extract the impulsive component of the seismic records, and the corresponding estimates of the pulse period are in good agreement with the results obtained by means of existing methods. Input power and energy as well as acceleration, displacement, velocity and energy spectra have been calculated for the original seismic record and the modes extracted by means of the Variational Mode Decomposition technique.
1 INTRODUCTION

Near-fault pulse-like earthquakes often induce disproportionate damage to building and infrastructures with respect to far-field ground motions because of the higher displacement and velocity demands as well as the transmission of large amounts of energy in a relatively short time. A further issue is the occurrence of unusual peaks in the spectral values even for low frequencies, which might be an unpredictable but relevant threat for the structural safety of very deformable buildings or base-isolated structural systems. Within this framework, several researches have investigated the effects of near-fault pulse-like earthquakes on the seismic response of structures [1–7] and studied the effectiveness of common seismic protection strategies [8–14]. The analysis of near-fault pulse-like earthquakes has been also addressed in several studies. For instance, Baker [15] has applied a wavelet decomposition technique to detect the velocity pulse. An algorithm to extract the pulses based on the repetitive smoothening of the accelerogram is developed in Ref. [16]. Amiri and Moghaddam [17] have proposed a modified version of the S-Transform for the decomposition of seismic signals in order to identify the pulse-like part of near-fault velocity records. Chang et al. [18] have identified the dominant impulsive mode from the original record by minimizing the difference between a numerical pulse model and the velocity time-history of the seismic ground motion. On the other hand, Mimoglou et al. [19] have illustrated a direct method to estimate the pulse period: it assumes that the pulse period corresponds to the period value for which the product of velocity and displacement elastic response spectra for 5% damping attains its maximum value. The analysis of signal components other than the dominant one also deserves proper consideration. Actually, it has been found that higher signal modes can play an important role on the dynamics of structural systems under near-fault earthquakes because a single dominant pulse can lead to non-conservative estimates of the maximum interstorey drift [20]. Multiple pulses are identified in Ref. [21] through the wavelet analysis.

The present paper is intend to contribute to the characterization of near-fault pulse-like earthquakes by investigating the feasibility of a recent signal processing technique, namely the Variational Mode Decomposition [22]. In this perspective, some fault-normal horizontal seismic signal components have been analyzed by means of this technique in order to evaluate its effectiveness in extracting the mode that embeds the dominant pulse and estimating the associated pulse period. The relevance of secondary modes accompanying the dominant one is also discussed in terms of spectral values.

2 ANALYSIS OF NEAR-FAULT PULSE-LIKE SEISMIC SIGNALS

2.1 Variational Mode Decomposition of pulse-like seismic signals

The Variational Mode Decomposition (VMD) technique proposed by Dragomiretskiy and Zosso [22] is here employed for the analysis of near-fault pulse-like seismic ground velocity component in the strike-normal direction. Some concepts about the VMD technique are provided hereafter whereas more details can be fund in Ref. [22].

According to the VMD technique, the bandwidth of the mode is determined through the following procedure:

- compute the analytical signal for each mode using the Hilbert transform in such a way to obtain an unilateral frequency spectrum;
- shift the mode’s frequency spectrum to baseband for each mode, by mixing with an exponential tuned to the respective estimated center frequency;
• estimate the bandwidth using the H1 Gaussian smoothness of the demodulated signal (i.e., squared L2-norm of the gradient).

Formally, this leads to the following constrained variational problem:

\[
\begin{aligned}
\min_{v_k(t), \omega_k} & \left\{ \sum_{k=1}^{N} \left\| \frac{\partial}{\partial t} \left[ \delta(t) + \frac{j}{\pi t} \right] * v_k(t) \right\|^2 e^{-j\omega_k t} \right\}^2 \\
\text{s.t.} & \sum_{k} v_k(t) = v(t)
\end{aligned}
\]  

(1)

where \(v(t)\) is the signal to be decomposed (i.e., the ground motion velocity), \(v_k(t)\) is the \(k\)th mode and \(\omega_k\) the corresponding center pulsation \((k = 1, \ldots, N, \text{in which } N \text{ is the number of modes})\), \(\delta(\cdot)\) is the Dirac delta operator and \(*\) is the convolution operator \((t \text{ is the time variable})\).

A convenient way to solve this mathematical programming problem is based on the use of Lagrangian multipliers and quadratic penalty term, in such a way to transform the original constrained optimization into an unconstrained one. So doing, the augmented Lagrangian \(L\) is:

\[
L(v_k(t), \omega_l, \lambda) = \alpha \sum_{k=1}^{N} \left\| \frac{\partial}{\partial t} \left[ \delta(t) + \frac{j}{\pi t} \right] * v_k(t) \right\|^2 e^{-j\omega_k t} \\
+ \left\| v(t) - \sum_{k=1}^{N} v_k(t) \right\|^2 + \left\langle \lambda, v(t) - \sum_{k=1}^{N} v_k(t) \right\rangle,
\]  

(2)

where \(\lambda\) is the Lagrangian multiplier and \(\alpha\) determines the data fidelity. The solution of the problem given in Eq. (1) is thus obtained as the saddle point of \(L\) through a sequence of iterative sub-optimizations named alternate direction method of multipliers. The procedure is stopped once the following convergence criterion is fulfilled:

\[
\sum_{k=1}^{N} \left\| v_{i+1} - v_i \right\|^2 < \epsilon,
\]  

(3)

where \(i\) is the iteration counter and \(\epsilon\) is the selected tolerance. From a computational standpoint, however, the complete final algorithm is more efficiently posed in the spectral domain.

Previous applications of the VMD have shown that it is able to precisely detect the embedded modes, largely irrespective of their relative amplitudes and how close their frequencies are. Such features are especially important for the analysis of near-fault ground motions since multiple modes characterized by similar amplitudes and frequencies can be embedded into the seismic record.

2.2 Pulse indicator and identification of the dominant impulsive mode

The pulse indicator proposed by Baker [15] is here adopted to identify the mode that embeds the dominant pulse. For each mode \(k\), this is defined as follows:

\[
PI_k = \frac{1}{1 + \exp\left(-23.4 + 14.6\rho_{PGV} + 20.5\rho_{E}\right)},
\]  

(4)

where \(\rho_{PGV}\) is the peak ground velocity of the residual record divided by the original record’s peak ground velocity whereas \(\rho_{E}\) is the energy of the residual record divided by the original record’s energy (herein, the energy is computed as the cumulative squared velocity).
residual record is obtained by removing the $k$th mode $v_k(t)$ from the original record $v(t)$. The dominant impulsive mode is $p = \text{argmax}_k \{ PI_k \}$ subjected to $PI_k \geq PI_{\text{min}}$, where $PI_{\text{min}}$ is a suitable threshold value of the pulse indicator. The pulse period of the dominant impulsive mode is thus defined as $T_p = \frac{2\pi}{\omega_p}$, where $\omega_p$ is the center pulsation of the dominant impulsive mode $v_p(t)$. According to Ref. [15], $0.15 \leq PI_{\text{min}} \leq 0.85$ where $PI_{\text{min}} = 0.15$ and $PI_{\text{min}} = 0.85$ can be interpreted as “soft threshold” and “hard threshold”, respectively.

2.3 Calibration of numerical pulse-like models

The parameters vector $\theta$ of two well-known numerical pulse-like waveforms $\hat{v}_p(t; \theta)$ have been also calibrated to simulate the identified impulsive mode $v_p(t)$ of the signal by solving the following optimization problem:

$$\min_{\theta} \{ \text{MSE}(\hat{v}_p(t; \theta), v_p(t)) \}, \tag{5}$$

where $\text{MSE}(\cdot, \cdot)$ is the mean squared error operator. The numerical pulse-like waveform considered in the present study are the ones proposed in Ref. [23] and Ref. [24]. Since the objective function in Eq. (5) can exhibit multiple local optima [23], this problem is resolved by using soft computing based optimizers with global search capability. Specifically, a suite of numerical methods comprising genetic algorithms, differential evolution algorithms and particle swarm optimization techniques described in Refs. [25, 26] has been adopted to look for the optimal set of model parameters.

3 NUMERICAL RESULTS

3.1 Analysis of seismic signals

A detailed review of the results carried out from the analysis of some seismic signals is provided hereafter. The time histories of seismic acceleration $a$, seismic velocity $v$ and seismic displacement $d$ are first given. Therefore, the modes extracted by means of the VMD technique are shown. After a preliminary survey, it has been found that consistent conclusions can be drawn by extracting three modes, i.e. $v_1$, $v_2$ and $v_3$. The $k$th mode having the highest $PI_k$ value – provided that $PI_k \geq PI_{\text{min}}$, with $PI_{\text{min}} = 0.85$ – is labeled as $v_p$, and the corresponding central period denotes the pulse period $T_p$. Moreover, the absolute value of the relative input power per unit of mass $P_r/m$ and the relative input energy per unit of mass $E_r/m$ are given for each mode. In this regard, it is assumed a linear elastic single-degree-of-freedom (SDOF) system whose natural period $T_n$ is equal to the central period of the considered component $T_k$ and constant viscous damping $\xi = 0.05$. Such results are shown together with the energy of the original seismic signal $E_v$ (which is computed as the cumulative squared velocity of the original seismic signal). Finally, total acceleration spectrum $S_a$, velocity spectrum $S_v$ and displacement spectrum $S_d$ are illustrated for the original seismic signal and the extracted modes (they are given in dimensionless form by dividing the corresponding spectral values by the peak ground acceleration $PGA$, peak ground velocity $PGV$ and peak ground displacement $PGD$ of the original seismic signal, respectively). The relative input energy spectrum per unit of mass $S_{E_r}/m$ is also included. All these spectra are evaluated for a linear elastic SDOF system with viscous damping $\xi = 0.05$ and assuming 150 period values $T$ equally spaced between 0.01 s and 15 s. Here and henceforth, the SI unit system is adopted and the time parameter $t$ is omitted for the sake of conciseness.
3.2 Analysis of the Imperial Valley earthquake (1979)

The analyzed record of the Imperial Valley earthquake (1979) is shown in Figure 1 whereas the obtained results are given in Figure 2, Figure 3 and Figure 4.

The velocity time record exhibits a clear single pulse, which is embedded into the first mode extracted by means of the VMD technique ($PI_1 \approx 1$ while $PI_2 \approx 0$ and $PI_3 \approx 0$). The corresponding pulse period value is $T_p = 4.23$ s, and it is in good agreement with the estimates obtained according to the approach presented by Baker [15] ($T_p = 4.61$ s), Chang et al. [18] ($T_p = 4.40$ s) and Mimoglou et al. [19] ($T_p = 4.34$ s). Numerical pulse-like waveforms proposed in Ref. [23] and Ref. [24] simulates very well the impulsive mode of the signal. The spectral displacement values only are well approximated by the impulsive mode of the signal over the full range of period values. On the other hand, spectral acceleration, spectral velocity and spectral relative energy values of the original signal are larger than the ones of the impulsive mode if the considered period is less than $T_p$.

3.3 Analysis of the Coalinga earthquake (1983)

Figure 5 shows the considered record of the Coalinga earthquake (1983). Results obtained from the application of the VMD technique are given in Figure 6, Figure 7 and Figure 8.

The pulse of this seismic record can be easily identified in the velocity time history (see Figure 5). The VMD recognizes the mode with the dominant pulse, as shown in Figure 6. It coincides with the second extracted mode ($PI_2 \approx 1$ while $PI_1 \approx 0$ and $PI_3 \approx 0$). The corresponding estimate of the pulse period is $T_p = 0.75$ s, which is in excellent agreement with the estimate obtained according to Mimoglou et al. [19] ($T_p = 0.71$ s). It is also close to the estimates obtained according to Baker [15] ($T_p = 0.92$ s) and Chang et al. [18] ($T_p = 0.90$ s). Moreover, it can be observed in Figure 6 that both numerical pulse models proposed in Ref. [23] and Ref. [24] approximates very well the dominant pulse. Finally, Figure 7 and Figure 8 demonstrates that the mode with the dominant pulse and its best numerical approximation lead
Figure 2: Imperial Valley earthquake (1979): time-histories of the extracted modes together with the best-fitting numerical pulse models proposed in Ref. [23] (dashed line) and Ref. [24] (dash-dot line).

Figure 3: Imperial Valley earthquake (1979): relative input power (left side) and relative input energy (right side) corresponding to the extracted modes (red line: first mode, blue line: second mode, green line: third mode), together with cumulative squared velocity of the original seismic record (black line). Relative power and energy values obtained by means of the best-fitting numerical pulse model proposed in Ref. [24] are also shown (dash-dot line).
Figure 4: Imperial Valley earthquake (1979): damped elastic spectra of original seismic record and extracted modes (black line: original seismic record, red line: first mode, blue line: second mode, green line: third mode). The spectra corresponding to the best-fitting numerical pulse model proposed in Ref. [24] are also shown (dash-dot line).

Figure 5: Coalinga earthquake (1983): time-histories of ground acceleration, velocity and displacement.
Figure 6: Coalinga earthquake (1983): time-histories of the extracted modes together with the best-fitting numerical pulse models proposed in Ref. [23] (dashed line) and Ref. [24] (dash-dot line).

Figure 7: Coalinga earthquake (1983): relative input power (left side) and relative input energy (right side) corresponding to the extracted modes (red line: first mode, blue line: second mode, green line: third mode), together with cumulative squared velocity of the original seismic record (black line). Relative power and energy values obtained by means of the best-fitting numerical pulse model proposed in Ref. [24] are also shown (dash-dot line).
Figure 8: Coalinga earthquake (1983): damped elastic spectra of original seismic record and extracted modes (black line: original seismic record, red line: first mode, blue line: second mode, green line: third mode). The spectra corresponding to the best-fitting numerical pulse model proposed in Ref. [24] are also shown (dash-dot line).

to a rather satisfactory reproduction of the whole seismic signal, with the possible exception of the spectral accelerations for very low period values (where the third mode contributes significantly) and the spectral displacements for period values larger than \( T_p \) (where the contribution due to the first component is significant).

3.4 Central Italy earthquake (2016)

The last seismic record analyzed in the present study is the one recorded in the municipality of Amatrice during the Central Italy earthquake (2016), see Figure 9. The whole set of results is presented in Figure 10, Figure 11 and Figure 12.

Despite its apparent simplicity, a deeper analysis of Figure 9 reveals the complexity of the velocity time history. In fact, a first positive half-cycle occurs in the velocity time history before \( t = 9 \) s. Its period is larger than that of the second negative half-cycle, which occurs in the velocity time history after \( t = 9 \) s. It should be remarked that this variation takes place in a very short time, which is a significant challenge for every time-frequency methods of signal processing. The evidences of the visual inspection are confirmed by the results of the VMD. The whole velocity time history is basically the sum of two modes, each one of them with a pulse. The first mode \( v_1 \) is the dominant impulsive component of the velocity time history (\( PI_1 \approx 0.99 \)), but the second mode \( v_2 \) is also pulse-like (\( PI_2 \approx 0.95 \)), and it mainly contributes to the negative peak velocity. The third component \( v_3 \) is a non-pulse residue (\( PI_3 \approx 0 \)). As regards the numerical approximation of the extracted impulsive components, good results are obtained for the second mode whereas the accuracy is less satisfactory for the first mode. The pulse period corresponding to the dominant impulsive component is \( T_p = 1.66 \) s, and it results in fairly good agreement with the result obtained according to Mimoglou et al. [19] (\( T_p = 1.99 \) s). The estimate obtained according to Chang et al. [18] is \( T_p = 0.70 \) s, which is close to the period of the second impulsive mode extracted by means of the VMD technique. The estimate of the pulse period obtained according to Baker [15] is \( T_p = 0.90 \) s, which lies between the period of the first impulsive mode and that of the second impulsive mode identified through the VMD technique. Such complexity of the seismic record reflects into the spectral values. Specifically, maximum acceleration, velocity and relative energy spectral values are mainly due to the second and the third component of the signal for periods less than \( T_p \). In the case of
Figure 9: Central Italy earthquake (2016): time-histories of ground acceleration, velocity and displacement.

Figure 10: Central Italy earthquake (2016): time-histories of the extracted modes together with the best-fitting numerical pulse models proposed in Ref. [23] (dashed line) and Ref. [24] (dash-dot line).
Figure 11: Central Italy earthquake (2016): relative input power (left side) and relative input energy (right side) corresponding to the extracted modes (red line: first mode, blue line: second mode, green line: third mode), together with cumulative squared velocity of the original seismic record (black line). Relative power and energy values obtained by means of the best-fitting numerical pulse model proposed in Ref. [24] are also shown (dash-dot line).

Figure 12: Central Italy earthquake (2016): damped elastic spectra of original seismic record and extracted modes (black line: original seismic record, red line: first mode, blue line: second mode, green line: third mode). The spectra corresponding to the best-fitting numerical pulse model proposed in Ref. [24] are also shown (dash-dot line).
velocity, displacement and relative energy spectra, the dominant impulsive component plays an important role starting from periods close to $T_p$.

4 CONCLUSIONS

A variational approach has been employed in the present study to decompose near-fault pulse-like seismic signals, thereby allowing the identification of the mode that embeds the dominant pulse as well as the estimation of the corresponding pulse period. The analysis of a small set of earthquakes has demonstrated that the VMD technique can be an effective tool for the analysis of pulse-like seismic signals. Moreover, the obtained results have confirmed that a single pulse-like waveform can lead to non-conservative estimates of the seismic demand.

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References


COMPARISON OF ENERGY-BASED RESPONSES OF STRUCTURAL SYSTEMS TO REAL AND SIMULATED GROUND MOTION RECORDS

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Abstract

Energy-based design and assessment approaches usually require use of ground motion datasets. Previous researchers attempted to use solely actual records, however, simulated records can also be used as an alternative in the absence of a homogenous ground motion database. The main objective of this study is to examine the differences in energy-based responses of structures subjected to real and simulated datasets. For this purpose, the obtained input energy spectra based on ‘simulated’ and ‘real’ records are compared. The simulated dataset is generated by the stochastic finite-fault method, considering the regional seismicity parameters that correspond to Duzce city located on the western segments of North Anatolian Fault zone in Turkey while the real dataset is formed by selection of regionally compatible records from NGA-West2 ground motion database. Next, time history analyses are performed for both elastic and inelastic SDOF structural systems to investigate the differences in terms of input energy, resulting from alternative ground motion datasets. The analysis results present variability in terms of energy-based structural demand parameters when alternative ground motion datasets, real or simulated records, are employed.

Keywords: Energy-Based Response, Single-Degree-of-Freedom Models, Time History Analysis, Stochastic Finite-Fault Method, Simulated Ground Motions.
1 INTRODUCTION

Energy-based seismic design and analysis approaches have been investigated for many decades by previous studies [e.g.: 1-10]. All these studies have used actual ground motion records corresponding to worldwide past earthquakes. However, in general it is not simple to form a homogeneous ground motion dataset having well-distributed ground motion parameters that are compatible with the regional seismology of the study area. As an alternative, homogeneous and well-distributed simulated ground motion datasets generated based on the regional seismological parameters for each study area can be utilized in order to estimate the energy-based seismic demand parameters of the structures.

The aim in this study is to investigate the efficiency of simulated ground motion records on the estimation of the elastic and inelastic input energy levels for single-degree-of-freedom (SDOF) structural systems. For this purpose, the 1999 Duzce (Western Turkey) earthquake (Mw=7.1) which occurred on the right-lateral strike-slip active North Anatolian fault zone is selected. In this study, first, the 1999 Duzce event is simulated using the previously validated source, path, and site parameters [11]. For simulations, the stochastic finite-fault methodology which can efficiently model the frequencies of engineering interest is employed [12]. Next, seismic input energy levels for elastic and inelastic SDOF systems with varying periods are assessed through time history analysis. Then, real and estimated input energy levels for both elastic and inelastic SDOF models are evaluated and compared under the real and simulated motions of the event at selected five stations. In the next part, a scenario event of magnitude 7.0 is simulated at different nodes with varying source-to-site distance levels in Duzce region. For all simulations soil class D is utilized, which is mostly compatible with the soil class of the study area. In addition to the simulated records, the real dataset is formed by the selection of regionally compatible records from the NGA-West2 ground motion database. In the selection process, the parameters of concern are considered as the soil class, moment magnitude, fault type and source-to-site distance. Then, time history analyses are performed for both elastic and inelastic SDOF structural systems using the real and simulated record sets to investigate the differences in terms of input energy levels.

2 INPUT GROUND MOTION DATASETS

In this study the seismic input energy levels are compared for two alternative sets of ground motion records: Simulated and real records. For this purpose, Duzce region, which is located in the western part of North Anatolian fault zone, is considered as the study area. In 1999, a major earthquake with Mw=7.1 occurred in Duzce region on the strike-slip fault zone. In the first part of this study, the real and simulated records of the 1999 Duzce event at a total of five stations are used in order to validate the simulated records of this event in terms of seismic input energy evaluation. The simulated records at these stations are prepared using the stochastic finite-fault ground motion simulation methodology based on a dynamic corner frequency approach [12]. It is noted that the stochastic finite fault simulation method simulates only one random horizontal component at each station. For ground motion simulation of the 1999 Duzce event at selected 5 stations, the validated source, path, and site parameters that were first proposed by [13] and then modified by [11] are used.

In the second part of this study, simulations are performed for a scenario event of magnitude 7.0 at a total of 370 blind nodes within the Duzce region with boundaries of 30°- 32° longitudes and 40°- 41° latitudes. For ground motion simulations at these nodes, the validated source, path, and site parameters that were first proposed by [13] and then modified by [11] are used. For simulation at all nodes, generic site amplification factors corresponding to NEHRP soil class D that are compatible with the study region are employed. The simulated
record dataset is prepared for different Joyner and Boore distance (RJB) intervals as 0-10 km, 10-20 km, and 20-40 km. Next, real ground motion dataset is constituted from the past earthquakes by considering the regional seismological characteristics of the study area (Duzce, Turkey). The seismological characteristics considered in selection process include the moment magnitude of Mw=7.0, fault mechanism of strike slip, source-to-site distance of 0-40 km, and NEHRP soil class of D. As a result, a suitable set of records are randomly selected for distance intervals of 0-10 km, 10-20 km, and 20-40 km.

Tables 1 and 2 illustrate the variations of peak ground acceleration (PGA) and peak ground velocity (PGV) with respect to RJB for the selected real and simulated datasets. When the results in terms of the simulated ground motion intensity levels are compared with the real ones, generally more scatter is observed for the real record dataset. This is mostly due to the fact that the simulated record set is region-specific with less variation.

<table>
<thead>
<tr>
<th>PGA</th>
<th>µReal [cm/s²]</th>
<th>σReal [cm/s²]</th>
<th>µSyn [cm/s²]</th>
<th>σSyn [cm/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0≤RJB&lt;10</td>
<td>280.09</td>
<td>204.69</td>
<td>280.65</td>
<td>82.53</td>
</tr>
<tr>
<td>10≤RJB&lt;20</td>
<td>200.20</td>
<td>30.27</td>
<td>153.39</td>
<td>39.75</td>
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<tr>
<td>20≤RJB≤40</td>
<td>90.99</td>
<td>33.31</td>
<td>93.98</td>
<td>19.85</td>
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</table>

Table 1: Mean and standard deviations of PGA distributions

<table>
<thead>
<tr>
<th>PGV</th>
<th>µReal [cm/s]</th>
<th>σReal [cm/s]</th>
<th>µSyn [cm/s]</th>
<th>σSyn [cm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0≤RJB&lt;10</td>
<td>37.84</td>
<td>27.08</td>
<td>36.29</td>
<td>12.37</td>
</tr>
<tr>
<td>10≤RJB&lt;20</td>
<td>20.40</td>
<td>8.65</td>
<td>20.55</td>
<td>5.85</td>
</tr>
<tr>
<td>20≤RJB≤40</td>
<td>12.09</td>
<td>6.68</td>
<td>12.48</td>
<td>4.13</td>
</tr>
</tbody>
</table>

Table 2: Mean and standard deviations of PGV distributions

3 ASSESSMENT OF SEISMIC INPUT ENERGY

The governing differential equation of motion for a general nonlinear SDOF system excited at the base by a horizontal translation from earthquake ground motion is given by:

$$m\ddot{u}(t) + c\dot{u}(t) + f_r(u(t)) = -m\ddot{u}_g(t)$$  \hspace{1cm} (1)

where m and c are mass and viscous damping coefficients; \(\ddot{u}\), \(\dot{u}\) and \(u\) are acceleration, velocity and displacement relative to the moving base respectively, and the term \(\ddot{u}_g\) corresponds to the horizontal ground acceleration at time \(t\). The restoring force is denoted as \(f_r\), and it can be computed as \(f_r(u) = ku(t)\) for linear elastic systems. In case of nonlinear response, force-displacement relationship is determined by the hysteretic characteristics of the system. Through the numerical integration of Equation 1 in time domain, the energy balance equation is obtained and defined as follows:

$$E_h + E_c + E_u = E_i$$  \hspace{1cm} (2)
where the three terms on the left-hand side of the formulation are kinetic energy $E_k$, viscous damping energy $E_\xi$, and absorbed energy $E_a$, respectively. Absorbed energy term represents the total amount of energy that structure absorbs either through recoverable elastic or unrecoverable inelastic deformations, thus it can be expressed as sum of the two where the former is called elastic strain energy $E_s$, and the latter is called hysteretic damping energy $E_h$. On the right-hand side of the formulation, the input energy, $E_i$ is obtained. Throughout the ground motion excitation, the energy imparted to the structure is temporarily converted into the $E_k$ and $E_s$, which eventually fades away, and is dissipated as either of the damping energy terms, $E_\xi$ and $E_h$. In elastic systems, the only way to dissipate the input energy is the viscous damping since there are no inelastic deformations. Uang and Bertero [4] derived two alternative forms of energy balance equation (absolute and relative), where the definition of input energy and kinetic energy terms differ. Essentially, both formulations are mathematically equivalent, although their physical definitions are different. In fact, Uang and Bertero [4] demonstrated that the computed input energy values by the two formulations are nearly equal for a wide period range, 0.1 to 5 sec., which covers most of the building structures of engineering interest. Absolute input energy is evaluated based on the work done by the total base shear on ground displacement while the relative input energy is evaluated based on the work done by equivalent seismic load applied on the equivalent fixed-based system. In other words, the rigid body translation is not considered in calculation of the latter. In this study, the relative energy equation is utilized where the input energy is calculated as follows:

$$ E_i^r = - \int m \ddot{u}_g(t) du $$

### 4 RESULTS

In the work presented herein, the main goal is to evaluate the efficiency of a simulated ground motion dataset by comparing with the corresponding observed ones in terms of seismic input energy demands in structures. Yet, the simulations must be verified before making such a comparison in order to avoid any bias. For this purpose, the input energy spectra are obtained first for both the real and synthetic records of the 1999 Duzce (Mw=7.1) earthquake at five stations as presented in Table 3 for the sake of comparison. Next, set of simulated records are compared with regionally compatible ground motion records from the NGA-West2 database. Recently, Karimzadeh et al. [13] have conducted a parametric study and discussed validity of the simulated dataset employed herein from the aspect of spectral characteristics for varying moment magnitude, source-to-site distances and soil conditions. All the dynamic analyses are performed on SDOF systems with 5% damping ratio. Two different structural models are considered; elastic linear model and piece-wise linear inelastic model that was proposed by Clough and Johnston [14] and exhibits stiffness degradation during cyclic behavior. In the latter model, the strength ratio, which is the normalized yield strength with respect to the weight, is considered as $\eta=0.1$. The reason for selecting a low level of strength ratio is due to the known fact that the building structures in the region are highly vulnerable to seismic action, which was unfortunately revealed by the massive destruction and death toll during the 1999 Duzce earthquake. In addition, low level of strength ratio means more inelastic action and a good base for comparing the inelastic input energies for real and synthetic ground motion records. The results are presented in terms of $E_i'(T)$ per unit mass.
Table 3: Information on the strong motion stations that recorded the 1999 Duzce earthquake.

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Duzce</td>
<td>DZC</td>
<td>40.844</td>
<td>31.149</td>
<td>D</td>
<td>9.3</td>
<td>0.0</td>
<td>520.41</td>
<td>328.03</td>
<td>86.54</td>
<td>54.53</td>
</tr>
<tr>
<td>Mudurnu</td>
<td>MDR</td>
<td>40.442</td>
<td>29.717</td>
<td>D</td>
<td>34.1</td>
<td>38.5</td>
<td>20.06</td>
<td>21.25</td>
<td>1.97</td>
<td>2.27</td>
</tr>
<tr>
<td>Goynuk</td>
<td>GYN</td>
<td>40.396</td>
<td>30.783</td>
<td>D</td>
<td>55.2</td>
<td>34.3</td>
<td>22.17</td>
<td>25.79</td>
<td>5.84</td>
<td>4.49</td>
</tr>
<tr>
<td>Yarımca Petkim</td>
<td>YPT</td>
<td>40.764</td>
<td>29.762</td>
<td>D</td>
<td>116.9</td>
<td>67.9</td>
<td>16.15</td>
<td>23.47</td>
<td>4.08</td>
<td>8.38</td>
</tr>
<tr>
<td>Izmit</td>
<td>IZT</td>
<td>40.463</td>
<td>31.182</td>
<td>C</td>
<td>100.7</td>
<td>80.9</td>
<td>16.41</td>
<td>18.73</td>
<td>2.27</td>
<td>1.73</td>
</tr>
</tbody>
</table>

4.1 Verification of simulated seismic input energy levels for the 1999 Duzce (Turkey) earthquake (Mw=7.1)

Figure 1 shows the comparison of input energy spectra for linear elastic systems subjected to real and synthetic (labelled as Syn in the figures) ground motion records for the selected stations in Duzce. As it can be observed in the plots, station DZC is the only one that yields a significant amount of input energy when compared to the other stations due to the fact that it is the closest and the most affected station from the 1999 Duzce earthquake. It can also be stated that the input energy variations of real and synthetic ground motion records at both DZC station and the other four stations are in comparable grounds. The variations of the fits between the synthetic and the real records can be attributed to complex source, path and site effects.

Figure 2 shows the same input energy comparison for the inelastic SDOF systems using the Clough Johnston hysteresis model with a strength ratio $\eta=0.1$. The spectral variations at DZC station reveal that the input energy obtained from the synthetic ground motion lies in between the input energies obtained from the two horizontal components of the real ground motion records. In other stations, the variation is more due to the low levels of input energy. Actually, the elastic and the inelastic input energy plots for GYN, YPT and IZT stations in Figures 1 and 2 are the same due to the fact that the SDOF systems remains in the elastic range during the 1999 Duzce earthquake.

Overall, the results in Figures 1 and 2 indicate that the real input energy demand and the corresponding simulated values closely match. This observation confirms that simulated ground motions generated by stochastic finite-fault method are appropriate for this particular region and can be used in regions with similar characteristics for energy-based seismic design and assessment purposes.
Figure 1. Comparison of elastic input energy spectra for real and simulated records of the 1999 Duzce (Turkey) earthquake (Mw=7.1)
Volkan Ozsarac, Shaghayegh Karimzadeh, M. Altug Erberik and Aysegul Askan

Figure 2. Comparison of inelastic input energy spectra for real and simulated records of the 1999 Duzce (Turkey) earthquake (Mw=7.1) using the Clough Johnston model.
Figure 3. Comparison of input energy spectra for the selected real and simulated record sets
4.2 Comparison of input energy spectra for the selected real and simulated record sets

Figure 3 illustrates the mean elastic and inelastic input energy spectra together with their ±1 standard deviations for the aforementioned ground motion datasets using three different distance intervals. It is observed that the mean curves are generally closer to each other in the given period range. From the plots it also appears that the spectral variations are relatively more in the case of real ground motions for closer distance range. On the other hand, the variations for the real and synthetic record sets get smaller for longer distances due to the attenuation effects. These trends are also consistent with the values given in Tables 1-2 in terms of ground motion intensity parameters. Overall, it can be clearly seen that the mean elastic and inelastic spectra levels are consistent with the simulated ones for all distance ranges.

5 CONCLUSIONS

This study focuses on the efficiency of ground motion simulations for energy-based design and assessment purposes. For this purpose, the simulations are performed by considering the regional characteristics of Duzce city which is located in a shallow alluvial basin. The simulations are initially validated based on the observed event in the area in terms of both elastic and inelastic input energy spectra. It is observed that simulations provide mostly good fits. Next simulated records for a scenario event of Mw=7.0 are investigated against a selected number of real records set compatible with the regional seismicity. Results are compared in terms of the mean values together with the standard deviations from the mean. The following conclusions can be stated based on the results of this study:

- Validations for the selected case study reveal a good match for input energy variation in 5 different stations. This is due to the use of input parameters in the ground motion simulations of the 1999 Duzce event validated in terms of seismological parameters.
- Overestimation of the real responses can be observed in most of the cases, which encourages the use of ground motion simulations as an alternative conservative approach in seismic design.
- Selection of real records due to seismological parameters like magnitude, distance, soil condition and faulting type results in higher variations in terms of ground motion intensity parameters. This trend is also reflected in energy calculations. Hence region-specific ground motions should be used in order to minimize this variation in seismic design and analysis of structures at any specific site. The only way to achieve this is to employ simulated records which are extensively validated.

Finally, more investigations should be carried out using larger ground motion sets of real ground motions records. In addition, further evaluations should be performed for alternative scenario events before reaching to solid conclusions in this research field.
REFERENCES


ON THE IMPORTANCE OF ENERGY-BASED PARAMETERS

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Abstract

In the Energy Based Design methodology damage is directly related to the input energy. The potential for an earthquake ground motion to damage a specific structure is closely associate with the energy input to that structure and its energy dissipative capacities. In Performance-Based Seismic Engineering (PBEE), seismic hazard is computed through a parameter that is usually indicated as Intensity Measure (IM), which should comprehensively define seismic input to the structure. The development of an energy-based framework for PBEE requires, among other steps, the characterization of the relationships between IMs and with suitable engineering demand parameters (EDPs). A promising parameter for assessing the damage potential of these motions is the input energy that takes into account the contribution of amplitude, frequency content and duration of ground motion. Moreover, the effects of inelasticity and ground motion duration that are overlooked by the more conventional spectral parameters are instead implicitly captured by input energy, that is directly related to the number and amplitudes of the cycles of oscillator response: hence, it conveys information on the ground motion duration, reflecting cumulative effects by virtue of the integration over time that is involved in their computation. In this paper, by using energy-based parameters, we intend to highlight the contribution of energy IMs in describing several effects of seismic action in simple and more complex structures. To this purpose IMs in predicting strength, energy and deformation levels for a structure will be established, in order to identify parameters providing the best levels of correlation to structural damage measures also in presence of site effects.

Keywords: Instructions, ECCOMAS Thematic Conference, Structural Dynamics, Earthquake Engineering, Proceedings.
1 INTRODUCTION

Usually, the intensity of ground shaking and the demand on structures have been characterized using parameters such as peak ground acceleration as well as strength- or displacement-based parameters such as response spectrum ordinates (e.g., pseudo-spectral acceleration at fundamental period. However, various studies suggested that improved performance parameters based on the energy approach could be considered explicitly during seismic design [1-13].

Energy-based methodologies, beyond the potentiality of designing earthquake-resistant structures by balancing energy demands and supplies, allow to characterize the different types of time histories (impulsive, periodic, with long-duration pulses, etc.) which may correspond to an earthquake ground shaking properly, considering the dynamic response of a structure simultaneously [14].

Since energy is a cumulative measure of ground shaking, it also captures duration effects. In fact, it is well known that a certain amount of seismic damage can be due not only by the maximum response such as force or lateral displacement, but also by inelastic excursions below the maximum response. Therefore, energy demand parameters can be considered as reliable tools to use in seismic hazard analysis, for selecting earthquake scenarios, and establishing design earthquake for many type of engineering analyses [15-21]. Finally, using the energy concept and the energy balance equation allows to optimize the design and detailing and to select strategies and techniques for innovative control or protective systems, such as base isolation and passive energy dissipation devices, in the earthquake-resistant design of new structures or in the seismic retrofitting of existing buildings [22].

Ground motion time histories, which may include both ground motion records (comprising accelerograms recorded in both ordinary and special site conditions such as on soft soil, in the near field, with long-duration pulses), and broad-band synthetic accelerograms, all characterized by an appropriate set of ground motion parameters, are used to perform dynamic analyses on SDOF and/or MDOF systems. Systems response to the energy and deformation demands imposed by the dynamic excitation and dependent on the hysteretic behavior, involving strength deterioration and stiffness degradation effects, is therefore analyzed in order to relate appropriate energy quantities to significant kinematic and deformation parameters. From the comparison of the energy/displacement relationships derived for SDOF and MDOF systems, and suitably calibrated on real structures, simplified models validation and/or refinement ensue; furthermore complex models, envisaging a more accurate description of higher-mode effects, failure mechanism effects and strength/stiffness distribution, can be defined. From the energy demand it is then possible to develop an explicit energy-based design methodology. As already underlined by Fardis [23], there is a significant deficiency in evaluating the structural capacity in energy terms. Only few works, e.g. [24, 25], were oriented in the direction of filling this gap.

2 GENERAL CONSIDERATION ABOUT ENERGY-BASED INTENSITY MEASURES

Intensity measures (IMs) are fundamental for describing the important characteristics of ground motion in compact, quantitative form. Many parameters have been proposed to characterize the amplitude, frequency content and duration of ground motions. The most commonly used measure of the amplitude of ground motions, obtained directly from the acceleration time histories, are the peak ground acceleration (PGA), and the peak ground velocity (PGV). However, firstly it was shown that do not exist a good correlation between PGA and the damage potential of earthquake ground motion. In fact, high PGA might be associated with high fre-
quency pulses which do not produce significant damage because most of the impulse is absorbed by the inertia of the structure with little deformation. Secondly, even though the velocity is less sensitive to the higher-frequency components of the ground motion, it represents another parameter only useful for the characterization of ground motion amplitude. From the point of view of damage potential, the area under the acceleration pulse, which represents the incremental velocity (IV), can be considered a more suitable parameter.

A parameter that takes into account duration and amplitude is the Arias Intensity, $I_A$ [26]:

$$ I_A = \frac{\pi}{2g} \int_0^t a_g(t) \, dt $$(1)

where $a_g(t)$ is the ground motion acceleration at time $t$, $t_o$ is the total duration of the ground motion, and $g$ is the acceleration of gravity. It has units of velocity and has been used to evaluate damage potential, even though it does not reflect the frequency content and tends to overestimate the intensity of long duration motions with high amplitude and a broad range of frequency content.

Cumulative absolute velocity (CAV), represented by the integral over time of the absolute value of acceleration, is another instrumental parameter that differs from Arias Intensity by integrating over the absolute value rather than the square of acceleration [27]

$$ CAV = \int_0^{t_o} |a_g(t)| \, dt $$

(2)

where $v_g(t)$ is the ground motion acceleration at time $t$. An important instrumental parameter, that takes into account the duration of ground motion is the Specific Energy Density (SED)

$$ SED = \int_0^{t_o} \left(v_g(t)\right)^2 \, dt $$

(3)

Even though this parameter does not include the contribution of the frequency content, it gives a better insight into the characterization of the most damaging pulses of the considered records. This aspect will be more detailed later.

Frequency content is taken into account by Housner Intensity ($I_H$) [28], that corresponds to the area below the pseudo velocity spectrum in the period range 0.1-2.5 sec:

$$ I_H = \int_{0.1}^{2.5} S_{PV}(\xi, T) \, dT $$

(4)

where $S_{PV}$ is the pseudo velocity spectrum, $T$ is the vibration period, and $\xi$ is the damping ratio. It captures important aspects of the amplitude and frequency content (in the range of primary importance of structures) in a single parameter, and it can be also considered as a measure of the damage potential in energy terms under the assumption that elastic response spectra may be used to estimate the energy available to cause damage. In this case the pseudo velocity spectrum $S_{PV}$ can be used as a measure of the elastic energy demand according to the following relation:

$$ E_v = \frac{1}{2} m \left(S_{PV}\right)^2 $$

(5)

It is necessary to underline that the pseudo velocity spectrum represents the lower bound of the Input Energy spectrum [2, 6]. Also, the pseudo velocity spectrum coincides approximately (except in the short period range) [1] with the so-called “energy that contributes to damage”, defined by Housner as the total input energy $E_I$ minus the energy dissipated by damping $E_\xi$, converted to equivalent velocity by $V_D = \sqrt{2(E_I - E_\xi) / M}$. Anyway, Housner Intensity does not take into account the duration of motion that could be important for demands that push the structure into the non-linear range.
Earthquake Destructiveness Power, $P_D$, of Araya and Saragoni [29], that is quite well correlated with observed damage expressed in terms of observed macroseismic intensity, takes into account at the same time duration, frequency content and amplitude. It is defined by:

$$
P_D = \frac{\pi \int_{t_0}^{t_0} a^2(t) dt}{2g} = \frac{I_A}{\nu_0} \quad \nu_0 = \frac{N_0}{t_0}$$

where $\nu_0$ is the number of zero crossing in unit of time of the acceleration time history, $N_0$ is the number of crossings, $t_0$ is the duration of the time history, $I_A$ is the Arias Intensity. Duration and amplitude are considered by means of the Arias Intensity, while $\nu_0$ [sec$^{-1}$] represents an average index of the frequency content.

It is also believed that damage to structures is more closely related to the duration of strong ground motion. As structural damage strongly depends on the number of load reversals that may occur during an earthquake, a motion of short duration could not generate enough load reversals to damage a structure, even if the amplitude of the motion is high. Conversely, a motion with moderate amplitude but a long duration can produce enough load reversals to cause significant damage. Several procedures have been proposed to define the strong motion part in an acceleration record. However, comparisons between records with similar acceleration or energy indicate that significantly different effects on structural systems may occur. Although for equal accelerations, greater duration is usually more damaging, for equal energy, shorter duration presents a greater seismic hazard.

It was shown [30] that, to account for the influence of the structural characteristics, IMs derived from response spectra appeared more efficient in predicting structural damage. Perhaps the more simple to obtain is the elastic spectral acceleration at the first mode-period $S_a(T_1)$. Nonetheless, for tall and long-period buildings as well as for structures subjected to near-source ground motions, $S_a(T_1)$ may not be neither efficient nor sufficient because of the limited spectral shape information. This is in part due to the fact that $S_a(T_1)$ accounts neither for contribution of higher modes nor for period lengthening owing to structural nonlinearity.

Several alternative IMs were proposed as adjustments of $S_a(T_1)$ in order to explicitly overcome the aforementioned drawbacks [31, 32]. The objective of these proposals is not only to improve the predictive efficiency of the IM for all damage levels of a given structure, but also to account for the IM computability through a ground-motion hazard analysis without the need of any new attenuation relationships.

However, recent studies have shown that a reliable characterization of the engineering demand parameters (EDPs) distribution given the IM is not a straightforward task especially for complex three-dimensional structural models under multi-directional excitations [33]. Moreover, there are building components that are sensitive to more than one EDPs, such as in case of in-plane and out-of-plane collapse of infill masonry, base-isolated structures, retrofitted structures with special dissipative devices, non structural components, etc.

Therefore, as opposed to a scalar IM, vector-valued IMs have been evaluated so as include more parameters. The vector-valued IM consisting of $S_a(T_1)$ and spectral values at other periods was shown to be a good predictor for ordinary ground motions as well as for pulse-like ground motions. A detailed analysis about the goodness of vector-valued IMs can be found in [34, 35]. However, the question is that spectral acceleration represents a parameter that gives information mainly about the amplitude of ground motion. The frequency content is only indirectly taken into account. For this reason, it is necessary to add the contribution of other frequencies by means of a vector of parameters.
Numerous spectrum-based scalar IMs including energy-derived ones to explicitly account for higher-mode effects as well as period lengthening due to structural softening, were investigated, and studies showed that velocity-based IMs are in general better correlated to deformation demands for different kind of structures [36-42].

The question is that a rational methodology for the assessment of the earthquake destructiveness potential, particularly in the case of inelastic behaviour, should take into account simultaneously the relevant characteristics of the ground motion (amplitude, frequency content, duration, etc.) and of the earthquake-resisting structure. Effects of inelasticity and ground motion duration that are overlooked by the more conventional spectral parameters are instead implicitly captured by Input Energy, that is directly related to the number and amplitudes of the cycles of oscillator response. Hence, it conveys information on the ground motion duration, reflecting cumulative effects by virtue of the integration over time that is involved in their computation. Energy parameters allow to characterize properly the different types of time histories (impulsive, periodic with long durations pulses, etc.) which may correspond to an earthquake ground shaking.

The energy balance equation, obtained by integrating the equation of motion of a linear or non linear SDOF system subjected to ground excitation, can be written as [3] is:

$$E_I = E_k + E_\xi + E_e + E_H$$

where $E_I$ is the absolute input energy, $E_k$ is the absolute kinetic energy, $E_\xi$ is the damping energy, $E_e$ is the recoverable elastic strain energy and $E_H$ is the irrecoverable hysteretic energy that can be directly associated with the damage.

Absolute input energy, $E_{Ia}$ represents the work done by the total base shear at the foundation displacement. The input energy can be expressed by:

$$E_{Ia} = \int \dddot{u}_t du_g = \int \dddot{u}_t \dddot{u}_g dt$$

where $m$ is the mass, $u_t = u + u_g$ is the absolute displacement of the mass, and $u_g$ is the earthquake ground displacement. From now on the input energy per unit mass, i.e. $E_I/m$, will be denoted as $E_{Itr}$. These two energy parameters can be conveniently converted into equivalent velocities using the following equation:

$$V_{E_{Ia}} = \sqrt{2E_{Ia}/m} \quad V_{E_{Itr}} = \sqrt{2E_{Itr}/m}$$

Rearranging terms in Eq.(7), the following expression is obtained:

$$E_D = E_I - E_\xi = E_e + E_H$$

where $E_D$ was referred to by Housner [1] as the energy that contributes to damaging the structure and $E_e$ ($=E_e+E_H$) is the elastic vibration al energy. The input energy is a reliable parameter in selecting the most demanding earthquake and, at a given site, the input energy permits the selection of the possible critical motions for the response of the structure. In previous works [6, 7], the area enclosed by the elastic input energy spectrum, $AE_{I}$, in the interval of periods between 0.05 and 4.0 seconds was proposed as IM:

$$AE_{I} = \int_{0.05}^{4.0} \bar{E}_{I}(\xi = 5\%, T) \ dT$$

This parameter, unlike the peak energy spectral value, which generally corresponds to a narrow band of frequencies, takes account of the global energy structural response amount, and it can also be seen as the energy version of the Housner Intensity, with some differences due to the fact that the pseudo-velocity spectrum constitutes the lower bound of the input energy spectrum [7, 9]. $AE_{I}$ has been used to assess the seismic capacity of existing structures [43, 54, 55].

Modifications of the period range to be considered were proposed by Mollaioli et al. [40] that suggested the use of new integral IMs, which were obtained by modifying the existing ones
in order to get better correlation with the predicted demands. Integral-based structure-specified IMs were evaluated by integration of the energy spectral values over a given period range in order to explicitly account for higher-mode effects as well as period lengthening due to structural softening [40].

3 EFFECTIVENESS OF ENERGY-BASED INTENSITY MEASURES

In the following, some comparison between common IMs and energy-based IMs will be illustrated to the purpose of highlighting the effectiveness of the energy-based approach in the characterization of the damage potential of earthquake ground motion.

In Table 1 peak ground acceleration (PGA), peak ground velocity (PGV), maximum spectral acceleration (\(S_a\)), Arias intensity (\(I_A\)), effective duration (\(t_\Delta\)) according to Trifunac and Brady [44], \(AE_I\) factor, and maximum input energy (\(\mathcal{E}_{\text{imax}}\)), derived from strong motion records, obtained during severe earthquakes in different conditions of magnitude, source-to-site distance and soil, are shown.

It was already recognized that damage is inadequately correlated with PGA or \(I_A\), and this aspect is still reflected by the poor correlation existing between those parameters and energy parameters. For example, for the SCT 127 record, to a low PGA value corresponds perhaps the highest energy ever recorded. PGA is often controlled by the high frequency content in the ground motion (i.e., it is not associated with a narrow range of frequencies). Sometimes records can show isolated short-duration, high-amplitude spikes with little engineering significance.

<table>
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<tr>
<th>Earthquake</th>
<th>Station</th>
<th>PGA (cm/s²)</th>
<th>PGV (cm/s)</th>
<th>(I_A) (cm/s)</th>
<th>(t_\Delta) (s)</th>
<th>(S_a) (g)</th>
<th>(AE_I) (cm²/s)</th>
<th>(\mathcal{E}_{\text{imax}}) (cm²/s²)</th>
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<td>32934</td>
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<tr>
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<td>501</td>
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<td>1.97</td>
<td>10736</td>
<td>7175</td>
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<td>43</td>
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<td>1.01</td>
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<td>31826</td>
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<td>33.6</td>
<td>1.98</td>
<td>53819</td>
<td>67045</td>
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<tr>
<td>San Salvador 1986</td>
<td>CIG 90</td>
<td>681</td>
<td>81</td>
<td>242</td>
<td>4.3</td>
<td>1.95</td>
<td>26612</td>
<td>24124</td>
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<tr>
<td>Messico 1985</td>
<td>SCT 127</td>
<td>168</td>
<td>60</td>
<td>243</td>
<td>36.5</td>
<td>0.99</td>
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<tr>
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<td>827</td>
<td>8.3</td>
<td>2.67</td>
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<td>Newhall 360</td>
<td>578</td>
<td>96</td>
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<td>5.9</td>
<td>2.39</td>
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<td>27548</td>
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</table>

Table 1: Comparison between different IMs.

Duration is not correlated with energy demand as short duration time histories could give high energy values for pulse-like records (Rinaldi S48W, Kobe JMA NS, etc.). Duration definitions, such as other demand measures, ignore both the strength of structural systems (elastic vs inelastic response) as well as the period sensitivity of the spectral response quantities. Looking at the Lucerne valley station, the two components presents the very close effective duration (and also very close values of \(S_a\) and \(I_A\)) but complete different energy demands. It is possible to observe, that the Lucerne N80W is characterized by a large value of PGV due to the presence of a significant long duration pulse that is clearly highlighted by its energy content. In another case, Joshua tree vs SCT, to similar values of \(S_a\) and \(I_A\) corresponds very different values of \(AE_I\) and \(\mathcal{E}_{\text{imax}}\).

Even for a single recording station (Newhall) it is possible to obtain for the two horizontal components similar values of PGA, \(I_A\), \(t_\Delta\), \(S_a\), but different values of \(\mathcal{E}_{\text{imax}}\) and \(AE_I\). Highest
energy value in the component N360 are due to the presence of a long duration pulse due to the forward directivity effect.

Although $I_A$ is also a measure of energy input to an elastic system, it tends to overestimate the intensity of an earthquake with long duration, high acceleration and broad band frequency content (Llolleo N10E).

To similar PGA and $S_a$ values (e.g., Llolleo N10E vs CIG 90) may correspond completely different values of $\tilde{E}_{i,\text{max}}$ and AEI values, which correctly reflect the considerable different magnitude levels of the Chile and San Salvador earthquakes.

Figure 1 – Comparison between $S_a$ spectra (a) and input energy $E_{i,a/m}$ spectra (b) for the 1986 San Salvador earthquake CIG record and 1985 Chile earthquake Llolleo record.

Figure 2 – Acceleration time histories. (a) Llolleo record (1985 Chile earthquake); (b) CIG record (1986 San Salvador earthquake).
As already underlined, one significant disadvantage of the classical response spectra in terms of acceleration, $S_a$, (also in presence of nonlinear behaviour) to the respect of energy spectra is that the effect of strong motion duration is not considered. In the following, an interesting example in which the influence of duration is highlighted only by energy spectra, already reported in 1988 by Uang & Bertero [2], will be illustrated. The constant ductility $S_a$ spectra, corresponding to the 1986 San Salvador earthquake (CIG record) and 1985 Chile earthquake (Llolleo record) are reported in Figure 1 a,b, respectively. By comparing these spectra, it seems that the damage potential of these ground motions is quite similar, even though the CIG and Llolleo are recorded in different condition of magnitude, 5.4 and 7.8 respectively, and distance. Similar results were also obtained for displacement spectra.

Re-examining the comparison of the damage potential of the CIG and Llolleo records in terms of input energy, a completely different picture is obtained. In fact, the $E_{ia}$ of the Llolleo record is considerably higher than that of the CIG record, both in the elastic and inelastic cases.

Moreover, looking at Figure 2, where the acceleration time-history of the two records is illustrated, it is possible to observe the different frequency content and duration. In the first case (Llolleo N10E), the behavior is characteristics of a signal recorded during a large earthquake at distances between 30-40 km. In the second case (CIG 90), the signal is obtained from a moderate earthquake in near-fault conditions. In fact, it is possible to observe in the velocity time-history shown in Figure 3 the presence of a distinct pulse. This pulse was due to the forward directivity effect encountered in this case.

3.1 Pulse-like energy demands

As already stated, the acceleration response spectra do not provide sufficient information to account for the energy transmission mechanisms attributable to the presence of pulse-like near-
field ground motions. It is therefore to argument the necessity of considering information related to the number, sequence and characteristics of pulses, usually associated with large velocity increments. A comparison of the different effects produced by impulsive and harmonic excitations has been illustrated in Bertero et al. [45] and Mollaioli et al. [14] with a series of analyses on SDOF elastic and inelastic systems subject to simple idealized ground motion.

It was shown that while high frequency motions are critical essentially for elastic systems with fundamental period close to the resonance period, medium-to-long duration pulses appear to be critical for structures with fundamental periods belonging to a significantly wider interval.

Moreover, due to the fact that the totality of the energy demand is concentrated in the time interval corresponding to the duration of the pulse, this time interval is not long enough for the structure to efficiently utilize the structural damping to reduce the vibrations. Consequently, most of the energy is dissipated through hysteresis, which implies the development of structural damage. It was recognized that that the intensity and distribution of the seismic energy demand depend upon the duration of the pulses and the importance of energy-based parameters in the characterization of the seismic demand of structures subject to near-fault pulses was emphasized in [14].

As already underlined, duration was also considered by the Arias Intensity parameter. However, as it is possible to observe in Figure 4, where the normalized time histories of $I_A$, the Energy Flux are compared with that of acceleration, velocity and displacement, a slow increase

![Normalized time-histories of Arias Intensity ($I_A$), Energy Flux (SED), acceleration, velocity and displacement corresponding to the fault normal component of the Gilroy #6 record (1984 Morgan Hill Earthquake, $M_w=6.2$).](image)
of the Arias Intensity was observed with two larger increases in correspondence of the maximum acceleration spikes. A different behavior is observed for the Energy Flux (SED parameter) that is a measure of energy demand. In this case, a significant jump in the time history is associated with the late pulse corresponding to both maximum velocity and displacement. This aspect, differently to what can be seen in the acceleration spectrum, is also highlighted by the maximum spectral values of equivalent velocity $V_{ElA}$ shown in Figure 5, whose period corresponds to the pulse duration.

Figure 5 – Acceleration ($S_a$) and Equivalent input energy velocity ($V_{ElA}$) spectra corresponding to the fault normal component of the Gilroy #6 record (1984 Morgan Hill Earthquake, $M_w=6.2$).

Figure 6 – Normalized time-histories of Arias Intensity ($I_A$), Energy Flux (SED), acceleration, velocity and displacement corresponding to the fault normal component of the TCU075 record (1999 Chi Chi Taiwan Earthquake, $M_w=7.6$).
A similar situation is observed in Figure 6 where the normalized time histories of $I_A$, the Energy Flux, of acceleration, velocity and displacement of the TCU075 record (1999 Chi Chi Taiwan Earthquake, $M_w=7.6$) are shown. The acceleration time history presents apparently a broadband frequency content that is reflected by the Arias Intensity the increases slowly and continuously during the whole duration of the record. Looking at the velocity time history a different picture is shown. In this case, it is possible to observe a distinct velocity pulse associated to the maximum displacement demand. Corresponding to this pulse the energy flux suddenly increases and then remains more or less constant.

This feature is also highlighted by the equivalent velocity input energy spectrum compared with the acceleration spectrum (Figure 7). While it is quite evident that the maximum spectral $V_{EIa}$ value occurs at a period corresponding to the duration of the pulse the maximum acceleration demand is concentrated in the high frequency range.

![Figure 7 – Acceleration ($S_a$) and Equivalent input energy velocity ($V_{EIa}$) spectra corresponding to the fault normal component of the TCU075 record (1999 Chi Chi Taiwan Earthquake, $M_w=7.6$).](image)

Considering that the period of the velocity pulse, $T_p$, is essential in evaluating the structural response under pulse-like ground motions, the information gained from the energy spectrum seems fundamental. The definition of the pulse period is not unique in the literature. It was defined as the interval of time so that the pseudo-velocity response spectra of their equivalent and recorded motions display their maximum value, or as the period associated with the maximum Fourier amplitude of a wavelet (when the periods achieved from the wavelet and the velocity spectrum methods disagree appreciably, the wavelet period is assumed as a robust measure of pulse period), or as the period at the peak of the response velocity spectrum evaluated with 5% damping ratio, etc. [46]

As already shown, since the duration of the near-fault ground motion pulses characterizes the intensity and the distribution of the input energy, $T_p$ was defined as the period corresponding to the peak in the elastic input energy (absolute $E_{Ia,max}$, or relative $E_{Ir,max}$) spectrum with 5% damping ratio [46]. Table 2 contains a comparison between energy-based pulse durations, $T_p(E_{Ia,max})$ and $T_p(E_{Ir,max})$, pulse periods evaluated at the peak of the pseudo-velocity response spectrum, $T_p(S_{pv,max})$, and at the peak of the Fourier spectrum, $T_p(S_{a,Fourier})$, with $T_p$ values determined by using other methods [47-49]. It is possible to observe in Figure 8 in terms of normalized spectra, that the input energy spectra ($E_{Ia}$ and $E_{Ir}$) can be quite well represented by the Fourier amplitude spectrum with difference that in the latter case the spectrum is not smoothed. A similar trend is also detected with the pseudo-velocity spectrum.

It is possible to observe that the periods obtained by using the different approaches are quite similar for near-fault motions recorded during earthquakes of moment magnitude in the range between 5.7 to 7.0, with few exceptions. For more severe events like 1999 Chi-Chi, Taiwan Earthquake, $M_w=7.6$, the durations achieved from the input energy (absolute and relative) and $T_p$ in [49] differ significantly. In the latter case, the wavelet pulse period is associated with the velocity pulse because the purpose in [49] has been to identify in a qualitative way ground
motions containing velocity pulses. In contrast, by using an energy-based procedure taking into account simultaneously acceleration and velocity time histories the pulse period based on energy spectra corresponds to the most damaging oscillatory portion of the motion.

![Figure 8](image_url)

**Figure 8** – Normalized absolute and relative input energy (EIₐ and EIₐ), Pseudo-velocity (Sₚᵥ) and Fourier Amplitude spectra corresponding to the fault normal component of the El Centro #8 record (1979 Imperial Valley Earthquake, Ms=6.5).

<table>
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<td>1.31</td>
</tr>
<tr>
<td>FN674</td>
<td>2.30</td>
<td>2.30</td>
<td>2.00</td>
<td>2.16</td>
<td>2.70</td>
<td>2.19</td>
</tr>
<tr>
<td>FN912</td>
<td>3.80</td>
<td>3.80</td>
<td>4.50</td>
<td>3.90</td>
<td>5.88</td>
<td>2.30</td>
</tr>
<tr>
<td>FN928</td>
<td>1.65</td>
<td>1.60</td>
<td>6.50</td>
<td>1.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FN943</td>
<td>2.50</td>
<td>2.60</td>
<td>2.50</td>
<td>1.33</td>
<td>-</td>
<td>3.85</td>
</tr>
<tr>
<td>FN952</td>
<td>3.90</td>
<td>3.90</td>
<td>4.50</td>
<td>3.81</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FN959</td>
<td>4.00</td>
<td>4.00</td>
<td>4.75</td>
<td>4.00</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2: Pulse periods Tₚ, evaluated with different procedures. FN179 (Gilroy array #6, 1979 Coyote Lake Eqk.); FN290 (Downey—Maint Bldg, 1987 Whittier Narrows Eqk.); FN360 (North Palm Springs, 1986 N. Palm Springs Eqk.); FN596, FN598, FN599, FN600 (El Centre Array #4, #6, #7, #5, Imperial Valley 1979 Eqk.); FN646 (Erzican, 1992 Erzincan Eqk.); FN647, FN674 (Rinaldi Receiving Sta, Newhall, 1994 Northridge Eqk.); FN912, FN928, FN943, FN952, FN959 (TCU075, TCU054, TCU102, TCU087, TCU128, 1999 Chi-Chi, Taiwan Eqk.).

For a selected group of near-fault ground motions relative to the 1999 Chi-Chi, Taiwan Earthquake, described in Table 3, consider the strength demand (represented by the seismic coefficient Cₛ, equal to the ratio between maximum base shear and weight of the building) corresponding to the pulse periods which are identified by the energy and Baker method [49].
The ground motions in Table 3 correspond to $T_p$ values that differ significantly with each other. The result is that the pulses obtained from the energy-based wavelet analysis produce the largest strength demand ($C_y$) just at the considered pulse periods.

<table>
<thead>
<tr>
<th>Station</th>
<th>$C_y$ (g)</th>
<th>$T_p$($E_{Ia\text{,max}}$)</th>
<th>$C_y$ (g)</th>
<th>$T_p$($E_{Ir\text{,max}}$)</th>
<th>$C_y$ (g)</th>
<th>$T_p$ (Baker, 2007)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FN928</td>
<td>0.30</td>
<td>0.30</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FN943</td>
<td>0.42</td>
<td>0.38</td>
<td>0.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FN952</td>
<td>0.16</td>
<td>0.16</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FN959</td>
<td>0.25</td>
<td>0.26</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Strength demands, $C_y$ (g), for 1999 Chi-Chi, Taiwan Earthquake.

Moreover, a significant difference in terms of inelastic displacement demands between the various methodologies appears. For example, consider an inelastic SDOF system, an elastic-perfectly plastic oscillator characterized by a yielding limit strength of 0.15 g, subjected to the pulses obtained with the energy-based and Baker approaches for the FN928 (TCU054, 1999 Chi-Chi Taiwan earthquake) record. For energy-based analyses, at a pulse period of about 1.65 s, a displacement ductility demand ($\mu$) of 3.7 is detected. By using the procedure indicated in [49], where the pulse period is equal to 10.5 s, the SDOF systems remains elastic. Similar results have been obtained for the other records characterized by prominent differences in pulse-periods.

3.2 Correlation between energy and displacements

It was previously underlined that there is a correlation between the velocity and displacement time histories for the selected examples presented. However, global and local deformations of the structure are effectively correlated with energy parameters for various structural systems, showing the possibility of integrating them within a performance-based seismic design methodology.

In Figure 9, for two completely different records, the normalized elastic spectra of Interstory Drift ratio ($ID_{I\text{max}}$), Input Energy ($E_i$) and Acceleration ($S_a$) are compared for the Erzincan and Hachinoe records. The first one (Figure 9a) is pulse-like, while the second (Figure 9b) is a typical long duration signal.
obtained during a large magnitude earthquake. As it is possible to observe, the spectral shapes of \( \text{IDI}_{\text{max}} \) and \( E_I \) are perfectly correlated, differently from \( S_a \). Generally, the shapes of the spectra of the inter-story drift index, \( \text{IDI} \), are more influenced by the signal energy content than the displacement spectra. This situation arises from the comparison between energy and drift spectra, which indicate both spectral coincidence of periods where the maximum values are attained and agreement in the general trend. In contrast, similar correspondences are not recognizable in the displacement spectral shapes. This should mean that a strong energy demand imposed to a structural systems does not necessarily cause a strong top displacement, while a strong local deformation demand is quite likely to occur [50]. For SDOF systems, Fajfar & Vidic [4] proposed a non-dimensional parameter, \( \gamma \), that connects the dissipated hysteretic energy, \( E_H \), with the maximum displacement of the system, \( \delta \). As also shown by Mollaioli et al. [50], there exists a more stable quantity that can be yielded by setting a relation between the square root of the input energy and the displacement. As a matter of fact input energy, \( E_I \), represents an effective tool in the characterization of the seismic demands; scarcely depending on the hysteretic properties of the structure. Therefore, the parameter, \( \zeta \), was then defined as [5, 50]:

\[
\zeta = \sqrt{\frac{E_I/m}{\omega \delta}}
\]

(12)

where \( E_I \) is the input energy, \( m \) is the mass of the system, \( \omega \) is the natural circular frequency and \( \delta \) is the maximum displacement of the system. In order to extend the characterization of the relationships between energy and displacement demands to multi-degree-of-freedom (MDOF) systems, in Mollaioli et al. [50] two further pairs of parameters were defined, one that relates input energy with global displacements, the other that correlates energy quantities with inter-story drifts. The first two parameter, \( \zeta' \), depending on the input energy, \( E_I \), can be directly derived by utilizing modified expressions of the formula derived for the SDOF systems:

\[
\zeta' = \sqrt{\frac{E_I/m}{\omega_1 \delta_{\text{roof}}}}
\]

(13)

where \( m \) is the total mass of the system, \( \omega_1 \) is the fundamental frequency, and \( \delta_{\text{roof}} \) is the top displacement. The definition of the second parameter, \( \zeta'' \), is defined by substituting the global displacement, \( \delta \), with the inter-story drift, \( \text{IDI}_{\text{max}} \), leading to the following expression:

\[
\zeta'' = \sqrt{\frac{E_I/m}{\omega_1 \cdot h \cdot \text{IDI}_{\text{max}}}}
\]

(14)

where \( N \) is the number of stories, \( h \) is the story height and \( \text{IDI}_{\text{max}} \) is the maximum interstory drift index. The quantities above have proved to be capable of yielding the best results in terms of statistical stability with regard to the characteristics of the ground motion records considered [50]. Generally, the maximum drift is greatly influenced by both the amount of energy input and the way the energy is imparted to the structure. A greater rate of input in the energy demand time history influences strongly the drift demand for pulse-like time histories in comparison with long duration motions, which of course could impart the same amount of energy to the structure, but in a longer time. As a matter of fact, near-fault records could induce a lower number of cycles in the structure than far-field ones, but with a higher energy content. This circumstance can account for a larger concentration of the drift demand. Figures 10, refer to the parameters \( \zeta' \) and \( \zeta'' \) for the inelastic case corresponding to a ductility ratio of 2, for different distance interval and fixed soil class and magnitude interval.

It is possible to observe that, curves relevant to the parameter \( \zeta'' \), though decreasing beyond the low-period range, appear to assume a stable trend as the fundamental period of the structure increases, indicating that, input energy is well correlated to the damage measure represented by
maximum interstory drift. Generally, the degree of correlation between energy and displacement quantities is noticeably more stable for the case of input energy rather than dissipated energy.

![Figure 10](image_url)

Figure 10 – Median inelastic spectra of the parameters $\zeta$, $\zeta'$, and $\zeta''$ for various distance intervals. Degrading Hysteretic model (DGR), Soil D according to Eurocode 8.

### 4 DESIGN PARAMETERS FOR THE ENERGY-BASED DESIGN METHODOLOGY

The energy-based design methodology requires to quantify the total plastic strain energy dissipation demand $E_H$ on the overall structure. This can be done from the energy balance equation Eq.(10), once $E_e$ and $E_\xi$ are determined. Moreover, it also necessary to estimate how $E_H$ distributes at the local (story, member etc.) level in order to compare the plastic strain energy dissipation demand in each structural part or member with the corresponding plastic strain energy dissipation capacity.

Several empirical equations have been proposed in the literature to estimate $E_\xi$. Recalling Eq.(10) and based on analyses of SDOF systems with elastic-perfectly-plastic restoring force characteristics, Akiyama [1] proposed the following equation that relates $E_I-E_\xi$ with $E_I$, expressed in terms of equivalent velocities, i.e. $V_D = \sqrt{2(E_I - E_\xi)}/M$ and $V_E = \sqrt{2E_I/M}$:

$$V_D/V_E = 1/(1 + 3\zeta + 1.2\sqrt{\xi})$$

(15)

In this equation the dependence of $V_D/V_E$ on the level of plastification of the system is neglected for simplicity. In systems experiencing large plastic deformations the dependence of $V_D/V_E$ on on the level of plastification is negligible, but it is not for low levels of plastification. Kuwamura and Galambos [51] proposed the following equation based on analyses of elasto-plastic SDOF systems subjected to four seismic events (El Centro NS 1940, Parkfield Station No 2 record N65E 1966, Pacoima Dam S16E 1971 and Hachinoe EW 1968):

$$V_D/V_E = \left\{\frac{\eta}{\eta + 0.15}\right\} /\left\{1 + [20(3\zeta + 1.2\sqrt{\xi})/(\eta + 10)]\right\}$$

(16)

Where $\eta$ indicates the level of plastification in terms of an adimensional cumulative plastic deformation ratio defined by:

$$\eta = E_H/(Q_\gamma \delta_y)$$

(17)
Here \( Q_y \) and \( \delta_y \) are the yield shear force and the yield shear displacement, respectively, of the SDOF system. Kuwamura’s equation takes into account the level of plastification through the parameter \( \eta \) but it was derived from a limited number of ground motions. Fajfar and Vidic [4] carried out parametric studies with non-linear SDOF systems subjected to ground motion records from California 1979, Montenegro 1979, Friuli 1976, Banja Luka 1981 and Chile 1985 and proposed the following equation for systems with \( \xi = 0.05 \):

\[
V_D/V_E = \sqrt{c_E (\mu - 1)^\eta / \mu}
\]

(18)

Here \( c_E \) and \( c_H \) are constants depending on the restoring force characteristics of the system. For stiffness-degrading models, \( c_E \) and \( c_H \) should be taken as \( c_E = 1.13 \) and \( c_H = 0.8 \). The parameter \( \mu \) is the ratio of the maximum displacement to \( \delta_y \). Using over 100 ground motion records obtained from 48 earthquakes, Benavent-Climent et al. [10] proposed the following expression that modifies Akiyama’s equations to take into account the dependence of \( E_c \) on \( \eta \) for low levels of plastification:

\[
V_D/V_E = 1.15 \eta / \{(0.75 + \eta) / \left(1 + 3 \xi + 1.2 \sqrt{\xi}\right)\}
\]

(19)

Focussing on buildings, \( E_c \) and the distribution of \( E_H \) among the stories (i.e. \( E_H/E_H \)) can be estimated as follows. Low-to-middle rise building structures can be represented with an equivalent shear-strut model of uniform mass \( m(x)=m \) and linearly changing shear rigidity \( G(x)=G_0+(G_T-G_0)/(xH) \) — where \( G_0 \) and \( G_T \) are values of \( G \) at the bottom and top of the strut. The analysis of the undamped elastic response (see Appendix A) of this strut model applying the modal superposition method gives the following expression for the elastic vibration energy:

\[
W_c = e \frac{M \hat{a}(0)^2 g^2}{2a^2 \omega^2} = \frac{M g^2 T_1^2}{4\pi^2} \left( \frac{e}{a^2} \right) \hat{a}(0)^2 / 2
\]

(20)

Here \( T_1 \) is the fundamental period of the structure, \( M \) is the total mass of the structure, \( g \) is the gravity acceleration, \( \hat{a}(0) \) is the maximum base shear coefficient of the strut, and \( a \) and \( e \) are two parameters that depend on the dynamic properties of the structure and the characteristics of the ground motion. Akiyama (1985) computed \( e/a^2 \) for strut systems of different mass \( m \) and rigidity distribution \( G/G_T \) subjected to ground motions characterized by a bilinear energy input spectrum (expressed in terms of equivalent velocity \( V_D-T \) ) defined by:

\[
\begin{align*}
&\text{for } T<T_G : \quad V_D = V_{D,\max} T/T_G \\
&\text{for } T\geq T_G : \quad V_D = V_{D,\max,}
\end{align*}
\]

(21)

(22)

Here, \( T_G \) represents the characteristic period of the ground. Akiyama [1] found that for realistic values of \( T_1/T_G \) ranging from 1 to 5 and \( G_0/G_T \) from 1 to 10, the quotient \( e/a^2 \) is only slightly above 1. Based on these findings and for design purposes he proposed to estimate \( E_c \) with Eq.(23) making \( e/a^2=1 \), that is:

\[
E_{c_0} = \frac{M g^2 T_1^2 Q_{H1}}{4\pi^2 a^2}
\]

(23)

where \( Q_{H1} \) is the base yielding shear strength of the structure.

The estimation of how \( E_H \) distributes among the stories (i.e. \( E_H/E_H \)) is a key aspect in the energy-based design. It is also cumbersome because \( E_H/E_H \) is affected by the characteristics of the ground motion and of the building. A simple approach to estimate \( E_H/E_H \) is the one proposed by Akiyama (1985) and adopted by the Building Standard Law of Japan (2009) for designing structures with energy dissipation systems. This approach is based on the following considerations. The first one is that there is an “optimum” distribution of the yield shear force
coefficient $\tilde{\alpha}_i$, referred hereafter as $\tilde{\alpha}_{i,\text{opt}} = \tilde{\alpha}_{i,\text{opt}} / \tilde{\alpha}_{1,\text{opt}}$, that makes the cumulative plastic deformation ratio of each story defined by $\eta_i = E_{Hi} / (Q_{yi} \delta_{yi})$, equal in all stories. For a $N$-storey building, $\alpha_i$ is defined as $\tilde{\alpha}_i = Q_{yi} / \sum_{i=1}^{N} m_i g$ where $m_i$ is the mass and $Q_i$ the yield shear strength of the $i$-th storey. The exact “optimum” distribution of $\tilde{\alpha}_i$ can be calculated for a given earthquake and for a given structure, but it can be only established in an approximated way for general buildings structures and earthquakes. The second consideration is that the “optimum” distribution of the yield shear force coefficient can be approximated with the distribution that would experience the building in case of an elastic response. Using the elastic shear-struct

model of uniform mass $m(x)=m$ and linearly changing shear rigidity $G(x)=G_o+(G_f-G_o)/(x/H)$
described above, the distribution of the shear force coefficient $\bar{\alpha}(x) = \tilde{\alpha} (x) / \tilde{\alpha} (0)$ when the system is subjected to an earthquake characterized by the energy input spectrum defined with Eq.(16) and (17) can be easily obtained (see Appendix A). It is represented with solid lines in Figure 11 for different values of $T_i/T_G$ and $G_i/G_f$. The solid lines can be approximated with the dot lines shown in Fig.11. For a lumped-mass system, $\bar{\alpha}$ will be recalled $\alpha$ hereafter, and these curves are expressed by:

$$
\bar{\alpha}_i = \frac{\alpha_i}{\bar{\alpha}_i} = \exp \left[ \left( 1 - 0.02 \frac{k_i}{k_N} - 0.16 \frac{T_i}{T_G} \right) \bar{\alpha} - \left( 0.5 - 0.05 \frac{k_i}{k_N} - 0.3 \frac{T_i}{T_G} \right) \bar{\alpha}^2 \right]
$$

(24)

where $\bar{x} = (i-1)/N$, $k_i$ and $k_N$ are the lateral stiffness at the base and uppermost $N$-th storey. Thus, the second assumption is $\tilde{\alpha}_{i,\text{opt}} = \tilde{\alpha}_i$. Alternative expressions for $\tilde{\alpha}_{i,\text{opt}}$ have been proposed in the literature with the aim of further simplifying Eq.(19). One of them is the one adopted in the Building Standard Law of Japan [52]:

$$
\tilde{\alpha}_i = 1 + \left[ \frac{1}{\sqrt{\bar{\alpha}_i}} - \tilde{\alpha}_i \right] \frac{2T_i}{1+3T_i}
$$

(25)

where $\tilde{m}_i = \sum_{k=1}^{N} m_k / M$. The distribution of hysteretic energy along the height of the building, $E_{Hi}/E_{H1}$, for a structure with the “optimum” yield shear force coefficient distribution, i.e. $s_i=E_{Hi}/E_{H1,\text{opt}}$, can be easily expressed in terms of the yield shear force coefficient and lateral stiffness $k_i$ of each story noting that $\eta=\eta_i$ is the same in all stories, $\eta_i = E_{Hi} / (Q_{yi} \delta_{yi})$, $\alpha_i = \alpha_{i,\text{opt}} = \tilde{\alpha}_{i,\text{opt}} \alpha_1$ and $k_i=Q_{yi}/\delta_{yi}$, giving:

$$
\begin{align*}
\bar{s}_i = \frac{E_{Hi}}{E_{H1,\text{opt}}} = \frac{\eta_i Q_{yi} \delta_{yi}}{\eta_1 Q_{y1} \delta_{y1}} = \frac{Q_{yi} \delta_{yi}}{Q_{y1} \delta_{y1}} = \frac{\left( \tilde{\alpha}_{i,\text{opt}} \alpha_1 \tilde{m}_i \right)^2 \bar{M}^2 g^2}{\left( \alpha_1 \right)^2 \bar{k} \bar{M} g^2} = \frac{\tilde{m}_i^2}{\bar{M}} \tilde{\alpha}_{i,\text{opt}} \frac{2 \bar{k} \bar{k}_1}{k_i} 
\end{align*}
$$

(26)

In practice, it is impossible to make the lateral strength distribution of an actual multi-storey building agree completely with the optimum distribution. The reasons are found in the scatter in mechanical properties of materials, the rearrangement of geometrical shapes of members in order to simplify the fabricating processes etc. Deviations of lateral strength with respect to the “optimum” value causes concentration of damage (in terms of $E_{hi}$) in some stories (those with relatively smaller strength). The severity of this concentration of damage depends mainly on the extent of this deviation, on the characteristics of the structural system (e.g. a strong-column weak-beam frame is less prone to damage concentration than a strong-beam weak-column frame), and can be enhanced by torsional effects. The third consideration is that $E_{Hi}/E_H$ is governed by the deviation of the actual yield shear force coefficient of the structure with respect to the “optimum” value, as indicated by the following expression:

$$
\frac{E_{Hi}}{E_H} = \frac{s_i (p x t)^{-n}}{\sum_{k=1}^{N} k (p x t)^{-n}}
$$

(27)
Here \( p_i = \alpha_i / \alpha_{i, opt} \) quantifies the deviation of lateral strength with respect to the optimum value. \( p_t \) reflects the torsional effects. More precisely, the proneness of a story to torsional effects can be characterized by the ratio \( e_{ox} / r_x \), where \( e_{ox} \) is the distance between the center of stiffness and the center of mass, measured along the direction perpendicular to the direction of analysis considered, and \( r_x \) is the torsional radius. The effect of torsion in a given storey \( s \) is measured by an equivalent decrease in the yield-shear force that is characterized herein by \( p_{ts} \). The value of \( p_{ts} \) has been calibrated in past studies \([1]\) and expressed as follows:

\[
 p_{ts} = \begin{cases} 
 1 & \text{for } (e_{ox} / r_x) \leq 0.15 \\
 1.15 - (e_{ox} / r_x) & \text{for } 0.15 < (e_{ox} / r_x) < 0.30 \\
 0.85 & \text{for } (e_{ox} / r_x) \geq 0.30 
\end{cases} 
\]  

(28)

The coefficient \( n \) characterizes the proneness of the structure to concentrate damage. The adoption of a larger value results in overestimation of damage concentration. In stories with \( (p_s p_{ts}) > 1 \), \( (p_s p_{ts})^n \) becomes smaller as \( n \) increases and tends to be little damage. On the other hand, in the stories with \( (p_s p_{ts}) < 1 \), \( (p_s p_{ts})^n \) becomes larger as \( n \) increases, and there is a concentration of damage. The Building Law of Japan \([52]\) adopts \( n=4 \) for strong column-weak beam systems, and \( n=8 \) otherwise. Further investigation is needed to refine the estimation of \( E_{Hi} / E_H \) and the dependence of the damage concentration factor \( n \) on the characteristics of the structure.

Fig. 11: Calculated and approximated distributions of shear-force coefficient for:
(a) \( G_s / G_i = 1 \); (b) \( G_s / G_i = 2 \); (c) \( G_s / G_i = 4 \); (d) \( G_s / G_i = 10 \).

Another key aspect in the energy based-design methods is the prediction of the maximum deformation \( \delta_{max,i} \) or the apparent plastic deformation \( (\delta_{max,i} - \delta_{p,i}) \). At the storey level the apparent plastic deformation is characterized by the ductility ratio, \( \mu_i \) defined by:

\[
 \alpha(x) = \frac{\delta(x)}{\delta(0)} 
\]
Several expressions have been developed in the literature to relate the apparent plastic deformations with cumulative hysteretic energy in terms of $\mu_i$ and $\eta_i$. The ratio $\eta_i/\mu_i$ is a key parameter for energy-based design methods. Estimating $\eta_i/\mu_i$ enables one to take into account not only the cumulative damage expressed in terms of $E_{Hi}$, but also the damage associated with maximum plastic deformation. The ratio $\eta_i/\mu_i$ is closely related to the concept of “equivalent number of cycles” that has been defined with the following expression:

$$n_{eq,i} = \frac{E_{k,i}}{Q_{y,i}(\delta_{max,i} - \delta_{y,i})} = \eta_i / \mu_i$$  \hspace{1cm} (30)$$

$n_{eq}$ is influenced by the elastic and plastic deformation capacity of the structure, as well as by the seismological characteristics of the earthquake. For this reason, it is essential to distinguish between near- and far-field ground motions.

Rearranging Eq. (30) for a SDOF, the following expression is obtained:

$$\frac{E_{HI}}{n_{eq}} = Q_y \delta_{max} - \frac{Q_y^2}{k}$$  \hspace{1cm} (31)$$

Thus, $E_{HI}/n_{eq}$ represents the monotonic plastic strain energy associated to the maximum displacement $\delta_{max}$. The second member of the Eq. (31) represents, in turn, the monotonic plastic strain energy in terms of capacity for a given $\delta_{max}$, $(E_{HI}/n_{eq})_{cap}$, which depends only on $Q_y$ and $k$. Therefore, a designer disposes of a variety of values for $Q_y$ that lead to dissipate $E_{HI}$ achieving $\delta_{max}$ at most (Fig. 12.a). Further, the second member of Eq. (31) depends only on the properties of the structure ($Q_y, k$) and the maximum displacement allowed $\delta_{max}$, and can be interpreted as a parameter characterizing the capacity of the structure in terms of the ratio $E_{HI}/n_{eq}$; let call it $(E_{HI}/n_{eq})_{cap}$. Nonetheless, the maximum value of $(E_{HI}/n_{eq})_{cap}$ can be achieved only for $Q_y = k\delta_{max}/2$. Hence, if the demand $E_{HI}/n_{eq}$ is higher than this maximum value of $(E_{HI}/n_{eq})_{cap}$, the displacement required by the structure to dissipate the energy introduced by the earthquake will exceed the design value $\delta_{max}$ and vice. $n_{eq}$ is lower for near-field earthquakes than for far-field earthquakes (especially for pulse-like ground motions) and this leads to larger values of $Q_y$. This explains the greater displacements of the structures subjected to pulse-like ground motions than that obtained for ordinary ground motions, when $E_{HI}$ is similar in both of them.

Fig. 12. Relationship among $E_{HI}$, $Q$, $\delta$ and $n_{eq}$. (a) Dissipated energy vs $Q$, for a fixed $\delta_{max}$. (b) Capacity curve for $E_{HI}/n_{eq}$.
For MDOF systems, the same analysis can be carried out when $E_H$ and $n_{eq}$ are known for each story. In this case, the values of $Q_y$ and $\delta_{\text{max}}$ are referred to the yield-shear force and the interstory drift for the $i$-th story, respectively. Nonetheless, more studies are needed to solve key questions such as the distribution of $n_{eq}$ over the structure, $n_{eq,i}$.

5 CONCLUSIONS

This paper has identified and discussed key issues of the energy-based methodology that need to be investigated within a probabilistic framework for the further development of the seismic design approach based on the energy concept. The probabilistic framework makes it possible to explicitly account for the uncertainties of the parameters involved in the design process and can thereby make energy-based seismic design methods the preferable design tool to attain the goals of Performance Based Earthquake Engineering. The following key issues calling for a probabilistic characterization for a further characterization of energy-based methods are identified and discussed: the input energy $E_I$, the amount of input energy to be dissipated through hysteretic action $E_H$, the distribution of damage among the stories, and the number of equivalent cycles of plastic deformation $n_{eq}$. Ongoing research explores these key issues under a probabilistic approach.

6 ACKNOWLEDGEMENTS

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APPENDIX A

A low- to medium-rise multi-story building of can be represented by a shear strut of height $H$ and the undamped elastic response under a ground motion acceleration $z_g(t)$ can be obtained with the mode superposition method as follows. The differential equation that controls the response is:

$$m \frac{d^2 y}{dx^2} - \frac{d}{dx} \left( G \frac{dy}{dx} \right) = -m \ddot{z}_g$$

(A.1)

where $x$ is the height at an arbitrary point; $t$ is the time; $y(x,t)$ is the horizontal displacement; $m(x)$ is the mass, and $G(x)$ is the shear rigidity. The equation of the free vibration is:

$$m \frac{d^2 y}{dx^2} - \frac{d}{dx} \left( G \frac{dy}{dx} \right) = 0$$

(A.2)

whose solution have the following form

$$y = \sum q_j \phi_j$$

(A.3)

where $q_j(t)$ and $\phi_j(x)$ are the time function and the mode function for the $j$-th mode of vibration. The summatory $\Sigma$ is extended from $j=1$ to $\infty$. $q_j(t)$ is given by

$$q_j(t) = a_j \sin(\omega_j t + \alpha_j)$$

(A.4)
Here, $a_j$ is the amplitude, $\omega_j$ the frequency and $\alpha_j$ the phase-angle, that are constant for each mode $j$. $\phi_j(x)$ is an orthogonal function that satisfies:

$$\int_0^H m\phi_i \phi_j dx = 0 \text{ if } i \neq j$$  \hspace{1cm} (A.5)

and can be normalized so that

$$\int_0^H m\phi_j^2 dx = M$$  \hspace{1cm} (A.6)

where $M$ is the total mass of the system. Substituting Eqs.(A.3) and (A.4) in Eq.(A.2) gives:

$$\sum q_j \left[ m\omega_j^2 \phi_j + \frac{d}{dx} \left( G \frac{d\phi_j}{dx} \right) \right] = 0$$  \hspace{1cm} (A.7)

Eq. (A.7) must be satisfied for arbitrary values of $q_j$, therefore:

$$\frac{d}{dx} \left( G \frac{d\phi_j}{dx} \right) = -m\omega_j^2 \phi_j$$  \hspace{1cm} (A.8)

The solution of Eq.(A.1) and Eq.(A.3) takes the same form, thus substituting Eq.(A.3) in Eq. (A.1) and taking into account Eq.(A.8) gives:

$$m\sum \phi_j \ddot{q}_j + m\sum \omega_j^2 \phi_j q_j = -m\ddot{z}_g$$  \hspace{1cm} (A.9)

Multiplying Eq.(A.9) by a given mode function $\phi_s$, integrating and recalling Eqs.(A.5), (A.6) gives:

$$M\ddot{q}_s + M\omega_s^2 q_s = -\ddot{z}_g \int_0^H m\phi_s dx$$  \hspace{1cm} (A.10)

Eq.(A.11) is the equation governing the vibration in the $s$-th mod. Multiplying the second member of Eq.(A.9) by $ydt = \sum \phi_j \ddot{q}_j dt$ and integrating over $O-H$ and $0-t_0$ gives the total energy input $E$:

$$E = \int_0^H \left[ \int_0^H \ddot{z}_g \sum \phi_j \ddot{q}_j dt \right] dx = -\sum \int_0^H \ddot{z}_g \ddot{q}_s \int_0^H m\phi_s dx dt$$  \hspace{1cm} (A.11)

Multiplying the second member of Eq.(A.10) by $\dot{q}_s dt$ and integrating over $0-t_0$, the energy input in a single mode of vibration $s$, $E_s$, is:

$$E_s = -\int_0^t \ddot{z}_g \ddot{q}_s \left( \int_0^H m\phi_s dx \right) dt$$  \hspace{1cm} (A.12)

It follows from Eqs.(A.11) and (A.12) that:

$$E = \sum E_j$$  \hspace{1cm} (A.13)

On the other hand, the energy input in a fictitious undamped SDOF of mass $M$ and frequency $\omega_2$ subjected to the ground motion acceleration $\ddot{z}_g(t)$, $E_{ij}$, is given by:

$$E_{ij} = -\int_0^t M\ddot{z}_g y dt$$
where $y$ is the fictitious displacement. For a given vibration mode $j=s$, the force represented by the second member of Eq.(A.10) is \( \int_0^H m \phi_j(x) \, dx / M \) times larger than that acting on the fictitious SDOF system \(-M\ddot{z}_g(t)\). Therefore, the energy \( E_s \) is \((\int_0^H m \phi_s(x) \, dx / M)^2\) times larger than \( E_{1j} \):

\[
E = \sum E_{ij} \left( \frac{\int_0^H m \phi_j(x) \, dx}{M} \right)^2
\]  

(A.14)

For convenience \( E \) can be expressed in terms of new coefficients \( D_j \) which satisfy

\[
\sum D_j \phi_j = 1
\]  

(A.15)

Multiplying both sides of Eq.(A.15) by \( m \phi_j \) and integrating gives

\[
D_j = \frac{\int_0^H m \phi_j(x) \, dx}{M}
\]  

(A.16)

Raising Eq.(A.15) to the second power, multiplying by \( m \), integrating over the 0-\( H \) and recalling Eqs.(A.5),(A.6) gives:

\[
\sum D_j^2 = 1
\]  

(A.17)

From Eq.(A.16) and Eq.(A.14), the total energy input \( E \) can be rewritten:

\[
E = \sum E_{ij} D_j^2
\]  

(A.18)

For convenience, \( E_{ij} \) is expressed in form of an equivalent velocity \( V_{ij}(\omega_j) \) as follows:

\[
E_{ij} = \frac{MV_{ij}^2}{2}
\]  

(A.19)

and then the total input energy \( E \) given by Eq.(A.18) can be expressed by:

\[
E = M \sum (V_{ij} D_j)^2
\]  

(A.20)

For \( t \geq t_0 \), the system is oscillating in an undamped free oscillation, thus the term \( V_{ij} D_j \) represents the maximum velocity of the system vibrating in mode \( j \). The maximum absolute acceleration can be obtained multiplying the maximum velocity by the frequency \( \omega_k \), then the maximum force at height \( x \) is \( \omega_j V_{ij} D_j \phi(x)m(x) \) and the corresponding shear force \( \dot{Q}_j \) is:

\[
\dot{Q}_j(x) = \omega_j V_{ij} D_j \int_x^H m \phi_j \, dx
\]  

(A.21)

Combining the \( \dot{Q}_j \) of each mode with the rule of the square root of sum of the squares, the maximum total shear at height \( x \) normalized by the weight over \( x \), expressed in terms of a shear-force coefficient \( \hat{\alpha} \), is:

\[
\hat{\alpha}(x) = \sqrt{\sum \dot{Q}_j^2(x)} = \sqrt{\sum (\omega_j V_{ij} D_j \int_x^H m \phi_j \, dx)^2}
\]  

(A.22)
Recalling Eq.(A.16) and making $x=0$ in Eq.(A.22) base shear force coefficient is:

$$\hat{\alpha}(0) = \frac{\sqrt{\sum (\omega V_0 D_j^2)}}{g}$$  \hfill (A.23)

$E$ and $\hat{\alpha}(0)$ can be further expressed as a fraction of the corresponding values in a fictitious SDOF system of mass $M$ and frequency equal to that of the first mode $\omega_1$ of the real MDOF system, subjected to the same round motion $\ddot{z}_g(t)$ by two new coefficients $e$ and $a$ as follows:

$$\hat{\alpha}(0) = a \frac{\omega_1 V_1}{g} \quad ; \quad E = e \frac{MV_1^2}{2}$$  \hfill (A.24)

Here $V_I$ denotes the input energy, in form of equivalent velocity, in the fictitious SDOF system of frequency $\omega_1$. In case of an undamped elastic system, $E$ coincides with the elastic vibration energy $W_e$ for $t \geq t_0$. Thus, noting that $\omega_1=2\pi/T_1$ and using Eq.(A.24), $W_e (=E)$ is expressed by:

$$W_e = e \frac{M^2 \omega_1^2 g^2}{2a^2} + \frac{Mg T_1^2}{4\pi^2} \left( \frac{e}{a^2} \right) \hat{\alpha}(0)^2$$  \hfill (A.25)

Using above equations, it is obtained next the response of a shear strut of uniform mass $m(x)=m$ and linearly changing shear rigidity $G(x)=G_o+(G_T-G_o)(x/H)$ —where $G_o$ and $G_T$ are values of $G$ at the bottom and top of the strut— subjected to an earthquake characterized by a bilinear input energy spectra (expressed in terms of equivalent velocity $V_D-T$) by:

for $T<T_G$ : $\quad V_D = V_{D_{\text{max}}}T/T_G$ \hfill (A.26a)

for $T \geq T_G$ : $\quad V_D = V_{D_{\text{max}}}$ \hfill (A.26b)

Here $V_{D_{\text{max}}}$ is the maximum $V_D$ in the spectra and $T_G$ is the predominant period of the ground motion. Substituting $G(x)=G_o+(G_T-G_o)(x/H)$ in Eq.(A.2) and performing a variable transformation $X=G_o+(G_T-G_o)(x/H)$, Eq.(A.2) relates to Bessel's equation and the $\alpha_i$ and $\phi_i(x)$ can be obtained through numerical calculations. Substituting these $\phi_i(x)$ in Eq.(A.16) provides the $D_i$'s and the corresponding $V_{ij}$ are calculated from the $V_D-T$ spectra defined by Eq.(A.26). Substituting $\omega_i$, $\phi_i(x)$ and these $D_i$'s, $V_{ij}$'s in Eqs.(A.20), (A.22) and (A.23), the values of $E$, $\hat{\alpha}(x)$, $\hat{\alpha}(0)$ and the shear-force coefficient distribution $\tilde{\alpha}(x) = \hat{\alpha}(x)/\hat{\alpha}(0)$ is obtained.

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REFERENCES


CONSIDERATION OF POUNDING AND SSI IN ENERGY-BASED SEISMIC DESIGN OF BUILDINGS

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Abstract

The energy demand on a given construction is commonly determined, in terms of equivalent velocity, from smoothed design spectra that are ordinarily derived after dynamic analyses on SDOF systems. The application to actual multi-story buildings is far from trivial, given that their behavior must be characterized with MDOF models; in this context, a relevant issue is the distribution of energy along the building height. In other words, additional research is required to further promote the development of the energy-based approach. Thus, this work addresses two issues not sufficiently covered in past studies: the influence of soil-structure interaction (SSI), and pounding between adjoining buildings. The study begins by selecting four low-rise (3 and 5-story) RC frame buildings and four representative severe ground motion records (i.e. historic accelerograms). Such inputs are selected with respect to their most relevant characteristics: frequency content (indirectly represented by the soil type) and relevance of velocity pulses (forward-directivity effects). Then, 2-D nonlinear dynamic analyses are performed on single (lone) buildings and on pairs of colliding buildings. These calculations provide the total input and hysteretic energy and their distribution among the building stories. Such results are compared with those from a more simplified approach (i.e. the individual energy spectra of the considered inputs), and relevant conclusions are derived. Preliminary results show that the comparison between the energies determined with the two abovementioned approaches shows a reasonable fit, and that the influence of SSI proves significant. Regarding pounding, it does not alter significantly the total input and hysteretic energy, but their distribution among the floors changes considerably, concentrating in the colliding floors.

Keywords: Energy-based design, hysteretic energy, seismic pounding, soil-structure interaction
INTRODUCTION

The seismic demand, on a given construction, in form of input and hysteretic energy, is commonly determined after smoothed design spectra; in these spectra, energy is expressed in terms of equivalent velocity. Such spectra are ordinarily derived after dynamic analyses on nonlinear SDOF systems. The application of this approach to actual multi-story buildings is far from trivial, given that their behavior cannot be characterized with SDOF but requiring MDOF models (at least one DOF per floor in symmetric buildings or three in asymmetric buildings); for example, in this context, a relevant issue is the distribution of energy along the building height. In other words, additional research is required to further promote the development of the energy-based design approach.

In the context of seismic energy-based design of buildings, this work addresses two issues that have not been sufficiently covered in past studies: the influence of soil-structure interaction (SSI), and pounding between adjoining buildings. The study begins by selecting four low-rise (3 and 5-story) RC frame buildings, and four representative severe ground motion records (i.e. historic accelerograms). The buildings are designed for high seismicity regions as intermediate moment frames; regarding the inputs, are selected with respect to their most relevant characteristics: frequency content (indirectly represented by the soil type) and relevance of velocity pulses (forward directivity effects). Then, 2-D nonlinear dynamic analyses are performed on two cases: single (lone) buildings and pairs of colliding buildings. These calculations provide the total input and hysteretic energies and their distribution among the building stories. Such results are compared with those from a more simplified approach (i.e. the individual energy spectra of the considered inputs), and relevant conclusions are derived. Preliminary results show that the comparison between the energies determined with the two abovementioned approaches (i.e. using the smoothed spectra of a SDOF system and directly with nonlinear time-history analysis of MDOF systems) shows a reasonable fit, and that the influence of SSI proves significant. Regarding pounding, preliminary results show that it does not alter significantly the total input and hysteretic energy, but their distribution among the floors changes (although not as considerably as might be expected), concentrating in the colliding floors.

This work is a part of a wider research effort oriented to investigate the seismic pounding between pairs of buildings with aligned slabs. This research involves deep discussion on existing pounding models [Kharazian, Lopez-Almansa 2017], proposing new criteria for estimating the damping parameter of the Kelvin-Voigt model [Kharazian 2017; López Almansa, Kharazian 2018], performing a parametric study on the effects of pounding between short-to-mid height RC buildings [Kharazian 2017], and conducting experiments aimed to clarify the most controversial and less studied issues [Kharazian et al. 2018].

SEISMIC DESIGN BASED ON INPUT ENERGY SPECTRA

This section briefs the basic principles of the hysteretic energy spectra, which are commonly considered for energy-based seismic design.

The nonlinear equation of motion of a SDOF system subjected to a horizontal ground motion is given by:

\[ m \ddot{x} + c \dot{x} + Q(x) = -m \ddot{x}_g \]  

In equation (1), \( x \) is the relative displacement, \( m \) is the mass, \( c \) is the viscous damping coefficient, \( Q(x) \) is the restoring force, and \( \ddot{x}_g \) is the driving ground acceleration. Multiplying (1) by \( dx = \dot{x} dt \) and integrating along the input duration, the following energy balance relation is obtained:

\[ E_k + E_c + E_a = E_i \]
In equation (2), $E_k$ is the relative kinetic energy, $E_\zeta$ is the energy dissipated by the inherent damping, $E_a$ is the energy absorbed by the spring, and $E_I$ is the relative input energy:

$$E_k = \int_0^t m \dddot{x} \, dt$$

$$E_\zeta = \int_0^t c \ddot{x}^2 \, dt$$

$$E_a = \int_0^t Q(x) \dot{x} \, dt$$

$$E_I = - \int_0^t m \dddot{x} \dot{x} \, dt$$

(3)

$E_a$ comprises both the recoverable elastic strain energy $E_s$ and the irrecoverable hysteretic energy $E_H$ that generates the structural damage:

$$E_a = E_s + E_H$$

In its turn, the sum of $E_k$ and $E_s$ constitutes the elastic vibration energy, $(E_e = E_k + E_s)$, so that equation (2) can be rewritten as:

$$E_e + E_\zeta + E_H = E_I$$

(4)

The difference between $E_I$ and $E_e$ is denominated [Housner 1956] as the energy that contributes to damage $E_D$:

$$E_D = E_I - E_\zeta = E_e + E_H$$

(5)

At the ground motion end, $E_e$ is almost zero; consequently, equations (4) and (5) show that $E_H$ can be taken as equal to $E_D$, i.e. $E_H \approx E_D$. Further, $E_I$ and $E_D$ can be normalized by the mass $m$ and expressed in terms of equivalent velocities $V_E$ and $V_D$:

$$V_E = \sqrt{\frac{2 E_I}{m}}$$

$$V_D = \sqrt{\frac{2 E_D}{m}} \approx \sqrt{\frac{2 E_H}{m}}$$

(6)

For a given ground motion, the relationship between $V_E$ and the natural period of the system is defined as the energy input spectrum [Akiyama 1985] (Figure 5). Noticeably, the elastic energy input spectra (i.e. obtained by assuming that the structure behaves linearly), are also valid for inelastic systems. This is because the total energy input is scarcely affected by the strength and plastification level of the system, as pointed out in the Introduction.

$V_D$ can be determined by multiplying $V_E$ by the $V_D / V_E$ ratio. Past studies [Akiyama 1985; Kuwamura, Galambos 1989; Kuwamura et al. 1994; Fajfar, Vidic 1994; Manfredi 1995; Lawson, Krawinkler 1995; Teran-Gilmore 1996; Decanini, Mollaioli 2001; Benavent et al. 2002 and 2010] showed that $V_D / V_E$ depends mainly on damping and ductility, and put forth empirical expressions of the ratio $V_D / V_E$ in terms of damping and ductility parameters. Later [Yazgan 2012; López-Almansa et al. 2013] considered as well the influence of the period of the SDOF system, and thus provided expressions depending also of such parameter.

3 REPRESENTATIVE PROTOTYPE BUILDINGS

The main characteristics of the selected prototype buildings are depicted in this section; a more detailed description is available in [Kharazian 2017]. Such buildings are selected to represent the most common situations in developing countries, where pounding is more feasible to occur. The buildings are assumed to be correctly designed for high seismicity regions; conversely, their gap is insufficient. This situation is frequent in developing countries, since the design commonly fulfills all the legal requirements (to obtain the construction permits) but the construction control is not completely strict. The considered buildings have plan symmetric and uniformity along their height. The reason is that most of the actual buildings fulfill such regularity conditions; moreover, irregular situations are difficult to categorize. This regularity implies that the columns are uniformly distributed and are not interrupted (i.e. continuous down to foundation), and that the story height is the same in all the floors. The cooperation of the masonry infill walls is not taken into consideration because of its lack of reliability, and because frequently the walls are separated from the main frame to allow for relative motion. Since in
developing countries most buildings have moderate height, only short to mid-height edifices are considered. Regarding the buildings use, housing and administrative is contemplated.

After the above considerations, four prototype buildings have been selected. The buildings have RC structure with square columns, two-way solid slabs and rectangular cast-in-situ beams joining the columns, see Figure 1.

![Figure 1: Prototype buildings](image)

As shown in Figure 1, the buildings have uniformity in elevation and symmetry in plan, with rectangular plan layout. There are no basements. There are six frames (i.e. five bays) in the direction parallel to the joint between the buildings (y); in the other direction (x) the number of bays of each building ranges in between two and five, to account for the differences in mass between both colliding buildings. The story height is 3.2 m, and the span length is 5 m in both directions. The beams section is 40 cm × 50 cm, and the slabs are 15 cm deep. Inside each story, all the columns are alike, even their reinforcement. Table 1 depicts the column cross section, and the building height and seismic weight (corresponding to $D + 0.2L$).

<table>
<thead>
<tr>
<th>Building</th>
<th>Height (m)</th>
<th>1st floor columns (cm)</th>
<th>2nd floor columns (cm)</th>
<th>3rd floor columns (cm)</th>
<th>4th floor columns (cm)</th>
<th>5th floor columns (cm)</th>
<th>Weight (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-story 5-bay</td>
<td>9.6</td>
<td>60 × 60</td>
<td>55-story 5-bay5</td>
<td>50 × 50</td>
<td>-</td>
<td>-</td>
<td>3709</td>
</tr>
<tr>
<td>3-story 2-bay</td>
<td>9.6</td>
<td>60 × 60</td>
<td>55-story 5-bay5</td>
<td>50 × 50</td>
<td>-</td>
<td>-</td>
<td>1526</td>
</tr>
<tr>
<td>5-story 5-bay</td>
<td>16</td>
<td>60 × 60</td>
<td>55-story 5-bay5</td>
<td>50 × 50</td>
<td>45 × 45</td>
<td>40 × 40</td>
<td>6067</td>
</tr>
<tr>
<td>5-story 2-bay</td>
<td>16</td>
<td>60 × 60</td>
<td>55-story 5-bay5</td>
<td>50 × 50</td>
<td>45 × 45</td>
<td>40 × 40</td>
<td>2486</td>
</tr>
</tbody>
</table>

Table 1. Representative buildings
The seismic design of the buildings is performed for 0.4 g zero period spectral acceleration (PGA), corresponding to 475 years return period (10% probability of being exceeded in 50 years). The seismic design category is D, the structure in an intermediate RC frame, and \( R = 5 \) has been assumed. The foundation consists either in isolated (pad or spread) footings (connected with ties) or a mat (slab).

4 NUMERICAL MODELING

4.1 Numerical model of pounding

The pounding effect is described with concentrated linear Kelvin-Voigt models (parallel combination of a spring and a dashpot) [Anagnostopoulos 1988; Kharazian, Lopez-Almansa 2017; López-Almansa, Kharazian 2018]. Figure 2.a displays such a model; \( m_1 \) and \( m_2 \) represent the lumped colliding masses, \( x_1 \) and \( x_2 \) refer to their displacements, and \( k \) and \( c \) are the stiffness and damping coefficients of the model, respectively. The gap \( d \) represents the initial separation between the frames. In this analysis, the Kelvin-Voigt models are placed in each pounding story; Figure 2.b depicts an example of two 3 and 5-story colliding frames.

\[
\begin{align*}
\begin{array}{c}
\text{(a) Distribution of Kelvin-Voigt models} & \quad \text{(b) Kelvin-Voigt model during impact} \\
\end{array}
\end{align*}
\]

Figure 2: Lumped Kelvin-Voigt models for pounding simulation

The stiffness coefficient \( k \) does not have a deep effect; as suggested in [Muthukumar, Des Roches 2006] it is selected to be higher than the axial stiffness of the longest colliding slab. Conversely, the damping coefficient \( c \) has proven extremely high influence in the impact simulation. It is commonly selected after the coefficient of restitution \( r \), being defined as the ratio between the post-impact and initial relative velocities between the colliding masses:

\[
r = \frac{v'_2 - v'_1}{v_1 - v_2}
\]

In equation (7), \( v_1 \) and \( v_2 \) are the initial velocities of the left and right slabs, respectively, and \( v'_1 \) and \( v'_2 \) are the corresponding after-impact velocities. Equation (7) shows that \( r \) ranges between 0 (plastic impact) and 1 (elastic impact). The physical meaning of the coefficient of restitution is more obvious than the one of the damping parameter; therefore, commonly it is preferred to start the process by selecting the value of \( r \). In this study, two approaches are considered, the traditional formulation of [Anagnostopoulos 2004] and a more recent one [Kharazian 2017; López-Almansa, Kharazian 2018]. The traditional approach is based on neglecting,
during impact, the influence of the structures of the colliding buildings and of the seismic excitation. The following inverse relations between $r$ and the damping ratio $\zeta$ of the Kelvin-Voigt model are obtained

$$\zeta = -\ln r / (\pi^2 + \ln^2 r)^{1/2}$$

$$r = e^{(1-\zeta^2)^{1/2}}$$

The expression linking $\zeta$ and $c$ is $\zeta = c / 2 \sqrt{m_1 + m_2 \over k m_1 m_2}$. This formulation has been used by many researches, showing repeatedly its efficiency. The works [Kharazian 2017; López-Almansa, Kharazian 2018] propose another approach that releases some of the assumptions in the traditional one; in that case, the relation between $r$ and $\zeta$ is given by a simple algorithm.

4.2 Soil-Structure interaction

Soil-Structure Interaction (SSI) is relevant to this study, mainly given its significant influence on the pounding effects [Kharazian, Lopez-Almansa 2017; Kharazian 2017]. Next paragraphs describe the considered models of interaction for spread footings and mat foundations, respectively. In both cases, SSI is represented by linear models, and two soil types are considered, namely B and C [EN-1998 2005].

SSI for isolated foundation is represented by an uncoupled spring model [FEMA 356 2000] consisting of elastic springs and dashpots linking the foundation (pad) footings and the underlying soil. In this context, “uncoupled” refers to the lack of relation between the stiffness and the damping of the springs and the dashpots that correspond to different degrees of freedom (directions). The foundation members are infinitely rigid compared to the soil. Given that this study is 2-D, each spring has horizontal, vertical and rotational (rocking) stiffness coefficients [Kharazian 2017].

In the mat foundation, SSI is represented by a coupled spring model [Harden 2003]; “coupled” refers to indirect consideration of rotational stiffness by increasing the corresponding parameter of the springs located at slab ends. The foundation slab stiffness is infinitely higher than the soil one (rigid mat). The equivalent width of the foundation slab that actually cooperates with the frame is determined according to [ACI-318-11 2011] as 1.5 $h$ at both column sides, where $h$ is the slab depth. The horizontal and vertical stiffness coefficients are the same than for isolated foundation; the rotational (rocking) stiffness coefficient is given by another expression [Kharazian 2017].

4.3 Modal analysis

Linear modal analyses have been conducted. Two types of analyses are performed: in the first case, the bending stiffness of beams and columns refers to the moments of inertia of the gross section, and in the second case, such stiffness are reduced to account for the cracking effect [FEMA 356 2000]; the reducing factors are 0.7 and 0.5 for columns and beams, respectively. Table 2 displays the fundamental periods of each prototype building in three types of situations: by neglecting the SSI, and by considering SSI with isolated and mat foundation,
respectively. When SSI is taken into consideration, b and c refer to stiff and soft soil type, respectively [EN-1998 2005].

<table>
<thead>
<tr>
<th>Building</th>
<th>SSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NO</td>
</tr>
<tr>
<td>3-story 5-bay</td>
<td>0.293 / 0.374</td>
</tr>
<tr>
<td>3-story 2-bay</td>
<td>0.280 / 0.359</td>
</tr>
<tr>
<td>5-story 5-bay</td>
<td>0.524 / 0.667</td>
</tr>
<tr>
<td>5-story 2-bay</td>
<td>0.501 / 0.642</td>
</tr>
</tbody>
</table>

Table 2: Fundamental periods (s) of the representative buildings without / with cracking

The figures in Table 2 show that the simplified consideration of cracking generates a significant elongation of the fundamental period. Table 2 shows also that the consideration of the SSI slightly elongates the fundamental period of the buildings; as expected, the softer the soil, the higher the lengthening. Comparison between isolated and mat foundation reveals negligible or no influence.

5 SEISMIC INPUTS CONSIDERED IN THE STUDY

5.1 Selection criteria

This study is oriented to provide general remarks on buildings undergoing strong ground motions; therefore, the inputs should be selected to represent the actual conditions in any type of high seismicity regions. The inputs are selected based on two issues: (i) the forward-directivity (near-fault) effects, and (ii) the soil type. The near-fault effects are relevant, given the presence of velocity pulses; they are important, since the sudden delivery of input energy in a short time interval magnifies the input damaging potential [Zhai et al. 2013]. Regarding the soil type, it has a direct effect on the frequency content of the accelerogram.

5.2 Selected inputs

Given the considerations in the previous paragraph, four representative inputs are selected; are obtained combining the presence or absence of velocity pulses, and stiff and soft soil conditions. Table 3 depicts the most relevant characteristics of the four chosen inputs [PEER 2017]. $I_A$ is the Arias Intensity [Arias 1970] given by $I_A = \frac{\pi}{2 \ g} \int \ddot{x}_g^2 \ dt$ where $\ddot{x}_g$ is the input ground acceleration (equation (1)); $I_A$ is an estimator of the input severity. $PI$ and $E_p$ quantify the relevance of the velocity pulses. $PI$ is the pulse index [Baker 2007], ranging between 0 and 1; records scoring above 0.85 and below 0.15 are classified as pulses and non-pulses, respectively. $E_p$ is the relative pulse energy [Zhai et al. 2013], representing the portion of the total energy of the ground motion that corresponds to the pulse; the pulse is extracted by the peak-point method [Dickinson, Gavin 2010]. Values of $E_p$ greater than 0.3 correspond to pulse-like records and values equal to or below 0.3 are ambiguous. The Trifunac duration is defined as the time between the 5% and the 95% of the Arias Intensity [Trifunac, Brady 1975]. The hypocentral distance corresponds to the straight separation between the hypocenter and the recording station. The closest distance corresponds to the shortest way to the rupture surface [PEER 2017]. $v_{s,30}$ is the weighted harmonic average shear wave velocity in the top 30 m. The soil type corresponds to the classification of the Eurocode 8 [EN-1998 2005].
Table 3: Selected input records

<table>
<thead>
<tr>
<th>Earthquake / Year / ( M_w )</th>
<th>Hypo-center depth (km)</th>
<th>Station Comp.</th>
<th>PGA (g)</th>
<th>( I_A ) (m/s)</th>
<th>( PI )</th>
<th>Trifunac duration (s)</th>
<th>Hypo-central / closest distance (km)</th>
<th>Soil type (EC8) ( / v_s,30 ) (m/s)</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northridge / 1994 / 6.7</td>
<td>17.5</td>
<td>Sylmar - Olive View</td>
<td>CDMG24514</td>
<td>0.84</td>
<td>5.01</td>
<td>1.0</td>
<td>0.61</td>
<td>5.32</td>
<td>24.24 / 5.3</td>
</tr>
<tr>
<td>Northridge / 1994 / 6.7</td>
<td>17.5</td>
<td>Newhall - W Pico</td>
<td>UCS90056</td>
<td>0.42</td>
<td>1.54</td>
<td>1.0</td>
<td>0.78</td>
<td>6.25</td>
<td>27.76 / 5.48</td>
</tr>
<tr>
<td>Victoria / 1996 / 6.3</td>
<td>11</td>
<td>Cerro Prieto</td>
<td>UCSD6604</td>
<td>0.64</td>
<td>1.99</td>
<td>0.0062</td>
<td>0.17</td>
<td>8.25</td>
<td>35.48 / 14.37</td>
</tr>
<tr>
<td>Northridge / 1994 / 6.7</td>
<td>17.5</td>
<td>Saticoy St</td>
<td>USC90003</td>
<td>0.46</td>
<td>4.63</td>
<td>0.0104</td>
<td>0.21</td>
<td>10.62</td>
<td>17.83 / 12.09</td>
</tr>
</tbody>
</table>

Table 3 shows that the first two inputs are pulse-like, and that the last two ones are not. Next in this paper, the four inputs in Table 3 are termed P B, P C, NP B and NP C, respectively; in this notation, “P” and “NP” account for Pulse and Non-Pulse, respectively, and “B” and “C” refer to soil type.

To highlight the major characteristics of the four selected ground motion records, Figure 3 displays their time histories.

![Figure 3: Considered input accelerograms](image)

The accelerograms displayed in Figure 3 confirm that those in the top plots are pulse-like, while those in the bottom plots are not. As well, the left plots exhibit greater high-frequency contents than those in the right plots, this being coherent with the soil type. Deeper discussions can be found in [Kharazian 2017].

### 5.3 Response spectra of the selected inputs

To point out the period-dependent characteristics of the selected accelerograms, their absolute acceleration and input energy response spectra are displayed in Figure 4 and Figure 5,
respectively. The acceleration spectra are determined using the software SeismoSignal [SeismoSoft 2017]. The input energy spectra are computed after equation (3); then, such energy is expressed in terms of equivalent velocity according to equation (6). In equation (3), the structural behavior is assumed to be linear; noticeably, according to the Housner-Akiyama theory [Akiyama 1985], the obtained results are also somehow valid for nonlinear behavior, given that the input energy is roughly independent on the constitutive law. The plots in Figure 5 correspond to damping 5%.

Figure 4: Absolute acceleration response spectra of the considered inputs

Figure 5: Input energy response spectra of the considered inputs
The spectra in Figure 4 confirm the considerations stated after the accelerograms in Figure 3. Comparison among the spectra in Figure 4 and Figure 5 show that the spectra corresponding to the same input exhibit peaks for the same periods, although those of energy spectra are higher for longer periods; this similarity among peaks is coherent with the broad likeness among energy and velocity spectra [Akiyama 1985].

As discussed in section 2, the $V_D$ spectrum is commonly obtained by multiplying the $V_E$ spectrum by a convenient value of the $V_D / V_E$ ratio; such ratio depends mainly on the damping factor $\zeta$, the displacement ductility $\mu$ (i.e. the ratio between the maximum and yield displacements), and the building fundamental period $T_F$. References [Yazgan 2012; López-Almansa et al. 2013] contain linear regression studies providing average expressions $V_D / V_E = b - a T_F$ where coefficients $a$ and $b$ depend on $\zeta$ and $\mu$. In this study, it is assumed that $\zeta = 0.05$; then, the available values of $a$ and $b$ in terms of $\mu$ are listed next. For $\mu = 2$, $a = 0.042$, $b = 0.67$, for $\mu = 3$, $a = 0.045$, $b = 0.77$, for $\mu = 5$, $a = 0.049$, $b = 0.84$, for $\mu = 10$, $a = 0.054$, $b = 0.88$, for $\mu = 15$, $a = 0.055$, $b = 0.88$, for $\mu = 20$, $a = 0.052$, $b = 0.87$. For $\mu = 1$, obviously $V_D / V_E = 0$, i.e. $a = 0$, $b = 0$. For values of $\mu \geq 2$, linear interpolation provide enough accuracy. Conversely, for values of $\mu$ between 1 and 2, linear interpolation would deliver too small $V_D / V_E$ ratios; alternative interpolation criteria [Yazgan 2012, López-Almansa et al. 2013] are suggested instead.

Table 4 displays values of the $V_D / V_E$ ratio for the four selected inputs (Table 3) and the four chosen buildings (Table 1). In Table 4, the yield displacements are obtained after the capacity curves) derived from pushover analyses. Obviously, if $\mu < 1$, the hysteretic energy is zero and no values of $a$ and $b$ are provided. Noticeably, in Table 4 only soil C is considered in the cases with SSI, given that for stiff soil (B) the effect of SSI is less relevant and no dynamic analyses have been performed (Section 6).

<table>
<thead>
<tr>
<th>Building</th>
<th>Input</th>
<th>NO SSI</th>
<th>SSI FOR ISOLATED FOUNDATION</th>
<th>SSI FOR MAT FOUNDATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$</td>
<td>$a$</td>
<td>$b$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-story 5-bay</td>
<td></td>
<td>1.612</td>
<td>0.041</td>
<td>0.631</td>
</tr>
<tr>
<td>3-story 5-bay</td>
<td></td>
<td>0.127</td>
<td>0.042</td>
<td>0.624</td>
</tr>
<tr>
<td>3-story 5-bay</td>
<td></td>
<td>0.555</td>
<td>0.047</td>
<td>0.580</td>
</tr>
<tr>
<td>3-story 5-bay</td>
<td></td>
<td>0.394</td>
<td>0.047</td>
<td>0.585</td>
</tr>
<tr>
<td>3-story 5-bay</td>
<td></td>
<td>0.436</td>
<td>0.047</td>
<td>0.585</td>
</tr>
<tr>
<td>3-story 5-bay</td>
<td></td>
<td>1.193</td>
<td>0.040</td>
<td>0.589</td>
</tr>
<tr>
<td>5-story 5-bay</td>
<td></td>
<td>1.273</td>
<td>0.041</td>
<td>0.634</td>
</tr>
<tr>
<td>5-story 5-bay</td>
<td></td>
<td>1.034</td>
<td>0.040</td>
<td>0.597</td>
</tr>
<tr>
<td>5-story 5-bay</td>
<td></td>
<td>0.631</td>
<td>0.040</td>
<td>0.597</td>
</tr>
</tbody>
</table>

Table 4: $V_D / V_E$ ratio in terms of displacement ductility and period for the considered buildings

The $V_D / V_E$ ratio is little influenced by the approximate consideration of cracking in the determination of the building fundamental period; this is expected, given the little influence of the period in the $V_D / V_E$ ratio. Thus Table 4 show the average between the ratios for the cracked and uncracked periods.
6 DYNAMIC ANALYSIS OF THE LONE AND COLIDING PROTOTYPE BUILDINGS

This section describes the nonlinear dynamic response of the prototype buildings in section 3 to the accelerograms in section 5; the analyses employ the models in section 4. Two types of analyses are conducted: single (lone) buildings and pairs of colliding buildings. In this last case, four pairs are contemplated: 3-story 5-bay vs. 5-story 2-bay, 5-story 5-bay vs. 5-story 2-bay, 5-story 5-bay vs. 3-story 2-bay and 5-story 2-bay vs. 3-story 2-bay; noticeably, the pounding among two alike buildings is not analyzed, because collision is not feasible to occur. Four each building or pair of buildings, three situations regarding soil-structure interaction are analyzed: neglecting such interaction (no SSI), and interaction for isolated (ISOL.) and mat foundation (MAT). In the cases with SSI (ISOL. and MAT), only the inputs for soft soil (P C and NP C, see Table 3) are considered, given that, for stiff soil (P B and NP B), the effect of SSI is rather negligible [Kharazian 2017]. Briefly, 32 and 32 analyses are performed for the lone buildings and the pairs of buildings, respectively.

The time integration is carried out using the Newmark algorithm with $\gamma = 0.5$ and $\beta = 0.25$ (constant acceleration interpolation). Inside each time step, the maximum number of iterations is 300 and the convergence is verified based on displacement and rotation; the displacement and rotation tolerances are $10^{-4}$ m and $10^{-5}$ rad, respectively. The damping matrix is generated, with a classical Rayleigh model, as proportional to the tangent stiffness [Petrini et al. 2008]; the assumed damping ratio is 5%.

In any nonlinear analysis, time step selection is a key issue, since the usual criteria for linear analysis do not apply, and instabilities might arise; noticeably, this might happen even in the supposedly unconditionally stable Newmark algorithm. In pounding analysis, this subject is still more crucial, since pounding generates sudden changes in extremely short time intervals, thus leading to important accelerations and involving higher-mode response. In this research, the time increment is selected starting with a coarse time discretization ($\Delta t = 0.01$ s) and then refining it progressively until obtaining similar results regardless of the considered sampling period. Satisfactory performance has been achieved with $\Delta t = 0.0005$ s [Kharazian 2017].

7 RESULTS FOR ENERGY

This section presents and discusses the results of input ($E_I$) and hysteretic ($E_H$) energy that constitute the main output of this paper. Two approaches are employed: (i) $E_I$ and $E_H$ are obtained from the dynamic analyses described in section 6, and (ii) $E_I$ is determined from the energy spectra in Figure 5, and then $E_H$ is obtained according to the formulation described in the paragraph before Table 4 (Section 5).

In the first approach, the input energy is determined after equation (3). However, in this work that equation cannot be applied directly, because it corresponds to a SDOF system; given that the dynamic behavior of the multi-story analyzed buildings can be broadly described with MDOF lumped masses models, the input energy for the whole building can be obtained as the sum of the contribution of each story:

$$E_I = - \int_0^t \dot{x}^T \mathbf{M} \dot{\mathbf{r}}_g \, dt = - \int_0^t \sum_{i=1}^N \dot{x}_i \, m_i \ddot{x}_g \, dt = - \sum_{i=1}^N \int_0^t \dot{x}_i \, m_i \ddot{x}_g \, dt \tag{9}$$

In this expression, $\dot{x}_i$ and $m_i$ are the relative velocity and the mass of each story, respectively, $\ddot{x}_g$ is the input (driving) ground acceleration (equation (1)), and $N$ is the number of floors of the building under consideration; the relative velocity $\dot{x}_i$ is obtained after the conducted nonlinear analyses (section 6). Analogously, the hysteretic energy is calculated as the sum of the energy...
that is dissipated at each story (i.e. area encompassed by the shear force-drift displacement hysteresis loops):

\[ E_H = \int_0^t \sum_{i=1}^N V_i \, d\delta_i = \sum_{i=1}^N \int_0^t V_i \, d\delta_i \] (10)

In this expression, \( V_i \) and \( \delta_i \) are the story shear force and the drift displacement, respectively.

The first approach (equations (9) and (10)) is allegedly more accurate, since the second one involves several relevant simplifications: (i) representation of the buildings as SDOF systems, (ii) lack of consideration of the building fundamental period elongation due to its nonlinear behavior, (iii) representation of the building nonlinear behavior with an elastic-perfectly plastic model, and (iv) approximate determination of the \( V_D / V_E \) ratios (Table 4).

One of the main issues in energy-based design of multistory buildings is the distribution among stories of the total hysteretic energy. Regarding this question, Figure 6 displays a comparison between the input and hysteretic energy for each story for a pair of colliding and lone buildings, respectively. These energies have been obtained with the 1st approach (integration).

According to this figure, the distribution of hysteretic energy among stories is scarcely affected by pounding. This conclusion should be taken as tentative, given the limited number of buildings and ground motions considered in this study.

8 CONCLUSIONS

This paper discusses the influence of SSI (Soil-Structure Interaction) and pounding between adjoining buildings in their input and hysteretic energies; this study is conducted on two pairs of low-rise (3 and 5-story) RC colliding buildings. Such energies are determined by direct integration of the dynamic response (accurate formulation) and after individual energy spectra of the seismic inputs (simplified formulation); the agreement between both strategies is only reasonable. Preliminary results indicate that the input energy shows high scattering, probably due to difficulty in estimating the equivalent nonlinear fundamental period. The distribution among stories of the total input and hysteretic energy is being investigated.

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REFERENCES

Abstract

In energy-based seismic design approach, effect of ground motions on structures is considered as an energy input to structures. The earthquake input energy spectra are created combining the maximum input energies of single-degree-of-freedom (SDOF) systems having a certain damping ratio for different natural vibration periods. The determination of input energy spectra is of great importance for the energy-based seismic design since the total energy input to structural systems can be practically obtained via these graphs. This study presents the investigation of elastic input energy spectra for selected actual earthquake ground motions at stiff soil sites. Accelerogram set is selected from Pacific Earthquake Engineering Research (PEER) database for the specific range of average shear wave velocities in the top thirty meters of soils. Time history analyses are conducted for linear elastic SDOF systems having viscous damping ratio of 5% and energy-time histories are computed. Then the elastic input energy spectra for selected actual earthquake ground motions are obtained. The mean of energy spectra is investigated together with the mean plus one and two standard deviations of the energy spectra. The aim of the present study is to evaluate the earthquake input energy demand spectra of SDOF systems for stiff soil site classes. The results show that the elastic design input energy spectrum can be proposed for selected ground motions at stiff soil sites.

Keywords: Single-Degree-of-Freedom System, Input Energy Spectra, Energy-Based Seismic Design, Earthquake Ground Motion, Soil Site Class.
1 INTRODUCTION

Earthquake design acceleration spectra are directly used in conventional force-based seismic design and analysis methods to determine the earthquake demand. These spectra are generated to design structures to resist earthquakes considering past seismic activities of a region and they are created for different site conditions, damping ratios, safety levels and earthquake magnitude. Seismic design codes generally define strong ground motion in the form of a response spectrum of acceleration. Equivalent Static Lateral Force method which takes part in many seismic codes such as UBC-97, Eurocode 8, NBCC (2005), IBC (2006), TSDC (2007), ASCE/SEI 7-10 and TBEC (2018) [1-7] directly uses the design acceleration spectra. Response Spectrum Analysis procedure which is a linear-dynamic statistical method computes the maximum seismic response of structures directly considering the design acceleration spectra as in the Equivalent Static Lateral Force method. By brief explanation, the design acceleration spectra are significant graphs which are needed for force-based structural design and therefore they take part in many seismic design codes as standardized graphs [1-7].

In displacement-based seismic design approaches, the structural designer can dimension the structure with required strength and ductility according to a target displacement. The primary design quantity is the target displacement and the inelastic behavior of structures are directly considered in contrast with the force-based procedures [8, 9]. Direct displacement-based design was first introduced by Priestley [10] and after then it received great deal of attention by countries such as Europe, North America and New Zealand [8, 11, 12]. The fundamental philosophy of this approach is that the structures are designed by using a specified performance level under a specific seismic intensity level. Accordingly, the direct displacement-based design approach is based on the performance-based seismic design which considers direct and indirect displacement of structures to gain the performance objectives [13]. The design displacement response spectra are constructed in direct displacement-based seismic design and then structures are dimensioned to give an effective period. Finally, the total base shear force is determined and it is distributed along the height of the structure [9, 13]. In the course of time, alternative performance-based procedures which use the acceleration response spectra are comprised [9, 14]. The Capacity Spectrum approach, a performance-based seismic analysis technique, establishes the point of structural capacity-earthquake demand balance [14]. Nonlinear static pushover analyses are performed for structures, the pushover (base shear-roof displacement capacity) curves are obtained and they are converted to spectral acceleration and spectral displacement form (ADRS curve). The nonlinear pushover curves become capacity spectra with this conversion process. Then the earthquake demand is represented by acceleration response spectrum and it is reduced with an increasing damping. The intersection of the ADRS curve and the reduced demand spectrum gives the performance point of the structure [15, 16, 17]. Consequently, the design acceleration spectra are directly used to define the demand of earthquake in performance-based seismic design procedures. It can be concluded that the earthquake effect is characterized as a function of the design acceleration spectra both in the force-based and displacement-based (nonlinear static) procedures. In dynamic time history analyses, the design acceleration spectra are indirectly considered [18]. Each ground motion record has different response spectrum depending upon their origin type of earthquakes and local site conditions and target response spectrum is designed in seismic codes considering the large number of ground motions. Therefore, the scaling procedure is applied to ground motions considering the target spectrum in time history analyses and design acceleration spectra have been indirectly used.
Energy concept in seismic design of structures has been widely studied over a half-century period and energy-based methods have always been considered more rational and reliable for the design and assessment of structures under seismic effects when compared to conventional force-based and displacement-based methods [19-21]. In the energy-based methods earthquake effect is considered as an energy input to structures and this energy input expresses the total energy demand of the earthquake. Making a structure safe is considered as a balance of energy dissipation capacity and earthquake energy demand in these approaches. However, an important question of the energy-based seismic design is to determine the energy input to structures with earthquake motion. There have been many previous researches on literature about the energy-based seismic design of structures and seismic input energy. Energy-based earthquake resistant design was first proposed by Housner [22]. Housner studied the seismic energy input to structures using the velocity spectra of elastic systems [22]. Energy-based design parameters were first defined in his researches and these formed a basis for earthquake resistant energy-based design. Some other researchers also made many previous estimations about the input energy concept and they considered the input energy as an effective tool in earthquake-resistant design [19, 23, 24]. Zahrah and Hall [25], Akiyama [26], Kuwamura and Galambos [27], Fajfar et al [28], Uang and Bertero [19] and Manfredi [24] made pioneer studies like Housner [22] about seismic energy concepts and they proposed useful analytical and empirical equations for the seismic input energy. Energy approaches in performance-based seismic design have been studied for nearly two decades by many researchers [29-34]. Practical design procedure based on conventional plastic design concept was proposed by Leelataviwat et al. [32] and the energy-based design base shear was derived considering a predefined yield mechanism. A parametric study to determine the acceptability criteria in structural response parameters corresponding to selected performance levels for specified levels of ground motions was conducted by Akbas and Shen [30]. An energy method for earthquake-resistant design of RC structures was presented in Terapathana’s thesis [35]. Nonlinear time history analyses were performed on frame type structures to obtain the required hysteretic energy demands used for the design [35]. Enderami et al. [36] presented a new energy-based approach for predicting seismic demands of steel structures at the near-fault sites. Seismic demands of steel frames were researched by using the concept of dissipated hysteretic energy in their study [36].

The usage of energy spectra is an effective tool in energy-based seismic design methods, such as the use of design acceleration spectra in force-based and displacement-based methods. The obtention of input energy spectra offers an important advantage to determine the energy input to structures with the effect of ground motions. Peak ground acceleration, peak ground velocity, the ratio of peak ground acceleration to peak ground velocity, the duration and predominant periods of ground motions, fault type and mechanism, distance to the fault, the magnitude of ground motions and soil conditions were referred in many previous studies as specific parameters to obtain the seismic input energy spectra [19, 25, 28, 37-44]. Fajfar and Vidić [37], Sucuoglu and Nurtug [38], Decanini and Mollaioli [39], Manfredi [24], Benavent-Climent et al. [40], López-Almansa et al. [41], Dindar et al. [42], Quinde et al. [43], Alici and Sucuoglu [44] and Özsarac et al. [45] made pioneer and further investigations about the earthquake input energy spectra for both elastic and inelastic systems.

This study presents the development of elastic design input energy spectra using 100 real ground motion records having strike-slip focal mechanism. Accelerograms were obtained from Pacific Earthquake Engineering Research (PEER) database [46]. Selected near-fault and far-fault ground motions are on site class D according to National Earthquake Hazards Reduction Program (NEHRP; Building Seismic Safety Council [47]) site classification. The average shear-wave velocities in the top thirty meters of the soil ($V_{S30}$) were selected between...
the value of 180 m/s and 360 m/s (For Site Class D: Stiff soil with 180 m/s < $V_{S30}$ ≤ 360 m/s [47]). Time history analyses were conducted using selected accelerograms for SDOF systems having damping ratio of 5% and the energy time histories were obtained for different natural vibration periods. The elastic earthquake input energy spectra were created using the data of energy time histories. A computer algorithm was generated by the author to speed up the calculation process and it was used while creating the input energy spectra graphs. The mean elastic input energy spectrum for selected ground motions on the same site class was obtained within the study. The earthquake input energy spectrum graph was also investigated for the mean plus one and two standard deviations of the energy spectra. Then the elastic design input energy spectrum was proposed for selected ground motions and the spectral shape of the design spectrum was obtained. Three main regions of the design spectrum were characterized by equations. The effect of the average shear-wave velocity in the top thirty meters of the soil ($V_{S30}$) and the effect of the Joyner-Boore distance ($R_{JB}$) on the elastic design input energy spectrum were also investigated. The variation of the proposed design input energy spectra was obtained for four ranges of $V_{S30}$ (for ranges of 190 m/s < $V_{S30}$ ≤ 235 m/s, 235 m/s < $V_{S30}$ ≤ 280 m/s, 280 m/s < $V_{S30}$ ≤ 325 m/s and 325 m/s < $V_{S30}$ ≤ 370 m/s, respectively) and for four ranges of $R_{JB}$ (for ranges of 0 km < $R_{JB}$ ≤ 46 km, 46 km < $R_{JB}$ ≤ 92 km, 92 km < $R_{JB}$ ≤ 138 km and 138 km < $R_{JB}$ ≤ 184 km, respectively). A comprehensive evaluation was performed in terms of the smoothed mass-normalized earthquake input energy spectra for real ground motions on the stiff soil site. The elastic design input energy spectra were developed for selected accelerograms and the effect of the site class (or $V_{S30}$ velocity) and the effect of the shortest distance from a site to a rupture surface (or $R_{JB}$) on the design input energy spectra were investigated.

2 ENERGY EQUATION AND BASIC CONCEPT

The energy-balance equation for a SDOF system can be derived by integrating the equation of motion with respect to the relative displacement of the system. A fixed-based SDOF system subjected to a strong ground motion is illustrated in Figure 1. The energy-balance equation of the SDOF system under the effect of strong ground motion can be written as (Uang and Bertero [19]):

$$\int_0^{u(t)} m \cdot \ddot{u} \, du + \int_0^{u(t)} c \cdot \dot{u} \, du + \int_0^{u(t)} f_s(u) \, du = - \int_0^{u(t)} m \cdot \ddot{u}_g(t) \, du$$

(1)

where $u$ is the relative displacement of the SDOF system with respect to the ground, $\dot{u}$ is the relative velocity, $\ddot{u}$ is the relative acceleration, $m$ is the total mass, $c$ is the damping coefficient, $f_s(u)$ is the resisting force and $f_g(u)$ is the strong ground acceleration. In Figure 1, $u_t$ shows the total displacement of the structure under the effect of strong ground motion and it is equal to the sum of relative displacement and displacement of the ground ($u_t = u + u_g$). $k$ is the stiffness of the SDOF system and the resisting force $f_s(u)$ is equal to $k \cdot u$ for linear elastic systems. Equation (1) may be restated in general types of energy components as below:

$$E_K + E_\xi + E_S = E_I$$

(2)

On the left-hand side of Equation (1) the first integral term is the kinetic energy, the second integral term is the damping energy and the last integral term represents the total absorbed energy. In Equation (2), in which the symbolic expressions of the energy-balance equation are involved, these integrals are written as in the form of representative energy terms. $E_K$ represents the relative kinetic energy, $E_\xi$ represents the damping energy, $E_S$ represents the total absorbed energy and $E_I$ stands for the total input energy.
The energy $E_S$ is composed of two parts as elastic strain energy ($E_{Se}$), and hysteretric (irrecoverable plastic) energy ($E_H$) which is the main energy component in nonlinear behavior. Equation (2) may be rewritten as:

$$E_K + E_\xi + (E_{Se} + E_H) = E_I$$

(3)

The energy-balance equation for a SDOF structure based on relative motion can be rewritten in terms of time integral writing a derivative equality for $du$ ($du = \dot{u} dt$) as ([19]):

$$\int_0^t m \cdot \ddot{u} \cdot \dot{u} dt + \int_0^t c \cdot \dot{u} \cdot \dot{u} dt + \int_0^t f_s(u) \cdot \dot{u} dt = -\int_0^t m \cdot \ddot{u}_s(t) \cdot \dot{u} dt$$

(4)

The energy input to SDOF structures can be computed by using the right-hand side term of Equation (4). This is the total input energy of SDOF systems under the effect of strong ground motion. The energy $E_I$ can be rewritten independent of the mass (per unit mass) as:

$$\frac{E_I}{m} = -\int_0^t \ddot{u}_s(t) \cdot \dot{u} dt$$

(5)

The representative energy-time history graph of an inelastic SDOF structure under a strong ground motion effect is given in Figure 2. The sum of the elastic strain energy ($E_{Se}$), kinetic energy ($E_K$), damping energy ($E_\xi$) and hysteretric energy ($E_H$) is equal to the total input energy ($E_I$) as can be seen from the figure.

Figure 2: The representative energy-time history graph of an inelastic SDOF system.
The substantial components of the earthquake input energy are the damping energy \( E_{\delta} \), and the hysteretic energy \( E_{H} \) which contributes the most to structural damage. The input energy \( E_{I} \) generally tends to be constant towards the end of the earthquake duration.

## 3 INPUT ENERGY SPECTRA

In the field of energy-based earthquake-resistant design of structures, the energy demand of an earthquake should be less than (or, in limit, should be equal to) the energy dissipation capacities possessed by the structure (Decanini and Mollaioli [39]). It is of utmost importance for structural and earthquake engineers that the seismic input energy demand transmitted to structures is computed exactly. However, this process involves detailed dynamic analyses and therefore many researchers have sought to obtain practical ways for computation of the input energy. In literature, many approximate formulas have been proposed for obtaining the maximum seismic input energy [19, 22, 24-28]. Moreover, the seismic input energy spectra have been created for a specific strong ground motion and the input energy values of both elastic and inelastic systems have been obtained practically via these graphs [24, 37-45]. In brief definition, the seismic input energy spectra are the graphs which combine the maximum input energy values corresponds to different natural vibration periods of SDOF systems. There are many factors (such as earthquake magnitude, closest distance from the seismic source, soil type, focal mechanism, damping ratio, type of hysteresis for the inelastic spectra, and etc.) which have an impact on the seismic input energy spectra [39, 42, 43].

![Flowchart to obtain the input energy spectra graphs.](image-url)
The flowchart to obtain the input energy spectra is presented in Figure 3. The procedure begins with the selection of an accelerogram and continues with the calculation of velocity-time history (\( \ddot{u}(t) \)) relation for SDOF structure. After computing the energy input-time history graph in per unit mass, the maximum input energy value \((\frac{E}{m})_{\text{max}} \) can be readily determined for the constant period of structure. When adequate number of \((\frac{E}{m})_{\text{max}} - T \) points are obtained, the combination of these values in a diagram gives the input energy spectra graph. If the SDOF structure is linearly elastic the input energy spectra may be called as the elastic input energy spectra, and if the nonlinear behavior of the structure is taken into consideration in dynamic analyses the input energy spectra may be called as the inelastic input energy spectra.

4 STRONG GROUND MOTION RECORDS

100 real ground motion records having strike-slip focal mechanism have been selected within the study. The average shear-wave velocities in the top thirty meters of the soil (\( V_{S30} \)) have been selected between the value of 180 m/s and 360 m/s. Soil class is D (stiff soil with 180 m/s < \( V_{S30} \) ≤ 360 m/s) according to National Earthquake Hazards Reduction (NEHRP) Program site classification [47]. Moment magnitudes (\( M_w \)) of earthquakes are between 6.0 and 7.5. Joyner-Boore distances (\( R_{JB} \)) are between 0 km and 175 km and values of closest distance to rupture surface (\( R_{\text{rupture}} \)) are between 0.34 km and 175 km. The peak ground accelerations (PGA) are between the values of 0.011g and 0.777g. All of the selected accelerograms in the study are obtained from the database of Pacific Earthquake Engineering Research Center, PEER [46].

The selection of accelerograms is an important issue among structural and earthquake engineers to assess the structural performance. Real earthquake records may be selected in accordance with the specific characteristics of the ground motion [48, 49]. The selection is generally executed considering an elastic response spectrum, an earthquake scenario or seismological parameters etc. Many seismic codes present criteria compatible with response spectrum for the appropriate ground motion selection. Magnitude of earthquake is another important parameter for the ground motion selection because it influences the frequency content of the motion, so the appropriate magnitudes should be selected in an earthquake data set [48]. Strong ground motion parameters such as peak ground acceleration (PGA), peak ground velocity (PGV) and peak ground displacement (PGD) are the other parameters that should be taken into consideration in the selection of accelerogram with fault-site distances and soil site conditions [48-50]. If the selected accelerograms will be used to perform dynamic time history analyses, they should be scaled according to the suitable design acceleration spectrum using an appropriate scaling method (to match the spectra) [48-50].

In this study, the elastic earthquake input energy spectra for selected real accelerograms have been investigated and the elastic design input energy spectra has been proposed. The effect of the average shear-wave velocity in the top thirty meters of the soil (\( V_{S30} \)) and the effect of the Joyner-Boore distance (\( R_{JB} \)) on the design input energy spectra have been studied. The primary aim of the study is only to propose the elastic design input energy spectra for real and unscaled ground motions on the specific soil site. The analyses were carried out by using unscaled ground motion data, because the proposed energy spectra would not be used in any assessment of structural performance (or analysis). In the selected accelerogram set, there are some records having the source-to-site distance less than 15 km (\( R_{\text{rupture}} < 15 \text{ km} \)) but most part of the set have the source-to-site distance value greater than 40 km (\( R_{\text{rupture}} < 40 \text{ km: far-fault earthquake} \)). In other words, the near fault earthquakes were not sorted out from the set, because the aim is not to obtain the near fault effects to the energy spectra but the aim is only
to investigate the effect of $V_{S30}$ (soil class) and $R_{sb}$. The effects of near faults on elastic or inelastic energy spectra can be investigated in another studies.

Selected earthquake records are given in Table 1, Table 2 and Table 3, in three parts. Earthquakes between the number of 1 and 34 are presented in Table 1, earthquakes 35-67 are presented in Table 2 and earthquakes 68-100 are presented in Table 3. $I_A$ is the Arias Intensity, as defined by Arias [51], and it is proportional to the square of the ground acceleration integrated over time. Arias Intensity may be written as:

$$I_A = \frac{\pi}{2g} \int_0^t a^2(t) \, dt$$

(6)

where $a(t)$ is the strong ground acceleration, $t_A$ is the total earthquake duration, $g$ is the acceleration of gravity and $I_A$ is the Arias Intensity in m/s unit.

<table>
<thead>
<tr>
<th>Event Name (*1)</th>
<th>Station</th>
<th>Year</th>
<th>$M_o$</th>
<th>$I_A$ (m/s)</th>
<th>$R_{sb}$ (km)</th>
<th>$V_{S30}$ (m/s)</th>
<th>PGA (g)</th>
<th>PGV (cm/s)</th>
<th>PGD (cm)</th>
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<td>0.101</td>
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<td>213.44</td>
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<td>14.60</td>
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<td>Erzincen</td>
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<td>6.69</td>
<td>1.8</td>
<td>0</td>
<td>352.05</td>
<td>0.496</td>
<td>78.10</td>
<td>28.01</td>
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<td>Duzce</td>
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<td>Yermo Fire</td>
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<td>0.71</td>
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<td>353.6</td>
<td>0.152</td>
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<td>76.06</td>
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<td>Parachute Test Site</td>
<td>1979</td>
<td>6.53</td>
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<td>12.69</td>
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<td>6.53</td>
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</tr>
</tbody>
</table>

Table 1: Selected earthquake records [Part I: EQ 1-34], [46].

Moment magnitudes ($M_o$) of selected earthquakes can be seen from Figure 4. Moment magnitude of the large majority of records is $M_o=7.28$ (41 earthquake records). In Figure 5, the average shear-wave velocity ($V_{S30}$ values) in the top thirty meters of the soil for 100 selected earthquakes are given (180 m/s/$V_{S30}\leq360$ m/s, Soil Class D (NEHRP, [47])). Figure 6a shows the moment magnitude ($M_o$) versus distance ($R_{sb}$) distribution of selected earthquake data set and Figure 6b gives the moment magnitude ($M_o$) versus PGA (g) distribution of the same data set.
Distance variables are expressed in logarithmic form. The Joyner-Boore relation is given in Figure 7a and PGV-R$_{JB}$ relation is given in Figure 7b. Distance variables are expressed in logarithmic form.

Table 2: Selected earthquake records [Part II: EQ 35-67], [46].

<table>
<thead>
<tr>
<th>Event Name (*°)</th>
<th>Station</th>
<th>Year</th>
<th>$M_w$</th>
<th>$I_a$ (m/s)</th>
<th>$R_m$ (km)</th>
<th>$V_{S80}$ (m/s)</th>
<th>PGA (g)</th>
<th>PGV (cm/s)</th>
<th>PGD (cm)</th>
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<td>Victoria HS. Sotano</td>
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<td>219.8</td>
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Figure 5: The average shear-wave velocities ($V_{S30}$ values) in the top thirty meters of the soil for 100 selected earthquakes.

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<th>Year</th>
<th>$M_c$</th>
<th>$I_A$ (m/s)</th>
<th>$R_{DP}$ (km)</th>
<th>$V_{S30}$ (m/s)</th>
<th>PGA (g)</th>
<th>$PGV$ (cm/s)</th>
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Table 3: Selected earthquake records [Part III: EQ 68-100], [46].

5 DETERMINATION OF ELASTIC EARTHQUAKE INPUT ENERGY SPECTRA USING SELECTED RECORDS

Input energy spectra of elastic SDOF systems having damping ratio of 5% were obtained using the selected real earthquakes. To that end, at first, the velocity-time histories ($\ddot{u}(t)$) were computed by using the software PRISM [52]. Input energy-time histories ($\langle E/\nu \rangle m^t$)
graphs) were then computed for the specific natural vibration period \((T)\) of SDOF system. Period values were taken from \(T=0.1\) sec. to \(T=3.0\) sec. using the increment of \(\Delta T=0.1\) sec. For an accelerogram, 30 response time history (RTH) analyses were carried out. Totally, 3000 RTH analyses were performed and the maximum mass normalized earthquake input energy values \((E/m)_{\text{max}}\) for all records were computed. The maximum energy-time histories for selected 100 earthquake records can be seen from Figure 8. Earthquakes were separated to 4 groups according to \(V_{S30}\) velocities \((190\ \text{m/s}<V_{S30}\leq235\ \text{m/s},\ 235\ \text{m/s}<V_{S30}\leq280\ \text{m/s},\ 280\ \text{m/s}<V_{S30}\leq325\ \text{m/s}\) and \(325\ \text{m/s}<V_{S30}\leq370\ \text{m/s})\), and the input energy-time histories were computed for 4 earthquake groups (Figure 8). A computer algorithm was created in Excel by the author to combine the maximum input energies \((E/m)_{\text{max}}\) and to obtain the input energy spectra. Velocity-time histories were taken from PRISM software [52].

The mass-normalized input energy spectra of elastic SDOF systems having 5% viscous damping ratio is shown in Figure 9. The energy spectra were created using the selected 100 earthquake records in Table 1, Table 2 and Table 3. The mean of input energy spectra and the mean+1 and +2 standard deviations were indicated in Figure 9 and also separately in Figure 10. From the variation coefficients \((V_C)\) of elastic input energy spectra which are shown in Figure 10, it can be seen that the dispersion is generally considerable. Variation coefficients were also computed for 4 groups of earthquakes which were separated according to \(V_{S30}\) velocities (Figure 10).
Figure 8: The maximum energy-time histories ($E_{f/m}$-t graphs) for selected 100 earthquake records (for 4 groups of $V_{S30}$ velocities).

Figure 9: Mass-normalized elastic input energy spectra for Soil D and for 5% viscous damping ratio ($E_{f/m}$-T graphs), (with Mean, Mean+1 St.Dev. and Mean+2 St.Dev. of 100 selected ground motion records).

Figure 10: a) The mean of input energy spectra (The mean of 100 selected records), b) The input energy spectra graphs for: Mean, Mean+1 St.Dev. and Mean+2 St.Dev. values, c) Variation coefficients of the total selected records and variation coefficients of the separated 4 groups (according to $V_{S30}$ velocities).
6 CLASSIFICATION OF EARTHQUAKE RECORDS ACCORDING TO VS30 VELOCITIES AND RJB DISTANCES AND CREATING THE ELASTIC INPUT ENERGY SPECTRA OF DIFFERENT GROUPS

Earthquake records were classified according to VS30 velocities and RJB distances within the study. Four ranges of VS30 were taken into account as: 190 m/s < VS30 ≤ 235 m/s, 235 m/s < VS30 ≤ 280 m/s, 280 m/s < VS30 ≤ 325 m/s and 325 m/s < VS30 ≤ 370 m/s, respectively. RJB distances were separated to four groups as: 0 km < RJB ≤ 46 km, 46 km < RJB ≤ 92 km, 92 km < RJB ≤ 138 km and 138 km < RJB ≤ 184 km, respectively. VS30 and RJB histograms may be shown in Figure 11. Number of records for VS30 and RJB intervals can be obtained from Figure 11.

![VS30 and RJB histograms for selected earthquake records.](image)

The elastic input energy spectra of earthquake groups according to VS30 velocities are shown in Figure 12. The mean of the spectra and the mean+1 and +2 standard deviations were indicated in the figure. With the increase in VS30 velocities, it was observed that the values of elastic input energy spectra tended to decrease gradually (Figure 12).

![Mass-normalized input energy spectra of earthquakes.](image)

Figure 12: Mass-normalized input energy spectra of earthquakes: a) the group of VS30 between [190, 235] m/sec, b) the group of VS30 between (235, 280] m/sec, c) the group of VS30 between (280, 325] m/sec, d) the group of VS30 between (325, 370] m/sec.
Figure 13: Mass-normalized input energy spectra of earthquakes: a) the group of $R_{JB}$ between [0, 46] km, b) the group of $R_{JB}$ between (46, 92] km, c) the group of $R_{JB}$ between (92, 138] km, d) the group of $R_{JB}$ between (138, 184] km.

The elastic input energy spectra of earthquake groups according to $R_{JB}$ distances are shown in Figure 13. The mean of the spectra and the mean+1 and +2 standard deviations were indicated in the figure. With the increase in $R_{JB}$ distances, it was observed that the values of elastic input energy spectra tended to decrease gradually (Figure 13). The maximum input energies were obtained for the interval of 0 km<$R_{JB}$≤46 km (Figure 13a).

7 ELASTIC DESIGN INPUT ENERGY SPECTRA FOR SELECTED EARTHQUAKES

Linear elastic design earthquake input energy spectrum was proposed for selected ground motions on site class D (on stiff soil with 180 m/s<V$_{S30}$≤360 m/s). The design energy demand spectrum was created for viscous damping ratio of 5% and the lower period was considered as $T_0$=0.05 sec. and the maximum period was considered as $T_{max}$=3.0 sec. The design spectral shapes were associated with simple mathematical expressions and smoothened curves as:

$$
T_0 \leq T \leq T_1 \rightarrow \frac{E_i}{m} = a \cdot T + b
$$

$$
T_1 < T \leq T_2 \rightarrow \frac{E_i}{m} = c \left( \frac{E_i}{m} \right) \left( \frac{m^2}{s^2} \right)
$$

$$
T_2 < T \rightarrow \frac{E_i}{m} = k \cdot T^n
$$

(7)

where $a$, $b$, $c$, $k$ and $n$ are parameters of mass-normalized design input energy spectrum. Parameters depend on the soil type. In Equation (7); $E_i/m$ is in unit of m$^2$/s$^2$. Parameter “$a$” represents the coefficient of period $T$ in linear considered region of normalized spectrum and “$b$” is the constant parameter in region of $T_0$<$T$≤$T_1$. “$c$” is the maximum spectral value relative to the constant value of the spectrum. “$n$” is the exponential parameter governing the decrease in normalized spectrum for larger period values ($T_2$<$T$) and “$k$” is the multiplier of $T_2$ (Figure 14).
The fundamental design input energy spectrum shape is shown in Figure 14. $T_1$ is the period corresponding to the beginning of the constant value of the spectrum and $T_2$ is the period corresponding to the ending of the constant value of the spectrum (corner periods of the spectrum). Input energy spectra is the function of vibration period $T$, soil class and viscous damping ratio $\xi$ [39]. $\xi$ is assumed to be 5% in this study and the design input energy spectra were constructed for $\xi=5\%$.

The elastic design input energy spectrum for selected 100 earthquake records on site class D and for $\xi=5\%$ is shown in Figure 15. Statistical analyses were conducted and curve fitting which is the process of specifying the model that provides the best fit to the specific curves in the dataset was used. The results of linear regression analyses with $R$-squared values may be shown in Table 4 and Table 5. From Table 4; it can be concluded that the first region of the design spectrum ($T_0<T\leq T_1$ region) can be well-defined by linear model in Equation (7) ($b.a+T+b$). However, $R$-squared values showed that there is not a strong relation with data in the region of $T_2<T$ as there is in the first linear-considered region of the spectrum. It can be seen from Table 4 and Table 5 that the $R$-squared values are greater for “the mean of input energy spectrum”.

![Figure 14: The spectral shape of elastic design input energy.](image1)

![Figure 15: Elastic design input energy spectrum for selected 100 earthquake records on site class D and for $\xi=5\%$.](image2)
\[
\frac{E_I/m}{T} \text{ (m}^2/\text{s}^2) \\
\begin{array}{|c|c|c|c|c|}
\hline
\text{Mean} & 0.3184 & -0.0124 & 0.599 & 0.9918 \\
\text{Mean+1 St.Dev.} & 1.0982 & -0.0529 & 0.597 & 0.9835 \\
\text{Mean+2 St.Dev.} & 1.878 & -0.0934 & 0.601 & 0.9811 \\
\hline
\end{array}
\]

Table 4: Values of parameters introduced in Figure 14, for the elastic design input energy spectrum in Figure 15 (parameters for region of \(T_0 < T \leq T_1\)).

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\frac{E_I/m}{T} \text{ (m}^2/\text{s}^2) & c & T_2 (s) & k & n & R^2 \\
\hline
\text{Mean} & 0.1786 & 1.389 & 0.2508 & -0.662 & 0.8371 \\
\text{Mean+1 St.Dev.} & 0.5423 & 1.198 & 0.5373 & -0.420 & 0.7309 \\
\text{Mean+2 St.Dev.} & 0.9046 & 1.202 & 0.8575 & -0.401 & 0.6579 \\
\hline
\end{array}
\]

Table 5: Values of parameters introduced in Figure 14, for the elastic design input energy spectrum in Figure 15 (parameters for region of \(T_1 < T \leq T_2\) and \(T_2 < T\)).

The elastic design input energy spectra for earthquake groups which were separated according to \(V_{S30}\) velocities and \(R_{JB}\) distances (see intervals in histograms of Figure 11) are shown in Figure 16. Parameters of design input energy spectra \((a, b, c, k, n, T_1\) and \(T_2\) are presented by Table 6, Table 7, Table 8 and Table 9. In the group with lower \(V_{S30}\) velocities (especially the group: 190 m/s < \(V_{S30}\) ≤ 235 m/s), the design input energy spectrum was obtained greater for SDOF systems having damping ratio of 5%. The design input energy spectrum was obtained greater for the earthquake group having \(R_{JB}\) distances between 0 km < \(R_{JB}\) ≤ 46 km. The elastic design input energy spectra of different groups were shown by different colors. All design energy spectra in Figure 16 were created considering the mean of input energy spectra of different groups (from the mean spectra in Figure 12 and Figure 13).

Figure 16: Elastic design input energy spectra for earthquake groups (in Figure 11) according to \(V_{S30}\) velocities and \(R_{JB}\) distances (for \(\zeta = 5\%\)).
### Table 6: Values of parameters introduced in Figure 14, for the elastic design input energy spectrum of different $V_{S30}$ groups (parameters for region of $T_0 < T_1$).

<table>
<thead>
<tr>
<th>Design $E_I/m - T$ (m²/s²)</th>
<th>$a$</th>
<th>$b$</th>
<th>$T_1$ (s)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$190 \text{ m/s} &lt; V_{S30} \leq 235 \text{ m/s}$</td>
<td>0.5179</td>
<td>-0.0152</td>
<td>0.512</td>
<td>0.9905</td>
</tr>
<tr>
<td>$235 \text{ m/s} &lt; V_{S30} \leq 280 \text{ m/s}$</td>
<td>0.2080</td>
<td>-0.0083</td>
<td>0.761</td>
<td>0.9778</td>
</tr>
<tr>
<td>$280 \text{ m/s} &lt; V_{S30} \leq 325 \text{ m/s}$</td>
<td>0.4295</td>
<td>-0.0258</td>
<td>0.485</td>
<td>0.9316</td>
</tr>
<tr>
<td>$325 \text{ m/s} &lt; V_{S30} \leq 370 \text{ m/s}$</td>
<td>0.1500</td>
<td>-0.0022</td>
<td>0.975</td>
<td>0.9662</td>
</tr>
</tbody>
</table>

### Table 7: Values of parameters introduced in Figure 14, for the elastic design input energy spectrum of different $V_{S30}$ groups (parameters for region of $T_1 < T_2$).

<table>
<thead>
<tr>
<th>Design $E_I/m - T$ (m²/s²)</th>
<th>$a$</th>
<th>$b$</th>
<th>$T_2$ (s)</th>
<th>$k$</th>
<th>$n$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$190 \text{ m/s} &lt; V_{S30} \leq 235 \text{ m/s}$</td>
<td>0.2662</td>
<td>1.389</td>
<td>0.2825</td>
<td>-0.587</td>
<td>0.7271</td>
<td></td>
</tr>
<tr>
<td>$235 \text{ m/s} &lt; V_{S30} \leq 280 \text{ m/s}$</td>
<td>0.1598</td>
<td>1.720</td>
<td>0.3335</td>
<td>-1.079</td>
<td>0.7527</td>
<td></td>
</tr>
<tr>
<td>$280 \text{ m/s} &lt; V_{S30} \leq 325 \text{ m/s}$</td>
<td>0.1578</td>
<td>1.709</td>
<td>0.4658</td>
<td>-1.479</td>
<td>0.9177</td>
<td></td>
</tr>
<tr>
<td>$325 \text{ m/s} &lt; V_{S30} \leq 370 \text{ m/s}$</td>
<td>0.1475</td>
<td>1.715</td>
<td>0.2215</td>
<td>-0.505</td>
<td>0.5167</td>
<td></td>
</tr>
</tbody>
</table>

### Table 8: Values of parameters introduced in Figure 14, for the elastic design input energy spectrum of different $R_{JB}$ groups (parameters for region of $T_0 < T_1$).

<table>
<thead>
<tr>
<th>Design $E_I/m - T$ (m²/s²)</th>
<th>$a$</th>
<th>$b$</th>
<th>$T_1$ (s)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \text{ km} &lt; R_{JB} \leq 46 \text{ km}$</td>
<td>0.4914</td>
<td>-0.0165</td>
<td>0.585</td>
<td>0.9937</td>
</tr>
<tr>
<td>$46 \text{ km} &lt; R_{JB} \leq 92 \text{ km}$</td>
<td>0.1718</td>
<td>-0.0126</td>
<td>0.594</td>
<td>0.9691</td>
</tr>
<tr>
<td>$92 \text{ km} &lt; R_{JB} \leq 138 \text{ km}$</td>
<td>0.0544</td>
<td>-0.0060</td>
<td>1.405</td>
<td>0.9525</td>
</tr>
<tr>
<td>$138 \text{ km} &lt; R_{JB} \leq 184 \text{ km}$</td>
<td>0.0672</td>
<td>-0.0083</td>
<td>1.697</td>
<td>0.9246</td>
</tr>
</tbody>
</table>

### Table 9: Values of parameters introduced in Figure 14, for the elastic design input energy spectrum of different $R_{JB}$ groups (parameters for region of $T_1 < T_2$).

<table>
<thead>
<tr>
<th>Design $E_I/m - T$ (m²/s²)</th>
<th>$c$</th>
<th>$T_2$ (s)</th>
<th>$k$</th>
<th>$n$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \text{ km} &lt; R_{JB} \leq 46 \text{ km}$</td>
<td>0.2681</td>
<td>1.198</td>
<td>0.3101</td>
<td>-0.581</td>
<td>0.9212</td>
</tr>
<tr>
<td>$46 \text{ km} &lt; R_{JB} \leq 92 \text{ km}$</td>
<td>0.090</td>
<td>1.980</td>
<td>0.2173</td>
<td>-1.340</td>
<td>0.8915</td>
</tr>
<tr>
<td>$92 \text{ km} &lt; R_{JB} \leq 138 \text{ km}$</td>
<td>0.080</td>
<td>2.150</td>
<td>0.2279</td>
<td>-1.547</td>
<td>0.9142</td>
</tr>
<tr>
<td>$138 \text{ km} &lt; R_{JB} \leq 184 \text{ km}$</td>
<td>0.108</td>
<td>2.155</td>
<td>0.2498</td>
<td>-1.149</td>
<td>0.8640</td>
</tr>
</tbody>
</table>
8 CONCLUSIONS

This paper presents the development of elastic design input energy spectra using real ground motion records. The effect of the average shear-wave velocity in the top thirty meters of the soil ($V_{S30}$) and the effect of the Joyner-Boore distance ($R_{JB}$) on the elastic input energy spectra are also investigated. Actual ground motion records taken from many different earthquakes all over the world are selected to investigate the elastic input energy spectra for damping ratio of 5%. Selected ground motions are all on the same site class according to National Earthquake Hazards Reduction Program site classification. Elastic input energy spectra of the selected ground motions are computed and the proposal of design input energy spectra are carried out. The study mainly focuses on the development of elastic design input energy spectra of real ground motions on stiff soil. Ground motion records are separated into four groups considering the $V_{S30}$ velocities and $R_{JB}$ distances and the design input energy spectra of each group are also investigated independently. The design spectral shapes are expressed with simple mathematical expressions and idealized lines and curves. Coefficient of determination values are computed for each region of the design spectra to measure of how close the data are to the fitted regression lines. The effect of the change in $V_{S30}$ velocities and $R_{JB}$ distances on the elastic design input energy spectra of real earthquakes is investigated. The results of the study are restricted to the selected ground motions which are on stiff soil with 180 m/s < $V_{S30}$ ≤ 360 m/s. The notable findings of the study are as follows:

- The mean input energy spectrum of selected earthquakes has a very good correlation with the proposed design elastic input energy spectra. Coefficient of determination ($R$-squared) value for the first region of the design input energy spectrum (for $T_0 < T ≤ T_1$ region) is obtained as 0.9918 so, the region can be well-defined by linear model. For larger period values (for $T_2 < T$), the coefficient of determination ($R$-squared) value is obtained as 0.8371. The region can also be well-defined with the curve $k.T_n$. However, the linear regression in $T_0 < T ≤ T_1$ region shows that there is stronger correlation between the real input energy spectra and the proposed design input energy spectra in that region than the region of $T_2 < T$.

- The type of soil (so $V_{S30}$ velocity) has a significant influence on the design input energy spectra. Elastic input energy spectra decrease as $V_{S30}$ velocity increases (from soft soil to stiff soil). However, the decrease is obtained more significant from the first selected earthquake group to the second selected earthquake group (from the group having 190 m/s < $V_{S30}$ ≤ 235 m/s to the group having 235 m/s < $V_{S30}$ ≤ 280 m/s). The results of the study are valid for selected earthquake records.

- Elastic input energy spectra are obtained maximum for the earthquake group with 0 km < $R_{JB}$ ≤ 46 km. It can be seen from the study that the input energy spectra tend to decrease with the increase in $R_{JB}$ distance. As the shortest distance from a site to the surface projection of the rupture surface ($R_{JB}$) becomes very smaller, the values of elastic input energy spectra becomes higher.

- In the 3rd earthquake group having 280 m/s < $V_{S30}$ ≤ 325 m/s, the coefficient of variation for the elastic input energy spectra is obtained smaller when it is compared to other groups. The input energy spectra of this group scattered around the mean spectra better than other earthquake groups.
It is observed from the study that the elastic input energy spectra of near fault earthquakes are higher than that of the far fault records. The records which have the source-to-site distance is less than 15 km show an instantaneous jump in the input energy spectrum graph. If the near fault earthquakes were extracted from the selected earthquakes within the study, the proposed design input energy spectra graph could be smoother and lower.

The elastic design input energy spectra may be used in seismic design of structures if the energy capacity of structures can be determined. The main purpose of this study is to propose demand design energy spectra for selected earthquakes. Further studies can be done using wide range of ground motion records which are on different soil sites. The study can be extended for SDOF systems having different damping ratios. Inelastic SDOF systems having different ductility ratios can be analyzed to obtain the inelastic input energy spectra. Different cyclic models can be used to define the nonlinear force-displacement relation. It should be noted that each earthquake reflects its own characteristics in dynamic analyses and in seismic input energy computations.

REFERENCES


Abstract

The main objective of the energy-based design (EBD) is by controlling of the dissipative capacity and the ductility of the potential post-elastic mechanisms to reduce the risk of losing global stability and collapse of the structures during the limit design event. Following this design philosophy, investigations of the influence of different parameters of the hysteretic models on nonlinear performance of SDOF expressed via energy indicators are carried out. The most important parameters which are the subject of the study are: the stiffness after the yielding point $\beta$, the factor of stiffness degradation at unloading USDP (unloading stiffness degradation parameter), the FYRP (force yield relation parameter) parameter, the initial stiffness and the level of the first cracks in concrete, as well as some specific parameters that are specific for the shear models only. Selected results concerning influence of these parameters on the maximum values of dissipated hysteretic energy (Emax) and its index (Eh) are presented in the paper.

Keywords: Seismic performance, Energy-based Method, Hysteretic models, Maximum Dissipated Energy, Index of Hysteretic Energy.
1 INTRODUCTION

Since the introducing of the "Performance Based Seismic Design" (PBSD) and "Capacity Design" (CD), the concept of performance have replaced the concept of resistance. Today the performance assessed in function of energy consideration represents the most notable side of the latest progress in Seismic Engineering. This has brought a new energy-based philosophy which is not intended to replace the traditional force-based or the more sophisticated displacement-based design, but to supplement the current design methods. This could make the performance-based seismic design more complete.

The main objective of the energy-based design (EBD) is by controlling of the dissipative capacity and the ductility of the potential post-elastic mechanisms to reduce the risk of losing global stability and collapse of the structures during the limit design event.

Within the framework of performance based design, the Vision 2000 report on future seismic design codes [1] identified displacement-based design (DBD) and energy-based design (EBD) as promising approaches. Although at the begging both were very promising, with time EBD lacks behind DBD.

The fundamentals of an energy-based method was proposed by Housner and further elaborated by Uang and Bertero, [2]. This design method is based on the premise that energy demand during an earthquake can be predicted and the energy capacity of a structural element (or system) can be established.

The energy approach is based on the fact that damage degree of a structure exposed to real seismic effects, depends on the earthquake energy that is transmitted to the structure and the capacity of the structure and its elements to absorb this energy. The total input energy of the earthquake is dissipated through vibrations of the structure (kinetic energy), the mechanism of viscous damping (damping energy) and through the energy absorbed by the structure (consisting of elastic energy and hysteretic energy which is dissipated through controlled nonlinear deformations).

The input energy is a parameter that refers to the total destructive earthquake potential, whereas the hysteretic energy is a structural parameter related to the damage degree of structures. In order that the structure has adequate seismic performance, it is necessary that its energy capacity for absorption and dissipation of energy be greater than the energy demand posed by the earthquake.

The damage of the structure as a result of the earthquake actions do not depend only from the maximum achieved deformations (strains, rotations, displacements) but also from the total number of the nonlinear excursions.

Experience has shown that the great majority of well designed and constructed buildings survive strong ground motions, even if they were in fact designed for only a fraction of the forces that would develop if the structure behaved entirely as linearly elastic. A reduction of seismic forces is possible thanks to the beneficial effects of energy dissipation in ductile structures and inherent overstrength. Although the influence of the structural system and its capacity for energy dissipation has been recognized already in late 1950s, the force reduction factor (or simply R factor) in the current format was first proposed in ATC-3-06. Since then, R factor has been present, in various forms, in all seismic regulations (in the European standard EC8 it is called the behaviour factor q), [3].

The comprehensive state-of-the-art in EBD is given in [4]. According to this review, most of the research covers mainly the total seismic energy input, however the energy flow and particularly its distribution during the nonlinear performance of the structure is still open issue which is correlated with the energy capacity of structure and its elements. Fundamental research in this topic was done by Gaspercic, Fajfar and Fischinger [5] who developed simplic-
fied method for nonlinear structural analysis where the structural damage is correlated with the level of the dissipated hysteretic energy by the structural system.

This paper re-visited research carried out at the Institute of Earthquake Engineering and Engineering Seismology, IZIIS from Skopje within the frame of the scientific project financed by Ministry of Education and Science. Presented further will be selected results from the parametric analysis related to the influence of several hysteretic model parameters on the total dissipated hysteretic energy and hysteretic energy index, [6, 7].

2 NONLINEAR PERFORMANCE OF RC BUILDINGS AND HYSTERETIC ENERGY

When structures are exposed to strong earthquakes they behave nonlinear. The most accurate and the same time the most complex method for deterministic structural analysis is nonlinear time history analysis. Dynamic characteristics of structure changed when it is exposed to the earthquakes. Their response is characterized by multiply excursions in inelastic range followed by deterioration of stiffness, strength or both. Analytical definition of structural response to earthquake effect must contain within itself the mechanism of nonlinear behavior, i.e., the force – displacement relationship. This relationship represents, in fact, the nonlinear variation of stiffness and strength (or both parameters) of a structure during its exposure to earthquakes. Such performance of elements/structures is defined as hysteretic and it is accompanied by energy dissipation.

The phenomenon of degradation/deterioration of the stiffness and strength is rather complex and depends on a number of factors such as type of structure and structural elements, amount and quality of built-in material, current state of deformations as well as previous loading history, cracks and failure of concrete, yielding and hardening of reinforcement, adhesion of concrete and reinforcement, bar sliding effect etc.

The most accurate relationships, at element level (force-displacement), at cross-section level (moment-rotation) and at material level (stress-strain) are developed as a result of experimental investigations. Since such type of relationships (hysteretic diagrams) are too complex to be used in analytical investigations, therefore for practical application different researchers proposed idealized hysteretic diagrams (models). Such idealizes hysteretic models of structural elements or entire storey of a structure represent the main indicators of nonlinear performance. They are expected to simultaneously respond to two basic functions, namely, to provide stiffness and strength under the effect of a random displacement history on one hand and, on the other hand, to be described by simple mathematical relationships (rules).

According to the type of primary curve all models are classified into: (1) bilinear, in which the variation of stiffness takes place at two points, namely, tensile reinforcement yielding point “Y” and concrete failure point “U” and (2) trilinear, in which the variation of stiffness is defined at three points, first crack in the tensile concrete, point “C”, prior to points “Y” and “U”. Related to the dominant mode of structural/element performance during earthquakes they are classified into two groups, hysteretic models for simulation of flexural and shear behavior.

The most important parameters by which the different hysteretic models are described and which affect the nonlinear response of the system and the shape of the hysteretic loops, i.e., the maximum dissipated energy (Emax) and its index (Eh) are, [6,7]:

- the stiffness after the yielding point β,
- the factor of stiffness degradation at unloading USDП (unloading stiffness degradation parameter),
• the FYRP (force yield relation parameter) parameter,
• the initial stiffness and the level of the first cracks in concrete
• parameters specific for shear models only

For the needs of this research, the maximum hysteretic energy ($E_{max}$) is calculated by summing up the surfaces enclosed by the hysteretic loops from the force-displacement relationship, during the nonlinear vibrations of the system under real seismic excitation. The energy dissipation index ($E_h$) is obtained using the below equation, (Figure 1),

$$E_h = \frac{\Delta W}{2*\pi*F_m*D_m}$$  \hspace{1cm} (1)

where:
\begin{itemize}
  \item $\Delta W$ is the area enclosed by the hysteretic curve
  \item $D_m$ is the achieved max. displacement
  \item $F_m$ is the achieved max. force
\end{itemize}

Figure 1: Parameters of the primary curve of hysteretic models and index of energy dissipation

Expressions for calculation of the index of hysteretic energy ($E_h$) for selected models, [8] are given in the Table 1.

<table>
<thead>
<tr>
<th>Hysteretic model</th>
<th>Index of hysteretic energy, $E_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilinear model</td>
<td>$E_h = 2*(1 - \beta) * (\mu - \mu^\alpha * (1 - \beta + \mu\beta))$</td>
</tr>
<tr>
<td>Clough model</td>
<td>$E_h = \frac{1}{\pi} * \left{ 1 - \frac{D_{Fy}}{D_{Fm}} * \mu^\alpha * (1 - \beta + \mu\beta) \right}$</td>
</tr>
<tr>
<td>Takeda model</td>
<td>$E_h = \frac{1}{\pi} \left{ 1 - \frac{1 + \frac{D_{Fy}}{D_{Fm}}^\mu}{1 + \frac{F_{Fy}}{F_{Fm}}} * \mu^\alpha * (1 - \beta + \mu\beta) \right}$</td>
</tr>
<tr>
<td>Trilinear degrading model</td>
<td>$E_h = \frac{2}{\pi} \left( 1 - \frac{K_y}{K_c} \right) * \frac{F_{Fy}}{F_c}$</td>
</tr>
</tbody>
</table>

Table 1: Index of the hysteretic energy
All nonlinear dynamic analysis necessary for calculation of Emax and Eh were performed by applying an in-house developed software package. The main dynamic model of the RC building structure represents "shear-type" model. The nonlinear variation of stiffness and strength (force-displacement relationship) at the story level, which is considered as a "macro element", is defined through corresponding hysteretic models. The story force – displacement relationship is obtained by summing up the force-displacement relationships of all RC vertical elements (columns and shear walls), [9]. The selected earthquake record is imposed in the form of time history of ground acceleration.

For the purpose of the investigations, six models simulating a bending behaviour (bilinear, Clough model, Takeda, bilinear Takeda, Hisada, trilinear model with degradation) and two models simulating shear behaviour (bilinear-slip and Takeda-slip) were chosen. The parametric analyses refer to single-degree-of-freedom systems exposed to the Ulcinj-Albatros (Montenegro) earthquake, with maximum accelerations of 0.3g and 0.5g.

The results obtained from the parametric analyses are presented through its maximum values of the hysteretic energy as well as energy indexes. Selected part of these results are given further in the paper.

3 INFLUENCE OF HYSTERETIC MODEL PARAMETERS ON DISSIPATED HYSTERETIC ENERGY

3.1 Post-yielding Stiffness Parameter $\beta$

Post-yielding stiffness parameter $\beta$ is associated with "strain hardening" effect of steel and results with positive stiffness (Ku) after yielding of the reinforcement. It is mathematically defined as a ratio between stiffnesses Ku/Ky and for the needs of the study was varying in the range between 0.05 - 0.2.

This parameter has the strongest influence upon the shape of the hysteretic curves, i.e., the capacity for energy dissipation. With the increase in this parameter, the shape of the hysteresis loops changed from "fat" to "thin" due to the fact that the area they enclosed is smaller. The only exception is the Hisada's model that is characterized by stable hysteresis loops. With increasing of $\beta$, the index of hysteretic energy, Eh in all the analyzed models (with the exception of the trilinear model) is decreased (Figure 2).

![Hysteretic energy, Emax [kNcm]](image.png)

Figure 2: Effect of $\beta$ parameter on energy dissipation and energy index

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3.2 Unloading Stiffness Degradation Parameter - USDP

Generally, the parameter USDP is used to model the stiffness degradation at unloading due to some of the following phenomena: Baushinger's effects, cracks in the tensile concrete, disturbance of the connection at the concrete - steel contact, sliding or loss of effective anchorage and shear strains. Its determination on the basis of known characteristics of materials and geometry of structures is not possible. For the needs of the analyses, this factor has been varied at interval of 0.0 - 0.5.

The USDP factor has the greatest influence upon the nonlinear response of the system. The increase in this parameter leads to a decrease in the hysteretic energy and its index in all the models, (Figure 3) (with the exception of the trilinear model due to the fact that Eh does not depend on the ductility but only on the force and displacement values at cracking and yielding points, (Table 1).
3.3 Force Yield Relation Parameter – FYRP

The FYRP parameter represents a ratio between yielding force $F_y$ (at random ductility) and yielding force $F_{ys}$ corresponding to the design ductility value i.e. $FYRP = F_y/F_{ys}$. The parameter has been varied at an interval of 0.6 to 1.5. Its effect upon the energy characteristics of the system is considerable. Its increase causes pinching of the hysteretic curves leading to a decrease in hysteretic energy and its index (Figure 4).

3.4 First crack at concrete and Initial stiffness

The level of force $F_C$ is varied at interval of 0.1 to 0.7 $F_Y$, whereas the initial stiffness is taken twice higher than the yielding stiffness ($K_y$). The effect of the change of the $F_C/F_Y$ ratio is most significant for the trilinear model (Figure 5). The increase in this ratio leads to an increase in dissipated hysteretic energy and its index.

The initial stiffness is taken 1.5-4.0 times higher than the yielding stiffness. The effect of the change of the $K_C/K_Y$ ratio of the nonlinear response of the system is again most significant for the trilinear model (Figure 6). The increase in this ratio leads to an increase in dissipated hysteretic energy and its index.
Figure 4: Effect of FYRP parameter on energy dissipation and energy index, (cont.)

Figure 5: Effect of FC/FY (first crack in concrete) on energy index

Figure 6: Effect of KC/KY (initial stiffness) on energy index
3.5 Specific parameters of Bilinear-slip and Takeda-slip models

Two characteristic parameters arising from the very nature of the Bilinear-slip model itself are the B0 parameter (ratio between the sliding stiffness and initial stiffness, i.e., KL/KY) and parameter B1 (ratio between displacement at which the sliding starts and displacement at which the sliding ends i.e., DU/DL), (Figure 7). The Takeda-slip model is obtained by modification of the original Takeda model, introducing two new parameters (B2, which defines the "softening" of the stiffness due to the pinching effect and B3, which defines the increase in stiffness after closing of the cracks in the compressed concrete, (Figure 7).

![Figure 7: Specific parameters of Bilinear-slip and Takeda-slip models](image)

At the bilinear model, with the increase of B0 (0.0-0.3), the angle between the sliding axis and the x-axis is increased leading to a decrease in hysteretic energy and its index, (Figure 8). The increase of B3 at Takeda-slip model (0.5-2.0) leads to decreasing of energy index, Eh, (Figure 8).

![Figure 8: Effect of specific parameters of slip models on energy dissipation and energy index](image)

4 CONCLUSIONS

The paper presents part of the performed analytical investigations of the effect of different parameters of hysteretic models on the nonlinear response of structures. The parametric study covers the following parameters: the stiffness parameter obtained after yielding point $\beta$, the factor of stiffness degradation at unloading (USDP), the parameter defined as a ratio between the yielding force at random ductility value and yielding force corresponding to the designed ductility value (FYRP), the initial stiffness and the level of the first cracks and two parameters specific for he slip hysteretic models.

The results obtained from the analyses expressed via maximum values of dissipated hysteretic energy and its indexes are considerably different. Out of all analyzed parameters, the
stiffness parameter obtained after yielding point $\beta$ has the greatest effect upon the shape of the hysteretic curves and hence the size of the dissipated hysteretic energy. With the increase of this parameter, the hysteretic curves change their shape from thick to thin hysteretic curves. The USDP parameter has the greatest effect upon the maximum values of displacements, namely with its increasing, the maximum displacements are increased, also. The FYRP parameter is an important indicator of the strength characteristics of a system. Very low values of this parameter (0.70 or less) point to weak, underdesigned systems, whereas FYRP values higher than a unity are characteristic for strong overdesigned systems. The effect of variation of parameters of initial stiffness and the point defining the level of the first cracks is mostly expressed in the trilinear model, whereat the increase of these parameters leads to an increase of the energy index.

Hence, a general conclusion is drawn that for a successful prediction of the performance of RC structures exposed to strong earthquake effects, it is necessary to perform nonlinear dynamic analysis using hysteretic models that will represent, most appropriately, the characteristics of the analyzed structure. Understanding the influence of their parameters on the overall structural performance, including energy indicators, is initial step to further investigations toward energy based seismic design.

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ENERGY DISSIPATION CAPACITY OF RC COLUMNS SUBJECTED TO UNIDIRECTIONAL AND BIDIRECTIONAL SEISMIC LOADING

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Abstract

One of the gaps of knowledge for the application of the energy-based approach to conventional structures concerns the evaluation of the energy dissipation capacity of structural elements and systems. Addressing this issue calls for experimental data on structural members, structures or substructures subjected to realistic seismic loadings. The best source for such information is the dynamic shake table test. It can reproduce complex effects such as cumulative damage and rate-of-loading effects, that play an important role in the response of the structure. However this type of tests are very costly and time consuming. Therefore, it is necessary to complete the test information with numerical simulations. This paper provides information on the energy dissipation capacity of columns subjected to one and to two components of the seismic action. This information comes from shake table test conducted on RC waffle flat plate structures supported on isolated columns. The energy dissipated at the base of the columns until collapse is estimated applying different approaches.

Keywords: energy dissipation capacity, RC columns, shake table tests, unidirectional seismic loading, bidirectional seismic loading
1 INTRODUCTION
The most straightforward way to characterize the loading effect of earthquakes on structures is in terms of energy dissipation demand. In contrast to forces or displacements, which are vectors, energy is a scalar measure that can synthesize in a single value the complex 3D loading effects caused by earthquakes on structures [1]. Thus, comparing the energy dissipation demand with the energy dissipation capacity of the structure becomes the most rational way to evaluate structural safety, which in turn constitutes the basis of the energy-based approach. The energy dissipation demand at the global (overall structure) or local (structural members) level can be readily estimated from the total energy input by the earthquake. This estimation is facilitated by the fact that the energy input by an earthquake is a very stable quantity that depends almost exclusively on the mass and fundamental period of the structure, and it is scarcely affected by the viscous damping ratio, the post-yield hardening ratio, the degree of inelastic action or the number of degrees of freedom [2]. Past research on the energy-based approach focused on the energy dissipation demand side of the problem, addressing two aspects with different extent: (i) the estimation of the total amount of energy input by an earthquake, and (ii) its distribution among the structural components, that is the energy dissipation demand at the storey or structural member level. Most research effort has been devoted to the former while the later needs further development. In contrast to the energy dissipation demand, research on the energy dissipation capacity of structural elements and systems is almost inexistent and constitutes a fertile field for research.

Estimating the energy dissipation capacity of structural elements under seismic-type dynamic loadings is a cumbersome and challenging issue. One of the reasons for that is the path-dependency of the amount of energy that a structural element can dissipate until failure (i.e. the ultimate energy dissipation capacity). That is, it is influenced by the distribution of the plastic cycles during the deformation history [3]. The number and amplitude of cycles of deformation imposed by earthquakes has a very random nature. They can hardly be converted into a sequence of closed cycles of constant amplitude, as counting methods such as the rain-flow or the range-pair methods do for histories with few reversals to failure. Dynamic shake tests of complete or partial structures that include the structural component under study, are the most realistic way to reproduce the response of a structural component subjected to a real earthquake. This kind of tests consider phenomena such as the variations of boundary conditions on the structural component within the structure (i.e. changes of the inflexion point in columns) or the simultaneous effects of bidirectional bending moments and axial forces. However, evaluating the energy dissipation capacity of the structural component from its results is cumbersome due to the limited information provided by the instrumentation. In addition shake table tests are expensive and time consuming, therefore it is necessary to complete the information provided by this type of test with numerical simulations.

This paper discuss several approaches to estimate the energy dissipation capacity of reinforced concrete (RC) column bases from unidirectional and bidirectional shake table tests conducted on waffle-flat plate structures subjected to one and two horizontal components of ground motion.

2 DESCRIPTION OF SHAKE TABLE TESTS

2.1 Prototype structure and test specimen
A prototype structure consisting of a three-story RC waffle-flat-plate system supported on isolated columns was designed to sustain gravity loads and lateral seismic loads prescribed in the
current Spanish Seismic Code NCSE-02. The depth of the slab was 0.35m with voids forming an orthogonal grid of ribs separated 0.83 m to each other. The plate had a solid head around the columns. Along the perimeter, the plate was surrounded by transverse beams of the same depth (0.35m). The loads considered were: 3.13 kN/m2 and 3.46 kN/m2 as dead loads for floors and roof; and 2 kN/m2 and 1 kN/m2 as live loads for floors and roof, respectively. The structure corresponded to an ordinary building located in Granada (Spain) on soil type C (180<vs,30<360, where vs,30 is the average shear wave velocity in m/s). A behavior factor $\mu=2$ was adopted to reduce the elastic response spectra, as prescribed by the NCSE-02 for flat-plate systems. The mechanical properties of materials were $f_c=25$ MPa for the concrete compressive strength and $f_y=500$ MPa for the yield strength of steel. A capacity design and ductile detailing was used for the columns to prevent shear failure before flexure failure, resulting in a design base shear force coefficient of 0.33.

From the prototype, a partial structural model consisting in three columns and one story and a half was isolated. The test specimen was defined from the partial structural model applying scale factors of $\lambda_L=2/5$ for length, $\lambda_a=1$ for acceleration and $\lambda_s=1$ for stress. Other physical quantities were scaled to satisfy similitude requirements. Figure 1 shows the geometry and reinforcing details of the test specimen. The ratios of longitudinal reinforcement area, $A_{sl}$, to the gross section area, $A_g$, were $A_{sl}/A_g=0.031$ and $A_{sl}/A_g=0.024$ for the exterior and interior columns of the first floor, and $A_{sl}/A_g=0.018$ for all columns of the second floor. The volumetric ratio of shear reinforcement $\rho_s = V_{st}/V_{cc}$ was 0.03 for interior columns of the first floor and 0.02 for the rest. Where $V_{st}=nA_{st}l_s$ is the volume of the shear reinforcement ($n$, $A_{st}$, $l_s$ are the number, cross-section area and length of the stirrups) and $V_{cc}$ is the volume of concrete confined by the shear reinforcement. The waffle-flat plate was provided with an upper and lower base reinforcement consisting of a mesh of Ø6 bars separated 332 mm. Additional longitudinal rebars of Ø6 mm and Ø8 mm were located on the solid head of the plate around each column. A more detailed description of the prototype and test specimen can be found in references [4,5].

2.2 Test setup and instrumentation

A general view of the test setup and instrumentation is shown in Figure 2. The reacting mass and gravity loads were imposed through steel blocks attached at each floor (Figure 2) so as to satisfy similitude requirements. The instrumentation consisted of uniaxial accelerometers, displacement transducers and 472 strain gauges fixed to the longitudinal reinforcement. Strain gauges were located at column ends and in the critical regions of the solid heads of the plate around the columns. Figure 3 shows a detail of the position and number of strain gauges at the base of the exterior column (C1) that is the focus of this study. Two strain gauges, diametrically opposite each other and on the same cross section, were attached to each rebar. The average value of the two measurements was used as strain $\varepsilon_{Sr}$ of the rebar.
Figure 1: Test specimen
The displacement transducers measured in-plane translations, interstory drifts, and curvatures in columns and plate. The sampling rate of the data acquisition system was 600 Hz. Video cameras recorded the tests, and especially the movements at column bases.

Figure 2: Test set-up and instrumentation

Figure 3: Position of strain gauges (x) at base of exterior column (C1)

2.3 Seismic tests

Two identical specimens with the geometry and reinforcement described in section 2.1 were built in the Laboratory of Structures of the University of Granada (Spain) in different periods of time. They will be referred to as BS and BS1 hereafter. Steel of the same batch was used for both specimens. The yield stress of longitudinal rebars of diameters 8 (Ø8) and 6mm (Ø6) were 525 and 543MPa respectively. The yield stress of the steel used for stirrups was 656MPa. The same concrete mix was used for both specimens with different casting dates. Compression tests conducted on samples from concrete casted in specimen BS resulted in compressive strengths of 39 MPa at 28 days and 43 MPa the day of the tests. For specimen BS1 the results in compressive strengths of 43 MPa at 28 days and 44 MPa the day of the tests.

Both specimens were subjected to dynamic tests using an MTS 3×3 m² shake table. In both cases, the horizontal components (NS and EW) of ground acceleration recorded at Calitri during the Campano Lucano (1980) earthquake were used. Specimen BS was subjected to the NS component of the ground motion (unidirectional loading), while specimen BS1 was subjected simultaneously to the two components, NS and EW, of the ground motion. The original acceleration records were scaled in time by the scaling factor $\lambda_t=\left(\frac{\lambda_a}{\lambda_s}\right)^{0.5}=0.63$. The original acceleration records were scaled also in amplitude to 100%, 200%, 300% and 350% in case of specimen BS, and to 35%, 50%, 100%, 200% and 300% in case of specimen BS1. Due to a
problem with the control system of the shaking table, the 200% test was repeated twice. The first attempt to apply the records scaled by 200% to specimen BS1 (referred to as test C200i hereafter) was interrupted at half of its original duration. The test was then repeated with the same amplitude (test C200) and completed successfully. The amount of energy input to the specimen by the failed test C200i was only one third of that input in the successful test C200. In the case of specimen BS subjected to unidirectional loading, the four seismic simulations will be referred to as C100, C200, C300 and C350 hereinafter and the corresponding peak ground accelerations, PGAs, were 0.16g, 0.31g, 0.47g and 0.55g, where \( g \) is the acceleration of gravity. In the case of specimen BS1, subjected to bidirectional loading, the PGA does not necessarily occur in the NS or EW direction and it was calculated as follows. Calling \( G_{NS}(t) \) and \( G_{EW}(t) \) the horizontal acceleration at instant \( t \) in the NS and EW directions, the acceleration at instant \( t \) in a direction forming a counterclockwise angle \( \theta \) with EW-axis is \( G_A(t, \theta) = G_{NS}(t) \sin \theta + G_{EW}(t) \cos \theta \). The value of \( G_A(t, \theta) \) was calculated for different \( t \) and \( \theta \), and the maximum was taken as PGA. The PGAs obtained are 0.07g, 0.10g, 0.19g, 0.39g, 0.39g and 0.59g for the tests referred to herein as C35, C50, C100, C200i, C200 and C300 respectively.

3 ANALYSIS OF TEST RESULTS

For simplicity the following discussion will only focus on the response, in terms of energy dissipation capacity, of the exterior column C1 base. When necessary, reference will be done to the total amount of energy dissipated by the test specimen. A detailed description of the overall performance of the specimens can be found in references [4,5].

3.1 Overall performance of column bases

Specimen BS

During test C100 the strains in longitudinal bars at the column base remained below 0.75\( \varepsilon_y \). Early in test C200, a plastic hinge with ductile flexural yielding developed at the base of column C1, showing maximum strains of 1.13\( \varepsilon_y \). During test C300 cracks and concrete spalling was observed at column base, and the maximum strains of the longitudinal reinforcement reached 2.2\( \varepsilon_y \). During test C350 the base of column was heavy damaged, involving severe concrete crushing (see Figure 4) and strains up to 7.8\( \varepsilon_y \) were measured in the longitudinal reinforcement. Figure 4 shows the state of the base of column at the end of the tests.

![Figure 4: Specimen BS. Final state of exterior column C1](image)
Specimen BS1

At the end of tests C35 and C50 hairline flexural cracks were visible at column base. During these simulations, the maximum strains in longitudinal rebars reached $0.5 \varepsilon_y$. Strains increased during simulation C100 reaching $5.1 \varepsilon_y$. A visual inspection of the column base after tests C200i and C200 revealed concrete crushing and gauges attached to the longitudinal reinforcement measured strains up to $6.8 \varepsilon_y$. Concrete spalling occurred during test C300, and the maximum strains in longitudinal rebars reached $7 \varepsilon_y$.

![Figure 5: Specimen BS1. Final state of exterior column C1](image.png)

3.2 Procedure for evaluating the energy dissipated at column bases

The elastic strain energy stored, $W_{es,k}$, and the plastic strain energy dissipated, $W_{p,k}$, in a plastic hinge $k$ located at a column base from instant $t=0$ to a given instant $t=t_i$ is the sum of the energy stored/dissipated by concrete, $W_{c,k}$, and by steel, $W_{s,k}$. The term “plastic hinge” refers to the region of the column end delimited by two parallel planes perpendicular to the longitudinal reinforcement and spaced a distance $l_p$. $W_{c,k}$ and $W_{s,k}$ can be estimated as follows. The cross section of the plastic hinge, of depth $h$ and width $b$, is divided in $N \times N$ fibers of depth $h/N$ and width $b/N$. A material (steel or concrete) is assigned to each fiber, depending on the position of the rebars in the RC section. It is assumed that each fiber experiences the same strain along its length $l_p$. Based on these assumptions, the strain at any instant $i$ of any fiber $i$ of the cross section $\varepsilon_i(t)$, was estimated from the strains measured at the gauges adhered to the longitudinal rebars $\varepsilon_{gi}$ (see Figure 3) applying two different approaches:

a) Plane approximation. A plane defined by the equation $(x=ay+bz+c)$ was obtained at each time step using the method of least squares. The method minimizes the sum of the squared difference between the actual strain $\varepsilon_{gi}$ measured by the strain gauge $i$ located at coordinates $(y_i, z_i)$ of the cross section, and the estimated value $x_i$ on the plane. That is, the function to minimize is $E=\sum_{i=1}^{n}(\varepsilon_{gi} - (ay_i + bz_i + c))^2$, where $n$ is the number of strain gages in the cross section. Coefficients $a, b, c$ were obtained imposing $dE/da=0$, $dE/db=0$ and $dE/dc=0$. It is worth noting that when the plane approximation is applied, the strains used for computing the energy dissipated by the steel at points where data provided by gauges are available, are not the actual strains provided by the gauges, but
the strains estimated with the plane approximation. The degradation of anchorage conditions of the longitudinal reinforcement under cyclic loading is not necessarily the same in all rebars of the section; this can cause significant differences in rebars strains due to different bond degradation that are not captured with the plane approximation.

b) **Surface approximation.** The function “scattered interpolant” implemented in the software MATLAB [6] was used to determine the strain at each fiber and time step. Inside the perimeter defined by the rebars instrumented with strain gages the interpolation is done with a function with continuity type C1 [7]. Outside this perimeter the extrapolation is done using the gradient calculated in the perimeter [6]. It is worth noting that when the surface approximation is used, the strains used for computing the energy dissipated by the steel at points where data provided by gauges are available are always the actual strains provided by the gauges. That is, the surface passes through the points where the actual strain is measured during the test. This means that if all rebars of the section are instrumented with gauges, the energy dissipated by the steel with the surface approximation is “exact”. In this case, the possible differences in rebars strains due to different bond degradation are captured.

Figure 6 shows an example for each approach. While rebars remained in the elastic range, the surface obtained with both approaches was very similar. However, notorious differences among both approaches can be found when rebars undergoes plastic deformations, as seen in the figure.

![Figure 6: Cross section deformation. Approximation with plane (left) and with surface (right)](image)

Once the strains of the $N \times N$ fibers of the section were determined the corresponding stresses were estimated as follows. The stress on steel fibers $\sigma_{Sr}(t)$ were estimated using an energy conservative steel constitutive model (the Menegotto-Pinto model) [8]. The model incorporates strain-hardening and Bauschinger effects. Figure 7 shows the stress-strain curve obtained for this constitutive model and a loading history consisting of cycles of incremental amplitude.

![Figure 7: Constitutive law used for steel](image)
The stress on concrete fibers $\sigma_{C}(t)$ were estimated using four different constitutive models proposed in the literature and referred to hereafter as C01, C02, C07 and M. The stress vs strain plot for each model are shown in Figure 8. Models C01 and C02 were developed by Mohd [9] and model M is a simplified version of the model developed by Maekawa [10]. As can be observed in the figure, the main difference between model C01 and the other two models (C02 and M) is the shape of the unloading and reloading branches. In model C01 the unloading and reloading branches follow the same path, hence no energy is dissipated when reloading. In models C02 and M unloading and reloading branches follow different paths and some amount of energy is dissipated when reloading. Model C07 is the well known Mander model [11] and differs with the other three models in the shape of stress-strain curve when the concrete is reloaded after having experienced some permanent deformations in previous cycles. The simplification made in the Maekawa model consisted on neglecting the resistance of the concrete in tension and the re-contact effect. The common mechanical properties adopted for concrete in all models are: Young’s modulus $E_c=30000$ MPa, compressive strength $f_c=43$ MPa, strain at maximum compressive stress $0.28\%$. The four models were calibrated so that the area enclosed by the stress-strain curve under monotonic loading was the same. This was attained modifying slightly the strain at failure, that resulted in all cases very close to $1.5\%$.

![Figure 8: Constitutive laws used for concrete: a) C01; b) C02; c) C07; d) Maekawa](image)

Once the stress in each steel fiber and at each instant $t$ of a given plastic hinge $k$ is obtained, the energy dissipated by the steel reinforcement up to this instant, $W_{S,k}(t)$ is calculated as follows:

$$W_{S,k}(t) = \sum_{r=1}^{R} \int_{\varepsilon_{SR}(0)}^{E_{SR}(t)} \varepsilon_{SR} A_{SR} d\varepsilon_{SR}$$

where $A_{SR}$ the area of rebar $r$ and $R$ is the number of longitudinal rebars in the plastic hinge. Similarly, once the stress in each concrete fiber $\sigma_{C}(t)$ is determined, the energy dissipated by concrete $W_{C,k}$ until a given instant $t$ is obtained as follows:
\[ W_{C,k}(t) = \sum_{\text{concrete fibers}} \int_{\varepsilon_{CR}(t)}^{\varepsilon_{CR}(0)} l p b h \sigma_{Cr} d\varepsilon_{Cr} \]  

(8)

where the summation extends to all concrete fibers of the cross section. The total amount of energy stored and dissipated by a plastic hinge \( k \), \( W_k \), is then:

\[ W_k = W_{es,k} + W_{p,k} = W_{S,k} + W_{C,k} \]  

(9)

Figure 9 shows a typical history of \( W_k \). The recoverable elastic strain energy stored in the plastic hinge, \( W_{es,k} \), causes the oscillating part of the \( t-W_k \) curve. This part can be easily removed by taking the minimum envelope of the history of \( W_k \). The resulting curve is shown in Figure 9 with bold lines and represents the total dissipated energy \( W_{pk} \).

3.3 Energy dissipated at column bases of BS1 applying different approaches

Influence of plane or surface approximation

Figure 10 compares the total energy dissipated \( W_{pk} \) at column C1 base depending on the approximation used to estimate the deformation of the cross section. The comparison is made for the four constitutive laws for concrete (C01, C02, C07 and M) investigated. The abscissa indicates the time measured from the onset of the first test until the end of last test. The intervals of time between tests are not included. It can be seen that: (i) both approximations provide similar results irrespective of the constitutive law used, and (ii) the differences between the plane approximation and the surface approximation tend to be larger as the level of damage increases and the number of cycles of imposed deformations is larger (i.e. with time).
Influence of constitutive law for concrete

Figure 11 compares the total energy dissipated $W_{pk}$ at of column C1 base for the different constitutive laws considered for concrete. The comparison is made for both approaches (plan and surface) used to estimate the deformation of the cross section. As can be seen, constitutive models C01, C02 and M provide similar results; while model C07 estimates a much higher amount of dissipated energy. The reason can be easily found in the shape of the hysteretic loops of each model shown in Figure 8. The area enclosed by the reloading branch up to the stress reached in previous cycle of loading, when unloading in previous cycle reached positive strains is notably higher in model C07 than in models C01, C02 and M. For the columns tested in this study, the use of the constitutive model C07, either using the deformation of the cross section obtained with a plane or with a surface, is not appropriate because it would lead to absurd results as explained next. From measurements provided by the instrumentation during the test, it is possible to estimate the total energy input in the specimen $E_I$, the energy dissipated by damping $W_\zeta$ and the kinetic energy $W_k$. A detailed calculation can be found in [5]. Figure 12 shows the histories of $E_I$ and $W_\zeta + W_k$. The difference $E_I - (W_\zeta + W_k)$ must equal to the energy stored/dissipated by the structure (including columns and plate). However, the sum of $(W_\zeta + W_k)$ and the energy stored/dissipated by columns estimated with the constitutive law C07 and the plane approximation, curve named $(W_\zeta + W_k) + W_{col,plane}$ in Figure 12, or with the surface approximation,
curve named \((W_x + W_k) + W_{col, surface}\) in Figure 12, would yield values of total energy that exceed the total input energy, that is meaningless.

Figure 11: Total energy dissipated at base of column C1 approximating the deformation of the section with a: (a) plane; or a (b) surface

3.4 Energy dissipated at base of column C1 under uni- and bidirectional loading

This section compares the energies dissipated by the plastic hinge \(k\) located at the base of column C1 in specimens BS and BS1. The energies were estimated with the surface approach and the Maekawa constitutive law for concrete described above. Figure 13 shows the energy dissipated by the longitudinal reinforcement, \(W_{Sk}\), by the concrete \(W_{Sk,c}\), and the total dissipated energy at the plastic hinge, \(W_k\). It is worth recalling that the plastic hinged exhausted its energy dissipation capacity at the end of the test. Therefore, the ordinate of the final point of the \(W_k\) curve represents the ultimate energy dissipation capacity of the plastic hinge under unidirectional shaking (left plot of figure 13), and under bidirectional shaking (right plot of Figure 13). It is worth noting that, even though the distribution of dissipated energy between steel and concrete and the history of energies are different, the total amount of energy dissipated up to failure under unidirectional and bidirectional loading are quite similar (2083 Nm for BS and 1879 Nm for BS1); the difference is less than 10%. In other words, these results suggest that the ultimate
energy dissipation capacity of the column is basically the same if the column is subjected to
one or to two simultaneous components of the ground motion up to failure.

![Unidirectional and Bidirectional Energy Dissipation](image)

Figure 13: Energy dissipated at the base of column C1 in specimen (left) BS and (right) BS1

### 3.5 Moment-rotation and axial force-displacement curve at base of column C1 in BS1

For convenience in the following discussion, three axes are defined in the cross section: axis X is perpendicular to the cross section and passes through the centroid; perpendicular axes Y and Z are contained in the cross section and pass through its centroid. When the deformation of the cross section is approximated with a plane, the strain $\epsilon$ in a fiber of coordinates $(s_y, s_z)$ relates to the curvatures $\phi_Y$ and $\phi_Z$ and to strain $\epsilon_X$ of the center of gravity by the following expression:

$$
\epsilon = \epsilon_X + \phi_Y s_y - \phi_Z s_z
$$

(10)

The increment of axial force $\Delta N$ and bending moments $\Delta M_Y$ and $\Delta M_Z$ are related to the increments of stress $\Delta \sigma$ and strain $\Delta \epsilon$ in each fiber by the following equations:

$$
\Delta N = \int \frac{\Delta \sigma}{\Delta \epsilon} \left( \Delta \epsilon_X - s_Z \Delta \phi_Z + s_Y \Delta \phi_Y \right) dA
$$

(11)

$$
\Delta M_Y = \int \frac{\Delta \sigma}{\Delta \epsilon} \left( \Delta \epsilon_X - s_Z \Delta \phi_Z + s_Y \Delta \phi_Y \right) s_y dA
$$

(12)

$$
\Delta M_Z = \int \frac{\Delta \sigma}{\Delta \epsilon} \left( \Delta \epsilon_X - s_Z \Delta \phi_Z + s_Y \Delta \phi_Y \right) s_z dA
$$

(13)

Where $dA$ represents the area of the fiber. If the cross section is divided in a discrete number of fibers, the integral symbol is replaced by a sum, and the moment-curvature and axial force-strain relationships can be obtained with an incremental analysis that is summarized in Figure 13.
Figure 13: Incremental procedure to obtain the M-\(\phi\) and N-\(\varepsilon\) of the cross section

Recalling that it has been assumed that the strain in each fiber remains constant along a length \(l_p\) (length of the plastic hinge), multiplying \(\phi_Y\), \(\phi_Z\) and \(\varepsilon_X\) by \(l_p\) provides the rotations \(\theta_Y\), \(\theta_Z\) between the two outermost cross sections of the plastic hinge, and the elongation of the central fiber of the plastic hinge. Figure 14 shows the \(N-\Delta\), \(M_Y-\phi_Y\) and \(M_Z-\phi_Z\) relationships obtained for the base of column C1 in specimen BS1, using the plane approximation and the constitutive law M.

Figure 14: Moment-curvature and axial force-axial strain curves
4 CONCLUSIONS

The energy dissipated until failure at the base of two identical RC columns is investigated experimentally through shake table tests. The columns were tested within two partial structures (specimens) consisting of RC waffle flat plate systems. One specimen was subjected to one horizontal component of the ground motion, and the other specimen to two horizontal components. The energy dissipated by the columns was estimated from the readings provided the strain gauges attached to the longitudinal reinforcement by applying two approaches for estimating the deformation of the cross section (plane approximation and surface approximation), and using four different constitutive laws for concrete. The main conclusions can be summarized as follows:

1. The plane and the surface approximations provide similar results (irrespective of the constitutive law used for concrete) for low levels of damage (i.e. low number of cycles of imposed deformations on the plastic hinge). However, as the level of damage increases the differences become significant. One possible explanation if that the surface approximation can capture differences in steel strains due to different degrees of degradation of the anchorage conditions of the rebars under a large number of cyclic reversals. The plane approximation cannot capture these effects.

2. For the columns tested, the constitutive laws developed by Mohd and implemented in OpenSees as concrete type C01 and C02, and the model developed by Maekawa provide similar results (i.e. similar values of the energy dissipated by concrete). However the model developed by Mander and implemented in OpenSees as concrete type C07 is not appropriate for the particular column tested because it yields amounts of dissipated energy that exceed the total energy input during the tests to the overall specimens.

3. The ultimate energy dissipation capacity of the column is basically the same if the column is subjected to one or to two simultaneous components of the ground motion up to failure.

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ENERGY-BASED SEISMIC DESIGN: NEEDS OF ENERGY DAMAGE INDEX VALUES FOR SERVICEABILITY AND ULTIMATE LIMIT STATES FOR GRAVITY DESIGN BUILDINGS?

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Abstract

During the past earthquakes, different low ductile failure modes are observed in the gravity design structures and thus, the most of existing damage indices may fail to assess the damage of gravity design structures accurately in referring to the two main performance levels: immediate occupancy and ultimate limit state. Therefore, this study investigates the energy dissipated by the brittle structures and the possible damage indices based on energy for the damage assessment of gravity design frames. In the framework of an Energy-Based Seismic Design Approach, we need the assessment of the Demand and on the Capacity, both expressed in Energy. A methodology for the assessment of the seismic energy demands imposed on structures is already proposed, but not such methodology that makes consensus is proposed for the calculation of the Energy dissipation Capacity avoiding the Hysteretic models. The estimation of the energy expended by the building during an earthquake excitation is a tricky issue. For this purpose, this study considers the wavelet based energy estimation and compares it with different approaches for measuring the damages of a structure: the dominant inelastic period of a building and the more classical measure, the inter-story drift. IDA analysis are performed in energy, drift and inelastic period. Furthermore, the damage assessment results based on the expended energy for three gravity design buildings are compared and discussed relatively to the results expressed in inelastic period and drift. Finally, this study concludes that no significant effects of number of inelastic cycles to the damage assessment results for low ductile structures. However, this study also highlights the effects of number of inelastic cycles to the damage for medium and high ductile structures.

Keywords: Energy-Based Seismic Design, Damage indices, wavelet energy, inter-storey drift, dominant inelastic period, gravity design buildings.
1 INTRODUCTION

During the past earthquakes, different low ductile failure modes were observed in gravity design concrete frame structures. Joint failures, flexural failures, shear failures and combined failure of shear and flexure of mostly the column elements are common types of failure modes ([1] Saatcioglu et al., 2001). In particular, shear failures are observed in the short columns as shown in Figure 1. Such short columns are formed due to the opening placed to accommodate the windows. Therefore, the challenge is which damage index proposed in the literature is suitable to quantify the damage state of such structures more accurately. In the following text, it is briefly discussed the proposed damage indices in the literature.

Figure 1: Diagonal cracks in the short columns

A various damage indices have been proposed in the literature to assess the damage state of a structure subjected to a seismic excitation. The studies by [2] Cosenza et al. (1993) and [3] Bozorgnia and Bertero (2001) have summarized the many of the damage indices proposed in the literature. Basically, all the damage indices can be categorized into two different groups depending upon the damage index parameter or parameters used to define the index. They are called non-modal and modal parameter based damage indices. Furthermore, the common feature of many of those damage indices is that they are equal to zero when a structure remains in the elastic range during a seismic event while they are equal to 1 at the complete collapse of a structure.

Non-modal parameters based damage indices are defined by either using a damage parameter such as ductility, which can be defined in terms of curvature, rotation or displacement, inter-storey drift and energy or combination of few of those parameters. The ductility and the interstorey drift are the most commonly used non-modal damage parameters. [4] Powell and Allahabadi (1988) proposed a damage index based on the ductility defined in terms of displacement as expressed in Eq1.

\[
DI = \left( \frac{u_{\text{max}} - u_y}{u_{\text{mon}} - u_y} \right) = \left( \frac{\mu_{\text{max}} - 1}{\mu_{\text{mon}} - 1} \right)
\]

where the \( u \) and \( u_y \) are the maximum and yield displacement, respectively. 
\( \mu_{\text{max}} = u_{\text{max}}/u_y \) is the displacement ductility imposed by an earthquake and \( \mu_{\text{mon}} = u_{\text{mon}}/u_y \) is the monotonic ductility capacity of the structure.

When the ductility is defined in terms of the top displacement of a multi degree of freedom frame, this damage index fails to identify the concentration of damage in a single story.
Therefore, the inter-story drift damage index is used as a better non-modal parameter based damage index to quantify the damage of a structure. It is defined as the ratio of maximum inter-story drift at the center of mass to the ultimate inter-story drift, which usually corresponds to the 30% strength drop of the whole story, as given in Eq 2.

\[ DI = \left( \frac{ID_{u}}{ID_{n}} \right) \]  

(2)

Since the ductility and the drift based damage parameters do not account themselves the accumulation of damage due to the number of inelastic cycles that the structure is subjected and the energy dissipation demand, they could not estimate the actual damage state of a structure ([5] Mahin and Bertero (1981); [6] Mahin and Lin (1983)). However, it should be noted that drift based damage index could yield good results when assessing the damage state of a structure subjected to a near field seismic event which usually produces a single plus of loading causing the large plastic deformation in structural members or short duration events causing less amount of plastic deformations in the members but with few number of inelastic cycles. Moreover, they are the most commonly used damage indices by engineers and researchers due to their simplicity in the estimation of the global damage state of a structure.

Another widely used damage index is the [7, 8] Park and Ang (1985, 1987) damage index which is the linear combination of the ductility defined in terms of displacement and the hysteretic energy dissipation as expressed in the following form.

\[ DI = \left( \frac{\delta - \delta_{y}}{\delta_{u} - \delta_{y}} \right) + \beta \frac{E_{h}}{F_{y} \left( \delta_{u} - \delta_{y} \right)} \]  

(3)

\( \beta \) parameter is calibrated using the experimental data.

Since this damage index considers the hysteretic energy dissipation, it includes the cumulative effect of repeated cycles of inelastic response to the damage. However, experimental determination of the \( \beta \) parameter is difficult and the methodology is not well described as well. Later, [9] Kunnath et al. (1992) have modified the Park and Ang damage index basically by referring the moment-curvature response of plastic hinge region instead of the force-deformation response of a structural member. The modified damage index is expressed in Eq 4.

\[ DI = \left( \frac{\varphi - \varphi_{y}}{\varphi_{u} - \varphi_{y}} \right) + \beta \frac{E_{h}}{M \varphi_{u}} \]  

(4)

Even though, both Park and Ang damage index and the modified damage index by [9] Kunnath et al. are calibrated for the concrete member experimentally, they might not be appropriate for assessing the damage state of only gravity design structures without proper calibration of \( \beta \) parameter for poorly confined reinforced concrete members.

[3] Bozorgnia and Bertero (2001) have introduced two improved damage indices for generic inelastic single degree of freedom (SDOF) system in combining of displacement ductility and the hysteretic ductility \( \mu_{H} \) which is defined by [5] Mahin and Bertero (1981) as the ratio of hysteretic energy \( E_{h} \) to energy capacity \( E_{mom} \) under monotonically increasing lateral deformation. They are given in Eqs 5 and 6.
\[ DI_1 = (1 - \alpha_1) \left( \frac{\mu - 1}{\mu_{\text{mon}} - 1} \right) + \alpha_1 \left( \frac{\mu_H - 1}{\mu_{H\text{mon}} - 1} \right) \]  

\[ DI_2 = (1 - \alpha_2) \left( \frac{\mu - 1}{\mu_{\text{mon}} - 1} \right) + \alpha_2 \left( \frac{\mu_H - 1}{\mu_{H\text{mon}} - 1} \right)^{1/2} \]

where \( \alpha_1 \) and \( \alpha_2 \) are constants.

Modal parameters such as natural periods, mode shapes and modal damping ratios can also be used as damage parameters for seismic damage assessment of civil engineering structures. They are widely used for structural health monitoring of large civil engineering structures. Damage indices proposed in the literature using a modal parameter is referred in this paper as modal parameter based damage indices. However, the authors could find very few modal parameter based damage indices in the literature. [10] Di Pasquale and Cakmak (1990) have proposed a damage index based on the natural period of the vibration at the undamaged and damaged state of the structure. The damage index is expressed in the following form:

\[ DI = 1 - \frac{T_e}{T_d} \]  

where \( T_e \) and \( T_d \) are the natural period of undamaged and damaged the structure, respectively.

Out of the damage indices discussed before, this study considers only the drift and the natural period based damage indices (as given in Eqs. 2 and 7) for the seismic damage assessment of the three buildings selected in this study as the most suitable damage indices. In addition, two new damage indices are also considered in this study based on the wavelet based energy and the dominant inelastic period of the buildings. Furthermore, the damage assessment results of the three buildings from the four damage indices are compared and discussed. It is important note that the changes of the wavelet based energy and the dominant inelastic period as the damage is progressed are estimated using the continuous wavelet transform (CWT) with complex Morlet wavelet. The CWT method is discussed briefly in the following section.

This paper is organized in the following form that in Section 2, the continuous wavelet transform method and the new damage indices are introduced. In Section 3, the building description and the numerical modelling of the building are presented. In section 4, the seismic damage assessment results from the four different damage indices are discussed. This paper is briefly concluded in Section 5.

2 CONTINUOUS WAVELET TRANSFORM METHOD AND NEW DAMAGE INDICES

This section introduces a brief description of the wavelet transform and the wavelet energy based damage index. However, authors strongly recommend to readers to refer the key references to understand the theoretical background of the method ([11] Chui C.K., 1992).

CWT can be used to decompose of a function \( x(t) \) into frequency-time domain as defined in the following form:

\[ W_{(a,b)} = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^* \left( \frac{t-b}{a} \right) dt \]
where \( \psi^*(t) \) and \( b \) are the complex conjugate of \( \psi(t) \) and the parameter localizing the wavelet function in the time domain, respectively and \( W(a,b) \) are the CWT coefficients that represent the measure of the similitude between the function \( x(t) \) and the wavelet at the time \( b \) and the scale \( a \).

The complex Morlet wavelet which is commonly used for continuous wave transform as a basic function can be expressed as in Eq. 9 and its Fourier transform can be expressed as in Eq. 10. The band with parameter \( F_b \) is selected in order to optimize the time and frequency resolution.

\[
\psi(t) = \frac{1}{\sqrt{\pi F_b}} e^{2\pi if(t)} e^{-\frac{t^2}{F_b}}
\]  
(9)

\[
\tilde{\psi}(af) = \frac{1}{\sqrt{\pi F_b}} e^{iF_b \pi^2 (af-f_c^2)}
\]  
(10)

where \( f \) and \( f_c \) are Fourier frequency and central wavelet frequency.

2.1 Damage index based on wavelet energy

Furthermore, using the CWT method, which decomposes the signal \( x(t) \) into time-frequency resolution, wavelet energy for each scale \( a_i \) can be estimated as ([12] Minh-Nghi and Lardiés, 2006):

\[
E_{a_i} = \sum_j |W_{(a_i,b_j)}|^2
\]  
(11)

In other words, the \( E(a_i) \) is the summation of square modulus of wavelet coefficients over the number of translations \( j \) for a given value of scale \( a_i \). As a consequence, the total wavelet energy can be obtained as given in Eq. 12.

\[
E_t = \sum_i E_{a_i}
\]  
(12)

Therefore, this study proposes a new damage index based on the wavelet energy as expressed in Eq. 13 to take into account the effects of number of inelastic cycles to the damage:

\[
DI = \left( \frac{E_t}{E_u} \right)
\]  
(13)

where \( E_t \) is the total wavelet energy associated with the acceleration response at the top storey of a structure during the seismic excitation and it is estimated using Eqs. 11 and 12. It is important to note that the top storey response is selected to take into account the effects of the maximum applied response to the damage index. \( E_u \) is the ultimate energy of the structure. In order to estimate the ultimate energy of a structure, this study performed the incremental dynamic analyses for 14 real ground motions and the ultimate state of the structure is defined as 30% drop of the strength at any storey level. Therefore, \( E_u \) is estimated averaging of fourteen values of total wavelet energy \( E_t \) corresponding to the failure of the structure.
2.2 Damage index based on dominant inelastic period

The wavelet ridges are formed at an instantaneous period and time when the period of the response at a time is equal to the period of the dilated mother wavelet. Therefore, the periods of the vibration can be evaluated at the wavelet ridges where the CWT coefficients reach their maximum values. Since the CWT method is capable to decompose a non-stationary response into the time-period domain, the changing of the dominant inelastic period of a structure due to different level of damage can also be evaluated. Furthermore, it is important to note that the damage index given in Eq. 7 is inversely proportional to the damage period and subsequently, it results that damage index reaches 1 at the period of infinity. This may leads to a under estimation of the damage state of a structure. As a consequence of that this study proposes a new damage index based on the dominant inelastic period $T_d$ of the structure as given in Eq. 14:

$$DI = \frac{T_d - T_e}{T_u - T_e}$$ (14)

Where $T_e$ is the elastic period of the first mode and $T_u$ is the inelastic period of the structure at the ultimate limit state of the structure. This relation gives the damage index is linearly proportional to the dominant inelastic period.

3 BUILDING DESCRIPTION AND MODELLING

Three different gravity design buildings are investigated in this study. The first building is a six story reinforced concrete structure including one under-ground story as shown in Figure 2(a). It could be considered to be symmetric in plan and elevation. The floor plan is approximately rectangle with the dimensions of 45m and 14.5m in length and width, respectively. In the transverse direction, the building has two bays with the equal bay width of 7m while in the longitudinal direction, it has 16 bays with the equal bay width of 2.6m. The height of each story is 3.1m. Furthermore, it is worth to note that there are some interior in-fill walls for partitioning. The second building is 3 story reinforced concrete wall building shown in Figure 2(b). Initially, it was a reinforced concrete frame building but later, it has been retrofitted with lightly reinforced concrete walls. It could also be considered to be symmetric in plan and elevation. The building has one bay of 6.7m width in the transverse direction and the 12 bays with equal width of 4.4m in the longitudinal direction. Altogether, there are six reinforced concrete walls with equal cross section of 4.1x0.2m in the longitudinal direction. They are continuous to the roof with the same cross section. In-fill walls for interior partitioning are mainly in transverse direction.

![Figure 2: The 3D view of the buildings selected for this study.](image-url)
Third building is reinforced concrete frame building as shown in Figure 2(c). This building has a single bay of 7.49m width in the transverse direction while it has 10 bays with equal bay width of 4.22m in the longitudinal direction. This could also be considered to be symmetric in the plan and the elevation. It is important to note that there are series of short columns along the longitudinal direction and they are formed due to the presence of openings in the in-fill walls. This building was slightly damaged due an earthquake excitation and all the damages are concentrated in the short columns. The initiation of diagonal cracks in those short columns can be observed as shown in Figure 1(a). However, there is no other damage observed in in-fill walls in either the longitudinal or transverse directions.

A 3-dimensional (3-D) finite element model is developed for each of the building using [13] OpenSees finite element program in order to investigate the performance under seismic loadings. Figure 3(a) illustrates the longitudinal and transversal elevation of the first building. It consists of frame elements and truss elements to represent all the beams and columns, and masonry in-fill walls, respectively. Figure 3(b) illustrates the longitudinal and transversal elevation of the model of the second building which consists of frame elements to represent the beams, columns and the concrete walls. This model consists of the truss elements only in transverse direction. It is important to note that in the first and second building, the masonry in-fill walls are completely filled. Therefore, the truss elements are spanned over the full story height. However, the masonry in-fill walls in the longitudinal direction of the third building are partially filled for accommodating the windows and the doors. Therefore, in the third model shown in Figure 3(c) truss elements are spanned only over the height of the in-fill wall forming the short columns. Furthermore, similar to the first and the second model, it also consists of frame elements to represent the beams and the columns.

All frame (beams and columns) and wall elements in the three models are inelastic beam-column elements available within OpenSees framework. They are based on the force formulation.
The material nonlinearity of the concrete represents a uniaxial Kent-Scott-Park concrete material model with degraded linear unloading/reloading stiffness according to the work of Karsan-Jirsa and no tensile strength. Since there are no adequate shear reinforcements provided in the columns and the beams in all three buildings, the confinement effect of the core concrete is minimized.

The masonry in-fill walls in all the three models are conveniently modeled as diagonal struts along its compressed diagonal. The properties of the material and the parameters required to define the geometry of the compressed diagonals are taken as given in [14] Eurocode 8. Figure 4(a) shows the axial force-axial deformation hysteretic response of the truss element during an earthquake. Figure 4(b) indicates the backbone curve used for the concrete short columns with cross section of 0.7x0.2m and the hysteretic response.
Figure 4. Hysteretic response of (a) axial force-axial deformation (b) shear force-shear deformation.

It is important to note that all the short columns are modeled in taking into account the axial, bending and shear effects. A nonlinear shear force-shear deformation response is attached to the sectional response of the fiber section which accounts the axial-moment interaction. Therefore, the shear deformation is uncoupled from the axial-moment interaction in the section stiffness. However, the shear and bending forces are coupled at the element level because the equilibrium is enforced along the beam element ([15] Marini and Spacone, 2006). The hysteretic response of the shear force-shear deformation is represented by the Pinching4 Material available in OpenSees finite element program. The backbone curve is defined for the concrete section using the modified compression field theory which is implemented in the Response 2000 ([16] Bentz, 2001). Using the Response 2000, the member analysis is performed assuming the fixed support at the bottom of the column while the load on the continuous column. The floor slabs are represented by rigid floor diaphragms ignoring the effect of the flexibility and all the degrees of freedom at the base nodes are assumed to be restrained.

4 RESULTS AND DISCUSSION

This study compares the damage assessment results by the four damage indices: the inter-story drift, wavelet energy and inelastic period based damage indices as discussed in the section 1 and 2. As indicated in Eqs. 2, 13 and 14, the three damage indices require the calibrated damage parameters corresponding to the ultimate limit state of the structure. Since they are defined as the ratio of maximum damage parameter imposed by an earthquake to the ultimate damage parameter, the damage indices are equal to 1 at the ultimate limit state of the structure. The first part of this section discusses the adopted procedure for estimating the serviceability and ultimate damage parameters.

4.1 Incremental dynamic analysis curves

The promising tool that can be used to calibrate the damage parameters corresponding to the different performance levels is the incremental dynamic analysis (IDA) that has been developed by [17] Vamvatsikos and Cornell (2002). IDA involves nonlinear dynamic analysis of a structural modal under a selected set of ground motions. For this study, 14 real ground motions are selected from the [18] PEER data base. IDA is performed for several scaling levels of each ground motion in order to force the structure to behave all the way from elasticity.
to its global failure. Subsequently, the IDA curves of structural response are generated as measured by a damage parameter versus the scale factor of the ground motion. The serviceability limit state is defined as an elastic limit of the structure while the ultimate limit state is defined based on the type of the failure mode observed in the critical elements in which larger plastic deformation is expected.

Figure 5. IDA curves for the first building (a) scale factor versus inter-story drift (b) scale factor versus wavelet based energy.

Figure 6. IDA curves for the second building (a) scale factor versus inter-story drift (b) scale factor versus wavelet based energy.
Figure 7. IDA curves for the third building (a) scale factor versus inter-story drift (b) scale factor versus wavelet based energy.

It is clear from Figure 5(a), 6(a) and 7(a) that IDA curves start as straight line in the elastic range and then shows the softening by displaying a tangent slope less than the elastic. Figure 5(a) illustrates significant softening after the initial straight branch of each curve due to compression failure of the diagonal struts which represent in-fill walls at the first storey level. However, some of the IDA curves harden displaying the slope almost equal to the elastic slope even at the higher drift level. Finally, all the IDA curves start softening again showing the larger record-to-record variability due to different characteristics of the ground motions and their effect on the inelastic response. Furthermore, Figure 6(a) and 7(a) also indicate the significant softening displaying the effect of yielding. They also display the record-to-record variability but it is not significant as in Figure 5(a). This could be due to the fact that both the second and the third buildings are subjected to a lower amount of plastic deformations before the failure. IDA curves generated as measured by wavelet based energy versus scale factor shown in Figure 5(b), 6(b) and 7(b) display the gradual increment of the amount of energy dissipation as increasing the scale factor beyond the elastic limit. This proves that wavelet based energy could be a good damage index parameter. Furthermore, it should be noted that the last point of each of the IDA curves corresponds to the failure of the structure. In the next section, it is discussed in details how the failure point on the IDA curve is defined incorporating element failure to the global failure of the structure.

4.2 Defining serviceability and ultimate limit state

As discussed before, the serviceability limit state of each structure is defined as the limit of the elastic limit. For the first building, serviceability limit point on each IDA curve is defined corresponding to the failure of the first story in-fill wall while for the second and the third building it is corresponding to the flexural yielding of the first story columns. Table 1 summarizes the average values of the wavelet energy and the inert-story drift based damage parameters corresponding to the serviceability limit state. It is important to note that the elastic period of each of the three buildings are estimated from the ambient vibration measurements using the CWT method as reported in paper by [19] Wijesundara et al. (2012). Furthermore, each of the numerical models is validated by comparing the estimated first mode period from ambient
vibration measurements to the first mode period obtained from the Eigen value analysis. Table 1 also gives the measured and the numerical 1st mode periods for the three buildings considered in this study.

Table 1: Damage Parameters at Serviceability limit state.

<table>
<thead>
<tr>
<th>Bld. No</th>
<th>Wavelet energy ($E_t$)</th>
<th>Inter-story Drift, DR (%)</th>
<th>Measured 1st mode elastic Period, T (s)</th>
<th>Numerical 1st mode elastic Period, T (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11978695</td>
<td>0.58</td>
<td>0.330</td>
<td>0.320</td>
</tr>
<tr>
<td>2</td>
<td>10359584</td>
<td>0.29</td>
<td>0.230</td>
<td>0.235</td>
</tr>
<tr>
<td>3</td>
<td>17780476</td>
<td>0.46</td>
<td>0.303</td>
<td>0.312</td>
</tr>
</tbody>
</table>

According to the study by [17] Vamvatsikos and Cornell (2002), the ultimate limit state point on an IDA curve is defined as a point where the IDA slope is equal to 20% of the elastic while it also belongs to a softening branch. However, this study incorporates the element performance to define the ultimate limit state point of the structure on the IDA curve. From the numerical investigation, it is evidenced that the global failure of the first building results in the failure of first story column elements in flexure due to the formation of soft-story mechanism. As the result of the gravity design of the frames, effective depths of the beams are higher than the columns and, in turns, this results beam sections have more strength and stiffness than the corresponding column sections. Therefore, plastic deformations are concentrated at the first story columns forming the soft story mechanism. The failure of the second building results in the failure of wall elements in flexure at their bases. In both the cases, the strength drop of the flexural elements results mainly due to the crushing of core concrete. Therefore, in this study, the failure of individual frame element in flexure is defined by the 30% drop from the peak load at the peak rotation ductility. The cumulative effect of strength drop of the individual elements in any story causes the significant drop of the story shear capacity. The drop of the story shear capacity is approximately equal to the drop of the moment capacity of individual element based on the assumption that all the columns in the story level reach their peak deformation simultaneously. As a consequence of this, the global failure points on IDA curves of the first and the second buildings corresponds to the 30% drop from the peak shear capacity of any story level.

Due to opening placed for the windows and doors as shown in Figure 1, the short columns in the third building has already subjected to an initiation of diagonal cracks leading to the shear failure. An individual element failure in shear is defined as stating of the negative incremental stiffness of the shear force-shear deformation response of the element. Thus, the global failure point of the structure on the IDA curve is defined as all the short columns in the story level reach their individual element failure. Table 2 summarizes the average values for the three damage parameters corresponding to the ultimate limit state.

Table 2: Damage Parameters at Ultimate Limit State.

<table>
<thead>
<tr>
<th>Bld. No</th>
<th>Wavelet energy ($E_t$)</th>
<th>Inter-story Drift, DR (%)</th>
<th>1st mode inelastic Period, $T_u$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42330660</td>
<td>2.68</td>
<td>1.13</td>
</tr>
<tr>
<td>2</td>
<td>48091036</td>
<td>1.20</td>
<td>0.789</td>
</tr>
<tr>
<td>3</td>
<td>31397630</td>
<td>0.97</td>
<td>0.702</td>
</tr>
</tbody>
</table>
As specified in [20] FEMA 356 (2000), the inter-story drift limits corresponding to the collapse prevention performance level for well design reinforced concrete frame and concrete wall buildings for seismic loading are 4% and 2%, respectively. The collapse prevention performance level, which is defined as the post-earthquake damage state in which building is on the verge of partial or total collapse, corresponds to the ultimate limit state of the structure. However, the inter-story drift limits in Table 2 is significantly lower than the specified limits in [20] FEMA 356 (2000). This indicates the vulnerability of gravity design structures against a seismic loading.

Table 3 present the damage levels obtained from the wavelet energy and the inter-story drift based damage indices corresponding to the serviceability and ultimate limit states of the three buildings.

<table>
<thead>
<tr>
<th>Bld. No</th>
<th>Serviceability Limit State</th>
<th>Ultimate Limit State</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wavelet energy damage index</td>
<td>Inter-story damage index</td>
</tr>
<tr>
<td>1</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>2</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>3</td>
<td>0.57</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 3: Damage levels corresponding to serviceability and ultimate limit states.

It is clear from Table 3 that the damage levels obtained from the wavelet energy and the inter-story drift based damage indices are not significantly different and they are correlated well with the calibration of minor damage state (0.1 < Damage Index < 0.25), which is proposed by [7] Park et al. (1985) based on observations of post-earthquake damage of reinforced concrete buildings, for the first and the second buildings for which the ultimate limit state is defined in failure of element in flexure. However, the higher damage level is observed for the third building due to the brittle failure of element in shear. As a consequence, the damage levels for the assessment of a structure using the either of wavelet energy or inter-story drift based damage index must be specified according to the mechanism which leads to the global failure of a structure.

4.3 Comparison

In the first part of this section, the correlation between the wavelet energy based damage index and the drift based damage index is investigated based on the results of the IDA. For this purpose, each IDA curve is normalized by its ultimate value of the damage parameter. Figure 8 (a), (b) and (c) illustrate variation of wavelet energy based damage index against the inter-story drift based damage index for the first, second and third building, respectively. The gray solid line in each figure indicates the linear correlation between the damage indices while the black solid line indicates the linear regression curve fitted to the data points.

It is clear from Figure 8(b) and (c) that strong linear correlation between the wavelet energy based damage index and the inter-story drift based damage index can be observed in the second and third structures which have relatively low drift capacity. Therefore, it is proven that the effects of number of inelastic cycles to the damage are not significant for relatively lower deformation levels. However, Figure 8(a) shows highly scattered variation of wavelet energy based damage index against the inter-story drift based damage index due to the fact that the wavelet energy based damage index can account for the effects of number of inelastic
cycles and the level of maximum displacement to the damage of a structure. Therefore, this highlights the effects of number of inelastic cycles to the damage for medium level of inelastic deformations. However, the linear correlation between the two damage indices can still be assumed.

![Figure 8](image1.png)  
**Figure 8.** Correlation between wavelet energy based damage index and the inter-story drift based damage index (a) first building (b) second building and (c) third building.

In the second part of this section, the damage assessment results by the four damage indices expressed in Eqs. 2, 7, 13 and 14 are compared. For this comparison, the three structural models are subjected to 50 real ground motions. It must be noted that the ground motion records that have been already used in the previous section are not included in this set of ground motions. Since the structure is symmetric in plan and the elevation, the unidirectional loading in the longitudinal direction (weaker direction) is considered. Figure 9 illustrates the assessment results. Each damage indices used for this comparison is equal to 1 at the ultimate limit state of a structure, therefore the resultant damage index above 1 are considered to be equal to 1.

![Figure 9](image2.png)  
**Figure 9.** Comparison of damage assessment results by the four damage indices (a) first building (b) second building and (c) third building.

Figure 9 (a), (b) and (c) show variation of damage assessment results by the four equations against the PGA values. However, the main objective of this study is to compare the damage assessment results by the four equations. As discussed before, the damage index based on the inter-storey drift is widely used for the damage assessment of the building by the engineers and the researchers. Therefore, this study further investigates the correlation between the drift based damage index and the other damage indices.

The plots of (a), (b) and (c) in Figure 10, 11 and 12 illustrate the correlation between the inter-story drift based damage index and the wavelet energy based damage index, inelastic period based damage index as proposed in this study and the inelastic period based damage index as proposed by [10] Di Pasquale and Cakmak (1990), respectively while Figure 10(d),
11(d) and 12(d) illustrate the correlation between the period based damage index proposed in this study and the wavelet energy based damage index for the three different buildings considered in this study.

Figure 10. Correlation between different damage indices reference to the first building.
Figure 11. Correlation between different damage indices reference to the second building.
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Figure 12. Correlation between different damage indices reference to the third building.

The gray solid line in each figure indicates the linear correlation between the damage indices while the black solid line indicates the linear regression curve fitted to the data points. Figure 10(a), 11(a) and 12(a) show that the inter-story drift based damage index correlates linearly with the wavelet energy based damage index similar to the results in the first section. However, a significant scatter can be observed at higher damage levels for the first building. Some earthquakes induce higher level of wavelet energy based damage while they induce lower inter-story drift based damage and conversely, some earthquakes induce low level of wavelet based damage while they induce higher inter-story drift based damage. This is because the wavelet energy based damage index can account for the effects of number of inelastic cycles and the level of maximum displacement to the damage of a structure while inter-story drift based damage index can only account for the effects of maximum inter-story displacement. However, linear regression lines matched with linear correction lines highlights that the effects of number of inelastic cycles to the damage can be minimized for gravity design buildings considering the average response. Furthermore, it is important to note that the scatter is insignificant at lower inelastic deformation levels highlighting the fact that the effects of number of inelastic cycles to the damage are not significant at lower inelastic deformation levels.

Similar to the wavelet energy based damage index, Figure 10(b), 11(b) and 12(b) also show that a good correlation exists between the inter-story drift based damage index and the inelastic period based damage index proposed in this study for gravity design frames. But,
Figure 10(c), 11(c) and 12(c) indicate significant under estimation of the damage at higher damage levels by the inelastic period based damage index proposed by [10] Di Pasquale and Cakmak (1990) compared to the inter-story drift based damage. This is due to the fact that a structure reaches the damage level 1 at the infinity period of the structure, if the damage level 1 is considered as the ultimate limit state of the structure. However, this predicts the lower and medium levels of damage quite similar the other damage indices for the frame structures.

5 CONCLUSIONS

This study proposed new two damage indices based on the wavelet energy and the inelastic period to account the effects of number of inelastic cycles to the damage of a structure. From the damage assessment results of gravity design buildings, linear correlations between the inter-story drift based damage index and the wavelet energy based damage index, and inelastic period are observed. Therefore, it can be concluded that the drift based damage index can be used for the damage assessment of gravity design building in which relatively lower inelastic deformation capacity exist with adequate accuracy ignoring the effects of number of inelastic cycles.

The damage levels for the assessment of a gravity design building using either of wavelet energy or inter-story drift based damage index must be specified according to the failure modes of element which lead to the global failure. However, this study considers only two damage levels: serviceability and ultimate limit state. Therefore, if the element failure in low ductile flexure mode leads to the global failure of a gravity design frame, the wavelet energy or inter-story drift based damage index at the serviceability limit state can be taken to be equal to 0.25 while it is equal to 1 at the ultimate limit state. Furthermore, if the element failure in brittle shear or combination of flexure and shear mode leads to the global failure of a gravity design frame, they can be taken to be equal to 0.5 at the serviceability limit state while they are equal to 1 at the ultimate limit state.

Furthermore, this study highlights the importance of accounting the effects of the number of inelastic cycles to the damage assessment for the structures which have high ductile failure mode as in the case of ductile concrete or steel moment resisting frames. However, further study is required to establish the correlation between the damage parameters. In order to apply these issues in the framework of Energy-Based Design, the range of values of the Energy dissipation Capacity should be put in perspective with respect to the range of values of Input Energy spectra.

REFERENCES


ACCURATE COMPUTATION OF FREQUENCY RESPONSE FUNCTIONS OF DUAL CRAIG-BAMPTON REDUCED SYSTEMS

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Abstract

The dual Craig-Bampton method (DCBM) employs modes with free interfaces to build the reduction bases of the substructures, but assembles the substructures using interface forces, which is called dual assembly. The DCBM preserves the sparsity of the final reduced mass and stiffness matrices similar to the classical Craig-Bampton reduced matrices, which makes the reduced matrices simpler and less populated than the reduced matrices of other free interface methods.

However, the DCBM is not a Rayleigh-Ritz transformation only on the displacement degrees of freedom, but reduces the dually assembled system. Thereby, interface kinematic conditions are transformed, allowing incompatibilities between the substructures unlike other substructuring techniques. This interface kinematic transformation enforces only a weak displacement compatibility between the substructures. As a consequence, the reduced-order model always has as many negative eigenvalues as Lagrange multipliers used for the dual assembly. When applying a DCBM reduction, the reduced system is no longer positive (semi)definite. This means that the reduced system is unstable. Although rendering direct time integration impossible, this does not affect the computation of frequency response functions (FRFs), which is demonstrated in this paper.

The feasibility of a reliable and accurate FRF computation of DCBM reduced systems is investigated in detail. Hereby, DCBM reduced system means that a reduced system after a DCBM reduction is used without any modifications. It is demonstrated that the unstable behavior of the DCBM reduced system does not negatively influence the computation of the FRFs. Following this, methods for the further reduction are evaluated: modal reduction and interface reduction are applied to the DCBM reduced system. Moreover, the reduced system is enhanced by incorporating modal truncation vectors. This guarantees static correctness and enables a very accurate approximation of antiresonances. This is illustrated by three different examples. Thereby, the resulting FRFs of the DCBM reduced systems are compared to the FRFs of the unreduced systems.

Keywords: Dynamic Substructuring, Component Mode Synthesis, Model Order Reduction, Dual Craig-Bampton Method, Frequency Response Functions.
1 INTRODUCTION

The Craig-Bampton substructuring method [1] is one of the most popular approaches used for dynamic analysis and reducing the order of large finite element models. The success of the method is based on its accuracy, its simplicity, its ease of implementation, and its efficient use of computer resources [2]. The Craig-Bampton method (CBM) is classified as a fixed interface method, since it is based on fixed interface vibration modes and constraint modes: after dividing a large model into a certain number of substructures, each substructure is analyzed and reduced separately.

In [3], a free interface method was suggested as dual counterpart to the CBM, which is called the dual Craig-Bampton method (DCBM). The DCBM employs free interface vibration modes, (residual) attachment modes, and rigid body modes to build the reduction basis of the substructures, but uses interface forces to assemble the substructures, which is referred to as dual assembly. It was shown that such a method can present some advantages, although its application requires non-classical assembly and solution techniques. The dual Craig-Bampton method leads to simpler reduced matrices compared to other free interface methods [4, 5, 6] and preserves the sparsity of the reduced matrices similar to the classical Craig-Bampton matrices [3]. Comparing the approximation accuracy of the lowest eigenvalues, the DCBM often outperforms the CBM using the same number of normal modes per substructure as a reduction basis with comparable computational effort [7]. Approaches for improving the accuracy of the DCBM are presented in [8].

However, the dual Craig-Bampton reduced system always has as many non-physical negative eigenvalues as interface coupling conditions due to the weak interface compatibility between the substructures. If a very small reduction basis is used for the substructures (using only a very small number of free interface vibration modes), the negative eigenvalues of the reduced problem show up in the low frequency range [3, 9]. If the reduction basis is large enough, the negative eigenvalues are shifted in the high frequency range and do not show up if only the lowest eigenvalues of the original system are approximated, which is commonly the case. This is not a problem if only eigenvalues and eigenfrequencies have to be approximated, as was done in most studies in the past.

Nevertheless, the non-physical negative eigenvalues of the reduced dually assembled problem are intrinsic to the reduction process using the DCBM. Thus, when applying a DCBM reduction, the reduced system is not positive definite anymore. Obtaining a reduced system that is not positive definite means that this reduced system is unstable. This renders a straightforward time integration of the DCBM reduced system impossible. In [10], the unstable behavior when time-integrating DCBM reduced systems without further modifications is illustrated and an approach to overcome this instability is suggested: a modal analysis of the reduced system is performed as a subsequent step to the dual Craig-Bampton reduction. Only modes corresponding to positive eigenvalues are retained for transient analysis. This allows for stable time integration, but necessitates a subsequent modal reduction of the DCBM reduced system.

Since direct time integration is impossible, computation of frequency response functions (FRFs) of dual Craig-Bampton reduced systems is investigated in this paper. The feasibility of a reliable and accurate FRF computation of DCBM reduced systems is demonstrated in detail. Hereby, DCBM reduced system means that a reduced system after a DCBM reduction is used without any modifications. It is illustrated that the unstable behavior of the DCBM reduced system does not negatively influence the computation of the FRFs. Following this, methods for the further reduction of DCBM reduced systems are evaluated: modal reduction and modal inter-
face reduction are applied. Moreover, the reduced system is enhanced by incorporating modal truncation vectors: this guarantees static correctness and enables a very accurate approximation of antiresonances.

In Section 2, the original formulation of the dual Craig-Bampton method for undamped systems is concisely surveyed and the terminology and notations used throughout this paper are outlined. The dual assembly procedure (Section 2.1) and the reduction of the dually assembled system (Section 2.2) are recalled. Following this, both application of the modal truncation augmentation method (Section 2.3) and interface reduction (Section 2.4) for dual Craig-Bampton reduced systems are briefly outlined. A strategy for removing the negative eigenvalues of a dual Craig-Bampton reduced system is presented Section 2.5. Section 3 demonstrates the theory for computation of FRFs either by direct computation (Section 3.1) or by modal superposition (Section 3.2). Computation of FRFs of dual Craig-Bampton reduced systems is illustrated in detail by way of example in Section 4. Thereby, dual Craig-Bampton reduced systems with and without modal truncation augmentation, with and without interface reduction, as well as with and without removing negative eigenvalues are considered. Three different examples are investigated: the Benfield truss in Section 4.1, a three-dimensional beam frame in Section 4.2 and a two-dimensional solid in Section 4.3. The resulting FRFs of the DCBM reduced systems are compared to the FRFs of the unreduced systems. Finally, a brief summary of findings, recommendations for the computation of FRFs of dual Craig-Bampton reduced systems, and conclusions are provided in Section 5.

2 DUAL CRAIG-BAMPTON METHOD

We briefly summarize the dual Craig-Bampton method (DCBM) for undamped structures [3]. Consider a domain that is divided into \( N \) non-overlapping substructures such that every node belongs to exactly one substructure except for the nodes on the interface boundaries. The undamped equations of motion of one substructure \( s \) are

\[
M^{(s)} \ddot{u}^{(s)} + K^{(s)} u^{(s)} = f^{(s)} + g^{(s)}, \quad s = 1, \ldots, N.
\]

The substructure’s equations of motion (1) have \( n^{(s)} \) DOFs. \( M^{(s)}, K^{(s)}, \) and \( u^{(s)} \) are the substructure’s mass matrix, stiffness matrix, and displacement vector, respectively. \( f^{(s)} \) is the external force vector and \( g^{(s)} \) is the vector of internal forces connecting adjacent substructures at their boundary DOFs.

2.1 Dual assembly

One way to enforce interface compatibility between the different substructures is to consider the interface connecting forces \( g^{(s)} \) in Eq. (1) as unknowns. These interface forces \( g^{(s)} \) must be determined to satisfy the interface compatibility condition (interface displacement equality) and the local equations of motion of the substructures \( s = 1, \ldots, N \):

\[
\sum_{s=1}^{N} B^{(s)} u^{(s)} = 0 \tag{2}
\]

\[
M^{(s)} \ddot{u}^{(s)} + K^{(s)} u^{(s)} + B^{(s)T} \lambda = f^{(s)} \tag{3}
\]

\( B^{(s)} \) is a signed Boolean matrix expressing the interface displacement compatibility for the substructure DOFs \( u^{(s)} \) in Eq. (2). Vector \( B^{(s)T} \lambda \) represents the interconnecting forces between
substructures, which corresponds to the negative interface reaction force vector \( g^{(s)} \) in Eq. (1). This means \( g^{(s)} = -B^{(s)^T} \lambda \), where \( \lambda \) is the vector of all Lagrange multipliers acting on the interfaces, which are the additional unknowns. Using the block diagonal matrices

\[
M_{bd} = \begin{bmatrix} M^{(1)} & 0 \\ \vdots & \ddots \\ 0 & M^{(N)} \end{bmatrix}, \quad K_{bd} = \begin{bmatrix} K^{(1)} & 0 \\ \vdots & \ddots \\ 0 & K^{(N)} \end{bmatrix},
\]

(4)

the corresponding partitioned, concatenated vectors and Boolean matrix

\[
u_{bd} = \begin{bmatrix} u^{(1)} \\ \vdots \\ u^{(N)} \end{bmatrix}, \quad f_{bd} = \begin{bmatrix} f^{(1)} \\ \vdots \\ f^{(N)} \end{bmatrix}, \quad B = \begin{bmatrix} B^{(1)} & \cdots & B^{(N)} \end{bmatrix},
\]

(5)

substructure Eqs. (2) and (3) can be assembled as

\[
\begin{bmatrix} M_{bd} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_{bd} \\ \lambda \end{bmatrix} + \begin{bmatrix} K_{bd} & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} u_{bd} \\ \lambda \end{bmatrix} = \begin{bmatrix} f_{bd} \\ 0 \end{bmatrix}.
\]

(6)

\( B \) is referred to as the constraint matrix and Eq. (6) is referred to as the dually assembled system. In the hybrid formulation of Eq. (6), the Lagrange multipliers \( \lambda \) enforce the interface compatibility constraints and can be identified as interface forces. Constraint matrix \( B \) is a signed Boolean matrix if the interface degrees of freedom match, i.e., for conforming mesh conditions on the interfaces between the substructures.

### 2.2 Reduction of dually assembled system

Considering the equations of motion (3) of substructure \( s \) without external forces \( f^{(s)} \), every substructure can be seen as being excited by the interface connection forces \( g^{(s)} = -B^{(s)^T} \lambda \). The displacements \( u^{(s)} \) of each substructure are expressed in terms of local static solutions \( u_{\text{stat}}^{(s)} \) and in terms of eigenmodes associated with the substructure matrices \( K^{(s)} \) and \( M^{(s)} \):

\[
u^{(s)} = u_{\text{stat}}^{(s)} + \sum_{j=1}^{n^{(s)}-n_{r}^{(s)}} \theta_{j}^{(s)} \eta_{j}^{(s)}, \quad \text{with:} \quad u_{\text{stat}}^{(s)} = -K^{(s)^+} B^{(s)^T} \lambda + \sum_{j=1}^{n_{r}^{(s)}} r_{j}^{(s)} \alpha_{j}^{(s)}
\]

(7)

\( n^{(s)} \) is the dimension of the local substructure problem. Matrix \( K^{(s)^+} \) is equal to the inverse of \( K^{(s)} \) if there are enough boundary conditions to prevent the substructure from floating when its interface with adjacent substructures is free. If a substructure is floating, then a generalized inverse \( K^{(s)^+} \) has to be used, and \( R^{(s)} = \begin{bmatrix} r_{1}^{(s)} & r_{2}^{(s)} & \cdots & r_{n_{r}^{(s)}}^{(s)} \end{bmatrix} \) is the matrix containing the \( n_{r}^{(s)} \) rigid body modes as columns, which are assumed to be orthonormalized with respect to the mass matrix \( M^{(s)} \). Vector \( \alpha^{(s)} \) contains the amplitudes \( \alpha_{j}^{(s)} \) of the rigid body modes \( r_{j}^{(s)} \). The flexibility matrix \( G^{(s)} \) in inertia-relief format is computed from any generalized inverse \( K^{(s)^+} \) by projecting out the rigid body modes \( R^{(s)} \) using the inertia-relief projection matrix \( P^{(s)} \), which is defined as \([2][11]\)

\[
P^{(s)} = I^{(s)} - M^{(s)} R^{(s)} R^{(s)^T} \quad \text{and gives} \quad G^{(s)} = P^{(s)^T} K^{(s)^+} P^{(s)}.
\]

(8)
The amplitudes $\eta_j^{(s)}$ of the local eigenmodes $\theta_j^{(s)}$ in Eq. (7) are grouped in the vector $\eta^{(s)}$. The local eigenmodes $\theta_j^{(s)}$ satisfy the generalized eigenproblem

$$\begin{pmatrix} -\omega_j^{(s)} M^{(s)} + K^{(s)} \end{pmatrix} \theta_j^{(s)} = 0. \quad (9)$$

The eigenmodes $\theta_j^{(s)}$ are orthonormalized with respect to the mass matrix $M^{(s)}$. An approximation of the general solution in Eq. (7) is obtained by retaining only the first $n_\theta^{(s)}$ free interface normal modes $\theta_j^{(s)}$ corresponding to the $n_\theta^{(s)}$ lowest eigenvalues $\omega_j^{(s)}$. Calling $\tilde{\Theta}^{(s)}$ the matrix containing these eigenmodes in its columns, the approximation of the displacements $u^{(s)}$ of the substructure is written as

$$u^{(s)} \approx -G^{(s)} B^{(s)^T} \lambda + R^{(s)} \alpha^{(s)} + \tilde{\Theta}^{(s)} \eta^{(s)}. \quad (10)$$

Matrix $\tilde{\Theta}^{(s)}$ of kept eigenmodes satisfies the orthogonality conditions

$$\tilde{\Theta}^{(s)^T} M^{(s)} \tilde{\Theta}^{(s)} = I, \quad \tilde{\Theta}^{(s)^T} K^{(s)} \tilde{\Theta}^{(s)} = \Omega^{(s)^2} = \text{diag} \left( \omega_1^{(s)^2}, \omega_2^{(s)^2}, \ldots, \omega_n^{(s)^2} \right) \quad (11)$$

$\Omega^{(s)}$ is a diagonal matrix containing the $n_\theta^{(s)}$ eigenvalues $\omega_j^{(s)}$ corresponding to the kept free interface normal modes $\theta_j^{(s)}$. Since a part of the subspace spanned by $\tilde{\Theta}^{(s)}$ is already included in $G^{(s)}$, a residual flexibility matrix $G_{\text{res}}^{(s)}$ can be used instead of flexibility matrix $G^{(s)}$. The residual flexibility matrix $G_{\text{res}}^{(s)}$ is defined by

$$G_{\text{res}}^{(s)} = \sum_{j=n_\theta^{(s)}+1}^{n^{(s)}} \frac{\theta_j^{(s)} \theta_j^{(s)^T}}{\omega_j^{(s)^2}} = G^{(s)} - \sum_{j=1}^{n_\theta^{(s)}} \frac{\theta_j^{(s)} \theta_j^{(s)^T}}{\omega_j^{(s)^2}}. \quad (12)$$

Note that, by construction, $G_{\text{res}}^{(s)} = G_{\text{res}}^{(s)^T}$, which is computed using the second equality in Eq. (12). For further properties of $G_{\text{res}}^{(s)}$, see [3, 12]. As a result, the approximation of one substructure is

$$u^{(s)} \approx -G_{\text{res}}^{(s)} B^{(s)^T} \lambda + R^{(s)} \alpha^{(s)} + \tilde{\Theta}^{(s)} \eta^{(s)}. \quad (13)$$

Assembling all $N$ substructures in a dual fashion according to Eq. (9) by keeping the interface forces $\lambda$ as unknowns, the DOFs of the entire structure can consequently be approximated by

$$\begin{bmatrix} u_{\text{bd}} \\ \lambda \end{bmatrix} \approx \begin{bmatrix} R_{\text{bd}} & \tilde{\Theta}_{\text{bd}} & -G_{\text{res, bd}} B^{(s)^T} \end{bmatrix} \begin{bmatrix} \alpha \\ \eta \end{bmatrix} \quad (14)$$

with

$$R_{\text{bd}} = \begin{bmatrix} R^{(1)} & 0 & \cdots \\ 0 & R^{(N)} \end{bmatrix}, \quad \alpha = \begin{bmatrix} \alpha^{(1)} \\ \vdots \\ \alpha^{(N)} \end{bmatrix}, \quad (15)$$

$$\tilde{\Theta}_{\text{bd}} = \begin{bmatrix} \tilde{\Theta}^{(1)} & 0 & \cdots \\ 0 & \tilde{\Theta}^{(N)} \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta^{(1)} \\ \vdots \\ \eta^{(N)} \end{bmatrix}, \quad G_{\text{res, bd}} = \begin{bmatrix} G_{\text{res}}^{(1)} & 0 & \cdots \\ 0 & \cdots & G_{\text{res}}^{(N)} \end{bmatrix}. \quad (16)$$
The approximation of the dually assembled system’s dynamic equations (6) is
\[
M_{\text{DCB}} \ddot{q}_{\text{DCB}} + K_{\text{DCB}} q_{\text{DCB}} = f_{\text{DCB}}
\]  
(17)
with:
\[
M_{\text{DCB}} = T_{\text{DCB}}^T \begin{bmatrix} M_{\text{bd}} & 0 \\ 0 & 0 \end{bmatrix} T_{\text{DCB}}, \\
K_{\text{DCB}} = T_{\text{DCB}}^T \begin{bmatrix} K_{\text{bd}} & B^T \\ B & 0 \end{bmatrix} T_{\text{DCB}},
\]  
(18)
\[
q_{\text{DCB}} = \begin{bmatrix} \alpha \\ \eta \\ \lambda \end{bmatrix}, \\
f_{\text{DCB}} = T_{\text{DCB}}^T \begin{bmatrix} f \\ 0 \end{bmatrix}
\]  
(19)

The dual Craig-Bampton reduced system is quasi-diagonal similar to the reduced matrices of the Craig-Bampton method [1] and has the final size of \( n_{\text{DCB}} = \sum_{s=1}^{N} n_r^{(s)} + \sum_{s=1}^{N} n_\theta^{(s)} + n_\lambda \) with \( n_r^{(s)} \) rigid body modes of substructure \( s \), \( n_\theta^{(s)} \) kept free interface normal modes of substructure \( s \), and the total number \( n_\lambda \) of all Lagrange multipliers [3,7].

### 2.3 Modal truncation augmentation

The modal truncation augmentation (MTA) method (or modal truncation vector (MTV) method) tries to correct for the incomplete representation of the spatial loads in the reduced subspace by adding additional modes [13]. These modes are often referred to as *pseudoeigen-vectors* [13], *modal truncation vectors* [14, 15], or *load-dependent Ritz vectors* [16, 17, 18]. Dickens and Stroeve [15] extended the concept of the MTA method to the Craig-Bampton substructuring method. Rixen [19] generalized the method to higher-order modal truncation vectors. The essence of extending the MTA method to a dynamic substructuring procedure, i.e., to the Craig-Bampton method in [15], is to represent the truncated force on the substructure that is not represented by the modes retained in the substructure reduction. This concept is applied to the DCBM in the following. To this end, the MTA method improves the approximation by augmenting the reduction basis in Eq. (15) with an additional set of MTA vectors \( \Phi^{(s)} \):

\[
u^{(s)} \approx -G_{\text{res}}^{(s)} B^{(s)T} \lambda + R^{(s)} \alpha^{(s)} + \hat{\Theta}^{(s)} \eta^{(s)} + \Phi^{(s)} \xi^{(s)}
\]  
(20)

Vector \( \xi^{(s)} \) contains the amplitudes of the MTA vectors \( \Phi^{(s)} \). Assume the general case that a substructure is loaded by an external force \( f^{(s)} \) with the force application point not being an interface DOF. The DCBM reduced system is not able to represent the static deformation due to this external force \( f^{(s)} \) correctly, since \( \hat{\Theta}^{(s)} \) does not capture the spatial parts needed to represent the correct static solution due to that arbitrary external force \( f^{(s)} \). The modal truncation vectors \( \Phi^{(s)} \) enrich the dual Craig-Bampton reduction basis in such a way that a representation of the spatial parts not captured by Eq. (10) is given to obtain statically correct solutions. The modal truncation vectors \( \Phi^{(s)} \) are computed by solving the static problem

\[
K^{(s)} \Phi^{(s)} = f^{(s)}
\]  
(21)

If only one external force vector \( f^{(s)} \) is applied to substructure \( s \), \( \Phi^{(s)} \) contains only one column vector. The substructure approximation in Eq. (20) gives rise to the augmented DCBM reduction basis \( T_{\text{DCB, aug}} \) as

\[
\begin{bmatrix} R_{\text{bd}} & \Phi_{\text{bd}} & -G_{\text{res, bd}} B^{T} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \eta \\ \xi \\ \lambda \end{bmatrix} = T_{\text{DCB, aug}} q_{\text{DCB, aug}}
\]  
(22)

\[\text{2356}\]
with
\[
\Phi_{bd} = \begin{bmatrix}
\Phi^{(1)} & 0 \\
\vdots & \ddots \\
0 & \Phi^{(N)}
\end{bmatrix}, \quad \xi = \begin{bmatrix}
\xi^{(1)} \\
\vdots \\
\xi^{(N)}
\end{bmatrix}.
\] (23)

Eq. (22) is the augmented version of Eq. (14) and the augmented DCBM reduced system is obtained by analogy with Eqs. (18) to (19) by using \( T_{DCB,\text{aug}} \) instead of \( T_{DCB} \). The dynamic equations of the augmented DCBM reduced system are written as
\[
M_{DCB,\text{aug}} \ddot{q}_{DCB,\text{aug}} + K_{DCB,\text{aug}} q_{DCB,\text{aug}} = f_{DCB,\text{aug}}.
\] (24)

### 2.4 Interface reduction

In order to reduce the interface problem for the dual Craig-Bampton method, we consider the interface problem, which is obtained by not keeping any vibration modes and using the flexibility matrix instead of the residual flexibility [20]. As a result, Eq. (14) changes to
\[
\begin{bmatrix}
u_{bd} \\
\lambda
\end{bmatrix} \approx \begin{bmatrix} R_{bd} & -G_{bd} B^T \\ 0 & I \end{bmatrix} \begin{bmatrix} \alpha \\ \lambda \end{bmatrix} = T_{\text{int}} q_{\text{int}}
\] (25)

and gives the interface problem
\[
M_{\text{int}} \ddot{q}_{\text{int}} + K_{\text{int}} q_{\text{int}} = f_{\text{int}}
\] (26)

with:
\[
M_{\text{int}} = T_{\text{int}}^T M_{bd} T_{\text{int}}, \quad K_{\text{int}} = T_{\text{int}}^T K_{bd} B^T T_{\text{int}}, \quad f_{\text{int}} = T_{\text{int}}^T f
\] (27)

A modal analysis of Eq. (26) yields interface modes, which are used to form the interface mode matrix \( X_{\text{int}} \). Calling \( \tilde{X}_{\text{int}} \) the matrix containing a certain number of these eigenmodes as its columns, the approximation (also called modal interface reduction) of the interface parameters \( q_{\text{int}} \) writes
\[
q_{\text{int}} \approx \tilde{X}_{\text{int}} \tilde{q}_{\text{int}}
\] (28)

with reduced interface parameters \( \tilde{q}_{\text{int}} \). The DCBM approximation of Eq. (14) with interface reduction results in
\[
\begin{bmatrix}
u_{bd} \\
\lambda
\end{bmatrix} \approx \begin{bmatrix} \Theta_{bd} \\ 0 \end{bmatrix} \eta + T_{\text{int}} \tilde{X}_{\text{int}} \tilde{q}_{\text{int}}.
\] (29)

Further details regarding interface reduction of DCBM reduced systems can be found in [20, 21].

### 2.5 Removing negative eigenvalues of dual Craig-Bampton reduced systems

A strategy to eliminate the negative eigenvalues of DCBM reduced systems is based on a second reduction (modal reduction) of the reduced quantities \( M_{\text{red}}, K_{\text{red}}, \) and \( f_{\text{red}} \). \( M_{\text{red}}, K_{\text{red}}, \) and \( f_{\text{red}} \) are obtained by a first reduction, i.e., either the DCBM reduction of Section 2.2 or the augmented DCBM reduction of Section 2.3. The reduced equations of motion of the system after the first reduction are
\[
M_{\text{red}} \ddot{q}_{\text{red}} + K_{\text{red}} q_{\text{red}} = f_{\text{red}}
\] (30)
with vector $q_{\text{red}}$ of reduced coordinates of size $n_{\text{red}}$. Associated with the reduced system of Eq. (30) is the eigenvalue problem

$$
\left( -\tilde{\omega}_j^2 M_{\text{red}} + K_{\text{red}} \right) x_j = 0
$$

with eigenvalue $\tilde{\omega}_j^2$ and corresponding eigenvector $x_j$ for $j = 1, \ldots, n_{\text{red}}$. Solving Eq. (31) for eigenvectors $x_j$ gives the modal matrix

$$
X = \begin{bmatrix} x_1 & x_2 & \ldots & x_{n_{\text{red}}} \end{bmatrix},
$$

which is normalized with respect to the reduced mass matrix $M_{\text{red}}$. This allows removing the negative eigenvalues $\tilde{\omega}_j^2$ from the reduced system of Eq. (30) by truncating the modes corresponding to negative eigenvalues. Therefore, a second reduction with reduction matrix $X_{\oplus}$, which contains all the vectors of the modal matrix $X$ corresponding to positive eigenvalues $\tilde{\omega}_j^2$, is performed:

$$
M_{\oplus} = X_{\oplus}^T M_{\text{red}} X_{\oplus}, \quad K_{\oplus} = X_{\oplus}^T K_{\text{red}} X_{\oplus}, \quad f_{\oplus} = X_{\oplus}^T f_{\text{red}}
$$

$M_{\oplus}, K_{\oplus}$ and $f_{\oplus}$ are the quantities of the reduced system after the second reduction. This reduced system does not have negative eigenvalues anymore and the equations of motion with vector $q_{\oplus}$ of reduced coordinates are

$$
M_{\oplus} \ddot{q}_{\oplus} + K_{\oplus} q_{\oplus} = f_{\oplus}.
$$

3 COMPUTATION OF FREQUENCY RESPONSE FUNCTIONS

The following dynamic problem is given:

$$
M \dddot{u} + C \dot{u} + K u = f
$$

with mass matrix $M$, damping matrix $C$, stiffness matrix $K$, displacement vector $u$, and external force vector $f$. For the computation of frequency response functions (FRFs), harmonic excitation

$$
f = f_0 e^{i\omega t}
$$

and harmonic displacement response

$$
u = u_0 e^{i\omega t}
$$

are assumed. The FRF can be computed in two different ways, either through the inversion of the dynamic stiffness matrix or by the incorporation of a modal superposition.

3.1 Direct computation

Combining Eqs. (35) to (37) gives

$$
\left( -\omega^2 M + i\omega C + K \right) x_0 e^{i\omega t} = f_0 e^{i\omega t}
$$

and the dynamic stiffness matrix $Z(\omega)$ is defined as

$$
Z(\omega) = -\omega^2 M + i\omega C + K.
$$
The inverse \( Z(\omega)^{-1} \) of the dynamic stiffness matrix is called flexibility matrix \( Y(\omega) \). Using the flexibility matrix \( Y(\omega) \), the amplitude \( u_0 \) of the displacement is obtained by

\[
u_0 = Z(\omega)^{-1} f_0 = Y(\omega) f_0.\] (40)

Therefore, the FRF that corresponds to the amplitude of the displacement of DOF \( i \) as a result of an excitation of DOF \( j \) is the entry \( Y_{ij}(\omega) \) of matrix \( Y(\omega) \). Note that for each \( \omega \), Eq. (40) has to be solved.

### 3.2 Modal superposition

After performing a modal analysis, the FRFs of dynamic systems can be computed by a modal superposition. The modal equation for mode \( i \) is

\[
m_{jj} \ddot{q}_j + c_{jj} \dot{q}_j + k_{jj} q_j = p_j \quad \text{for } j = 1, \ldots, n\] (41)

with modal mass \( m_{jj} \), modal damping \( c_{jj} \), modal stiffness \( k_{jj} \), and modal force \( p_j \). All modal equations are independent from each other and can be solved separately. With harmonic excitation and displacement analogous to Eqs. (36) and (37), the scalar equation

\[
(-\omega^2 m_{jj} + i\omega c_{jj} + k_{jj}) q_{0,j} = p_{0,j} \quad \text{for } j = 1, \ldots, n
\] (42)

determines the amplitude \( q_{0,j} \) of the modal parameter \( q_j \) only by a scalar division. Eq. (42) is used for all modes and the total FRF is obtained by superposition of the solutions of all modes. The damping factor \( \zeta_j \), which is later used to define the damping properties, of mode \( j \) is

\[
\zeta_j = \frac{c_{jj}}{2m_{jj} \omega}.
\] (43)
4 NUMERICAL EXAMPLES

In this Section, we want to demonstrate the computation of FRFs for dual Craig-Bampton reduced systems on three different examples. Rayleigh damping $C = \alpha M + \beta K$ is assumed for all examples. Damping factors $\zeta_1$ and $\zeta_2$ of the first two modes are used to define the damping properties and to determine $\alpha$ and $\beta$ in the following. In Section 4.1, the general properties of FRFs of DCBM reduced systems are illustrated by the Benfield truss, which is rather an academic example. In order to apply the FRF computation to a more realistic example, a three-dimensional beam frame is investigated in Section 4.2. In Section 4.3, interface reduction is illustrated by a two-dimensional solid plane stress problem for a larger interface compared to the beam frame. In this paper, all FRFs are normalized such that $|Y(\omega = 0)| = 1$.

4.1 Benfield truss

The Benfield truss in Figure 1 (taken from [22] and investigated in detail in [23]) is used to demonstrate the general properties of the computation of FRFs of DCBM reduced systems. The planar truss consists of two substructures having uniform bay sections whereas all members have constant area and uniform stiffness and mass properties. Further details of the systems can be found in [22]. In the following, DOF $a$ is considered as shown in Figure 1. Thereby, the absolute value $|Y_{aa}|$ of the frequency response function $Y_{aa}$ is considered, i.e., the displacement of DOF $a$ due to a harmonic excitation with circular frequency $\omega$ at DOF $a$.

4.1.1 Influence of negative eigenvalues

The eigenvalues $\omega^2$ associated to the Lagrange multipliers of the eigenvalue problem of Eq. (6), which can be written as

$$\omega^2 M_{\text{dual}} q_{\text{dual}} = K_{\text{dual}} q_{\text{dual}},$$

are plus and minus infinity due to the dual assembly [24]. Since the dual Craig-Bampton method does not strive for a strong interface compatibility, the eigenvalues $\omega^2$ associated to the Lagrange multipliers of the eigenproblem

$$\omega^2 M_{\text{DCB}} q_{\text{DCB}} = K_{\text{DCB}} q_{\text{DCB}}$$

corresponding to the DCBM reduced equations of motion in Eq. (17) become finite and negative, as explained in [20]. The system’s eigenfrequencies $\omega$ are the roots of the eigenvalues $\omega^2$. If $\omega^2$ is negative, the corresponding eigenfrequencies $\omega$ is imaginary.
This property is illustrated in Figure 2. For demonstration purposes, we chose very small damping parameters to make the effects more clearly visible. Thereby, the complex circular frequency $\omega \in \mathbb{C}$ of excitation is assumed to be a complex number. The absolute value $|Y_{aa}|$ of the FRF $Y_{aa}$ is plotted in the third dimension above the complex plane for $\omega$. The resonances

![Graph showing the FRF for a complex circular excitation frequency $\omega$ for Benfield truss of Figure 1. The dual Craig-Bampton method (DCBM) uses five free eigenmodes for both substructures. To make the effect clearer, the chosen damping parameters are very small. The modal damping factors of the first two eigenmodes are $\zeta_1 = \zeta_2 = 0.01\%$.](image)

and antiresonances of the eigenmodes with positive eigenvalues (called physical eigenmodes in the following) can be seen on the real $\omega$-axis. Three peaks show up on the imaginary $\omega$-axis, i.e., for $\Re(\omega) = 0$ in Figure 2. They represent three of the six negative eigenvalues (called unphysical eigenmodes in the following). The peaks of the other three eigenvalues cannot be visualized properly either because they are too small (in two cases) or because they are too close to another peak (in one case).

This implies that the existence of negative eigenvalues $\omega^2$ does not have a negative impact on the calculation of the FRF since only the real $\omega$-axis is of interest. Figure 3 shows the FRF for the unreduced and for the dual Craig-Bampton reduced Benfield truss using realistic damping parameters compared to the unrealistically small damping used to emphasize the effects in Figure 2. The direct computation of the FRF via the dynamic stiffness matrix (see Section 3.1) works without any problems concerning the negative eigenvalues of the reduced stiffness matrix of Eq. (45). The discrepancies in regions of high frequencies are the outcome of the reduction.
Figure 3: Absolute value $|Y_{aa}(\omega)|$ of FRF $Y_{aa}(\omega)$ for real circular excitation frequency $\omega$ for Benfield truss of Figure 1. The dual Craig-Bampton method (DCBM) uses five free eigenmodes for both substructures. The modal damping factors of the first two eigenmodes are $\zeta_1 = 0.5\%$ and $\zeta_2 = 0.2\%$.

4.1.2 Removing negative eigenvalues of DCBM reduced system

Since the unphysical eigenmodes do not influence the computation of the FRF, they can be disregarded. Eigenmodes can be removed by computing the FRF via modal superposition as described in Section 3.2. Figure 4 shows the FRFs for different scenarios for either $\omega$ being only real or $\omega$ being only imaginary: besides the FRF of the unreduced system, the FRF for the DCBM reduced system (five free eigenmodes for each substructure) is computed via direct computation. In contrast, the FRF for the DCBM reduced system is computed via modal superposition, whereby modal superposition is used to remove the negative eigenvalues. This is denoted DCBM$\oplus$ in the following. For this purpose, a modal analysis of the DCBM reduced system has to be performed and all single eigenvalues with corresponding eigenmodes are available. For clarity, the FRFs of all the single eigenmodes are additionally depicted: the FRFs of physical and unphysical are distinguished by different colors. DCBM$\oplus$ is the superposition of the FRFs of all physical eigenvalues, i.e., of the positive eigenvalues only: a DCBM$\oplus$ reduced system is a DCBM reduced system where the negative eigenvalues are removed.

For $\omega$ being real, Figure 4a shows small discrepancies between the unreduced system and the two reduced systems when regarding the position of the antiresonances. Larger differences occur for $\omega > 0.1$ rad/s. There is no recognizable discrepancy among the FRF of the DCBM reduced system and the FRF of the DCBM reduced system where all unphysical eigenmodes are removed. This is due to the low values that the FRFs of the unphysical eigenmodes attain. As a consequence, it can be stated that it does not matter if the unphysical eigenmodes are kept or removed. For practical application, this is exactly the case of interest.

For $\omega$ being imaginary, Figure 4b shows that around 0.2 to 0.3 rad/s, the DCBM reduced system has three resonances with related antiresonances, which are caused by the peaks in the FRFs of the unphysical eigenmodes. These are the same resonances that can be seen on the imaginary axis of Figure 2. By disregarding the unphysical eigenmodes, the FRF computed with modal superposition does not possess those resonances.
Figure 4: Absolute value $|Y_{aa}(\omega)|$ of FRF $Y_{aa}(\omega)$ for complex circular excitation frequency $\omega$ for Benfield truss of Figure 1. The dual Craig-Bampton method (DCBM) uses five free eigenmodes for both substructures. The modal damping factors of the first two eigenmodes are $\zeta_1 = 0.5\%$ and $\zeta_2 = 0.2\%$. 

(a) $\Re(\omega)$ in rad/s; $\Im(\omega) = 0$

(b) $\Im(\omega)$ in rad/s; $\Re(\omega) = 0$
4.2 Beam frame

For further investigations, the three-dimensional beam frame (taken from [3]) depicted in Figure 5 is considered. It is composed of cells and consists of steel with Young’s modulus $E = 210 \text{kN/mm}^2$, Poisson’s ratio $\nu = 0.3$, and density $\rho = 7500 \text{kg/m}^3$. The beams aligned with the coordinate axis are hollow cylinders with an inner diameter of 18 mm and outer diameter of 20 mm. The diagonal beams are solid cylinders with a diameter of 8 mm. The height of each cell is 0.35 m and the side length of the quadratic base is 0.5 m. The cells of substructures 2, 3, and 4 are turned by 90°, 180°, and 270°. Substructure 5 is a cube with a side length of 0.5 m. In total, the unreduced system has 2958 DOFs.

4.2.1 Further reduction of DCBM⊕ reduced system

The unphysical (negative) eigenmodes do not provide any relevant information for the calculation of FRFs. This fact also holds for some physical (positive) eigenmodes, i.e., not all modes kept in the DCBM⊕ reduced system are relevant for some problems. The number of DOFs can be reduced further by considering only the important physical eigenmodes. The importance of eigenmodes shall be evaluated by the percentage of participation in the static displacement.

Therefore, the absolute value of FRF $Y_{ca}(\omega)$ is considered in Figure 6 for the beam frame of Figure 5; the dual Craig-Bamton reduced system (DCBM) using four free eigenmodes for each of the substructures 1, 2, 3, and 4 and three eigenmodes for substructure 5 perfectly reproduces the FRF of the unreduced system. DCBM is a superposition of all physical eigenmodes and all unphysical eigenmodes. This does not change when all unphysical eigenmodes and the physical eigenmodes with small static participation are removed from the reduction basis, which is called DCBM⊕ModRed in Figure 6 (DCBM⊕ModRed is a superposition of only important physical eigenmodes). Therefore, the importance of the physical eigenmodes is evaluated by the partic-
Figure 6: Absolute value $|Y_{ca}(\omega)|$ of FRF $Y_{ca}(\omega)$ for circular excitation frequency $\omega$ for beam frame of Figure 5. The dual Craig-Bampton method (DCBM) uses four free eigenmodes for each of the first four substructures and three eigenmodes for the fifth substructure. DCBM $\oplus$ ModRed uses only physical eigenmodes with a participation greater than 0.001 to the static solution (green) and physical eigenmodes with lower participation are ignored (black). The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.25\%$.

This is very different if the absolute value of FRF $Y_{dd}(\omega)$ is considered in Figure 7. DOF $d$ is a rotational DOF. For the DCBM reduction, four free eigenmodes are kept for substructure 3, which obviously do not properly represent the torsion (rotational DOF $d$). Therefore, the static torque at DOF $d$ is not properly represented by the reduced system. Considering the value of the FRF for the very small frequency $\omega = 1$ rad/s, it can be seen that the displacement of the DCBM reduced system is far away (46%) from the displacement of the unreduced system. This results in a very poor approximation of the FRF over the whole frequency range, except for the position of most of the resonances. It should be highlighted again here that nothing changes when only the physical eigenmodes with a static participation greater than 0.001 are used for the reduction matrix (DCBM $\oplus$ ModRed in Figure 7).

There are two possibilities to improve the FRFs: The reduction basis can be enriched with additional free eigenmodes. Depending on the case, it can take many additional eigenmodes to get a good static representation. Alternatively, it can be much more efficient to use a modal truncation vector (MTV) as presented in Section 2.3. Both options are subjects of the investigations in the subsequent Sections 4.2.2 and 4.2.3.
Unreduced system Important phys. eigenm.
DCBM Unimportant phys. eigenm.
DCBM⊕ModRed Unphysical eigenmode

Figure 7: Absolute value $|Y_{dd}(\omega)|$ of FRF $Y_{dd}(\omega)$ for circular excitation frequency $\omega$ for beam frame of Figure 5. The dual Craig-Bampton method (DCBM) uses four free eigenmodes for each of the first four substructures and three eigenmodes for the fifth substructure. DCBM⊕ModRed uses only physical eigenmodes with a participation greater than 0.001 to the static solution (green) and physical eigenmodes with lower participation are ignored (black). The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.25\%$.

4.2.2 Variation of number of kept free interface normal modes

The effect of changing the number of kept free eigenmodes will now be examined. This will be done first with DOF $a$ of the beam frame in Figure 5. Figure 8 shows the absolute value of the FRF $Y_{aa}$ of the unreduced system and the DCBM reduced system with three different combinations of kept eigenmodes for the substructures. Every version contains the three rigid body modes for each of the substructures 2, 3, 4, and 5. Obviously, the version with no eigenmodes ([0,0,0,0,0] means zero kept eigenmodes for each of all five substructures) performs much worse than the version with a total number of 15 eigenmodes ([4,4,4,4,3] means that 4 eigenmodes are kept for substructure 1, 4 eigenmodes are kept for substructure 2, etc.), which is to be expected. The interesting part is the case with only four eigenmodes for the substructure 2 (case [0,4,0,0,0] in Figure 8): though having less than a third of the eigenmodes compared to the system with 15 eigenmodes, it is similarly good at reproducing the FRF of the unreduced system. Consequently, the eigenmodes of substructure 2 are more important than those of the other substructures. This is because the considered DOF $a$ belongs only to substructure 2 (DOF $a$ is input and output for $Y_{aa}$).

When input and output DOF of the desired FRF do not belong to the same substructure, unlike for $Y_{aa}$ in Figure 8, the question might be which substructure then seeks then for additional eigenmodes in the reduction basis. Due to the symmetry of the dynamic stiffness matrix $Z(\omega)$, the flexibility matrix $Y(\omega) = Z(\omega)^{-1}$ is symmetric as well, i.e., $Y_{jk} = Y_{kj}$. Thus, the FRFs from input $k$ to output $j$ and from input $j$ to output $k$ are the same. Therefore, it does not make sense to increase the number of kept modes only for one of the substructures. Figure 9 exemplarily shows the FRF $Y_{ed}$ for input and output DOF not being on the same substructure. Substructures 3 and 4 are involved in the FRF, since DOF $d$ belongs to substructure 3 and DOF
Figure 8: Absolute value $|Y_{aa}(\omega)|$ of FRF $Y_{aa}(\omega)$ for circular excitation frequency $\omega$ for beam frame of Figure 5. The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.25\%$.

e belongs to substructure 4. The DCBM reduced system with 15 free eigenmodes generates a proper FRF. Only the DCBM reduced system that uses four eigenmodes for substructure 3 and four eigenmodes for substructure 4 (case $[0,0,4,4,0]$ in Figure 9) achieves a comparatively good approximation of the FRF of the unreduced system. All of the three other reductions – the DCBM with no free eigenmodes (case $[0,0,0,0,0]$ in Figure 9), the DCBM with only four eigenmodes for substructure 3, and the DCBM with only four eigenmodes for substructure 4 – perform much worse.

Figure 9: Absolute value $|Y_{ed}(\omega)|$ of FRF $Y_{ed}(\omega)$ for circular excitation frequency $\omega$ for beam frame of Figure 5. The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.25\%$. 
4.2.3 Modal truncation augmentation

In contrast to the variation of the number of kept free interface normal modes, the effect of modal truncation augmentation (MTA), i.e., adding load-dependent static modes (also called modal truncation vector MTV), is evaluated in this Section. Two types of FRFs are considered: FRFs $Y_{jj}$ for DOF $j$ being input and output DOF and FRFs $Y_{jk}$ for different input and output DOFs $k$ and $j$.

Investigation of $Y_{jj}$: Figure [10] shows the FRF $Y_{aa}$ of the beam frame for the unreduced system, the DCBM reduced system and – indicated with MTV$_a$ – the DCBM reduced system with a modal truncation vector (MTV) for DOF $a$. The DCBM reduced system with 15 free eigenmodes approximates the FRF of the unreduced system very accurately. This has already been demonstrated in Figure [8]. There, the static approximation is very good (small value for $\omega$). Figure [10] indicates that the additional MTV does not improve the FRF in the small frequency range since the static approximation is already pretty good. Nevertheless, the position of the antiresonances are slightly improved if the MTV is added. Besides that, an additional resonance for a high frequency (about $10^{16}$ rad/s) can be observed. This is a result of the additional DOF that comes with the static mode.

In this example, the improvement of the FRF due to the MTV is not very significant. Nevertheless, it can be expected that for an example with a worse static representation of the external force, the effect of the additional MTV is more evident. Therefore, the FRF $Y_{dd}$ known from Figure [7] which has a poor static approximation, is further investigated. In Figure [11] an additional MTV is added to improve $Y_{dd}$ from Figure [7]. The positive effect of this MTV is obvious: the FRF with additional MTV coincides with the FRF of the unreduced system over most of the frequency range. Small deviations only occur for higher frequencies.

Also compared to Figure [10], the MTV has a tremendous positive effect in this example: for $Y_{aa}$ in Figure [10] the static representation is already good without MTV, which is not the case...
The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 25\%$. Therefore, the MTV improves $Y_{dd}$ significantly in this example. As this case shows, the FRF of the DCBM reduced system without MTV is not useful at all. However, at the cost of one additional MTV, which is only one single additional DOF, the FRF can be enhanced to match the FRF of the unreduced systems nearly flawlessly.

**Investigation of $Y_{jk}$:** When considering FRFs that involve different input and output DOFs, MTVs could be added either only for the input DOF or only for the output DOF or for both DOFs. As in Section 4.2.2, the FRFs $Y_{jk}$ and $Y_{kj}$ between two DOFs are equal due to the symmetry of the dynamic stiffness matrix. Both FRFs cannot be distinguished. It does not make sense to make a statement about the DOFs that is excited (input) and the DOF that is measured (output), because they can be interchanged arbitrarily.

Figure 12 shows the absolute values of the FRF $Y_{ba}$ of the beam frame. Since there are no free eigenmodes used for the reduction basis for demonstration purposes in this example, the static representation as well as the whole FRF is not very accurately approximated by the DCBM. The static mode for DOF $b$ (MTV$_b$) enhances the FRF for $\omega < 150$ rad/s, whereas the static mode for DOF $a$ (MTV$_a$) enhances the FRF for $\omega < 220$ rad/s. There is no recognizable difference between using MTV$_a$ only and using both the static mode for DOF $a$ and DOF $b$ (MTV$_{ab}$). Since it cannot be easily predicted which one of the static modes improves the FRF more, it is recommended to always use both.

**4.2.4 Interface reduction**

To further reduce the number of DOFs of a DCBM reduced system, an interface reduction is performed. The interface problem of the beam frame consisting of the interface DOFs and the rigid body modes has 144 DOFs. Figure 13 shows how the DCBM reduced system with interface reduction performs. The term “int.red.(29)” means that the interface problem of the DCBM reduced system (144 DOFs in this case) is further reduced by 115 DOFs to a final number of 29 DOFs. Since no free eigenmodes are added, this is the number of DOFs of the
The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.25\%$.

The very small discrepancy between the DCBM reduced system with and without interface reduction implies that the reduction of almost 80% of the DOFs of the interface problem does not deteriorate the FRF. Nonetheless, the representation of the antiresonances compared to the unreduced system is unsatisfying. This can once again be enhanced by the incorporation of the static mode MTV$_a$. The additional resonance is at a position where the approximation of the unreduced system is already poor, which means the additional resonance does not deteriorate the FRF.

The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.25\%$.
4.3 Two-dimensional solid

Since the beam frame has a very small interface due the connection of beam elements, the modal truncation augmentation and the interface reduction will now be investigated for larger interfaces. Therefore, a two-dimensional solid is used, as shown in Figure 14. It consists of steel with Young’s modulus $E = 210 \text{kN/mm}^2$, Poisson’s ratio $\nu = 0.3$, and density $\rho = 7500 \text{kg/m}^3$. Its thickness is 0.1 m, the height 1.8 m, and the length 3.6 m. The finite elements are cubes with a side length of 0.1 m. The stress is assumed to be a plane stress which makes the finite elements squares. The system is divided into six substructures as depicted in Figure 14 by different colors. The interface of this system is much larger compared to the relatively small interface of the beam frame of Section 4.2. The displacements of the upper-right corner are considered in the following: DOF $a$ denotes the vertical displacement and DOF $b$ denotes the horizontal displacement.

4.3.1 Modal truncation augmentation

The absolute value of the FRF $Y_{ba}$ of the two-dimensional solid is shown in Figure 15. Five cases are plotted: the FRF of the unreduced system and the FRF of the DCBM reduced systems, where 8 free interface normal modes per substructure are kept (DCBM [8,8,8,8,8,8]), are plotted for reference. In the other three cases, additional MTVs are added: MTV$_a$ means that one static mode is added corresponding to DOF $a$, MTV$_b$ means that one static mode is added corresponding to DOF $b$, and MTV$_{ab}$ uses both static modes. In this example, any of the two possible static modes MTV$_a$ and MTV$_b$ can be used to enhance the FRF significantly. Since this cannot be predicted in advance, i.e., before computing the FRF of the unreduced system, in the general case both static modes should be used. Figure 15 also shows the additional resonances for each static mode: one additional peak if one MTV is added (MTV$_a$ and MTV$_b$) and two additional peaks if two MTVs are added (MTV$_{ab}$).

4.3.2 Interface reduction

The interface problem of the two-dimensional solid has 164 DOFs, which makes a total number of 204 DOFs for the DCBM reduced system with eight free eigenmodes kept for each substructure. Interface reduction is used to reduce the 164 DOFs by 131 DOFs to a remaining
Figure 15: Absolute value $|Y_{ba}(\omega)|$ of FRF $Y_{ba}(\omega)$ for circular excitation frequency $\omega$ for two-dimensional solid of Figure 14. The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.5\%$.

number of 33 DOFs. The corresponding FRFs can be seen in Figure 16 for $Y_{aa}$ and in Figure 17 for $Y_{ba}$. Both examples make it clear that the exactness of the FRF does not change significantly through the interface reduction, even if almost 80% of the interface DOFs are reduced. Furthermore, the FRF can be enhanced by adding one (Figure 16) and two (Figure 17) corresponding MTVs, respectively.

Figure 16: Absolute value $|Y_{aa}(\omega)|$ of FRF $Y_{aa}(\omega)$ for circular excitation frequency $\omega$ for two-dimensional solid of Figure 14. The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.5\%$. The interface problem has 164 DOFs.
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Figure 17: Absolute value $|Y_{ba}(\omega)|$ of FRF $Y_{ba}(\omega)$ for circular excitation frequency $\omega$ for two-dimensional solid of Figure 14. The modal damping factors of the first two eigenmodes are $\zeta_1 = 1\%$ and $\zeta_2 = 0.5\%$.

5 CONCLUSIONS

In this paper, we investigated the feasibility of the computation of frequency response functions (FRFs) of dual Craig-Bampton (DCBM) reduced systems and its accuracy. The inherently unstable behavior of DCBM reduced systems does not have a negative influence on the computation of FRFs, which is in contrast to time integration. Direct computation via the dynamic stiffness matrix or modal superposition can be used to obtain accurate FRFs. The negative eigenvalues of the reduced system, which are intrinsic to the DCBM reduction, do not show up on the real $\omega$-axis of the FRFs. They appear on the complex $\omega$-axis, since the negative eigenvalues correspond to complex excitation frequencies, which is commonly not of practical interest. All in all, FRF computation of DCBM reduced systems is possible without any modifications of the DCBM reduced system.

No difference can be observed in the resulting FRFs between the DCBM reduced system without modifications (i.e., with negative eigenvalues) and the DCBM reduced system, where the negative eigenvalues are removed by modal superposition. This modal superposition is a subsequent step to the DCBM reduction, which does not improve the accuracy of the FRFs, but further reduces the size of the reduced system. If the external load, i.e., the force excitation for the FRF, is only incompletely represented by the DCBM reduction basis, the computed FRFs can be very inaccurate. Modal truncation augmentation is used to correct for the incomplete representation of the spatial loads in the reduced subspace by adding additional modes, which are called modal truncation vectors (MTVs). They aim to represent the truncated force on the substructure that is not represented by the modes retained in the substructure reduction. This improves the FRFs particularly well if the static deformation due to the exciting force is not captured by the DCBM reduction basis. All resonances and antiresonances coincide up to a certain cut-off frequency. An additional peak in the FRFs is introduced by each additional MTA vector, which does not influence the approximation quality of the FRF in the lower frequency range, since the additional peaks always appear in the high frequency range where the approximation accuracy is always poor due to the reduction. Hence, if the points of application of external forces are known, it is highly recommended to include MTA vectors related to the points of
application of the forces in the reduction basis. This guarantees that the results will be statically correct in any case.

Moreover, interface reduction is applied to further reduce the number of DOFs of the DCBM reduced system. The exactness of the FRF does not change significantly through the interface reduction, even if a high percentage of the interface DOFs are reduced. Therefore, interface reduction can decrease the number of remaining DOFs significantly while keeping a very good approximation accuracy.

REFERENCES


A NOVEL DERIVATION FOR MODAL DERIVATIVES BASED ON VOLterra SERIES REPRESENTATION AND ITS USE IN NONLINEAR MODEL ORDER REDUCTION

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\textbf{Abstract.} This paper presents a novel derivation for modal derivatives based on the Volterra series representation of nonlinear structural systems. After reviewing the classical derivation, new modal derivatives are proposed based on the employment of the Volterra theory and the variational equation approach. It turns out that the gained new derivatives are almost identical to the conventional ones, except for the fact that a sum/subtraction of eigenfrequencies results in our definition. In addition to the novel derivation, some possible impacts and applications of the new derivatives are presented and discussed, pursuing the aim that the conceptual results are also useful for practical purposes.
1 INTRODUCTION

Model order reduction for large-scale nonlinear structural systems has gained an increased attention over the past ten years. This is mainly due to the rising number of engineering applications, where detailed finite element models are employed for the design and optimization of complex mechanical structures undergoing large deformations. Since the numerical analysis of such high-dimensional models is too demanding, model reduction is usually applied to construct a reduced order model with substantially less degrees of freedom.

Simulation-based dimensional reduction techniques such as Proper Orthogonal Decomposition (POD), often combined with hyper reduction methods like e.g. the Energy-Conserving Sampling and Weighting (ECSW) [4], are established nonlinear approaches. POD, however, requires expensive training simulations for the computation of the reduction basis needed for projection. On the other hand, simulation-free reduction procedures have been also studied. Particularly in nonlinear structural dynamics, the concept of basis augmentation with so-called modal derivatives (MDs) has been successfully applied in numerous applications [8, 1, 25, 28, 27, 19], in order to reduce the computational effort involved in the numerical analysis. Recently, MDs have been also applied for the efficient computation of nonlinear normal modes (NNMs) [10] and the nonlinear frequency response function (NLFRF) of structural systems [26, 22, 23].

The key idea of this simulation-free concept is to first compute some dominant modes of the linearized, second-order system, and then to augment the reduction basis with perturbation derivatives capturing the nonlinear behavior. In [8], MDs were originally derived by perturbing the linearized eigenvalue problem w.r.t. the linearization point. This derivation yields a singular linear system of equations (LSE) under mass consideration, and also involves the derivatives of eigenfrequencies. However, the authors neglected these eigenfrequencies derivatives and the mass terms. These simplifications lead to a regular LSE to compute the so-called static modal derivatives (SMDs) [7, 8, 21].

We propose here a novel derivation for modal derivatives using the Volterra series representation for nonlinear structural systems. The Volterra series representation [18, 16, 2] is a useful system-theoretic tool for the approximation of nonlinear systems, which has been recently used for the reduction of bilinear and quadratic bilinear state-space models. From our perspective, the Volterra theory shares several similarities with the multiple-scale analysis and perturbation methods described in [13, 14]. In any case, it turns out that our derivation using the Volterra series representation yields a calculation formula for the MDs which is almost identical to the conventional definition, except for the fact that a sum/subtraction of eigenfrequencies results. Thus, the resulting LSE is only singular, if the sum/subtraction of eigenfrequencies is again an eigenfrequency. Moreover, if the eigenfrequencies cancel out, the definition for the SMDs results, without having to explicitly neglect the mass terms.

In addition to the novel derivation, we discuss the possible implications and applications of the gained new derivatives for nonlinear structural dynamics: the explanation of nonlinear effects like internal resonances, the gained analytical solution and its comparison with the nonlinear normal modes (NNMs), as well as the application of some novel quadratic manifold approaches for model order reduction.

Finally, it is emphasized that this paper constitutes a theoretical and conceptual contribution. The focus lies on proposing new relationships/concepts and bridging them to established material, in order to offer an integrated/unifying framework. The numerical validation of the presented new derivatives and their applications is current work-in-progress and subject to further research for a future paper.
2 FUNDAMENTALS

2.1 System representation

We consider a large-scale, nonlinear time-invariant, second-order model of the form
\begin{equation}
M\ddot{q}(t) + D\dot{q}(t) + f(q(t)) = BF(t), \quad q(0) = q_0, \quad \dot{q}(0) = \dot{q}_0,
\end{equation}
with nonsingular mass matrix $M \in \mathbb{R}^{n \times n}$, displacements $q(t) \in \mathbb{R}^n$, input forces $F(t) \in \mathbb{R}^m$, outputs $y(t) \in \mathbb{R}^p$ and the smooth mapping $f(q): \mathbb{R}^n \rightarrow \mathbb{R}^n$. Note that the considered equations of motion represent a special case of the more general system representation
\begin{equation}
M\ddot{q}(t) + f^{\text{int}}(q(t), q(t)) = f^{\text{ext}}(q(t), q(t), t), \quad q(0) = q_0, \quad \dot{q}(0) = \dot{q}_0,
\end{equation}
with $f^{\text{int}}(q(t), q(t)) = D\dot{q}(t) + f(q(t))$ denoting the internal forces, $f^{\text{ext}}(q(t), q(t), t) = BF(t)$ representing the time- and sometimes even displacement-dependent external forces, and $h(q(t), q(t)) = Cq$ as the output mapping. Note that the modeling of damping in nonlinear dynamic analysis is not a trivial task. Thus, zero damping ($D = 0$) or a linear Rayleigh damping $D = \alpha M + \beta K(q_{eq})$ with $\alpha, \beta \geq 0$ and $K(q_{eq}) = \|f(q_{eq})\|_{q_{eq}}$ are often assumed.

2.2 Linear Galerkin projection

The goal of model reduction is to approximate the full order model (FOM) by a reduced order model (ROM) of much smaller dimension $r \ll n$. A common and well-established way to reduce nonlinear second-order systems is to apply the linear Galerkin projection ansatz $q(t) \approx Vq_r(t)$, where the full displacements are approximated by a linear combination of the basis vectors stored in the reduction matrix $V \in \mathbb{R}^{n \times r}$. Inserting this ansatz in (1) and premultiplying the overdetennined system with $V^T$ yields the well-determined (squared) ROM
\begin{equation}
M_r\dddot{q}_r(t) + D_r\ddot{q}_r(t) + V^T f(Vq_r(t)) = B_rF(t), \quad q_r(0) = V^T q_0, \quad \dot{q}_r(0) = V^T \dot{q}_0,
\end{equation}
with reduced matrices $\{M_r, D_r\} = V^T \{M, D\} V$, $B_r = V^T B$, $C_r = C V$ and the reduced nonlinear forces $f_r(q) = V^T f(Vq_r)$ with $f_r(q) : \mathbb{R}^n \rightarrow \mathbb{R}^r$. Note that the reduction of second-order systems is usually performed by an orthogonal (Galerkin) projection rather than by an oblique (Petrov-Galerkin) projection, since the former fulfills the principle of virtual work and, thus, preserves the symmetry and definiteness of the original matrices as well as the stability of the FOM. In this projective framework, the main task is to efficiently compute a reduction basis $V \in \mathbb{R}^{n \times r}$ that comprises the most dominant nonlinear dynamics to ensure a good approximation, measured e.g. point-wise by $\|q(t) - Vq_r(t)\|_1$ or $\|y(t) - y_r(t)\|_1$ with a suitable vector norm $(\cdot) = \{1, 2, \infty, \text{Fro}, \ldots\}$.

3 ORIGINAL DERIVATION OF MODAL DERIVATIVES

As mentioned in the introduction, different approaches exist for the computation of the reduction basis $V$ needed for projection. One simulation-free technique that has lately gained a lot of attention in nonlinear structural dynamics is based on the concept of basis augmentation with modal derivatives. The key idea is to first compute some dominant modes of the linearized,
second-order system, and then to augment the reduction basis with perturbation derivatives capturing the nonlinear behavior.

In the following, the original derivation of modal derivatives based on the perturbation of the linearized eigenvalue problem w.r.t. the linearization point is reviewed. Hereby, different strategies to handle the singularity under mass consideration, the concept of static derivatives and the resulting analytical solution are discussed.

3.1 Power series expansion of state-dependent projection ansatz

First of all, we start by assuming a nonlinear projection mapping \( q(t) \approx \nu(q(t)) \). To be more precise, we assume the following projection ansatz with the state-dependent basis \( V(q(t)) \)

\[
q(t) \approx V(q(t))q_r(t) = \sum_{i=1}^{r} v_i(q(t))\eta_i(t)
\]

and reduced coordinates \( q_r(t) = [q_{r,1}(t), \ldots, q_{r,r}(t)]^T = [\eta_1(t), \ldots, \eta_r(t)]^T \). This state-dependent projection ansatz can be expanded in a Taylor series at an equilibrium or operating state \( q_{eq} \): [28]

\[
q(t) = q(t)|_{eq} + \sum_{i=1}^{r} \frac{1}{1!} \frac{\partial q(t)}{\partial \eta_i(t)}|_{eq} \eta_i(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \frac{1}{2!} \frac{\partial^2 q(t)}{\partial \eta_i(t) \partial \eta_j(t)}|_{eq} \eta_i(t) \eta_j(t) + \cdots
\]

\[
= q_{eq} + \sum_{i=1}^{r} v_i(q_{eq})\eta_i(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \frac{1}{2} \left( \frac{\partial v_i(q_{eq})}{\partial \eta_j(t)} + \frac{\partial v_j(q_{eq})}{\partial \eta_i(t)} \right) \eta_i(t) \eta_j(t) + \cdots
\]

where \( \frac{\partial v_i(q_{eq})}{\partial \eta_j(t)} = \frac{\partial v_j(q_{eq})}{\partial \eta_i(t)} \) is assumed in the last step. Truncating the series after the second order term and assuming that \( q_{eq} = 0 \) yields the approximation

\[
q(t) \approx \sum_{i=1}^{r} v_i(q_{eq})\eta_i(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \frac{\partial v_i(q_{eq})}{\partial \eta_j(t)} \eta_i(t) \eta_j(t).
\]

Based on this, the following projection ansatz can be defined

\[
q(t) \approx V_{aug} q_{r,aug}(t),
\]

with the constant, augmented reduction basis \( V_{aug} \) and the corresponding reduced vector \( q_{r,aug}(t) \):

\[
V_{aug} = \begin{bmatrix} v_1(q_{eq}) & \cdots & v_r(q_{eq}) \\
\frac{\partial v_1(q(t))}{\partial \eta_1(t)}|_{eq} & \cdots & \frac{\partial v_r(q(t))}{\partial \eta_1(t)}|_{eq}
\end{bmatrix},
\]

\[
q_{r,aug}(t) = [\eta_1(t) \cdots \eta_r(t) \mid \eta_{11}(t) \cdots \eta_{rr}(t)]^T.
\]

The augmented basis \( V_{aug} \) contains both basis vectors from the linearized system \( v_i(q(t))|_{eq} \) and their derivatives \( \frac{\partial v_i(q(t))}{\partial \eta_j(t)}|_{eq} \). These derivatives describe the change of the basis vector \( v_i(q_{eq}) \), when the system is perturbed in the direction of \( v_j(q_{eq}) \). Thus, they capture the behavior of the quadratic term of the series. The augmentation of the reduction basis with
basis vector derivatives leads to an augmented reduced vector $q_{r,aug}(t)$ with additional reduced coordinates $\eta_{ij}(t)$.

The presented framework for basis augmentation can be applied along with any linear(ized) reduction technique. In the following, we apply it to modal truncation with vibration modes, i.e. $v_i(q_{eq}) = \phi_i(q_{eq})$.

3.2 Vibration modes of the linearized model

The nonlinear structural system $\{1\}$ is first linearized around a linearization or equilibrium point $(q_{eq}, F_{eq})$.

**Linearization point** A possible choice for the linearization point is to use the static solution of the nonlinear system $q_c$ for a given static input force $F_s$. To this end, the nonlinear system of equations $0 = -f(q_c) + BF_c$ has to be solved for $q_c$. Another popular approach is to select $q_{eq} = 0$ and $F_{eq} = 0$, which leads to $f(q_{eq}) = 0$ and $\dot{q}_{eq} = 0$, $\ddot{q}_{eq} = 0$.

The Taylor series approximation of $f(q(t))$ around $(q_{eq}, F_{eq})$ yields the linearized model

$$M \ddot{\Delta q}(t) + D \dot{\Delta q}(t) + f(q_{eq}) + K_{eq} \Delta q(t) = B \Delta F(t), \quad \Delta q(0) = \Delta q_0, \quad \Delta \dot{q}(0) = \Delta \dot{q}_0, \quad \Delta y(t) = C \Delta q(t),$$

(8)

where $\Delta q(t) = q(t) - q_{eq}$, $\Delta F(t) = F(t) - F_{eq}$ and $\Delta y(t) = y(t) - y_{eq}$ represent the deviation from the linearization point. The tangential stiffness matrix is given by

$$K_{eq} = K(q_{eq}) = \left. \frac{\partial f(q(t))}{\partial q(t)} \right|_{q_{eq}}.$$

(9)

To derive the quadratic eigenvalue problem, the damping is set to zero ($D = 0$) and the homogeneous system is considered ($\Delta F(t) = 0$). In this undamped free motion case, the ansatz for the displacements and accelerations is given by

$$\Delta q(t) = \sum_{i=1}^{n} c_i \phi_{i,eq} e^{i \omega_i t}, \quad \Delta \dot{q}(t) = \sum_{i=1}^{n} -i \omega_i c_i \phi_{i,eq} e^{i \omega_i t},$$

(10)

where the scalings $c_i$ are determined through the initial conditions. Inserting the above ansatz into the linearized, undamped, homogeneous system (8) with $f(q_{eq}) = 0$ yields

$$-i \omega_i^2 M \sum_{i=1}^{n} c_i \phi_{i,eq} e^{i \omega_i t} + K_{eq} \sum_{i=1}^{n} c_i \phi_{i,eq} e^{i \omega_i t} = 0,$$

(11)

and cancelling the time-dependent term $e^{i \omega_i t}$ finally delivers the quadratic eigenvalue problem

$$(K_{eq} - \omega_i^2 M) \phi_{i,eq} = 0 \quad \iff \quad K_{eq} \Phi_{eq} - M \Phi_{eq} \Omega_{eq} = 0.$$

(12)

Hereby, the undamped eigenfrequencies $\{\omega_{i,eq}\}_{i=1}^{n}$ and the eigenmodes $\{\phi_{i,eq}\}_{i=1}^{n}$ are encoded in the matrices $\Omega_{eq} = \text{diag}(\omega_1, \ldots, \omega_n) \in \mathbb{R}^{n \times n}$ and $\Phi_{eq} = [\phi_{1,eq}, \ldots, \phi_{n,eq}] \in \mathbb{R}^{n \times n}$. Typically, the eigenmodes are normalized using the $M$-weighted inner product. Moreover, they are orthogonal to each other in both the $M$- and $K$-norm:

$$\phi^T_{i,eq} M \phi_{j,eq} = \delta_{ij}, \quad \phi^T_{i,eq} K_{eq} \phi_{j,eq} = \omega_{i,eq} \delta_{ij},$$

$$\Phi^T_{eq} M \Phi_{eq} = I, \quad \Phi^T_{eq} K_{eq} \Phi_{eq} = \Omega^2_{eq}.$$  

(13)
3.3 Perturbation of eigenmodes

The vibration modes of the linearized system \( \phi_j(q_{eq}) \) depend on the chosen linearization point \( q_{eq} \). Hence, the change of the modes with respect to the linearization point is of interest now. To this end, the perturbation of the modes \( \phi_j(q_{eq}), \ j = 1, \ldots, r \) with respect to the amplitude \( \eta_j(t) \) of the modes \( \phi_j(q_{eq}), \ j = 1, \ldots, r \) is considered in the following. Please note that the amplitudes \( \eta_j(t) \) correspond to the reduced coordinates \( q_i(t) = [q_{i,1}(t), \ldots, q_{i,r}(t)]^T = [\eta_1(t), \ldots, \eta_r(t)]^T \) in the projection ansatz from Section 3.1.

The perturbation of the linearized eigenvalue problem (12) with respect to the amplitude \( \eta_j(t) \) of mode \( \phi_j,eq \) yields

\[
\frac{\partial}{\partial \eta_j(t)} \left( K(q_{eq}) - \omega_i^2(q_{eq}) M \right) \phi_i(q_{eq}) = 0 \tag{14}
\]

\[
\left( \frac{\partial K_{eq}}{\partial \eta_j(t)} - \omega_i^2 \frac{\partial^2 \phi_i}{\partial \eta_j(t)} M \right) \phi_i,eq + \left( K_{eq} - \omega_i^2 M \right) \frac{\partial \phi_i,eq}{\partial \eta_j(t)} = 0. \tag{15}
\]

**Derivative of eigenfrequencies**

The derivative of the squared \( i \)-th eigenfrequency \( \frac{\partial \omega_i^2}{\partial \eta_j(t)} \) can be obtained, if we premultiply the equation (15) by \( \phi_i,eq^T \). This yields (cf. [5, Sec. 6.11])

\[
\phi_i,eq^T \left( \frac{\partial K_{eq}}{\partial \eta_j(t)} - \omega_i^2 \frac{\partial^2 \phi_i}{\partial \eta_j(t)} M \right) \phi_i,eq + \phi_i,eq^T \left( K_{eq} - \omega_i^2 M \right) \frac{\partial \phi_i,eq}{\partial \eta_j(t)} = 0. \tag{16}
\]

Using the eigenvalue problem in its transposed form \( \phi_i,eq^T \left( K_{eq}^T - \omega_i^2 eq^T M \right) = 0^T \) with symmetric matrices \( M = M^T, K_{eq} = K_{eq}^T \) together with the normalization condition \( \phi_i,eq^T M \phi_i,eq = 1 \), one finally obtains

\[
\frac{\partial \omega_i^2}{\partial \eta_j(t)} = \phi_i,eq^T \frac{\partial K_{eq}}{\partial \eta_j(t)} \phi_i,eq. \tag{17}
\]

Rearranging (15) – with \( \frac{\partial \omega_i^2}{\partial \eta_j(t)} \) given as in the equation above – yields the system

\[
\left( K_{eq} - \omega_i^2 eq^T M \right) \frac{\partial \phi_i,eq}{\partial \eta_j(t)} = \left( \frac{\partial \omega_i^2}{\partial \eta_j(t)} M - \frac{\partial K_{eq}}{\partial \eta_j(t)} \right) \phi_i,eq. \tag{18}
\]

The so-called modal derivative \( \theta_{ij} = \partial \phi_i,eq / \partial \eta_j(t) \) represents the derivative of mode \( \phi_i,eq \) with respect to the amplitude \( \eta_j(t) \) of mode \( \phi_j,eq \). Note, however, that the obtained system constitutes a singular linear system of equations (LSE), since the matrix \( (K_{eq} - \omega_i^2 eq^T M) \) is singular. Therefore, special care has to be taken to be able to solve the singular system (18). Different methods to tackle the singularity are discussed in the following section, in order to compute the modal derivatives under mass consideration.

The right-hand side of the above equation is known, since \( \frac{\partial \omega_i^2}{\partial \eta_j(t)} \) is computed according to (17) and \( \frac{\partial K_{eq}}{\partial \eta_j(t)} \) can either be calculated analytically within the finite element assembly procedure, or numerically via a finite difference scheme applied to the tangential stiffness matrix \( K(q_{eq}) \) with the step width \( h \):

- **Forward difference:** \( \frac{\partial K(q)}{\partial \eta_j(t)} \bigg|_{q_{eq}} = \frac{K(q_{eq} + \phi_i,eq^T h) - K(q_{eq})}{h} \),

- **Backward difference:** \( \frac{\partial K(q)}{\partial \eta_j(t)} \bigg|_{q_{eq}} = \frac{K(q_{eq}) - K(q_{eq} - \phi_i,eq^T h)}{h} \),

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3.3.1 Computation of modal derivatives including mass consideration

In order to compute the modal derivatives under mass consideration, the singular system (18) has to be solved. One way to achieve this consists in imposing an additional constraint to the modal derivative \( \theta_{ij} = \partial \phi_{i,eq}/\partial \eta_j(t) \) to obtain a regular LSE with a unique solution. The additional condition for the modal derivatives requires that the norm of the vibration mode \( \phi_{i,eq} M \phi_{i,eq} = 1 \) remains unchanged w.r.t. the amplitude \( \eta_j(t) \):

\[
\frac{\partial}{\partial \eta_j(t)} \left( \phi_{i,eq}^T M \phi_{i,eq} \right) = 0 \implies \left( \frac{\partial \phi_{i,eq}}{\partial \eta_j(t)} \right)^T M \phi_{i,eq} + \phi_{i,eq}^T M \frac{\partial \phi_{i,eq}}{\partial \eta_j(t)} = 0. \tag{19}
\]

With symmetric \( M \), this imposition leads to the following constraint for the modal derivatives

\[
\left( \frac{\partial \phi_{i,eq}}{\partial \eta_j(t)} \right)^T M \phi_{i,eq} = 0 \iff \phi_{i,eq}^T M^T \frac{\partial \phi_{i,eq}}{\partial \eta_j(t)} = 0, \tag{20}
\]

meaning that the derivative \( \theta_{ij} \) should be orthogonal to the vibration mode \( \phi_{i,eq} \) w.r.t. the \( M \)-weighted inner product.

Nelson’s method The first procedure to compute the modal derivatives is based on Nelson’s method [15], which has been further described in [7][8][21][19]. The idea is to fix one component of the solution vector \( \partial \phi_{i,eq}/\partial \eta_j(t) \) to make the coefficient matrix \( (K_{eq} - \omega_{i,eq}^2 M) \) regular. A good choice is to fix the component at the location “\( \ell \)” with the maximum absolute value of the mode \( \phi_{i,eq} \). Then, the system (18) is partitioned as

\[
\begin{bmatrix}
(K_{eq} - \omega_{i,eq}^2 M)_{11} & \cdots & 0 & \cdots & (K_{eq} - \omega_{i,eq}^2 M)_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 1 & 0 \\
\vdots & \ddots & \cdots & 0 & 0 \\
(K_{eq} - \omega_{i,eq}^2 M)_{n1} & \cdots & 0 & \cdots & (K_{eq} - \omega_{i,eq}^2 M)_{nn}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_\ell \\
v_n
\end{bmatrix}
= 
\begin{bmatrix}
r_{hs1} \\
rhs_\ell \\
rhs_n
\end{bmatrix}, \tag{21}
\]

where \( (K_{eq} - \omega_{i,eq}^2 M)_{\ell i} = 0 \) and \( (K_{eq} - \omega_{i,eq}^2 M)_{\ell \ell} = 0 \) for \( i = 1, \ldots, n, \ i \neq \ell, \) and \( (K_{eq} - \omega_{i,eq}^2 M)_{\ell \ell} = 1, (r_{hs})_\ell = 0. \) This system is nonsingular and can be solved for \( v \) using a direct or iterative solver. The solution \( \theta_{ij} \) of (18) consists of the particular solution \( v = [v_1, \ldots, 0, \cdots, v_n]^T \) and the homogeneous solution \( c_i \phi_{i,eq}, \) i.e. \( \theta_{ij} = v + c_i \phi_{i,eq}. \) The scalings \( c_i \) are determined using the constraint (20) and the normalization condition \( \phi_{i,eq}^T M \phi_{i,eq} = 1: \)

\[
(v + c_i \phi_{i,eq})^T M \phi_{i,eq} = 0 \implies c_i = -v^T M \phi_{i,eq}. \tag{22}
\]

Direct method This method has been introduced in [5 Sec. 6.11.3] and applied in [9][23]. The idea is to augment the singular LSE (18) with the derivative of the normalization condition (20) and to include \( \partial \omega_{i,eq}^2/\partial \eta_j(t) \) in the unknown vector:

\[
\begin{bmatrix}
(K_{eq} - \omega_{i,eq}^2 M) & -M \phi_{i,eq} \\
-M \phi_{i,eq}^T & 0
\end{bmatrix}
\begin{bmatrix}
\theta_{ij} \\
\frac{\partial \omega_{i,eq}^2}{\partial \eta_j(t)} \phi_{i,eq}
\end{bmatrix}
= 
\begin{bmatrix}
-\frac{\partial K_{eq}}{\partial \eta_j(t)} \phi_{i,eq} \\
0
\end{bmatrix}. \tag{23}
\]
This augmented linear system of equations can be solved directly using a dense/sparse direct solver (e.g., factorization routines from libraries like BLAS, LAPACK, UMFPACK, PARDISO (Intel MKL), MUMPS, ...), or iteratively using an iterative solver (e.g., via Krylov subspace methods (CG, GMRES, etc.), matrix splitting (Jacobi, Gauss-Seidel) or multigrid approaches).

**Pseudoinverse and Least-Squares approaches** Another method to handle the singularity of a linear system of equations $Ax = b$ is a pseudoinverse approach $x = A^+ b$, where $A^+$ denotes the Moore-Penrose pseudoinverse. The pseudoinverse computation (e.g., `pinv` in MATLAB) is based on a singular value decomposition (SVD) of the coefficient matrix $A$. Mathematically, the pseudoinverse approach delivers the approximate solution $\tilde{x}$ to the least-squares minimization problem $\tilde{x} = \arg\min_x \|Ax - b\|_2$. If the underdetermined equation $Ax = b$ has several solutions, one could e.g., get the one with the minimum norm $\|\tilde{x}\|_2$ using the minimum norm least-squares algorithm `lsmilnor` in MATLAB. This method is based on the complete orthogonal decomposition (COP) of $A$, which is a generalization of the QR decomposition with column pivoting for rank deficient matrices. For more information, cf. [5, Sec. 6.6-6.8].

### 3.3.2 Static modal derivatives excluding mass consideration

The computation of the modal derivatives under mass consideration involves the solution of a singular LSE for each $i = 1, \ldots, r$, $j = 1, \ldots, r$. In many applications, however, the mass terms included in (18) are neglected, leading to the simplified LSE

$$K_{eq} \frac{\partial \phi_{i,eq}}{\partial \eta_j(t)} \bigg|_{s} = - \frac{\partial K_{eq}}{\partial \eta_j(t)} \phi_{i,eq}, \quad (24)$$

where $\theta_{s,ij} = \partial \phi_{i,eq}/\partial \eta_j(t)$ denotes the so-called static modal derivative (SMD). Please note that the omission of the shift-term $-\omega^2_{i,eq} M$ in the coefficient matrix leads to a regular linear system of equations, since the left-hand side of (24) is only composed of the tangential stiffness $K_{eq}$. This matrix is constant and independent of the eigenfrequency $\omega_{i,eq}$ of mode $\phi_{i,eq}$, meaning that only one factorization of $K_{eq}$ is needed to compute all SMDs.

The right-hand side of (24) becomes simpler than in (18) due to the omission of the derivative of eigenfrequencies $\partial \omega^2_{i,eq}/\eta_j(t)$. What is more, the right-hand side can be rewritten as

$$\frac{\partial K_{eq}}{\partial \eta_j(t)} \phi_{i,eq} = \frac{\partial (q(t))}{\partial \eta_j(t)} \bigg|_{q_{eq}} \phi_{i,eq} := \frac{\partial^2 f(q)}{\partial q \partial q} \bigg|_{q_{eq}} \frac{\partial q}{\partial \eta_j(t)} \bigg|_{q_{eq}} = \frac{\partial^2 f(q)}{\partial \eta_j(t)} \bigg|_{q_{eq}}$$

using the relation (9) for the tangential stiffness matrix and expressing the displacement field at $q_{eq}$ as $q(t)|_{q_{eq}} = \phi_{i,eq} \eta_i(t) + \phi_{j,eq} \eta_j(t)$. Since $\partial^2 f/\partial \eta_i \eta_j = \partial^2 f/\partial \eta_j \eta_i$ holds, the right-hand side, and consequently the SMDs, become symmetric with respect to the indices $i$ and $j$:

$$\frac{\partial K_{eq}}{\partial \eta_j(t)} \phi_{i,eq} = \frac{\partial K_{eq}}{\partial \eta_i(t)} \phi_{j,eq} \quad \Longrightarrow \quad \theta_{s,ij} = \frac{\partial \phi_{i,eq}}{\partial \eta_j(t)} \bigg|_{s} = \frac{\partial \phi_{j,eq}}{\partial \eta_i(t)} \bigg|_{s} = \theta_{s,ji}. \quad (26)$$

Hence, the SMDs can be finally given as

$$\theta_{s,ij} = -K_{eq}^{-1} \frac{\partial K_{eq}}{\partial \eta_j(t)} \phi_{i,eq} \quad (27)$$

and interpreted as the negative static response to the second derivative of the nonlinear forces.
3.4 Augmentation of the reduction basis

In many publications, e.g. [27][19][23][3], it has been shown that the reduction of a nonlinear structural system with a pure linear reduction basis \( V_{\Phi} = [\phi_{1,eq}, \ldots, \phi_{r,eq}] \) containing only vibration modes may yield unsatisfactory results. In order to construct a reduction basis that also captures the nonlinear behavior of the FOM, the linear basis \( \Phi_r = [\phi_{1,eq}, \ldots, \phi_{r,eq}] \in \mathbb{R}^{n \times r} \) is augmented with (S)MDs in the following.

In the non-symmetric case of the MDs \( \theta_{ij} = \partial \phi_{i,eq} / \partial \eta_{j}(t) \), the reduction basis is composed of \( r \) vibration modes \( \Phi_r \in \mathbb{R}^{n \times r} \) and \( r^2 \) modal derivatives \( \Theta_{ij} = [\theta_{11}, \ldots, \theta_{rr}] \in \mathbb{R}^{n \times r^2} \), yielding an augmented basis \( V_{\text{aug}} = [\Phi_r, \Theta_{ij}] \in \mathbb{R}^{n \times r+r^2} \).

In the symmetric case of the SMDs, where \( \theta_{s,ij} = \theta_{s,ji} \), only \( o = r(r + 1)/2 \) distinct derivatives are included in the basis, yielding \( \Theta_{s,o} \in \mathbb{R}^{n \times o} \) and \( V_{\text{aug}} = [\Phi_r, \Theta_{s,o}] \in \mathbb{R}^{n \times r+o} \).

3.4.1 Normalization and orthogonalization via deflation

The raw reduction basis composed of both vibration modes and (S)MDs is generally not full rank, since the column vectors are usually redundant or linear dependent to each other. Thus, a Gram-Schmidt orthogonalization or, preferably, a deflation of the raw basis is often employed, in order to obtain a full rank, orthogonal basis that yields a numerically well-conditioned ROM.

Firstly, the basis vectors are normed:

\[
V_{\text{raw}} = \left[ \phi_{1,eq} / \| \phi_{1,eq} \|, \ldots, \phi_{r,eq} / \| \phi_{r,eq} \|, \theta_{11} / \| \theta_{11} \|, \ldots, \theta_{rr} / \| \theta_{rr} \| \right].
\]  

(28)

Then, a rank-revealing QR decomposition (RRQR) or a singular value decomposition (SVD) of \( V_{\text{raw}} \) is performed to deflate the basis. The latter yields \( V_{\text{aug}} = U \Sigma N^T \), where \( U \) and \( N^T \) are orthonormal matrices containing the singular vectors, and \( \Sigma \) is a diagonal matrix with the singular values arranged in descending order. The deflated basis is finally constructed with \( u_{\text{defl}} \) left singular vectors corresponding to the leading singular values: \( V_{\text{aug}} = [u_1, \ldots, u_{\text{defl}}] \). The deflated reduced order may be chosen at wish, or selected according to a given tolerance \( \varepsilon \).

3.4.2 Selection criteria for modes and modal derivatives

As we have seen before, the reduction basis may be augmented with up to \( r^2 \) MDs corresponding to \( r \) modes. This quadratic growth of the basis can be alleviated with a deflation technique to capture the most relevant subspace and reduce the dimension to \( r_{\text{defl}} \). Nevertheless, from a methodological point of view, the question raises how to choose the \( r \) dominant vibration modes and (preferably systematically) rank the (S)MDs.

For large-scale systems, only a few eigenvectors are generally computed using power iteration methods or the Arnoldi/Lanczos iteration. In the first-order case, only the eigenvectors associated to the eigenvalues fulfilling a certain criterion (e.g. smallest magnitude, largest real part, etc.) or closest to a given complex shift are calculated. In the second-order case, the eigenmodes associated to the smallest eigenfrequencies or to a specific, relevant frequency range are often computed.

A possible (rather heuristic) way to reduce the number of MDs is to restrict the choice to the derivatives \( \theta_{ij} = \partial \phi_{i,eq} / \partial \eta_{j}(t) \), describing the change of mode \( \phi_{j,eq} \) when the system is perturbed in the same direction \( \phi_{j,eq}^\prime \). Another (more systematic) approach is to make the selection based on the nonlinear normal modes (NNMs) and modal interactions of interest [23]. The relevant modal interactions could be identified based on the proximity of the ratios \( \omega_{i,eq} / \omega_{j,eq} \).
to an integer number. A further method, inspired by [1,19], is to stronger weight the low-frequency modes and their derivatives rather than the high-frequency ones. Thus, the weightings $W_{ij} = \frac{1}{\omega_{ij}^2}$ are calculated. Then, the MDs $\theta_{ij}$ with higher weights are selected. Please note that further weighting schemes exist, e.g. [24,29], that are not explained here.

4 NEW DERIVATION OF MODAL DERIVATIVES

In this section, we present a novel derivation for modal derivatives based on a polynomial/tensorial representation of the nonlinear forces and the subsequent exploitation of the Volterra theory.

4.1 Polynomial system representation

The aim is to approximate the nonlinear forces $f(q)$ in [1] by a Taylor series at the equilibrium point $q_{eq}$:

$$f(q) = f(q_{eq}) + \frac{\partial f(q_{eq})}{\partial q}(q - q_{eq}) + \frac{1}{2!} \frac{\partial^2 f(q_{eq})}{\partial q^2}(q - q_{eq})^2 + \frac{1}{3!} \frac{\partial^3 f(q_{eq})}{\partial q^3}(q - q_{eq})^3 + \cdots$$

Setting $q_{eq} = 0$ and assuming $f(q_{eq}) = 0$, we can write

$$f(q) = K^{(1)} q + K^{(2)} (q \otimes q) + K^{(3)} (q \otimes q \otimes q) + \cdots , \quad (29)$$

where the polynomial matrices are defined here as $K^{(k)} = \frac{1}{k!} \frac{\partial^k f(q_{eq})}{\partial q^k} \in \mathbb{R}^{n \times n}$, i.e. including the coefficients $1/k!$. Truncating the series after the cubic term yields the approximated system

$$M \ddot{q}(t) + D q(t) + K^{(1)} (q(t) + K^{(2)} (q(t) \otimes q(t)) + K^{(3)} (q(t) \otimes q(t) \otimes q(t)) = BF(t) . \quad (30)$$

Please note that the nonlinear forces of any geometric nonlinear system exhibiting a linear St. Venant-Kirchhoff material are a cubic function of the displacements, i.e. $f(q) = O(q^3)$. Thus, in this case the Taylor series converges after the third term, so that the representation (30) is exact and not an approximation.

The symmetric tensors $K^{(1)} \in \mathbb{R}^{n \times n}$, $K^{(2)} \in \mathbb{R}^{n \times n \times n}$ and $K^{(3)} \in \mathbb{R}^{n \times n \times n \times n}$ can be obtained from different strategies [11,19]: they can be calculated analytically within the finite element assembly procedure, numerically via finite differences, or identified via e.g. the Implicit Condensation and Expansion (ICE) method [50] or the Enforced Displacement (ED) method [12,17]. The tensors are symmetric, since they originate from the elastic potential $V = O(q^4)$ as follows:

$$f(q) = \frac{\partial^2 f}{\partial q^2}, \quad K^{(1)} = \frac{\partial f}{\partial q}, \quad K^{(2)} = \frac{1}{2} \frac{\partial^2 f}{\partial q^2} = \frac{1}{2} \frac{\partial^3 V}{\partial q^3}, \quad K^{(3)} = \frac{1}{6} \frac{\partial^3 f}{\partial q^3} = \frac{1}{6} \frac{\partial^4 V}{\partial q^4}.$$

$$f_a = \frac{\partial f}{\partial q_a}, \quad K^{(1)}_{ab} = \frac{\partial f}{\partial q_b}, \quad K^{(2)}_{abc} = \frac{1}{2} \frac{\partial^2 f}{\partial q_a \partial q_b} = \frac{1}{2} \frac{\partial^3 V}{\partial q_a \partial q_b \partial q_c}, \quad K^{(3)}_{abcd} = \frac{1}{6} \frac{\partial^3 f}{\partial q_a \partial q_b \partial q_c \partial q_d} = \cdots \quad (31)$$

4.2 Variational equations

The Volterra series representation [18] allows to describe a nonlinear system by an infinite series of cascaded, linear subsystems. The solution of the nonlinear system is then given by the sum over all sub-solutions: $q(t) = \sum_{k=1}^{\infty} q_k(t)$. To obtain the subsystem state-equations, the so-called variational equation approach [18] is employed. To this end, it is assumed that the response of the system to an input of the form $\alpha F(t)$ can be written as a sum of sub-responses:

$$q(t) = \alpha q_1(t) + \alpha^2 q_2(t) + \cdots ,$$

$$\dot{q}(t) = \alpha \dot{q}_1(t) + \alpha^2 \dot{q}_2(t) + \cdots . \quad (32)$$
Inserting the assumed input and the assumed response into (30) yields (for \( D = 0 \))

\[
M(\alpha \ddot{q}_1(t) + \alpha^2 \ddot{q}_2(t) + \ldots) + K^{(1)}(\alpha q_1(t) + \alpha^2 q_2(t) + \ldots) + K^{(2)}(q(t) \otimes q(t)) + K^{(3)}(q(t) \otimes q(t) \otimes q(t)) = B \alpha F(t).
\]

(33)

Note that, with the calculation rules for Kronecker products, it holds:

\[
q(t) \otimes q(t) = (\alpha q_1(t) + \alpha^2 q_2(t) + \ldots) \otimes (\alpha q_1(t) + \alpha^2 q_2(t) + \ldots)
\]

\[
= \alpha^2 q_1(t) \otimes q_1(t) + \alpha^3 (q_1(t) \otimes q_2(t) + q_2(t) \otimes q_1(t)) + \ldots,
\]

(34)

Since the above differential equation must hold for all \( \alpha \), coefficients of like powers of \( \alpha \) can be equated. This yields the variational equations (subsystem state-equations):

\[
\alpha : \quad M \ddot{q}_1(t) + K^{(1)} q_1(t) = B F(t),
\]

\[
q_1(0) = q_0,
\]

(35a)

\[
\alpha^2 : \quad M \ddot{q}_2(t) + K^{(1)} q_2(t) = -K^{(2)}(q_1(t) \otimes q_1(t)),
\]

\[
q_2(0) = 0,
\]

(35b)

\[
\alpha^3 : \quad M \ddot{q}_3(t) + K^{(1)} q_3(t) = -K^{(2)}(q_1(t) \otimes q_2(t) + q_2(t) \otimes q_1(t)) - K^{(3)}(q_1(t) \otimes q_1(t) \otimes q_1(t))
\]

\[
q_3(0) = 0,
\]

(35c)

\[
\vdots
\]

Note that each \( k \)-th subsystem state-equation is linear in \( q_k(t) \), but depends nonlinearly on the solution of the previous subsystems \( q_{k-1}(t) \), etc. (cf. (35b) and (35c)).

4.3 Derivation of modes and modal derivatives

The variational equations are exploited in the following to derive the modes and the new modal derivatives.

4.3.1 Modes (First subsystem)

The undamped (\( D = 0 \)), homogeneous (\( F(t) = 0 \)) subsystem (35a) is considered first. The ansatz for the solution (with \( \dot{q}_1(0) = 0 \)) is given by a free oscillation with the scalings \( c_i \), the eigenmodes \( \phi_i \) and the eigenfrequencies \( \omega_i \):

\[
q_1(t) = \sum_{i=1}^{n} c_i \phi_i \cos(\omega_i t).
\]

(36)

Inserting this ansatz (together with \( \ddot{q}_1(t) \)) into (35a) and cancelling the time-dependent term \( \cos(\omega_i t) \), yields the quadratic eigenvalue problem

\[
\left( K^{(1)} - \omega_i^2 M \right) \phi_i = 0, \quad \forall i = 1, \ldots, n.
\]

(37)

The scalings \( c_i \) are determined through the initial condition of the first subsystem (\( \dot{q}_1(0) = 0 \))

\[
q_1(0) = \sum_{i=1}^{n} c_i \phi_i \quad \iff \quad \Phi c = q_1(0) \quad \iff \quad c = \Phi^{-1} q_1(0)
\]

(38)

using the fact that \( \Phi^T M = \Phi^{-1} \) (due to \( \Phi^T M \Phi = I \)) to solve the equation for \( c = [c_1, \ldots, c_n]^T \).
4.3.2 Modal derivatives (Second subsystem)

The second subsystem (35b) is considered now. Note that the right-hand side is composed of the quadratic coupling \( q_1(t) \otimes q_1(t) \). Employing the ansatz for the first subsystem (36), we obtain (for \( n = 2 \) and the trigonometric product-to-sum identities[1]):

\[
q_1(t) \otimes q_1(t) = (c_1 \phi_1 \cos (\omega_1 t) + c_2 \phi_2 \cos (\omega_2 t)) \otimes (c_1 \phi_1 \cos (\omega_1 t) + c_2 \phi_2 \cos (\omega_2 t))
\]

\[
= c_1^2 (\phi_1 \otimes \phi_1) \cos^2(\omega_1 t) + c_2^2 (\phi_2 \otimes \phi_2) \cos^2(\omega_2 t) \\
+ c_1 c_2 (\phi_1 \otimes \phi_2 + \phi_2 \otimes \phi_1) \cos(\omega_1 t) \cos(\omega_2 t)
\]

Looking at the form of the obtained right-hand side and using the method of undetermined coefficients, we choose the following ansatz for the (particular) solution of the second subsystem (for \( n = 2 \)):

\[
q_2(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{1}{2} c_i c_j \left( \vartheta_{ij} \cos ((\omega_i + \omega_j) t) + \bar{\vartheta}_{ij} \cos ((\omega_i - \omega_j) t) \right)
\]

\[
= \frac{1}{2} c_1^2 \left( \vartheta_{11} \cos (2\omega_1 t) + \bar{\vartheta}_{11} \right) + \frac{1}{2} c_2^2 \left( \vartheta_{22} \cos (2\omega_2 t) + \bar{\vartheta}_{22} \right)
\]

\[
+ \frac{1}{2} c_1 c_2 \left( \vartheta_{12} + \bar{\vartheta}_{21} \right) \cos ((\omega_1 + \omega_2) t) + \left( \bar{\vartheta}_{12} + \bar{\vartheta}_{21} \right) \cos ((\omega_1 - \omega_2) t)
\]

Inserting this ansatz (together with \( \dot{q}_2(t) \)) into (35b) and ordering the terms, leads to \(^2\)

\[
0 = \frac{1}{2} c_1^2 \cos (2\omega_1 t) \left( \left( - (2\omega_1)^2 M + K^{(1)} \right) \vartheta_{11} + K^{(2)} (\phi_1 \otimes \phi_1) \right)
\]

\[
+ \frac{1}{2} c_1^2 \left( K^{(1)} \vartheta_{11} + K^{(2)} (\phi_1 \otimes \phi_1) \right)
\]

\[
+ \frac{1}{2} c_2^2 \cos (2\omega_2 t) \left( (2\omega_2)^2 M + K^{(1)} \right) \vartheta_{22} + K^{(2)} (\phi_2 \otimes \phi_2) \)
\]

\[
+ \frac{1}{2} c_1 c_2 \cos ((\omega_1 + \omega_2) t) \left( \left( - (\omega_1 + \omega_2)^2 M + K^{(1)} \right) \vartheta_{12} + \bar{\vartheta}_{21} \right)
\]

\[
+ K^{(2)} (\phi_1 \otimes \phi_2 + \phi_2 \otimes \phi_1)
\]

\[
+ \frac{1}{2} c_1 c_2 \cos ((\omega_1 - \omega_2) t) \left( \left( - (\omega_1 - \omega_2)^2 M + K^{(1)} \right) \bar{\vartheta}_{12} + \vartheta_{21} \right)
\]

\[
+ K^{(2)} (\phi_1 \otimes \phi_2 + \phi_2 \otimes \phi_1) = 0
\]

\[
0 = \left( \left( - (2\omega_1)^2 M + K^{(1)} \right) \vartheta_{11} + K^{(2)} (\phi_1 \otimes \phi_1) \right)
\]

\[
0 = \frac{1}{2} c_1^2 \cos (2\omega_1 t) \left( (2\omega_1)^2 M + K^{(1)} \right) \vartheta_{22} + K^{(2)} (\phi_2 \otimes \phi_2)
\]

\[
0 = \frac{1}{2} c_1 c_2 \cos ((\omega_1 + \omega_2) t) \left( \left( - (\omega_1 + \omega_2)^2 M + K^{(1)} \right) \vartheta_{12} + \bar{\vartheta}_{21} \right)
\]

\[
0 = K^{(2)} (\phi_1 \otimes \phi_2 + \phi_2 \otimes \phi_1)
\]

\[
0 = \left( \left( - (\omega_1 - \omega_2)^2 M + K^{(1)} \right) \bar{\vartheta}_{12} + \vartheta_{21} \right)
\]

\[
0 = K^{(2)} (\phi_1 \otimes \phi_2 + \phi_2 \otimes \phi_1)
\]

1\(^{\text{cos x cos y = 1/2 (cos (x + y) + cos (x - y)) and therefore cos}^2 x = \cos x \cos x = 1/2 (\cos(2x) + 1)}\)

2\(^{\text{Actually, there should be eight brackets for eight unknowns, instead of six brackets for eight unknowns. However, multiplying (41e) and (41f) out, respectively leads to the same LSE and thus yields} \)
Since the previous equation must hold for all \( t \), only the six brackets should be equal to zero.

Without loss of generality, assuming \( \tilde{\theta}_{ij} = \tilde{\theta}_{ji} \) and \( \tilde{\theta}_{ij} = \tilde{\theta}_{ji} \), it follows from all brackets:

\[
\begin{align*}
\left( K^{(1)} - (\omega_i + \omega_j)^2 M \right) \tilde{\theta}_{ij} &= -K^{(2)} (\phi_i \otimes \phi_j), \quad \forall i, j = 1, \ldots, n, \\
\left( K^{(1)} - (\omega_i - \omega_j)^2 M \right) \tilde{\theta}_{ij} &= -K^{(2)} (\phi_i \otimes \phi_j), \quad \forall i, j = 1, \ldots, n.
\end{align*}
\]  

(42a, 42b)

Equivalent description for the right-hand side

The right-hand side of equation (42) can be equivalently represented by the right-hand side of (24). First, we rewrite the right-hand side of (42) on the LSE (45) which is almost identical to the conventional equation (18) or rather (24), the factor \( 1/2 \) of eigenfrequencies is again an eigenfrequency. This fact could be related to the occurrence of internal resonances in nonlinear structural dynamics.

Interestingly, our novel derivation yields a calculation formula (45b) which is almost identical to the conventional equation (18) or rather (24), except for the fact that a sum/subtraction of eigenfrequencies results (the factor \( 1/2 \) on the right-hand side is only a definition issue).

The modal derivatives \( \tilde{\theta}_{ij} \) and \( \tilde{\theta}_{ij} \) describe the vibration shapes of the second subsystem associated to the oscillations \( \cos((\omega_i + \omega_j)t) \) and \( \cos((\omega_i - \omega_j)t) \) (cf. \( q_2(t) \) in (40)).

Comments on the gained new derivatives

Interestingly, our novel derivation yields a calculation formula (45) which is almost identical to the conventional equation (18) or rather (24), except for the fact that a sum/subtraction of eigenfrequencies results (the factor \( 1/2 \) on the right-hand side is only a definition issue).

The modal derivatives \( \tilde{\theta}_{ij} \) and \( \tilde{\theta}_{ij} \) are equal, if the mass term is neglected, yielding the well-known SMD equation (24). More importantly: the derivatives \( \tilde{\theta}_{ii} \) (where the eigenfrequencies cancel out) correspond to the SMDs, without having to explicitly neglect the mass term.

The new modal derivatives \( \tilde{\theta}_{ij} \) and \( \tilde{\theta}_{ij} \) are symmetric (\( \tilde{\theta}_{ij} = \tilde{\theta}_{ji}, \tilde{\theta}_{ij} = \tilde{\theta}_{ji} \)), even if the mass term is not neglected. This seems to be more plausible than the non-symmetric MDs (18).

The LSE (45) is regular, if the sum \( \omega_i + \omega_j \) or subtraction \( |\omega_i - \omega_j| \) of eigenfrequencies is not an eigenfrequency of the linearized system, yielding regular matrices \( (K^{(1)} - (\omega_i + \omega_j)^2 M) \) or \( (K^{(1)} - (\omega_i - \omega_j)^2 M) \). Note that in the conventional definition (18) the matrix \( (K^{(1)} - \omega_i^2 M) \) is always singular. Furthermore, the LSE (45) is singular, if the sum/subtraction of eigenfrequencies is again an eigenfrequency. This fact could be related to the occurrence of internal resonances in nonlinear structural dynamics.
5 IMPACT OF THE NEW MODAL DERIVATIVES ON STRUCTURAL DYNAMICS

5.1 Analytical solution via truncated Volterra series

One possible application of the new modal derivatives could be to assess the approximation quality of the solution of the first and second subsystem

\[ q_{1,2}(t) = \sum_{i=1}^{n} c_i \phi_i \cos(\omega_i t) + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} c_i c_j \left( \tilde{\theta}_{ij} \cos\left((\omega_i + \omega_j) t\right) + \overline{\theta}_{ij} \cos\left((\omega_i - \omega_j) t\right) \right) \]  \( \text{(46)} \)

in comparison to the solution of the whole nonlinear system. The latter could be computed e.g. (i) using the Volterra model by taking a higher (but finite) number of subsystems into account

\[ q_{1..N}(t) = \sum_{i=1}^{n} c_i \phi_i \cos(\omega_i t) + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} c_i c_j \left( \tilde{\theta}_{ij} \cos\left((\omega_i + \omega_j) t\right) + \overline{\theta}_{ij} \cos\left((\omega_i - \omega_j) t\right) \right) + \ldots \]

\[ = \sum_{k=1}^{\infty} q_k(t) \approx \sum_{k=1}^{N} q_k(t), \]  \( \text{(47)} \)

(ii) by superposing the nonlinear normal modes (NNMs) computed via shooting and path continuation methods yielding \( q_{\text{NNM}}(t) \) or (iii) via an expensive simulation of the nonlinear system to obtain the true (numerical) solution \( q_{\text{sim}}(t) \).

Moreover, each nonlinear mode of the first and second subsystem (here e.g. the first one)

\[ q_{1,2}^1(t) = c_1 \phi_1 \cos(\omega_1 t) + \frac{1}{2} c_1^2 (\tilde{\theta}_{11} \cos(2\omega_1 t) + \overline{\theta}_{11}) \]  \( \text{(48)} \)

could be compared with the corresponding nonlinear normal mode (e.g. the first one NNM1).

5.2 Reformulated analytical solution and equivalence of modal derivatives

We now define some new modal derivatives

\[ \tilde{\theta}_{ij} = \frac{1}{2} (\theta_{ij} + \theta_{ij}) \quad \text{and} \quad \overline{\theta}_{ij} = \frac{1}{2} (\theta_{ij} - \theta_{ij}), \]  \( \text{(49)} \)

to reformulate the solution \( q_2(t) \) in \( \text{(40)} \) a little differently and also for later purposes.

Using the trigonometric product-to-sum identities\(^3\), the solution of the second subsystem can be rewritten as:

\[ q_2(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} c_i c_j \left( \tilde{\theta}_{ij} \cos\left((\omega_i + \omega_j) t\right) + \overline{\theta}_{ij} \cos\left((\omega_i - \omega_j) t\right) \right) \]

\[ = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} c_i c_j \left( \tilde{\theta}_{ij} + \overline{\theta}_{ij} \right) \left( \cos\left((\omega_i + \omega_j) t\right) + \cos\left((\omega_i - \omega_j) t\right) \right) - \left( \tilde{\theta}_{ij} - \overline{\theta}_{ij} \right) \left( - \cos\left((\omega_i + \omega_j) t\right) + \cos\left((\omega_i - \omega_j) t\right) \right) \]

\[ = \sum_{i=1}^{n} \sum_{j=1}^{n} c_i c_j \left( \tilde{\theta}_{ij} \cos(\omega_j t) \cos(\omega_i t) - \theta_{ij} \sin(\omega_i t) \sin(\omega_j t) \right). \]  \( \text{(50)} \)

\(^3\cos x \cos y = 1/2 (\cos(x + y) + \cos(x - y)) \) and \( \sin x \sin y = 1/2(-\cos(x + y) + \cos(x - y)) \)
This reformulated analytical solution, including products rather than sums of eigenfrequencies, will show its advantages later on.

Note that the modal derivatives (49) can be computed indirectly from \( \tilde{\theta}_{ij} \) and \( \tilde{\tilde{\theta}}_{ij} \). Nevertheless, they can be also obtained directly by solving the following, rather complicated LSEs (given without derivation):

\[
\begin{align*}
(AB^{-1} - B) \hat{\theta}_{ij} &= -b, \\
(-B + AB^{-1} A) \bar{\theta}_{ij} &= -AB^{-1} b,
\end{align*}
\]

(51)

with \( b = K^{(2)} (\phi_i \otimes \phi_j) \), the (regular) matrix \( A = (K^{(1)} - (\omega_i^2 + \omega_j^2)M) \) and the regular matrix \( B = 2\omega_i \omega_j M \). Obviously it is easier to compute \( \theta_{ij} \) and \( \hat{\theta}_{ij} \) from \( \tilde{\theta}_{ij} \) and \( \tilde{\tilde{\theta}}_{ij} \) than solving the above LSEs.

Next, we want to discuss the equivalence of the modal derivatives \( \tilde{\theta}_{ij} \) and \( \tilde{\tilde{\theta}}_{ij} \). Let us assume that \( \tilde{\theta}_{ij} = \tilde{\tilde{\theta}}_{ij} \). Then, (42a) − (42b) yields

\[
-4 \omega_i \omega_j M \tilde{\theta}_{ij} = 0 \quad \Rightarrow \quad \tilde{\theta}_{ij} = \tilde{\tilde{\theta}}_{ij} \iff \omega_i = 0 \lor \omega_j = 0,
\]

(52)

meaning that the derivatives are only equal, if and only if the eigenfrequency \( \omega_i = 0 \) or \( \omega_j = 0 \). If the system is kinematically determinate, i.e. without rigig-body motions, then \( \omega_i, \omega_j \neq 0 \) and, hence, the derivatives are not equal.

**Case 1:** \( \tilde{\theta}_{ij} \neq \tilde{\tilde{\theta}}_{ij} \)

The nonlinear solution of the first and second subsystem is given in this case by

\[
q_{1,2}(t) = \sum_{i=1}^{n} c_i \phi_i \cos(\omega_i t) + \sum_{i=1}^{n} \sum_{j=1}^{n} c_i c_j \left( \tilde{\theta}_{ij} \cos(\omega_i t) \cos(\omega_j t) - \hat{\theta}_{ij} \sin(\omega_i t) \sin(\omega_j t) \right).
\]

(53)

**Case 2:** \( \tilde{\theta}_{ij} = \tilde{\tilde{\theta}}_{ij} \)

If \( \tilde{\theta}_{ij} = \tilde{\tilde{\theta}}_{ij} \), then \( \tilde{\theta}_{ij} = \tilde{\tilde{\theta}}_{ij} \) and \( \hat{\theta}_{ij} = 0 \) holds. Consequently, the nonlinear solution of the second subsystem simplifies, yielding

\[
q_{1,2}(t) = \sum_{i=1}^{n} c_i \phi_i \cos(\omega_i t) + \sum_{i=1}^{n} \sum_{j=1}^{n} c_i c_j \tilde{\theta}_{ij} \cos(\omega_i t) \cos(\omega_j t).
\]

(54)

5.3 **Different projection approaches for model order reduction**

Another possible application of the new modal derivatives could be to formulate novel quadratic manifold projection approaches for model order reduction.

**Common quadratic manifold approach**

The quadratic manifold approach that has been commonly used in several publications, e.g. [9] [20], is given by

\[
q(t) \approx \sum_{i=1}^{r} \phi_i q_{r,i}(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_{ij} q_{r,i}(t) q_{r,j}(t) = \Phi_r q_r(t) + \Theta_{r,2} (q_r(t) \otimes q_r(t)),
\]

(55)
using the conventional, **symmetrized** MDs $\theta_{ij}$ from (18), or rather by

$$q(t) \approx \sum_{i=1}^{r} \phi_i q_{r,i}(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_{s,ij} q_{r,i}(t) q_{r,j}(t)$$

$$= \Phi_r q_r(t) + \Theta_{s,r^2} (q_r(t) \otimes q_r(t)),$$

using the popular, symmetric SMDs $\theta_{s,ij}$ from (27).

**Novel quadratic manifold approach (1)**

We propose a first novel quadratic manifold approach based on equation (54). Using the ansatz $q_{r,i}(t) = c_i \cos(\omega_i t)$, $i = 1, \ldots, r$ for the reduced coordinates and employing the derivatives $\theta_{ij} = \frac{1}{2}(\tilde{\theta}_{ij} + \hat{\theta}_{ij})$, it follows

$$q(t) \approx \sum_{i=1}^{r} \phi_i q_{r,i}(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_{ij} q_{r,i}(t) q_{r,j}(t)$$

$$= \Phi_r q_r(t) + \Theta_{r^2} (q_r(t) \otimes q_r(t)).$$

Note that this approach looks similar as the previous ones (55) and (56). However, they are not the same, since $\theta_{ij} \neq \bar{\theta}_{ij} = 1/2(\tilde{\theta}_{ij} + \hat{\theta}_{ij})$ and $\theta_{s,ij} \neq \bar{\theta}_{ij} = 1/2(\theta_{ij} + \bar{\theta}_{ij})$.

**Novel quadratic manifold approach (2)**

We propose a second novel quadratic manifold approach based on equation (53). We again use the ansatz $q_{r,i}(t) = c_i \cos(\omega_i t)$, $i = 1, \ldots, r$ for the reduced displacements, but now together with the corresponding reduced velocities $\dot{q}_{r,i}(t) = -c_i \omega_i \sin(\omega_i t)$. This leads to

$$q(t) \approx \sum_{i=1}^{r} \phi_i q_{r,i}(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \bar{\theta}_{ij} q_{r,i}(t) q_{r,j}(t) - \frac{1}{\omega_i \omega_j} \hat{\theta}_{ij} \dot{q}_{r,i}(t) \dot{q}_{r,j}(t)$$

$$= \Phi_r q_r(t) + \Theta_{r^2} (q_r(t) \otimes q_r(t)) - \hat{\Theta}_{r^2} (\dot{q}_r(t) \otimes \dot{q}_r(t)),$$

where the factor $1/\omega_i \omega_j$ is included to exactly obtain the same solution as in (53). Interestingly, this factor is often used for weighting and ranking the conventional MDs (cf. Section 3.4.2). The (until now) rather heuristic employment of this factor could be perhaps explained more system-theoretically by the analytical solution (53) and the quadratic manifold approach (58).

Further note that the above ansatz also considers the quadratic coupling of the velocities $\dot{q}_i \otimes \dot{q}_i$, in contrast to the other approaches.

6 **CONCLUSIONS**

This conceptual paper presents a novel derivation for modal derivatives using the Volterra series representation for nonlinear structural systems. After reviewing the original derivation based on the perturbation of the linearized eigenvalue problem, we present new modal derivatives obtained from the exploitation of the Volterra theory and the variational equation approach. Interestingly, our novel derivation yields a calculation formula which is almost identical to the conventional definition, except for the fact that a sum/subtraction of eigenfrequencies results.
This allows, among others, to retrieve the static modal derivatives when eigenfrequencies cancel out, without having to explicitly neglect mass terms.

Besides the novel derivation, some possible implications of the new modal derivatives are proposed and discussed. These include: (i) the system-theoretic explanation of nonlinear effects like internal resonances, (ii) the assessment of the approximation quality of the gained analytical solution in comparison to nonlinear normal modes, the harmonic balance method or a full simulation of the nonlinear system and (iii) the application of novel quadratic manifold approaches for model order reduction.

The numerical validation of the presented new modal derivatives, together with their promising impacts and applications, is subject of ongoing research. We are currently comparing the new derivatives with the conventional ones and using all them for model order reduction purposes (basis augmentation and quadratic manifold). In addition, we want to evaluate the differently obtained ROMs both in time- (via simulation runs) and frequency-domain (via NNMs and NLFRFs) in the following, in order to confirm the expected performance advantage of the new modal derivatives. Our goal is that the concepts presented here are not only useful from a theoretical perspective, but also beneficial for practical purposes in nonlinear structural dynamics.

REFERENCES


REDUCED ORDER MODELING FOR THE DYNAMIC ANALYSIS OF STRUCTURES WITH NONLINEAR INTERFACES

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Abstract

In the present paper, linear substructures with nonlinearities localized at their interfaces, such as the joints in a beam structure, are studied. By subdivision of the total structure into substructures, reduced subsystems are obtained by component mode synthesis. Nonlinear elements are introduced at supports or between substructures. A numerical example is presented where a beam subjected to blast loading is studied. The influence of the nonlinear behavior as well as the number of retained fixed-interface normal modes in the reduced subsystems are evaluated. The response is also compared to the response of equivalent single-degree-of-freedom systems, which are frequently employed in blast load design calculations. For the load cases studied, the displacement computed from an equivalent single-degree-of-freedom system correspond fairly well to the displacement given by a refined two-dimensional beam model, reduced by substructuring. In contrast, the shear force differs significantly due to that higher order modes are neglected in the single-degree-of-freedom system.

Keywords: Substructuring, Component Mode Synthesis, Blast Loading, Structural Dynamics
1 INTRODUCTION

Design of structures subjected to accidental loading, such as impact and blast loading, can be challenging compared to the design of static loading. As for dynamic loading in general, the structure mass, stiffness and strength affect the response and must be considered to determine whether a certain design fulfill the design code requirements. Consequently, it is often necessary to consider accidental loads in both the conceptual and detailed design phase and, therefore, it is important to employ simplified, conservative and computationally efficient models to estimate the structure response in a time-efficient manner. Moreover, the response computed from a large complex nonlinear model can be difficult to interpret and verify, hence, a smaller and less complex model simplifies the result evaluation.

In the present paper, linear substructures with nonlinearities localized at their interfaces, such as the joints in a beam structure, are studied. By subdivision of the total structure into substructures, reduced subsystems are obtained by dynamic substructuring [1]. Nonlinear elements are introduced at supports or between substructures [3]. The concept is presented in a numerical example in which a simply supported beam subjected to blast loading is studied. The influence of the nonlinear behavior as well as the number of retained fixed-interface normal modes in the reduced subsystem are evaluated. The response is also compared to the response of equivalent single-degree-of-freedom (SDOF) systems which are frequently employed in blast load design calculations.

2 REDUCED ORDER MODELING OF BEAMS SUBJECTED TO BLAST LOADING

The response of a linear structural dynamic system can be analyzed in a computationally efficient manner by considering an approximate reduced order model. For example, the response of a few important eigenmodes can form a reduced model. However, in analyses related to blast loading it is important to include the nonlinear behavior to ensure a model that predicts a realistic structural response. The material nonlinearities are often localized to certain areas such as plastic hinges in heavily loaded beams and plates. Hence, the structure can be subdivided into substructures with a linear response, connecting the nonlinear elements introduced at the supports or between substructures. Since each subsystem is linear, dynamic substructuring can be employed to form a reduced model.

2.1 Impulse pressure due to unconfined explosion

An unconfined explosion results in a shock wave that moves radially away from the center of the explosion [6]. Upon impact, the initial wave is reinforced and reflected. The reflected impulse acting on the structure is characterized by a very large pressure and short duration. For design purposes, the reflected impulse can, in general, be replaced by an equivalent triangular pulse where the actual duration is replaced by a fictitious duration, calculated based on the peak reflected pressure and reflected impulse. Moreover, if the explosion is unconfined and the explosion center is reasonable far from the structure considered, the pressure acting on a structure member can, in general, be approximated by a uniform pressure.

2.2 Single-degree-of-freedom systems

Equivalent SDOF systems are frequently employed for the design of the semi-global response of structural members subjected to blast loading, e.g. as proposed in [4]. This is a well-established approach compatible with the requirements in several design codes, e.g. UFC [6].
Equivalent SDOF systems can be derived for structures idealized as either beams or plates. In the study presented here only beams are considered.

The main assumption when developing an equivalent SDOF system is that the member deforms according to an assumed shape, \( \phi(x) \), which is constant through time. Hence, the member deflection \( u(x,t) \) can be expressed as \( \phi(x)u_s(t) \), where \( u_s \) is the displacement of a reference point, e.g. the point of maximum displacement, see Figure 1. The shape function \( \phi(x) \) is often chosen as a Ritz vector corresponding to the static deflection of the external pressure. Note that, as for the deformed shape, the load distribution is assumed constant through time. The beam model is then transformed into a SDOF system by calculating an equivalent mass, stiffness and load in terms of the reference point displacement.

The equation of motion for an equivalent SDOF system can be expressed as:

\[
\kappa_m m \ddot{u} + \kappa_k R(u) = \kappa_F p(t)
\]  

(1)

where \( \kappa_m, \kappa_k \) and \( \kappa_F \) are dimensionless transformation factors for the mass \( (m) \), resistance \( (R) \) and load \( (p) \), respectively. The mass factor, by which the total distributed mass of an element is multiplied to obtain an equivalent lumped mass, is derived by assuming conservation of kinetic energy [4]. If the mass is uniformly distributed, the mass factor for a beam with length \( L \) is given by:

\[
\kappa_m = \frac{1}{L} \int_0^L \frac{\phi(x)^2}{u_s^2} \, dx
\]  

(2)

The load factor, by which the total pressure on the element is multiplied to obtain an equivalent concentrated force, is derived by assuming conservation of external work [4]. If the external pressure is uniformly distributed, the load factor for a beam with length \( L \) is given by:

\[
\kappa_F = \frac{1}{L} \int_0^L \frac{\phi(x)}{u_s} \, dx
\]  

(3)

The resistance factor, by which the resistance of the structural element is multiplied to obtain the equivalent resistance of the SDOF system, is derived by assuming conservation of strain energy for the structural member, computed based on the assumed deformed shape. According to [4], it can be shown that the resistance-factor must always be equal to the load-factor, i.e.:

\[
\kappa_k = \kappa_F
\]  

(4)

Hence, the equation of motion (1) for the SDOF system can be rewritten as

\[
\frac{\kappa_m}{\kappa_F} m \ddot{u} + R(u) = p(t)
\]  

(5)

Consequently, the beam can be transformed into an equivalent SDOF system by scaling the mass only and, therefore, it is convenient to define a load-mass factor \( \kappa_mF = \kappa_m/\kappa_F \).
When the ultimate moment capacity of a beam member is reached, in general, a plastic hinge is developed, which indeed affect the shape of the deflection. Hence, the assumed deflection shape considered for an elastic response, reasonable in the initial stage, differ significantly from the assumed plastic deflection shape which is reasonable in the second stage where a plastic hinge has been developed, cf. Figure 1. Moreover, the transformation factors given by Eqs. (2) and (3), which are constant through time, depend on the assumed shape \( \varphi(x) \). Consequently, the equivalent SDOF system must be derived based on either an elastic or plastic deflection shape. For example, the \( \kappa_{mf} \) factors corresponding to a simply supported beam subjected to a uniform pressure are 0.787 and 0.667 for an elastic and plastic deflection shape respectively.

For a simply supported beam subjected to a uniform pressure the ultimate resistance can be calculated as:

\[
R_u = \frac{8M_u}{L} \tag{6}
\]

where \( M_u \) is the ultimate moment capacity, which implies that the maximum shear force at the supports can be calculated as \( V = R_u/2 \). Hence, the ultimate moment capacity has, in general, a large impact on the shear force magnitude and clearly a larger moment capacity is not beneficial (however, the moment capacity must be sufficiently large to ensure that the plastic rotation is smaller than the rotation capacity). For example, if the amount of bending reinforcement in a concrete member is increased the amount of shear reinforcement due to blast loading must be increased accordingly. Since both the mass, stiffness and ultimate capacity affect the response, the design often requires iterative design calculations where simplified models, as the equivalent SDOF system, are important.

However, it should be noted that the shear force given by an equivalent SDOF system is computed with the assumption that the beam deflection shape is constant through time and, consequently, higher order modes are neglected. Furthermore, the shear forces are in general large at the initial stage, due to that higher order modes are excited. As e.g. observed in [7], the neglect of higher order modes indeed affect the precision of the shear force computed from an equivalent SDOF analysis.

For concrete members, a shear failure at the initial stage is referred to as a direct shear failure and is characterized by a rapid propagation of a vertical crack, located at the supports. Unlike diagonal shear failure, shear reinforcement perpendicular to the beam axis does not prevent this type of failure, instead inclined bars may be needed to ensure an adequate design. However,
it is in general not possible to determine the actual magnitude of the maximum shear force based on the response computed from an equivalent SDOF system.

2.3 Substructuring of beam model with plastic hinges

The response of a beam subjected to blast loading is, in general, elastic in a first stage and a mixture of both elastic and plastic in a second stage. Furthermore, in contrast to the response computed from an equivalent SDOF system, the fundamental mode as well as higher order modes are excited. To fully capture the structural behavior, it is therefore necessary to employ a nonlinear multi-degree-of-freedom (MDOF) model. Nonlinear analyses of large systems are, however, time consuming and might not be suitable in a design calculation. Nevertheless, a more refined model, compared to an equivalent SDOF system, might be necessary to enable accurate predictions of both the maximum displacement and shear force.

If the material nonlinearities are localized to certain areas, such as plastic hinges in heavily loaded beams, the total structure can be subdivided into substructures. Each substructure then consists of a subsystem with a linear response, connecting the nonlinear elements introduced at interfaces, i.e. at the supports or between substructures. Since each subsystem is linear, it is straightforward to employ component mode synthesis (CMS) to form a reduced model [3]. Hence, a reduced model that captures a combined elastic and plastic response as well as including higher order modes can be derived. This procedure can be extended further to include both material and geometrically nonlinearities, i.e. to allow for large translations and rotations of the substructures. However, in the study presented here only material nonlinearities are considered.

The substructures can for example be reduced by condensation methods, such as Guyan reduction [2], where only physical DOFs are involved or by hybrid methods, such as component mode synthesis by Craig-Bampton or Krylov subspace component mode synthesis, where both physical and generalized DOFs are considered [8].

A finite element formulation of a subsystem leads to a linear equation of motion of the following form:

\[
\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p}
\]  

Neglecting damping the partitioned mass and stiffness matrices can be written as:

\[
\begin{bmatrix}
\mathbf{M}_{ii} & \mathbf{M}_{ib} \\
\mathbf{M}_{bi} & \mathbf{M}_{bb}
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{u}}_i \\
\ddot{\mathbf{u}}_b
\end{bmatrix}
+ 
\begin{bmatrix}
\mathbf{K}_{ii} & \mathbf{K}_{ib} \\
\mathbf{K}_{bi} & \mathbf{K}_{bb}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_i \\
\mathbf{u}_b
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{p}_i \\
\mathbf{p}_b
\end{bmatrix}
\]

where the subscripts \(i\) and \(b\) denotes the interior and interface boundary DOFs, respectively. If assumed force-free, the interior DOFs can be expressed as:

\[
\mathbf{u}_i = -\mathbf{K}_{ii}^{-1}(\mathbf{M}_{ii}\ddot{\mathbf{u}}_i + \mathbf{M}_{ib}\ddot{\mathbf{u}}_b + \mathbf{K}_{ib}\mathbf{u}_b)
\]

By neglecting the inertia terms this leads to the following transformation matrix:

\[
\begin{bmatrix}
\mathbf{u}_i \\
\mathbf{u}_b
\end{bmatrix}
= 
\begin{bmatrix}
-\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} \\
\mathbf{I}_{bb}
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{u}}_b \\
\mathbf{u}_b
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{\Psi}_{ib} \\
\mathbf{I}_{bb}
\end{bmatrix}
\mathbf{u}_b
= 
\mathbf{T}_G\mathbf{u}_b
\]

where \(\mathbf{T}_G\) is the Guyan transformation matrix. By applying the transformation matrix to Eq. (7) a reduced system is given by:

\[
\mathbf{M}_G\ddot{\mathbf{u}}_b + \mathbf{C}_G\dot{\mathbf{u}}_b + \mathbf{K}_G\mathbf{u}_b = \mathbf{p}_G
\]
where,

\[
\begin{align*}
M_G &= T_G^T M T_G \\
C_G &= T_G^T C T_G \\
K_G &= T_G^T K T_G \\
p_G &= T_G^T p
\end{align*}
\]

The Craig-Bampton method combines the retained physical DOFs with fixed-interface normal modes, obtained by the generalized eigenvalue problem:

\[
(K_{ii} - \omega_j^2 M_{ii}) \{ \phi_j \} = 0
\]

The eigenvectors are then normalized in order that

\[
\Phi_{ii}^T M_{ii} \Phi_{ii} = I_{ii}
\]

where \(\Phi_{ii}\) is the complete set of fixed-interface normal modes. The physical coordinates can be represented as:

\[
\begin{bmatrix}
\mathbf{u}_i \\
\mathbf{u}_b
\end{bmatrix}
= 
\begin{bmatrix}
\Phi_{ik} & \Psi_{ib} \\
0_{bi} & I_{bb}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_k \\
\mathbf{u}_b
\end{bmatrix}
= T_{C-B} \begin{bmatrix}
\mathbf{q}_k \\
\mathbf{u}_b
\end{bmatrix}
\]

where the subscript \(k\) denotes the retained (kept) fixed-interface normal modes, \(T_{C-B}\) is the Craig-Bampton transformation matrix, \(\mathbf{q}_k\) is the generalized DOFs and \([\Psi_{ib} \ I_{bb}]^T\) is the interface constraint mode matrix, equal to the Guyan transformation matrix. Hence, the Craig-Bampton method can be interpreted as an extension of the Guyan reduction where the neglected inertia terms are compensated by including a set of fixed-interface normal modes.

By applying the transformation matrix to Eq. (7) a reduced system is given by:

\[
M_{C-B} \begin{bmatrix}
\dot{\mathbf{q}}_k \\
\dot{\mathbf{u}}_b
\end{bmatrix}
+ C_{C-B} \begin{bmatrix}
\dot{\mathbf{q}}_k \\
\dot{\mathbf{u}}_b
\end{bmatrix}
+ K_{C-B} \begin{bmatrix}
\mathbf{q}_k \\
\mathbf{u}_b
\end{bmatrix}
= p_{C-B}
\]

where,

\[
\begin{align*}
M_{C-B} &= T_{C-B}^T M T_{C-B} \\
C_{C-B} &= T_{C-B}^T C T_{C-B} \\
K_{C-B} &= T_{C-B}^T K T_{C-B} \\
p_{C-B} &= T_{C-B}^T p
\end{align*}
\]

Note that each constraint mode is the deflection shape due to a unit displacement of a boundary DOF, while the interior DOFs are force-free and the other boundary DOFs are held fixed, i.e.

\[
\begin{bmatrix}
K_{ii} & K_{ib} \\
K_{bi} & K_{bb}
\end{bmatrix}
\begin{bmatrix}
\Psi_{ib} \\
I_{bb}
\end{bmatrix}
= \begin{bmatrix}
0_{ib} \\
R_{bb}
\end{bmatrix}
\]

where \(R_{bb}\) is the reaction forces acting on the substructure.

By using a similar procedure as presented above, the fixed-interface normal modes employed in the Craig-Bampton method can be replaced by other Ritz vectors, e.g. Krylov subspace vectors derived from a suitable load distribution, see for example [5].
3 NUMERICAL EXAMPLE: SIMPLY SUPPORTED CONCRETE BEAM SUBJECTED TO BLAST LOADING

3.1 Reduced two-dimensional beam model

The effect of higher order modes on the displacement and shear force is studied by evaluating the response for a simply supported concrete beam subjected to a uniform distributed impulse pressure. The beam length is \( L = 3 \) m and the cross-section width and height is 1000 mm and 200 mm respectively. The load consists of a uniform reflected impulse pressure of 1500 Pa\( \cdot \)s. Two load cases are studied with a peak reflected pressure of 1000 kPa in Load Case 1 and 300 kPa in Load Case 2, respectively. The pulse is approximated by an equivalent triangular pulse, hence, a fictitious duration can be calculated to 3 ms and 10 ms for Load Cases 1 and 2 respectively.

The beam is assumed to consist of concrete C30/37 with reinforcement Ø16s200 K500C-T. The modulus of elasticity for concrete and reinforcement steel is 32 GPa and 200 GPa respectively and the density for reinforced concrete is set to 2500 kg/m\(^3\). The ultimate moment capacity is set to \( M_p = 80 \) kNm. The response is calculated with a two-dimensional beam model with a total of 20 Euler-Bernoulli two-node beam elements, as shown in Figure 2. Due to symmetry, only half of the beam is included in the FE model. Furthermore, small deformations are considered and the axial DOFs of the beam elements are neglected. It is assumed that a plastic hinge can appear at the beam midspan only. The plastic hinge is modelled by adding a rigid-perfectly plastic rotational spring to the rotational DOF at the symmetry line, as shown in Figure 2. Several effects are neglected in the model, e.g. catenary effects, reduced stiffness due to concrete cracking, concrete spalling etc. Nevertheless, the beam model is appropriate for evaluating the influence of higher order modes on the shear force and midspan displacement.

The beam model consists of a linear elastic subsystem, namely the beam element assemblage, which is connected to a nonlinear element. As discussed in Section 2, a reduced model can therefore be obtained by substructuring, e.g. by Guyan reduction or CMS by Craig-Bampton. Only one boundary DOF is kept, i.e. the rotational DOF at the beam symmetry line, denoted with superscript \( b \) in Figure 2. All other DOFs are internal DOFs and denoted with superscript \( i \) in Figure 2. Accordingly, the fixed-interface normal modes are calculated with fixed boundaries, i.e. with fixed rotation at the symmetry line. Hence, the substructure normal modes correspond to the symmetric eigenmodes for a simply supported beam, which are also the only modes that are excited by a uniform load. Thus, for an elastic response the system response is equivalent to the response of a linear elastic simply supported beam analyzed with modal truncation.
3.2 Structural analysis and results

In Figure 3 the first four fixed-interface normal modes are shown, which correspond to the first, third, fifth and seventh eigenmodes for a simply supported beam. Hence, by employing a symmetry model instead of a full model both the number of physical DOFs and modal coordinates are halved.

The constraint mode is shown in Figure 4 and correspond to a unit rotation of the boundary DOF, i.e. a unit rotation of the rotational DOF at the symmetry line. Hence, the constraint mode corresponds to a rigid body mode of the beam element assemblage which in turn correspond to the plastic deflection shape considered for an equivalent SDOF system, cf. Figure 1b.

The nonlinear dynamic response is calculated using the Newmark β-method with $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{4}$ (constant average acceleration) combined with the modified Newton-Raphson method. The total analysis time is 50 ms and the time-stepping is performed with very fine time increment < 0.01 ms to ensure sufficient resolution of the shear force.
The midspan displacement and shear force at the supports are evaluated for the beam model reduced by both Guyan reduction and the Craig-Bampton method for Load Cases 1 and 2 respectively. The response is compared to the response computed from an equivalent SDOF system. The stiffness of the SDOF system is calculated based on an uncracked cross section and the ultimate resistance is computed from the ultimate moment capacity, according to Eq. (6). A plastic deflection shape in accordance with Figure 1b is considered when determining the load-mass factor.

![Figure 5: Midspan displacement vs. time for Load Case 1.](image1)

![Figure 6: Midspan displacement vs. time for Load Case 2.](image2)
As shown in Figure 6, the midspan displacement for Load Case 2 calculated by the two-dimensional beam model is close to the displacement computed from an equivalent SDOF system. However, for Load Case 1 the response somewhat differs due to a larger influence of higher order modes, see Figure 5. Furthermore, as shown in Figures 5 and 6, only two fixed-interface normal modes need to be retained to obtain a response very close to the response for the full unreduced model.

**Figure 7:** Shear force at supports for Load Case 1.

**Figure 8:** Shear force at the supports for Load Case 2.
The response obtained with Guyan reduction is included for comparison only. Additional boundary (or master) DOFs must be added to increase the precision of the Guyan reduction. As shown in Figures 5 and 6, a Guyan reduction where only the rotational DOF at the symmetry line is kept correspond to an equivalent rigid-plastic SDOF system, i.e. only the rigid mode of the beam assemblage is activated and the external work is dissipated by plastic deformation of the rigid-perfectly plastic rotational spring alone.

The shear force at the supports for Load Cases 1 and 2, computed from both an equivalent SDOF system and the two-dimensional beam model, are shown in Figures 7 and 8, respectively. As shown in the figures, the shear force computed from the two-dimensional model is, as expected, much larger due to that higher order modes are considered. The difference is greater for Load Case 1, where the impulse duration is shorter. As shown in Figure 7, at least four fixed-interface normal modes need to be included to capture the peak shear force. For Load Case 2, however, the shear force computed from a reduced model with two to three fixed-interface normal modes is fairly close to the peak shear force given by the full model.

Note that the shear force is \( V \neq 0 \) at \( t = 0 \). This is due to the discretization of the beam substructure. Half of the pressure on the beam element connected to the vertical support is instantaneously transferred to the support. Hence, the shear force/reaction force at \( t = 0 \) due to discretization can be calculated as \( V(0) = p(0) \cdot L/(2 \cdot 2 \cdot n) \), where \( n \) is the number of beam elements in the symmetry model. Since a Guyan reduction only includes a rigid body mode of the beam elements the shear force should clearly be equal to zero, thus, the shear force shown in the diagrams is only due to the discretization of the beam assemblage. For Load Cases 1 and 2 the shear force due to discretization is calculated to 38 kN and 11 kN, respectively, i.e. in accordance with the response shown in Figures 7 and 8.

4 CONCLUSIONS

In the present paper, linear substructures with nonlinearities localized at their interfaces, such as plastic hinges in a beam member, are studied. By subdivision of the structure into substructures, reduced subsystems are obtained by use of the Craig-Bampton method. A numerical example is presented where a simply supported beam subjected to blast loading is studied.

For the Load Cases studied, the midspan displacement computed from an equivalent SDOF system, which are frequently employed in blast load design calculations, correspond fairly well to the displacement computed from a refined two-dimensional beam model, reduced by substructuring. In contrast, the shear force computed from an equivalent SDOF systems differ significantly from the peak shear force given by a refined two-dimensional beam model. This is due to that higher order modes are neglected in the equivalent SDOF system. As expected, the difference is greater for Load Case 1, where the beam is subjected to a pulse with higher peak pressure and shorter duration. To capture the peak shear force at least the first three to four fixed-interface normal modes need to be included in the reduced model. However, one boundary DOF and three to four generalized DOFs result in a MDOF system with five DOFs, which is still a very small system appropriate for time efficient iterative design calculations in both the conceptual and detailed design phase.

For a simply supported beam with a plastic hinge at the midspan the boundary DOFs in the Craig-Bampton method can be selected so that the linear response is equivalent to a linear elastic beam analyzed by modal truncation. Furthermore, the fixed-interface normal modes employed in the Craig-Bampton method can be replaced by other Ritz vectors, such as Krylov subspace vectors, which are derived from the current load configuration. Krylov subspace component mode synthesis can be expected to be efficient if the load configuration does not match the first normal modes.
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THE OUT OF PLANE SEISMIC DEMAND OF INFILL WALLS IN THE NONLINEAR FIELD

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Abstract

Field observations after seismic events evidenced that, in many cases, the damage of infill walls is due to out-of-plane actions that pose a risk to human life (falling of heavy parts). The out-of-plane behaviour of the infill walls depends on many factors including the type of infill wall, the in-plane damage and the efficiency of their connection to the RC frame. The evaluation of the performance of the out-of-plane infill walls requires the evaluation of both seismic demand (i.e. seismic action on the infill) and capacity (strength). The main aim of this paper is to study the effect of the nonlinear behaviour of the structure on the out-of-plane seismic demand of the infill walls, also taking in account their nonlinearity and flexibility. In particular, 3D RC frames are studied considering a lumped plasticity model for the RC elements and adopting a recent nonlinear model that implements the in-plane and out-of-plane interaction behaviour for the infill walls.

The analyses highlight that the nonlinear behaviour of the infill wall influences the variation of acceleration along the height of the wall; this effect is amplified when also the nonlinearity of the RC structure is considered. Moreover, the effect of the infill nonlinearities varies along the height of the building.

Keywords: Infill Wall, Out-Of-Plane, Nonlinear Behaviour, Acceleration Amplification.
1 INTRODUCTION

Even though any structural function is attributed to the infill walls and usually they are not considered in the models of the seismic resisting system, they are able to interact with the frame structure influencing the seismic response of the buildings. Moreover, field observations after seismic events evidenced that out-of-plane collapse frequently depends on the type of wall and efficiency of its connection to the RC frame and can be influenced by the in-plane damage due to the interaction with the resisting frame. Currently, the subject is further complicated by the innovation of the market with an ever-increasing number of materials and coatings needed to meet functional requirements such as thermal insulation.

The evaluation of the performance of the out-of-plane infill walls requires the evaluation of both seismic demand (i.e. seismic action on the infill) and capacity (strength) of the panel. In this paper, the evaluation of the seismic demand of the infill walls in terms of out-of-plane acceleration is focused. In a previous work [1] a parametric analysis on the infill walls characteristics allowed to obtain quite general results for the dynamic behavior of single walls with a wide range of elastic properties and for the interaction between the dynamic behavior of the framed structure and its infill walls in the elastic field. Surely, the knowledge of the demand in acceleration in the elastic field is of paramount importance when the design or safety of a structure is based on the elastic spectrum for defining the seismic input (as in the case of linear dynamic analysis) because the demand and capacity can be independently evaluated.

However, the linear modeling neglects the damage effect of the in-plane behavior on the out-of-plane in contrast with the current and recent knowledge of several authors, e.g. [2; 3; 4; 5] demonstrating the important effect of the in-plane damage on the out of plane response.

Furthermore the out-of-plane demand and consequently the resistance verification should be conducted at life safety limit state (SLV), i.e. in non linear field for dissipative structures, because this collapse represent a severe risk for life safety. Therefore, the Authors extended their theoretical and numerical analysis on the out-of-plane seismic demand of the infill walls in the non linear field. A 3D RC frame was studied considering the lumped plasticity of the RC elements and adopting a recent nonlinear model that implements the in-plane and out-of-plane interaction behavior of the infill walls. Moreover, in the nonlinear field the RC detailing, that governs the structure ductility i.e. its deformation at the ultimate condition, surely influences the demand of the infill walls. For this reason, as case study a 3-story RC framed structure was chosen considering two design procedures: a design under seismic actions and a design under gravity loads only.

2 CASE STUDY OF BUILDINGS AND MODELS

The 3D RC frame assumed as case study has 3 bays and 3 storeys. The length of each bay is l=5m and the height of each story is h=3m. The first structure (M1) is representative of a new construction designed according to the capacity design provisions of EC8 [7] that are the same of the Italian code NTC2018 [6]. The class B450C and C25/30 were used for steel and concrete, respectively; the dead loads were calculated assuming a height of the RC lightened floor of 25 cm; the weight of the floor considering the self-weight and dead load was assumed equal to 5.4 kN/m², with a live load of 2 kN/m². The second building (M2), instead, is a reinforced concrete building designed according the same codes but considering only gravity loads. This building is in general representative of existing structures under designed for the seismic actions. In Figure 1 the key dimensions in plan of the structure are depicted showing a double symmetry, moreover, the dimensions and the longitudinal reinforcement of the columns are shown in Table 1.
Two sets of models were implemented of each one structure: bare structure (N) and infilled structure (T). For each set both linear (L) and nonlinear model (NL) were developed.

In all models the structural elements were implemented as elastic frame elements in the linear field, i.e. considering the gross moment of inertia of the elements and assuming an elastic modulus of 31476 MPa, according to the C25/30 class concrete. In the nonlinear model, instead, the beams and columns were modelled with lumped plasticity introducing the plastic hinges at the ends of the elements. An elastic perfectly plastic envelope of the moment-curvature law was assigned to the plastic hinges with maximum moment and ultimate curva-
ture depending on the reinforcement and concrete characteristics. The cyclic moment curvature degradation was governed by a Takeda law [8]. The response of the infill walls was introduced by the macro-model proposed by Mosalam and Gunay [9], originally implemented in OpenSees and here implemented in SAP2000 software [10]. In the model, each infill wall was represented by a single diagonal, Figure 2a, composed of two frame elements and connected at a midpoint node with an assigned lumped mass in the OOP direction. The cross-section of the beam-column element was modelled by nonlinear fibers for flexure out of the plane as shown in Figure 2b.

In this way, the beam-column element acts as truss and flexural elements in the IP and OOP directions, respectively, considering the interaction between the IP axial strength and the OOP bending strength. Location of the fibers and their nonlinear material properties are set to satisfy the intended strength interaction and the IP axial and OOP bending stiffness values. For this purpose, FEMA-356 [11] equations were used to calculate the axial stiffness and unidirectional strength in the IP direction. The OOP mass, stiffness and unidirectional bending strength were calculated such in order to obtain: (a) the same natural frequency as the infill wall, (b) the same restoring force as the infill wall in the OOP direction for a given support motion (story acceleration) and (c) the yielding at the same motion that causes the yielding of the infill wall. Relevant equations and their derivation can be found in Kadysiewski and Mosalam [12].

The IP axial and OOP bending strength interaction is the 3/2-power curve reported in equation 1:

\[
\left(\frac{P_H}{P_{H0}}\right)^{3/2} + \left(\frac{P_N}{P_{N0}}\right)^{3/2} = 1.0
\]  

Figure 2: a) Infill wall model for IP-OOP interaction; b) fiber layout in the cross-section (from Kadysiewski and Mosalam, 2009).

where \(P_H\) and \(P_{H0}\) are IP axial strength with and without OOP force, respectively, and \(P_N\) and \(P_{N0}\) are OOP bending strength with and without IP force, respectively.

Because of its simple formulation, the aim of the macro-model is to investigate the response of infill panels in a global analysis of the building allowing to consider IP/OOP interaction. In this work the model was used to evaluate the out-of-plane acceleration at the center of the infill wall, where the OOP mass is concentrated.
3 SELECTION OF THE ACCELEROMETERS

The first step of this analytical study is to assume a seismic input for the structural time history analysis. The seismic input consists of two sets of seven accelerograms matching the elastic response spectrum at SLV (Life Safety Limit State) of the Italian code NTC2018 [6] for an assigned type of soil (type C). The SLV spectrum is representative of a moderate Italian hazard and was considered more appropriate for the analysis of the infill walls, but the results are significant in general for seismic input because the acceleration amplification in the walls is estimated. The accelerograms were selected from the European Strong Motion Database [13] and scaled to guarantee the compatibility between their mean spectrum and the code spectrum according to the Eurocode 8 recommendations [7], that are adopted also in Italy, through the software REXEL 3.4 BETA [14]. In particular, the mean elastic spectrum of the selected ground motions has ordinates equal to 90% or 130% of the spectrum target elastic response spectrum proposed by the Italian code NTC2018 in the range of periods between 0.15 s and 2 s, that is valid for all the considered models of 3D frame (bare and infilled).

Figure 3 shows the elastic spectra of the 14 records, along with their average spectrum (blue thick line), the reference elastic design spectrum (black thick line) and the limit curves (black thin lines).

![Figure 3: Combination selected for the dynamic analysis.](image)

4 DYNAMIC BEHAVIOUR OF THE CASE STUDY BUILDINGS

A modal analysis for the two case study buildings designed considering the bare model, was performed in order to identify the dynamic behavior of the structures. In both cases the first and second modes are translational in Y and X direction, respectively while the third mode is torsional.

The main dynamic properties (period, mode shapes and mass participating ratio) are reported in Table 2 for the first three modes.
Table 2: Modal properties

The first three modal shapes along the height of the building are shown in Figure 4; from these plots it can be noted that for the seismic designed building (M1) the first and second mode contribute more to the dynamic response at the roof level, the third mode appears significant at the first floor. Instead, for the gravity load designed building (M2), the first and second mode contribute more to the dynamic response at the first floor while the third mode appears more significant at the upper floors. It is clear that the design approach defines structures with a very different dynamic response.

Figure 4: First three mode shapes of the buildings M1 (left) and M2 (right).

5 ANALYSIS OF THE BARE FRAMES

In this section, the results of the dynamic analyses carried out on the bare frame buildings are discussed. In particular, the PFA amplification factor \( f_a \) obtained from the selected input ground motions was monitored. The acceleration amplification \( f_a \) at a given storey is defined as the ratio between the absolute value of the peak of the acceleration time history at that storey and PGA. The results obtained with both sets of linear and nonlinear models are summarized and shown side by side in Figure 5, where the distribution of peak floor acceleration amplification \( f_a \) for the building (M1) designed for seismic loads (left) and the building (M2) designed for gravity loads (right), obtained in X direction (the structure is equal along the two directions), are displayed. The solid and dashed curves represent the mean (over all the records considered) \( f_a \) distribution along the height of the building of the linear and the nonlinear models of the buildings, respectively.
In the linear models, the $f_a$ distributions along the building’s height (solid curves) are characterized by a similar profile in both buildings, i.e. the accelerations tend to increase linearly with the floor level, because only small differences of the dimensions of the resistant elements influence the stiffness of the buildings. When the non-linear model is introduced (dashed curves in Figure 5) $f_a$ reduces and tends to become constant along the height for the case M1, because a global mechanism can occur according to the seismic design provisions; in the case M2, designed for gravity loads, the average $f_a$ profile tends to be constant but the lowest value occurs at the second floor. It can also be underlined that the $f_a$ trend follows the mode shape profiles reported in Figure 4.

6 ANALYSIS OF THE INFILLED FRAMES

For the infilled buildings, two aspects were studied: the effect of the nonlinearity of both the infill walls and the structure and the effect of the infill walls flexibility. Regarding the infill walls features, the following two cases were considered:
(a) In the first case strong infill walls were studied; they are made of 250 mm thick hollow bricks and are characterized by a compressive and shear strength equal to 2.40 and 0.35 MPa, respectively, and a Young modulus of 1440 MPa.
(b) In the second case a weak (flexible) infill walls were studied. In particular, the infill walls are made of 80 mm thick hollow bricks, characterized by a compressive and shear strength equal to 1.9 and 0.35 MPa, respectively, and a Young modulus of 900 MPa.

For each case, linear and non linear models of the structures and infill walls were considered for both the buildings M1 and M2; a total of 16 models were considered obtained as resumed in Table 3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Seismic design (M1)</th>
<th></th>
<th>Gravity load design (M2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Structure</td>
<td>Infills</td>
<td>Model_ID</td>
<td>Structure</td>
</tr>
<tr>
<td>a, b</td>
<td>Linear</td>
<td>Linear</td>
<td>M1-L-L</td>
<td>Linear</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>Non linear</td>
<td>M1-L-NL</td>
<td>Linear</td>
</tr>
<tr>
<td></td>
<td>Non linear</td>
<td>Linear</td>
<td>M1-NL-L</td>
<td>Non linear</td>
</tr>
<tr>
<td></td>
<td>Non linear</td>
<td>Non linear</td>
<td>M1-NL-NL</td>
<td>Non linear</td>
</tr>
</tbody>
</table>

Table 3: Models considered for the analyses
For the strong infill walls (case a) the effect of the nonlinearity of the infill walls and the structure were studied separately. In Figure 6 it is displayed the distribution of peak floor accelerations amplification \( f_a \), obtained in X direction, for a linear (left) and nonlinear (right) model of the infill wall; the solid line represents the trend of the floor acceleration amplification, the dashed line represents the prediction of Eurocode 8 while the triangular markers represent the acceleration amplification at the barycentre of the infill walls.

![Figure 6: Peak floor acceleration amplification \( f_a \) for the seismic design structure M1 and linear (left) and nonlinear (right) strong infill wall (a).](image)

In case of infill walls with high strength (type a) the walls are practically undamaged and their non-linear behaviour don’t influence the response of the structure. The linear and non-linear models of the RC structure give results not much different because the high strength and stiffness of the walls govern the structural response; however, as expected, when the non-linearity of the frame is introduced a reduction of the acceleration amplification \( f_a \) can be observed both at the floor levels and at the barycentre of the infill walls. Anyway the acceleration amplification at the barycentre of the infill wall could be estimated with good approximation as the average of the accelerations of the two floors that include the infill wall.

![Figure 7: Peak floor acceleration amplification \( f_a \) for the gravity load design structure M2 and linear (left) and nonlinear (right) strong infill wall (a).](image)
The same trend with little bit different values was obtained for the structure (M2) designed for gravity loads, as reported in Figure 7. Moreover, in both cases the EC8 prediction is safe.

The effect of the infill wall flexibility and strength was studied in the case (b) introducing the weak infill walls. In particular, in Figure 8 the distribution of the peak floor accelerations amplification ($f_a$) is depicted for a linear (left) and nonlinear (right) model of the infill walls considering a linear and nonlinear behaviour of the RC structure (M1).

![Figure 8: Peak floor acceleration amplification $f_a$ for the seismic design structure M1 and linear (left) and nonlinear (right) weak infill wall (b).](image1)

Again in this case the non linearity of the infill walls isn’t influent albeit the walls attain an high level of damage, because the structural response is governed by the RC structure due to their low stiffness. Conversely the trend of $f_a$ at the barycentre of the infill walls doesn’t follow the acceleration along the frame height. Therefore, assuming the floor acceleration as action on the infill wall a considerable underestimation of the demand occurs. Despite all, the EC8 prediction is still safe in both linear and nonlinear cases.

Moving to the gravity load designed structure (M2) the same trend of seismic designed structure (M1) was detected for $f_a$ but the values are significantly lower (up 27%), as shown in Figure 9. Moreover, in both cases the EC8 prediction is safe.

![Figure 9: Peak floor acceleration amplification $f_a$ for the gravity load design structure M2 and linear (left) and nonlinear (right) infill wall Type c).](image2)
7 CONCLUSIONS

In this paper, the procedure for evaluating the seismic demand for the out-of-plane response of infill walls in RC frames was analyzed. The seismic demand was defined as the amplification factor of the acceleration applied at the base of the RC structure and the acceleration demand was measured at the barycentre of the wall.

The study was developed considering two types of infill walls located in a tridimensional frame building with three floors considering the linear and nonlinear behavior of both the infill walls and the RC structure. The main results of the analyses are the following:

- For a typical 250mm thick hollow bricks infill wall (case a) the damage of the infill walls is negligible under a seismic action at ultimate limit state therefore the trend of peak floor accelerations amplification (f_a), is the same for the linear and nonlinear model of the infill wall, both in the case of linear and nonlinear model of the structure. In this case the acceleration amplification at the barycentre of the infill wall could be estimated with good approximation as the average of the accelerations of the two floors that include the infill wall. In case of nonlinear behavior of the structure, as expected, a reduction of the acceleration amplification f_a can be observed both at the floor levels and at the barycentre of the infill walls. The application of the EC8 prediction for the evaluation of the acceleration amplification is safe.
- Using infill walls with greater flexibility and lower strength (case b), the walls are damaged but the effect of nonlinearity is again negligible as in the first case (case a) because the response is governed by the RC structure. The amplification of the acceleration at the barycentre of the panel increases becoming much higher than that at the adjacent floors. Despite all, the EC8 prediction is still safe in both linear and nonlinear cases.
- The design of the RC elements (seismic or only for vertical load) seems not influent if strong and stiff infill walls are used because the effect of the infill walls is prevalent.

The simple analyses proposed in the paper confirm the effect of the infill walls on the response of the RC framed structures, but especially evidence the importance of the strength and flexibility of the infill walls on the acceleration at the barycentre of the panels, that can largely overcome the acceleration at the adjacent floors.

REFERENCES


DISTRIBUTION OF SHEAR RESISTANCE AMONG COMPONENTS OF R. C. FRAMES WITH MASONRY INFILL WALLS CONTAINING CONFINED DOOR AND WINDOW OPENINGS

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Abstract

In earthquake resistant design of r. c. frame structures with unreinforced masonry infill walls, containing different in size and position window and door openings, confining elements (tie-columns) are crucial component of seismic detailing of the structure. Having construction of the masonry infill wall without confining elements along opening edges, seismic response prediction becomes unreliable, due to uncontrolled sequenced failure mode of masonry infill walls, its out-of-plane instability, unfavorable crack distribution and premature and total disintegration. Confining elements are not subdued to design as moment-resisting r. c. frames and their construction details are based on simple recommendations. The aim of this study is, by usage of calibrated computational micromodel in computer program ATENA 2D Eng, to determine the shear resistance distribution among components of r. c. frames with masonry infill walls, containing confined door and window openings, at damage grades in compliance with EMS-98 scale.

Keywords: R. C. Frame, Unreinforced Masonry Infill Walls, Door and Window Opening, R. C. Confining Elements, Partial Shear Resistance.
1 INTRODUCTION

In earthquake resistant design of r. c. frame structures with unreinforced masonry infill walls, containing different in size and position window and door openings, confining elements (tie-columns) constitute an essential part of seismic detailing of the structure [1–3]. Construction of vertical r. c. confining elements along opening edges can significantly improve the seismic performance of the structure [4, 5]. On the other hand, having construction of the masonry infill wall without confining elements along opening edges, seismic response prediction becomes unreliable due to sequenced failure of masonry infill wall, out-of-plane instability, unfavorable crack distribution and premature and total collapse [6–13]. Confining elements are not subdued to design as moment-resisting r. c. frames and their construction details are based on simple recommendations.

The basis for this study were tested 1/2.5 scaled physical models of r. c. frames with masonry infill walls, containing centrically or eccentrically positioned medium size windows and door openings (opening to masonry infill wall area ratio $A_o / A_i \leq 15\%$ [14]), and walls without openings (see Figures 1 to 3 and Table 1), designed and constructed in compliance with [1–3] provisions, as moment-resisting frames by considering the medium ductility form of seismic construction detailing [4, 5]. Masonry infill walls were made of clay block masonry units that belonged to Group 2 and general purpose masonry mortar of M5 class which satisfied the seismic design requirements for unreinforced structural masonry walls. Model structures were divided in three groups (see Table 1), namely I, II and III. Group I models were same as models of Group II but without confining elements. Tests under cyclic in-plane shear action revealed the attaining of a very heavy damage of a masonry infill wall (DG 4 i.e. damage grade 4 in compliance with EMS-98 scale [15, 16]) at a drift ratio of about 1.25\%, compared to 0.5\% in a case without confining elements. Furthermore, confining elements contend the influence of opening and enabled the resistance of structure to horizontal seismic shear force equal as in the case without opening.
Figure 2: Tested 1/2.5 scaled r. c. frame structures with masonry infill walls containing confined openings and walls without openings [4,5]

<table>
<thead>
<tr>
<th>Specimen Group</th>
<th>Mark</th>
<th>Appearance of the specimen</th>
<th>Type, area and area ratio</th>
<th>Opening Position</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>Door</td>
<td>Centric</td>
<td>l/h w=0.35/0.90 m</td>
<td>e_o = l/2 = 0.90 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A_o/A_i=0.14; A_o=0.32 m²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Window</td>
<td>Centric</td>
<td>l/h w=0.50/0.60 m</td>
<td>e_o = l/2 = 0.90 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A_o/A_i=0.13; A_o=0.30 m²</td>
<td></td>
<td>P=0.40 m</td>
</tr>
<tr>
<td>II</td>
<td>3</td>
<td>Door</td>
<td>Eccentric</td>
<td>l/h w=0.35/0.90 m</td>
<td>e_o = h/5+l_o/2 = 0.44 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A_o/A_i=0.14; A_o=0.32 m²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Window</td>
<td>Eccentric</td>
<td>l/h w=0.50/0.60 m</td>
<td>e_o = h/5+l_o/2 = 0.44 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A_o/A_i=0.13; A_o=0.30 m²</td>
<td></td>
<td>P=0.40 m</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>-</td>
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<td>Reference specimens</td>
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<tr>
<td></td>
<td>2</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notations:**
- $A_o$ is the area of an opening and is equal to the height of the opening ($h_o$) multiplied by the length of the opening ($l_o$); $A_i$ is the area of the masonry infill wall and is equal to the height of the masonry infill wall ($h_i$) multiplied by the length of the masonry infill wall ($l_i$); $h_i$ is equal to 1.3 m; $l_i$ is equal to 1.8 m; $e_o$ is the eccentricity of the opening; $t_i$ is the masonry infill wall thickness and is equal to 0.12 m; $P$ is the parapet wall height.

Table 1: Classification and description of the specimens tested in the laboratory [4, 5]
At heavy damage of masonry infill wall (DG 3), occurring at drift ratio of 0.75 to 1 %, the confining elements along opening edge were vulnerable to shear failure in the vicinity of opening corners (see Figures 2 and 3).

The aim of this study was to determine the shear resistance distribution among components of tested r. c. frames with masonry infill walls, containing confined door and window openings, and walls without openings, at damage grades in compliance with EMS-98 scale [15, 16] by using computational micromodels and by employing the nonlinear static analysis.

A particular attention was given to the shear resistance contribution and design of vertical r. c. confining elements constructed along opening edges.

2 COMPUTATIONAL MICROMODEL

A 2D computational simplified micromodel was developed in computer program ATENA 2D Eng [17–19] and calibrated against previously described 1/2.5 scaled tested physical models [4, 5], as described in detail in [20] (see Figure 4).

The adopted modelling approach, compared to other available modelling solutions e.g. [21–27], had the ability to fully simulate tests, to take into account opening type, size and position and confining elements in a straightforward manner, to simulate the complex failure mechanism precisely and to distinguish the shear resistance distribution among the structural members [20, 28–33].
The model was limited to 2D actions, as it was in the tests. The geometry characteristics of the model were adopted the same as of the tested 1/2.5 scale physical models (see Figures 1 to 4). The model was built by using iso-parametric plane FEs (9-node quadrilateral and 6-node triangular) for concrete and masonry units, and truss elements (3-nodes) for reinforcement and gap elements for the interface. The finite element mesh size, based on convergence tests, corresponded to one quarter of the structural element size as shown in Figure 4.

The masonry units and masonry units and masonry mortar interface (zero thickness interface) were modelled separately. The normal and tangential stiffness of the interface were estimated based on the expressions $K_{nn} = E/t$ and $K_{nt} = G/t$ respectively, where $E$ and $G$ are the modulus of elasticity and the shear modulus of the masonry unit, and $t$ is the thickness of the mortar joint [17,18].

The adopted constitute laws for each individual material and its properties are given in Tables 2 to 6. Additionally, the special effect of masonry unit and masonry mortar joint interlocking was considered for the bed joints [34] by inclusion of the cohesion hardening–softening function (see Figure 5). All the material properties were determined by standard tests or by theoretical expressions.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus*</td>
<td>$E$</td>
<td>41000</td>
<td>MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\mu$</td>
<td>0.2</td>
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</tr>
<tr>
<td>Tensile strength</td>
<td>$f_t$</td>
<td>4</td>
<td>MPa</td>
</tr>
<tr>
<td>Compressive strength*</td>
<td>$f_c$</td>
<td>-58</td>
<td>MPa</td>
</tr>
<tr>
<td>Specific fracture energy</td>
<td>$G_f$</td>
<td>1.20·10^{-4}</td>
<td>MN/m</td>
</tr>
<tr>
<td>Critical compressive displacement</td>
<td>$w_c$</td>
<td>-1.0·10^{-3}</td>
<td>m</td>
</tr>
<tr>
<td>Eccentricity, defining the shape of the failure surface</td>
<td>$E_{xy}$</td>
<td>0.52</td>
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</tr>
<tr>
<td>Multiplier for the direction of the plastic flow</td>
<td>$\beta$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Crack model coefficient (1.0 for Fixed, 0.0 for Rotated)</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Plastic strain at compressive strength</td>
<td>$\varepsilon_{CP}$</td>
<td>-1.417·10^{-3}</td>
<td>-</td>
</tr>
<tr>
<td>Reduction of compressive strength due to cracks</td>
<td>$f_{c,LIM}$</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>Crack shear stiffness factor</td>
<td>$x_f$</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>Aggregate size*</td>
<td>-</td>
<td>0.016</td>
<td>m</td>
</tr>
<tr>
<td>Crack spacing</td>
<td>$x_{max}$</td>
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<td>m</td>
</tr>
<tr>
<td>Tension stiffening</td>
<td>$e_{ts}$</td>
<td>0.4</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: *designates experimentally obtained value as described in [4,5]

Table 2: Concrete properties for material model NonLinCementitious2

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>Elastic modulus*</td>
<td>$E$</td>
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<td>MPa</td>
</tr>
<tr>
<td>Yield strength*</td>
<td>$\sigma_y$</td>
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<td>MPa</td>
</tr>
<tr>
<td>Ultimate strength*</td>
<td>$\sigma_t$</td>
<td>650</td>
<td>MPa</td>
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<tr>
<td>Strain at ultimate strength*</td>
<td>$\varepsilon_{lim}$</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>Bauschinger effect exponent</td>
<td>$R$</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>Menegotto–Pinto model parameter</td>
<td>$C1$</td>
<td>0.925</td>
<td>-</td>
</tr>
<tr>
<td>Menegotto–Pinto model parameter</td>
<td>$C2$</td>
<td>0.15</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: *designates experimentally obtained value as described in [4,5]

Table 3: Reinforcement properties for cycling reinforcement

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Davorin Penava, Filip Anić, Vasilis Sarhosis and Lars Abrahamczyk

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus parallel to the head joints*</td>
<td>$E_{ih}$</td>
<td>5650</td>
<td>MPa</td>
</tr>
<tr>
<td>Elastic modulus parallel to the bed joints*</td>
<td>$E_{ib}$</td>
<td>850</td>
<td>MPa</td>
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<tr>
<td>Poisson’s ratio</td>
<td>$\mu$</td>
<td>0.1</td>
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</tr>
<tr>
<td>Tensile strength*</td>
<td>$f_t$</td>
<td>1.8</td>
<td>MPa</td>
</tr>
<tr>
<td>Compressive strength parallel to the head joints*</td>
<td>$f_c$</td>
<td>-17.5</td>
<td>MPa</td>
</tr>
<tr>
<td>Compressive strength parallel to the bed joints*</td>
<td>$f_c$</td>
<td>-2.8</td>
<td>MPa</td>
</tr>
<tr>
<td>Type of tension softening</td>
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<td>Exponential</td>
<td></td>
</tr>
<tr>
<td>Specific fracture energy</td>
<td>$G_f$</td>
<td>$0.45 \times 10^{-4}$</td>
<td>MN/m</td>
</tr>
<tr>
<td>Crack model</td>
<td></td>
<td>Rotated</td>
<td></td>
</tr>
<tr>
<td>Compressive strain at compressive strength in the uniaxial compressive test*</td>
<td>$\varepsilon_C$</td>
<td>$-1.358 \times 10^{-4}$</td>
<td>-</td>
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<tr>
<td>Reduction of compressive strength due to cracks</td>
<td></td>
<td>0.8</td>
<td>-</td>
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<tr>
<td>Type of compression softening</td>
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<td>Crush Band</td>
<td></td>
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<tr>
<td>Critical compressive displacement</td>
<td>$w_d$</td>
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<td>m</td>
</tr>
<tr>
<td>Shear retention factor</td>
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<td>Variable</td>
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<tr>
<td>Tension–compression interaction</td>
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<td>Linear</td>
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</table>

Note: *designates experimentally obtained value as described in [4,5]

Table 4: Properties of the clay block masonry unit for model SBeta

<table>
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<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
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<tr>
<td>Normal stiffness</td>
<td>$K_{nn}$</td>
<td>$5.65 \times 10^5$</td>
<td>MN/m$^3$</td>
</tr>
<tr>
<td>Tangential stiffness</td>
<td>$K_{tt}$</td>
<td>$2.57 \times 10^5$</td>
<td>MN/m$^3$</td>
</tr>
<tr>
<td>Cohesion*</td>
<td>$c$</td>
<td>0.35</td>
<td>MPa</td>
</tr>
<tr>
<td>Tensile strength*</td>
<td>$f_t$</td>
<td>2</td>
<td>MPa</td>
</tr>
<tr>
<td>Friction coefficient*</td>
<td></td>
<td>0.24</td>
<td>-</td>
</tr>
<tr>
<td>Minimum normal stiffness</td>
<td>$K_{nn,\min}$</td>
<td>$5.65 \times 10^1$</td>
<td>MN/m$^3$</td>
</tr>
<tr>
<td>Minimum tangential stiffness</td>
<td>$K_{tt,\min}$</td>
<td>$2.57 \times 10^1$</td>
<td>MN/m$^3$</td>
</tr>
<tr>
<td>Function tension softening–hardening</td>
<td></td>
<td>Not used</td>
<td></td>
</tr>
<tr>
<td>Function cohesion softening–hardening</td>
<td></td>
<td>Used</td>
<td></td>
</tr>
</tbody>
</table>

Note: *designates experimentally obtained value as described in [4,5]

Table 5: Initial properties of bed joints

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal stiffness</td>
<td>$K_{nn}$</td>
<td>$8.50 \times 10^4$</td>
<td>MN/m$^3$</td>
</tr>
<tr>
<td>Tangential stiffness</td>
<td>$K_{tt}$</td>
<td>$3.86 \times 10^4$</td>
<td>MN/m$^3$</td>
</tr>
<tr>
<td>Cohesion*</td>
<td>$c$</td>
<td>Adopted from the bed joints</td>
<td></td>
</tr>
<tr>
<td>Tensile strength*</td>
<td></td>
<td>Adopted from the bed joints</td>
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<tr>
<td>Friction coefficient*</td>
<td></td>
<td>Adopted from the bed joints</td>
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<tr>
<td>Minimum normal stiffness</td>
<td>$K_{nn,\min}$</td>
<td>$8.50 \times 10^1$</td>
<td>MN/m$^3$</td>
</tr>
<tr>
<td>Minimum tangential stiffness</td>
<td>$K_{tt,\min}$</td>
<td>$3.86 \times 10^1$</td>
<td>MN/m$^3$</td>
</tr>
</tbody>
</table>

Note: *designates experimentally obtained value as described in [4,5]

Table 6: Initial properties of head joints

The concrete of the confining elements and the lintel were represented by NonLinCementitious2concrete constitutive law. Its compressive strength was equal to 30 MPa, and other parameters were evaluated by theoretical expressions given in [17, 18].
3 ANALYSIS AND RESULTS

In order to determine the shear resistance among structure components, namely r. c. frame $V_{R,f}$ (kN), masonry infill wall $V_{R,if,i}$ (kN) and r. c. confining element $V_{R,if,c}$ (kN), a displacement controlled nonlinear static (pushover) analysis was employed up to the displacement $d=28$ mm i. e. drift ratio $d_r=2\%$. A $d_r=2\%$ was adopted as the point of very heavy structural damage and destruction of the r. c. frame. The displacement controlled approach adopted, enabled observation of structure up to the drift ratio of 2 % (1.25 % was maximal drift in force controlled approach in tests) and of the shear resistance after maximum resistance was reached, compared to the force controlled approach exercised in tests and in calibration procedure.

The shear resistance of each component was compared with the shear resistance of the r. c. frame without masonry infill wall (bare frame) $V_{R,f}$ (kN) at drift ratios $d_r$ (%) 0.10, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75 and 2%. The named drift ratios correspond to the damage grades 1 to 5 based on EMS-98 damage scale [15,16]. The shear resistance ratio was expressed as $V_R / V_{R,f,max} \times 100$ (%), where $V_{R,f,max}$ is the maximal shear resistance of the r. c. frame without masonry infill wall. The shear resistance values were observed at the feet of the r. c. column, r. c. confining element or at the base of the masonry infill wall, as the sum of internal forces along the length of each individual component. In case of the eccentric opening the analysis was performed separately from left (positive) and right (negative) side.

The results are presented in Figures 6 and 7 and in Tables 7 to 9.
The damage grades in compliance with the EMS-98 damage scale and corresponding drift ratio considered in this study were: Grade 1: Negligible to slight damage (no structural damage, slight non-structural damage) at \( d_r \) equal to 0.1%; Grade 2: Moderate damage (slight structural damage, moderate non-structural damage) at \( d_r \) ranges from 0.2 to 0.4%; Grade 3: Substantial to heavy damage (moderate structural damage, heavy non-structural damage) at \( d_r \) equal to 0.5%; Grade 4: Very heavy damage (heavy structural damage, very heavy non-structural damage) at \( d_r \) ranges from 0.75 to 1.0%; Grade 5: Destruction (very heavy structural damage) at \( d_r \) equal to 2.0%.
Table 8: Shear resistance contribution of masonry infill wall component

<table>
<thead>
<tr>
<th>Group</th>
<th>III</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>d_r (%)</td>
<td>d (mm)</td>
<td>V_R,if (kN)</td>
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<td>0.0</td>
<td>0</td>
</tr>
<tr>
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<td>1.4</td>
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<td>0.25</td>
<td>3.5</td>
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<tr>
<td>0.5</td>
<td>7.0</td>
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<tr>
<td>0.75</td>
<td>10.5</td>
<td>235</td>
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<tr>
<td>1.0</td>
<td>14.0</td>
<td>223</td>
</tr>
<tr>
<td>1.25</td>
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<tr>
<td>1.5</td>
<td>21.0</td>
<td>121</td>
</tr>
<tr>
<td>1.75</td>
<td>24.5</td>
<td>121</td>
</tr>
<tr>
<td>2.0</td>
<td>28.0</td>
<td>121</td>
</tr>
</tbody>
</table>

Table 7: Shear resistance contribution of r. c. frame component

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>d_r (%)</td>
<td>d (mm)</td>
<td>V_R,if (kN)</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>1.4</td>
<td>80</td>
</tr>
<tr>
<td>0.25</td>
<td>3.5</td>
<td>133</td>
</tr>
<tr>
<td>0.5</td>
<td>7.0</td>
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<td>0.75</td>
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<td>1.0</td>
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<tr>
<td>1.25</td>
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<tr>
<td>1.5</td>
<td>21.0</td>
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<tr>
<td>1.75</td>
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<td>121</td>
</tr>
<tr>
<td>2.0</td>
<td>28.0</td>
<td>121</td>
</tr>
</tbody>
</table>

Table 9: Shear resistance contribution of r. c. confining element component
4 DISCUSSION OF RESULTS

Distribution of shear resistance among components of r. c. frames with masonry infill walls containing confined door and window openings, and walls without openings obtained by computations using calibrated micromodel, is given in Tables 7 to 9 and Figures 7 and 8, at selected drift ratios i.e. damage grades in compliance with EMS-98 scale. The shear resistance contribution of r. c. frame and masonry infill wall component reached its maximum at drift ratio of in either 0.75 % or 1 % in all cases (DG 4). In general, the opening type, presence and position influence the shear resistance distribution among components.

In Figure 8 given is the overall comparison of ratio of maximum shear resistance of individual component and maximum shear resistance of r. c. frame without masonry infill wall (bare frame) $V_{R,max} / V_{R,f,max} \times 100$ (%).

As observed in Figure 8, the shear resistance of the r. c. frame components is different from case to case and is influenced by opening presence. It is lower up to 40 % or equal in case of walls with window openings and wall without openings and higher of about 12 to 16 % in the case of walls with door openings irrespective of the opening position. It is not significantly affected by loading direction in case of walls with eccentric door or window openings, as in case without r. c. confining elements [13].

The contribution of the masonry infill wall component is influenced by opening type, presence and position. The lowest contribution of 44 and 46 % is obtained in the cases of walls with door openings. The highest contribution of 87 % is in the case of wall without openings. It is not significantly affected by loading direction in case of walls with eccentric window openings. In the case of walls with door openings the difference in contribution caused by loading direction is 12 %.

Figure 8: Shear resistance of components of r. c. frame with masonry infill wall comparison
The r. c. confining element shear resistance contribution is in average 11 %, and it is more pronounced in case of centrically positioned opening (up to 15 % in case of door and 12 % in case of window opening). As shown in Figure 7, the contribution of r. c. confining elements in case of walls with door opening, is up to drift ratio of 0.5 % (DG 2 or DG 3). On the other hand, in case of window openings, it is up to the drift ratio of the structure collapse.

As shown in Figure 7, in cases of walls with openings, after reaching the maximum shear resistance (after $d_r=0.75$ or 1 %), r. c. confining elements prevent the rapid drop of shear resistance of masonry infill wall component.

5 CONCLUSIONS

A calibrated computational micromodel of a tested 1/2.5 scaled r. c. frame with masonry infill wall, containing eccentrically and centrically positioned medium size confined window and door openings, and wall without openings, was used to determine the distribution of shear resistance among the structure components. The model had the ability to fully simulate tests by taking into account opening type, size and position and confining elements in a straightforward manner.

The following observations and conclusions about the distribution of shear resistance among the structure components, namely r. c. frame, masonry infill wall and r. c. confining elements, were drawn, with respect to opening type and position:

- In case of walls with window opening, and wall without opening, the shear resistance distribution among r. c. frame and masonry infill component is approximately equal, while in the case of wall with door opening, it is dominant for r. c. frame component.

- In case of walls with eccentrically positioned door openings the shear taken by the r. c. frame component is higher than in the case of r. c. frame without masonry infill wall (bare frame) i.e. design value of shear resistance capacity.

- The r. c. confining elements mitigated the influence of the loading direction in case of walls with eccentrically positioned openings and enabled for the structure to hold shear resistance to higher drift ratio values.

In general, the shear resistance distribution among the structure components was different from case to case, and was influenced by opening type and position and the presence of r. c. confining element. The construction and design of r. c. confining elements for shear resistance is fully recommended, in order to provide them with sufficient resistance up to the design drift ratio values.

Further research is needed to understand the effect of increase of internal shear force in r. c. frame columns in case of masonry infill walls with confined eccentric door opening, compared to the shear force in r. c. frame columns without masonry infill wall (bare frame) i.e. the designed shear resistance value for the frame.

REFERENCES


[27] Laughery L, Pujol S. Compressive strength of unreinforced struts. ACI Struct J. 2015;112(5).


A DISCRETE MACRO-ELEMENT FOR SIMULATING THE NON-LINEAR IN-PLANE BEHAVIOUR OF RC INFILLED FRAMES

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Abstract

The seismic performance of Reinforced Concrete Infilled Frame Structures (RCIFS) is strongly influenced by the presence of non-structural infills. Even though rigorous approaches are available in the literature to simulate both the non-linear behaviour of infills and its interaction to the RC frame, such approaches remain unsuitable to be applied by practitioners to perform seismic assessments of large structures. To overcome this limit, in the latest decades many researchers have developed simplified engineering oriented macro-models, able to predict with sufficient accuracy and reduced computational burden, either the cyclic static or dynamic behaviour of RCIFS.

This paper presents a new 2D plane macro-element, which provides a refined simulation of the non-linear cyclic response of RCIFS. The proposed model is geometrically consistent with the infill, also in the presence of regular or irregular distributions of openings. This model has been implemented into the open-source software platform OpenSees and validated against experimental tests performed by other authors on single storey, single-span RC infilled frame prototypes characterized by different masonry units and full or open geometric configurations.

Keywords: Discrete Macro Model, Existing RC Buildings, Infilled Frame Structures, Seismic Assessment, Non-linear Static Analysis, OpenSees structural software.
1 INTRODUCTION

The seismic response of reinforced concrete (RC) framed buildings is significantly influenced by the presence of non-structural masonry infills, because of the modification in the lateral stiffness, strength and ductility capacity of the structure [1][2]. Moreover, an irregular in plan or in elevation distribution of infills may drastically reduce the seismic performance of buildings [3][4]. Numerous experimental tests have been performed to understand the response of infill frame prototypes subjected to cyclical lateral loads [5]-[7]. Several experiments have also been performed on structural assemblages, such as multi-storey 2D infilled frames or spatial infilled framed structures [8][9].

A rigorous simulation of the seismic response of infill frame structures requires refined micro-models (FE models), able to simulate the high non-linear interaction between masonry and frame elements [10]. However, these methods are rarely employed for large structures in the common engineering practice. To overcome this drawback, numerous simplified macro-models have been proposed. Within this framework, two approaches are worth mentioning: the 1D strut model and the Discrete Macro Element model (DMEM). In the first approach, the infill is simulated by a single unidirectional element (strut) working on axial force, interacting with the frame through nodes [11]-[14]. The second approach simulates the infill by means of a mesh of plane discrete macro-elements (panels) consisting of regular articulated quadrilateral. Each panel simulate the shear non-linear response of a macro portion of masonry while, the axial/rocking masonry response are simulated by non-linear zero-thickness interfaces [15][16]. The interfaces simulate the normal and tangential infill-frame contact behavior along beams and columns. The DMEM model is particularly attractive, compared to the equivalent strut model, in the presence of openings because it is able to replicate the actual geometry of infills with any regular or irregular distribution of openings by means of an appropriate mesh of macro-elements [17][18]. On the other hand, the DMEM model requires the formulation of special beam elements able to interact with the interfaces [19].

This paper presents a new parsimonious macro-model aiming at guaranteeing an easier coupling with standard finite element elements, if compared to the DMEM model, while guaranteeing a more refined description of the infill-frame interaction, compared to that provided by strut model approaches. The model is composed of a shear deformable macro-element consisting of an articulated quadrilateral with a single diagonal non-linear link. A set of eight concentrated 2D non-linear links, located at the edges of the quadrilateral, simulates the flexural and sliding masonry behavior. The proposed macro-model derives this 2D simplified kinematics from the DMEM. However, the new model differs from the previous for the use of contact links instead of spread interfaces (typical of distinct element micro-modeling approaches) [20]-[22]. The great advantage of the proposed model is that it can be easily implemented in general finite element software and coupled with a large class of non-linear beam elements. In this study, the proposed model has been coupled with forced-based non-linear beam finite elements with fiber cross section discretization and implemented in the structural program OpenSees [23]. The model is used to simulate the results of cyclic tests performed by other researchers on one-storey infilled frame prototypes characterised by weak or strong masonry, with or without openings. Subsequently, a model of a multi-storey 3D building is developed and analyzed by push-over analysis. The results highlight that the model is able to simulate the non-linear response of infilled frame structures both in terms of hysteresis loops and damage pattern. The limited time spent for the analyses performed in the present study, confirms the suitability of the macro-model to be used for static and dynamic analyses of large multi-storey structures.
2 THE PROPOSED MODEL

The mechanical scheme of the proposed model (Fig. 1) consists of an articulated quadrilateral (panel), with an internal 1D diagonal link and eight 2D perimeter links. The shear response of the macro-model is governed by the panel, consisting of four rigid elements hinged at their ends ($v_1, ..., v_4$ in Figure 1) and the diagonal link, located from the vertex $v_1$ to the vertex $v_3$. On each rigid element of the panel, two 2D links connect the panel to the adjacent macro-elements, beam/column elements or external supports. The 2D links consist of zero-length 2D links plus rigid offsets ($\Delta_1, ..., \Delta_4$ in Figure 1) and are located at 1/4 of the rigid element length ($b_p$ or $h_p$). The rigid offsets are used to connect the panel to the centerline of the frame elements in the respect of the effective size of the infill. The response of the link in the normal direction simulates the axial and bending responses of masonry while, the response in the tangential direction simulates the sliding motion at mortar joints or between the infill and the adjacent frame members. The eight external nodes corresponding to the end-nodes of the 2D links ($n_1, n_2, ..., n_8$) are used to enforce the compatibility between the macro-model and the other interacting elements (RC members or other macro-models) in accord with the standard Finite Element approach.

The proposed macro-element allows a geometric consistent modeling of infills, also in the presence of regular or irregular openings. As an example, a 3x3 discretization of the full-infilled single span frame is reported in Figure 2. In any case, the values of the mechanical parameters of the macro-element do not vary because of the presence of openings, but only depend on the geometry of the panels composing the mesh and the mechanical properties of the masonry by means of the calibration procedures described later.

![Figure 1: Mechanical scheme of the proposed macro-model](image)

![Figure 2: qualitative representation of a single span of full-infilled frame (a) and its numerical modeling by means of a 3x3 mesh of macro-element (b).](image)
3 THE MODEL CALIBRATION

The response of the 2D contact links is characterized by two different one-dimensional constitutive laws in the normal and tangential directions: the normal behavior of the link is assumed independent to the tangential direction response. Conversely, the tangential behavior depends on the normal force of the link. The envelope of the normal response is defined by means of the following mechanical parameters of masonry (considered as a homogenised continuous non-isotropic material): Young modulus ($E_m$), compression and tensile strengths ($f_c, f_t$) and fracture energies in compression and in tension ($g_c, g_t$). Starting from the tributary volume of the link and from the above mechanical properties of masonry, the parameters characterizing the envelope of the normal response are easily evaluated by means of an equivalence between a continuous model and the discrete model (Fig 3). The cyclic response is simulated by means of the Kent and Park model [24] (Fig. 3b) as implemented in OpenSees (model Concrete02). The tangential response of the links connecting a macro-element to a frame element (or external support) is simulated by means of an elasto-plastic model associated with a Mohr-Coulomb plasticity criterion with zero tension cut-off. The tangential response of the links connecting different macro-elements is simulated, instead, by an elastic constitutive law, because any plastic sliding between parts of masonry is assumed as considered in the calibration of the diagonal link of the panel. The axial response of the (1D) diagonal link of the panel is simulated by means of a pinching constitutive law, already available in OpenSees [23]. It depends on the shear deformation of the articulated quadrilateral by means of a quadri-linear envelope curve that is defined by eight parameters (Fig. 4) which are calibrated as reported in Table 1 where the $G_m$ is the masonry shear modulus, $(\tau_{cr}, \gamma_{cr}), (\tau_y, \gamma_y), (\tau_0, \gamma_0)$ and $(\tau_R, \gamma_R)$ the shear stress and deformation at the cracking, yielding, peak-strength and residual strength.

![Figure 3](image1.png)  
**Figure 3:** tributary area of the contact link (a); normal constitutive law (b).

![Figure 4](image2.png)  
**Figure 4:** Central panel: (a) mechanical scheme and (b) cyclic response of the diagonal link.
Three parameters ($r_{\text{lim}}$, $r_{\text{re}}$ and $r_{\text{d,rec}}$) rule the cyclic response of the link while two damage indexes ($D_k = 1 - \alpha_k \left[ E_{\text{cum}} / (\eta_k E_{\text{mon}}) \right] / D_{k,\text{lim}}$ and $D_R = 1 - \alpha_R \left[ E_{\text{cum}} / (\eta_R E_{\text{mon}}) \right] / D_{R,\text{lim}}$) rule the reloading stiffness and strength reduction, where $\alpha_k$, $\beta_k$, $\eta_k$, $\alpha_R$, $\beta_R$, and $\eta_R$ are constitutive parameters of the damage model, $D_{k,\text{lim}}$ and $D_{R,\text{lim}}$ are the limit values of $D_k$ and $D_R$, $E_{\text{cum}}$ is the cumulated energy and $E_{\text{mon}}$ is the cumulative energy corresponding to a monotonic loading.

4 SIMULATION OF INFILL FRAME PROTOTYPES

The proposed macro-element is validated by comparison with results of laboratory tests, performed on single-storey, single-span RC infilled frames [5][7] by means of a cyclic lateral displacement history of increasing amplitude at the top beam. Four benchmarks are considered in this paper. The masonry infill is built with weak hollow clay blocks (TA2-TA4), solid calcarene bricks (S1A1) or strong hollow clay blocks (S1B1). TA2, S1A1 and S1B1 are full infilled frames while TA4 is a partial infilled frame. Two frame geometry layouts characterise the specimens (Fig. 5). In particular, Figure 5a shows the layout referring to TA2, and TA4 specimens, while Figure 5b shows those referring to S1A1 and S1B1 specimens. The main mechanical masonry parameters are evaluated from the results of specific laboratory tests carried out on the same masonry typologies [5][7][25]. The other mechanical parameters are calibrated so as to achieve an accurate simulation of the experimental response of the infilled frames. The masonry parameters adopted in the analyses are reported in Table 2.

<table>
<thead>
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<th>Prototype</th>
<th>Load direction</th>
<th>$E_m$ [MPa]</th>
<th>$f_c$ [MPa]</th>
<th>$f_t$ [MPa]</th>
<th>$\varepsilon_0$ [%]</th>
<th>$E/E_m$ [%]</th>
<th>$G_m$ [MPa]</th>
<th>$\tau_0$ [MPa]</th>
<th>$\tau_0$ [%]</th>
<th>$\gamma_0$ [%]</th>
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</thead>
<tbody>
<tr>
<td>TA2-TA4</td>
<td>Horizontal</td>
<td>494</td>
<td>1.08</td>
<td>0.35</td>
<td>0.44</td>
<td>11.9</td>
<td>377</td>
<td>0.359</td>
<td>0.27</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>Vertical</td>
<td>5299</td>
<td>4.64</td>
<td>0.23</td>
<td>0.18</td>
<td>4.7</td>
<td>1348</td>
<td>0.73</td>
<td>0.63</td>
<td>1.90</td>
</tr>
<tr>
<td>S1A1</td>
<td>Horizontal</td>
<td>7408</td>
<td>3.08</td>
<td>0.25</td>
<td>0.083</td>
<td>0.4</td>
<td>2547</td>
<td>1.07</td>
<td>0.92</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Vertical</td>
<td>3933</td>
<td>2.67</td>
<td>0.13</td>
<td>0.135</td>
<td>2.0</td>
<td>2547</td>
<td>1.07</td>
<td>0.92</td>
<td>0.50</td>
</tr>
<tr>
<td>S1B1</td>
<td>Horizontal</td>
<td>5038</td>
<td>4.18</td>
<td>0.49</td>
<td>0.166</td>
<td>0.3</td>
<td>2547</td>
<td>1.07</td>
<td>0.92</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Figure 5: Frame geometry and steel reinforcement details of the TA2-TA4 (a) and S1A1-S1B1 (b) prototypes.

Table 1: Calibration of the diagonal link.

Table 2: Masonry mechanical parameters.
Table 3: Concrete and steel mechanical parameters.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>$f_{ck}$ [MPa]</th>
<th>$f_y$ [MPa]</th>
<th>$f_{ck}$ [MPa]</th>
<th>$f_y$ [MPa]</th>
<th>$\varepsilon_0$ [%]</th>
<th>$\varepsilon_u$ [%]</th>
<th>$E$ [GPa]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [MPa]</th>
<th>$\varepsilon_u$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TA2, TA4</td>
<td>28.22</td>
<td>11.29</td>
<td>40.20</td>
<td>36.18</td>
<td>0.20</td>
<td>0.80</td>
<td>200</td>
<td>538</td>
<td>600</td>
<td>7.5</td>
</tr>
<tr>
<td>S1A1, S1B1</td>
<td>25.00</td>
<td>10.00</td>
<td>35.63</td>
<td>32.06</td>
<td>0.20</td>
<td>0.80</td>
<td>200</td>
<td>450</td>
<td>540</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Figure 6: Comparison between the numerical and experimental results for the TA2 (a); TA4 (b); S1A1 (c) and S1B1 (d) prototype.

The normal stress-strain response of concrete is simulated by means of the Kent and Park model [24] whereas, the response of the reinforcement steel bars is simulated by means of the Chang and Mander model [26] implemented in the uniaxial material ReinforcingSteel of OpenSees. The deformation and strength mechanical parameters of concrete and steel, adopted in the analyses, are summarised in Table 3. The frame members do not consider shear failure [27].

The numerical and experimental results are compared here in terms of base shear vs lateral top displacement. In particular, Figures 6a and 6b show the response of TA2 and TA4 prototypes whereas Figures 6c and 6d show the results of S1A1 and S1B1 prototypes. In keeping with the experimental results, the numerical responses of TA2 and TA4 prototypes are significantly different. Owing to the aspect ratios of the infill panels, the response of TA2 is strongly influenced by the shear deformation of the infill whereas the TA4 model is strongly influenced by the flexural behaviour of the infill panels. The comparison between numerical and experimental results proves the ability of the model to simulate the main features of the lateral response of the prototypes (i.e. initial lateral stiffness, peak resistance and post-peak softening behaviour) with a satisfying level of accuracy. The largest discrepancies are observed with reference to model S1B1 in the range of positive drifts from 0.015 to 0.025m. It is likely to be caused by unexpected interlocking effects between frame and infill not taken into account in the numerical model.
5 MULTI-STOREY BUILDING SIMULATION

In this section, the proposed model is applied to simulate the lateral response of a medium-rise RC infilled building in order to evaluate the capability of the model to simulate large structures and the corresponding time of analysis required. The building under investigation has six levels and four RC frames in two orthogonal directions (named x- and y-directions). The span length is equal to 4.5 m in the x-direction and 5.0 m in the y-direction. The interstorey height is equal to 3.3 m. The outer frames of the building are fully infilled (without opening) whereas the inner frames are without infills. A simple design procedure, based on gravity loads only, is applied to define the member cross-sections and reinforcements. The beam cross-sections are equal to 30X50 cm at all floors. The column cross-sections are selected equal at the single storey, namely 30X70 cm² at the first floor, 30x60 cm² at the second floor and 30x50 cm² at all the other storeys. The area of the longitudinal reinforcement of the columns is approximately equal to 1% of the concrete cross-section area. The steel reinforcement area of the beams is equal to 3ϕ20 at the top and bottom sides of the cross-section. Each infill panel is modeled by means of a 2x2 mesh of the proposed macro-model. The mechanical properties of the material models are those calibrated for the tests in Pavia.

The model is subjected to a pushover analysis, along the x-direction, with an invariant lateral force pattern, proportional to the fundamental mode of vibration. Despite the symmetry of the building, the whole structure has been modelled to test the computational effectiveness of the macro-element on a quite large multistorey structure model. The analysis has been carried out up to a maximum drift ratio larger than 1.5%. The capacity curves in terms of base shear force vs floor displacements or storey drifts are reported in Figure 8a and b. The time of analysis (about 1700 steps) was approximately 45 minutes.

To highlight the effects of the presence of the infills, the same analysis has been repeated on the model without infills. The capacity curves of this model are plotted in Figure 8c and d. As is evident from all the capacity curves in this latter figure, the infills cause an increment of the lateral strength, approximately equal to 65%. In addition, the floor displacements of the infilled frame structure appear to be more uniformly distributed along the height of the building. The computational time of the analysis without infills was about 8 minutes and, therefore, the analysis time ratio between mixed and bare-frame structures resulted equal to about 5.6. This latter result is deemed to be still compatible with engineering applications of medium-rise infilled frame structures.

6 CONCLUSIONS

In this paper, a new 2D discrete macro-element is proposed to simulate the in-plane non-linear cyclic response of masonry infills. The macro-element consists of an articulated quadrilateral panel, a single 1D diagonal link and eight 2D links. The panel - including the internal 1D diagonal link - simulates the shear behaviour of a large (macro) portion of infill and the corresponding failure mechanism; the eight 2D links simulate the axial/flexural response of the infill and its flexural/sliding interaction with the surrounding frame. The proposed macro-element presents important advantages over classical strut models since it can be used in mesh to obtain a geometric consistent representation of the infill, also in presence of openings. Unlike the previous DMEM model, which use non-linear interfaces to interact with frame elements, the proposed element allows a simpler connection with FE beam elements.
Figure 8. Results of the pushover analysis: (a) floor displacements of the infilled structure; (b) storey drifts of the infilled structure; (c) floor displacements of the bare structure; (d) storey drifts of the bare structure.

The proposed macro-element has been implemented into the open source software platform Opensees and used to simulate the response of single-storey, single-span RC infilled frame prototypes tested in laboratory by other researchers. The infill of these frames was full or with openings and built with weak hollow clay blocks, solid calcarenite bricks or strong hollow clay blocks. The numerical analyses have highlighted the satisfying accuracy of the simulated response and the ability of the model to reproduce the distinguishing aspects of the cyclic lateral response of infilled frames, including the degradation of the response with the number of cycles extending over an equal range of displacements. The computational time has revealed to be sufficiently low and consistent with the static or dynamic analyses of multi-storey infilled structures.

REFERENCES


B. Pantò, P.P. Rossi


COMPARISON OF EXPERIMENTAL AND ANALYTICALLY
PREDICTED OUT-OF-PLANE BEHAVIOR OF FRAMED-MASONRY
WALLS CONTAINING OPENINGS

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Keywords: Comparison, experimental, analytical, out-of-plane, infilled frames, openings

Abstract. During an earthquake, structures are loaded in both in-plane and out-of-plane direction. This paper investigates the behaviour of load-bearing frames with infill walls that contain openings. As when they are subjected to out-of-plane, inertial loads. In the experimental campaigns of like structures, it was found that even with openings, the beneficial arching-action was able to develop. However, its effectiveness was limited. Namely, the deformation capabilities in all cases were significantly lowered. Same can not be stated for the load-bearing capacities, as some researches found no reduction while others did. Additionally, this paper analyses the existing equations that can calculate the load-bearing capacity of such structures. Low correlations were found between the experimental and analytical capacities. Hence, further research endeavours should be addressed in order to gain a reliable analytical model.
1 Introduction

Various countries around the world are located on seismically active areas. Globally, common structural systems of multi-storey buildings are assembled of load-bearing frames with masonry infill walls. Generally, hollow clay masonry blocks are used as infill unit, and reinforced-concrete (RC) or structural steel (SS) frames. During an earthquake, ground motions excite such structures in both in-plane (IP) and out-of-plane (OoP) direction. Thus, the field of seismic engineering specialised in the analysis of those general and their combined direction.

During ground motions, frames in such structures interact with infills. This however, is a topic greatly researched, as a way of implementing the interaction is still not provided within the European seismic codes [5].

Various conditions affect the interaction, and with it, the overall behaviour of the structure. Those conditions include the influence of: infill type, slenderness, openings, frame stiffness, gravity load, boundary conditions and etc.

This paper investigates the OoP relation between experimental and analytical models of frames with infills that contain openings. The field of IP loading was researched in greater extend when compared to the OoP field [3]. This is especially true in the case of openings. Namely, in the IP studies openings had a profound effect on the overall behaviour [15, 14]. However, the same is not clear in the case of OoP behaviour.

2 Experimental endeavours

When structures are excited by earthquakes ground motions, the inter-storey drift and inertial forces act upon them. The majority of research done in the field of OoP loading was conducted with the inertial methods. Namely, with the use of air-bags. In like manner, all OoP experiments that included openings were done using inertial methods with air-bags. Prior to loading, the openings were covered with plywood and frames were restrained from translation. Hence, such test procedures damage the infill while frames are more or less intact (Fig. 1). When tested with inertial methods, infills had transverse bearing capacity substantially higher than what would be expected from flexural theory. This is however, due to the effects of arching-action. Arching-action is well observed phenomena of developing additional compressive that resist transversal forces. In detail, when infill is loaded it bends as a beam would. With an increase of load, infill cracks and separates in two parts. Those parts, on one end clamp and on the other open. Points that clamp make the compression arch. If the infill is fully bounded by all sides, the horizontal and vertical arching forms a characteristic “X” like failure pattern (Fig. 1a).

From Figure 1, it is evident that, even with the presence of openings, arching-action was able to develop. The only difference can be found with door opening, as it has cracks nearly all vertical. This can be attributed to the fact that due the door opening, boundary conditions
developed such as having only horizontal arching-action. As it is case with columns-infill gaps [7, 17]. Furthermore, all authors that studies the effects of openings [1, 6, 17, 13, 16] observed a significant reduction of deformation capabilities (Fig. 2). The same can not be stated for the case of load-bearing capabilities, as different authors had different outcomes. For instance, [6, 1] (Fig. 2b) found no decrease with window, while, [17, 13, 16] (Fig. 2a) found a significant drop in bearing capacity of window, door and full height opening.

Likewise, in the case of initial stiffness, openings in some cases did lower it (Fig. 2a) and in others did not (Fig. 2b).

In Table 1, the geometrical and mechanical properties of specimens with openings are shown. It is to be noted that [9, 16] tested a specimen with full wall height opening. However, data provided from authors is scarce.

Table 1: Geometrical and mechanical characteristics of specimens with openings

<table>
<thead>
<tr>
<th>#</th>
<th>Author (year)</th>
<th>Specimen</th>
<th>thickness (mm)</th>
<th>length (mm)</th>
<th>height (mm)</th>
<th>Frame</th>
<th>Opening</th>
<th>Lintel</th>
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<tr>
<td>1</td>
<td>Wang (2017)</td>
<td>IF-RC-DO</td>
<td>90</td>
<td>1350</td>
<td>980</td>
<td>RC Door</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Sepasdar (2017)</td>
<td>IF-W</td>
<td>90</td>
<td>1350</td>
<td>980</td>
<td>RC Window</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Dawe &amp; Seah (1989)</td>
<td>WE9</td>
<td>190</td>
<td>3600</td>
<td>2800</td>
<td>SS Window</td>
<td>n/a</td>
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<tr>
<td>4</td>
<td>Akhoundi et al. (2015)</td>
<td>SIF-B</td>
<td>110</td>
<td>2415</td>
<td>1635</td>
<td>RC Window</td>
<td>Yes</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>#</th>
<th>Column size (mm×mm)</th>
<th>Beam size (mm×mm)</th>
<th>$f_m$ (MPa)</th>
<th>$E_m$ (MPa)</th>
<th>$E_F$ (MPa)</th>
<th>$A_o/A_i$ (%)</th>
<th>Capacity $w$ (kPa)</th>
<th>Displacement @ $w$ (mm)</th>
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<tbody>
<tr>
<td>1</td>
<td>180×180</td>
<td>180×180</td>
<td>9.0</td>
<td>7650</td>
<td>16911</td>
<td>17</td>
<td>36.2</td>
<td>7.9</td>
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<tr>
<td>2</td>
<td>180×180</td>
<td>180×180</td>
<td>9.0</td>
<td>7650</td>
<td>16911</td>
<td>17</td>
<td>43.7</td>
<td>4.3</td>
</tr>
<tr>
<td>3</td>
<td>W250 × 58</td>
<td>W200 × 46</td>
<td>24.3</td>
<td>17575</td>
<td>210000</td>
<td>19</td>
<td>22.3</td>
<td>n/a</td>
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<tr>
<td>4</td>
<td>160×160</td>
<td>270×160</td>
<td>1.0</td>
<td>1000*</td>
<td>32000*</td>
<td>20</td>
<td>9.9</td>
<td>25.0</td>
</tr>
</tbody>
</table>

* Estimated

![Figure 2: Force vs. displacement graph of infilled frames with openings](image)
3 Analytical models

All developed analytical models are based on inertial failures and with it, arching-action. There are no specific equations developed for infilled frames with the implementation of openings. However, Mays et al. (1998) [11] developed an equation (Eq. 6) that can be used to modify an arbitrary equation of infilled frame as to address the effects of openings. Hence, one can use equations made for infilled frames, and modify it with the equation of Mays et al. (1998) [11]. Note that the equation was developed for RC walls with openings (no frame).

Equation by Angel et al. (1994) [2] Authors developed an equation to answer the problem of OoP capacity due to previous IP damage. Ways of calculating both $R_1$ and $R_2$ were omitted as all specimens with openings did not contain previous IP damage nor did infill had any connection gap with the frame.

$$w = R_1 R_2 \frac{2\lambda f_m}{h/t}$$

No previous IP damage $R_1 = 1$, full frame and infill contact $R_2 = 1$, $\lambda$ see Tab. 2

<table>
<thead>
<tr>
<th>$h/t$</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
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<tbody>
<tr>
<td>$\lambda$</td>
<td>0.129</td>
<td>0.060</td>
<td>0.034</td>
<td>0.021</td>
<td>0.013</td>
<td>0.008</td>
<td>0.005</td>
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</tbody>
</table>

Equation by Dawe & Seah (1989) [6] Authors also developed an equation to evaluate the one-way (gapped) arching action. However, as all specimens with openings have their infill restrained by all sides; hence, the equation was omitted from this paper.

$$w = 0.8 f_m^{0.75} t^2 \left( \frac{\alpha}{t^{2.5}} + \frac{\beta}{h^{2.5}} \right)$$

Where: $\alpha = \frac{1}{h} (E_f I_c h^2 + G_t J_c t h)^{0.25}$; $\beta = \frac{1}{l} (E_f I_b l^2 + G_t J_b t l)^{0.25} \leq 50$

Equation by Flanagan & Bennett (1999) [8] Authors here modified Eq. 2, by changing the front constant and by eliminating torsional effects from parameters $\alpha$ and $\beta$

$$w = 0.73 f_m^{0.75} t^2 \left( \frac{\alpha}{l^{2.5}} + \frac{\beta}{h^{2.5}} \right)$$

Where: if $h/t < 8 \rightarrow t = h/8$; $\alpha = \frac{1}{h} (E_f I_c h^2)^{0.25} \leq 50$; $\beta = \frac{1}{l} (E_f I_b l^2)^{0.25} \leq 50$

Equation by Moghaddam & Goudarzi (2010) [12] Authors differentiated failure of slender and thick infills. Thick infill’s suffered crushing at supports (frame); while slender ones had transverse instability failure, due to large deflections. The transverse instability failure is withal,
a favourable one due to magnification of arching-action effects.

\[ w = \min \left\{ \frac{w_{cr}}{w_{max}}, \left( \frac{0.85 f_m}{(h/t)^2} - \left( \frac{0.12 + 0.45}{\alpha} \right) \frac{f_m^2}{E_m} \right) \right\} \quad (4) \]

Where: \( w_{cr} \) crushing failure, \( w_{max} \) transverse instability failure, \( \alpha = \frac{384 E_l h l}{E_m t^4} \)

**Equation by Klingner et al. (1996) [10]** Authors here developed their equation based on the work by [4].

\[ w = 8 \frac{M_{yv}}{h} (l - h) + 8 \frac{M_{yh}}{h} \ln(2) \left( \frac{x_{yv}}{x_{yh}} \right) \ln \left( \frac{l}{l - h/2} \right) l \quad (5) \]

Where: for calculation of \( x_{yv} \) replace \( h \) with \( l \), and for calculation of \( M_{yh} \) replace \( x_{yv} \) with \( x_{yh} \),

\[ x_{yv} = \frac{tf_m}{1000E_m} \left( 1 - \frac{h}{2\sqrt{(h/2)^2 + t^2}} \right) \]

**Equation by Mays et al. [11]**

\[ w_o = w + wF_r \left( \frac{A_o}{A_i} \right) \quad (6) \]

Where \( F_r \) is obtained by using Tab. 3

<table>
<thead>
<tr>
<th>Panel type</th>
<th>Opening location</th>
<th>( F_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>One window</td>
<td>Central and offset</td>
<td>-1.00</td>
</tr>
<tr>
<td>One door</td>
<td>Central</td>
<td>+1.36</td>
</tr>
<tr>
<td>One door</td>
<td>Offset</td>
<td>-0.13</td>
</tr>
<tr>
<td>Two windows</td>
<td>Evenly distributed</td>
<td>-0.05</td>
</tr>
<tr>
<td>One window + one door</td>
<td>Evenly distributed</td>
<td>-0.41</td>
</tr>
</tbody>
</table>

For the analysis of equations a mixture of Eq. 6 and others (Eq. 1 - 5) was used. In detail, plain masonry’s load-bearing capacity \( w \) was calculated by Eq’s. 1 - 5. Than it was modified in order to address the openings \( w_o \) by the use of Eq. 6. Data that was used for the calculations was obtained from Tab. 1.

**4 Results**

By the use of Eq. 6, the reductions of each openings are shown on Tab. 4. The reductions were calculated with an excerpt from Eq. 6: \( F_r(A_o/A_i) \).

The differences between experimentally and analytically obtained bearing capacities are presented in Tab. 5 and Fig. 3.
5 Discussion and conclusion

From Tab. 4 it is clear that there was an obvious mismatch between the analytical and experimental outputs. For instance, specimens WE9 and PIF-A from [6, 1] analytically had a 20% reduction of bearing capacity. Both authors observed no reduction in their experimental investigations. Furthermore, with the door opening IF-RC-DO specimen [17], equation resulted in an increased capacity of 24%. This was also inconsistent with data obtained experimentally, as
there was also a drastic decrees in the bearing capacity.

From Tab. 5 and Fig. 3 it is clear that the best experimental to analytical correlation was with Eq’s. 2 - 4. Certainly, the best fitting was with Eq. 4. Furthermore, IF-W model had the greatest correlation with all equations.

From the literature review and results analysis the following points can be drawn:

1. Openings do not prevent the development of arching-action; rather, they limit its effectiveness. In all cases the deformation capabilities were significantly lowered. However, load-bearing capacities and initial stiffness’s in some instances were lowered and in others staid the same;

2. Window openings tend to develop characteristic “X” shaped yield lines as plain masonry infills do. However, door opening developed more or less vertical yield lines. This can be attributed to type of opening in that it changes the boundary conditions;

3. Analytical models showed great aberration between experimental data and between other models. Hence, they could be rendered non-reliable;

4. The Eq. 4 by Moghaddam & Goudarzi (2010) [12] had the best correlation with all the specimens, followed by Eq’s. 3, 2, 5 & 1. Furthermore, window specimen IF-W [13] had the best overall correlation with analytical models.

In summation, more research effort should be made to address the effects of openings in structural systems of infilled frames. Also, there is a need to address the effects of out-of-plane, inter-storey drift forces on plain infilled frames and those with openings.

**Annotation**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$h$</td>
<td>Height</td>
</tr>
<tr>
<td>$l$</td>
<td>Length</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness</td>
</tr>
<tr>
<td>$f_m$</td>
<td>Masonry compressive strength</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Masonry elastic modulus</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Frames elastic modulus</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Columns moment of inertia</td>
</tr>
<tr>
<td>$I_b$</td>
<td>Beams moment of inertia</td>
</tr>
<tr>
<td>$J_c$</td>
<td>Columns torsional constant</td>
</tr>
<tr>
<td>$J_b$</td>
<td>Beams torsional constant</td>
</tr>
<tr>
<td>$G_1$</td>
<td>Frames shear modulus</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Previous IP damage coeff.</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Boundary condition coeff.</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Area of opening</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Area of infill</td>
</tr>
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</table>

**REFERENCES**


MACRO-MODELLING OF COMBINED IN-PLANE AND OUT-OF-PLANE SEISMIC RESPONSE OF THIN STRENGTHENED MASONRY INFILLS

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Abstract

Some strengthening techniques, aimed to increase the infill Out-Of-Plane (OOP) resistance and its safety against OOP collapse, have been recently tested at the University of Padova. Based on this experimental work, a new infill macro-model consisting of two nonlinear fibre struts for each diagonal direction, able to predict the combined In-Plane/Out-Of-Plane (IP-OOP) seismic response of the panels, is proposed in this paper and calibrated on the above-mentioned experimental results for thin clay masonry panels strengthened by two external plaster layers, directly applied on the masonry surface, containing a biaxial basalt fibre grid. These macro-models were then used to carry-out parametric nonlinear static analyses on typical reinforced concrete infilled frames, designed both traditionally and seismically, with the aim of evaluating their lateral response both in the case of IP forces only (by assuming two force distributions applied to the frame) and in the case of IP-OOP combined forces (by applying OOP equivalent static forces to the panels). The main effects of the OOP forces are presented through pushover curves and drift profiles associated to the achievement of the various infill and frame limit states. Numerical results show the effectiveness of the infill strengthening in terms of improvement of the infills OOP performance and thus of the overall structural response.

Keywords: infilled RC frames; In-Plane and Out-Of-Plane interaction; masonry infill; strengthened masonry; macro-modelling; non-linear analysis.
1 INTRODUCTION

Thin unreinforced masonry (TURM) infill, generally used as internal partition walls and classified as non-structural elements, are often neglected in the current design procedures. TURM walls made of clay units with horizontal holes have been the typical light enclosure system used since 1960s and are widespread in existing buildings. These infills were studied, still through full-scale tests, by Calvi et al. [1], 2004. These panels have a great influence on the seismic response of structures in terms of stiffness and strength, despite being their thickness low in the region of 10÷15 cm (Dolšek and Fajfar [2], 2008). Due to the brittle nature of masonry and to the limited thickness, a possible combined In-Plane/Out-Of-Plane (IP/OOP) seismic action can bring to a quickly collapse of the panel increasing the risk for human safety. As consequence, a first-storey mechanism can occur also in that buildings with a regular distribution of masonry infills (Dolšek and Fajfar [3], 2001). The recent seismic events have highlighted the high seismic vulnerability of weak traditional unreinforced masonry, above all related to the lack of a suitable and effective design procedure (Hak et al. [4], 2012). Braga et al. [5], (2011) studied the common damages on masonry infills to identify the causes of failure and linking them to the common construction rules.

Considering the intrinsic vulnerability of weak masonry infills, different strengthening techniques can be adopted to improve the seismic performances in terms, especially, of Out-Of-Plane resistance. This issue triggered great interest in the scientific world in the last decades. The first experimental campaigns focused on the application of external reinforcing layers applied directly on masonry infills: Fibber-Reinforced Plastic (FRP) layers (Tumialan et al. [6], 2003, Saatcioglu et al. [7], 2005) and strengthening meshes, embedded in one or more layers of plaster, were experimentally studied to increase the OOP capacity. In particular, the use of Textile Reinforced Mortars (TRM) on non-load bearing masonry panels has been studied by Calvi et al. [8], 2001, Papanicolaou et al. [9], 2007, Valluzzi et al. [10], 2014, Minotto et al. [11], 2019. Furthermore, European standard EN 1998-1-1 [12] recommends adequate interventions on infill panels (slenderness > 15), proposing the use of light wire meshes.

The macro-modelling of masonry walls is a modelling strategy useful to investigate the overall response of the infill wall and the global behaviour of structure. In recent years, numerous studies concerning macro-modelling of infilled Reinforced Concrete (RC) frame structures have been proposed (Asteris et al. [13], 2011, Jeselka et al. [14], 2013 and Tarque et al. [15], 2015) studying different macro-models (single diagonal strut and more complex multi-strut models). The researchers have addressed their attention to the development of IP equivalent strut macro-models for the simultaneous prediction of the IP/OOP interaction effects on infill panels (Shing et al. [16], 2016, Ricci et al. [17], 2017). Mosalam & Gunay [18], (2015) proposed a macro-model (MG-model) based on a single diagonal element which consists of two elastic beams with a plastic hinge in the contact point, where an inelastic fibre section with fibres spaced in the OOP direction is defined. In this way, the fibres withstand to axial force as well as to bending moment, allowing to take into account the IP/OOP interaction. In this contest, a large numerical models state of art review is presented in Asteris et al. [19], (2017).

The aim of this work is to develop a new macro-model (derived from MG-model) for strengthened thin masonry infills based on two nonlinear fibre struts for each diagonal direction able to consider simultaneously the IP and OOP behaviour. The numerical model is calibrated on the experimental results obtained by combined IP/OOP tests (Minotto et al. [11], 2019) on strengthened masonry walls. In detail, three different types of strengthened thin clay masonry panel were numerically studied; the first strengthening type is characterized by the application of a bi-directional basalt mesh embedded in a special geo-polymeric plaster. The
other two strengthening solutions consist of applying a fibre-reinforced lime-based plaster, and one of them is also provided with an additional bi-directional basalt mesh. Subsequent parametric nonlinear static analyses are performed on typical traditionally and seismically designed RC frame buildings with two In-Plane force distributions and simultaneous static forces acting on the panels in the OOP direction, defined according to the current Italian Standard [20] and [21].

2 MACRO-MODELLING OF REINFORCED MASONRY INFILLS

2.1 Description of the F.E. macro-model

The combined IP/OOP behaviour of thin reinforced masonry infills (TRM) was modelled through the macro-model (implemented in OpenSees) shown in Figure 1 and was derived from that proposed by Mosalam et al. [18], 2015. The macro-model was characterized by two equivalent struts for each diagonal direction in order to better evaluate the stress distribution on RC member (Crisafulli [22], 1997). The diagonal equivalent struts are connected to the RC frame (hinge connection) defining a contact length appropriately calculated according to Stafford Smith [23], 1966.

![Figure 1. Macro-model proposed by the authors for thin reinforced masonry infills.](image)

Each diagonal equivalent strut has a central fibre section composed of 120 fibres symmetrically placed Out-Of-Plane with respect to the strut axis. The fibres are defined through their area $A_i$, location $z_i$ related to the central axes, yielding stress $f_y$, and yielding deformation $\varepsilon_y$. Unlike the MG-model, the geometric position of the fibers and the yielding condition is given by an IP/OOP (Axial Force/Bending Moment) diagram based on the following domain:

$$\left( \frac{P_N}{P_{NO}} \right)^{3/2} + \left( \frac{M_N}{M_{NO}} \right)^{3/2} \leq 1$$

(1)

where $P_{N0}$ and $M_{N0}$ are respectively the In-Plane and Out-Of-Plane capacities derived from experimental tests results. The calculation of the “yielding point” (maximum strength) of all fibres was done in accord to MG-model, considering the same masonry Young’s modulus. In order to represent the masonry strength and stiffness degradations, the “Hysteretic material” (OpenSees material library) was implemented to simulates the infill experimental behaviour. In particular, to take into account the contribution of the external reinforcement and calibrate the macro-model on OOP test results, the more external fibres present different material properties (but the same type of constitutive law) respect to the internal ones which represent the contribution of the masonry. This was necessary to increase the controllability of the model for the following calibrations. According to the negligible tensile behaviour of masonry, the constitutive laws of internal fibres are represented only by a compression envelope branch.
According to the MG-model, a removal domain is necessary to describe the real Out-Of-Plane collapse of the infill wall at the Collapse Limit State (CLS). This corresponds to a sudden reduction of the infilled-frame IP resistance (residual IP strength of the panel). The constitutive laws of masonry fibers simulate the stiffness and strength degradation up to zero, thus the IP removal condition is not used. To assess the damage level reached by the masonry panels during the numerical analyses (for combined IP/OOP action), similar displacement domains were experimentally calibrated for Damage and Ultimate Limit States (DLS and ULS), also shown in Figure 4. Each IP-drift/OOP-displacement domain, is obtained by combined tests results interpolating the OOP displacements to achieve the specific LS, which are function of the in-plane damage (Figure 3), and limiting it to the maximum IP drift $\theta_{IP}$ of that Limit State LS (see Table 1).

The IP inertial masses include both the frame masses and the infill ones, and they are represented as lumped masses at each beam-column joint. The Out-Of-Plane mass is considered only for the masonry infill, because of the bi-dimensional model, and this is concentrated only in the central point of the two struts, equally subdivided between the struts. In the OOP direction, the two central nodes of the struts are rigidly connected.

Furthermore, a more detailed RC frame model was developed to consider the effects of the different stirrups spacing between critical and not critical regions along beams and columns. All RC members were modelled using force-based beam-column elements with non-linear fiber sections. The concrete behaviour was modelled through the Kent-Scott-Park [24] constitutive law (Concrete02, linear tension softening) and the effects of the confinement given by reinforcing steel was taken into account thanks to the Mander’s model [25]. Reinforcing steel bars were modelled with an elasto-plastic constitutive law with strain hardening (Steel02, Menegotto & Pinto [26]). Concrete and steel material laws were calibrated on the experimental test results conducted on bare frame (BF). The 2D RC frame model presents OOP elastic springs with OOP stiffness placed on the frame joints to confer a realistic OOP stiffness. The stiffness values of these fictitious springs were iteratively calibrated for each model so as to obtain similar first periods of vibration in both OOP and IP directions.

### 2.2 Calibration on experimental results (IP/OOP tests)

A large experimental campaign was carried out at the Laboratory of the University of Padova concerning combined IP/OOP tests conducted full-scale, one-bay and one-story, RC frame specimen fully infilled by thin masonry walls made of clay units variously externally strengthened. The experimental campaign involves a total number of 8 specimens that were reinforced with three different strengthening solutions identified as F, FB and RBB. Specimen
characteristics and tests results of the three tested strengthening solutions are available in Minotto et al. [11], 2019. The RC frame clear span and height were 4.15 m and 2.65 m respectively and the infill wall dimensions coincide with those values. All specimens were designed following the criteria described in da Porto et al. [27], 2013. A progressive stiffness and strength degradation is observed in the Out-Of-Plane direction increasing the IP damage (Figure 3) and the following Limit States can be identified on the capacity curves.

- a Damage Limit State (DLS) corresponding to the peak strength on the IP undamaged curve;
- an Ultimate Limit State (ULS), corresponding to the peak strength on the IP damaged curve;
- a Collapse Limit State (CLS), which identifies the moment (determined also by test observations) of sudden strength degradation, which anticipates the out-of-plane collapse of the panel.

The IP Limit States were derived by the experimental capacity curve of the only infill obtained by IP cyclic tests of infilled specimens less the contribution of the BF configuration. These values are reported in the following Table 1. Finally, Figure 4 shows the IP-drift/OOP-displacement domains used in the following parametrical analysis to identify the infill limit states (DLS, ULS and CLS). The results of the model calibration are reported from Figure 5 to Figure 7.

<table>
<thead>
<tr>
<th>Limit State</th>
<th>F</th>
<th>FB</th>
<th>RBB</th>
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<td>DLS</td>
<td>0.30%</td>
<td>0.30%</td>
<td>0.25%</td>
</tr>
<tr>
<td>ULS</td>
<td>0.50%</td>
<td>0.50%</td>
<td>0.50%</td>
</tr>
<tr>
<td>CLS</td>
<td>1.50%</td>
<td>1.50%</td>
<td>1.50%</td>
</tr>
</tbody>
</table>

Table 1: In-Plane limit drifts $\theta_{IP}$ at DLS, ULS and CLS.
Figure 5. Numerical calibration of specimen with panel type F: hysteresis loops of the BF, b) hysteresis loops of the infilled frame up to 1.2% IP drift, c) OOP capacity curve without and with IP damage.

Figure 6. Numerical calibration of specimen with panel type FB: hysteresis loops of the BF, b) hysteresis loops of the infilled frame up to 1.2% IP drift, c) OOP capacity curve without and with IP damage.

Figure 7. Numerical calibration of specimen with panel type RBB: hysteresis loops of the BF, b) hysteresis loops of the infilled frame up to 1.2% IP drift, c) OOP capacity curve without and with IP damage.

3 PARAMETRIC NON-LINEAR ANALYSIS

After the calibration of the proposed model for all strengthening solutions, a widespread parametric non-linear static analysis was performed in OpenSees in order to investigate the seismic response of RC infilled frames considering the combined IP/OOP response of infills. At this scope, several frame configurations (squat, regular and slender) were specifically designed. These frame configurations are named as “n x m” (n=number of bays, m=number of storeys). The following configurations have been studied: 4x2, 2x3, 1x3, 2x6. The bay length and the storey height are calculated considering the same geometry of the testes infills and
considering the geometry of column and beam sections derived from the design procedure. The set of frames, representing some of the most common Italian R.C. buildings, was thought with two different types of design: only for gravity loads, or traditional design (TD), and for both gravity and seismic forces, or seismic design (SD). The material characteristics and sections of these frames, with the longitudinal and transverse reinforcement, are reported in Table 3. For SD frames a steel B450C and a concrete C30/37 were used according to Italian Code NTC18 [20]; for TD frames a steel AQ50-60 and a concrete C20/25 were chosen following some common Italian design criteria of the 50-80s (Cristofaro et al. [28], 2011). The design of the RC frame was performed considering the variability of the column section every three columns.

Pushover analyses were initially carried out on the BF models, as reference, and then on the infilled frame (IF) configurations, considering both infill types previously calibrated. For the IP load pattern, two distributions of inertial forces were used (Gr1_a and Gr2_a) according to the Italian Standard NTC18 [20]. Distribution Gr1_a corresponds to a force distribution proportional to masses and heights while distribution Gr2_a corresponds to a uniform distribution of accelerations along the height of the building. In the OOP direction, all infills are characterized by the presence of a pattern of forces applied on the central nodes of the same. This distribution of forces is determined in accordance with the formulation of the seismic demand on non-structural elements (NTC18 [20] and [21]). In order to apply the formulation of the spectral acceleration $S_a$ proposed in [21], a reduction of the masonry OOP infill period $T_a$ was considered as function of the IP damage reached by the structure at each floor and at each analysis step. Three different values of PGA were chosen for the definition of the OOP pattern: 0,30g (SZ1), 0.20g (SZ2) and 0.10g (SZ3). Furthermore, the analysis with OOP distribution were conducted considering two different intensities of forces equal to 100% (IP+100%OOP) and 30% (IP+30%OOP) of the maximum PGA value.

During numerical analysis, step by step, the evaluation of the structural damage level was possible thanks to the definition of some Performance Levels (PLs) and Limit States defined also for the RC frame. In particular, the PLs considered are:

- $beamYM$ (beam Yield bending Moment), calculated from a parametric moment-curvature analysis where the parameter is the vertical axial load $N$ on the section; a function of the nominal yield curvature $\chi_y$ depending on $N$ was derived;
- $beamUM$ (beam Ultimate bending Moment), calculated as $beamYM$; a function of the ultimate curvature $\chi_u$ depending on $N$ was derived;
- $colYM$ (column Yield bending Moment), calculated as for beams;
- $colUM$ (column Ultimate bending Moment), calculated as for beams;
- $colSF$ (column Shear Failure), calculated according to the R.C. shear strength formulation of Sezen [29],2002, which takes into account the section ductility;
- $colNF$ (column Nodal Failure), obtained by the node rotation capacity defined in NTC08 [21].

The Limit States of the frame $(fr)$, Yielding and Ultimate, are therefore defined as:

$$frDLS = \min \{colYM; \ ISDR = 0.5\%\}$$
$$frULS = \min \{beamUM; colUM; colSF; colNF; ISDR = 2.0\%\}$$

[2]
3.1 Capacity curves

The pushover curves (Figure 8-10) show the base total In-Plane shear force versus the maximum displacement reached at the top of some frame configurations analysed. Each graph plots the pushover curves of bare frame (BF) and infilled frame considering different load patterns: IP, IP+30%OOP and IP+100%OOP, where the number is the percentage of the reference PGA described before. Furthermore, each curve reports the Performance Levels and Limit States reached by both frame and masonry infills. The principal outcomes from the pushover curves are briefly summarized below.

1. The type of frame design, seismic (SD) or traditional (TD), significantly influences both its ultimate strength and the position of Performance Levels. In the case of traditional design, the yield of the RC vertical resistant elements occurs very quickly and for lower strength values, depending on the configuration, from 2.5 to 5 times compared to the same seismically designed configurations. In case of SD frames, the strength corresponding to the frame Ultimate Limit State ($frULS$) is 2-2.5 times higher than in the traditional case. In conclusion, the yield and the last displacement of the bare frame are anticipated in the case of traditional design, whose pushover curve has a more sudden passage from the elastic to the plastic branch with respect to the seismic case.

2. The maximum strength contribution of the panel to the infilled frame system in-plane (IP) (i.e. the maximum distance between the IP curve and the BF one):
   - is not very affected by the number of bays, but becomes greater as the number of bays increase (squat configurations);
   - is greater in the case of a seismic frames (SD) than in the traditional ones (TD), with the same panel, with a ratio between ~ 1.0 and 1.2. This last observation is justified by the fact that the SD frame guarantees a better distribution of the seismic action between the various floors of the building, taking into account the resistance of the upper floor panels. In relation to this behaviour, it is possible to observe on the capacity curves of the buffered frames a "residual" contribution of resistance of the panels for high displacements, compared to the naked frame, which is greater for the slenderer (globally more deformable) frame configurations.

3. The extension of the pushover branch given by the resistance contribution of the panels (excluding the subsequent "residual" contribution to large displacements) is greater in the case of distribution Gr1_a type, while it is reduced with distribution Gr2_a due to the greater concentration of IP forces at the ground floor level which leads to a more rapidly collapse of the infill.

4. Concerning the Performance Levels of the RC frame, the presence of the infill anticipates both colYM columns and the frame Ultimate Limit State ($frULS$). Traditional frames show the columns brittle failure, therefore the SLU of the frame is reached for limited displacements on the elastic branch of the capacity curve.

5. The OOP actions acting on infill walls with an external reinforcement solution does not induce any effect in terms of anticipated collapse and, therefore, no reduction in the maximum resistance. The FB type infill shows an anomalous behavior different from what has just been described with a reduction in the capacity curve linked to the OOP action (although much more contained than in the case of an unreinforced panel). As already observed, in fact, the FB-type reinforcement solution is characterized by a high experimental stiffness with peak strength values comparable with the other strengthening solution (see Figure 3), with a subsequent anticipation of the collapse due to the IP/OOP interaction. Finally, the OOP action does not produce relevant effects of variation in the position of the
Limit States of the panels along the capacity curves except for FB-type reinforcement solution for the reasons explained above.

Figure 8. Capacity curve of 2x3 configuration, seismic design (SD) and infill type F.
Figure 9. Capacity curve of 2x3 configuration, seismic design (SD) and infill type FB and FB (Gr1_a).
3.2 Inter-storey profiles

In the following Figure 11 and 12 are shown the IP inter-storey drift profiles along the building height, for different case studies, at the following Limit States: damage and ultimate limit states of the infill (infDLS and infULS) and ultimate limit states of the frame (frULS). These profiles are calculated as an envelope of the profiles corresponding to the two distributions Gr1_a and Gr2_a considering at each floor the most relevant drift. In the case of reinforced panels (RBB, FB and F), the OOP action has almost negligible effects. In fact, the reinforcement solution avoids an anticipation of the damage and expulsion of the panel ensuring that the response is linked to the IP behaviour of the infill. As consequence, all structural configurations, both seismic and traditional, equipped with reinforced panels, the drift profiles are not affected by OOP actions.
The OOP action does not affect the frame Limit States. At this regard, it is interesting to observe that in case of slender configurations (2x6) the drift profile due to IP actions shows a concentration of deformation at the second level and increasing the OOP seismic actions the behaviour remains the same. In case of BF configuration, it is underlined also a great deformation at the 4th floor associated to the reduction of column section, as explained before. In case of traditional frame (TD), the drift profile at the frULS Limit State, is very squashed towards the ordinate axis due to the breaking of the columns.

Figure 11. Inter-storey drift profiles: 2x6 configuration, seismic design (SD) and infill type F.
4 CONCLUSIONS AND OBSERVATIONS

The paper discusses about a new F.E. macro-model developed to simulate the interaction between the In-Plane and Out-Of-Plane behaviour of strengthened thin masonry infills in RC frames. The aim of the work is to evaluate the performances of the mentioned solutions starting from the calibration of the macro-model on combined test results obtained from previous experimental campaigns on real scale infilled frame. To investigate the seismic response of RC infilled frames, the model has been implemented in a parametric pushover analysis (OpenSees). Four frame configurations (squat, regular and slender) and, for each of these, two types of design (for gravitational loads only and for seismic actions) were specially designed and implemented in the analysis. The analyses were conducted applying wo different IP force distributions (Gr1_a and Gr2_a) and an OOP force pattern associated to three different PGAs (PGA=0.30g, 0.20g and 0.10g). The main influences of the OOP forces are presented through the capacity curves (indicating the principal performance levels and limit states), and the inter-storey drifts. The principal outcomes are listed below.

- The infill contribution on the global capacity of the infilled frame is much greater and extended in case of SD squat frames and for solutions F and RBB.
- Traditional frames, bare and infilled, reach their yielding and ULS before than the seismic frames and a brittle shear failure of the columns may occur.
- The OOP action, also in case of high values of PGA, does not induce effects of early collapse of the panels and, therefore, no reduction in the maximum resistance.

Figure 11. Inter-storey drift profiles: 2x6 configuration, traditional design (TD) and infill type F.

Marco Donà, Massimiliano Minotto, Enrico Bernardi, Elisa Saler, Nicolò Verlato, Francesca da Porto
REFERENCES


EXPERIMENTAL AND NUMERICAL ANALYSIS OF RC FRAMES WITH DECOUPLED MASONRY INFILLS

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Abstract

Masonry infill walls are commonly used in reinforced concrete (RC) frame structures, also in seismically active areas, although they often experience serious damage during earthquakes. One of the main reasons for their poor behaviour is the connection to the frame, which is usually constructed using mortar. This paper describes the novel solution for infill/frame connection based on application of elastomeric material between them. The system called INODIS (Innovative Decoupled Infill System) has the aim to postpone the activation of infill in in-plane direction and at the same time to provide sufficient out-of-plane support. First, experimental tests on infilled frame specimens are presented and the comparison of the results between traditionally infilled frames and infilled frames with the INODIS system are given. The results are then used for calibration and validation of numerical model, which can be further employed for investigating the influence of some material parameters on the behaviour of infilled frames with the INODIS system.

Keywords: Earthquake, In-plane, Out-of-plane, Isolation, Seismic, Modelling.
1 INTRODUCTION

Masonry construction is a very common construction technique over the whole world. Older buildings mostly consist of unreinforced masonry (URM) walls, while in modern construction, masonry walls are widely used to fill concrete or steel internal and external frames. These walls are called masonry infill walls and they represent the most traditional enclosure system since they have demonstrated reasonable performance and durability with respect to temperature, noise, moisture and fire. They are easy to build, attractive for architecture and have a very efficient cost-performance. Therefore, the use of masonry infill walls in RC frame structures is common in many countries and is a highly used constructive technology for residential and commercial buildings. Due to that fact, masonry infill walls are a popular form of construction in seismic regions worldwide. Due to the complexity of the problem and the absence of a realistic, yet simple analytical model, infill walls within frame buildings have been generally considered as non-structural elements and thus have been typically neglected in the design process. Nevertheless, their presence changes the stiffness of the frame structures under lateral loads, and thus influences the dynamic behaviour of the overall system [1-4]. Furthermore, the deformation of the frame in in-plane direction activates infills, which leads to the damage of the infill wall, and in some cases of the frame components too. Additionally, acceleration perpendicular to the wall plane can cause out-of-plane failure of infill walls. Many examples of damages after earthquakes are reported [5-7] showing catastrophic behaviour of masonry infilled frame structures (Figure 1).

![Figure 1: a) Damage of a RC structure with infill walls [8] and b) out-of-plane failure of ground infill walls [9].](image)

Therefore, the behaviour of infill masonry walls subjected to seismic loads has been intensively researched for decades. In-plane behaviour was mostly examined [9-15], whereas the behaviour of infills when subjected to out-of-plane loading was presented in the papers [16-20] among others. Recently, the influence of previous in-plane damage on out-of-plane behaviour is highly investigated [21-25], while simultaneous action of in- and out-of-plane loads was investigated only by few authors [18, 26-28].

As it can be seen, a huge effort has been made in order to investigate, both experimentally and numerically, the behaviour and influence of infill walls on RC frames. In that sense, also different solutions for improvement of the behaviour of infills have been developed and pro-
posed. They can be classified into the three approaches depending on the degree of confinement of the masonry wall within the frame. The first approach goes into the direction of making a stronger infill having rigid connection to the frame, thus being a load-bearing element [29-32]. Second approach is based on keeping the infill walls rigidly attached to the frame, or slightly disconnected, but at the same time increasing the deformability of the infill wall using special construction measures in the wall [33-36]. And the third approach is to separate the frame and infill so that deformations in the frame do not generate high forces in the infill, thus allowing relative displacements between the wall and the frame to occur without interactions [37-39]. These systems are providing decoupling of the infill wall and the frame.

This publication presents a new decoupling system for the seismic connection of masonry infills in RC frames developed within the framework of the European research project INSYSME (Innovative Systems for Earthquake Resistant Masonry Enclosures in RC Buildings) and which is patented in Europe. The behaviour of the system is examined on small specimen tests on components and on an infilled RC frames for separate and combined cyclical in-plane and out-of-plane loads.

2 DESCRIPTION OF THE DECOUPLING SYSTEM

Within the framework of the European research project INSYSME system INODIS (Innovative Decoupled Infill System) was developed, with the aim to improve the seismic behaviour of masonry infilled RC frames. The system INODIS aims to raise the in- and out-of-plane resistance by means of elastomeric U-Profiles placed along the circular contact areas of the infill to the RC frame. The system decouples the infill wall and RC frame with U-shaped elastomeric bearings at the top and along the vertical edges, while three strips of elastomer are placed at the bottom with the middle one glued to the frame and side ones glued to the bottom of the first row of bricks.

Figure 2: a) Sketch of the front view of the infilled frame with the system INODIS; b) vertical cross section; c) detail of installed elastomeric U-profile at the column; d) horizontal cross section; e) attaching plastic profiles to the frame; f) placing of elastomeric U-profile; g) bottom corner detail [41].
At the top and the sides, plastic profiles are attached by screws to the surrounding frame while elastomeric bearings are glued to the masonry infill on one side and placed around plastic profiles on the other side, thus preventing the out-of-plane failure. In this way, deformation of the frame is compensated with the compression of the elastomers, thus significantly reducing loads load transfer to the infill. The deformation capacity of the elastomer is chosen according to the design needs in order to separate the infill wall from imposed in-plane deformations of the RC frame. Figure 2 shows the arrangement of the system with the connection details. More detailed description is given in [40, 41].

3 EXPERIMENTAL TESTS

3.1 Characteristics of system components

In order to evaluate the mechanical properties of the components of the infill wall with the INODIS system, a series of characterization tests on single materials as well as masonry specimens have been conducted in accordance with European standards and codes, when available. A summary of these results is reported in Table 1, while detailed description of these tests is given in [40].

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<tr>
<th>Mortar type</th>
<th>Compressive strength $f_{cm} [\text{N/mm}^2]$</th>
<th>Modulus of elasticity $E_{cm} [\text{N/mm}^2]$</th>
<th>Flexural tensile strength $f_{\text{int,flex}} [\text{N/mm}^2]$</th>
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<table>
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<th>Joint tensile bond strength $f_{\text{t,b}} [\text{N/mm}^2]$</th>
<th>Compressive strength $f_{cm} [\text{N/mm}^2]$</th>
<th>Modulus of elasticity $E_{cm} [\text{N/mm}^2]$</th>
<th>Flexural strength $f_{\text{int}}$ [N/mm$^2$]</th>
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<td>3.1</td>
<td>4870</td>
<td>parallel to the bed joints: $f_{\text{t,b}}$, 0.23, Perpendicular to the bed joints: $f_{\text{t,b}}$, 0.14</td>
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<th>Dimensions L / T / H [mm]</th>
<th>Density [g/cm$^3$]</th>
<th>Static elastic modulus [MPa]</th>
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<td>0.229</td>
<td>0.25 – 0.45</td>
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<td>0.6 – 1.0</td>
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<tr>
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<td>0.460</td>
<td>1.1 – 1.7</td>
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<tr>
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<td>0.561</td>
<td>2.6 – 2.7</td>
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<tr>
<td>Regufoam® 680</td>
<td>249.7 / 249.9 / 25.1</td>
<td>0.637</td>
<td>2.0 – 2.9</td>
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</tbody>
</table>

Table 1: Mean values of the material properties and strengths.

3.2 Experimental tests on infilled frames

Overall in the test campaign, one bare frame and seven infilled frame specimens were tested. Since the focus of this paper is numerical analysis of the infilled frames in in-plane direction, here tests on in-plane loaded specimens are presented. In all the tests, vertical forces of
200 kN have been applied on each column at the start and kept constant until the end of the test. In-plane load is applied as sinusoidal cyclic displacement. Detailed description of the test set-up, instrumentation and all the tests with the results is given in [40].

In order to identify the contribution of the RC frame to the resistance capacity, first a reference test without masonry infill has been performed (bare frame-specimen A) by applying displacement up to 3.5% of inter-storey drift (Figure 3a). Then, masonry infill traditionally connected to the surrounding RC frame with a mortar connection has been tested as described in Figure 3b. First in-plane in-plane sine load up to an interstorey drift of 1.25% has been assigned. The in-plane load is then stopped and a gradual out-of-plane loading and unloading takes place up to an equivalent surface load of 3 kN/m². After removing the out-of-plane load, a second in-plane loading phase up to a maximum interstorey drift of 2.1% is applied.

In contrast to the above-mentioned tests A and BI, for the infilled frame with the INODIS system all load combinations (in-plane, out-of-plane and simultaneous in- and out-of-plane) are investigated on one specimen (DIO). The load protocol can be divided into the five phases with amplitude steps as depicted in Figure 3c.

In order to better understand the behaviour and positive benefits of the application of the system INODIS, a comparison between the tested specimens is shown on Figure 4. Since BI and DIO specimens were subjected up to 1.25% of drift in the first loading phase, for the purpose of comparison, for the test A the part of the curves just until 1.25% is shown (Figure 4a). It can be seen that the infill with the INODIS system has much lower stiffness and horizontal force of 130 kN in comparison to 225 kN for the traditionally infilled frame. The hysteresis curves for the DIO test are wide and exhibit a constant stiffness in the loading and unloading...
stages. In the contrast to this, the hysteresis curves for the traditional infill (Test BI) are narrower and exhibit a decreasing stiffness. Figure 4b shows the envelope curves of the hystereses for tests A, BI, and DIO together with the contribution curves for the BI, and DIO tests. The comparison shows that the envelope curves for tests A and DIO are very close to each other and that the load supported by the infill is only 20.6 kN.

It is important to mention that first part of the in-plane cyclic load caused reduction of stiffness due to initial cracks that occur in the wall and opening of bed joints at 110 kN. Afterwards, the restoring forces increase steadily but with decreased stiffness up to force of around 225 kN (Figure 4b) at 1.25% of in-plane drift. At this point notable cracks in the infill wall are noticed. Then out-of-plane load was applied producing a maximum deformation of 29 mm in the center of the wall. Due to the fact that in-plane load damaged mortar connection of the infill wall and concrete beam, out-of-plane load produced tilting of the infill wall and moving out-of-plane as a rigid body. This proves that the weakness of the traditional system is in the contact between RC frame and infill wall. On contrary, in the phase 1 of the DIO test, no damage and cracks appeared in the bricks and bed joints. The masonry infill remained completely intact while the test was carried out, reaching steadily the maximum force of 125 kN at 1.25% of drift (Figure 3a).

In the phase 2 of the DIO test, out-of-plane load has been increased until reaching 5 kN/m² (35 kN) causing no damage at all and negligible deformations, while phase 2 of the BIO tests produced high out-of-plane displacements of the infill wall.

In Figure 5a complete hysteresis, presenting results of both first and second in-plane loading phase of the BI test, is shown. The restoring forces increased steadily but with decreased stiffness until reaching a plateau with a maximum resisting force of approximately 240 kN at an interstorey drift of 1.5 %. At the maximum drift of 2.1 %, the resistance still amounts to about 180 kN. The deformation behaviour as well as the failure is being dominated by opening and closing of bed joints as well as failure of the bricks. In contrast, in the phase 3 of the DIO test, where out-of-plane surface load of 1.5 kN/m² (10.5 kN) was applied simultaneously with in-plane displacement ranging from 0 to 1% of drift. Linear behaviour of the specimen was noticed with negligible out-of-plane displacements. The same was for the fourth loading phase, where out-of-plane load varying from 2.5 to 5.0 kN/m² was applied together with increasing in-plane load. Until in-plane drift of 1.8 % specimen behaved linearly (Figure 5a), due to the elastic deformations of U-shaped elastomer connection. During the first cycle of in-plane drift of 1.8 % in combination with an out-of-plane loading of 5.0 kN/m², together with the first sound of cracking, a horizontal crack was noticed in the third bed joint from the bottom of the infill. During the third cycle, the vertical crack arose on the right side of the wall, through the units. From this point onwards the out-of-plane deformation started to increase. Until this point displacements of the wall were less than 10 mm. After the load is removed, some of the deformations caused by the formation of cracks remain in the wall, but the deformations in the hyperelastic elastomers reverse for the most of the wall. Afterwards, in the fifth loading phase, increasing in-plane sinusoidal load starting with a interstorey drift of 1.0% and increasing up to a maximum interstorey drift of 3.25% was applied together with the out-of-plane load that started at an initial load of 6.25 kN/m2 and is then reduced to the constant value of 1.5 kN/m2. Maximum in-plane resistance of 150 kN was reached at an interstorey drift angle of 2.2 %, followed by the slow fall of the in-plane resistance to 135 kN at the maximum drift angle of 3.25 % (Figure 5a). During this phase damage of the wall increased but the wall remained stable until the very end of the test reaching the equipment limit of 3.25% of in-plane drift.
Figure 5b shows that even under combined loading, level of infill activation for the infilled frame with the INODIS system is low, which enables reaching of high in-plane drifts, in contrast to the traditionally infilled frame.

![Graph showing hysteresis curves and envelopes with contribution curves for the Test A, BI and DIO up to 1.25% of drift.]

Figure 4: a) Hysteresis curves and b) envelopes with the contribution curves for the Test A, BI and DIO up to 1.25% of drift (just in-plane loading)

Figure 5: a) Hysteresis curves and b) envelopes with the contribution curves for the Test A, BI and DIO

4 NUMERICAL ANALYSIS

In order to extend the results obtained in the experimental campaign, a three dimensional finite element numerical model has been developed in software package Abaqus [42] using the simplified micro-modelling technique. First, a calibration of micro model using small specimen tests on system components has been carried out. Afterwards, these material parameters were used for simulations of experimental tests on infilled frames in order to validate the numerical model.

4.1 Description of the numerical model

Since explicit dynamic analysis is employed, according to the recommendations [43], three-dimensional 8-node hexahedral continuum finite elements, with the reduced integration (C3D8R) are used as most appropriate for this type of analyses to model concrete frames, masonry blocks and elastomer. For reinforcement, truss elements (T3D2) have been used and they are embedded in solid concrete elements.

RC frame is represented with its actual geometry and size, with each reinforcement bar and stirrup modelled and incorporated in an assembly as a separate part with its exact length and position as shown. The simplified micro-modelling technique is used for the numerical model, where masonry units are extended from all directions by half the thickness of the mortar joint.
Interaction between the units representing the mortar joint behaviour is defined. This interaction is used as the crack source as well as the units modelled using smeared crack approach. All the joints between the bricks, both vertical and horizontal, as well as joints between the frame and infill have been defined using general contact with the specified interaction properties defined through “hard” contact, penalty friction assignment and surface-based cohesive interaction based on traction-separation model in Abaqus [43]. Masonry units were modelled as solid elements without holes, although blocks used in experimental campaign are hollow blocks. Effect of holes is taken into account through material definition, by assigning material characteristics of masonry assembly to the units. More detailed description of the numerical model is given in Marinković [40].

4.2 Definition of materials

For material definition, built-in material models in Abaqus [42] were used to describe the behaviour of concrete, masonry units, reinforcing steel and elastomer. For reinforcement an elasto-plastic material using Von Mises criterion was employed, while hyperfoam material suitable to describe hyperelastic behavior of elastomer is used. In the following sections description with the recommendations for concrete and masonry materials are presented.

4.2.1. Concrete

The constitutive model adopted for definition of concrete material is the concrete damage plasticity (CDP) model implemented in Abaqus [42]. The elastic response is assumed to be linear and isotropic and it can be assessed through the use of modulus of elasticity and Poisson’s ratio. Modulus of elasticity is taken from the experimental tests, while the value of 0.2 is used for Poisson’s ratio as suggested in Eurocode 8 [44]. As for the plastic part, in compression the stress-strain curve is first linear until 0.4f'cm according to Eurocode 2 [45]. Beyond this point, concrete is in the plastic region and for this part, expression given in Eurocode 2 [45] is used. However, this curve is defined only up to the nominal ultimate strain. To reach experimental levels of deformation, the curve is extended with respect to the equation proposed by Pavlović (2013). The relationship between the crack width and the corresponding tension stress is based on an equation proposed by Hordijk [46]. This model for reinforced concrete considers independently the concrete behaviour from the rebar. Effects associated with the rebar/concrete interface, such as bond slip and dowel action, are modelled approximately by introducing “tension stiffening” into the concrete material. To account for different damage characteristics in tension and compression, the degradation of the elastic stiffness due to the cracking is characterized by two damage variables, namely dt and dC, which are assumed as functions of the inelastic strains. The damage parameters are equal to zero for undamaged material and equal to one for completely damaged material. The damage parameters are calculated using an exponential function as proposed by Lee and Fennes [47]:

\[ d_c = 1 - e^{-a_c \varepsilon} \]  \hfill (1)  
\[ d_t = 1 - e^{-a_t \varepsilon} \]  \hfill (2)

Where \( a_c \) and \( a_t \) are parameters for the uniaxial compression and tension respectively, and can be calibrated to match the best fit with the experimental results.

4.2.2. Masonry

The same as for concrete material, the constitutive model for masonry used in this study is concrete damage plasticity (CDP) model. Stavridis and Shing [48] recommended that the ma-
terial characteristics of masonry units based on that of masonry prisms should be used, rather than that of individual brick units. This approach has been accepted in this study. Also, a modified approach according to Stavridis and Shing [48] is used to generate the required stress curves in both compression and tension, where linear elastic behaviour is assigned up to intersection point of linear elastic curve and curve proposed by Stavridis and Shing [48] as defined in the following equation:

$$\sigma = f_m \left( 2 \frac{E}{\varepsilon_1} \frac{\varepsilon_1^2}{\varepsilon_1^2} \right)$$

(3)

Where $\varepsilon_1$ is strain at maximal stress, $f_m$. After this point, plastic behaviour starts and until point D is defined with this equation. From point D, stress-strain curve is defined with the following equation:

$$\sigma = \sigma_D + \left\{ \left[ \sin \left( 1 - e^{\frac{f_m}{E_c} (\varepsilon_c - \varepsilon_D)} \right) \right] \pi \cdot \alpha \right\}$$

(4)

Where $\sigma_D$ and $\varepsilon_2$ are stress and strain at point D, and they are together with the coefficient $\alpha$ determined in calibration process to provide best fit with the experimental results. This equation is very suitable for calibration process, because increase/decrease of residual strength of bricks i.e. infill can be easily accomplished. The curve for tension is defined using the approach from Stavridis and Shing [48]:

$$\sigma = f_m \left[ r_t + \left( 1 - r_t \right) e^{\frac{\alpha_1(e_1 - \varepsilon_c)}{f_m}} \right]$$

(5)

Where $\varepsilon_c$ is the strain at the maximum tensile strength, $f_m$. The parameters $r_t$ and $\alpha_1$ define the slope of the applied curve and have to be calibrated using experimental results.

4.3 Numerical simulations of experimental tests

4.3.1. Material calibration

Experimental tests on small specimens were used for calibration procedure of FE model in software package Abaqus [42]. All parameters for definition of concrete material for numerical model were calibrated on the bare frame specimen (Test A). Since reinforcement is modelled using truss elements, there is no need for calibration on one bar. Values for material characteristics, needed to define steel material for reinforcement in Abaqus [42], are calibrated on the bare frame specimen too.

Vertical compression test on masonry assembly was used for calibration of masonry material assigned to the bricks in the model and the interaction between the bricks. Compressive strength and modulus of elasticity of masonry assembly are assigned to brick units as recommended by Stavridis and Shing [48], while the tensile strength of the masonry units is assumed to be 10% of its compressive strength [49]. Material property values used for masonry units and the interaction are taken directly from the experiment, calculated according to the recommendation from the literature or calibrated as described in detail in [40].

Model representing vertical test on masonry assembly was made as similar as possible to the experimental test. It is consisted of three bricks connected and two rigid steel plates, with the lower one used to represent fixed bottom and upper one as a loading plate. Figure 6a shows the comparison of the simulation and experimental results, which clarifies that the simulation results are satisfactory.
Also static compression tests on elastomer were modelled with two strong steel plates. One representing base of the compression machine being fixed at the bottom and second plate presents upper plate with the all degrees of freedom fixed except vertical displacement, which was assigned as a loading. Reaction and displacement at upper plate were measured in order to get force-displacement i.e. stress-strain curves. Elastomer material and its hyperelastic behaviour are described with the use of hyperfoam material available in Abaqus (2013). Figure 6b present diagrams showing a good match between experimental and numerical curves.

4.3.2. Bare frame (Test A) simulation

Loading steps are divided in three phases: gravity, vertical load and horizontal displacement (in-plane load). First, gravity load has been defined in order to take into account the weight of the model components. Afterwards, vertical load of 200 kN per column has been applied on the columns top surface. In the third step, in-plane loading has been applied as a horizontal displacement of the upper beam of the frame. All the loads were applied as monotonically increasing, using smooth amplitude in order to avoid sudden ramp loading effects.

In order to calibrate concrete and reinforcement material models, the bare frame (Test A) was simulated first. As already explained, full bonding between concrete and steel is considered and simulated using embedded element constraint. This formulation has been proven satisfactory for monotonic loading of reinforced concrete [50], that was applied in the simulations.

As shown in Figure 7a, the obtained force-displacement curve is close to the one of experimental test results. A very good matching is succeeded in terms of stiffness decrease. The differences for the forces and drifts when infill cracking started, at maximum force and at the maximal displacement are quite small.
4.3.3. Simulation of the traditional infill (Test BI)

The bare frame numerical model was used as a base for infilled frame model. Each brick is added as a separate part and assembled in an infill wall, by defining interaction between the bricks and infill panel and frame.

Loading protocol applied was the same as in experimental test, first vertical load of 200 kN per column has been applied followed by in-plane displacement up to 1.25% of drift. When infilled frame is returned to zero position, the out-of-plane pressure load of 3 kN/m² is applied and then after removing it second in-plane loading phase up to 2.1% of drift is performed. Figure 8a shows comparison of pure in-plane loading results up to 1.25% of drift, while Figure 8b shows in-plane force-displacement curves for the whole test. It can be seen that difference in force and displacement values between experimental and numerical results is negligible.

![Figure 8a: Comparison of force-displacement curves for the first in-plane loading phase up to 1.25% of drift](image1)

![Figure 8b: Force-displacement curves for the whole test](image2)

4.3.4. Simulation of the infilled frame with the INODIS system (Test DIO)

The traditionally infilled frame was used as a base by adding U-Profile of elastomer together with the plastic profiles attached to the frame at the sides and the top. Also three strips elastomer were added to the bottom of the infill.

In experiment, DIO specimen was loaded in couple of phases combining in- and out-of-plane loading, where the first loading phase was pure in-plane loading, and it was used to validate numerical model of the INODIS system (Figure 9a). It can be seen that numerical model predicts very well stiffness and strength of an infilled frame with the INODIS system.

Afterwards, model was simulated up to around 5.70% of in-plane drift (Figure 9b) in order to investigate what would happen if in experiment test specimen was loaded up to this level. This high level of in-plane drift reached can be explained with the contribution curve showing that activation of the infill is postponed to the high drift levels. The cracking of an infilled frame with the INODIS system is initiated at the same time when maximum load capacity (195 kN) was reached, which happen at 2.8% of in-plane drift (Figure 9b). Damage of infill wall occurs due to the crack openings, which leads to the abrupt drop of the resistance force. Afterwards, residual strength of an infill is activated due to the fact that contact between frame and infill is soft and not stiff as in traditional infill. Elastomeric U-Profiles around infill allow for reconsolidation of infill bricks after occurrence of cracks which brings additional capacity to the infill.
5 CONCLUSIONS

This study presents the efficiency of a recently proposed construction detail, employed to improve the seismic performance of masonry infilled RC frames. In contrast to traditional solutions with stiff mortar connection between frame and infill, the proposed system INODIS decouples infill from the frame with the application of U-Profiles made of elastomers. The connection detail include plastic profiles attached to the columns and upper beam with elastomeric U-Profiles placed around them, thus at the same time providing decoupling in in-plane direction and stable and constant restrain in out-of-plane direction.

The performance of the proposed system has been evaluated with experimental results which showed relatively strong resistance of the infill in a case of traditional specimen (BI), which led to the appearance of shear cracks in the RC columns, caused by the activation of the infill strut. This proved that possibility for occurrence of shear failure mechanisms is high in the case of strong infills and it should be prevented through adequate measures, where one of them can be decoupling through elastomers. Also, sudden and brittle failure of the traditional specimen was observed in contrast to the masonry infill with INODIS system, where damage was postponed to high drift levels and even after crack appearance and further increase of in-plane displacement together with out-of-plane load wall remained stable not showing brittle behaviour. When compared to the bare frame, infilled frame with the INODIS system has similar behaviour due to the decoupling and stiffness reduction through elastomers.

Experimental results on system components were used for calibration of numerical model, which is then validated using tests on bare frame and infilled frames. Comparison of numerical and experimental results shows that model is able to satisfactorily represent behaviour of traditionally infilled frames and infilled frame with the INODIS system under monotonic loading. Both detailed and global analyses were successfully performed using the developed model. Therefore, numerical model can be used for investigating influence of different geometry and material characteristics on the behaviour of infilled frames with INODIS system. This is part of the on-going work that will be presented in future publications.

REFERENCES


FEM SIMULATION OF THE IN-PLANE SEISMIC EXPERIMENTAL RESPONSE OF R.C. FRAMES WITH UNREINFORCED AND BED-JOINT REINFORCED AAC MASONRY INFILLS

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Abstract

Unreinforced masonry infills in RC structures are used worldwide and commonly built in complete adherence with the RC frame without any reinforcement and/or fastening. According to post-seismic surveys and experimental studies, due to the intrinsic vulnerability of these non-structural elements, they can cause considerable economic and downtime losses, and even threat human lives. Although several researchers have studied the seismic behaviour of masonry infills, the response of AAC masonry and the solution with the bed-joint reinforcement need further studies starting from the available experimental results. The present paper introduces preliminary results of a numerical research where in-plane experimental tests carried out on unreinforced and bed-joint reinforced AAC masonry infills have been simulated through nonlinear FEM analysis. The numerical simulation has been anticipated by a calibration of the materials according to tests of characterization. Moreover, the influence of the bed-joint reinforcement on the in-plane seismic behaviour has been numerically studied, and the results have been compared to the unreinforced solution.

Keywords: Seismic response, numerical simulation, FEM modelling, masonry infill, bed-joint reinforced masonry infill
1 INTRODUCTION

Masonry infills are worldwide adopted as non-structural elements in newly designed structures. Although several studies have been accomplished during the past decades and several post-seismic inspections have been conducted on buildings with masonry infills, their seismic behaviour is not fully identified yet due to the variety of constructive and design processes. The strong influence that these non-structural elements may play on the global and local behaviour of the building points out the need to consider both the seismic response of the infill and the interaction with the structure and/or with the structural members. Moreover, due to the seismic vulnerability of the masonry infills, many innovative systems have been proposed recently to improve the seismic response of these non-structural elements. Among different innovative systems, the most adopted solution is the enhancement of the masonry infill to increase the in-plane and out-of-plane seismic resistance through the inclusion of reinforcement in the masonry.

In particular, the insertion of bed-joint reinforcement in the mortar joints represents a viable practical solution to improve the resistance of the masonry. Such system has been studied by Penna and Calvi [1] through a wide experimental campaign. Moreover, recent studies on load-bearing walls with bed-joint reinforcement (see, e.g., Penna et al. [2]) have drawn the attention of this specific solution, albeit it has not been deeply studied for infill applications. The experimental and numerical researches on bed-joint reinforced masonry infills seem to be restricted to very few studies and further investigations on this topic are needed.

To define the influence of the bed-joint reinforcement as respect to a traditional unreinforced masonry (URM) infill, that is one of the aims of the present study, the in-plane pseudo-static cyclic tests conducted by Penna and Calvi [1] have been numerically simulated through a micro detailed model, where both the masonry units and the mortar joints have been explicitly modelled.

In the present paper, the literature review on numerical and experimental past studies is discussed and a brief presentation of the reference experimental campaign is reported. Subsequently, the modelling criteria are described, along with the model calibration. Finally, the results of the numerical simulation of the reference RC bare frame, of the URM infill, and of infilled RC frame specimen with bed-joint reinforced masonry are illustrated and compared, in order to estimate the possible improvement given by the reinforcement.

2 LITERATURE REVIEW

As recently defined by Morandi et al. [3], there are three different approaches to improve the seismic response of masonry infills: subdivide the infill through weak/sliding joints ([3]), uncouple di infill from the surrounding frame ([4]), or enhance the infill through reinforced plaster ([5], [6]) or steel reinforcement. The latter approach can be distinguished between two cases: one where the infill is made of vertical and horizontal reinforced masonry ([7]), the second where the panel resistance is enhanced with bed-joint reinforcement only ([1]).

Numerically, as defined by Milanesi et al. ([8]) and Tarque et al. ([9]), masonry infills can be modelled with different modelling approaches developed in the past. Crisafulli et al. ([10]) classified the models used for RC infilled structures into two groups: micro-model, also called local models, where the structure is divided into numerous elements to take into account the local effects in details; and the macro-models, also called simplified models, which are based on a physical understanding of the behaviour of the infill panel ([9]). Furthermore, Lourenço ([11]) proposed to classify the micro-model depending on the level of refinement. The first possibility is to represent the masonry as a homogenous material and, sometimes, these models are defined as “meso-models” ([8]). It is also possible to model the masonry as composed
by masonry unit continuous elements and interface elements as mortar joints ([12]). A third approach consists in modelling the masonry as two-phase materials; masonry units and mortar joints are modelled as two different continuous elements and their mechanical behaviour is specified separately. For the present study, the latter detailed micro-model approach has been used. Although it has not been deeply adopted yet for masonry infills, the detailed micromodels have been used recently for load-bearing masonry walls in many approaches such continuum approaches ([13]), or discrete element methods ([14], [15]).

3 REFERENCE EXPERIMENTAL CAMPAIGN

In 2006, several tests on full-scale AAC masonry infills have been conducted by Penna et al. [1] at the Department of Civil Engineering and Architecture of the University of Pavia and at EUCENTRE. Different typologies of AAC infills were tested: unreinforced, reinforced with a RC beam at mid-height, reinforced with steel bars and flat trusses in the bed-joints and with and without opening. Within this paper, the main experimental outcomes of the in-plane cyclic tests are presented. Herein the focus is on the traditional unreinforced AAC infill and on the AAC masonry reinforced with steel flat-truss in the bed-joints.

In all cases, the masonry was made of AAC units 300 mm thick with thin layer bed- and head-mortar joints, whereas a traditional mortar has been adopted for the interface between the panel and the RC elements.

The in-plane tests were performed applying horizontal displacement cycles, according to predefined targets. Three cycles were performed at each target displacement.

3.1 In-plane cyclic test on a RC bare frame

The RC bare frame has been tested in the late ’90s, when experimental tests on different weak clay masonry infills in reinforced concrete frames have been executed in an extensive research performed by Calvi and Bolognini [16]. Full scale, one-storey, one-bay frame specimens were designed according to modern seismic codes and subjected to in-plane and out-of-plane tests.

The experimental loading scenario consisted of a vertical constant load of 400 kN applied at the top of each column and a horizontal displacement imposed to the beam in order to perform a cyclic pseudo-static in-plane test up to 3.6% drift for the bare frame. The point of application of the loads during the tests and the Force-Displacement curve of the bare frame obtained from the in-plane test is reported in Figure 1.

![Figure 1: Force-Displacement curve of the bare frame obtained from in-plane cyclic pseudo-static test and application points of the loads during the test [16].](image)
3.2 In-plane cyclic test on “traditional” URM AAC infill

The in-plane cyclic test on the “traditional” URM AAC infill has been conducted with the same experimental loading protocol adopted by Calvi and Bolognini [16] for the bare frame test.

In Figure 2 the cracking pattern at the end of the test (Figure 2a) and the corresponding Force-Displacement curve (Figure 2b) are reported. The specimen has been tested up to a drift of 1.2%, albeit a much lower maximum drift was reached at ultimate conditions. Moreover, the crack pattern shows the creation of two diagonal struts (one for push loading direction and one for the pull loading direction) that are located within both the bed mortar joints and the AAC units. Furthermore, minor cracks at the corner and at the edges with the RC frame have been observed.

Figure 2: RC frame with URM AAC infill. (a) Specimen at the end of the in-plane test; (b) Force-Displacement curve (Penna and Calvi, [1]).

3.3 In-plane cyclic test on AAC masonry infill with steel flat-truss in the bed-joints

The in-plane cyclic test on AAC masonry infill with steel flat-trusses in the bed-joints have been conducted with the same experimental loading protocol adopted by Calvi and Bolognini [16] for the bare frame test.

The bed-joint reinforcement (Figure 3) was made of a steel flat-truss where the longitudinal reinforcement was composed by two steel rebars with an area of 12 mm² each, with an internal longitudinal spacing of 190 mm between the two. The reinforcement was located every two bed-joints, at about 400 mm vertical spacing.

In Figure 4 the cracking pattern at the end of the test (Figure 4a), and the corresponding Force-Displacement curve (Figure 4b) are reported; the specimen has been tested up to a drift of 3.6%, albeit, also in this case, a much lower maximum drift was reached at ultimate condition. Moreover, the crack pattern shows the creation of two diagonal struts (one for push loading direction, and one for the pull loading direction) that are located within both the bed mortar joints and the AAC units. Some horizontal cracks along the bed-joint have been observed, being often located within the unreinforced bed-joint. Furthermore, minor cracks at the corner and at the edges with the RC frame have occurred. Although a detailed study is currently ongoing, the bed-joint reinforcement improves the seismic behaviour of the masonry infill.
4 DESCRIPTION OF THE MODEL

The analyses have been accomplished with a software (Abaqus [19]) that allows the use of the explicit module, suitable to solve highly nonlinear problems. The mesh has been defined through a sensitivity analysis. The typologies of the continuum elements have been selected by paying attention to minimize the artificial energy and viscous dissipation.

The modelling approach selected allows performing non-linear static analyses with high-distorted elements and high-displacement. Although the cyclic behaviour may play an important role in the seismic response of masonry infilled structures, non-linear pseudo-static analyses has been carried out due to the limited information available on the cyclic behaviour of the materials.

The concrete was modelled using an eight-node 3D brick element. The size of the mesh was constant and equal to 50 mm. The presence of reinforcement has been simulated using the “Embedded Constraint”. The longitudinal and transversal rebar have been modelled as 1D beam elements embedded within the mesh representing concrete. The technique assumes full bond between the reinforcement and the concrete, which is a common assumption in case of ribbed bars. Embedding options are typically available in major finite element codes for modelling rebars (see, e.g., Milanesi et al., [8]).

A nonlinear constitutive model for continuum elements has been used for the concrete. This model allows simulating the plastic behaviour with the specification of the compressive and tensile strength of concrete, giving also the possibility to specify the softening as function
of the plastic strain. If properly calibrated, this model is capable to reproduce the response of the concrete simulating the reduction of stiffness and strength with plastic strain. The mechanical response of the embedded steel bars has been simulated using an elastic-plastic model with hardening.

In order to reproduce the experimental test, the precompression provided in the laboratory to the RC beam to transfer the tension and thrust force of the actuator and the vertical pre-compression on the RC columns have been simulated.

Experimental data on strength were available (Calvi and Bolognini, [16]), while the Young’s modulus and the softening curves have been calibrated by means of comparison between the results of the cyclic experimental tests on the RC frame and the numerical pushover. In agreement with the experimental tests of characterization of the concrete, the beam and column have different concrete resistances, whereas the Young Modulus has been computed as a fraction of the Elastic uncracked modulus, as reported by Milanesi et al. [8].

In the model of the URM infill, the mortar joints, the autoclaved aerated concrete (AAC) units and the mortar interface between the masonry infill and the RC frame have been explicitly modelled as reported in Figure 5a and Figure 5b. The mortar joints of the masonry infill have a thickness of 1 mm. The mortar interfaces between the masonry infill and the columns have a thickness of approximately 10 mm, whereas the other two interfaces, in contact with the beam and the foundation, have a thickness of about 2.5 mm. In order to simulate accurately the typical “de-bonding” between RC frame and masonry infill, which characterizes the in-plane response of this kind of system, a cohesive interface with zero-thickness has been introduced. This approach allows to simulate the partial/complete separation between concrete and masonry infill, avoiding excessive distortions of the mesh that can reduce the quality of the results.

The AAC units and mortar joints are modelled with the same continuum elements used for concrete. The size of the mesh of mortar and units was 20 mm and 45 mm, respectively. The AAC units have been modelled with the same nonlinear constitutive law used for concrete, calibrated according to the test on mechanical characterization. The model selected for the mortar joints of the masonry infill is the Mohr Coulomb Plasticity Model as defined in Andreotti et al. ([13]).

Finally, the numerical model of the infill with bed-joint reinforcement has been implemented starting from the model of the RC frame with URM infill. In particular, the bed-joint reinforcement has been explicitly modelled within the bed-joints (see Figure 5d). The numerical technique adopted to model the bed-joint reinforcement follows the same approach used for the steel rebars of the RC frame model. Each steel rebar composing the bed-joint reinforcement has been explicitly modelled as 1D beam element embedded within the mesh of the bed-joint using the “Embedded Constraint”.

The geometrical disposition of the steel flat-truss was the same of the one in the specimen tested by Penna and Calvi [1], with two longitudinal bars with a section of 12 mm2, and a diagonal inner bar with a circular section with a diameter of 1.5 mm, the width of the flat-truss was of 190 mm.
5 RESULTS OF THE CALIBRATION OF THE MASONRY INFILL

The numerical model introduced in the previous section has been calibrated in agreement with the tests of mechanical characterization of the masonry.

5.1 Numerical calibration of the AAC unit

The numerical mechanical behaviour of the AAC units has been calibrated according to the results of the tests carried out on AAC cubes (100x100x100 mm) and AAC units, to determine the vertical compression resistance. The test procedures are in agreement with EN 771-4 and EN 772-1. Figure 6 (a and b) shows the stress distribution within the specimens, whereas Figure 6c reports the stress-strain numerical curves compared with the experimental behaviour of the AAC (Ferretti et al., [17]).
Figure 6 Numerical simulation of the compression tests on AAC (a) AAC cube; (b) AAC unit; (c) comparison between the numerical and the experimental stress-strain curves.

5.2 Numerical calibration of the unreinforced masonry

The numerical results obtained from the simulation of the vertical and horizontal compression tests of characterization of the unreinforced masonry are herein presented.

Figure 7a presents the stress distribution for the vertical compression, whereas Figure 7b reports the stress distribution for the horizontal compression. Moreover, Figure 8a and Figure 8b show the comparison between the experimental and numerical results in terms of stress-strain relationships for vertical and horizontal compression, respectively. The numerical analysis is capable to capture the initial stiffness and the average post-yielding behaviour. Although the peak resistance is not completely matched, the use of isotropic material for AAC units has been considered sufficiently accurate due to a limited orthotropic behavior in the experimental response.

Figure 7 Numerical simulation of the masonry. Stress distribution in the (a) vertical compression and (b) horizontal compression.
Figure 8 Comparison between numerical and experimental stress-strain curves. Numerical simulation of the (a) vertical compression test; (b) horizontal compression test.

6 NUMERICAL SIMULATION OF THE IN-PLANE TESTS

In the present section, the results of the numerical simulation of the in-plane tests are reported.

6.1 Numerical simulation of the RC bare frame

The numerical model matches very well the experimental results (see Figure 9) and it can be assumed as reliable for the next steps, where the masonry infill is placed within the same RC frame model.

Figure 9 Comparison between experimental and numerical results of the RC frame.

6.2 Numerical simulation of the RC frame infilled with unreinforced masonry

In Figure 10 the comparison of the Force-Displacement experimental and numerical curve is reported. The numerical model simulates correctly the initial stiffness, the first peak and the post-peak overall response. This calibration appears suitable for the aim of the future steps of the research, since the initial stiffness, the peak response and the damage propagation are the key points to study the influence of the bed-joint reinforcement in the seismic behaviour of the masonry infill.

Figure 11a reports the maximum principal stress distribution, where the formation of a diagonal strut is evident (from blue to orange colours), demonstrating that the model realistically reproduces the overall behaviour of the specimen. The vertical detachment along the column height has been well replicated, as shown in the maximum principal logarithmic strain distribution reported in Figure 11b (red and grey colours).
Finally, the numerical simulation has permitted to extrapolate the seismic response of the specimen at higher drifts than the maximum ones attained during the test (Figure 10).

![Figure 10 Comparison between experimental and numerical results of the RC frame with unreinforced masonry infill.](image)

**Figure 10** Comparison between experimental and numerical results of the RC frame with unreinforced masonry infill.

(a) Minimum principal stress distribution of the URM infill; (b) Maximum principal logarithmic strain distribution of the URM infill.

**Figure 11** (a) Minimum principal stress distribution of the URM infill; (b) Maximum principal logarithmic strain distribution of the URM infill.

### 6.3 Numerical simulation of the RC frame infilled with bed-joint reinforced masonry with steel flat trusses

Figure 12 reports the comparison of the Force-Displacement experimental and numerical curve. The numerical simulation fits pretty well the initial stiffness, the global behaviour and the post-peak overall response.

Figure 13a shows the maximum principal stress distribution, showing the formation of a diagonal strut (from blue to orange colours). Also in this case, the vertical detachment along the column height is consistent with the experimental outcomes and it has been correctly captured, as shown in the maximum principal logarithmic strain distribution reported in Figure 13b (red and grey colours).
Figure 12 Comparison between experimental and numerical results of the RC frame with bed-joint reinforced masonry infill with steel flat trusses.

Figure 13 (a) Minimum principal stress distribution of the bed-joint reinforced infill with steel flat-truss. (b) Maximum principal logarithmic strain distribution of the bed-joint reinforced infill with steel flat-truss.

7 COMPARISON OF THE RESULTS

The overall force-displacement curves of the three numerical simulations are reported together in Figure 14. In these plots, the force is measured where the displacement is imposed (i.e., the force where the actuator is placed during the test), and the displacement as the horizontal displacement of the centre of the RC beam.

To compare the influence of the infill only, it is common practice to consider the infill contribution by computing the difference between the infilled frame and the bare frame force-displacement curves. The infill contributions are reported with dashed lines in Figure 15a, and a zoom for an imposed displacement up to 25 mm is reported in Figure 15b.

Although the behaviour for large imposed deformation seems to be similar between the two infill systems, the presence of the reinforcement enhances the seismic response. According to Morandi et al. [18], the peak force may correspond to a level of damage classified as a Damage Limit State. As shown in Figure 14 and Figure 15, the peak of the infill contribution in the case of the specimen reinforced with bed-joint flat trusses is shifted on the right (larger displacement capacity) as respect to the URM panel, enhancing the deformation capacity of about 20%. This shift represents an improvement in the lateral response of the infill, therefore delaying the attainment of the Damage Limit State.

However, since the F-D curves are not significantly different in the two infill systems, more refined post-processing with a more proper comparison of the damage level between the two infills (unreinforced and reinforced) is needed, in order to improve the interpretation on the influence of the bed-joint reinforcement in the seismic response of the panel.
Figure 14 Comparison of the numerical force-displacement curves

Figure 15 (a) Comparison between the system by considering also the infill contribution. (b) Zoom related to the comparison of the infill contribution only.

8 CONCLUSIONS

The scientific and technical interest on innovative infills to the improvement of the seismic response of masonry infills is recently growing and, among different approaches and systems, the introduction of bed-joint reinforcement may represent a suitable solution. However, further research is needed to study the effect of reinforced bed-joints on the seismic response of masonry infills. The aim of the present research was to simulate numerically the in-plane tests conducted by Penna and Calvi [1], in order to investigate the seismic response of masonry infill with steel reinforcement in the bed-joints. In order to model the masonry infills with bed-joint reinforcement, a refined finite element model with explicit solver with a micro-detailed approach was used.

The masonry and the RC frame were firstly calibrated. Subsequently, the RC frames with URM infill and with steel flat-truss bed-joint reinforced masonry infill were numerically simulated through pushover analyses. The numerical simulations showed a good match with the experimental results, being able to replicate the overall global Force-Displacement response and to capture both the creation of the compressive strut and the detachment of the infill from the RC frame.

According to the experimental [1] and numerical evidences (i.e., the contribution of the infills in terms of Force-Displacement curves), the bed-joint reinforcement seems to improve the in-plane seismic performance of the masonry infill, postponing the complete activation of the diagonal strut and, consequently, delaying the attainment of a level of damage that may correspond to a Damage Limit State as defined in the seismic codes.

In conclusion, since the global behaviour is not extremely different because of the creation of the diagonal strut, a more refined post-processing with a comparison of the damage be-
tween the two infills (enhanced and URM) would improve the interpretation of the results and the study of the influence of the bed-joint reinforcement in the seismic response of masonry infills. However, since the F-D curves are not significantly different in the two infill systems (unreinforced and reinforced), more refined post-processing with a more proper comparison of the damage level between the two infills is needed, in order to improve the interpretation on the influence of the bed-joint reinforcement in the seismic response of the panel. Moreover, numerical simulation of the out-of-plane behaviour is also needed and it would allow a more complete evaluation of the seismic performance of bed-joint reinforced masonry infills.

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INFILL WITH SLIDING PANELS AND A FULL-HEIGHT OPENING

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Abstract

The innovative infill construction technique for seismic resistance, implementing horizontal sliding joints to partition the wall into subpanels, is here tested in presence of a full height opening. With the double aim of protecting the opening fixtures (window or door) from the infill sub-panels’ relative sliding and offer out of plane support to the infill, a post is placed at the opening side spanning between the top and bottom beam of the frame. The post stiffness and strength design is the object of the study. The role of the post deformability was studied by modifying the post’s stiffness with additional steel profiles coupled to the initial wooden post configuration, in different test phases. The shear action at the post ends was measured with specific load cells, to provide information for the proportioning of the post and its connection to the beams. The results showed the efficiency of the post in governing the sliding mechanism between the infill sub-portions and the role of the post’s stiffness in modifying the in-plane response of the infill. Thanks to the post’s deformability, the overall infill-frame interaction was reduced with respect to previously tested similar infills without openings.

Keywords: infill-frame interaction, masonry infill, ductile infill, sliding joints, enclosures, earthquake damage mitigation.
1 INTRODUCTION

In the last years, several innovative constructive solutions have been proposed to mitigate the issues related to the seismic interaction between reinforced concrete (RC) framed structures and the masonry infill walls, typically built within the frame bays to achieve the necessary insulation performance to the building. In fact, such an interaction demonstrated to be one of the main causes of the damage and collapses registered in the post-earthquake surveys of the last seismic events [1–4]. Among all the proposed solutions, many of them aimed at reducing the negative infill-frame interaction by providing deformability to the stiff masonry infill. To this aim some authors [5,6] proposed a sort of isolation for the infill, by detaching it from the frame; others [7,8] suggested the adoption of dry joints instead of the mortar ones to activate relative movements between the blocks composing the masonry; others proposed the introduction of sliding [9–12] or deformable [13] joints within the masonry to partition the infill wall into sub-panels capable of activate a relative movement, thus following the frame deformation.

The authors carried out previous studies [14–18] aimed at validating an innovative construction technique for the infills, based on the introduction of sliding joints (in horizontal or vertical direction) at selected mortar joints. In addition, to protect the masonry from local crushing, weaker elements were introduced at the infill-to-column interfaces and a small gap was maintained between the infill and the frame top beam, to avoid the transfer of vertical stresses to the infill during the lateral deformation of the structure. The performed numerical and analytical studies highlighted the superior performance of the “sliding joints” technique in terms of deformation capacity, damage limitation and strength and stiffness reduction. However, in case of horizontal sliding joints, it was observed that, especially for large drift levels, the infill sub-panels generate relevant trusts on the frame columns, with a consequent high shear demand on the structure that has to be taken into consideration in the frame design.

In the referenced studies the infill was considered to fill the entire frame bay, with the masonry spanning between two adjacent frame columns. Nevertheless, in typical buildings the infills are characterized by openings for doors or windows and their role still need to be quantified. In fact, the studies performed by several authors on traditional infills [19–22] showed the substantial role played by openings in modifying the seismic response of the infill, so that their presence cannot be neglected.

The present paper deals with an experimental study focusing on the efficiency of the previously proposed “sliding joints” technique for the infill, in presence of a full-height opening. To this aim, a full-scale specimen was built and tested under in-plane quasi-static cyclic actions. In detail, the test focused on (i) the influence of the vertical post deformability on the infill in-plane response and on (ii) the shear demand in the post, due to the in-plane interaction with the infill. The first aim was obtained by splitting the test into different phases characterized by a progressively increased stiffness of the vertical post. The information about the shear action at the post ends were obtained by measuring the horizontal component of the reaction at the post’ end connections. It is worth noting that the infill out-of-plane response was not investigated in the experimental program, however specific detailing was implemented to provide a sufficient retain against transversal actions, according to previous experiences [15].

2 DESIGN OF THE SPECIMEN

In order to create the opening in an infill partitioned into horizontal sliding sub-panels it results necessary the introduction of a vertical post, adjacent to the opening, spanning between the top and bottom frame beams. In the in-plane deformation mechanism, the post is conceived to confine the masonry and govern the re-centering of the infill sub-panels under load reversals (Figure 1). According to this mechanism, the activation of the relative sliding along the joints
could ensure the in-plane strength and stiffness reduction with respect to traditional infills, similarly to that obtained in [15] for the same construction technique without opening. However, as demonstrated in [14], the deformational mechanism activated in the full infill produced contact forces lumped along the columns height, leading to a high shear rates on the columns themselves. As a consequence, the same mechanism activated in presence of the opening would generate a similar shear demand on the vertical post confining the masonry sub-panels. Such a demand could be reduced by reducing the confinement effect produced by the post, as a consequence of its deformability. However, a too flexible post will undergo large deflections, invading the volume of the opening and causing damage to the opening fixtures (windows or doors). Concerning the connections of the post to the frame, they have to ensure a horizontal restraint in both the post ends, while an axial degree of freedom has to be allowed to avoid the risk of buckling in the post due to vertical compression imposed by the beams’ deflection during the in-plane sway mechanism. For all these reasons, the design of the post and of its connections to the frame deserves particular attention and the results of the present study could provide some important recommendations.

As for the out-of-plane response, the presence of the opening interrupts the continuity of the masonry sub-panels between the two frame columns. To provide a restraint for the infill against the out-of-plane actions the post has to be designed to collect a share of such actions and transfer them to the frame. To this end, the post surface in contact with the masonry can be shaped to create a shear key embedded in the masonry and the post-to-beam connections can be organized to supply a simple support at the post ends. In this resisting mechanism, a sufficient stiffness is required to the post in order to allow the activation of compressed arches in the masonry thickness, spanning between the frame column and the post itself (Figure 2).
3 DESCRIPTION OF THE SPECIMEN AND TEST SETUP

The infill specimen adopted for the experimental campaign aims at reproducing the portion of an infill included between the frame column and the full-height opening.

The specimen (Figure 3) was built inside a steel frame designed and already adopted for previous experimental campaigns [23]. The frame is made of rigid beams and columns, with hinges placed at the columns’ ends to activate an in-plane sway mechanism with a practically zero strength. As a result of this frame configuration, the lateral load measured during the test represents the net infill lateral strength.

The masonry infill was built with hollow clay blocks and M5 premixed cementitious mortar (average flexural strength 2.54MPa and average compressive strength 8.31MPa from tests according to [24]), adopting the same material used in [15], in order to perform a comparison between the two infills performances. The infill was built according to the construction technique presented in [15], with the partitioning of the masonry wall obtained by means of planks embedded in selected horizontal mortar joints to create three sliding joints. To limit the friction along the horizontal plank, polyethylene sheets were placed between the upper plank surface and the mortar layer above. It is worth noting that at the base of the infill a normal mortar joint was adopted, without creating any sliding surface. This aspect had a relevant influence on the infill response, as discussed in the following. On the top of the masonry, at the end of the construction, a 5cm gap was maintained between the infill and the top frame beam to avoid the mutual interaction during the in-plane deformation. Such a gap, in real applications can be smaller (10-20 mm) and is meant to be filled with soft insulating material. Before the infill construction, a 5cm thick plank was dowelled to one column to create a weak cushion protecting the masonry from local crushing. The creation of the openings was obtained by introducing a vertical wooden post adjacent to the opposite column, connected to the frame beams. It had a 150x150 cross section and was made of wood classified as C14, according to [25].

The connection of the post to the surrounding frame was conceived to allow rotation at the post ends: the vertical restraint to the upper and lower beams consisted in a point support, obtained by shaping the post ends and interposing a teflon plate on the post-to-beam surface; to provide the horizontal restraints to the post, two truss elements were inserted between the post ends and the adjacent columns. Such elements were made of HEB160 steel profiles, shaped to realize nearly ideal trusses in order to limit the bending moment transfer from the post to the
column. Such elements were instrumented and calibrated to work as load cells in order to measure their axial load. The adopted configuration of the connections allowed to consider the post as simply supported to the frame and, according to this scheme, the load measured in the top and bottom load cells can be assumed as the shear at the post end connections. More details of the geometry of such truss elements are reported in [26].

It is worth noting that the results reported in the following in terms of shear action in the post can be assumed as a reliable trend, better than an exact measure, focusing on the peak shear demands that are significant when dealing with the design of the post.

To investigate the role of the post deformability in the in-plane response of the infill, three different configurations were adopted for the post ("A", "B" and "C" in the following), characterized by a progressive increase of the post’s stiffness. With reference to Figure 3, configuration “A” was characterized by the sole wooden post, while configurations “B” and “C” were obtained by coupling to the post one or two additional HEB100 steel profiles, respectively. The contact between the additional steel profiles and the wooden post was localized at the level of the sliding joints and at the profile’s ends and it was forced by a pre-stressing system screwed to the post.

A first estimate of the ideal flexural stiffness (\(EJ\)) of the post can be obtained under the approximated assumption that the stiffening profiles work in parallel with the wooden element and is reported in Table 1 for the different test configurations, compared to that of the frame steel column. It is worth noting that passing from Phase A to Phase C the post stiffness was increased by one order of magnitude, but it remained ten times lower than the stiffness of the HEB240 steel column.

<table>
<thead>
<tr>
<th>Stiffness of the HEB240 steel column</th>
<th>Flexural stiffness of the post (EJ)</th>
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<tbody>
<tr>
<td></td>
<td>Phase A (wooden post)</td>
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<tr>
<td>2.36 ( \cdot ) 10^4 kN \cdot m^2</td>
<td>2.95 ( \cdot ) 10^2 kN \cdot m^2</td>
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Table 1: Ideal flexural stiffness of the post in the different test phases.

The steel frame was fixed to the strong floor of the lab and the in-plane load was applied by imposing pseudo-static cyclic displacements at the frame top beam. The experimental campaign was divided into three test phases ("A", "B", "C"), characterized by the different configurations of the post stiffening. For each test phase, cycles with progressively increased inter-story drift were applied, according to the load protocol reported in Figure 4a. In detail, during "Test B" the same loading protocol adopted in "Test A" was applied to the specimen, while the following phase started imposing a cycle to the maximum inter-story drift experienced in the previous phase to highlight the effect of the stiffness modification of the post.

The response of the specimen during the test was monitored in terms of overall lateral strength, load in the cells at the post’s ends and deformations obtained by means of several potentiometric gauges measuring the displacement of the top beam of the frame and the deformation of the post in several points along the height (at the sliding joints positions and at the post ends).
4 RESULTS

The obtained experimental response was characterized by a large deformation capacity with low lateral strength, provided by the activation of the sliding mechanism along the horizontal joints. The presence of the opening, in fact did not obstruct the sliding mechanism, which activated in all the sub-panels, limiting the damage suffered by the masonry. In the following paragraphs the experimental results for the different test phases are reported in terms of lateral force-vs.-drift, maximum deflection along the post height and horizontal reactions at the post end connections. For a more extended discussion of the results reference is made to [26].

4.1 Test A

The main aspects of the response of the specimen with the sole wooden post (“Test A”) are reported with light grey lines in figures from Figure 5 to Figure 7. The lateral force-vs.-drift response of Figure 5 shows an asymmetric behavior characterized by higher strength in positive loading direction (with a ratio between 0.5 and 0.7 for different drift levels). A similar asymmetry was observed also in the maximum deflection of the post, showing higher deflections for negative drift (Figure 6), when the post is the windward element in the sway mechanism of the infill. According to the internal contact forces distributions schematized in Figure 1 and recalling the analytical interpretation provided in [27] for infills without openings, the horizontal equilibrium of each infill sub-panel confined by two vertical elements (i.e. the column or a post, as in this case) imposes the contact forces on the windward element being equal to the sum of the friction forces along the sliding joints and the contact force on the leeward element. As a consequence, when the post is the windward element, it receives contact forces larger than when it works as leeward element (positive drift). This effect can then justify the asymmetry observed in the post deflection.

Also the asymmetry in terms of strength can be explained under the above assumptions: the infill strength is directly related to the intensity of the contact forces between the infill and the windward and leeward elements, in particular, the higher the contact forces, the higher the infill strength. As demonstrated in [27], such contact forces increase with the drift as the compression strain along the sub-panel diagonal is increased by the confining elements in the sway deformation. In this mechanism, a relevant role can be acted by the deflection of the vertical post confining the sub-panels, as it can relax the diagonal strain in the sub-panels and, consequently, the contact forces on the confining elements. In the tested case, being the column significantly
stiffer than the post, it remained basically straight while rotating, while the diagonal strains in the infill sub-panels were strictly related to the post deflection, so that the infill strength was smaller when the post deflection was larger.

Figure 5: Results of the test: lateral load-vs.-drift.

An asymmetric response varying the imposed drift was obtained also in terms of shear action measured at the post ends (Figure 7 shows the response in the post’s base end). According to the considerations reported above, higher shear demand was observed for negative drift, when the post was the windward element in the deformation mechanism. In this test phase the peak shear values for top and bottom ends were similar, with slightly higher values in the bottom one. It is worth noting that during this test phase the sliding activated along all the horizontal sliding joints, while no sliding occurred in the mortar joint at the base of the infill. As schematized in Figure 8, the lack of sliding slightly modified the distribution of the contact force between the infill and the confining elements. In detail, for positive drift, the lack of sliding produced: (i) a concentration of the trust on the column in correspondence of the upper corner of the base sub-panel and (ii) the absence of contact forces on the opposite side, at the post base, as the stresses are directly transferred to the base beam trough friction. For this reason, during

Figure 6: Results of the test: deflection of the post in the negative loading direction.
Test A, the shear action at the post base end was limited when loading imposing positive drift values.

Similarly, for negative drift values, the lack of sliding at the infill base caused a contact forces concentration in the bottom sub-panel corner in contact with the post (Figure 8b) thus increasing the shear action at the base of the post itself. Such high concentrated action was testified also by the asymmetric deflection of the post along the height measured during Test A (light grey in Figure 6), characterized by higher displacement in the lower part. It is worth noting that this configuration remained unaltered up to the activation of a sliding mechanism also in the base mortar joint (occurred in the following test phases). Thereafter the contact forces distribution modified, moving toward the ideal configuration reported in Figure 1.

4.2 Test B

The results of the Test B, carried out after the application of one stiffening HEB100 profile to the wooden post, are reported in figures from Figure 5 to Figure 7 with black dashed lines. The lateral force-vs.-drift curve (Figure 5) shows a response that remained asymmetric and characterized by a slight increase of the maximum strength at 1.5% drift, with respect to the
response obtained during Test A at the same imposed deformation. Such increase of strength can be related to the increased stiffness of the post, which is also testified in Figure 6b by the lower deflection (almost -20%) suffered by the post, compared to Test A.

During the loading cycles, the response specimen showed a marked pinching, as also observed in previous tests [15]. This effect is mainly due to the local non-linear deformation suffered by the lateral plank and the post at the contact interface with the masonry sub-panels. As a result, the lateral strength increment that could have been produced by the stiffening of the post was compensated by the progressive strength reduction related to such local non-linear deformations at the contact locations.

The effect of stiffening was more evident on the shear action measured at the post ends (Figure 7, limited to the bottom end), with a general increase of the maximum values of about 20%, compared with Test A.

4.3 Test C

The effect of the further increase of the post stiffness, achieved during Test C, on the specimen response is described in Figure 5 to Figure 7 with black lines. The addition of a further HEB100 steel profile to the post produced only a negligible increase of the lateral strength, compared to the previous test phases (Figure 5). The effect of stiffening became more evident during the first cycle at 2% drift in positive direction, with an increase of the slope of the response up to 1.8% drift, when a 50kN lateral strength was measured. Thereafter, some cracks started developing in the lower masonry sub-panel (Figure 9) and the load remained stable up to 2% drift. At the load reversal a sudden sliding along the base mortar joint was observed at around -1.5% drift, associated with a drop in both the lateral strength (Figure 5) and the shear action at the base post’s end (Figure 7).

The activation of such sliding mechanism can be observed also in terms of post deflection. In Figure 6 the response of the -1.5% drift cycle of Test C can be compared to those of the previous tests: the effect of the post’s stiffening is clearly testified by the further reduction of the maximum deflection with respect to Test B, however the deformed shape remained asymmetric, due to the contact force distribution described above. In the following cycles of Test C, the activation of the sliding in the base joint (occurred between -1.5% and -2% drift) restored the symmetry of the deformed shape of the post along the height. This modification of the

Figure 9: Damage pattern at the end of Test C.
deflection profile testified a modified contact forces distribution, being characterized by a more uniform intensity of the contact forces along the post height.

In the following cycles of the test, the mechanism remained stable, being characterized by the sliding also at the infill base and with only a slight extension of the crack pattern in the base sub-panel. Because of the modification of the resisting mechanism and the suffered damage, the peak lateral load in the positive loading direction slightly diminished, while in the opposite direction it slightly increased up to 33kN at -2.5% drift. As for the shear action at the post ends, it is worth noting that after the full sliding activation in the base joint, the shear in the post ends had its maximum at the top and bottom sections for the negative and positive drift, respectively. The peak values of the shear action at the two post ends, for each cycle of the test, are reported in Table 2. From the table it can be observed that the shear demand reached approximately 65kN in the top section and 56kN in the base section, for negative and positive drift, respectively.

<table>
<thead>
<tr>
<th>Drift</th>
<th>Max Post Base Shear</th>
<th>Max Post Top Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50%</td>
<td>3.60</td>
<td>5.10</td>
</tr>
<tr>
<td>-0.50%</td>
<td>14.50</td>
<td>21.05</td>
</tr>
<tr>
<td>1.00%</td>
<td>5.10</td>
<td>8.85</td>
</tr>
<tr>
<td>-1.00%</td>
<td>27.20</td>
<td>20.70</td>
</tr>
<tr>
<td>1.50%</td>
<td>7.00</td>
<td>14.10</td>
</tr>
<tr>
<td>-1.50%</td>
<td>34.20</td>
<td>29.95</td>
</tr>
</tbody>
</table>

Table 2. Maximum shear action in the connections of the post to the frame measured during the test.

4.4 Effect of the presence of the opening: comparison with previous results

As shown by the experimental results above described, the effect of the opening on the in-plane response of the infill with horizontal sliding joints is basically governed by the deformability of the post introduced adjacent to the infill to create the opening. To better appreciate the influence of the post deformability, the obtained experimental results are here compared with those obtained in a previous experimental campaign on the same infill typology, but without the opening. The test, described in [15] was performed on an infill wall built with the same materials and in the same steel frame. The only difference consisted in an alternative detailing of the beams-to-columns connections, being characterized in that case by an elastic-plastic response, providing a small but not negligible lateral strength to the frame. As a consequence, for a proper comparison between the results obtained with the two different testing frames, the net infill strength was obtained by subtracting the bare frame contribution from the total experimental lateral force-vs.-drift response. In Figure 10 is reported the envelope curves for the full infill obtained by this procedure, compared to the envelope of the experimental results above.
described for the infill with the opening. The comparison highlights that the presence of the opening allowed an evident strength reduction (70% when the post is the windward element and 45% in the opposite direction) ensured by the deformability of the post that ensured the relaxation of the internal stresses in the masonry infill.

In Figure 10, a further comparison is reported with the experimental results presented in [23] for a full infill, built with the same material properties and within the same testing frame, partitioned by means of vertical sliding joints, connected to the frame beams. The comparison shows that the presence of the opening combined to the horizontal partitioning of the infill results in an in-plane response similar to that of the infill with vertical sliding joints, thus characterized by reduced interaction with the surrounding frame.

![Figure 10: Infill with sliding joints with or without the full-height opening: comparison between the envelope curves of the experimental results.](image)

5 CONCLUSIONS

An experimental study was conducted on a real scale specimen of an infill with horizontal sliding joints and a full-height opening in order to assess the mechanism activated in the in-plane response. The study focused mainly on the role of the vertical post introduced adjacent to the masonry infill to create the opening and to govern the sliding of the sub-panels during the frame sway mechanism. The variability of the response with the stiffness of the post was investigated by coupling additional steel profiles the initial wooden post. The response of the infill was monitored in terms of overall lateral strength and deflection and shear demand on the post during the cyclic deformation of the specimen. From the analysis of the results the following conclusions can be drawn.

- The response of the infill was characterized by large ductility and limited damage, with the deformation mechanism governed by the relative sliding of the masonry sub-panels along the horizontal sliding joints.
- The response is affected by the deformability of the post: the higher the post’s stiffness, the higher the lateral strength and the lower the post deflection. The results showed that increasing the stiffness of the post by one order of magnitude, a 25% increase of the infill strength is obtained, while the deflection reduced by 25%.
- The overall lateral strength of the specimen reached about 50kN, a value that is significantly smaller than the 134kN obtained in a previous test on a similar specimen without openings. This comparison highlights the relevant role of the flexibility of
the post in reducing the in-plane interaction between the infill and the surrounding frame, producing a response similar to that obtained for a full infill partitioned with vertical sliding joints.

- The shear at the post ends increases with the post’s stiffness: the maximum shear demand measured in the post was about 65kN, suggesting the need of a particular care in the design of the connections of the post to the frame.

Based on these considerations, the test highlighted the efficiency of the horizontal partitioning of the masonry also in presence of an opening in the infill and suggested the positive role acted by the post’s deformability in limiting the infill-frame interaction. As a consequence, the choice of the post (in terms of cross section and material) should be a trade-off between the need of limiting the post deflection, to protect the opening fixtures, and of reducing the infill-frame interaction. The here reported conclusions are based on the evidences of the single test performed, however an extension to different infill geometries and materials, requires further studies, supported by numerical and analytical models, capable of providing more accurate design recommendations. A first parametric study has already been performed by the authors and the results are reported in [28].

6 ACKNOWLEDGEMENTS

The authors gratefully acknowledge the technicians of the P.Pisa Lab of the University of Brescia for their assistance in the experimental testing. The presented study was achieved within the research program funded by the “Presidenza del Consiglio dei Ministri Dipartimento della Protezione Civile”.

7 REFERENCES


ASSESSMENT OF ROBUSTNESS OF REINFORCED CONCRETE FRAME STRUCTURES WITH MASONRY INFILL WALLS

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Abstract

Robustness of frame structures is fundamental to limit progressive collapse of buildings in case of accidental loss of columns. The interest in robustness assessment is growing in recent years especially for reinforced concrete structures, which are commonly analyzed considering the bare frame configuration. This paper presents a numerical study highlighting the influence of masonry infill walls on the robustness of reinforced concrete frame structures. The main geometrical and mechanical parameters identifying an infilled frame (aspect ratio, seismic detailing, lateral constrain degree) are investigated by performing parametric push-down tests on reference two-bay frames extracted from different types of buildings. The tests are carried out by using a detailed finite element model of the infilled frame. Results show significantly different responses from bare and infilled frames in terms of resistance and displacement capacity under the different conditions tested. In a relevant number of cases infills have shown to be fundamental to limit or avoid multiple collapses.

Keywords: Robustness, Infilled frames, Reinforced concrete, Masonry, Pushdown, Catenary

1 INTRODUCTION

In the last years, the interest in structural robustness is rapidly increasing within the scientific community and in practice engineering. For civil structures having residential, commercial or public use, the limitation damage propagation in consequence of the accidental loss of a primary structural element, such as a column, is fundamental to prevent public safety and reduce economic losses. Robustness based design of buildings addresses solutions to avoid that damage suffered by a structure, due to an accidental event, would not be disproportionate with respect to the cause that has caused it, as several times occurred in the past (Fig. 1). In frame structures, the loss of a perimetral column due to impacts, explosions or advanced material degradation may result into serious consequences for the whole building which depend
on several conditions such as the location of the collapsed element, the shape of the bays, the
type of frame elements and the arrangement of the reinforcement.

Figure 1: Examples of progressive collapses: a) Alfred P. Murrah Federal Building, Oklahoma City, 1995; b) Ronan Point Building, London, 1968.

For reinforced concrete bare frame structures, the possibility of avoiding or limiting multi-
ple collapses as a consequence of a column loss, depends on the capacity of the connected
beams to switch from the first flexural and arching resistant mechanism, to a catenary tensile
mechanism under large displacements regime. In the recent past, several studies have ad-
dressed progressive-collapse response of frame structures. Among these [1,2] formulated a
simplified method for the determination of dynamic load-displacement demand, through a
pseudo-static procedure. Experimental tests on reduced scale and real RC structures have been
also carried out [3,4]. Further experimental investigations, accompanied by numerical studies,
were provided in [5-9]. The principal findings of these studies report that ductility of beam
end cross-sections has an important role on the possible activation of the catenary mechanism,
but many other factors condition the response as for example the horizontal constraint degree
the actual capacity of elongation rebars and the height to length ratio of beams.

It should be evidenced that the aforementioned studies did not consider the influence of in-
fills on the progressive collapse mechanism, although infill-frame interaction effects are well
known and have been widely documented for the case of lateral forces by a number of exper-
imental studies and numerical investigations [10-19]. Only in the last few years the influence
of masonry infills on the progressive collapse response has known the first numerical studies
regarding masonry infilled steel frames [20, 21], and two experimental investigations on in-
filled RC structures [22, 23]. These studies revealed the significant modification of the resist-
ing mechanisms due to the noticeable strength increase and the alternative load pattern which
develops. As a consequence of this, sufficient reliability [24-35] in estimating progressive col-
lapse response of infilled frames (IF) cannot be achieved without explicitly considering
frame-infill interaction. Further the role played by different geometrical and mechanical pa-
rameters remains unknown even in terms of dynamic load amplification and safety levels.

Based on this background, the paper presents a study investigating parameters influencing
the robustness of RC structures with masonry infills in terms of resisting capacity and dynam-
ic amplification of the response in comparison with bare frames. Three reference case study
building structures have been designed with and without considering seismic load and detail-
ing. The pushdown response is obtained from refined finite element models of 2D sub-
structures. Numerical tests regarded a significant number of parameters such as lateral con-
straint degree, aspect ratio of the infilled frame, seismic detailing). Finally, possible single
strut and multiple strut modelling configurations are tested and compared.
2 ANALYSIS PROGRAM AND STRUCTURAL MODELS DEFINITION

The case study of a five-storey RC frame residential building is considered as reference. It is supposed the accidental loss of a column placed at the center of a perimeter frame. The building structures have been designed hypothesizing two different arrangements of central bays, one with short spans and beam length ($l_b$) column length ($h_c$) aspect ratio $l_b/h_c=1$ (Building A) (Fig. 2a), the other having long spans and $l_b/h_c=2$ (Fig. 2b) (Building B). Buildings A and B have deep beams (30x50 cm). Design strength of concrete ($f_{ck}$) and steel ($f_y$) are 25 MPa and 450 MPa respectively. Design loads of inter-floor slabs are $G_{1k}=3.20$ kN/m, $G_{2k}=3.80$ kN/m, $Q_{k1}=2.00$ kN/m. Buildings A and B have been designed with and without considering seismic loads, seismic detailing and capacity design. In the first case the site hazard of Palermo (Italy) is selected in combination with a Type A (hard) soil, a reference life ($V_N$) of 50 years, and a return period ($T_R$) of 475 years. For the second case buildings A and B are designed considering only vertical loads and disregarding seismic detailing rules. Reinforcement detail of the extracted sub-frames is shown in Figs. 3-5. The influence of the horizontal constraint degree, provided by the adjacent frames, on the progressive collapse response is also investigated. Pushdown tests on the sub-frames are then carried out in the two boundary cases of: i) full lateral constraint; ii) no lateral constraint.

Masonry infill walls are supposed to be the same typology as the one tested by [12], arranged with clay hollow bricks. Experimental mechanical parameters are shown in Table 1, and are referred to direction 1 (parallel to mortar joints) and direction 2 (orthogonal to mortar joints). For the current case, the units are supposed being 30 cm thick, 32 cm long and 26 cm high. Summarizing, investigated parameters are the aspect ratio of the frames ($l_b/l_c$), the aspect ratio of the beams ($h_b/t_b$ of the beam), the type of reinforcement design (seismic / non-seismic), the effectiveness of the lateral constraints (rigid or free). For every case bare frame and infilled frame conditions are analyzed.

![Figure 2: 3D view of case study buildings and extracted sub-frames.](image)

![Figure 3: Sub-frames with seismic design extracted from.](image)
DEFINITION OF THE FE MODEL AND TEST MODALITIES

3.1 The FE model

The extracted sub-frames are reproduced with a refined nonlinear FE model realized in ATENA 2D [34]. The assembly of the model is depicted in Fig. 6. Concrete element portions and masonry blocks are modeled by using CCIsoQuad nonlinear finite elements [36]. The constitutive model governing the response of concrete elements and masonry units (SBeta material) is defined by a uniaxial stress-strain concrete-type law that is associated with a biaxial domain regulating the failure surface. Compressive strength of concrete elements is assigned to the different element portions taking into account the actual confinement exerted by transverse reinforcement. Nominal concrete mechanical parameters are reported in Table 2. As for the masonry units parameters are initially taken from experimental values reported in Table 1, however, in order to account for the anisotropic behaviour of masonry, conventional compressive strength $f_m$ and elastic modulus $E_m$ are obtained by averaging experimental values along the two orthogonal directions [35]. Shear strength ($f_{vm}$) is the same as that experimentally detected from the shear tests. Mechanical parameters used for masonry units and unconfined concrete are summarized in Table 2. Frictional response of mortar joints is simulated by using interface elements between masonry units and between masonry units and concrete. The 2D interfaces are governed by a Mohr-Coulomb failure domain which depends on friction coefficient ($\mu$) and cohesion ($c$). Interface response depends also on tangential and normal stiffness moduli $K_t$ and $K_n$ assumed for the interfaces. Longitudinal reinforcement is accounted for by using "embedded rebar" elements with a uniaxial elasto-plastic strain hardening constitutive model with yield stress $f_y=450$ MPa, ultimate stress $f_t=540$ MPa and ultimate deformation $\varepsilon_{su}=12\%$. Transverse reinforcement is considered within the by means of the smeared reinforcement material, which uniformly spreads shear reinforcement over the mesh of macro-elements. For all the elements described above, geometric nonlinearity is considered to allow carrying out large displacements the analysis.
Figure 6: FE model assembly for an extracted sub-frame.

<table>
<thead>
<tr>
<th>Unconfined concrete</th>
<th>Masonry unit macro-elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$ (MPa)</td>
<td>$f_{ct}$ (MPa)</td>
</tr>
<tr>
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</tr>
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<td>$m_{E}$ (MPa)</td>
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<tr>
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<td>6.42</td>
</tr>
<tr>
<td>5716.5</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table 2: Mechanical parameters of masonry and unconfined concrete macro-elements.

3.2 FE model calibration

Parameters subject to calibration are those associated with the major uncertainty, namely those regarding interface response. In detail these are the normal stiffness modulus ($K_{nn}$), the tangential stiffness modulus ($K_{nt}$), the cohesive strength ($c$), the tensile strength ($f_t$) and the friction coefficient ($\mu$). Since masonry infills are hollow clay infills of the same type of those tested in [12], it has been reasonably supposed that calibration parameters values should be the same as those obtained in [37] where the same model was used to simulate lateral response of infilled frames experimentally tested in [12]. Interface parameters used are reported in Table 4. It should be said that original experimental tests are lateral force- lateral displacement tests, simulating seismic load conditions, while current load condition refer to vertical loading of the infilled frames. However interface parameters are not significantly affected by loading direction, hence the assumption made of adopting calibration values obtained in [37] for the current models is considered reasonable also in the framework of this study.

Figure 7: Experimental positive and negative envelopes of specimens S1B [12] and force-displacement response by the FE model after the calibration [37].
### 3.3 Definition of pushdown tests and capacity / demand assessment

Pushdown tests are conducted in two phases. First, vertical loads are applied at the top of lateral columns (Fig. 6), then, increasing vertical displacement is then imposed to the central column. Vertical displacement and reaction forces are monitored during the tests. The obtained force-displacement curve is the pushdown capacity-curve of the system under the column removal scenario. The static load demand \( P_0 \) for each analyzed system is determined as the rate of axial force previously acting on the removed column \( P_s \):

\[
P_0 = \alpha_{SF} P_s
\]

where \( \alpha_{SF} \) is a distribution factor obtained as the ratio between the number of floors involved in the sub-frame system \( n_{f, SF} \) and the total number of floors \( n_f \) as follows:

\[
\alpha_{SF} = \frac{n_{f, SF}}{n_f}
\]

In the current cases, 2 storey sub-systems are extracted from 5 storey frames, then the distribution factor \( \alpha_{SF} \) is 0.4 (2/5). The static load demand to the sub-frames \( P_0 \) is then 260 kN for the systems having \( l_b/l_c = 1 \) \( (P_s = 650 \text{ kN}) \) and 480 kN for the systems having \( l_b/l_c = 2 \) \( (P_s = 1200 \text{ kN}) \). Under the instantaneous column removal the static load demand is amplified because of the dynamic effect occurring due to the abrupt variation of the equilibrium conditions. The load amplification factor can be estimated through a pseudo static-procedure [1], namely by equating the external work done by gravity loads and the internal work resulting as the area below the pushdown curve. The external work done by gravity loads is the function:

\[
W_e(u) = \alpha P_0 u
\]

where \( \alpha = 0.5 \) is coefficient considering the fact that loads are distributed over the beams. Eq. (3) represents a straight line passing through 0 and is defined in the interval \([0, u_u]\), where \( u_u \) is the ultimate displacement. The internal work is the integral function of the pushdown curve \( P(u) \) defined in the interval \([0, u_u]\) obtained as (Fig. 8a):

\[
W_i(u) = \int_0^u P(u) du
\]

After column removal, the system will achieve a new equilibrium condition if functions \( W_s(u) \) and \( W_i(u) \) have an interception point in correspondence of the vertical displacement \( u = u_d \) (Fig. 8a), representing the dynamic displacement demand to the system. Displacement \( u_d \) can be evaluated by setting (Fig. 8a, 8b):

\[
W_s(u) - W_i(u) = 0
\]

that is:

\[
\alpha P_0 u_d - \int_0^{u_d} P(u) du = 0
\]
After, the dynamic load demand $\alpha P_d$ is evaluated as:

$$\alpha P_d = P(u_d)$$

Therefore the static load amplification factor is:

$$\lambda_d = \frac{P_d}{P_0}$$

The capacity / demand ratio of a system can be also expressed as the ratio between the maximum static load to which the sub-frame could be subjected to ($P_{0_{\text{max}}}$) and the current static load ($P_0$), that is:

$$\beta_s = \frac{P_{0_{\text{max}}}}{P_0}$$

where $P_{0_{\text{max}}}$ is easily obtained by changing the slope of the line expressed by Eq. (3) in such a way that external work function becomes is tangent to the internal work function (Fig. 8a). In this case Eq. (6) becomes:

$$\alpha P_{0_{\text{max}}} u_{d_{\text{max}}} - \int_{0}^{u_{d_{\text{max}}}} P(u) du = 0$$

which allows also obtaining maximum dynamic displacement demand $u_{d_{\text{max}}}$ and maximum dynamic load demand $\alpha P_{d_{\text{max}}}$, which is theoretically coincident with the peak load $P^*$ (Fig. 8c).

The capacity/demand ratio $\beta_s$ can be evaluated even in the case in which a system is not able to achieve a new equilibrium condition. In this case $\beta_s$ will be lower than 1.

4 INFLUENCE OF ASPECT RATIO, SEISMIC DETAILS AND LATERAL CONSTRAINT DEGREE

4.1 Bare and infilled frames having $l_b/l_c=1$

Pushdown curves of bare and infilled frames having $l_b/l_c=1$ are shown in Figs. 9 and 10. For the case of rigid lateral constraints the pushdown response of bare frames (Fig. 9a) exhibited a resistance peak followed by a rapid loss of resistance. This behaviour is consistent with a significant compressive thrust occurring when arching mechanism develops after the cracking at the beam ends. This justifies the initial increase of flexural capacity of beams. The following bearing capacity loss is related to the achievement of high strain rate of concrete which starts crushing compression. Pushdown tests of infilled frames (Fig. 9b) showed on av-
verage double resistance with respect to bare frames, together with a significant stiffness increase. The increase in strength is strictly related to the modification of the overall resisting mechanism because of the interaction with the infills. This is clear from the comparison of the damage scenarios of the two systems (Figs. 11a-b). The resisting mechanism of bare frame involves 8 plastic hinges at the ends of the beams, as expected for a simple flexural collapse (Fig. 11a). The resisting mechanism of the infilled frame (Fig. 11b) is more complicated. First it can be clearly observed the formation of two compression fields characterized by diagonal cracks passing through masonry units. Second, slippage of mortar joints occurs in the central portion of the infill, which is less affected by confinement effect exerted by the frame at the corner regions. The compression fields conveying the diagonal compression stresses on the masonry also influence the local distribution of internal forces of beams. This results in a shifting of the position of plastic hinges (upper beams external hinges and lower beams internal hinges) toward the inner of the beam (at about 35% of the internal length). Steel rebars plastic strain diagrams of bare and infilled frames highlight the position of plastic hinges (Fig. 11a-b). The rapid post-peak strength decay occurs in correspondence of mortar joints sliding.

As for the influence of seismic detailing, it can been observed, that the major strength of seismically designed bare frames with respect to frames with seismic detailing (Fig. 9a) is related to the larger amount of flexural reinforcement arranged to agree code provisions. Conversely, the seismic design of the frame resulted to be not actually relevant in the case of infilled frames if one considers the large strength increment provided by the infills (Fig. 9b).

![Figure 9: Pushdown curves for specimens $l_d/l_c=1$ and rigid lateral constraint: a) bare frames with and without seismic detailing; b) infilled frames with and without seismic detailing.](image)

![Figure 10: Pushdown curves for specimens $l_d/l_c=1$ and no lateral constraint: a) bare frames with and without seismic detailing; b) infilled frames with and without seismic detailing.](image)
Some different observations can be done in case of no lateral constraint (Fig. 10), where the larger deformability of the systems has as effect the significant reduction of the compression action on the beams, which is reflected as a reduction of the flexural resistance, ranging between -40% for the infilled frames and -70% for the bare frames. The reduction of the axial force on beams is consistent with the more ductile behaviour observed form the capacity curves. The influence of infills results in a more relevant strength increase action especially for the case of non-seismically designed frames, where strength increment is about 3 times.

It should be finally observed that for all the investigated systems with aspect ratio \( l_b/l_c = 1 \) there was no evidence of activation of the catenary mechanism after the flexural phase. This can be first attributed to the reduced inelastic displacement capacity of the beams, because of the low span length / cross-section height ratio. For the infilled frame cases this is also associated with the overall modification of the resisting mechanism which results in an increase of shear damage to bottom beams at relatively low displacements because of the infills pushing action.

Figure 11: Cracking pattern on the FE model at the end of the pushdown test of specimens with \( l_b/l_c = 1 \) and rigid lateral constrains: a) bare frame; b) infilled frame.

4.2 **Bare and infilled frames having \( l_b/l_c = 2 \)**

Push-down curves of bare and infilled frames with shape factor \( l_b/l_c = 2 \) are shown in Figs. 12 and 13. In the case of rigid lateral constraints (Figs. 12a-b), both the systems exhibit the achievement of a resistance peak followed by a rapid strength decay. Bare frames have shown significantly slight regain of resistance at large deformations due to a moderate activation of the catenary mechanism (Fig. 12a). The final cracking pattern of bare frames test (Fig. 14a) clearly shows the position of plastic hinges at the ends of beams, and the presence of cracks orthogonal to the longitudinal axis of the beams, denoting a uniform tensile stress state.

Infilled frames have shown a resistance increment of about +50% with respect to bare frames (Fig. 12b). In comparison with square frames, the lower influence of infills to the systems resistance is recognized in this case. This is related the larger deformability of the frame, which can provide effective confinement on a relatively smaller portion of masonry (Fig. 14b).

Slippage and separation of mortar joints, is observed in the central portions of the infills. The migration of the position of plastic hinges toward the center of the beams is more pronounced in this case. The position of plastic hinges is at about 35% of the internal length of the beams. In absence of horizontal constraints, the strength reduction is approximately -40% (Figs. 13a-13b) with respect to the fully restrained case for both bare and infilled frames. The achievement of the maximum load capacity is followed by a post-elastic branch not showing losses or increases in resistance. This highlights on the one hand a more ductile behaviour due
to the reduced axial compression level on beams, and, on the other hand, the inhibition of the catenary mechanism, because of the reduced capacity of the system to support horizontal tension forces.

For all the investigated systems, the presence of seismic detailing resulted to not influence the overall response in a significant way. The small strength increments observable from Figs. 12 and 13 can be entirely associated with the larger amount of longitudinal reinforcement.

**Figure 12:** Pushdown curves of specimens with $l_b/l_c=2$ and rigid lateral constraints: a) bare frames with and without seismic detailing; b) infilled frames with and without seismic detailing.

**Figure 13:** Pushdown curves of specimens with $l_b/l_c=2$ and no lateral constraints: a) bare frames with and without seismic detailing; b) infilled frames with and without seismic detailing.

**Figure 14:** Cracking pattern recognized on the FE model at the end of the pushdown test of specimens with $l_b/l_c=2$ and rigid lateral constraints: a) bare frame; b) infilled frame.

4.3 **Quantification of and capacity and demand modification due to infills**

Fig. 15 shows the ratios between infilled frame and bare frame peak loads ($P_{\text{inf}}^*$ and $P_{\text{bare}}^*$) under the different investigated conditions (aspect ratio, lateral constraint (LC), seismic detail-
The aspect ratio of the frames plays the major role. In fact, square frames provide spread confinement on the masonry, which results in a major infill contribution to the overall strength due to the increased strength. This is much more evident for the frames not provided with seismic detailing and lateral constrains, where the modification of the overall capacity produces by the infills achieves +209%. For the frames having $l_b/l_c=2$ the influence of infills is less relevant but still significant. In this case the tests have highlight minor dependence of the strength increment with the lateral constraint and seismic detailing conditions, recording an average increment of +45%.

Dynamic load amplification factors ($\lambda_d$) obtained for the specimens are reported as bar charts in Fig. 16. Square bare frames and infilled frames have shown similar dynamic amplification factors, which stay in the range 3.5-4.7, apart from for the case of non-seismically designed frames without lateral constraint. The effect of masonry infills on the modification of $\lambda_d$ was more relevant when $l_b/l_c=2$. In these cases the increase of dynamic load demands ranged between +30% and +60% depending on lateral constraint conditions, although the increment of dynamic load demand is however generally accompanied by an increase of bearing capacity. Capacity / demand ratios are finally shown in Fig. 17. The presence of infills resulted in a noticeable increase of $\beta_s$ coefficients (Fig. 17a) especially for square specimens, where $\beta_s$ was more than double with respect to bare frame cases. $\beta_s$ coefficients of infilled frames with $l_b/l_c=1$ ranged between 7 and 10. On the contrary $\beta_s$ coefficients assumed values between 1.28 and 2.0 for $l_b/l_c=2$. 

Figure 15: Infilled frame / bare frame maximum strength ratios for the investigated conditions.

Figure 16: Load amplification factors: a) $l_b/l_c=1$; b) $l_b/l_c=2$. 

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In order to find a computationally effective modelling strategy to simulate progressive collapse response, a possible adaption of equivalent strut modelling approach is here tested, while acknowledging that: a) load direction in case of column loss is vertical instead of horizontal; b) observed collapse mechanisms are different from those typical of infilled frames subjected to seismic actions. Numerical tests previously presented are reproduced with OpenSees [38], using fiber-section approach to model beams and columns, and the equivalent strut model by Di Trapani et. al (2018) [37] to model infills. This model is based on a single concentric fiber-section strut (S1 strut) governed by a concrete-type stress-strain law. The definition of the equivalent strut properties follow the equation provided in [37]. This approach is tested in two possible configurations having different degree of complexity: 1-strut configuration (Fig. 18a) and 3-strut configuration (Fig. 18b). The 1-strut configuration is the original strut configuration of the model, where, for the determination of all the parameters, the length of the infill ($l_b$) is inverted with its height ($h_c$). In the 3-strut configuration, the same S1 strut is accompanied by two rigid struts (S2 struts) which start from the end of S1 strut and point toward top and bottom beams at a distance $\alpha l_b$ (Fig. 18b). S2 struts are included in the model to simulate in a more effective way the observed damage mechanism, in which, masonry at corners remains almost intact. The distance $\alpha l_b$ represents the position where the plastic hinge forms. From the damage patterns observed by the FE models pushdown tests it can be reasonably assumed that assumed $\alpha l_b=0.35$ in the case in which $l_b/h_c=1$ and $\alpha l_b=0.1$ if $l_b/h_c=2$.

The tests are carried out for four specimens among those previously tested, in detail these are seismically designed frames with and without lateral constraints and with square and rectangular aspect ratio ($l_b/h_c=1$ and $l_b/h_c=2$). Results of the comparisons are illustrated in Figs. 19 and 20. It can be observed that the prediction of the peak resistance results significantly improved in the case of 3-strut configuration. On the contrary 1-strut models generally show an understimation of the capacity. The better predictive capacity of 3-strut models is due to the effective identification of the reduced span length mechanism, which results in an increase of bearing capacity of the system. This can be also observed from the deformed shapes of the specimens reported in Figs. 21a and 21b, that highlight the consistency of 3-strut models deformed profiles with those recognized by the FE models.

5 TESTING EQUIVALENT STRUT MODELLING APPROACH FOR THE SIMULATION OF PUSHDOWN RESPONSE OF INFILLED FRAMES

![Figure 17: Capacity / demand ratios: a) $l_b/l_c=1$; b) $l_b/l_c=2$.](image)
Figure 18: Possible equivalent strut configurations: a) 1-strut model; b) 3-strut model.

Figure 19: Possible equivalent strut configurations: a) 1-strut model; b) 3-strut model.

Figure 20: Possible equivalent strut configurations: a) 1-strut model; b) 3-strut model.

Figure 21: Deformed shapes of 1-strut and 3-strut models of infilled frames with: a) 1-strut with rigid lateral constraints \( (l_b/h_c=1) \); b) 3-struts with rigid lateral constraints \( (l_b/h_c=1) \), c) 1-strut with rigid lateral constraints \( (l_b/h_c=2) \); d) 3-struts with rigid lateral constraints \( (l_b/h_c=2) \).
6 CONCLUSIONS

The paper has presented a numerical investigation regarding the influence of masonry in- 
fills on the progressive collapse response of reinforced concrete frame structures. Results have 
shown a primary role of infills on the response of the system to a perimetral column acci-
dental loss scenario. The major findings can be summarized in the following points:

- In the progressive collapse mechanism infills work as diagonal struts providing addi-
tional resistance and at the same time modifying the internal forces distribution on 
frame members;
- Infills can increase the overall resistance in a significant way but their presence result 
in an inhibition of the potential activation of catenary mechanism, which instead is typ-
ical in the case of bare frames;
- Frame aspect ratio \( \frac{l_d}{l_c} \) is one of the most important parameters for both bare and in-
filled frames. Masonry infills in square specimens have much more influence on 
strength increment because of the major confinement action exerted by the frame;
- Lateral constrains degree is decisive on the collapse mode of bare and infilled frames. 
In fact, rigid constrains induce large axial force on beams which consequently have 
less deformation capacity. Then hence restrained frames are generally more resistant 
but show limited ductility and capacity to activate catenary mechanism;
- Equivalent strut macro-modelling seems a possible simple solution to effectively sim-
ulate progressive collapse, although further generalization of the tested approach is 
necessary.

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NUMERICAL MODELLING OF INFILLED RC FRAMES: THE DETECTION OF COLUMN FAILURE DUE TO LOCAL SHEAR INTERACTION

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Abstract

Numerical and experimental studies highlighted that the presence of masonry infills in Reinforced Concrete (RC) frames certainly leads to the increase in their lateral strength and stiffness. Nevertheless, post-earthquake observed damage showed that infills can also cause potential brittle failures due to the local interaction with structural elements, thus producing a limitation of deformation capacity of the surrounding frame. This detrimental effect is particularly important for existing masonry-infilled RC buildings designed for gravity loads only without any capacity design requirements.

A proper numerical model both for infills and RC members is necessary to reliably detect the shear failure on columns due to the local interaction with the masonry infill. In the last years, several researches dealt with this topic, ranging from very simple models such as the equivalent strut model to more complex models like the double- and triple-strut models or FEM-approaches. Nevertheless, the choice of the proper modelling strategies and degrading shear strength model is still a frontier issue for the most recent research works, especially when the shear failure occurs in the degrading phase of the infilled frame response.

This paper presents a preliminary numerical investigation on column shear failure due to local interaction between structural and non-structural elements, starting from the results of two experimental tests on infilled frames performed by the Authors. Different shear strength models and different strategies of macro-modelling for infills are applied and discussed, in order to (i) to match the experimental response in terms of initial stiffness, peak strength and corresponding displacement, and softening behaviour and (ii) to capture (or not) the column shear failure exhibited (or not) during the test. The best modelling strategy is finally identified to provide a support towards future necessary numerical investigations.

**Keywords:** Masonry infills, Reinforced concrete frames, Local interaction, Column shear failure, Numerical simulations.
INTRODUCTION

Numerical and experimental studies highlighted that the presence of masonry infills in Reinforced Concrete (RC) frames leads to the increase in their lateral strength and stiffness. Nevertheless, post-earthquake observed damage (e.g. [1]-[3]) showed that infills can also cause potential brittle failures due to the local interaction with structural elements, thus producing a limitation of deformation capacity of the surrounding frame. This detrimental effect is particularly important for existing masonry-infilled RC buildings designed for gravity loads only without any capacity design requirements.

During last decades, a growing attention has been addressed to the behaviour of masonry infill walls in RC buildings under seismic action, also considering that damage to these elements largely affects repair costs of buildings and their loss of functionality after earthquakes [4]. Several experimental studies investigated the seismic behaviour of RC frames with infills. A significantly smaller number of experimental studies investigated the effects of the interaction between panel and surrounding elements resulting in brittle failure mechanisms such as shear failure in RC columns (e.g., [5]-[6]), especially for hollow clay bricks, very widespread in the Mediterranean region.

1.1 Columns-infill shear interaction modelling from codes and literature

Experimental data have been the support for analytical modelling efforts since late 1970s’ (e.g., [7]-[9]). Infills have been generally modelled by means of quite complex FEM micro-modelling approaches or simpler single- or multi-struts (reacting only in compression) approaches. Nevertheless, even recently, there is lack of unanimity about the best modelling approach among the various literature proposals (e.g., [10]-[12]), above all about the column-infill shear interaction modelling strategy.

More specifically, about the shear failure modelling in non-ductile RC frames, some standards propose simplified procedures aimed at taking into account the effects of interaction between panels and surrounding elements. These procedures usually consider a concentrated load on the column (or beam) equal to the horizontal (or vertical) component of the resultant of the stresses along the loaded diagonal of the panel. Among codes, some practice-oriented prescriptions are present in the American code ASCE/SEI 41-06 [13], which suggested to model the infill as a single eccentric strut in compression as shown in Figure 1a. On the contrary, the most recent Italian technical standards ([14]-[15]) do not provide any indication for the modelling and the assessment of the local interaction phenomena in the case of solid panels adjacent to column/beam elements.

About research studies from literature, the issue of shear failure modelling in non-ductile RC frames due to local interaction with infill elements has been investigated with different approaches (Figure 1) during the last years (e.g., [16]-[19], among others). Multiple-strut approaches are generally suggested in these studies, generally proposing to model the infill by a minimum of one-strut - eccentrically placed respect to the infill diagonal, like suggested in the ASCE-SEI/41[13] - to a maximum of two (Figure 1b,d) or three struts (Figure 1c,e,f), basically different for the position of the loading points on the adjacent columns, the ratio of the total infill lateral load adsorbed by each strut, and for the struts inclination. The number of struts, their positions, and their width have to be carefully selected to reliably reproduce the stress demand they induce and the strength and deformability contribution of the panel to the infilled RC frame. Currently, additional efforts seem to be still required to this aim.
Moreover, a key issue of the shear local interaction modelling is the adequacy and the applicability of the usually adopted shear capacity models in capturing the shear-controlled behaviour of a RC member adjacent to an infill panel. All the proposals from codes and literature mentioned above suggest to use the model by Sezen and Moehle [20], also adopted in the American code ASCE-SEI/41 (2017) [21], as explained in detail in Section 5 and shown in Figure 2. This model assumes a shear strength degradation due to inelastic ductility demand, as typical in “free” columns/beams. In the case of very “squat” portions of columns, as those generated by the limited infill-to-column contact length (as shown in Figure 1), a shear strength degradation due to an increasing flexural demand appears not totally meaningful. Therefore, the model by Sezen and Moehle [20] should be applied without any strength degradation (k=1 in Figure 2). Alternatively, a different “family” of shear strength models should be adopted, degrading with the shear crack opening demand or, from a predictive standpoint, with the strain demand, instead than with the flexural inelastic ductility demand.
Therefore, the topic certainly deserves a deeper investigation, both about the best infill modelling strategy and the proper shear strength model to be adopted.

1.2 Aims and objectives

This paper presents a preliminary numerical investigation on column shear failure due to local interaction between structural and non-structural elements, starting from the results of some experimental tests on infilled RC frames performed by the Authors. Two conforming and non-conforming infilled frames were tested, designed according to the current Italian seismic technical code (SLD), and according to an older Italian technical code in order to be representative of existing RC buildings constructed between 1970s and 1990s (GLD). Infill panels are made of hollow clay bricks, common in Mediterranean countries. Due to the differences in these two specimens, the GLD frame exhibited a column shear failure due to interaction with the infill; conversely, in SLD frame no shear failures were experimentally detected.

Different strategies of macro-modelling for infills and different shear strength models are applied and discussed, in order (i) to match the experimental response in terms of initial stiffness, peak strength and corresponding displacement, and softening behaviour and (ii) to capture (or not) the column shear failure exhibited (or not) during the test. The best modelling strategy is finally identified to provide a support towards further numerical investigations.

2 ANALYSED EXPERIMENTAL DATA

One-storey one-bay half-scaled RC frames shown in Figure 3 were tested and presented in Verderame et al. [22]. These specimens represent the subject of the modelling strategies presented in this paper, and their results are briefly recalled in this Section. The GLD specimen (hereinafter referred to as GI-80) was designed in order to be representative of the bottom storey of a five-storey gravity load designed RC frame, according to Italian technical codes in force between 1970s and 1990s ([23]-[24]). The SLD specimen (hereinafter referred to as SI-80) was designed according to the Italian seismic code [14], in compliance with all the capacity design requirements.

Mean value of 28-day cylindrical concrete strength was equal to 21.9 MPa. Deformed bars were used for longitudinal and transverse reinforcement in both the specimens. Commercial typology of reinforcing steel B450C [14] was used, characterized by a mean yielding strength equal to 507 MPa, 586 MPa, 490 MPa and 481 MPa, for bar diameters of 6 mm, 8 mm, 10 mm, and 12 mm, respectively. Hollow clay units with cement mortar were used as infill material. Dimensions of brick units were 250×250×80 mm³, with 66.3% void ratio, and the category of the mortar was M15, with a mean compressive strength of 14.03 MPa. Compression tests carried out on three-course masonry prisms, perpendicular and parallel to the holes, and
diagonal shear test carried out on a five-course masonry prisms, lead to the mechanical prop-
erties shown in Table 1.

<table>
<thead>
<tr>
<th>Dimension of the wallette</th>
<th>770×770×80 (mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength (∥ to holes), f_w-h</td>
<td>4.88 (MPa)</td>
</tr>
<tr>
<td>Compressive strength (⊥ to holes), f_w-v</td>
<td>3.19 (MPa)</td>
</tr>
</tbody>
</table>

**Table 1:** Mechanical properties of infill materials.

About the test setup, the foundation block of the specimen was anchored to the strong floor by means of vertical post-tensioned steel rods connected to stiff steel profiles. The lateral load is applied by means of a hydraulic actuator in displacement control. The actuator is fixed to a steel reaction wall anchored to the strong floor. The actuator was connected to the mid-span of the beam through steel profiles connected to steel rod passing through the transverse hole in the mid-span of the beam. The related loading protocol applied to infilled specimens consisted of 3-push-pull-cycles per each imposed drift levels, the latter assumed to be equal to 0.01%, 0.02%, 0.15%, 0.50%, 0.90%, 1.30%, 1.70%, 2.00%, 2.40%, 3.00%, and 3.6%.

The vertical load on columns is applied by hydraulic jacks in load control. The vertical load is kept constant during the tests and it corresponds to an axial load ratio equal to 10%. Further details about setup can be found in Verderame et al. [22].

Linear Variable Displacement Transducers (LVDTs) were used to measure cracks width and deformations at columns’ and beam’s ends. Wire potentiometers were placed along infill panel diagonals in infilled specimens GI-80 and SI-80.
2.1 Experimental results

The experimental results related to the infilled specimens, which represent the core of the following numerical simulations, are reported in brief in this Section. Note that the corresponding “bare” frames (without infills) were also tested by the Authors, and the main related results can be found in [22].

First, the lateral load-drift cyclic response of the SI-80 specimen is shown in Figure 4, together with the final damaged state of the specimen.

![Lateral load-drift response of infilled specimen SI-80](image)

Figure 4: Lateral load-drift response of infilled specimen SI-80 – adapted from [22].

Basically, a ductile, flexure-controlled post-elastic behaviour of the frame, without shear failures, was observed, thanks to the applied strength hierarchy prescriptions. The formation of a plastic mechanism involving beam’s ends and columns’ base was observed. A corner crushing failure of the infill, completed at the peak load of the response, was observed. The lateral response following the collapse of the infill panel (i.e., for drift higher than ±2.00%) perfectly matches the response of the corresponding bare specimen tested by the Authors (see [22]). This is consistent with the development of the same plastic mechanism in the RC frame, as described above.

Similarly to SI-80, the lateral load-drift cyclic response of the GI-80 infilled frame is shown in Figure 5, together with the most significant photos of the damaged specimen. For GI-80 specimen, diagonal cracking developed in the panel since very low drift values (i.e., between 0.15% and 0.50%). A drop in lateral force associated to the development of severe diagonal cracking at the top of the columns (with crack inclination quite close to 45°) was observed for an applied drift range between 0.50% and 1.30%. At 1.70% of drift, an abrupt increase in vertical displacement of the top of the columns also occurred, highlighting the potential for an imminent axial failure. The observation of the local behaviour showed the evidence of a shear failure due to the local interaction between RC columns and infill panel [22], as also marked on the cyclic response of the specimen in Figure 5.

3 ADOPTED NUMERICAL MODELLING APPROACH

Some preliminary calculations can been carried out for beams and columns of the analysed frames depending on their design typology. Table 2 reports the yielding moment and the flexural strength of beams and columns evaluated based on the simplified assumption of constant axial load, equal to the initial test value \(N_0\), i.e. \(N_{\text{col},0}=P=86.0 \text{ kN}\) in columns and \(N_{b,0}=0\) in beam. Axial load - bending moment interaction is not be taken into account in such a prelimi-
nary simple calculation. Moments for columns and beam at yielding and peak condition are referred to as $M_{c,y}$, $M_{b,y}$ and $M_{c,max}$, $M_{b,max}$, respectively.

For the following calculations, bending moments are reported at the intersection of the beam and columns centrelines and referred to as $M^*_{c,y}$, $M^*_{b,y}$ and $M^*_{c,max}$, $M^*_{b,max}$, respectively. Such values can be used for a preliminary classification of the failure mode of beams and columns, in a simplified approach, allowing the calculation of the plastic shear ($V_{pl}$) to be compared with the maximum-degraded shear strength ($V_{n,min}$) (evaluated according to ASCE-SEI/41 [21], with $K=0.7$ in Figure 2). The plastic shear of the column is equal to $V_{c,pl} = 2M_{c,max}/H_w$, and plastic shear of the beam is calculated as $V_{b,pl} = 2M_{b,max}/L_w$, where $H_w$ and $L_w$ are the infill panel height and length, respectively. Since, both for beams and columns in SLD and GLD frames, $V_{pl}$ results lower than $V_{n,min}$, the elements can be defined as “ductile elements”.

The expected yielding ($V_y$) and the maximum ($V_{max}$) lateral loads for the GLD and SLD frames are also reported in Table 2 and calculated by means of Eqs. (1) and (2), namely by assuming the simultaneous attainment of yielding and flexural strength, respectively, in members involved in the collapse mechanism.

Starting from these preliminary remarks, first the modelling strategy for the ductile behaviour of the frames is described here.

$$V_y = 2 \frac{M_{c,y}}{H} + \min(M_{b,y}^*, M_{c,y}^*)$$  \hspace{1cm} (1)
\[
V_{\text{max}} = 2 M_{c,\text{max}} + \min(M_{h,\text{max}}; M_{c,\text{max}}^*) \frac{V}{H}
\]  

\begin{table}[h]
\centering
\begin{tabular}{|l|l|c|c|c|c|c|c|c|c|}
\hline
Frame & Element & \(N_0\) (kN) & \(M_Y\) (kNm) & \(M_{\text{max}}\) (kNm) & \(M^{*}_{\text{max}}\) (kNm) & \(V_{\text{pt}}\) (kN) & \(V_{n,\text{min}}\) (kN) & \(V_Y\) (kN) & \(V_{\text{max}}\) (kN) \\ \hline
GLD & Column & 86.0 & 22.1 & 25.7 & 26.2 & 30.4 & 38.1 & 43.3 & 65.6 & 78.4 \\ & Beam & 0.0 & 24.2 & 24.9 & 26.5 & 27.3 & 23.7 & 52.9 & & & \\ SLD & Column & 86.0 & 33.4 & 40.2 & 38.0 & 45.7 & 59.6 & 90.6 & 81.0 & 99.9 \\ & Beam & 0.0 & 24.2 & 24.9 & 26.5 & 27.3 & 23.7 & 114 & & & \\
\hline
\end{tabular}
\caption{Yielding and maximum lateral load of GLD and SLD specimens.}
\end{table}

The infilled frames have been numerically reproduced by means of the numerical model shown in Figure 6. The considered loading direction is shown with a red arrow in these figures. However, if the opposite loading direction is considered, all the remarks reported in the following can be repeated in a specular way. Increasing horizontal displacements are imposed in the mid-point of the beam, as in the experimental setup, thus faithfully reproducing the variation of axial load acting in beam during the test. The fibre-type \textit{ForceBeamColumn element} has been adopted in \textit{OpenSees} [25] for each beam/column element. Mander et al. [26]’s concrete law, considering confining effect, if any, has been used (\textit{Concrete04 uniaxial material}). \textit{ReinforcingSteel uniaxial material} reproduces the experimental constitutive laws of the adopted reinforcing steel, characterized by the strength values reported in Section 2. Beams and columns have been considered as ductile members, as classified above. Beam-column joints are considered as supplied by infinite strength and stiffness.

The modelling approaches adopted for the infill panels, the core of this work, are investigated in detail in the next Section. Two different modelling approaches have been used to reproduce the response of the infill panel for specimens SI-80 and GI-80, based on the most common modelling strategies, namely, in a single- or three- (compressive only) strut approach. Compressive-only struts have been introduced in Figure 6 consisting with the considered loading direction.

![Figure 6: Numerical modelling approach adopted for infilled frames: single-(a) and three-(b) strut approaches.](image-url)
4 ADOPTED INFILL MODELLING STRATEGIES

First, the global infill response adopted for numerical simulations in terms of horizontal load ($V_w$) versus-horizontal displacement (D) (or, equivalently, Drift) reproduces the experimental response. The latter has been derived as the difference between the infilled frame and the corresponding bare one, thus assuming that the RC frame and the infill panel work as a parallel system. This procedure (see [12]) implies that the RC surrounding frame exhibit the same base shear-top displacement response in bare and infilled configurations. Such a hypothesis is not totally rigorous, but often suggested and adopted in literature (e.g. [12], [27]-[28]) to obtain the infill lateral response. This hypothesis will be “validated” by the numerical-versus-experimental comparisons themselves, as shown in Section 6. Force-drift infill response has been finally averaged between positive and negative loading directions for both the specimens (Figure 7a,b), by assuming that experimental results are approximately symmetric. The obtained averaged curve represents the experimental response of the infill panel adopted for the simulations.

![Figure 7: Average curve and multi (three)-linearized experimental response for infills in SI-80 (a) and GI-80 (b).](image)

The experimental responses obtained in such a way clearly show a high initial stiffness until first cracking occurrence and a subsequent stiffness degradation up to the peak load. After the achievement of the maximum lateral load, a degrading branch can be easily recognised. Therefore, some characteristic points have been recognised, namely: cracking point ($D_{cr}$, $V_{w,cr}$); peak load point ($D_{peak}$, $V_{w,peak}$); residual load point ($D_{res}$, $V_{w,res}$). Figure 7 shows the three-linearization of the experimental response for GI-80 and SI-80, based on the characteristic points mentioned above. The corresponding data are reported in Table 3 together with a comparison with the predictions by the well-known model proposed by Panagiotakos and Fardis (“P&F”) [29]. Such a model well predicts the cracking point, but slightly underestimates the peak strength (-15% on average) and significantly the corresponding displacement (-42% on average) for both tests (see Table 3). Figure 7 finally shows that the infill response is similar for the corresponding two tests; slight differences, at the peak point and in the softening branch, can be likely ascribable to the different failure mode exhibited by the infill panels.

<table>
<thead>
<tr>
<th></th>
<th>$V_{cr,h}$ (kN)</th>
<th>$D_{cr,h}$ (%)</th>
<th>$V_{max,h}$ (kN)</th>
<th>$D_{max,h}$ (%)</th>
<th>$a_{soft} = K_{soft}/K_{el}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GI-80 (exper.)</td>
<td>59.37</td>
<td>0.03</td>
<td>90.86</td>
<td>0.46</td>
<td>-1.56</td>
</tr>
<tr>
<td>SI-80 (exper.)</td>
<td>64.21</td>
<td>0.04</td>
<td>93.47</td>
<td>0.40</td>
<td>-3.43</td>
</tr>
<tr>
<td>P&amp;F [29]</td>
<td>60.48</td>
<td>0.03</td>
<td>78.62</td>
<td>0.25</td>
<td>-(0.5÷10)</td>
</tr>
</tbody>
</table>

Table 3: Characteristic points of infill responses for tests GI-80 and SI-80.
As anticipated before, the modelling of infills adopted herein is based on two strategies. The first one models the infill as a single compressive diagonal strut (as shown in Figure 6a). Such an approach is generally adopted to catch the global behaviour of infilled frames, being able to reproduce the contribution of the infill panel to the global strength and stiffness of the infilled frame. In this case, the horizontal response of the infill is simply projected along the diagonal direction to characterise the axial load-deformation behaviour of the compressive strut. Nevertheless, this model is not able to detect any eventual localised shear demand in the surrounding RC members, thus failing in the detection of eventual shear failures of the adjacent columns induced by the infill-column shear interaction.

Therefore, the second modelling approach reproduces the infill contribution by means of three compressive struts (as shown in Figure 6b) based on the proposals by Chrysostomou et al. [30] and Jeon et al. [19]. In this case, the global (V – drift) response obtained before is distributed among three compressive diagonal struts, as explained in the following. First, the contact length z between infill panel and column is computed by following the approach proposed by Stafford Smith [31]. In the critical section of column shear failure, assumed to be located at a distance equal to the column effective depth from the beam-column interface [19], the contact length z is divided into two portions: one in contact with the lower part of the central strut and the other in contact with the lower off-diagonal strut. The portion of the global lateral load absorbed by the central strut (γc) is determined based on the corresponding area of the bearing stress distribution with respect to the total area of the same bearing stress distribution, according to the proposal by Jeon et al. [19]. On the same bases, the end-point of the lower off-diagonal truss (zc in Figure 6b) is obtained. As a result, the central diagonal truss absorbs 35% of the global lateral load (γc = 0.35).

In addition, elastic stiffness, secant-to-peak stiffness and lateral loads of the off-diagonal struts have been reproduced according to the proposal by Chrysostomou et al. [30] depending on their eccentricity with respect to central diagonal strut, and, more specifically, depending on the parameter α = (zc /Hw)<1.

First, being K and ΔU the global stiffness of the infill panel and an assigned incremental displacement in horizontal direction, respectively, the axial stiffness of the central diagonal strut (kc) and related incremental displacement (Δuc) can be computed as in Eqs. (3) and (4):

\[
k_c = \frac{K}{\cos^2 \theta} \cdot \gamma_c
\]

\[
\Delta u_c = \Delta U \cdot \cos \theta
\]

Axial stiffness of the off-diagonal struts (koff) and related incremental displacement (Δuoff) can be obtained in the hypothesis of linear deformed shape of the columns, as suggested by Chrysostomou et al. [30] and shown in Eqs. (5) and (6):

\[
k_{off} = \frac{K - k_c \cdot \cos^2 \theta}{2(1 - \alpha)^2 \cdot \cos^2 \theta} = \frac{K(1 - \gamma_c)}{2(1 - \alpha)^2 \cdot \cos^2 \theta}
\]

\[
\Delta u_{off} = \Delta U \cdot \cos \theta \cdot (1 - \alpha)
\]
\[ \Delta p_c = k_c \cdot \Delta u_c = \left( \frac{K}{\cos^2 \theta} \cdot \gamma_c \right) \cdot (\Delta U \cdot \cos \theta) = \frac{K \cdot \Delta U}{\cos \theta} \cdot \gamma_c = \frac{\Delta V_w}{\cos \theta} \cdot \gamma_c \]  

(7)

\[ \Delta p_{off} = k_{off} \cdot \Delta u_{off} = \frac{K \cdot (1-\gamma_c)}{2(1-\alpha)^2 \cdot \cos^2 \theta} \cdot (\Delta U \cdot \cos \theta \cdot (1-\alpha)) = \frac{K \cdot (1-\gamma_c)}{2(1-\alpha) \cdot \cos \theta} \cdot \Delta U = \]

\[ = \frac{\Delta V_w \cdot (1-\gamma_c)}{\cos \theta} \cdot \frac{1}{2} \cdot \frac{1}{(1-\alpha)} \]  

(8)

In such a way, the global (horizontal) infill action \( \Delta V_w \) obtained by means of a three-strut model is equal to the infill lateral response obtained by means of a single concentric strut model, as expected, namely:

\[ \Delta V_w = \left[ k_c \cdot \Delta u_c + 2(k_{off} \cdot \Delta u_{off}) \cdot (1-\alpha) \right] \cdot \cos \theta = \]

\[ = \Delta V_w \cdot \gamma_c + 2 \left( \Delta H_w \cdot \frac{(1-\gamma_c)}{2(1-\alpha)} \right) \cdot (1-\alpha) = \Delta V_w \cdot (\gamma_c + (1-\gamma_c)) \]  

(9)

By assuming that \( \gamma_{off} = (1-\gamma_c)/2 \), it results: \( \gamma_c + 2 \gamma_{off} = 1 \).

For both the modelling approaches, the struts have been implemented in OpenSees by means of Truss elements defined with a Hysteretic uniaxial material and reacting only in compression.

5 COLUMNS SHEAR FAILURE DETECTION

The numerical response described until now does not account for a potential shear failure of the columns, particularly important for RC frames interested by a local shear interaction with the infill panel.

Shear failures in columns can be first detected starting from a post-processing of the numerical output, but a proper shear strength model should be selected, suitable for squat columns, as those generated by the column-to-infill interaction (see Figure 6b). Therefore, two models have been adopted herein as explained in the following.

First, shear strength is evaluated according to ASCE/SEI 41 [21] (see Eq. (10)), as proposed by Sezen and Moehle [20] (“S&M”) and very commonly adopted in previous studies from literature for similar applications (e.g. [17]-[19], [32]):

\[ V_n = k \left( \frac{0.50 \sqrt{f_c}}{a/d} \cdot \sqrt{1 + \frac{N}{0.50 \sqrt{f_c} A_g}} \cdot (0.80 A_g) + \frac{A_{sw} f_{yw}}{s} \right) \]  

(10)

In Eq. (10), \( a/d \) is the shear span-to-column effective depth ratio, \( A_g \) the gross area of the column, \( A_{sw}, f_{yw} \), and \( s \) are the transverse reinforcement area, yielding strength and stirrups spacing, respectively. \( N \) is the column axial load at the interface section with the joint and changes for increasing applied horizontal displacement (reaching a minimum value in the left column and a maximum value in the right column, given the assumed loading direction). Actually, ASCE/SEI 41 [21] suggests to conservatively assume \( N=0 \) in Eq. (10) to assess the shear capacity in columns subjected to the local shear interaction with the surrounding infills.
Such an assumption can result too much conservative for the numerical-versus-experimental comparison. Therefore, the actual axial load acting on the column (and varying step-by-step) is assumed herein in Eq. (10). Factor $k$ takes into account the strength degradation due to inelastic ductility demand. Since the lack of meaning of a strength degradation assessment due to an increasing flexural demand in such a squat columns, the strength model by ASCE/SEI-41 [21] is applied herein always with $k=1$. The shear span $a$ in Eq. (10) has been assumed as equal to the distance between the null point of the moment diagram in the squat portion of column and the beam-column interface (variable during the analysis), limiting the $a$-to-$d$ ratio between 2 and 4, as prescribed by Sezen and Moehle [20] and in ASCE/SEI-41 [21].

Additionally, as anticipated in Section 1.1, a different “family” of shear models could be adopted for the squat columns of the infilled frames. A model providing a strength value that degrades with the crack opening demand, namely, on the longitudinal strain demand ($\varepsilon_x$) (according to Eq.s (12) and (13)).

$$\varepsilon_x = \frac{1}{2A_sE_s} \left( \frac{M}{d^2} + V - 0.5N \right)$$

(14)

where $A_s$ and $E_s$ are tensile longitudinal steel area and Young modulus of steel, respectively; $V$, $M$ and $N$ represent the shear, moment and axial compression demand, respectively, acting on the (squat) column at the “critical section” (see Figure 8), and $d^*$ is assumed equal to 0.9 times the effective depth of the column. Therefore, looking at Figure 9, a shear failure is detected if

$$\beta = \frac{0.4}{1+1500\cdot\varepsilon_x} \cdot \frac{1300}{1000+s_{xe}}$$

(12)

$$\theta = \left( 29^\circ + 7000\varepsilon_x \right) \cdot \left( 0.88 + \frac{s_{xe}}{2500} \right) \leq 75^\circ$$

(13)

The parameter $s_{xe}$ in above equations – representing the expected horizontal distance among shear cracks – is assumed equal to 300 mm, according to Bentz et al. [34]. The longitudinal strain at the mid-depth of the web ($\varepsilon_x$) is evaluated as equal to one-half of the strain in the longitudinal tensile reinforcing steel [34] in a “critical section” located at a distance $d$ from the null point of the bending moment (see Figure 8), also taking into account the translation of the bending moment diagram, as suggested by Model Code [35], namely:
the column shear demand-drift curve (black curve) intersects the column shear capacity-drift curve (green curve), for a shear value hereinafter referred to as $V_{\text{col, SF}}$.

![Figure 8: Definition of the critical section for shear safety check.](image)

$V_{\text{col, SF}} = \beta \sqrt{f_c} (b_d \cdot d) + \frac{A_{sw} f_{yw}}{s} \cdot 0.9d \cdot \cot \theta$

![Figure 9: Shear failure detection.](image)

The shear failure could be also directly detected by properly adding to the numerical model an explicit shear spring in the squat column, as shown in Figure 10 (similarly to Jeon et al. [19], among others). This spring could be defined by means of a sort of “limit state material” (as defined in [32]), which monitors the strain demand in the critical section of the column (according to Bentz et al. [34]) instead of the inter-story drift demand (as originally suggested by Elwood [32]) during the analysis. Such a monitoring possibility is currently not implemented in OpenSees; therefore, herein, the strength of this spring is defined as the shear demand for which the shear failure is achieved ($V_{\text{col, SF}}$).

The shear spring introduced in the model is characterized by an infinite elastic stiffness up to $V_{\text{col, SF}}$, and by a softening constant stiffness up to a residual strength ($V_{c, \text{res}}$), if any (see Figure 11). The softening stiffness adopted is defined according to Elwood and Moehle [36] proposal (referred to as “E&M” in the following). On a mechanical base, Elwood and Moehle [36] proposed to estimate the displacement $\delta_{a, \text{E&M}}$ at which column begins to lose its axial load carrying capacity after the occurrence of a shear failure as in Eq. (15). Such a displacement coincides with the total loss of lateral load carrying capacity ($V_{c, \text{res}}=0$).

$$\frac{\delta_{a, \text{E&M}}}{L} = \frac{4}{100} \cdot \frac{1 + (\tan \theta)^2}{\tan \theta + \frac{N \cdot S}{A_{sw} \cdot f_{yw} \cdot d \cdot \tan \theta}}$$  \hspace{1cm} (15)
In Eq. (15), $L$ is imposed equal to $z_c$, defining the displacement $\delta_{a,E&M}$ as the relative displacement between the two end points of the squat column. Additionally, the column axial load $N$ is assumed equal to its initial value ($N_0$), in the hypothesis that, when the column axial load failure occurs, the infill does not provide anymore a strength contribution, so that no axial load variation is induced in the adjacent column with respect to $N_0$.

Additionally, the shear critical angle $\theta$ in Eq. (15) is assumed by Elwood and Moehle [36] equal to 65°. Nevertheless, the values of $\theta$ should be coherent with Eq. (13). Therefore, Eq. (15) is implemented by assuming both the values of $\theta$: the one suggested by Elwood and Moehle (2005) and $\theta_{SMCFT}$.

In both cases, the shear spring has been introduced in the numerical model in OpenSees as shown in in Figure 10, namely by means of a ZeroLength Element (between two nodes, in red, geometrically coincident) defined with a Hysteretic Uniaxial Material.

$$V_{Rd} = \sqrt{\frac{b_d}{A_{bw}}} (b_w \cdot d) + \frac{A_{dy} f_{yw}}{s} \cdot 0.9d \cdot \cot \theta$$

$$\delta_{a,E&M} = \frac{4}{100} \cdot \frac{1 + (\tan \theta)^2}{\tan \theta + \frac{N \cdot s}{A_{bw} \cdot f_{yw} \cdot d \cdot \tan \theta}}$$

Figure 11: Shear spring backbone.

### 6 NUMERICAL VERSUS EXPERIMENTAL COMPARISONS

The results of the described modelling approach shown in Figure 6, with a single- (dotted red line) or a three-strut (solid red line) infill model, is shown in Figure 12a and Figure 12b for test SI-80 and GI-80, respectively. Note that $V_b$ in Figure 12 is the base shear, which coincides with the applied lateral load.
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In both numerical simulations for SLD infilled frame (test SI-80), the resulting numerical-versus-experimental comparison shows a very good matching in terms of maximum load and also initial stiffness and frame deformability in the first loading steps. Note that the missing of the fixed-end-rotation and of the joint panel deformability contribution in the numerical modelling does not affect this comparison, because the weight of the infill strut lateral stiffness is clearly predominant over the numerically missing deformability contributions. The maximum lateral load finally achieved during the simulation is equal to 159.0 kN, very close to the corresponding experimental value (158.6 kN). Moreover, at about 2.00% drift, lateral load drops up to the expected flexural strength ($V_{\text{max}}$) of the bare frame only (calculated as in Eq. 2), in tune with the experimental evidence. Such an outcome can be useful to validate the assumptions that allowed the experimental definition of the infill panel response, as explained in Section 4. Additionally, the agreement between the results of the modelling approach based on a single concentric strut and a tri-strut modelling approach (shown in Figure 12a) confirms the validity of the proposals by Chrysostomou et al. [30] for the off-diagonal struts. It can be concluded that the linear deformed shape of columns, which is the base of this proposal, results realistic for this struts configuration.

Regarding test GI-80 (see Figure 12b), the agreement between modelling results and experimental findings appears very good in terms of maximum load and also initial stiffness and frame deformability, especially in the ascending branch of the lateral behaviour up to shear cracks in the column start to significantly increase their width (namely until the penultimate cycle of the response). The maximum lateral load achieved during the simulation is equal to 140.9 kN, that is very close to the corresponding experimental value (140.1 kN). Similarly to SI-80 test, the agreement between the results of the modelling approach based on a single concentric strut and a tri-strut modelling approach confirms the validity of the proposal by Chrysostomou et al. [30] and the assumption of the linear deformed shape of the columns that is the base of this proposal.

As stated before, the modelling approach shown in Figure 6, and resulting in Figure 12, is not able to detect any eventual shear failure. Therefore, first this eventuality is checked by means of a post-processing of the analysis results (as explained in Section 5). In particular, the attention will be focused on the left (L) side column, where axial load decreases during the simulation thus producing a reduction in shear strength, and which clearly reached the cracking condition during the test GI-80 (where local shear interaction exhibited the most signifi-
cant experimental evidence). Both shear strength models, by ASCE/SEI 41 [21] and by Bentz et al. [34] (SMCFT) are applied herein for both the infilled specimens (see Figure 13). As a results, no shear failure has been detected by ASCE/SEI 41 [21]’s strength model (see “S&M” curve in Figure 13a). Note that $V_{Rd}$ by “S&M” model is reducing for increasing drift levels due to the action of the lower off-diagonal strut that produces a progressive reduction of the column axial load. On the contrary, for test GI-80, Bentz et al. [34]’s proposal leads to a column shear failure at about 1.0% of applied lateral drift (red star in Figure 13a), namely on the descending branch of the global lateral response (red curve), as experimentally observed. $V_{col,SF}$ is equal to 52.4 kN in this case.

Note that Figure 13 also shows axial load (N) and shear demand (V) variation during the simulation (in grey and black solid line, respectively), together with the global ($V_{base}$-drift) response (red line). It is also noteworthy that, as expected, the same safety checks for SI-80 specimen do not provide any column shear failure, since shear strength capacity evaluated by both SMCFT theory and “S&M” proposal is higher than the maximum column shear demand (see Figure 13b).

In a second step of modelling, for the specimen GI-80, where the shear failure is detected, an explicit shear spring has been added to the model, as shown in Figure 10 and explained in Section 5. Such a spring is linear elastic up to $V_{col, SF}$, and then characterised by a softening constant stiffness up to a null residual strength ($V_{c, res} = 0$). The softening stiffness adopted is defined according to Elwood and Moehle [36] proposal, namely by means of Eq. (15), where the shear critical angle $\theta$ should be first defined. If $\theta$ is evaluated by Eq. (13) at the onset of the shear failure ($\theta_{SMCFT}$), the value of 41.2° is obtained. Otherwise, according to Elwood and Moehle [36] proposal, $\theta$ is equal to 65°. Both these hypotheses on $\theta$ (65° and 41.2°) have been analysed. The corresponding values of $\delta_{a,E&M}$ are equal to 30 mm and 8.2 mm, respectively. Note that $\theta_{SMCFT}$ value is closer to the experimental main shear crack angle shown in Section 2. Therefore, a better prediction of the numerical response is expected when $\delta_{a,E&M}$ is evaluated with $\theta = \theta_{SMCFT}$, as shown immediately below.

The results of these simulations for the specimen GI-80 are shown in Figure 14a and the related responses of the shear spring are reported in Figure 14b. It can be observed that shear failure – meant as the initiation of a shear-controlled softening response – begins at a drift value roughly equal to 1% (see Figure 14a). The global softening branch changes depending on the assumption adopted for $\theta$. The numerical response better reproduces the experimental response if the $\theta_{SMCFT}$ value is adopted (red solid lines in Figure 14a).
In conclusion, Figure 15 shows the axial load ($N_w$)-axial deformation ($\varepsilon_w$) response of the three compressive struts without (Figure 15a) and with (Figure 15b) the explicit shear spring in the numerical model. At the end of the test all the struts are still contributing to the lateral response of the frame ($N_w>0$) if no shear spring is modelled (Figure 15a). On the contrary, the lower off-diagonal strut starts unloading at the onset of the shear failure in the adjacent column, as expected, when the shear spring is explicitly modelled; after the column shear failure, the remaining struts still contribute to the lateral response since the axial displacement demand can still increase in these two (Diagonal and Upper Off-Diagonal) struts (Figure 15b).

7 CONCLUSIONS

A preliminary numerical investigation on column shear failure due to local interaction between structural and non-structural elements has been presented, starting from the results of some experimental tests on infilled frames performed by the Authors. Different shear strength models and different strategies of macro-modelling for infills have been applied and discussed, in order to (i) capture (or not) the column shear failure exhibited (or not) during the test and
(ii) to match the experimental response in terms of initial stiffness, peak strength and corresponding displacement, and softening behaviour.

The analysed experimental data have been briefly described and numerically reproduced by means of proper models carefully taking into account the local shear interaction between infill panel and RC columns observed during one of the analysed tests. It resulted that the shear demand on the surrounding columns due to the interaction with the infill panels could be well caught by means of a three-compressive struts modelling approach, properly identifying the location and the contribution of each strut according to the proposals by Jeon et al. [19] and Chrysostomou et al. [30]. On the other side, the column shear strength could be well estimated based on the modified compression field theory [34], more suitable model for the squat columns produced by this interaction with respect to shear strength models degrading due to the increasing cyclic ductility demand.

The numerical results shown in the paper appear to be very close to the experimental ones. Nevertheless, future further efforts will be devoted to the extension of this modelling strategy to other similar specimens experimentally tested in the literature, to finally provide a wider validation of the adopted numerical models.

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ESTIMATION OF BASIC DYNAMIC CHARACTERISTICS OF PLIABLE MASONRY INFILLS WITH HORIZONTAL SLIDING JOINTS FROM IN-PLANE TEST RESULTS

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Abstract

Many post seismic event inspections and studies conducted by different researchers have identified the threat to human lives and considerable economic losses related to the damage sustained by “traditional” masonry infills. Although several innovative infill solutions and new design approaches have been proposed, a viable practical and universally accepted solution has not been achieved yet. Among the very promising designs, according to the ongoing studies, are pliable infill panels with the in-plane stiffness reduced and the displacement ductility increased compared to “traditional” infills. One way to attain such a significant pliability of a masonry infill is to divide it into a number of segments interconnected through horizontal sliding joints. The research units at the University of Newcastle (Australia) and the University of Pavia have proposed and are studying two different infill systems of this type: one mortar-less made of specially shaped masonry units capable of sliding on all bed joints; another with infill masonry panel subdivided into several horizontal sub-panels using specially engineered sliding joints. Within this paper, two systems are compared and discussed with particular emphases on their basic dynamic characteristics required for seismic analysis.

Keywords: Innovative masonry infill, semi-interlocking masonry, sliding joint, in-plane seismic response, macro-modeling.
1 INTRODUCTION

Traditional masonry infill solutions, where the panels are built in complete contact with the surrounding RC frame without provision of any gap or connection around the boundaries and after the hardening of the RC members, have evinced a series of critical aspects related to in-plane and out-of-plane seismic response. These issues have been commonly observed both in post-seismic surveys, e.g. in L’Aquila 2009 (Braga et al. [1]), Emilia 2012 (Manzini et al. [2]) and Central Italy 2016/2017 (Fragomeli et al. [3]), with in-plane failures and out-of-plane collapses/expulsions of single leaf infills and partitions and/or external veneers in double leaf panels as well as from experimental outcomes. Reduced out-of-plane resistance due to uncontrolled levels of in-plane damage was detected for weak/slender infill panels (e.g., Calvi and Bolognini [4]), while local detrimental effects on RC members due to the thrust of the adjacent infill are known to be one of the most critical issue in the case of strong/thick masonry infills (e.g., Paulay and Priestley [5], Morandi et al. [6]). Most of these unfavourable effects can be associated to many factors, not only due to the intrinsic vulnerability of unreinforced masonry infills or the use of bad quality material and construction details, but also due to insufficient and unclear information in currently existing building codes for the seismic design of infilled frames (e.g., Fardis [7]).

To exceed the limitation of “traditional” infill, many innovative infill solutions have been proposed in the past years.

The systems that subdivide the infill through sliding or deformable joints have been studied separately by the University of Pavia and the University of Newcastle. These solutions, that provide two different pliable infills, have shown a promising experimental seismic response, and their lateral behaviour influence the global behaviour of the structure differently as respect to traditional infills.

Within the present paper, it is shown that a classical nonlinear single-strut macro-model can be calibrated to experimental results although, in some cases, the nature of the innovative pliable infill may require a strut model with some damping mechanism or tensile resistance. Moreover, the calibrated strut models allow to study the influence of the innovative infills in RC buildings and to evaluate their overall structural improvement in comparison with a “traditional” masonry infill.

2 LITERATURE REVIEW OF INNOVATIVE INFILL SOLUTIONS

The considerable economic and downtime losses and the serious threat to human lives have recently encouraged the research towards new systems for masonry infills.

Some of the objectives of innovative solutions aims at reducing the in-plane/out-of-plane seismic vulnerability of infills guarantying at the same time a sufficient thermal, acoustic and durability performance. A first category of such solutions is represented by systems where the infills enhance the in-plane/out-of-plane resistance (Figure 1a) through the inclusion, for example, of vertical and/or horizontal reinforcement (steel bars or light trusses) in the masonry panel, steel wire meshes (Calvi and Bolognini, [4]), or other types of fibre and/or cementitious materials, as CFRP (“Carbon Fibre Reinforced Polymer”) (Yuksel et al. [8]) or FRCM (“Fiber Reinforced Cementitious Matrix”) systems (Koutas et al. [9]). These interventions undoubtedly allow an increase of the in-plane and out-of-plane resistance, without however limiting the possible detrimental infill-frame interaction effects. A second category of modern solutions found in literature (e.g., FEMA [10], Tsantilis and Triantafillou [11]) aims at uncoupling the infills from the structure by using flexible joints around the wall-frame interface (see examples in Figure 1b), supplemented by suitable out-of-plane restraints, to ensure the out-of-plane stability of the panel. These systems reduce the negative infill-frame interaction and the in-plane damage
of the masonry. However, such solutions still remain to be experimentally validated and present several technological and design related complications for their practical implementation. Such complications include suitable joint dimensions to be used to prevent negative interaction between the infill and frame while still guaranteeing out-of-plane stability and allowing in-plane differential movement between the frame and the infill. Finally, a third group of innovative systems consists of reducing the infill-frame interaction through solutions that make use of “sliding” or “weak plane” joints with the aim of concentrating the in-plane deformation and damage in selected points.

With reference to the principles of this last typology, the masonry section research group of the University of Pavia, involved in the European FP7 research project “INSYSME” [12], and of the University of Newcastle (Australia) have developed and implemented two different innovative pliable masonry infill systems: one mortar-less made of specially shaped masonry units capable of sliding on all bed-joints (Lin et al. [13]); another with an infill masonry panel subdivided into several horizontal sub-panels using specially engineered sliding joints (Morandi et al. [14]). Other different solutions with “weak plane” joints have also been recently developed within the “INSYSME” project by other researchers (e.g., Verlato et al. [15] and Vintzileou et al. [16]) or in other studies (e.g., Mohammadi et al.[17], and Preti et al. [18], [19]).

Although the best option to analyse these latter typologies of infills is with the use of micro-models (e.g., Bolis et al. [20]), able to investigate the detailed response of the system and the local interactions with the structure, simplified macro-models that assume one single-strut along each diagonal, pin-ended at the RC member centreline intersections can also be adopted, as done in the past for traditional rigidly attached infills (e.g., Hak et al. [21], Di Trapani et al. [22]). Although such models are not able to capture possible local effects on RC members due to the interaction between structural frame elements and the masonry infill, since the focus here is on infills, and their in-plane seismic response is essentially governed by inter-story drift and “overall” frame behaviour, the single strut model is considered to be adequate for global structural analyses of infilled buildings with innovative systems, also for its low computational burden.

Within the present work, the calibration of a non-linear single-strut macro-element with lumped plasticity is presented as an efficient model technique to represent in-plane seismic response of the pliable masonry infills with horizontal sliding joints.
3 DESCRIPTION OF THE Pliable Masonry INFILLS

Within this section, a brief description of the deformable masonry infill solutions with sliding joints is presented. Further details are reported in Morandi et al. [14] for the system of the University of Pavia, and by Lin et al. [23] for the solution of the University of Newcastle.

3.1 Infill with sliding joints (University of Pavia)

Within the European FP7 Project “INSYSME”, the research unit of the University of Pavia has developed a seismic resistant masonry infill system with sliding-joints (Morandi et al. [14]) with original details on which a very wide experimental campaign has been conducted. The proposed engineered system aims to control the damage propagation in the masonry infill and to reduce the in-plane interaction between the RC frame and the panel, dividing the infill into four horizontal subpanels, able to slide one on each other through properly conformed sliding joints (corrugated male-female plastic elements, Figure 2c). Moreover, a deformable joint located at the frame/infill interfaces has the objective to reduce the local effects and the stress concentration in the proximity of the interface between the masonry panel and the RC elements. The unreinforced masonry used in the subpanels of the infill is realized with vertically perforated lightweight clay units (see Figure 2d) and general-purpose 1 cm thick mortar bed-and-head joints. The layout of the system is reported in Figure 2a, whereas the working principles and the construction details of the proposed system are presented in detail in Morandi et al. [14].

An extensive experimental campaign, composed by tests of characterization, in-plane cyclic tests on one-storey one-bay full scale RC bare frame and two different configurations of infilled frames (with and without a central opening) followed by out-of-plane shaking-table dynamic tests, and a dynamic test on a shaking table of a full scale two-storey building, has been performed in order to investigate the seismic performance of the proposed system. The combined use of deformable joints at the infill-frame interface and sliding horizontal joints within the infill has proved to be very efficient in terms of in-plane performance with a limitation of the damage in the masonry and of the detrimental infill-structure interaction, producing at the same time significant energy dissipation, as shown in the results of the in-plane cyclic tests on the infilled frames [14].

Moreover, the out-of-plane experimental response has been investigated through dynamic tests on shaking table on the same two infilled frames previously tested in-plane. The out-of-plane stability of the system is governed by the horizontal flexural/arching resistance of the masonry stripes and is guaranteed by suitable designed “shear keys” attached to the column, constituted by “omega” shaped steel profiles connected to the RC columns by means of nails shot with a nail gun (Figure 2e); moreover, the units at the edges of the infill adjacent to the
columns and to the openings are shaped with a recess (C-shaped units, Figure 2b) in order to accommodate the shear keys. The sliding joints, having a ribbed shape, allow obtaining a mechanical interlocking that, together with the plaster placed on both sides of the masonry, should ensure the out-of-plane stability of the panel.

Figure 2: (a) Details of the innovative masonry infill with sliding joints: 1. C-shape units (b); 2. mortar bed-joints; 3. sliding joints (c); 4. clay units (d); 5. interface joints; 6. shear keys (e); 7. plaster.

3.2 Semi-Interlocking Masonry (University of Newcastle)

To improve the seismic behaviour of framed masonry structures, a conceptually new type of framed masonry panel built of mortarless semi-interlocking masonry units, called SIM system, has been developed at the University of Newcastle. The major objective for developing this new masonry system was to improve the earthquake performance of framed structures by increasing the displacement ductility and the energy dissipation capacity of infill panels. In this system, all bed joints are sliding joints and masonry units slide along bed joints and courses while their interlocking features prevent relative out-of-plane movement. Consequently, stiffness and susceptibility to damage of the SIM panel are reduced and the capacity to dissipate earthquake energy increased. The SIM panels are considered as “non-gravity-load-bearing” structural elements and possess better capacity to resist the lateral loads compared with traditional masonry panels Totoev [24].

Two types of SIM units have been developed: with topological and mechanical semi-interlocking as shown in Figure 3b and 3d. Topological SIM units use the shape of the unit’s bed joint surfaces to allow the in-plane movement between units, while the mechanical SIM uses a series of slots and sockets for the same purpose (Lin et al., [13]). Forghani et al. [25] investigated the water penetration characteristics and thermal insulation properties of both mechanical and topological SIM panels and identified two types of joint fillers between SIM units, namely linseed oil-based putty and rubber foam tape (1.6mm thick and 12mm wide) which are suitable for the purpose. Hossain et al. [26] researched the effect of these joint fillers on the coefficient of friction of SIM units. They concluded in their research that the joint filler with putty possesses a higher coefficient of friction compared to the other tested surfaces.

A larger research project using mechanical and topological SIM units in SIM panels has been conducted recently to obtain a better understanding on the cyclic behaviour of SIM panels using quasi-static tests. SIM panels were constructed using the putty as a joint filler between the SIM units. Two types of gap filler were used in-between the steel frame and the top of the panel (self-expandable foam and cement grout), and the influence of the gap filler on the cyclic behaviour of the SIM panel was considered.
4 IN-PLANE EXPERIMENTAL RESULTS AND MASONRY INFILL SEISMIC PERFORMANCE

A brief description of the in-plane experimental results of the two deformable masonry infill solutions with sliding joints is presented in this section. Further details are reported by Morandi et al. [14] for the system of the University of Pavia, and by Hossain et al. [27] for the solution of the University of Newcastle. Information on the specimens and on the testing set-ups are briefly reported also.

4.1 Infill with sliding joints (University of Pavia)

In-plane pseudo-static cyclic tests have been carried out on fully and partially infilled (with a central opening) large-scale single-storey single-bay RC frames, designed according to the European and the Italian seismic code provisions. For the solid infill, besides the “low velocity”, also a “high velocity” test has been performed in order to investigate the response of the sliding joints subjected to dynamic actions, in particular if high temperatures and degradation of the mechanical performance of the plastic joints could occur due to loading velocity. In the present work, only the fully infilled specimen has been considered.

The dimensions of the single-storey single-bay RC frame specimen to be tested were chosen with the aim of realistically representing a part of a full-scale RC frame structure. A clear span of 4.22 m and a clear height of 2.95 m have been adopted. The design of the RC frame specimen was carried out following the European code provisions (EC 8-Part 1 [28] and the Italian national code NTC 2008 [29]), and all the reinforcement details are extensively described in Morandi et al. [6]. The fully infilled specimen, called TSJ1, is shown in Figure 3a.

Figure 3: (a) Details of the SIM infill: 1. SIM units: topological (b) or mechanical (d); 2. window frame (c) made of steel channel or angles; 3. perimeter out-of-plane restrain (c) made of steel angles attached to the frame; 4. sliding joints (e) with linseed oil based putty.
The in-plane cyclic tests have been carried out at the laboratory of EUCENTRE and of the Department of Civil Engineering and Architecture of the University of Pavia. The layout of the in-plane experimental setup is illustrated in Figure 4, further details on the cyclic pseudo-static in-plane testing procedures are reported by Morandi et al. [14].

For the in-plane tests, after the application of a vertical load of 400 kN per column, reverse cycles of horizontal in-plane loading (first pull, then push) have been imposed on the frame. In the case of pseudo-static tests, displacement-controlled loading cycles at increasing levels of in-plane drift were imposed up to 3.00% drift. For each level of loading, three complete reverse loading cycles have been carried out and the duration of load application has been kept approximately constant. Figure 3b reports the in-plane “low-velocity” testing protocol.

In order to measure the displacements and deformations of the specimens during the in-plane tests, displacement transducers (linear potentiometers) were adopted. In total, 45 potentiometers have been used for TSJ1. An optical acquisition system was installed to measure the in-plane displacements of optical markers on the RC frame and the masonry infills.

The results of the cyclic in-plane tests on the fully infilled frame TSJ1 (“low-” and “high-velocity”) are shown in Figure 5a, in terms of force-displacement hysteretic curve and corresponding envelopes for each cycle. The TNT specimen, which has been tested during a previous campaign (Morandi et al. [6]), has been taken as reference for a proper evaluation of the infill
contribution, since it has the same characteristics (overall dimensions, RC sections, details/amount of reinforcement and nominal concrete and steel strength values) of the RC frames of the infilled specimens. The test on the bare frame was carried out up to a drift of 3.50% with a testing protocol similar to the one described above; the F-D curve with the envelopes is reported in Figure 5b.

In the case of the fully infilled specimen (TSJ1) subjected to the “low-velocity” test, sliding in the three horizontal joints was observed from the first applied target drifts. Subsequently, starting from the first peak in the F-D curve (0.20% drift), no further damage was produced in the specimen up to a drift of 0.60%, when light horizontal cracks at the column-joint interface appeared. During cycles higher imposed drift, minor cracks occurred in the upper and bottom part of the RC columns. At a drift of 2.00%, some small areas of plaster located at the corners of the intermediate masonry subpanels fell down and flexural cracks formed at the beam ends. Finally, spalling of the concrete cover at the bottom of the columns was observed at 3.00% drift. The cracking pattern of the panel was predominantly concentrated in the plaster in proximity of the sliding and of the interface joints, whereas any damage in the remaining part of the infill did not occur, with the exception of some minor cracks in the units at the corners of the bottom three masonry subpanels. Such damages are however very easily repairable, and they appeared at very high levels of in-plane displacement demand (drift larger than 2.50%). The damage pattern at final stage is shown in Figure 6.

The activation of the sliding mechanism in each of the sliding joints was monitored through the measurement of the relative horizontal displacement between adjacent masonry subpanels coming from the processing of the potentiometers. Further details are reported by Morandi et al. [14].
Among different methods available to evaluate the dissipated hysteretic energy of tested infilled frames (e.g., Negro and Verzelletti [30]), a simplified criterion consists in the determination of the equivalent viscous damping $\xi_{eq}$. Given a single load-displacement cycle, $\xi_{eq}$ can be expressed as a function of the dissipated energy $W_d$ (area enclosed by each hysteretic loop) and the elastic energy at peak displacement $W_e$ (amount of elastic energy stored in the same loop), following the expression (in which signs + and – indicate the positive and the negative elastic branch, respectively):

$$\xi_{eq} = \frac{W_d}{2\pi \left[ W_e^+ + W_e^- \right]}$$

(1)

In accordance with Eq. (1), the equivalent viscous damping ratio evolution of the tested specimens has been evaluated and drawn against the imposed drifts in the tests estimated from the load-displacement loops considering the first, the second and the third cycle at each target displacement. In Figure 7 the curves of the equivalent viscous damping for TNT and TSJ1 (both for “low-” and “high-velocity” test) are reported for the first cycles. In the case of the RC bare frame, the damping begins to substantially increase from about 3% up to 10% starting from drifts larger than 1.50%. For the infilled specimen, the damping decreases from about 12% to about 7% up to 1.50% drift; after, the damping starts increasing up to about 11%. The “high-velocity” test on the fully infilled specimen (TSJ1_IPH) exhibited damping values similar to those of the “low-velocity” test (TSJ1_IPL). Finally, an estimation of net dissipation capacity of the two infills in terms of equivalent viscous damping applying Eq. (1) was also obtained, computing the area enclosed in the hysteretic cycles of the infilled frames after deduction of the area of the bare frame hysteretic curves at corresponding imposed drifts. The results, reported in Figure 7 in dashed lines together with those of the infilled frame, provide, after a drift of 0.60%, values of damping ranging between about 15% and 20% without a clear trend, with and average value $\xi_{eq,av.,drift_0.60-3.00\%} = 17.6\%$ that is much larger than that of the bare frame.

![Figure 7: Equivalent viscous damping of TNT, TSJ1 (for “low” and “high” velocity tests), and TSJ1-TNT at 1st cycles.](image)

In conclusion, the infill seismic performance has also been evaluated as specified in Morandi et al. [6], in terms of drift attained to a correspondent level of damage derived from in-plane cyclic tests on infills. An operational limit state (OLS), a damage limit state (DLS) and a life safety limit state (LSS or ULS) are defined specifically for infill performance. At OLS the infill is considered undamaged or slightly damaged, at DLS the infill is damaged, but can be effectively and economically repaired, whereas at LSS/ULS the infill is considered severely damaged and reparability is economically questionable, but lives are not threatened. For the case of the infill solution proposed by the University of Pavia, the following limits have been evaluated: OLS at 0.5% drift and DLS at 3.0% drift; the LSS (or ULS) has not been reached during the test, since at 3.0% the infill panel was still far from an ultimate condition.
4.2 Semi-Interlocking Masonry (University of Newcastle)

The testing of the panels was done with a special frame attached to the strong floor schematically depicted in Figure 8. The steel frame used in this research is made of Australian standard 310UC137 sections and T-sections. The lateral hydraulic jack cylinder body was mounted on the strong wall and the piston attached to the fixing point on the frame attachment plate using a single pivot pin. Four pin connections were introduced at the corners of the frame, as shown in Figure 8. The pin connected frame act as a mechanism which allows the applied force to be transferred completely to the masonry panel without significant resistance due to the frame stiffness. This system allows the masonry panel to be subjected to the in-plane shear displacement of up to 120 mm.

Instrumentation for the cyclic load testing included ten linear variable differential transformers (LVDTs) and four electrical strain gauges which were mounted to the steel frame as shown in Figure 8. LVDT1 to LVDT7 were used to monitor the lateral displacements at various locations, and LVDT8 and LVDT9 were used to measure diagonal displacements. LVDT7 represents the overall displacement of the panel and it was also used to determine the storey drift of the panel. LVDT10 was placed on the right top side pin joint to monitor the out-of-plane movement of the panel. The strain gauges were used to measure the strains in the steel members. Nine targets were also placed on the backside of the panels with a secondary camera, to determine the relative movements of the SIM layers. All transducers were wired to a data logger connected to a computer.

After set-up the bare frame with four pin connections, the SIM panel was constructed inside the bare frame. The joint filling putty was applied on all bed and head joints between the SIM units during panel construction.

The mechanical SIM units used in these tests were made of concrete, which has high compressive strength but significantly low tensile strength. The mean compressive strength of the
SIM unit used was 31.5 MPa (coefficient of variance 20%, using AS/NZS4456.4:2003 and average of 50 samples); the density of the units was 2250 kg/m³ (using AS3700-2011 and average of 30 samples). After construction of the SIM panels, there was a gap between the top of the panel and the steel frame of approximately 50mm at each corner and approximately 80mm gap in the middle of the SIM panels as shown in Figure 9. In Panel designated MO, the gap remained open during testing. In Panel designated MF, the gap was filled with self-expanding polyurethane foam (soft gap filler), and in Panel designated MG, the gap was filled with cement grout (hard gap filler) as shown in Figure 10.

The foam used was a self-expanding polyurethane foam filler that expands to about 2.5 times the initially dispensed foam after being sprayed, which allowed a complete seal to form around the panel. The grout used in this study was a mixture of cement and sand with a cement to sand ratio of 1:6. The mean compressive strength of the grout after 28 days of curing was 13.94 MPa with a coefficient of variance of 12.70% (using ASTM, 2016 and average of 6 samples).

The bare frame and frame with one panel are shown in Figure 11. A speckle pattern as shown in Figure 11 was applied on the panels so that the Digital Image Correlation (DIC) could be
used to obtain the displacement topology of the panel. The results obtained from DIC analysis are not presented herein.

Figure 11: Photos of the test - Bare Frame, Panel Zero (left) and Frame with SIM Panel (right).

The steel frame was then subjected to the lateral displacement history in cyclic form. Cyclic lateral displacements were applied at the left-hand side pin connection of the steel frame by a hydraulic jack. The cyclic lateral displacements consisted of a push cycle (frame goes rightward - positive) and a pull cycle (frame goes leftward - negative). The cyclic tests were conducted under displacement control using the horizontal LVDT7. The selection of the displacement history to simulate the seismic action has some relevance on the results, particularly in the post-peak region, where strength and stiffness degradation occurs. For this reason, the same displacement history shown in Figure 12 was used as an input seismic action for all the tested panels.

![Displacement history](image)

Figure 12: Applied displacement history for SIM.

Each displacement level was repeated three times in the cyclic form, and the results of the LVDTs, visual crack of the panel and strain gauges were monitored carefully throughout the whole test. The induced horizontal force and displacement measured by LVDT7 were recorded, and the latter represents the controlled displacement. The storey drift was calculated as it is an indication of the lateral displacement experienced at the top of the panel (LVDT7) in relation to the overall height of the Panel (2m). This factor can be critical in the construction of a multi-storey building. The duration of each test was varied from 450 to 500 min.

The hysteretic loops obtained for each wall from the cyclic testing protocol described before are depicted in Figure 13, as well as associated envelope curves of the 1st cycle at each displacement increment. Note that the vertical axis scaling in this figure is not the same for all graphs. In the Bare frame test, the hysteretic curves were almost similar at all levels of displace-
ment (Figure 13a). However, different behaviours were observed in the frame with mechanically interlocking SIM panel tests, as shown in Figure 13b to Figure 13d. This was caused by the uneven gap between the frame and the top of the panel.

Furthermore, there is no noticeable difference in the stiffness degradation between the three cycles at all levels of displacement of Panel zero (bare frame) which indicates that the steel frame does not suffer any damage during the cycling, even at the larger displacements. In addition, from all the hysteretic loops it is observed a reasonably symmetric and stable response for push and pull cycles up to a displacement of 20 mm.

5 NUMERICAL CALIBRATION

The numerical-macro models representing the innovative infills were calibrated taking, as reference, the in-plane cyclic tests previously described. The results of the experimental tests were compared to the response of the numerical macro-models in terms of force-displacement curves for the entire displacement history.

The purpose of the calibration was to reproduce numerically the in-plane experimental behavior of the frames with the pliable infill systems, which it was, in turn, necessary to obtain reliable models to be used in non-linear analyses at the “building scale”.

The infilled frames were modelled using a macro-element approach, using Ruamoko2D program ([31]). In the structural models, the RC members of the frames were defined as one-component Giberson elements ([32]) with concentrated plasticity at their ends. Assuming that the design of the frame was properly performed, the possibility of shear failures in the structural

![Figure 13: Force-displacement response hysteresis of the tested panels. (a) Panel Zero: Bare frame. (b) Panel MO: SIM infill panel with an open gap. (c) Panel MF: SIM infill panel with foam. (d) Panel MG: SIM infill panel with grout.](image-url)
members was not taken into account. In accordance with the recommendations by Priestley et al. [33], the region of intersection between the beam and the columns were characterized by perfectly elastic short elements. The hysteresis rule adopted to simulate the RC behavior of the section of the columns was the Schoettler-Restrepo rule ([31]), while the non-linear behavior of the beam was defined by the Fukada-rule ([34]), as shown in Figure 14.

![Figure 14: (a) backbone curve of Schoettler-Restrepo rule [31]; (b) Fukada degrading tri-linear hysteresis rule [34].](image)

Considering the infilled frame cases, two diagonal non-linear springs that represent the innovative infill system have been considered. These elements were pinned to the extreme nodes of the frame and work only in compression (“no tension” elements). The hysteresis rule associated to the non-linear behaviour of the two diagonal struts representing the infills was the one proposed by Crisafulli [35], as reported in Figure 15. According to the experimental results, the two diagonal spring defined by the Crisafulli hysteresis rule was capable to represent both the masonry infill solutions.

The spring elements, defined by the Crisafulli hysteresis rule, are defined by an elastic modulus \( E_{mo} \), a compressive strength \( f'_{m} \) and an initial stiffness \( K_D \) equal to \( (E_{mo}) \times (AREA1)/(element \ length) \). The area of the section of the element is assumed to be dependent on the deformation, \( AREA1 \) at displacement \( R1 \), and \( AREA2 \) at displacement \( R2 \). The shape of the envelope of the hysteretic cycles was assumed to be parabolic. The peak of the curve was defined by the point \( (\varepsilon'_{m}, f'_{m}) \) and the intersection with the x-axis occurs at the value of deformation \( \varepsilon_u \). The slope of the descending branch of the curve was determined by the factor \( \gamma_{un} \), while the reloading factor \( \alpha_{re} \) defines the point at which the reloading curves attain the strength envelope. The closing strain, \( \varepsilon_{cl} \), corresponds to the partial closing of the cracks where the compressive stresses could be developed.

The calibration of the macro-element of the University of Pavia system has been fulfilled by modelling the bare frame and the infilled one, being the influence of the RC frame not negligible. On the other hand, the tests conducted on the SIM allows a direct calibration of the macro-element by considering only the infill without modelling the hinged steel frame used during the experimental campaign since provides a very low contribution.
5.1 Infill with sliding joints (University of Pavia)

The final parameters of the calibrated diagonal macro-elements, representing the innovative infill system of the University of Pavia, are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_D$ (kN/m)</td>
<td>81371</td>
</tr>
<tr>
<td>$f_m$ (kPa)</td>
<td>-1297</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>-0.00047</td>
</tr>
<tr>
<td>$\varepsilon_u$</td>
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</tr>
<tr>
<td>$\varepsilon_c$</td>
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</tr>
<tr>
<td>$E_{mo}$ (MPa)</td>
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</tr>
</tbody>
</table>

Table 1: Parameters of the diagonal struts for infill with sliding of the University of Pavia.

Figure 16a and Figure 16b show the comparison, in terms of force-displacement curve, between the in-plane cyclic tests (black line) and the numerical models (red line) on the bare frames and on the frame infilled with the innovative system. The input of the numerical-model was given by the loading history of the displacement effectively applied at the half-height of the top-beam during the pseudo-static cyclic tests. The plots show that the force-displacement curves of the numerical models match very well the experimental ones, during all the cycles.
5.2 Semi-Interlocking Masonry (University of Newcastle)

The final parameters of the calibrated diagonal macro-elements, representing the mechanical interlocking SIM infill panel MO tested by the University of Newcastle, are listed in Table 2. The data are referred to the 2.0 x 2.0 m infill frame, as described previously.

<table>
<thead>
<tr>
<th>$K_D$</th>
<th>kN/m</th>
<th>1266</th>
<th>initial diagonal stiffness</th>
<th>$\gamma_{un}$</th>
<th>-</th>
<th>20</th>
<th>stiffness unloading factor</th>
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<td>kPa</td>
<td>-3861</td>
<td>compressive strength</td>
<td>$\alpha_{re}$</td>
<td>-</td>
<td>0.20</td>
<td>strain reloading factor</td>
</tr>
<tr>
<td>$\varepsilon'_m$</td>
<td>-</td>
<td>-0.00318</td>
<td>strain at $f'_m$</td>
<td>AREA1</td>
<td>m²</td>
<td>0.00348</td>
<td>initial strut cross-sectional area</td>
</tr>
<tr>
<td>$\varepsilon_{cl}$</td>
<td>-</td>
<td>-0.09546</td>
<td>ultimate strain</td>
<td>AREA2</td>
<td>m²</td>
<td>0.00348</td>
<td>final strut cross-sectional area</td>
</tr>
<tr>
<td>$\varepsilon_{cl}$</td>
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<td>2.00</td>
<td>closing strain</td>
<td>$R_1$</td>
<td>m</td>
<td>-0.01061</td>
<td>displacement at 1</td>
</tr>
<tr>
<td>$E_{mo}$</td>
<td>MPa</td>
<td>2059</td>
<td>initial masonry modulus</td>
<td>$R_2$</td>
<td>m</td>
<td>-0.1061</td>
<td>displacement at 2</td>
</tr>
</tbody>
</table>

Table 2: Parameters of the diagonal struts for SIM of the University of Newcastle.

Figure 17 shows the comparison, in terms of force-displacement curve, between the in-plane cyclic tests (black line) and the numerical models (red line) on the SIM infill. The input of the numerical-model was given by the loading history of the displacement effectively applied at the half-height of the top-beam during the pseudo-static cyclic tests. The plot shows that the force-displacement curve of the numerical model matches quite well the overall experimental response during all the cycles that have been considered relevant for the present study (up to 2.0% of drift). In this case, the nature of the innovative pliable infill may require an improved strut model with some damping mechanism or tensile resistance since the test results are not precisely captured when force inversion occurs.

Figure 17: Comparison between the force-displacement experimental and numerical curves of the SIM infill.

6 CASE STUDY: 6-STOREY INFILLED RC FRAME: NONLINEAR STATIC AND DYNAMIC TIME-HISTORIES ANALYSES

Non-linear static (“pushover”) and dynamic time-history analyses have been performed on a 6-storey case-study building (Figure 18a [36]), in order to evaluate the influence of the infills in the seismic response of the structures. The macro-modeling approach using the program Ruamumoko, as defined for the numerical calibration of the single infilled frames, was adopted. A case-study frame designed at a level of PGA equal to 0.35g-S according to EC8 was assumed. A “traditional” infill solution (Figure 18c) has been considered as reference (T2 as defined by Hak et al. [36]). The infills with sliding joints have been modeled according to the calibration described previously, being the dimensions of the tested panel very similar to the ones of the bay of the considered building. Instead, for the mechanical interlocking SIM panel MO, the calibrated values have been properly scaled according to the bay dimension of the frame. The
numerical outcomes have been compared with the bare frame configuration of the same building (Figure 18b).

![Figures](image-url)

Figure 18: (a) Elevation of two-dimensional prototype structure. (b) Different infilled configurations for the six-storey building. (c) Detail of the traditional masonry infill T2 considered.

In the non-linear static analyses, the horizontal action was given by means of equivalent horizontal loads applied at each mass of the structure. The intensity of the forces has increased monotonically, while their distribution was kept constant and proportional to the first modal shape of the structure. The results of the pushover curves (Figure 19) show how the innovative infills have a behavior that is almost parallel to the bare frame, with the solution of the University of Pavia having an initial stiffness as the one of the traditional infill. The building with SIM infill provides a lateral response very similar to the one of the bare frame. Both the cases with innovative infills have a better performance than the traditional infill. The traditional typology produces a strong strength and stiffness degradation after the peak, meaning a certain level of damageability, above all for large displacements (see the dashed blue line of Figure 19). On the other side, the innovative infills do not show any degradation and almost no damage up to very large displacement demands.

In addition, nonlinear time history analyses have been performed on the same structure with the three different infills. The selection of the earthquake records was done based on the recommendation of previous studies ([37]) on natural ground motions scaled to specified levels of seismicity.

The floor displacements and inter-storey drifts coming from the dynamic non-linear analyses have been evaluated. The selection of the input ground motion has a great influence in the response on the structures, also considering that different frame typologies, both infilled and bare, with different modes of vibrations, behave differently to the same earthquake. Nevertheless, considering that the sets of input ground motion adopted in the analyses were composed by ten spectrum compatible accelerograms, the quantities related to the response of the structure were analyzed in terms of average values. Figure 20 summarizes the displacement and drift profiles for each set of accelerograms, considering the bare frames and the infilled configurations with the three infills modelled. Displacement profiles are obtained for each frame type and for each ground motion record corresponding to the instant of a maximum displacements obtained in any of the storeys. The lines in the displacement-plots (Figure 20a and Figure 20b) represent the average values of the displacement for the specific set of accelerograms. Drift profiles are obtained for each frame type and for each ground motion record corresponding to the instant of a maximum drift obtained in any of the storeys.

Looking at the results of the nonlinear dynamic analyses the two innovative infills have shown different seismic response. While the SIM infilled structure has an overall deformation similar to the bare frame due to its small stiffness, the building infilled with the solution of the
University of Pavia provides a significant decrease of the inter-storey drift demand in comparison with both the bare frame and the traditional infill; this aspect is of paramount importance since, at the same level of global initial stiffness of the infilled structure (see pushover curves in Figure 19), the displacement demand from time histories is strongly reduced thanks to its large values of provided damping.

Figure 19: Capacity curves for the bare and infilled (innovative and traditional systems) frames.

Figure 20: (a) Displacement profiles at the DLS. (b) Displacement profiles at the LSS (ULS). (c) Inter-storey drift profiles at DLS. (d) Inter-storey drift profiles at LSS (ULS).
7 CONCLUSIONS AND FUTURE DEVELOPMENTS

Traditional rigidly attached masonry infills present a series of issues related to the seismic response that have been object of many post-seismic inspections and research studies. To exceed the well-known issues, many technological and theoretical solutions have been proposed and studied. One of the most promising belongs to the pliable infill category, where the infills can deform up to relevant drifts (i.e. 2.0%) with a limited interaction with the structure.

The University of Pavia and the University of Newcastle have separately studied and tested two different deformable infills with horizontal sliding and “deformable” joints. The experiments conducted have provided promising results, and within this paper the in-plane experimental pseudo-static cyclic seismic behaviour of fully (without opening) infills have been considered.

The scope of the present work was to define the basic dynamic characteristics of the two systems and to perform some preliminary study on the global behaviour of these two infill typologies through a calibrated non-linear macro-element. Their apparently complex in-plane seismic behaviour has been replicated through non-linear single-strut with lumped plasticity macro-models, as usually done for traditional infills. The calibration of the two solutions has been fulfilled by replicating the overall force-displacement experimental curves obtained from the cyclic in-plane tests.

Finally, the calibrated macro-models have been used to perform a case study on the global structural behaviour of infilled RC frame buildings. A comparison between the pliable infills and one traditional infill solution has shown that these new systems can modify the global behaviour of the structure differently from the traditional masonry infills. The SIM infilled structure has shown a seismic behaviour almost similar to a bare frame configuration, whereas the solution with the infill with sliding joints has provided an overall structural initial stiffness larger, and similar to the traditional considered “weak-medium” infill, with the capacity of significantly reduce the deformation of the structure thanks to its large value of damping.

In conclusion, the work herein presented still needs some future improvement and developments, for example about the modelling of SIM panels; moreover, parametric FEM analyses for such innovative infills changing the main characteristics (e.g., different aspect ratio, the properties of the boundary element materials, etc.) are still in process. Further research is also necessary to estimate the economical advantage of such systems within procedures for the evaluation of expected annual losses.

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PBEE ASSESSMENT OF RC FRAMES WITH TRADITIONAL AND SLIDING-JOINT INFILLS

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Abstract

In reinforced concrete (RC) multi-storey buildings, the important role of the seismic interaction of structural frames with masonry infills has been revealed by several earthquakes and investigated by many authors. Recently, several innovative infill solutions have been proposed to mitigate such interaction, which could result in widespread damage in both the masonry and the RC structure and sometimes jeopardize the building stability and the occupants’ safety. One solution consists in the partitioning of the masonry infill into several sub-panels, relatively sliding along specific joints. This paper investigates the seismic performance assessment of this technological solution in the framework of performance base earthquake engineering. A two-dimensional five-storey RC seismic-resistant frame is selected as case study and the performance is assessed by comparing the responses of the same structure infilled with different solutions, made of sliding joints or traditional masonry, or in the bare configuration. Incremental Dynamic Analyses (IDA) is used for the probabilistic determination of fragility curves of the structures. Results show the seismic fragility and reliability of the different investigated structures, especially addressing the probabilities of occurrence of damage at different limit states and quantifying the associated expected annual loss.

Keywords: seismic reliability, sliding-joints infills, expected annual loss, infilled frames, performance based earthquake engineering.
1 INTRODUCTION

Post-earthquake damage analyses have shown that a consistent part of the reparation costs of reinforced concrete (RC) buildings is related to reparation and/or strengthening of masonry infills and partition walls [1-3], which generally suffer significant damage even in the case of moderate earthquakes. In fact, despite their effectiveness in terms of thermal, acoustic, fire and durability performance, traditional masonry infills are characterized by a large in-plane strength and stiffness, combined with a marked brittleness. As a consequence, they could reach their peak strength for low deformation levels, typically induced by moderate intensity earthquakes, thereafter, as the imposed drift increases, infills show in-plane and out-of-plane response degradation, with diffuse cracking and local crushing. In several cases this may evolve into infills out-of-plane collapse, which significantly increases risk for human life [4,5]. Moreover, as shown in many studies [6, 9 among others], traditional infills entail large interaction with the surrounding frame, inducing localized trusts on the frame columns, which could jeopardize their local performance. Based on these observations, it is clear that the presence of traditional infills in the buildings seismic response could result in relevant reparation and downtime costs.

Several studies have been carried out in the last decade in order to develop innovative infill solutions capable of undergoing limited damage when subjected to different levels of interstorey drifts demanded by earthquakes. They can be summarized into two main categories, one providing infill-frame system strengthening (e.g. [10,11]), the other providing the reduction of infill-frame interaction [12-15]. Among the latter, the partitioning of masonry infills with horizontal sliding joints has shown to be an effective solution for reducing infill-frame interaction and limiting the damage to infills even in the case of severe earthquakes. Such technique has been experimentally confirmed [14, 16-18] and investigated in depth by parametric analyses [19] that allowed providing a simplified equivalent strut modelling approach effectively describing the in-plane sliding-joints infilled frame response [20].

In order to assess the potential of the proposed innovative construction technique for the infills in RC framed structures, in the present paper, its seismic performance is compared with that of a traditional masonry infill, within a probabilistic assessment framework merging seismic fragility, reliability and loss assessment during the service life. The study adopts a performance based earthquake engineering (PBEE) approach, which can provide a quantification of the actual gain obtainable by adopting such kind of technological solution. The structural assessment is based on incremental dynamic analysis (IDA) [21] for the determination of fragility curves, specifically defined in order to include limit states at structural and non-structural level. IDA are performed considering a selection of 30 ground motion records scaled, for the different systems, by assuming spectral acceleration at each specified vibration period, $S_a(T)$, as intensity measure (IM). The adopted procedure allows comparing fragilities of structural systems having substantially different fundamental vibration periods (e.g. traditionally infilled frames and sliding-joint infilled frames) by integrating the convolution between fragility and hazard functions. Once obtained the fragility curves for the different structural systems, the assessment is moved to reliability by evaluating probabilities of exceeding each limit state. The analysis results are finally used to extend the investigation in terms of expected annual loss associated to each specified limit state, thus allowing the estimation of post-earthquake restoration costs within the service life.
2 PERFORMANCE BASED EARTHQUAKE ENGINEERING ASSESSMENT FRAMEWORK

In the present paper, the PBEE framework is specifically designed to assess seismic performance of infilled frame systems, characterized by different infill configurations. As described by different authors [22-29], performance based earthquake engineering framework is generally made of four main steps: structural analysis, hazard analysis, damage analysis, and loss analysis.

The structural response is obtained by means of the IDA method, which has been recently widely employed by different authors (e.g. [30,31] among others) to obtain a statistical distribution of the intensity measures inducing a limit state, taking into account the ground motions variability. For the IDAs, a set of 30 spectrum-compatible ground motions is selected and scaled in amplitude up to the achievement of the specified limit states defined as: (i) achievement of structural collapses during the analyses or (ii) limit values of engineering demand parameters (EDPs) (e.g. maximum interstorey drifts). In the adopted framework, for each analyzed structure characterized by its own fundamental vibration period ($T_1$), the selected ground motions are scaled with respect to the spectral acceleration attained in correspondence of $T_1$, to obtain $S_a(T_1)$ as a common value for each spectrum. The obtained spectra, and the associated records, are then scaled to be adopted as input ground motion in time history analyses.

From IDA results fragility curves for each limit state (defined in the following) can be derived, which express the probability of exceeding a specified limit state as a function of a specified $IM$, quantified by the following expression:

$$P[C \leq D | IM = x] = \Phi\left(\frac{\ln(x) - \mu_{\ln x}}{\sigma_{\ln x}}\right)$$

where $P[C \leq D | IM = x]$ is the probability that a ground motion with $IM=x$ will cause the achievement of a limit state, $\Phi$ is the standard cumulative distribution function, $\ln(x)$ is the natural logarithm of the variable $x$ representing the intensity measure ($S_a(T_1)$) and $\mu_{\ln X}$ and $\sigma_{\ln X}$ are the mean and the standard deviation of the natural logarithms of the distribution of $x$, respectively.

Based on fragility curves, reliability analysis can be performed to evaluate the probability ($P_f$) of exceeding a given limit state in a reference time period (in years), as expressed in Eq. (2).

$$P_f = \int_{0}^{\infty} P[C \leq D | IM = x] \cdot P[x] dx$$

where $P[x]$ is the probability of exceeding an $IM=x=S_a(T_1)$ in a specific site in the reference period (50 years) described by a Poisson model as:

$$P[x] = 1 - e^{-\lambda \cdot x} \cdot dx$$

in which $\lambda(x)$ is a function describing the annual rate of exceeding the $IM=x=S_a(T_1)$.

According to Eq. (2), the probability $P_f$ is obtained by integrating the convolution of hazard curves and fragility curves. While the latter represent the probability of a specific structure of period $T_1$ to exceed a specified limit state, the former are the probability of exceeding the intensity $S_a(T_1)$ in a specific site in the reference service time period ($\Delta t$).

Hazard curves are obtained from the hazard analysis of the site, in which spectral ordinates at different vibration periods ($S_a(T_{1,i})$) are calculated for different annual rates of exceedance.
(λ), defined as the inverse of the return periods (λ = 1/T). The interpolation of results allows determining the hazard curves, which are site and period dependent. As shown in Fig.1a, since fragility curves are referred to structures with different fundamental periods, a higher fragility not necessarily means higher probability of failure. Under this observation, the evaluation of P_f allows making consistent comparison between structural systems characterized by different vibration periods.

![Figure 1: Samples of reliability assessment of two structures having periods T_1 and T_2 (a) and typical EAL curve achievable from the reliability assessment (b).](image)

The last stage of the PBEE framework consists in the evaluation of the expected annual loss (EAL) [32, 33]. EAL is determined starting from the performance of the structure for each limit state in terms of annual frequency of exceedance (λ_LS = 1/T_rC-LS, being T_rC-LS the capacity return period) and the associated repair costs, expressed as a fraction of reconstruction costs (%RC). In the proposed framework, the repair costs associated with each limit states have been assumed as those calibrated in [33]: the total loss or reconstruction limit state (R-LS) is assumed equal to 100%, while the %RC associated with operational limit state (O-LS), damage limit state (DL-LS), life safety limit state (LS-LS) and collapse limit state (CO-LS) are 7%, 15%, 50% and 80%, respectively. The initial damage limit state (ID-LS) is conventionally assumed having λ_ID=10% and %RC=0%. The EAL curve (Fig. 1b) can be obtained by connecting the points (λ_LS, %RC) representative of each limit state, and the area above the curve represents the EAL. The latter can be simply evaluated as:

\[
EAL = \sum_{i=2}^{5} \left[ \hat{\lambda}_{LS(i-1)} - \hat{\lambda}_{LS(i)} \right] \cdot \left( \%RC_{LS(i)} + \%RC_{LS(i-1)} \right) / 2 + \hat{\lambda}_{CO} \cdot \%RC_r
\] (4)

As a summary, in the current assessment framework, the following steps are provided:

a. determine the distributions of IM at each limit states on IDA curves;

b. determine fragilities for each limit state and the values of the intensity measures having 50% probability of occurrence \( S_a(T_i)_{LS} \) (mean values);

c. determine the corresponding annual rates of failure (\( \lambda_{LS} \)) through the hazard curves;

d. build EAL curve and evaluate the EAL.

3 THE REFERENCE CASE STUDY STRUCTURE

A 3-bays 5-stories RC frame, extracted from a typical Italian residential building (plan view in Fig. 2a) is selected as reference structure for the present study. The frame (Fig. 2b) is designed according to the Italian building code [34] meeting the design requirements for high
ductility class. Concrete is supposed having nominal strength $f_c = 25$ MPa, steel rebars have nominal yielding strength $f_y = 560$ MPa, while in Table 1 are reported the details of the structural frame elements. The design of horizontal seismic forces is carried out using the design response spectrum obtained for the city of Cosenza (Italy) (soil type C) scaled by a 5.85 behavior factor.

### Figure 2: Reference case study building: a) Plan view; b) Selected frame.

<table>
<thead>
<tr>
<th>BEAMS</th>
<th>Geometry</th>
<th>Longitudinal reinforcement</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b$ (mm)</td>
<td>$h$ (mm)</td>
<td>Bottom</td>
</tr>
<tr>
<td>Beam 1</td>
<td>300</td>
<td>450</td>
<td>$4\phi 20$</td>
</tr>
<tr>
<td>Beam 2</td>
<td>300</td>
<td>450</td>
<td>$4\phi 20$</td>
</tr>
<tr>
<td>Beam 3</td>
<td>300</td>
<td>450</td>
<td>$3\phi 20$</td>
</tr>
<tr>
<td>Beam 4</td>
<td>300</td>
<td>450</td>
<td>$2\phi 20$</td>
</tr>
<tr>
<td>Beam 5</td>
<td>300</td>
<td>350</td>
<td>$2\phi 20$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>COLUMNS</th>
<th>Geometry</th>
<th>Longitudinal reinforcement</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b$ (mm)</td>
<td>$h$ (mm)</td>
<td>End-zones</td>
</tr>
<tr>
<td>Column 1</td>
<td>450</td>
<td>450</td>
<td>$8\phi 20$</td>
</tr>
<tr>
<td>Column 2</td>
<td>450</td>
<td>450</td>
<td>$8\phi 22$</td>
</tr>
</tbody>
</table>

Table 1: Geometry and reinforcement details of beams and columns of the selected frame.

Three different configurations for the frame are considered in the following analyses: bare frame (BF), fully infilled frame with traditional masonry infills (TI) and fully infilled frame with infills partitioned by horizontal sliding joints (SJ). For the sake of simplicity no openings are assumed in the infills, whose effect would modify the response of both traditional [35] and sliding joints [36] solid infills. Details of the arrangement of traditional and sliding joint infills are illustrated in Fig. 3. Both the typologies of masonry infills are made of clay hollow blocks with a thickness ($t$) of 200 mm and 15 mm. Mechanical test results on materials are reported in Table 2. Sliding infills have horizontal sliding joints arranged as proposed by [14] with the introduction of wooden boards able to activate the sliding between two adjacent masonry sub-portions. Wooden boards are supposed to be inserted also at the column-to-infill interface, in order to provide a deformable contact joint preventing masonry from crushing at the sub-panel corners. In particular, in the reference structure the infill with horizontal sliding
joints is considered having lateral wooden boards with halved depth with respect to masonry thickness in order to reduce the contact forces exchanged between the infill and the frame [20].

![Image of infill types]

**Figure 3: Layout of traditional infills and sliding joint infills.**

<table>
<thead>
<tr>
<th>Material</th>
<th>Compressive Strength (MPa)</th>
<th>Elastic Modulus (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masonry prisms: Holes parallel to the load ($f_{m2} - E_{m2}$)</td>
<td>7.28</td>
<td>16148</td>
</tr>
<tr>
<td>Masonry prisms: Holes perpend. to the load ($f_{m1} - E_{m1}$)</td>
<td>2.4</td>
<td>4408</td>
</tr>
<tr>
<td>Mortar</td>
<td>12.24</td>
<td>18619</td>
</tr>
<tr>
<td>Wood perpend. to the grain</td>
<td>2.56</td>
<td>255</td>
</tr>
</tbody>
</table>

Table 2: Material properties from experimental tests [14].

### 4 MODELING OF THE STRUCTURE AND LIMIT STATES DEFINITION

#### 4.1. Modeling of the infilled RC frame

IDAs on the reference structure have been performed with the Opensees software platform [37]. A distributed plasticity approach is adopted to model the RC frame, using fiber-section beam-column elements characterized by the Concrete04 material stress-strain model for the cross section fibers. Confinement of concrete is accounted for by dividing cross-sections into effectively confined core fiber and unconfined cover fibers and elements into constant-confinement segments [38-40] in such a way to account for the different transversal reinforcement, while steel rebars are included by means of the Steel02 material model. The triggering of shear non-linear mechanism is not directly modeled, but possible shear damage or collapses in the frame elements are evaluated a-posteriori.

The in-plane interaction of masonry infills with the RC frame is modeled by adopting a simplified macro-element approach for both TI and SJ infills: equivalent diagonal struts are introduced within the frame bays and calibrated to simulate the infill contribution to the frame lateral strength and stiffness. The struts are compression only truss elements accounting for the degradation during the cyclic in-plane response and connected eccentric with respect to the frame joints in order to model the additional shear demand on the frame elements, induced by the infill-frame interaction. The calibration of the equivalent struts of TI and SJ infills is described in detail in the following sections.

For the traditional infill, a double strut configuration (Fig. 4) is adopted. It provides two parallel struts per each infill diagonal, which are eccentric with respect to the beam-columns joints. Such struts are connected to the frame columns and beams at specified distances from the joint nodes ($z_c$ and $z_{bs}$ respectively), which are quantified to reproduce both the infill lat-
eral response in terms of strength, stiffness and additional shear action induced in the frame elements. From the results of previous numerical studies [19, 20] such a distance can be set as 1/10 of the frame member length. The response in compression of the struts is defined by Concrete02 material stress-strain model (parabolic with linear compression softening). The calibration of the struts inelastic response is based on the procedure proposed by [41] for single equivalents strut modelling, which defines the stress–strain curve of the strut through four parameters, namely peak stress \( f_{md0} \), peak strain \( \varepsilon_{md0} \), ultimate stress \( f_{mdu} \), and ultimate strain \( \varepsilon_{mdu} \), which are directly linked to geometrical and mechanical properties of the infilled frame. The adopted correlation laws are presented in detail in [41]. A fundamental role in defining the infill in-plane response is acted by the width \( \omega \) of the compressed strut activating within the infill in the frame sway mechanism. In the proposed calibration of the single equivalent strut, the width of the strut is determined as proposed in [35].

Based on the stress-strain relationship obtained for the single equivalent the strut, the two eccentric struts in Fig. 4 are calibrated by imposing the equilibrium on the frame according to the following hypotheses:

- the frame is considered hinged in the joints;
- the overall horizontal force at the upper frame beam has to be equivalent to that provided by the single strut;
- the associated resisting forces \( F_{md0}, F_{mdu} \) in strut B is 80% of that in strut A. This hypothesis is based on the results of several numerical studies (e.g. [19]) demonstrating that shear action generated on the windward column is about the 80% of that on the leeward one.

\[
\begin{array}{ccccccc}
\text{Single strut} & f_{md0} & f_{mdu} & \varepsilon_{md0} & \varepsilon_{mdu} & \omega & F_{md0} & F_{mdu} \\
1170.54 & 362.40 & 111.43 \\
\text{Strut A} & 1.548 & 0.476 & 0.00078 & 0.00733 & 722.55 & 223.70 & 68.78 \\
\text{Strut B} & 578.04 & 178.96 & 55.02 \\
\end{array}
\]

Table 3: Calibration parameters for single and double equivalent strut model for the traditional infill.

As shown in Fig. 5a, infill with sliding joints are modeled with two alternative compression-only struts hinged on the columns at a specified distance \( z \) from the frame joint, as proposed in [20]. The calibration of the strut is based on expressions, allowing, at each deformation level, the simultaneous prediction of the infill lateral strength \( \Delta F_s \) and maximum shear in the columns \( V_{col}^{\text{max}} \), which can be estimated by means of simple equilibrium considerations based only on the geometric and material parameters of the infill. In detail, the
axial stress-strain law of each strut is obtained by means of three axial springs in parallel, calibrated in order to reproduce the analytically obtained force-displacement response (Fig. 5b) and the typical cyclic response of the considered infill typology. The distance $z$ is calculated as defined in Eq. (15), based on the analytical values of the maximum infill lateral strength and the maximum shear on the columns, at a selected drift level (in this case 2%).

$$z = h \frac{V_{\text{col}}^{\text{max}} - \Delta F_s}{2V_{\text{col}}^{\text{max}} - \Delta F_s}$$

(15)

The axial constitutive law of the springs is reproduced in the model by using an Elasto-Plastic Gap material, which also allows reproducing the cyclic degradation of the infill due to the plastic deformation of the lateral contact joint when increasing the drift level [20]. Table 4 summarizes the calibration parameters of the three springs composing the strut.

Fig. 6 reports the layout of the models of the reference structure adopted in the present study, characterized by different infill configurations.

Figure 5: Simplified equivalent strut model for the infills with horizontal sliding joints: a) equivalent strut model for the infill with horizontal sliding joints; b) force-drift relationships of springs.

<table>
<thead>
<tr>
<th></th>
<th>Spring 1</th>
<th>Spring 2</th>
<th>Spring 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial stiffness (kN/m)</td>
<td>8239827</td>
<td>345507</td>
<td>3900</td>
</tr>
<tr>
<td>Yielding Force (kN)</td>
<td>37.04</td>
<td>1E-05</td>
<td>53.47</td>
</tr>
<tr>
<td>Post-yielding stiffness (kN/m)</td>
<td>0</td>
<td>345.51</td>
<td>687.22</td>
</tr>
</tbody>
</table>

Table 4: Calibration parameters for the parallel spring elements modeling the infill with horizontal sliding joints.

Figure 6: Layout of the case study RC frame model: a) bare frame; b) traditionally infilled frame; c) sliding-joint infilled frame.
4.2. Definition of structural and non-structural limit states

In the assessment framework the four standard PBEE limit states (namely operational limit state (O-LS), damage limitation limit state (DL-LS), life-safety limit state (LS-LS) and collapse limit state (CO-LS)) are considered. Two additional limit states are added to better characterize the different damage states. One concerns the frame initial damage (FID-LS) due to first yielding or first shear cracking, the other considers the attainment of infills severe damage (ISD-LS). Among the considered limit states, O-LS, DL-LS and ISD-LS are referred to damage of non-structural components (the infills), while FID-LS, LS-LS and CO-LS identify damage of structural elements. The standard limit states (O-LS, DL-LS, LS-LS and CO-LS) are used for EAL assessment since the associated percentage losses are calibrated [33].

The criteria adopted to define the different limit states are summarized in Table 5. As regards non-structural limit states, O-LS, DL-LS and ISD-LS are defined as function of the interstorey drift, based on the results previous experimental studies on traditional [42] and sliding-joints infills [14, 16]. For what concerns structural limit states, collapse limit state (CO-LS) is achieved in correspondence of the first of the following conditions: i) achievement of ultimate chord-rotation (θ_u) of columns (evaluated according to Eurocode 8 [43]), ii) achievement of ultimate shear capacity (V_{R,u}) of columns, iii) achievement of 6.5% interstorey drift, when second order effects could jeopardize the stability of the structure. In the performed analyses, an axial force-chord rotation (N-θ) interaction domain is considered, in order to take into account the variation of chord rotation capacity as a function of the variation of axial load on columns, as proposed in [31]. The ultimate shear capacity V_{R,u} of column is evaluated according to the Model Code 2010 [44] expression, with unit safety coefficients for the materials to maintain the consistency with the material models used in the analysis.

Finally, the FID-LS is related to the first occurring condition between column rebars yielding and initial shear cracking. The former condition is associated with the achievement of the yielding rotation (θ_y) of frame column, according to Eurocode 8 [39], while the first shear cracking is associated with the achievement of the resistance V_{R,i}, evaluated using the expression proposed by Collins (1998) [45].

<table>
<thead>
<tr>
<th>Limit state</th>
<th>Limit state threshold</th>
<th>Considered for EAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-LS</td>
<td>IDR=0.20%</td>
<td>Yes</td>
</tr>
<tr>
<td>DL-LS</td>
<td>IDR=0.50%</td>
<td>Yes</td>
</tr>
<tr>
<td>ISD-LS</td>
<td>IDR=1.50%</td>
<td>No</td>
</tr>
<tr>
<td>FID-LS</td>
<td>θ_y or V_{1st crack}</td>
<td>No</td>
</tr>
<tr>
<td>LS-LS</td>
<td>0.80θ_u or 0.8V_{Rd}</td>
<td>Yes</td>
</tr>
<tr>
<td>CO-LS</td>
<td>θ_u or V_{Rd} or IDR=6.5%</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 5: Structural and non-structural limit state thresholds for traditional and sliding joint infills.

5 INCREMENTAL DYNAMIC ANALYSIS

As shown in Figure 7, a set of 30 natural ground motions is selected through the software REXEL [46] in order to get spectrum-compatibility with the design spectrum of the site of Cosenza (Italy) with soil type C and 457 years return period.

To perform IDAs, accelerograms are scaled in such a way that the respective spectra assume the same value of S_a(T_1) in correspondence to the first vibration period for each considered structure. The vibration periods of the three considered structures are evaluated on the bilinear equivalent capacity curves obtained by means of preliminary pushover analyses. Vi-
bration periods of each structure are derived from the stiffness of the elastic branch of bilinear curves, by averaging results obtained for modal and uniform distributions (Table 6).

<table>
<thead>
<tr>
<th></th>
<th>Uniform distribution</th>
<th>Modal Distribution</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare Frame</td>
<td>1.70</td>
<td>1.52</td>
<td>1.61</td>
</tr>
<tr>
<td>Frame with traditional infills</td>
<td>0.45</td>
<td>0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>Frame with sliding-joint infills</td>
<td>1.43</td>
<td>1.29</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Table 6: Determination of reference vibration period of the structures

IDA curves highlighting structural limit states are reported in Fig. 7. The overall trend shows that bare frame and sliding joint-infilled frame achieve collapse in correspondence of very similar spectral acceleration levels. Also maximum interstorey drifts recorded present similar magnitudes, ranging between 4.5% and 6.5%, which also demonstrate the trend of sliding-joint infilled frames to behave in a ductile manner with very few cases of shear collapse in the columns. A very different trend is observed for traditionally infilled frames, which present collapses at significantly higher spectral acceleration levels and noticeably reduced ultimate displacement values.

As regards frame initial damage limit state, the presence of the infill anticipates the damage activation in both the TI and the SJ case (occurred at about 0.5% and 1.5%, respectively), with respect to the BF one, for which the FID-LS is reached at about 3% drift. The difference between the performance of the two infilled configuration is related to the different stiffening effect acted by the two infill typologies.

Fig. 8 shows IDA curves highlighting non-structural limit states. For SJ and TI frames, it can be observed that O-LS and DL-LS occur at similar IM intensities, but for the SJ case they occur at significantly larger drift levels. As for the ISD-LS, it is achieved at about $S_a(T_1)=1.5g$ for the TI frame, while it is attained for a halved IM in case of SJ infilled frames. Also this LS, for the SJ case, is associated with larger IDR (4%), highlighting that severe damage of SJ infills occurs in correspondence of drift levels close to those inducing collapse.

![Figure 7: IDA curves and structural limit state points for: a) bare frame, b) traditionally infilled frame; c) sliding-joint infilled frame.](image-url)
HAZARD, FRAGILITY AND RELIABILITY ASSESSMENT

For the site under investigation (Cosenza, Italy) and the specified soil stiffness (type C according to EC8 classification), hazard curves, representing the annual rates of exceeding the $IM=S_a(T_1)$, are obtained for each vibration period associated with the three structural typologies. In detail, hazard curves are defined as interpolation functions of single $S_a(T_1)-\lambda$ points, representing the values assumed by spectral accelerations at a given period for different return period design spectra ($\lambda=1/T_R$). Hazard curves are then converted into probabilities of exceeding within a service life of 50 years by using the Poisson’s model equation provided in Eq. (3). The resulting hazard curves are superimposed with fragility curves of the three structural typologies (Figs. 9 and 10). The intersection areas between hazard and fragility curves are proportional to the probabilities of exceeding the different limit states, which are numerically determined by Eq. (2). Figs. 9 and 10 highlight the different amplitudes of the intersection areas between hazard and fragility curves, showing that, in the case of traditionally infilled frames, major intersection amplitudes can be recognized for both structural and non-structural limit states.
The obtained probabilities of occurrence \( P_f \) for both structural and non-structural limit states are reported in Table 7. From the structural point of view, noticeable differences can be observed for the FID-LS, where TI frames achieve a \( P_f \) of 15\%, which results 5 times and 10 times the same probabilities evaluated for SJ infilled frame and bare frame respectively. As regards LS-LS and CO-LS, the obtained probabilities of occurrence are in the same order magnitude for the three cases, with the traditionally infilled frame presenting slightly larger values. However, the largest differences are highlighted from non-structural limit states, which show a significantly reduced probability of occurrence in the cases of SJ infills with respect to TI for all the considered LS. Probabilities of occurrence of O-LS, DL-LS and ISD-LS for traditionally infilled frames are about 10 times, 13 times and 5 times the probabilities evaluated in the case sliding-joint infilled frames. This result can be justified by the reduced stiffness and shear interaction of SJ infills with the frame, which allows the attainment of non-structural LS at significantly larger drifts with respect to the case of TI frames. Moreover, the hazard for the SJ infilled frame is significantly lower than that of TI case, due to a longer vibration period, which is close to that of the bare frame.

<table>
<thead>
<tr>
<th>Probabilities of failure ( P_f ) (-)</th>
<th>Bare Frame</th>
<th>Traditional Infills</th>
<th>Sliding-joint infills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-structural</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O-LS</td>
<td>-</td>
<td>1.72 x 10(^{-1})</td>
<td>1.60 x 10(^{-2})</td>
</tr>
<tr>
<td>DL-LS</td>
<td>-</td>
<td>9.53 x 10(^{-2})</td>
<td>7.41 x 10(^{3})</td>
</tr>
<tr>
<td>ISD-LS</td>
<td>-</td>
<td>2.23 x 10(^{2})</td>
<td>4.41 x 10(^{3})</td>
</tr>
<tr>
<td>Structural</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FID-LS</td>
<td>1.48 x 10(^{-2})</td>
<td>1.50 x 10(^{-1})</td>
<td>2.50 x 10(^{-2})</td>
</tr>
<tr>
<td>LS-LS</td>
<td>3.92 x 10(^{-3})</td>
<td>4.77 x 10(^{-3})</td>
<td>4.45 x 10(^{-3})</td>
</tr>
<tr>
<td>CO-LS</td>
<td>2.49 x 10(^{-3})</td>
<td>3.95 x 10(^{-3})</td>
<td>2.61 x 10(^{-3})</td>
</tr>
</tbody>
</table>

Table 7: Probabilities of occurrence of limit states for the different structures.

7 LOSS ASSESSMENT

Expected annual loss assessment is carried out using the procedure by [33], updated as illustrated in section 2 and considering only the standard limit states for structural components (LS-LS and CO-LS) and non-structural components (O-LS and DL-LS). As described in section 2, the annual rates of exceeding the limit states are obtained from hazard curves \( \lambda(IM) \) using the spectral accelerations associated with the 50\% probability of exceeding the limit states \( \lambda_{LS} T_s(T_s) \) expressed by the respective fragility curves. \( \lambda_{LS} \) and \( \lambda_{LS}(T_s) \) values are reported in Table 8, while Fig. 11 illustrates the obtained \( \lambda-%RC \) relationships for TI and SJ.
cases. The obtained expected annual loss of the sliding-joint infilled frame (0.40%) is about half of the traditionally infilled frame (0.76%). This difference is entirely due to the gain in terms of reduced $\lambda$ for non-structural limit states, which is one order of magnitude lower with respect to the case of traditional infills. Both TI and SJ structures have EAL lower than the reference value of 1.13%, which is associated to the ideally code conforming building. This highlights that, traditionally infilled frames design according to seismic codes have adequate performance in terms of EAL, which allow assigning an A seismic risk class according to the Italian guidelines for seismic risk classification [33]. On the other hand, the adoption of sliding-joint infills allows the achievement of the most preforming risk class (A+).

Figure 11: $\lambda$-%RC relationships and EAL for TI frames, SJI frame and code compliant reference structure.

<table>
<thead>
<tr>
<th>Limit State</th>
<th>Traditional Infills</th>
<th>Sliding-joint Infills</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{S}_{\alpha-LS}(T_i)$ [g]</td>
<td>$\bar{S}_{\alpha-LS}(T_i)$ [g]</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{LS}$</td>
<td>$\lambda_{LS}$</td>
<td></td>
</tr>
<tr>
<td>EAL [%]</td>
<td>EAL [%]</td>
<td></td>
</tr>
<tr>
<td>CO-LS</td>
<td>1.73</td>
<td>0.66</td>
</tr>
<tr>
<td>LS-LS</td>
<td>1.64</td>
<td>0.59</td>
</tr>
<tr>
<td>DL-LS</td>
<td>0.52</td>
<td>0.49</td>
</tr>
<tr>
<td>O-LS</td>
<td>0.34</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 8: $\bar{S}_{\alpha-LS}(T_i)$ and $\lambda_{LS}$ values at the different limit states and EAL values for TI frames and SJI frames.

8 CONCLUSIONS

The paper presented a PBEE approach properly defined to assess and compare the performance of infilled frames with traditional infills and innovative infills with sliding-joint sub-panels. The adopted methodology is based on incremental dynamic analysis performed considering specific limit states defined to account for both structural and non-structural damage. The performances of the systems are compared through a reliability assessment carried out by accounting for both fragility and hazard to obtain probabilities of occurrence of each considered limit state. Finally, loss analysis provided the expected annual losses during the service life for the different infilled frame typologies.

The IDA curves show that the frame infilled with the innovative sliding joint technique tends to behave similarly to the bare frame in terms of strength, stiffness and failure modes. On the contrary, the response of traditionally infilled frame is characterized by significantly increased overall resistance but, in many cases, brittle shear failures due to the large shear demand related to the stronger infill-frame interaction. As for the non-structural limit states,
their attainment is significantly delayed in SJ frames (in terms of interstorey drift) because of the reduced tendency to undergo severe damage even in case of large interstorey drifts.

Although fragility curves of TI frames apparently show a significantly better performance, similar probabilities of occurrence ($P_f$) have been obtained for life safety and collapse limit states for the three considered cases. On the contrary, large reliability differences are observed for non-structural limit states, where $P_f$ of $O$-$LS$, $DL$-$LS$ and $ISD$-$LS$ for traditionally infilled frames was about 10 times, 13 times and 5 times the same probabilities evaluated in for SJI frames. Such reliability differences are due to the lower damage suffered by SJ infills with respect to TI frames, even for of large drifts, and also to the lower hazard associated with the longer vibration period of SJ infilled frames.

The performed expected annual loss assessment allows evaluating, for SJ infilled frames, an EAL equal to 0.40%, that is about a half of that of traditionally infilled frames (0.76%). This gain is entirely related to the reduced annual rates of exceeding evaluated for non-structural limit states in case of SJ infilled frames.

As a conclusion, the reported assessment highlights that frames infilled with the sliding joints technique results an effective design solution to improve reliability and reduce losses during the service life of masonry infilled RC structures.

9 ACKNOWLEDGEMENTS

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10 REFERENCES


OUT OF PLANE CAPACITY OF INFILLS AFTER IN PLANE LOADING: A PREDICTION ANALITICAL MODEL

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Abstract

Earthquakes produce on infills In-Plane (IP) and Out-Of-Plane (OOP) actions. The recent earthquakes have proved that the OOP collapse of infills is a diffused mechanism also in buildings designed to resist to seismic events in agreement to the most modern strategies. This fact makes the question arises about the safety of infills with respect OOP actions. The strong interaction between IP and OOP behaviour of infills traduces in a progressively reduction of the OOP strength. Further, only in few cases codes suggest adequate strategies to face this issue. For what above, in the paper the reduction of OOP strength because of the IP damage is studied by an extended numerical experimental campaign based on FE models to be considered as complementary to the very few laboratory experimental tests available in the literature.

Keywords: Infills, Out-of-Plane behaviour, In-Plane/Out-of-Plane interaction, Out-of-plane strength.
1 INTRODUCTION

In the last fifty years, the interaction between frame and infills deeply studied. Damage observed after earthquakes has highlighted the influence of infill panels on frames and the need to analyse in detail this phenomenon for a reliable assessment of buildings. A high number of studies refer to the in-plane (IP) behaviour (e.g. [1-12]). However, during earthquakes, the infills are subjected to out-of-plane (OOP) actions that frequently cause collapse. Further, the damage caused by IP actions reduces the OOP capacity of infills increasing the probability of OOP collapse with high risk for human health also in the case of building structures designed in agreement with the modern codes.

Different experimental studies have been carried out to observe the IP-OOP interaction. Dawe and Seah [13], Angel [14], Flanagan et Bennett [15]. Further, experimental studies were conducted by Komaraneni and Rai [16] and more recently Ricci et al [17]. A similar experimental campaign was carried out by Furtado et al [18]. Other tests can be found in [19-23].

Different codes face the problem of the OOP infills capacity (e.g. [24-26]). However, the need to have tools for the assessment of infilled frame structures of simple application has pushed different authors to modify the equivalent diagonal strut for the in-plane behaviour in such a way to assume a flexural behaviour suitable for the OOP behaviour. Different approaches have been proposed as explained in [26-32] that are the counterparts of much more classical strategies (e.g. [33-34]).

To date, the laws proposed for the reduction of the OOP capacity because of IP damage are based on very few tests. This study wants to increase the available data by a numerical experimentation and eventually to update the above laws. To this aim models have been formulated through the use of the Finite Element Method by using damage mechanical laws of the materials and an appropriate frame-infill interface as described in the next sections.

2 AVAILABLE OOP CAPACITY MODELS

The capacity models of infills subjected to OOP actions, under the hypothesis that the infill is in contact with the surrounding frame, are based on the arching action transmitted between frame and infill.

The arching actions and the dependence of the capacity on the compressive strength was first discussed by McDowell (1956) [19]. Dawe and Seah [13] developed a strength model based on virtual work concepts. The experimental studies carried out by them demonstrated a significant influence of the boundary conditions, in fact the infills everywhere in contact along the boundary with the frame had a capacity 4-5 times greater than others.

Bashandy et al. (1995) [35] extended the analytical method developed by McDowell et al. [19]. A simplification of the model by Angel [14] has been developed in FEMA 273 [25] and 356 [24].

The effect of the interaction between in-plane and out-of-plane responses is basic for a building safety assessment. First, experimental studies to recognize the IP-OOP interaction were carried out by Angel. (1994) [14]. As a result of the experimental campaign, the following out-of-plane capacity model was proposed:

\[ q_u = 2 \frac{f_{cm}}{h} R_1 R_2 A \]  

(1)
where $\lambda$ is a term that includes the effect of the maximum masonry compressive stress, the maximum strain and the ratio between width and height of the panel, all quantities being related to the slenderness ratio, while $R_1$ and $R_2$ are reduction factors.

The expressions of coefficients $R_1$ and $R_2$ are:

$$R_1 = \left( 1.08 + \left( \frac{h}{t} \right) \right) \left[ -0.015 + \left( \frac{h}{t} \right) \right] - 0.00049 + 0.000013 \left( \frac{h}{t} \right) \right] \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) 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\right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \rt
3 RESULTS AND DECAY LAWS

3.1 Numerical analyses

Once the model was validated, a numerical study was carried out. Three different types of frames were considered able to contain infills with height respectively 2400 mm and 3600 mm. The geometric characteristics are inserted in Fig. 2. Different types of masonry infills were studied having different thickness and different mechanical characteristics of the masonry. For each specimen different levels of the damage due to in-plane loading were caused before to be tested out-of-plane. The in-plane damage was obtained by making the infilled frames experience different levels of the drift during three loading cycles. In Tabs 1, 2 and 3 the specimens analyzed with the geometrical and mechanical characteristics of the infills and the frames are listed (the meaning of the symbols is obvious).

<table>
<thead>
<tr>
<th>model</th>
<th>$E_0$ [Mpa]</th>
<th>$\sigma_{mc}$ [Mpa]</th>
<th>$\varepsilon_{mc}$</th>
<th>$\sigma_{mcu}$</th>
<th>$\varepsilon_{mcu}$</th>
<th>$h/t$</th>
<th>$t$ [mm]</th>
<th>$h$ [mm]</th>
<th>$(\Delta_{ip})$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3000</td>
<td>3</td>
<td>0.002</td>
<td>$0.1\sigma_{mc}$</td>
<td>0.004</td>
<td>24</td>
<td>100</td>
<td>2400</td>
<td>0.1 0.35 0.5 1 2</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12</td>
<td>200</td>
<td>2400</td>
<td>0.1 0.35 0.5 1 2</td>
</tr>
<tr>
<td>6</td>
<td>3000</td>
<td>3</td>
<td>0.002</td>
<td>$0.1\sigma_{mc}$</td>
<td>0.004</td>
<td>8</td>
<td>300</td>
<td>300</td>
<td>0.1 0.35 0.5 1 2</td>
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<td>10</td>
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<td>3</td>
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<td>$0.1\sigma_{mc}$</td>
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<td>100</td>
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<td>300</td>
<td>300</td>
<td>0.1 0.35 0.5 1 2</td>
</tr>
</tbody>
</table>

Table 1. Properties of the numerical models and damage assigned
Figure 2. Geometric characteristics of infilled frames considered for the numerical analyses.
characteristics of infill material in tension (CDP model)  
frame geometry  
characteristics of concrete in compression (CDP model)

<table>
<thead>
<tr>
<th>model</th>
<th>( E_0 ) [Mpa]</th>
<th>( \sigma_{mt} ) [Mpa]</th>
<th>( \varepsilon_{mt} )</th>
<th>( \varepsilon_{tu} )</th>
<th>beam [cmxcm]</th>
<th>column [cmxcm]</th>
<th>( E_0 ) [Mpa]</th>
<th>( \sigma_{mc} ) [Mpa]</th>
<th>( \varepsilon_{mc} )</th>
<th>( \sigma_{mcu} )</th>
<th>( \varepsilon_{mcu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
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<td>0.05( \sigma_{mc} ) ( \sigma_{ml}/E_0 ) 4( \varepsilon_{mt} )</td>
<td>30x40</td>
<td>30x30</td>
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<td>25</td>
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<td>0.65( \sigma_{mc} )</td>
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</table>

Table 2. Properties of the infill materials in tension and of concrete in compression

characteristics of concrete in tension (CDP model)  
characteristics of steel rebars (elastic perfectly plastic model)

<table>
<thead>
<tr>
<th>model</th>
<th>( E_0 ) [Mpa]</th>
<th>( \sigma_{mt} ) [Mpa]</th>
<th>( \varepsilon_{mt} )</th>
<th>( \varepsilon_{tu} )</th>
<th>( E_s ) [Mpa]</th>
<th>( \sigma_s ) [Mpa]</th>
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<tbody>
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<td>20000</td>
<td>0.2( \sigma_{mc} ) ( \sigma_{ml}/E_0 ) 4( \varepsilon_{mt} )</td>
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<td></td>
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</table>

Table 3. Properties of the concrete in tension and of steel rebars

Each test was displacement controlled. In Fig. 3 a response due to in-plane loading is shown. The out-of-plane loading allowed to recognize the loosing of strength because of in-plane damage. In Fig. 4 some responses obtained by the FE analysis are inserted showing clearly a progressive reduction of strength for increasing in-plane damage.

---

Figure 3. In-plane loading cycles characterized by different maximum drifts.
3.2 OOP strength decays

From each curve of the type inserted in Fig. 4 the peak of strength was evaluated. Then, the decaying in the strength due to the damage produced by the in-plane loading was calculated. Considering the characteristics of the OOP load – OOP displacements curves, the peak of strength was obtained conventionally in correspondence of a strong variation (reduction) of the tangent stiffness.

Note that the hardening in each curve after the peak strength is the consequence of the arch effect perfectly reproduced by the FE model. This effect has not been always exhibited in the experimental tests probably for the low effectiveness of the infill surrounding supports (for example, the tests by Ricci et al. [17], do not highlight any hardening).

In Fig. 5 the numerical results obtained by the study here described and the (few) experimental results available in the literature are collected in terms of OOP strength decay ratio versus IP damage. The figure in question shows that the experimental results are in the cloud of the numerical results. If the results are differentiated associating at each of them the infill shape ratio h/t it is not possible to recognize a clear trend of the OOP strength decay depending on h/t. This fact is in contrast with the approach used by Angel - see Eq.2– that proposes a strong dependence on h/t. But this result validates the simpler approaches of Ricci et al. and Morandi et al. that are not dependant on the infill shape ratio h/t.

To this point a comparison of the experimental and numerical results with the models proposed by Ricci et al. and by Morandi et al. highlights that (observe Fig. 6 where the experimental results proposed in [38] are also included), while the model proposed by Morandi et al. is strongly not conservative between an IP damage, expressed in terms of IP drift, in the range 0%-1%, the model proposed by Ricci et al. is a good alternative, although it has been defined referring only to a low number of the experimental results.
Figure 5. Reduction of OOP strength versus IP damage: experimental and numerical results

Figure 6. Reduction of OOP strength versus IP damage: experimental and numerical results compared with the available models

Nevertheless, the model provided by Ricci et al. underestimates the mean loss of OOP strength for the IP drifts between 0% and 0.5% and overestimates the loss of OOP strength for the IP drifts between 0.5% and 2%. The overestimation of the loss of strength is evident both comparing the model proposed with the experimental and the numerical results. In Fig. 7 the lower bound curve, the curve fitting the results, the model by Ricci et al. and the experimental and numerical results are inserted. The lower bound curve and the fitting curve can be considered as two updated tools for the prediction of the OOP strength of infills that have experienced in-plane actions.
Figure 7. Reduction of OOP strength versus IP damage: experimental and numerical results compared with the proposed models.

The equations of the proposed curve fitting the experimental and numerical results is

$$F_f = \left[ \min \left( 1, 0.26 \, IDR^{-0.37} \right) \right]$$  \hspace{1cm} (7)

while the equation of the proposed lower bound curve is

$$F_{lb} = \left[ \min \left( 1, 0.15 \, IDR^{-0.49} \right) \right]$$  \hspace{1cm} (8)

The lower bound curve matches well the model of Ricci et al. after an IDR of 0.7% is reached.

4 CONCLUSIONS

In this paper the interaction IP-OOP behaviour of infills has been discussed starting from the strategies today available to predict the OOP response. The analysis of the literature highlights that the above prediction models are based on very few experimental tests, namely these models are not enough supported by real observations. For this reason, the paper provides a numerical analysis campaign based on FE models involving a number of infills with surrounding frames with different geometrical and mechanical characteristics in order to obtain data for a better definition of the decaying law for the OOP strength of infills to be used for the practical applications. The numerical investigation has shown that:

1) the models of the OOP infill capacity, at the moment available and based on very few experimental investigations, need to be improved;

2) the decaying of out-of-plane strength is strongly influenced by the damage undergone by infills because of in-plane loading and it is never negligible, not even for low IP drifts;

3) conversely of what proposed by Angel [14], the numerical investigation here discussed and the experimental data available in the literature show that there is not a
clear dependence of the decay of infill OOP strength on the shape ratio (height $h$ divided by width $t$); this observation is consistent with some of the approaches in the literature (e.g. Ricci et al. [17], Morandi et al [37]);

4) an updated model for the decaying of strength has been proposed in this paper fitting the data obtained by the numerical investigation carried out during this study and the data available in the literature from the laboratory tests;

5) also, a model for the lower bound OOP strength of infills has been proposed depending on the damage due to IP loading;

6) the proposed models update the available models in the sense of a more reliability of the prediction.

REFERENCES


SIMPLIFIED MODEL CALIBRATION FOR DYNAMIC RESPONSE ASSESSMENT OF INFILLED RC BUILDINGS

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Abstract

The calculation of engineering demand parameters (EDPs) such as interstorey drift ratios and peak floor accelerations due to earthquakes represents a fundamental step for the assessment of potential direct economic losses in existing buildings. Ideally, the performing of nonlinear time-history analyses on well-detailed finite element models would represent the most reliable procedure for the assessment of required EDPs. However, detailed procedures are not exploitable to large scale assessment due to the required computational effort. This paper presents a simplified model useful for the rapid assessment of EDPs in infilled moment resisting frames subjected to seismic loadings. The proposed simplified model consists of a system of joint masses concentrated in each storey, connected by means of nonlinear link elements. The procedure to construct the simplified model representing a multi-span, multi-storey frame is described. The interstorey nonlinear envelope is assembled starting from the single component behavior under the simplified hypothesis that the ends of the columns are restrained against rotation. The hysteretic behavior of nonlinear links is properly calibrated for an archetype 3D finite element building. The calibration is performed adopting a multi-objective Genetic Algorithm procedure that employs the results of nonlinear reverse pushover analyses. The results of proposed model are compared with those of a more refined finite element building model. The proposed model can be easily applied for simplified numerical analyses needed for the assessment of direct losses at the large scale.

Keywords: Infilled frame, Stick model, Parameter calibration, Genetic Algorithm, Engineering demand parameters.
1 INTRODUCTION

Computation of seismic risk is one of the primary steps towards investigation of the consequences of earthquakes in a region of interest, and for the planning and preparation of long-term risk reduction policies. The PEER performance-based earthquake engineering [1] sets up a framework for estimating seismic risk, where the economic losses are adopted as one of the metrics to measure it. The general approach to loss estimation in the PEER framework relies on structural analysis to calculate engineering demand parameters (EDPs) such as interstorey drift IDR and peak floor accelerations PFA throughout the structure during an earthquake, which are used to predict the damage in structural, non-structural components, and building contents. In [2]-[3] component-based damage assessment was performed to compute expected losses for case study buildings and nonlinear time history analyses are utilized for evaluation of EDPs. With the aim to simplify the estimation for practitioners, in [4] a simplified approach for component-based damage assessment, using pushover analyses and capacity spectrum method, was presented. Further simplification can be obtained relying on storey-based damage assessment, as proposed in [5]. However, the damage-loss relationships proposed by the authors are representative for building typologies that do not correspond to typical RC buildings in European-Mediterranean countries. The latter are often buildings not seismically designed or designed according to obsolete seismic codes; moreover, the presence of infills can significantly influence the seismic response and should be properly taken into account. Therefore, for the purpose of estimating seismic risk at the large scale, there is the need to develop simplified approaches for estimating IDR and PFA for existing RC buildings in European-Mediterranean countries, suitably accounting for the presence of infills [6]-[7].

This paper presents a simplified model useful for the rapid assessment of EDPs (IDR and PFA) in existing infilled moment-resisting frames subjected to seismic loadings. Starting from an idea already introduced in [8]-[9], a nonlinear simplified dynamic MDOF “stick” model is proposed. The proposed model consists of a system of joint masses concentrated in each storey, connected by means of nonlinear link elements. The “stick” model has been demonstrated to predict with sufficient accuracy the behavior of reinforced buildings at both building scale and large scale [10]. However, the method requires an appropriate calibration depending on the building typologies to be assessed.

In the next section the archetype building for which the calibration is performed is introduced. The geometrical and mechanical properties of all structural members is presented along with the definition of element envelope. In section 3, a refined finite element model that is adopted both to calibrate the hysteretic envelope of the nonlinear link elements and to compare the simplified model response is introduced. Section 4 describes the construction of the interstorey backbone curve and the calibration of the hysteretic behavior via Genetic Algorithm and presents the results of comparison between the simplified and the refined building model.

2 DESCRIPTION OF THE ARCHETYPE BUILDING

The archetype building consists of a 6-storey infilled moment-resisting frame with plan rectangular shape, regular both in plan and in elevation.

The structural and geometrical model for the frame is obtained by means of a simulated design procedure described in [11]. For this application a gravity load designed (GLD) buildings constructed in Italy before ’70s is considered. Complying with the building codes (e.g.,[12]-[13]) and the design practice in force at the time of construction (e.g.,[14]), the structural elements are firstly dimensioned. The longitudinal reinforcement in columns is de-
signed with reference to minimum longitudinal reinforcement geometric ratio prescribed by code for gravity load designed buildings, while the longitudinal reinforcement for beams is designed considering envelope moments deriving by limit load combination schemes according to construction practice. Note that, according to the Italian construction practice in force before ‘70s, the moment resisting frames are plane and formed by columns and deep beams only in one direction, since deep beams are mainly deputed to absorb gravity loads and are placed only in the direction perpendicular to the one-way slabs, while in the other direction the structure is formed by columns and beams that are embedded in the thickness of the horizontal slab; thus, only the perimeter frames are characterized by the presence of deep beams [14]. Finally, the perimeter frames are infilled, and it is supposed that the area of openings for the infills correspond to 20% of the infill area for each panel.

2.1 Geometrical and mechanical properties

The geometry for the building is assumed of 3 bays in the longitudinal and 2 bays in the transversal direction. The bay span in both the longitudinal and transversal direction are \( a_x = a_y = 4.0m \) and an interstorey height of \( a_z = 3.0m \) is assumed. Adopting the simulated design procedure described above, the structural model for the archetype building is obtained. Longitudinal frame columns dimensions are 30×30cm at each storey, except that for interior columns of the intermediate frame that are 40x30cm at first storey and 35x30cm at the second storey. Deep beams are 40x30cm at each storey for perimeter longitudinal and transversal frames and 45x30cm for the intermediate longitudinal frame, while flat beams (only in the transversal direction) are 20x35cm for every intermediate transversal frame.

Column reinforcement varies between 6\( \phi 16 \) for first storey interior columns and 4\( \phi 12 \) for first storey corner columns. For every column, shear reinforcement consisting of \( \phi 6 \) stirrups with 90-degree hooks and 30cm spacing is assumed. In every structural element, according to construction practice in force before ‘70s, plain rebars are adopted. Referring to the concrete
and steel properties, a compressive concrete stress $f_c = 25$ MPa and a steel tensile yielding stress $f_y = 399$ MPa are chosen as representative values for GLD buildings constructed in the decade '62-'71. Infill panels are realized with a double layer of hollow clay bricks having (80 + 120) mm thickness. These infill masonry configurations are widely used in European building practice [15]. The elastic shear modulus is assumed equal to $G_w = 1350$ MPa. The $E_w$ is assumed equal to $0.4G_w$, while cracking strength of the masonry is considered linearly dependent on $G_w$ according to boundary values indicated in [16].

### 2.2 Definition of element envelope

For RC columns, a multi-linear moment-rotation envelope is built with cracking and yielding as initial characteristic points. Moment at yielding ($M_y$) is calculated according to the simplified formulation proposed by Biskinis and Fardis [17], while the rotation at yielding ($\theta_y$) is identified by $M_y$ and the secant stiffness ($E_{ly}$) provided by Haselton et al. [18]. The brittle failure of non-conforming elements may significantly impact the global behavior of existing structures [19]. Thus, for each column, the expected failure mode is determined by comparing the yielding shear ($V_y$), calculated as the ratio between $M_y$ and the shear span of the column ($L_v$), and the shear strength ($V_n$) calculated according to EC8 [20]. The $L_v$ is assumed equal to one half of the column height. Despite $V_n$ may be significantly influenced by the column axial load variation due to horizontal loads, here only initial gravity loads are considered [21]. Depending on the ratio $V_y/V_n$, three possible failure modes are expected: flexure, flexure-shear or pure shear failure. If $V_y/V_n < 1$ for any value of ductility demand, RC column is expected to fail in flexure and a three-branch backbone is built, in which the ultimate rotation capacity corresponds to the ultimate chord rotation for ductile members [22]. If $V_y/V_n \geq 1$, the column is expected to fail in shear or flexure-shear. In the latter case a four-branch backbone is built that includes a degrading slope identified by the ultimate shear and the ultimate axial rotation capacities evaluated according to the simplified relationship proposed in Aslani and Miranda [2]. Due to the presence of plain reinforcement bars, longitudinal bar-slip effect in columns and beam-column joints cannot be neglected. Thus, the bar-slip effect is accounted adopting the model proposed in [23]-[24].

Another characteristic of existing GLD RC frames is the total lack of transverse reinforcement in the joint region, thus the possible joint failure should be considered. According to Pampanin [25], the joint shear capacity in beam-column joints with inadequate structural detailing can be directly related to principal tensile/compressive stresses. The beam-column joint backbone is thus constructed adopting the principal stresses and the panel rotation limits proposed in [25]-[26] transformed into the corresponding moment-rotation relationship directly derived from equilibrium considerations.

The behavior of infill panels adopts the force-displacement envelope evaluated according to the model proposed by Panagiotakos and Fardis [22], see Figure 2. In this model, the four branches lateral force-displacement relationship is constructed depending on the geometry of the surrounding frame, and on both mechanical and geometrical characteristics of the infill masonry.

The geometry of the infill panel is introduced in terms of gross length and height, $L$ and $H$, clear length and height, $L_w$ and $H_w$, clear diagonal length, $d_w$, and in terms of inclination of the diagonal strut $\theta = \arctan(h_w/l_w)$ with respect to the horizontal plane. The mechanical characteristics of the masonry are expressed in terms of elastic shear modulus, $G_w$, Young’s modulus, $E_w$, and shear cracking strength, $\tau_{cr}$. Finally, the equivalent strut width, $b_w$, is determined according to Mainstone’s formula [27] depending on quantities reported above and on the
moment of inertia, I_c, and Young’s modulus, E_c, of columns. When openings are present in the infill panel, e.g., to accommodate windows or balconies, both the stiffness and the strength of the infill panel are reduced.

\[
b_w = 0.175 \lambda h_w - 0.4 d_w
\]

\[
\lambda = \frac{h_w \sin(2\theta)}{4 I_c h_w}\]

\[
F_{cr} = \tau cr L_w L_h
\]

\[
b_w = \max(0; 1-1.8 A_o/A_p) \quad [28]
\]

In this study the presence of the opening is considered introducing a reduction factor \(\lambda_0 = \max(0; 1-1.8 A_o/A_p)\) [28] that modifies both the stiffness and the strength of the infill panel, where \(A_o\) is the area of openings and \(A_p\) the area of infill panels.

3 REFINED FINITE ELEMENT MODEL

A fixed-base three-dimensional finite element MDOF model developed using [29] is used to simulate the seismic response of the building. Figure 3 shows a schematic representation of the generic frame of the building modeled. The frame elements are modeled using lumped plasticity elements consisting of two inelastic rotational hinges connected in series by an elastic beam-column element. The inelastic behavior of beams and columns is conveniently characterized by a multilinear moment–rotation relationship in the plastic hinges, described by means of a set of characteristic points as indicated in §2. As noted in [19] and [30], since the frame members are modeled as an elastic element connected in series with rotational springs at either end, the stiffness of these components must be modified so that the equivalent stiffness of this assembly is equivalent to the stiffness of the actual frame member. Following the approach proposed in [30], the rotational springs are made “n” times stiffer than the rotational stiffness of the elastic element in order to avoid numerical problems. To ensure the equivalent stiffness of the assembly is equal to the stiffness of the actual frame member, the stiffness of the elastic element must be “(n+1)/n” times greater than the stiffness of the actual frame member. Similar considerations must be accounted for in the definition of the degrading branch slope of flexure-shear critical members [30]. The hysteretic behavior of column/beam elements is modeled adopting the Pinching4 material [31] with hysteretic rules proposed by [32] for old reinforced concrete columns.

To account for bar-slip effect, a zero-length bar-slip plastic hinge is added at the extremities of either ends, no hysteretic degradation is assumed for these elements.

The joint behavior is modeled using the “scissor” model [33], which includes the pinching hysteretic behavior to account for the non-linear shear deformation of the joint. The joint constitutive model adopts the backbone curve proposed in [26] with equivalent moment-rotation relationship obtained using the formulation proposed in [34]. The hysteretic behavior for
beam-column joints is simulated adopting the Pinching4 material with hysteretic rules proposed in Hassan [35].

The masonry infills are modelled by means in-plane equivalent diagonal struts carrying loads only in compression. The hysteretic behavior for these elements is simulated adopting a Pinching4 material with parameters defining the cycling degradation proposed in Lima et al. [36]. Note that such a simplified model is unable to simulate the local effects on the columns due to the presence of the masonry infills since it does not properly reproduce realistic moments and shear forces in the columns. For this reason, this model may lead to overestimate the strength and ductility capacity of the building [37].

Finally, the presence of RC one-way slabs is simulated by means of an elastic shell element. Classical Rayleigh damping is adopted with a value of 5%.

4 SIMPLIFIED MODEL

This section introduces the simplified model adopted to analyze the response of the arche-type building. To allow the prediction of the building response in large-scale seismic simulations, a multiple degree-of-freedom (MDOF) non-linear shear model (NLSM) is adopted (see Figure 4). In particular, each storey is discretized into a nonlinear shear spring with the following assumptions: 1) The model assumes that the mass of each storey is concentrated on its elevation and represented by lumped masses; 2) The seismic response of the structure is dominated by interstorey shear deformations; 3) The building has regular planar layout.
Some authors (e.g., [10], [38]) have shown that NLSM are suitable to describe the nonlinear characteristics and failure modes of multistorey buildings properly capturing the damage concentration in each storey.

![Non-linear Shear Model (NLSM)](image)

The elastic and inelastic properties of the shear springs are regulated by the properties of the infill wall and the frame. To determine the parameters in the interstorey hysteretic model, a method based on a simplified procedure to combine the behavior of multiple components is adopted [39].

### 4.1 Backbone curve for the simplified model

The interstorey backbone defined in terms of shear-displacement response is assembled considering that ends of the columns are restrained against rotation (Shear Type model), as already proposed in [39]. This simplifying hypothesis was already introduced in [8] and allows to reproduce the seismic response of existing buildings with a reasonable degree of approximation.

Once that force-displacement/moment-rotation relationships is defined for each relevant member, these are transformed in the corresponding shear-displacement curves and then, considering the RC frames and infill elements at the same storey as acting in parallel, a multi-linear interstorey shear-displacement relationship is constructed for each storey. The comparison in terms of Pushover curve (1st mode proportional load pattern) between the refined finite element model (FEM) presented in §3 and the NLSM, in which a multi-linear backbone for the interstorey shear-displacement envelope is adopted, is depicted in Figure 5.

As can be noted from Figure 5, the NLSM can accurately describe the global behavior of the FEM. The very low scatter between FEM and NLSM is related to assumption of shear-type model. However, this assumption to calibrate the interstorey shear-displacement relationship does not introduce significative bias. In fact, the presence of slab, that is explicitly simulated in the FEM, introduces a partial restrain against beam rotations limiting out-of-plane the inflection of slab making the shear-type assumption partially valid.
A tri-linear backbone curve is adopted to properly represent the nonlinear behavior of the multiple components at the same storey. The adoption of a tri-linear backbone curve model can accurately represent the interstorey behavior of a structure with acceptable modeling complexity and computational accuracy ([10],[40]). Figure 6 shows the backbone curve obtained with the proposed procedure and the corresponding tri-linear backbone adopted to approximate the interstorey shear-displacement (V-\(\Delta\)) curve in the NLSM.

In particular, the three points describing the interstorey backbone are defined as follows: 1) The first point corresponds to the attainment of first nonlinearity in the storey. Generally, due to the brittle behavior of the infill panels, this point corresponds to the cracking of the first infill panel. 2) The second point corresponds to the attainment of the maximum interstorey shear force. Due to the significant contribution to the lateral strength of the infill panels, this
point generally corresponds to the attainment of the maximum shear resistance of the infill panel. 3) The last point of the backbone curve corresponds to the collapse of the first infill panel. As can be noted by Figure 6, the tri-linear backbone is able to describe with enough accuracy the interstorey behavior only up to the failure of infill panels while the post infill failure zone is completely neglected. The latter part of the interstorey behavior could be important only when the complete interstorey behavior cannot be neglected (e.g. collapse simulation). However, since the prediction of collapse capacity is out of the scope of this paper, it is considered that a tri-linear backbone is enough accurate to describe the interstorey behavior.

4.2 Hysteresis loop calibration via Genetic Algorithm

The NLSM requires the calibration of a nonlinear interstorey hysteretic model for each storey of the building. While the backbone curve can be easily calibrated according to the simplified procedure indicated in §4.1, the calibration of hysteretic rules represents a more complex issue having to account for different degradation modes of multiple components in the same storey.

To simplify the hysteresis calibration problem, some authors (e.g. [10],[38]) proposed to adopt a single-parameter hysteretic model. However, the scope of this study is to validate the reliability and accuracy of the model and its calibration for a specific building. For this reason, a more complex hysteretic model is adopted. The Pinching4 model [31] is here employed to simulate the pinched response and cyclic degradation of strength and stiffness in each storey. The Pinching4 material requires the definition of 21 parameters that govern stiffness, strength and ductility degradation.

For each storey, the calibration is performed adopting Finite Element model composed of a single link element is created, with one end point fixed and a displacement loading history applied to the other end. For each storey the backbone defined in §4.1 is adopted as monotonic envelope, and the interstorey hysteretic response is calibrated in order to match the reverse cyclic pushover displacement history performed for the generic storey of the refined FEM model described in §3.

The calibration of Pinching4 parameters has been carried out adopting a Genetic Algorithm (GA) optimization procedure [41] with Matlab©. The GA allows the multi-objective optimization based on a selection process that mimics biological evolution. Starting from an initial population of individual solution, the GA repeatedly modifies the population randomly selecting individuals from the current population and using them as “parents” to produce “children” for the next generation. By simulating also “crossover” and “mutations”, the GA allows through successive generations to evolve toward an optimal solution, that for the multi-objective optimization is represented by a Pareto optimal solution. A solution is referred to as Pareto optimal if it is not dominated by any other solution. The scope of the GA is the minimization of the fitness function. In this study a multi-objective calibration problem is set considering two objective function: 1) Minimize the scatter between the cumulated energy of the FEM model and the single link element for each storey; 2) Minimize the scatter between the Force history of the FEM model and the link element for each storey.

4.3 Comparison of the NLSH with FEM

The main scope of this work is to establish a general framework for the calibration of the hysteretic behavior of a simplified model that is able to accurately represent the response of a
structure with a low computational effort and sufficient accuracy in prediction of EDPs. To validate the reliability and accuracy of the proposed model and the calibration method adopted, the response of the calibrated NLSM is compared with that of the refined FEM for a single earthquake ground motion selected within the set of the far field record set included in FEMA-P695 [42].

Figure 7 compares the response of the NLSM with that of the refined FEM subjected to the “21_68_PEL090” record scaled to 0.2g in terms of IDR for the first storey.

As it can be noted, the NLSM response in terms of IDR time-history shows a fairly good agreement with the more refined FEM. In particular, the response of two model is almost perfectly superimposed up to the IDR peak response. After 15s a higher scatter in the response can be evidenced. This scatter is mainly due to the impossibility for the NLSH to properly account for damping forces arising in the more refined FEM. The prediction of post-peak response is strongly influenced by the adopted model, as also evidenced in [43]. However, the model is sufficiently accurate to predict the maximum IDR and the localization of maximum deformations in the model. In fact, the maximum IDR occurs in the first storey, and is predicted with a 1% approximation.

Figure 7. IDR time-history for the NLSM and the FEM subjected to 21_68_PEL090 scaled to 0.2g (first storey).

Figure 8 shows the results in terms of floor acceleration time-history for the NLSM and the FEM. In this case, the scatter between the two models is higher. In particular, the PFA for the
FEM is 0.48g, while the PFA for the NLSH is 0.42g, resulting in a prediction error of 12% for the first storey.

Figure 9. Distribution of IDRmax (a) and PFA (b) along the height for the NLSM and the FEM subjected to 21_68_PEL090 scaled to 0.2g.

Figure 9(a)-(b) show the distribution of IDRmax and PFA along the height for the NLSM and the FEM. Figure 9(a) evidences that the maximum IDR occurs at the first storey, while IDRmax decreases along the height. In particular, the NLSM shows a good agreement compared to the more refined FEM, resulting in a maximum scatter in prediction that occurs at the fifth storey, in which IDR_{max,NLSM}=0.0406%, while the IDR_{max,FEM}=0.0454% for a scatter of 12%. Figure 9(b) shows that the PFA increase along the height up to a peak value of about 1.0g. Despite the shape distribution of PFA is well represented by the NLSM, a higher difference between NLSM and FEM occurs. In particular, the NLSM generally underestimate PFA, with a mean error of 11%. However, the maximum scatter between the two models occurs at the second storey, in which PFA_{NLSM}=0.51g, while the PFA_{FEM}=0.70g for a scatter of 12%.

Despite the calibration of the NLSM has been performed only for one structure, and the comparison performed only for a single record, the proposed model shows promising results in the prediction of IDR and PFA distribution along the height. In particular, the NLSM shows an appropriate level of accuracy to represent the response of a structure in terms of prediction of EDPs with a low computational effort and sufficient accuracy at large scale. The framework for the calibration of the hysteretic behavior of the simplified model presented herein could be adopted to calibrate the hysteretic behavior of building typologies for the assessment of large building portfolios at regional scale.

5 CONCLUSIONS

A simplified model and the associated parameter calibration procedure are introduced for the rapid assessment of engineering demand parameters in existing infilled reinforced concrete buildings. The proposed simplified model consists of a multiple degree-of-freedom nonlinear shear model.

The main scope of this work is to define a framework for the calibration of the hysteretic behavior of a simplified model to accurately represent the response of a structure in prediction of engineering demand parameters with a low computational effort and sufficient accuracy at large scale. The procedure to construct the simplified model for a multi-span, multi-storey re-
inforced concrete frame is described. In particular, the interstorey nonlinear envelope is assembled starting from the single component behavior under the simplified hypothesis that the ends of the columns are restrained against rotation. The hysteretic behavior of nonlinear links is performed adopting a multi-objective Genetic Algorithm procedure that employs the results of reverse static pushover performed on a more refined finite element model. The calibration procedure is demonstrated for an existing six-story building obtained via simulated design procedure. The results of the proposed method are compared with those of a more refined finite element model that properly accounts for the typical features of existing reinforced concrete buildings. The simplified model showed a good agreement in prediction of engineering demand parameters when compared to the more refined model.

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PRELIMINARY EXPERIMENTAL ASSESSMENT OF STRENGTHENED MASONRY INFILLS UNDER OUT-OF-PLANE ACTIONS

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Abstract

The characterization of masonry infills’ in-plane and out-of-plane response under seismic actions is nowadays deemed increasingly important for a reliable assessment of the seismic performance of buildings.
In this work, experimental tests performed at the Department of Structures for Engineering and Architecture of University of Naples Federico II to characterize the out-of-plane response of unreinforced and reinforced masonry infills are presented. Namely, an out-of-plane test is performed on an “as-built” specimen, whose performance is compared with the experimental response of specimens strengthened by applying two different techniques. The effectiveness of the strengthening techniques is evaluated. In addition, mechanical-based approaches for the evaluation of the out-of-plane strength of both unreinforced and reinforced infills are applied. The effectiveness of these approaches is validated based on the experimental tests’ results.

Keywords: Infill wall, out-of-plane, seismic, strengthening, composite, strength mechanism.
1 INTRODUCTION

In past and recent works, it has been demonstrated that unreinforced masonry (URM) infill walls in Reinforced Concrete (RC) are prone to damage and, potentially, collapse due to out-of-plane (OOP) seismic actions [1-9]. Unfortunately, the experimental and numerical outcomes presented in the literature in the last 30 years have been confirmed by the experience of past and recent earthquakes, during which OOP collapses of masonry enclosures have been registered.

For all these reasons, experimental and numerical research also investigated the effectiveness of strengthening and reinforcement techniques to improve the OOP seismic performance of masonry infills. More specifically, the efficiency of steel reinforcement (internal vertical and/or horizontal rebars or wire meshes plastered to the infill surface) was investigated, e.g., in [1-2, 10-13]. The effectiveness of Fiber Reinforced Polymers (FRP) was tested, e.g., in [14-17], while the OOP strengthening capacity of Fabric Reinforced Cementitious Matrices (FRCM) was assessed, e.g., in [18-20]. Modelling proposals for the evaluation of the OOP response and strength of reinforced/strengthened masonry infills were proposed in [10, 15, 21-23].

In this work, the preliminary results of OOP experimental tests on unreinforced and strengthened masonry infills in RC frames are presented. The main aims of the experimental programs are:

i. Investigating the strengthening effectiveness of two different techniques, a Glass FRCM and an FRP, by comparing the experimental response of two strengthened specimen with that exhibited by an as-built URM infill wall;

ii. Investigating the strength mechanism occurring in strengthened masonry infills under OOP actions, namely the occurrence of a flexural strength mechanism and of arching effect.

Especially to achieve the second aim, all the specimens tested were mortared to the confining structural elements only along the upper and the lower edges. In other words, the infill walls tested were detached from the RC columns of the confining frame. In this way, only one-way vertical arching effect can occur in the unreinforced specimen (OOP_2E_AB), while in the strengthened specimens (OOP_2E_RG and OOP_2E_EQ) only one-way flexural bending and/or one-way vertical arching are expected to occur.

2 CHARACTERISTICS OF SPECIMENS AND MATERIAL PROPERTIES

The RC frames tested were realized with class C28/35 concrete and reinforcement rebars with characteristic yielding strength equal to 450 N/mm2. The frame was 2:3 scaled and was designed according to the seismic provisions provided by the Italian building code NTC2008 [24]. Construction drawings of the RC frames are reproduced in Figure 1.

The experimental tests were performed on masonry infills realized with clay hollow bricks placed with horizontal holes and class M5 cementitious mortar. The clay hollow bricks used were 250 mm high, 250 mm wide and 80 mm thick. Infill walls were 1830 mm high, 2350 mm wide and 80 mm thick. The mechanical properties of masonry reported in Table 1 are referred to masonry wallets identical (for material, dimensions and workmanship) to those realized for the characterization of the specimens herein presented.
Specimen OOP_2E_AB was tested without any strengthening material.

Specimen OOP_2E_RG was strengthened by using a fiber-reinforced cementitious mortar with a bidirectional primed alkali-resistant fiberglass mesh with mesh size equal to 12.7 mm x 12.7 mm. More specifically, a 5 mm thick layer of plaster was applied on the infill surface; then, the fiberglass mesh was embedded in the plaster by covering it with a 5 mm thick layer of plaster. The fiberglass mesh was not connected to the RC frame elements. The mechanical properties of both materials are reported in Table 1.

Specimen OOP_2E_EQ was strengthened by using a bidirectional primed fiberglass fabric applied to the infill surface by using a water and polyurethane-based adhesive. The mechanical properties of the fiberglass fabric are reported in Table 1.

In all cases, the strengthening materials were applied only on one side of the infill, i.e., on the surface expected as in tension given the (monotonic) loading direction.

The experimental tests were performed by applying the OOP load in displacement control on four loading points. Further details on the loading system, on the experimental setup and on the instrumentation layout can be found in [25]. For what concerns the instrumentation layout, it is worth to mention that a Linear Variable Displacement Transducer (LVDT) was placed, in the vertical direction, to measure the outward displacement of the central point of the RC frame upper beam. Such a vertical displacement is due, potentially, to the deflection of the RC frame upper beam due to vertical arching thrusts forming in the infill thickness if arching action occurs.
<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Units</th>
<th>Average Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>Mean compressive strength</td>
<td>[N/mm²]</td>
<td>46.2</td>
</tr>
<tr>
<td>Reinforcement steel</td>
<td>Mean yielding stress</td>
<td>[N/mm²]</td>
<td>497</td>
</tr>
<tr>
<td>Masonry</td>
<td>Vertical compressive strength</td>
<td>[N/mm²]</td>
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</tr>
<tr>
<td>Masonry</td>
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<td>[N/mm²]</td>
<td>1090</td>
</tr>
<tr>
<td>Masonry</td>
<td>Crushing strain</td>
<td>[%]</td>
<td>0.17</td>
</tr>
<tr>
<td>Fiber reinforced mortar</td>
<td>Compressive strength</td>
<td>[N/mm²]</td>
<td>37.6</td>
</tr>
<tr>
<td>Fiber reinforced mortar</td>
<td>Elastic modulus in compression</td>
<td>[N/mm²]</td>
<td>10000*</td>
</tr>
<tr>
<td>Fiber reinforced mortar</td>
<td>Tensile strength (flexural)</td>
<td>[N/mm²]</td>
<td>8.46</td>
</tr>
<tr>
<td>Fiberglass mesh</td>
<td>Weight</td>
<td>[g/m²]</td>
<td>125*</td>
</tr>
<tr>
<td>Fiberglass mesh</td>
<td>Equivalent thickness of dry mesh</td>
<td>[mm]</td>
<td>0.024*</td>
</tr>
<tr>
<td>Fiberglass mesh</td>
<td>Tensile strength</td>
<td>[N/mm²]</td>
<td>1276*</td>
</tr>
<tr>
<td>Fiberglass mesh</td>
<td>Elastic modulus</td>
<td>[N/mm²]</td>
<td>72000*</td>
</tr>
<tr>
<td>Fiberglass mesh</td>
<td>Ultimate strain</td>
<td>[%]</td>
<td>1.8*</td>
</tr>
<tr>
<td>Fiberglass fabric</td>
<td>Weight</td>
<td>[g/m²]</td>
<td>286*</td>
</tr>
<tr>
<td>Fiberglass fabric</td>
<td>Equivalent thickness of dry mesh</td>
<td>[mm]</td>
<td>0.057*</td>
</tr>
<tr>
<td>Fiberglass fabric</td>
<td>Tensile strength</td>
<td>[N/mm²]</td>
<td>1620*</td>
</tr>
<tr>
<td>Fiberglass fabric</td>
<td>Elastic modulus</td>
<td>[N/mm²]</td>
<td>42000*</td>
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<tr>
<td>Fiberglass fabric</td>
<td>Ultimate strain</td>
<td>[%]</td>
<td>4.0*</td>
</tr>
</tbody>
</table>

*: nominal value declared by the producer

Table 1: Materials’ mechanical properties.

3 EXPERIMENTAL RESULTS

3.1 Test OOP_2E_AB

The OOP force-displacement response of the as-built URM specimen OOP_2E_AB is reported in Figure 2a. The first visible horizontal crack appeared near the infill mid-height, on the right side, for an OOP force equal to 5.7 kN and for an OOP displacement equal to 2.1 mm. At this point, the deflection of the upper beam, revealing the occurrence of arching action, begun, as shown in Figure 2b. A significant reduction of the infill OOP tangent stiffness was observed. Up to the attainment of the maximum load resistance, the first crack extended and covered the entire width of the infill. The maximum strength was equal to 9.9 kN and was registered for an OOP displacement equal to 13.9 mm. A steep softening branch was then registered, with a 20% strength degradation at an OOP displacement equal to 17.1 mm. During the last part of the test, the upper corners of the infill wall were crushing with detachment of bricks’ exterior tiles. A picture of the specimen at the end of test is reported in Figure 3.

![Figure 2](image-url)

Figure 2: Force-displacement response of specimen OOP_2E_AB: (a) OOP force-displacement, (b) OOP force-beam deflection.
3.2 Test OOP_2E_RG

The OOP force-displacement response of the specimen strengthened with mortar and fiberglass mesh, OOP_2E_RG, is reported in Figure 4a. The first visible horizontal crack appeared near the infill mid-height, in the central part of the infill, for an OOP force equal to 20.3 kN and for an OOP displacement equal to 2.8 mm. A drop in the load bearing capacity was observed up to a load equal to 17.0 kN registered for an OOP displacement equal to 3.6 mm. At this point, the deflection of the upper beam, revealing the occurrence of arching action, begun, as shown in Figure 4b. A significant reduction of the infill OOP tangent stiffness was observed. Up to the attainment of the maximum load resistance, the first crack extended and covered the entire width of the infill. The maximum strength was equal to 28.4 kN and was registered for an OOP displacement equal to 11.0 mm. A very steep softening branch was then registered, with a 20% strength degradation at an OOP displacement equal to 13.0 mm. During this phase, hairline horizontal cracks appear along the entire width of the infill, in its upper part. The test ended at an OOP force equal to 0.6 kN and at an OOP displacement equal to 16.1 mm. During the last part of the test, a wide horizontal crack appeared at the interface between the infill wall and the RC upper beam. A picture of the specimen at the end of test is reported in Figure 5.
3.3 Test OOP_2E_EQ

The OOP force-displacement response of the specimen strengthened with the fiberglass fabric, OOP_2E_RE, is reported in Figure 6a. Please note that, in this case, cracks in the infill during the test are not immediately visible, as the strengthening fabric cover up them. The first noticeable nonlinearity in the force-displacement graph, at which a significant reduction of the infill OOP tangent stiffness was observed, occurred for an OOP force equal to 8.2 kN and for an OOP displacement equal to 1.7 mm. At this point, the deflection of the upper beam, revealing the occurrence of arching action, begun, as shown in Figure 6b. The maximum strength was equal to 19.1 kN and was registered for an OOP displacement equal to 9.7 mm. A very steep softening branch was then registered, with a 20% strength degradation at an OOP displacement equal to 10.1 mm. During this phase, hairline horizontal cracks appear along the entire width of the infill, in its upper part. The test ended at an OOP force equal to 5.6 kN and at an OOP displacement equal to 11.8 mm. During the very last part of the test, a wide horizontal crack appeared in the lower part of the infill. A picture of the specimen at the end of test is reported in Figure 7.


4 DISCUSSION OF THE EXPERIMENTAL RESULTS

4.1 Comparison of the performance of URM and strengthened infills

The OOP response of the specimens is compared in Figure 8 and in Table 2. The ultimate displacement reported in Table 2 is conventionally defined as the OOP central displacement at an OOP strength degradation equal to 20%.

![Figure 8: Comparison of the responses of specimens: (a) OOP force-displacement, (b) OOP force-beam deflection.](image)

<table>
<thead>
<tr>
<th>specimen</th>
<th>strength [kN]</th>
<th>displacement at maximum [mm]</th>
<th>ultimate displacement [mm]</th>
<th>ductility</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOP_2E_AB</td>
<td>9.9</td>
<td>13.9</td>
<td>17.1</td>
<td>1.23</td>
</tr>
<tr>
<td>OOP_2E_RG</td>
<td>28.4 (+187%)</td>
<td>11.0</td>
<td>13.0</td>
<td>1.18</td>
</tr>
<tr>
<td>OOP_2E_EQ</td>
<td>19.1 (+93%)</td>
<td>9.7</td>
<td>10.1</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 2: Comparison of significant tests’ results.

It is observed that the strength of specimen OOP_2E_RG is nearly three times the strength of the as-built specimen, while the strength of specimen OOP_2E_EQ is nearly two times the strength of the as-built infill. However, there is no improvement of the ductility capacity of the infill wall. In other words, the strengthening techniques tested are effective in increasing the OOP strength of infills while they do not influence their ductility capacity.

The initial stiffness of the specimens is quite identical for specimens OOP_2E_AB and OOP_2E_EQ, as expected, as in the last case the strengthening fabric has a negligible thickness.
with respect to the masonry infill; on the contrary, a greater initial stiffness is observed for specimen OOP_2E_RG, in which the strengthening plaster layer is 10 mm thick.

As expected, the OOP load carried by the strengthened infills by a pure flexural strength mechanism, i.e., the load at which arching action forms, is higher than that carried by the URM specimen. Namely, the OOP load at the occurrence of arching action is equal to 5.7 kN for specimen OOP_2E_AB, to 17.0 kN (+198%) for specimen OOP_2E_RG, to 8.2 kN (+44%) for specimen OOP_2E_EQ.

It is worth to mention that the specimen strengthened with the fiberglass fabric (OOP_2E_EQ) exhibited a very slight damage during the test, with the formation of a noticeable crack (with no overturning of masonry parts or brick tiles) only in the very last part of the test, after the load-bearing capacity drop corresponding to conventional collapse. In addition, no visible damage was registered for the strengthening fabric.

4.2 Preliminary modification and application of Dawe and Seah’s model for the OOP strength prediction

In [10], Dawe and Seah proposed a model based on the application of the Principle of Virtual Works for the evaluation of the OOP response of URM infills. With a slight modification, the model is adapted to account for the potential presence of steel rebars for reinforcement. For this reason, for the calculation of the OOP strength of the reinforced infill, the model proposed by Dawe and Seah assumes that steel rebars have yielded. Of course, this assumption cannot be used when dealing with FRCM and FRP, given that these materials have an elastic-brittle behaviour, with no ductility.

Hence, the model by Dawe and Seah is herein modified to account for the specific response of the materials used: a parabolic behaviour is assumed for masonry up to the crushing strain, at which a drop in the compressive strength is assumed; a linear elastic behaviour is assumed for FRP and FRCM up to the fiberglass net or fabric ultimate strain, then, a brittle failure of the composite is assumed.

The strain at masonry outermost compressed fiber and in the composite material are calculated by applying Dawe and Seah’s proposal. However, the hypothesis of plane section is removed and two coefficients are introduced: the first, $\alpha_t$, is used to define the effective length for the composite strain calculation; the second, $\alpha_c$, is used to define an effective crushing strain for masonry, which is expected to be lower than that calculated for masonry wallets due to size effects. Namely, $\alpha_t$ is multiplied for the infill length in the vertical direction; $\alpha_c$ is multiplied for masonry crushing strain, $\varepsilon_{crush}$, determined by testing masonry wallets and reported in Table 1. Both are expected to be lower than the unit. Their value is determined based on the experimental results, in order to have the minimum error both in the prediction of the OOP strength of the strengthened specimen and, contemporarily, in the prediction of the OOP displacement at conventional collapse ($d_u$).

Dawe and Seah’s model is also applied, in its original version, for the prediction of the OOP strength of the as-built specimen OOP_2E_AB. Further details on the application of Dawe and Seah’s model are reported in [26].

The results of the application of Dawe and Seah’s model are reported in Table 3.
It is observed that $\alpha_t$ is quite different for specimens OOP_2E_RG and OOP_2E_EQ, which is expected for different strengthening materials. On the contrary, $\alpha_c$ seems to be not very different for the same specimens. Clearly, these are only preliminary considerations: further experimental efforts are needed to validate the potential effectiveness and replicability of the proposed modification of Dawe and Seah’s model.

5 CONCLUSIONS

In this study, out-of-plane experimental tests are performed on infill walls in RC frames. Three specimens are tested: an as-built unreinforced masonry specimen (OOP_2E_AB), a specimen strengthened by applying an FRCM with fiberglass mesh (OOP_2E_RG), a specimen strengthened by applying a fiberglass fabric (OOP_2E_EQ). The specimens are constructed in order to have only one-way vertical bending and/or one-way vertical arching action. The following conclusions can be drawn:

- Specimen OOP_2E_AB exhibited an OOP strength equal to 9.9 kN; specimen OOP_2E_RG exhibited an OOP strength equal to 28.4 kN; specimen OOP_2E_EQ exhibited an OOP strength equal to 19.1 kN. The application of the FRCM with fiberglass mesh increased the OOP strength by +187%; the application of the fiberglass fabric increased the OOP strength by +93%.

- A preliminary modification of Dawe and Seah’s model is proposed to predict the OOP strength of masonry infills strengthened with composite materials. However, the proposed model needs empirical correction coefficient which have been calibrated and should be validated in the future with a great number of further experimental tests.

- In future studies, the effectiveness of the proposed strengthening techniques will be assessed on infills with different boundary conditions (e.g., infills mortared along all edges to the confining frame), i.e., in presence of two-way arching action or in absence of arching action. In addition, the efficiency of the considered strengthening techniques will be assessed also with respect to the effects of the in-plane/out-of-plane interaction.

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AFTERSHOCK GROUND MOTION RECORD SELECTION: A NOVEL MAINSHOCK-CONSISTENT APPROACH

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Abstract

Recent advances in ground motion record selection for mainshock events have highlighted very clearly the critical importance of using record suites consistent with the seismic hazard at the site of interest. Nevertheless, this integral component for analytical derivation of fragility models remains underscrutinized when it comes to aftershock ground motions. Several authors have highlighted the caveats of repeating the same records as both mainshocks and aftershocks, but a conceptually sound methodology for such purposes is still lacking. Herein, we analyze many of the relevant hurdles and challenges involved in selecting ground motion records for aftershocks, and offer a discussion on potential strategies for overcoming them. Finally, we outline a novel procedure that enables a targeted pairing of records from a single database into mainshock-aftershock sequences, while ensuring that they represent realistic scenarios and that they respect the mainshock-aftershock correlation structure.

Keywords: aftershock risk assessment, ground motion record selection, aftershock sequences, seismic risk
INTRODUCTION

A necessary task for assessing the vulnerability of damaged structures under aftershock (AS) events is the selection of mainshock-aftershock ground motion recordings that best represent the likely ground motion sequences at the site of interest. Recent studies have shown the site-dependency of fragility functions developed for intact structures under mainshock (MS) ground motions and addressed it by means of different site-specific record selection alternatives [1–3]. Site-dependency is also present when evaluating the response of damaged buildings to AS events. Moreover, similarities between the MS and AS causal parameters are expected to be present given that the latter tend to generally occur on the same fault as the MS or in its vicinity. In addition, MS and AS ground motions are expected to share certain peculiarities, e.g. peaks or valleys in spectral shape or general deviations from the mean ground motion as given by a ground motion prediction equation (GMPE), as a result of similarities in terms of source characteristics, wave propagation paths and site conditions. This dependency of both AS causal parameters and ground motions characteristics (e.g. spectral acceleration residuals) on the MS event should in principle be accounted for when records are selected to represent AS ground motions.

Despite awareness over the caveats of the common practice of randomly combining records from the same database into MS-AS pairs, very few authors have tried to explicitly address the aforementioned issues and a conceptually sound methodology for AS record selection is still lacking. Herein, we offer a comprehensive discussion on several practical and theoretical hurdles for the task at hand. The record selection scheme should be consistent with the scope of its intended application (damage-dependent fragility functions or “lumping” aftershock effect into intact building fragility functions), the structural analysis scheme to be employed (multiple-stripe or cloud analysis, scalar of vector intensity measure), and the definition of the aftershock (one or multiple, if one then which?). We further touch upon the selection of appropriate aftershock causal parameters and discuss the relevant challenges. Taking into account the above, we discuss a series of simple and more complex potential strategies for selecting aftershock records, reflect on their suitability and point out likely pitfalls. Finally, leveraging upon the aforementioned discussion, we outline a pragmatic mainshock-consistent ground motion record selection procedure for mainshock-aftershock pairs, constructed with the explicit goal of achieving a sensible balance between practicality and conceptual soundness. The latter is expected to comprise a valuable resource for the analytical derivation of fragility functions for structures in damaged conditions exposed to aftershock sequences, and pave the way for further research.

TARGET RESPONSE SPECTRA FOR AS GROUND MOTIONS

2.1 Hazard-consistent record selection

Generally speaking, the selection of MS ground motion records in a manner that respects the characteristics of seismic hazard at the site of interest starts with the definition of a set of likely causal rupture parameters (typically given in the form of a seismic disaggregation table for a specific intensity level), such as magnitude and source-to-site distance, that need to be reflected in the characteristics of the eventual suite of records. The next step is to derive the joint distribution of the various intensity measures (IMs) of interest if any of the afore-defined causal ruptures were to occur (taking into account their relative likelihoods of occurrence). This distribution is typically computed with a condition on the value of a given IM (conditioning IM). The latter serves as the interface between the seismic hazard and structural response analyses and will be the variable as a function of which the fragility curves will be derived.
Finally, a set of records is identified from a given strong motion database to conform as much as possible to the joint distribution determined in the previous step. For more information regarding state-of-the-art procedures for MS record selection, the reader is referred to [2–6]. In this section, we introduce a few tools that could be used for the derivation of the joint AS IM distribution, while the next sections will discuss the definition of causal parameters and potential conceptual frameworks for selecting records.

2.2 Mainshock - Aftershock response spectrum conditional on a single MS spectral acceleration \( SA_{MS}(T^*) \) and given MS and AS ruptures

First of all, let us assume a given rupture sequence scenario, i.e. a sequence of mainshock and aftershock ruptures \( \text{rupSeq} = \{ \text{rupMS}, \text{rupAS} \} \). Each of these ruptures is described by a vector of causal parameters such as magnitude, distance, faulting style, soil conditions, etc which may coincide or vary between the MS and AS events. Given these ruptures, we are interested in modelling the joint distribution of a vector of MS and AS spectral accelerations at different periods of vibration, conditional on the occurrence of a MS spectral acceleration \( SA_{MS}(T^*) \) at a target period \( T^* \). For a specific ground motion, spectral accelerations at multiple vibration periods have been shown to follow a multivariate lognormal distribution [7], while Papadopoulos et al. [8] showed that extending this assumption to spectral accelerations of MS-AS pairs can be a reasonable strategy for practical purposes. The conditional mean and standard deviation vectors of the logarithmic spectral acceleration ordinates for the MS and AS ground motions can therefore be estimated as:

\[
\begin{align*}
\mu_{\ln SA_{\text{rupSeq}, \text{rupMS}}} & = \begin{bmatrix} \mu_{\ln SA_{\text{rupMS}}} \\ \mu_{\ln SA_{\text{rupAS}}} \end{bmatrix}, \\
\sigma_{\ln SA_{\text{rupSeq}, \text{rupMS}}} & = \begin{bmatrix} \sigma_{\ln SA_{\text{rupMS}}} \\ \sigma_{\ln SA_{\text{rupAS}}} \end{bmatrix}, \\
\mu_{\ln SA_{\text{rupSeq}, \text{rupMS}}} & = \begin{bmatrix} \mu_{\ln SA_{\text{rupMS}}} \mu_{\ln SA_{\text{rupAS}}} \end{bmatrix},
\end{align*}
\]

(1)

where \( \mu_{\ln SA_{\text{rupMS}}} \) and \( \mu_{\ln SA_{\text{rupAS}}} \) are the vectors of the unconditional logarithmic mean spectral accelerations for the MS and AS ground motions respectively, as obtained by a GMPE for the given MS and AS ruptures; \( \sigma_{\ln SA_{\text{rupMS}}} \) and \( \sigma_{\ln SA_{\text{rupAS}}} \) are the associated standard deviations; \( \rho_{\ln SA_{\text{rupMS}}} \) and \( \rho_{\ln SA_{\text{rupAS}}} \) are the correlation coefficient vectors between the MS spectral acceleration at \( T^* \) and the rest of MS and AS spectral ordinates respectively; finally \( \epsilon_{\ln SA_{\text{rupMS}}} \) denotes the number of standard deviations by which the logarithmic MS spectral acceleration at the conditioning period of \( T^* \) exceeds the mean as predicted by the employed GMPE. The conditional multivariate normal distribution \( \ln SA_{\text{rupSeq}, \text{rupMS}} \) can then be completely defined by the mean and standard deviation values calculated with Equations 1 and 2, along with the full correlation coefficient matrix.

2.3 Mainshock - Aftershock response spectrum conditional on a single MS spectral acceleration \( SA_{MS}(T^*) \) for multiple MS and AS ruptures

If now we seek to obtain a MS-AS conditional spectrum for a known MS rupture \( \text{rupMS} = k \), but considering alternative scenarios for the AS rupture, the equations given above can be easily extended. Assuming a set of mutually exclusive and collectively exhaustive AS rupture...
scenarios given the MS rupture, the conditional distribution $\ln \text{SA}|\text{rup}_\text{MS}$, $\text{SA}_y^{\text{MS}}$ can be obtained by means of the total probability theorem as:

$$f_{\ln \text{SA}|\text{rup}_\text{MS} - k, \text{SA}_y^{\text{MS}}} = \sum_{j=1}^{N_{\text{rup}}} f_{\ln \text{SA}|\text{rup}_\text{AS} - \text{rup}_{\text{AS}}, \text{SA}_y^{\text{MS}}} P_{\text{rup}_\text{AS}|\text{rup}_\text{MS} = k}$$

(3)

where $f_{\ln \text{SA}|\text{rup}_\text{AS} - \text{rup}_{\text{AS}}, \text{SA}_y^{\text{MS}}}$ denotes the probability density function of $\ln \text{SA}$ given the MS and AS ruptures as well as the MS SA at the target period $T^*$, i.e. the multivariate normal distribution for a given rupture sequence described in the previous section; $P_{\text{rup}_\text{AS}|\text{rup}_\text{MS} = k}$ is the probability that the AS ground motion that we are interested in is caused by the rupture $\text{rup}_{\text{AS}}$ given the occurrence of $\text{rup}_\text{MS} = k$. If the MS rupture is also unknown the only remaining condition is the value of $\text{SA}_y^{\text{MS}}$. From the seismic hazard disaggregation one can obtain the mutually exclusive and collectively exhaustive set of MS ruptures that can cause this level of spectral acceleration. Then, similarly with before, employing the total probability theorem the joint MS-AS conditional distribution of spectral acceleration ordinates considering all MS and AS rupture scenarios can be obtained as:

$$f_{\ln \text{SA}|\text{SA}_y^{\text{MS}}} = \sum_{i=1}^{N_{\text{rup}}} \sum_{j=1}^{N_{\text{rup}}} f_{\ln \text{SA}|\text{rup}_\text{AS} - \text{rup}_{\text{AS}}, \text{SA}_y^{\text{MS}}} P_{\text{rup}_\text{AS}|\text{rup}_\text{MS} = \text{rup}_{\text{MS}}} P_{\text{rup}_\text{MS}|\text{rup}_\text{AS} = \text{rup}_{\text{AS}}}$$

(4)

where $P_{\text{rup}_\text{MS}|\text{rup}_\text{AS} = \text{rup}_{\text{AS}}}$ is the probability of occurrence of $\text{rup}_\text{MS}$ given $\text{SA}_y^{\text{MS}}$.

2.4 Mainshock - Aftershock response spectrum conditional on multiple MS spectral acceleration ordinates $\text{SA}^{\text{MS}}(T_i)$ and given MS and AS ruptures

In this case, let us assume again a given rupture scenario $\text{rup}_\text{Seq}$, on top of which we now also fully know the MS response spectrum. To condition the AS spectrum on all the spectral ordinates of the MS ground motion, the unconditional mean $\mu_0$ and variance-covariance matrix $\Sigma_0$ of the joint MS-AS spectral acceleration distribution are first written as:

$$\mu_0 = \begin{bmatrix} \mu_{\ln \text{SA}(T_{1}^{\text{MS}})} & \mu_{\ln \text{SA}(T_{2}^{\text{MS}})} & \cdots & \mu_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})} \\ \mu_{\ln \text{SA}(T_{1}^{\text{AS}})} & \mu_{\ln \text{SA}(T_{2}^{\text{AS}})} & \cdots & \mu_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{AS}}}^{\text{AS}})} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{\ln \text{SA}(T_{1}^{\text{AS}})} & \mu_{\ln \text{SA}(T_{2}^{\text{AS}})} & \cdots & \mu_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})} \end{bmatrix}$$

(5)

$$\Sigma_0 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix} =

\begin{bmatrix}
\rho_{T_{1}^{\text{MS}}, T_{1}^{\text{AS}}} \sigma_{\ln \text{SA}(T_{1}^{\text{MS}})}^2 & \rho_{T_{1}^{\text{MS}}, T_{2}^{\text{AS}}} \sigma_{\ln \text{SA}(T_{1}^{\text{MS}})} \sigma_{\ln \text{SA}(T_{2}^{\text{AS}})} & \cdots & \rho_{T_{1}^{\text{MS}}, T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}}} \sigma_{\ln \text{SA}(T_{1}^{\text{MS}})} \sigma_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})} \\
\rho_{T_{2}^{\text{MS}}, T_{1}^{\text{AS}}} \sigma_{\ln \text{SA}(T_{2}^{\text{MS}})}^2 & \rho_{T_{2}^{\text{MS}}, T_{2}^{\text{AS}}} \sigma_{\ln \text{SA}(T_{2}^{\text{MS}})}^2 & \cdots & \rho_{T_{2}^{\text{MS}}, T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}}} \sigma_{\ln \text{SA}(T_{2}^{\text{MS}})} \sigma_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}}, T_{1}^{\text{AS}}} \sigma_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})}^2 & \rho_{T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}}, T_{2}^{\text{AS}}} \sigma_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})} \sigma_{\ln \text{SA}(T_{2}^{\text{AS}})} & \cdots & \rho_{T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}}, T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}}} \sigma_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})} \sigma_{\ln \text{SA}(T_{N_{\text{rup}}^{\text{MS}}}^{\text{MS}})} \\
\end{bmatrix}

(6)

where $\rho_{T_{i}^{\text{MS}}, T_{j}^{\text{AS}}}$ denotes the correlation coefficient between the log spectral accelerations at $T_i$ and $T_j$ of either mainshock or aftershock, as indicated by the apices. Next, if the MS ground motion is known, the AS spectral acceleration predictions can now be updated by conditioning on the MS observations. The conditional mean and variance-covariance matrix for the AS ground motion, $\mu_c$ and $\Sigma_c$, can be computed as:
\[ \mu_c = \mu_{01} + \Sigma_{12} \Sigma_{22} (x^{MS} - \mu_{02}) \]  

(7)

\[ \Sigma_c = \Sigma_{11} + \Sigma_{12} \Sigma_{22} \Sigma_{21} \]  

(8)

where \( x^{MS} \) denotes the vector of observed MS logarithmic spectral accelerations and the prime apex denotes the transposition of a matrix.

3 CAUSAL PARAMETERS AND CONCEPTUAL FRAMEWORK

While the previous section introduced different mathematical tools for obtaining potential target spectra given a set of causal rupture scenarios, the main challenge lies in integrating these tools into a pragmatic conceptual scheme for record selection and structural analysis and in identifying the said set of causal rupture scenarios. The following paragraphs aim to clarify certain concepts, reflect upon the many hurdles and challenges, as well as to discuss potential solutions.

3.1 Scope of fragility model

Before beginning the quest for a suitable record selection framework, a first fundamental question to be posed concerns the scope of the fragility model for the development of which the record selection needs to be carried out. Tesfamariam and Goda [9], for instance, “lump” the additional risk imposed by AS sequences within the conventional intact building fragility functions conditional on the MS \( IM \). This was accomplished by subjecting the building to back-to-back analyses with as-recorded MS-AS sequences, while using the MS spectral acceleration \( S_{A_{MS}} \) as the \( IM \) to characterize the entire sequence. This way AS hazard and damage accumulation are both indirectly taken into account within the fragility functions, and one can simply convolute the latter with the MS hazard curve. Of course, such an approach is overly simplistic (the AS hazard is essentially approximated in an arbitrary fashion via the characteristics of the employed MS-AS sequences), but could be seen as a pragmatic alternative. A record selection scheme oriented for such an application would then need to ensure, as much as possible, that for each MS intensity level, the selected sequences are representative of the AS hazard in terms of number of events, their potential magnitude and distance, order of occurrence, and distribution of ground motion residuals. On the other hand, if the AS hazard is explicitly modelled and one is interested in fragility functions of damaged buildings conditioned on the AS \( IM \), the AS hazard and vulnerability components are decoupled via the AS \( IM \), which serves as the interface between them. In that case, the purpose of the record selection is essentially to “correct” for the insufficiency of the AS \( IM \) in representing the various features of the ground motion that is expected at the site of interest. In the following paragraphs, only on the latter type of applications will be discussed.

3.2 Aftershock event definition and objective of record selection

In regard to the definition of the AS event, a seismic sequence typically involves numerous earthquakes, many of which may be able to induce a certain level of damage to a given structure. Ideally, an AS vulnerability model should reflect the damage and cyclic deterioration accumulated by multiple events, which would entail subjecting the structure to a series of AS accelerograms following each MS one. Nevertheless, the countless potential combinations of post-mainshock damage patterns, AS rupture scenarios, their number and order of occurrence currently impose a daunting obstacle for such treatments. A commonly used approach, that is more manageable from both a conceptual and a computational cost standpoint, is to employ a
single AS record, with the underlying assumption that the response of the structure given its damage state is independent to the number of ground shakings that it has experienced. In the latter case, the ground motion that is paired with the MS may represent various alternatives such as a random AS of the sequence, the AS with the largest magnitude, or rupture scenarios consistent with a specific AS or MS intensity level ($IM$). This choice is tightly connected to the adopted structural analysis scheme, the level of complexity that the analyst is willing to accept, as well as to how thoroughly AS hazard-consistency is going to be treated.

For instance, a multiple-stripe analysis (MSA), would require running nonlinear dynamic analyses with records that share a specific spectral acceleration $SA$ level at the conditioning period of interest $T^*$. The clear advantage of such a strategy is that for each level of $SA$ (stripe), ground motion records can be selected in a hazard-consistent manner by following the steps mentioned in section 2.1 and described in the references contained therein. By doing so, one will obtain a sample of structural responses for each $SA$ level that reflect the variability that is associated with the hazard at the site where the building is located. These structural responses can then be used to objectively estimate the probability of damage state exceedance at each $SA$ level, which is the information of interest for deriving fragility functions. In the case of AS ground motions, a target mean response spectrum (along with its dispersion) conditioned on the specific level of $SA^{AS}(T^*)$ would need to be derived based on the results of an aftershock probabilistic seismic hazard analysis [APSHA; 10]. An important aspect to note here is that the AS hazard is dependent on the MS that triggers it, hence APSHAs would need to be carried out separately for each MS rupture scenario, and then appropriately combined (if one wishes to estimate the AS hazard given a $SA^{MS}(T^*)$ level and not a MS rupture). Records selected to match this spectrum would represent likely aftershock event scenarios for the specific level of $SA^{AS}(T^*)$. Or alternatively, one could perform separate AS record selections for each different MS record, a choice which (even though it is more theoretically sound) would vastly increase the number of required MS-AS record sequences and consequently the computational time needed for performing all the back-to-back nonlinear analyses.

If on the other hand a cloud approach is employed for the AS structural analysis, the AS spectral accelerations do not need to be in stripes. This means that a condition on $SA^{AS}(T^*)$ is not strictly required, however hazard-consistency cannot be ensured anymore by selecting records to conform to target spectra. Note here that the cloud analysis comprises a simplified approach, which when applied for MS earthquakes, employs records typically selected on the basis of their causal parameters simply being plausible for the region of interest and with $IM$ levels that are expected to bring the building to exceedance of the damage state of interest or close to it. Keeping this in mind, we argue that even if AS hazard consistency is not actively pursued as in multiple stripe analyses, it is still important to select MS-AS pairs that are compatible. In other words, even if, for reasons of simplicity, we do not enforce the consistency between the empirical distribution of $SA^{AS} | SA^{AS}(T^*)$ within the sample of selected accelerograms with the theoretical one, it is important that the selected MS-AS sequences represent (i) realistic rupture scenarios (in terms of causal parameters) and (ii) statistically respect the MS-AS correlation structure that is known to be present [8]. The latter reflects the similar peculiarities such as peaks and valleys in the response spectra, as well as general deviations from the mean ground motion given by a ground motion prediction equation (GMPE) that MS and AS ground motions recorded at the same site are expected to share. We will henceforth call AS records that are selected taking into account such considerations as “mainshock-consistent”.

Coming back to the initial discussion, the definition of the AS event is clear that when the AS ground motion records are “conditionally” selected based on the level of AS hazard; they represent likely AS event scenarios for that level of AS hazard. On the other hand, when such
a “conditional” on $SA^{AS}(T^*)$ selection is not pursued and the objective of record selection is to 
simply ensure the use of consistent MS-AS record pairs, it is unclear what event the AS record 
will represent (given the numerous AS scenario possibilities) and hence how to accordingly 
select the causal parameters on which to base its selection. For the sake of clarity, let us 
focus on a single MS rupture and assume that we are interested in selecting one or multiple 
AS records compatible with that MS. If the AS rupture was known, or a set of likely AS rup-
tures was known, a target AS response spectrum could be generated conditional on the MS 
information, as shown in section 2. To determine the AS event that is being modelled and al-
low the identification of possible causal parameters, two potential options could be the fol-
lowing:

- Model the largest (in terms of magnitude) AS of the sequence. According to Båth [11], 
it's magnitude is on average 1.2 units smaller than that of the MS, independently of the size 
of the latter. The magnitude of the AS could be modeled either deterministically as 
$M_{AS}=M_{MS}-1.2$ or probabilistically by introducing a dispersion around the mean value.
- Model a random AS of the sequence. The magnitude of this event would be given 
probabilistically according to the Gutenberg-Richter (G-R) [27] law, i.e. smaller mag-
nitude events are more likely compared to higher magnitude ones according to an ex-
ponential distribution.

For the time being, let us assume that the location of the AS, which is going to determine the 
source-to-site distance $R$, is also determined by some simplifying assumptions. A further dis-
cussion on the location of the AS will be provided in a following section. In theory, using 
these sets of $M-R$ parameters, one could obtain the joint distribution of MS-AS spectral accel-
erations conditional on $SA^{MS}(T^*)$ via Equation (3) and use it to select MS-AS record pairs. Or 
select a MS record first and then use Equations (5)-(8) to get the conditional distribution of 
the AS spectral accelerations. Nevertheless, both options (largest or random AS) come with 
serious defects. In the former case, the largest in terms of magnitude AS is first of all not 
guaranteed to be neither the most damaging, but not even the one causing the highest ground 
acceleration at the site of interest within the sequence. Moreover, in both cases the joint 
distribution of $M-R$ parameters of the potential AS rupture would be particularly spread out, 
i.e. it would exhibit huge dispersion. That is because unlike the seismic hazard disaggregation 
plot that is typically used to determine the MS causal parameters, the aforementioned distribu-
tion is not conditional on the level of $SA^{AS}$. Instead the AS $M-R$ distribution in this case would 
reflect all the potential combinations of causal parameters given the MS rupture, which would 
result in a conditional spectrum with high dispersion that is difficult to satisfy. Moreover, a lot 
of weak records would need to be selected, even though they would be unlikely to cause any 
damage to the structure. This is even more problematic in the case of modelling a random AS, 
since the magnitude distribution is modelled via the G-R exponential distribution. This means 
that the small magnitude events are much more likely and the resulting conditional spectrum 
would have a very low mean. The latter, in fact, would essentially depend on the minimum 
magnitude chosen by the analyst. Essentially, in this case the compiled target spectrum re-

3.3 Joint or separate selection for MS and AS records?

Another choice that needs to be made is whether to select the MS-AS pairs jointly or sepa-
ately. If a selection conditional on both $SA^{MS}(T^*)$ and $SA^{AS}(T^*)$ based on a vector MS-AS 
PSHA is used, the two components of the sequence need to be selected together (i.e. based on 
the joint MS-AS distribution and not separately). A vector MS-AS PSHA and selection would 
solve many of the problems outlined in the previous section, however, such a procedure 
would need to be carried out with extreme caution, since due to the correlation between the
MS-AS spectral accelerations, retrieving the marginal distributions of the more critical MS spectral accelerations may depend on the binning resolution of the AS intensity levels. Furthermore, the complexity of such an approach is perhaps not justified, given the secondary impact of aftershock damage on risk estimates and especially the high level of approximation involved in the APSHA procedure (e.g. spatial distribution of aftershocks, declustering approach, etc). Another option would be to still select the records jointly but conditional only on $SA_{MS}(T^*)$. Such a strategy would not address the issues raised in the previous section, but would allow one to select AS records to conform to the full distribution of $SA_{AS} | SA_{MS}(T^*)$ using the methodology and equations given in section 2.3, rather than selecting multiple AS records for each MS rupture, e.g. using the equations of section 2.4. A joint selection in this sense, while perhaps a bit less “accurate” compared to a separate MS-AS selection (since even though agreement with the target AS spectrum is reached, this agreement does not take into account the obtained MS-AS combinations but just the final mean and dispersion of the suite of AS spectra), allows the selection of fewer records and thus the execution of fewer nonlinear analyses. On the other hand, selecting the MS and AS records separately allows for a bit more of flexibility, in the sense that the analyst can proceed and carry out the MS record selection with any methodology he wishes and then carry out the AS record selection at a later stage. Moreover, it also allows larger flexibility in devising a conceptual framework for the selection of AS records (e.g. see section 4).

3.4 Aftershock causal parameters

The definition of AS causal parameters is not a trivial task. Most ASs typically occur along or very close to the elongated MS rupture, while a number of other triggered earthquakes take place on nearby faults. With respect to the on- (or near-) fault aftershocks, integrating specific source information in the record selection procedure would require additional input regarding the geometry and orientation of the MS rupture, which is not readily available from a standard seismic hazard disaggregation output. Even worse, there is no clear-cut method for predicting the location of off-fault triggered events, at least without a sophisticated analysis of stress transfer and knowledge of the local fault system. A more approximate “directionless” approach could be adopted by lumping together aftershocks and triggered events and using an isotropic spatial distribution calibrated using an instrumental seismicity catalogue [e.g. 12]. However, most such models typically refer to direct offspring, i.e. ASs or triggered events that are triggered directly by the MS (or a specific AS). Hence, they do not describe the spatial distribution for the entirety of the seismic sequence that herein we are interested in. Moreover, the definition of the magnitude distribution of ASs is not straightforward either; most established models, such as ETAS [13] or Reasenberg-Jones [14], typically assume that the AS magnitude-frequency distribution is identical to that of the spontaneous seismicity, while several studies claim that significant differences exist between the two cases. Furthermore, it is unclear whether this magnitude distribution can be considered uniform across the considered region, as traditionally done, since recent studies support that the largest ASs occur further away from the MS rupture [15].

3.5 Damage dependence and record-to-record variability

Another relevant, issue is that when talking about damaged buildings, the initial damage pattern of the building given its damage state introduces additional variability. Say that, as commonly done, the criterion used for determining the damage state of a given multi-story building is whether the value of an engineering demand parameter (EDP), such as the max inter-story drift ration (MIDR), exceeds a certain threshold. Analysis realizations that have
exceeded this threshold, say 1%, would be placed in the corresponding DS, say DS=2; nevertheless, while all classified into DS=2, these damaged building realizations could potentially differ, e.g. in terms of residual deformations, spatial distribution of damage (e.g. the story where $MIDR=1\%$ was exceeded), number of yielded elements, etc. For a truly conceptually robust framework, this variability would perhaps need to be properly accounted for and propagated in combination with the record-to-record one. To better understand this, say that a set of $N$ records is selected to conform to a well-defined target $Sa$ distribution. If this set is imposed to an intact structure, it is then typically expected that the obtained distribution of structural response estimates would 1) be unbiased and consistent with the seismic hazard model (at least if ground motion characteristics that are not accounted for, such as duration, are not of importance), and 2) converge fast to the “true” distribution as the number $N$ of employed records increases. Now consider the case where each record of the set is imposed to a different post-MS building realization, always classified as being in a given DS but each time featuring a different damage pattern. Statistically robust response estimates would not be guaranteed if the matching of records-damage building realizations is done in an unsupervised fashion. While important to keep in mind, this issue seems difficult to address in a practical manner, while it is also unclear at the moment how much it could impact our fragility estimates. An informative damage criterion that reflects the potential drop in capacity of the building should help minimize such effects.

3.6 Other Implications

Some other notable considerations that need to be kept in mind and addressed in the future, concern the modeling of the seismic hazard and that of the ground motion. With respect to the former, it is standard practice to develop MS hazard models from the historical and instrumental seismicity catalogues after declustering them, i.e. removing foreshock/aftershock events. Most declustering algorithm entail a degree of subjectivity and are often seen to lead to very different outcomes [16]. The events that are identified as part of clusters are then used to model the AS hazard (note here that this is not always necessary, e.g. models such as ETAS are not predicated on declustering). In other words, some declustering approaches would classify more events as MSs and fewer as ASs, while others would do the opposite. Aside from the well-known effect of this on MS seismic hazard, it would essentially also result in differ causal parameter sets for different levels of $SA^{MS}(T^*)$ and hence in a different record selection and potentially fragility estimates. Moreover, there is an ongoing debate in the literature regarding the frequency content of AS ground motions compared to MS ones [e.g. 17,18]. Nearby (to the MS rupture) ASs tend to exhibit lower stress drops compared to MS events with similar magnitude. However, no sufficient evidence exists to support this for ASs occurring further away on different faults, which typically are the largest magnitude events triggered by the MS [15]. According to this idea, one would need to use a different GMPE for on- and near-fault AS, and a different one for those occurring further away. Nevertheless, this would not be currently straightforward, since as discussed in the causal parameters paragraph above, most models currently assume a uniform magnitude distribution for all triggered events (nearby and distant). Hence, without a sophisticated joint magnitude-distance distribution, we erroneously distribute the high magnitude events evenly (according to the weight of the spatial kernel) in the near and distant to the MS range. If different GMPEs are to be used, we would also erroneously model some of the largest magnitude ASs with lower stress drop.
4 A PRAGMATIC MAINSHOCK-CONSISTENT AS RECORD SELECTION PROCEDURE

Leveraging upon the discussion offered before, we offer herein a pragmatic AS ground motion record selection scheme that is designed to construct plausible seismic sequences, i.e. series of earthquakes with realistic causal parameters, and then select ground motion records for them that respect the statistical correlation between MS and AS spectral ordinates. The proposed framework can be essentially structured into three main components: Component 1: record selection for MS events; Component 2: generation of AS sequences; and Component 3: MSAS-CS based record selection for the generated AS events.

4.1 Component 1: MS record selection

The first component concerns the selection of MS records which constitutes the first step of the proposed procedure. Note that this task is carried out independently of the tasks pertaining to the selection of AS records. This adds flexibility in the proposed approach in the sense that the analyst can select the MS records according to his preference and then base the AS record selection on the recommendations provided herein. The only additional requirement that the MS selection needs to provide for is the association of each selected MS record with an earthquake rupture scenario characterized by a given magnitude-distance ($M-R$) pair. These MS scenarios will be used within the Component 2 of our methodology to generation AS sequence realizations. Depending on the MS record selection approach, the pair of $M-R$ values for each selected record can be obtained in different ways. For instance, in the case of a cloud analysis with natural (unscaled) MS records, the actual observed $M-R$ values associated with each recording should be used. If on the other hand the MS record selection is carried out via the GCIM approach [2], each record is anyways taken to represent a possible realization of a ground motion caused by a specific sampled $M-R$ causal rupture scenario. Finally, if a CS [3] approach is preferred, we outline below a modified version of the Greedy Optimization procedure proposed by [3] to accommodate this additional requirement.

The proposed CS-based selection follows the strategy proposed by Bradley [6], which we now also apply for a greedy optimization component. In the work of Jayaram et al. [3] the latter is performed by going through all the initially selected records and searching in the entire record database for replacements that would improve the match between the selected suite and the target spectrum. Herein, the greedy optimization is performed by re-sampling ruptures and random $\ln Sa|\text{rup}$ realizations to identify potential replacements for each of the initially selected ground motions. For a given MS intensity level, the MS record selection procedure can be summarized as follows:

I. Derive the theoretical multivariate conditional distribution $f_{Sa|\text{rup}(T^*)}$ of spectral accelerations $Sa(T_i)$ given $Sa(T^*)$ considering multiple causal scenarios via Equation (9) (for more details see Bradley, [6]).

$$f_{Sa|\text{rup}(T^*)} = \sum_{k=1}^{N_{rup}} f_{Sa|\text{rup}_k(\text{rup}_k),Sa(T^*)} P_{\text{rup}_k} P_{\text{rup}_k|\text{rup}_k(\text{rup}_k),Sa(T^*)}$$

where $f_{Sa|\text{rup}_k, Sa(T^*)}$ is the multivariate conditional distribution of MS spectral accelerations $Sa(T_i)$ given $Sa(T^*)$ and MS rupture $k$, while $P_{\text{rup}_k|\text{rup}_k(\text{rup}_k),Sa(T^*)}$ is the probability of MS rupture $k$ given $Sa(T^*)$.

II. For each of $N_{recs}$ ground motion records to be selected for the given intensity level, randomly draw and store a random rupture ($M-R$) scenario from the seismic hazard disaggregation probability mass function $P(\text{rup}|Sa(T^*))$. 

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III. For each of these randomly drawn ruptures assemble the associated single causal scenario conditional spectrum \( f_{\text{rup}^{\text{MS}_{T_{r_{\gamma}}}}} \), and generate a random realization by sampling correlated \( \ln Sa \) variables from the multivariate normal distribution.

IV. Match a ground motion recording from a database to each random realization by finding the (scaled) recording that returns the minimum sum of squared errors across periods [3]. Each selected ground motion record at this stage will be associated with the previously drawn M-R rupture scenario used to obtain the random realization.

V. Perform greedy optimization:
   a. Determine the goodness-of-fit of the selected suite of records \( S \) to the theoretical distribution by means of an error metric \( \text{Err} \) (e.g. KS statistic [6] or sum of squared errors in mean and standard deviation [3]).
   b. Set \( j = 1 \)
   c. Sample \( N^* \) (a number between 20–30 seems to be consistently sufficient) new rupture scenarios \( M-R \) from the disaggregation probability mass function, generate \( N^* \) associated \( \ln Sa | \text{rup}^{\text{MS}} \) realizations and match a new record \( \text{rec}_{jk} \) to every realization \( \text{real}_{jk} \) for \( k = 1, 2, \ldots, N^* \).
   d. For each new record \( k \), re-assess the goodness-of-fit to the theoretical distribution as in (a), if \( \text{rec}_{jk} \) was to replace the \( j \)th record \( \text{rec}_j \) of the record suite \( S \).
   e. If any of the \( N^* \) new records improve the error metric, replace \( \text{rec}_j \) in \( S \) with the \( \text{rec}_{jk} \) that leads to the best improvement. Then replace the stored rupture \( (M-R) \) associated with \( \text{rec}_j \) with the newly drawn rupture used to generate \( \text{real}_{jk} \).
   f. Set \( j = j + 1 \). If \( j \leq N_{\text{recs}} \), repeat steps (c)-(e).

This algorithm allows the association of explicit \( M-R \) causal parameters with each selected record, while also not requiring an assumption of multivariate lognormality for the joint distribution of \( Sa \) across periods and respecting the \( Sa \) correlation structure even during the greedy optimization phase. Figure 1 shows an example of MS record selection for a given intensity level and assuming a site in the city of Terni in Central Italy with latitude and longitude of \((12.643^\circ, 42.564^\circ)\) and over soil with an average shear-wave velocity in the top 30 m \( (V_{s30}) \) equal to 350 m/s. The MS hazard and disaggregation are obtained via the area source branch of the SHARE model [19] and using a single GMPE, the one of [20], henceforth referred to as ASK14. The CS illustrated in Figure 1b is derived using the ASK14 GMPE and the associated inter-period \( Sa \) correlation estimates given in Abrahamson et al. [20].

4.2 Component 2: Generation of AS Sequences

Upon knowledge of the magnitude and location of the MS event, one can simulate potential AS sequence scenarios via a suitable point process. Several such models can potentially be used, such as the generalized Omori’s law [21] used in [22] to simulate time series of earthquakes. Another option adopted in [23] could be the so-called BASS model [24]. A simpler alternative would be to consider only the largest AS of the sequence which according to [11] is on average 1.2 magnitude units smaller than the MS. Herein, we generate sequences by means of the triggering component of the space-time Epidemic-Type Aftershock Sequence (ETAS) model [13]. The simulation of stochastic sequence realizations via the ETAS model can be carried out via Equations (10-14). The number of direct offspring of the MS event with magnitude \( m_i \) is first sampled from a Poisson distribution with mean \( k(m_i) \) given by Equation (10). The inter-arrival time \( \Delta t \) between each parent and offspring event, the distance \( r \) between their epicentres, and the angle \( \theta \), which is the polar coordinate that along with the distance \( r \) defines the relative location of the offspring event with respect to its parent one, are simulated
through Equations (11-13), where $u_t$, $u_r$, and $u_\theta$ are uniformly distributed random variables over the range (0,1) and $A$, $a$, $p$, $c$, $D$, $q$, $\gamma$ are constant parameters. The G-R distribution is used to sampled the magnitudes $m_{AS}$ of the triggered events via Equation (14), where $u_m$ is again a uniformly distributed random variable over the range (0,1), $b$ is a constant parameter and $M_{min}$ is the minimum considered magnitude. Equation (14) refers to the case of untruncated G-R distribution; to sample magnitude values from its version that is truncated at a maximum magnitude, as typically done, the same equation can be used but only sampled values below the $M_{max}$ threshold would be accepted.

Following the generation of direct offspring (i.e. triggered by the mainshock), a second generation of earthquake events (triggered by direct aftershocks) is sampled by going through each of the former and repeating the same procedure. The second generation offspring are again used as seeds for a third one, and so forth until the sequence eventually dies out (zero offspring are sampled) or there are no more seed earthquakes within the assumed timeframe. The latter herein has been taken equal to 1 year which represents a time-span within which most aftershocks are expected to have occurred and the buildings not to have been repaired.

$$k(m_j) = A \cdot e^{-(m_j - M_{min})}$$

$$\Delta t = -c + e\left(1 - \mu_j\right)^{\frac{1}{1 - p}}$$

$$r(m_j) = d \cdot e^{\gamma(m_j - M_{min})} \sqrt{\mu_j^{\frac{1}{1 - q}} - 1}$$

$$\Theta = 2\pi u_\theta$$

$$m_{AS} = \frac{-\ln(1 - u_m)}{b \ln 10} + M_{min}$$

### 4.3 Component 3: AS Record Selection

Upon the completion of the tasks related to components 2 and 3, a series of MS records has been selected, each one of them associated with a specific $M-R$ rupture scenario and an AS.
sequence time-series \( \{ \text{rup}_{\text{AS1}}, \text{rup}_{\text{AS2}}, \ldots, \text{rup}_{\text{ASn}} \} \) defined by the timestamp, magnitude and location of each earthquake. Ground motion records for these ruptures can be selected using the MS-AS response spectrum conditional on multiple \( \text{SA}^{\text{MS}}(T_i) \) ordinates and the MS and AS rupture that was described in section 2.4.

A realization of the AS response spectrum can then be sampled from the joint AS \( \text{Sa} \) distribution (see example in Figure 2) and a ground motion record is subsequently selected from a given database on the basis of minimizing the sum of squared errors across periods, similar to what was done in the component 1 of the methodology for the MS records. It is worthwhile to note here that we select in a random fashion (as described above) only one AS ground motion record per AS rupture, unlike CS-based procedures used for MS record selection that involve the selection of record suits to conform to imposed target mean and variance spectra. As a consequence, we expect our response (and consequently fragility) estimates to converge to stable values only if a sufficiently large number of analyses is carried out. This is a conscious choice factoring in the various challenges in defining reliable target AS response spectra discussed previously.

![Figure 2](image-url)  

Figure 2. (a) A selected MS ground motion and the median prediction of the ASK14 GMPE for the associated MS rupture; (b) the AS response spectrum conditional on the MS rupture and ground motion of the left panel along with one random stochastic realization.

A problem that arises from the procedure described so far is that simulated AS sequences are more than likely to contain several events of small magnitude, events simulated at a long source-to-site distance, and/or events for which a small ground motion epsilon was sampled. “Weak” ground motions are associated to all such cases. These weak ground motions (and, therefore, the AS events associated with them) can be identified by the analyst based on criteria of his choice (e.g. intensity measure threshold below which the structure to be analyzed is expected to remain in the elastic range) and discarded from the suite to save computational time if they are not expected to induce any damage. Moreover, if a sequence does not contain any AS event with ground motion intensity sufficient to cause damage, a new AS sequence should be generated and new AS records should be selected.

Note here that, as explained earlier, we do not wish to embed the additional damage due to ASs into the MS fragility functions [e.g. see 9], but to derive AS fragility functions (conditional on the intensity measure of the aftershock) dependent on the damage level reached after the MS to be used along with a formal AS hazard model. As a consequence, the sample of AS sequences that we simulate does not need to be associated with an unbiased sample of ground motion intensities that will naturally comprise a number of weak ground motions along with stronger ones. However, we do wish to have an unbiased sample of AS ground motion char-
acteristics (herein only the spectral shape is explicitly considered) given the MS ground motion intensity level. While the latter is not formally verified, it is achieved asymptotically (when a sufficiently large number of records is used) via the sequence generation procedure that provides realistic causal parameters and the use of conditional AS target spectrum that ensures the consistency between the MS and AS spectral shapes. Any bias that may be introduced by forcing ruptures to cause “strong” ground motions (via discarding the “weak” ones and resampling) should be limited to the lower AS ground motion intensity levels and not of importance as long as the intensity threshold for discarding is kept reasonably low. This would ensure that small magnitude and/or large distance MS ruptures are not forced to produce outlier AS sequences and associated ground motions. Obviously, by adopting a very low threshold, one is then required to select more records to obtain a sample that would sufficiently populate all damage state bins and AS intensity levels, as is needed for computing the damage-state dependent AS fragility curves. The authors have carried out a preliminary investigation into this trade-off via a single-degree-of-freedom system. The latter, not shown here for conciseness (see [25]), confirmed that if the threshold is kept reasonably, the effect on fragility estimates is minimal. Nonetheless, exploring ways to more efficiently and consistently select a subset of the generated AS sequences is expected to be the topic of future research.

5 STRUCTURAL ANALYSIS SCHEME AND DERIVATION OF FRAGILITY FUNCTIONS

By following the procedure outline in the previous sections, one can obtain a suite of ground motion sequences for specific MS IM levels, each one consisting of a MS and one or multiple AS records that can be imposed in a back-to-back fashion to a structure. A sufficient gap need to be added between the records of each sequence depending on the period of vibration of the analyzed structure for allowing it to rest between successive time-history inputs. If we plot the associated responses of the structure obtained from the non-linear time-history analyses versus intensity, the MS data points will form “stripes” as these records were selected for specific MS IM levels, whereas the AS data points will appear as a “cloud” since the AS records correspond to random response spectrum realizations conditional on the MS event. The MS fragility functions can be easily computed by looking at the MS responses, assuming a lognormal cumulative distribution function and employing the maximum likelihood estimation approach [26]. For computing fragility functions for the structure in damaged conditions under the AS ground motions, we first assess the state of the building after each nonlinear dynamic analysis (MS or AS). The damaged building realizations are grouped according to the damage state they are found in, and their response to following ground shakings is studied separately. This can be visualized with an AS intensity versus AS response cloud, where the data points come only from analyses that started with the building having already reached the conditioning damage state in question due to the ground motions generated by the preceding earthquake(s) of the sequence. The sample of intensity-response pairs can then be used to derive the damage-dependent fragility functions via MLE.

6 SUMMARY AND CONCLUSIONS

A review of current challenges regarding the selection of AS ground motion records has been presented and discussed. Moreover, we offer herein a novel MS-AS ground motion record selection scheme intended for the analytical derivation of damage-dependent fragility models. The proposed methodology involves the combination of spectrally compatible ground motion records into MS-AS pairs by generating realistic AS ruptures and respecting the corre-
lation between the spectral ordinates of MS-AS events. In addition, it is flexible in the sense that the MS record selection can be performed individually with different approaches (CS, GCIM and cloud), and generally offers a favorable compromise between practicality and conceptual soundness. The newly developed framework is expected to comprise a valuable tool in the efforts for assessing the impact of aftershock events and damage accumulation in seismic risk estimates.

7 REFERENCES


NORCIA AND AMATRICE: A COMPARISON OF THE TWO HISTORIC CENTRES’ PERFORMANCE UNDER THE CENTRAL ITALY EARTHQUAKE SEQUENCE

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Abstract

The seismic sequence started on the 24th of August 2016 and concluded on January 2017 affected four region of Central Italy causing casualties, widespread damage to residential and heritage buildings and prolonged disruption. Among the many municipalities impacted by the seismic events, the towns of Amatrice and Norcia suffered the major losses. Although many similarities can be found in their urban layouts and comparison can be made in terms of building materials, techniques and periods of construction, epicentral distances from the fault of the relevant seismic events and categorization in terms of seismic zonation, the significant shaking alone cannot justify the severe damage extent observed in Amatrice in comparison to the very limited one recorded in Norcia.

The purpose of this paper is to investigate the different behavior of the two urban settlements of Norcia and Amatrice under the relevant shakings, and to discuss the different damage extents to their residential urban fabrics considering the specific structural features and vulnerability factors. To accomplish this, a set of building data collected in Norcia during the EEFIT individual research mission carried out in September 2017, is first analyzed using the vulnerability assessment method FaMIVE; an extrapolation of the same data set is then ‘re-adjusted’ in terms of material characteristics and strengthening elements to resemble the building stock of Amatrice: this is done on the basis of site observation collected by the authors during the 2016 Central Italy EEFIT mission. From the output of the FaMIVE procedure, capacity curves are derived and compared with the spectra of the main shocks of the seismic sequence. Cloud of performance points are generated for each event to be used to determine fragility curves, representative of the percentage of buildings undergoing certain damage levels under the specific seismic scenario. A discussion on the obtained results and the capability of the method to represent the observed damage extents concludes the paper.

Keywords: Seismic vulnerability, Historic Residential Masonry Structures.
1 INTRODUCTION

The seismic sequence that hit the Central Italy area within the four regions of Abruzzo, Marche, Lazio and Umbria from August 24th, 2016 to January 18th, 2017 was severely disruptive in terms of damage to historic residential buildings and heritage structures and very onerous in the consequent death toll. Within the time frame of the sequence, nine events have been recorded with magnitude greater than 5, three of which considered particularly devastating for the extensive damage they caused to sixteen municipalities which experienced a cumulative European Macroseismic Scale (EMS ’98, Grunthal, 1998) $I_{\text{EMS}} \geq \text{IX}$ [1].

The sequence started with a $M_w$ 6.2 event, with epicentre 5.6 km from the town of Accumoli: as a consequence of the shaking, the near town of Amatrice was almost completely razed to ground and nearly 300 people died. Two other main events occurred on 26th and 30th of October with epicentres located respectively 18.40 km and 7.8 km from the town of Norcia, the second of which, with a magnitude $M_w$ 6.5, was the stronger event ever occurred in Italy after the Irpinia earthquake in 1980 [2]. As a consequence, most of Norcia’s cultural heritage buildings were severely damaged, while many residential buildings experienced structural damage.

Several factors can be considered influential in the stark difference in response of the masonry building stock in the two towns. According to the published shake maps [2], and as discussed further in section 4, the seismic demand for the first event was considerably larger in Amatrice for stiffer structures, than in Norcia. Although the local soil is assigned the same class B according to EC8 [3] the topography of the two towns is substantially different, with Amatrice laid out at the top of a hill, while Norcia lays in a valley, contributing to substantial differences in amplification of the seismic shaking [4].

The 2016 sequence took place in a territory with a long history of destructive earthquakes, and historic events with similar patterns of shaking are recorded. Evidence proving that both Amatrice and Norcia have been heavily hit by severe shakings date back to the 14th century [5] and are documented through the numerous instances of repair and reconstruction that their historic centres have undergone [6]. Indeed, after the 1859 Lg 8-9 MCS, $M_w$ 5.7 Norcia earthquake a manual of “good a-seismic” building practice was produced for Norcia’s recovery. Following this, Norcia’s urban fabric and its buildings underwent other systematic strengthening projects, as a result of regional laws enacted after more recent seismic events [6]. A similar process did not occur in Amatrice, even though it was recognized as a seismic territory in 1915 and classified in zone 2 in 1927, but construction was not subject to any specific seismic building regulation until 1984 when, following the enactment of the first Italian seismic hazard map, the town was classified as seismic zone 2, to be later upgraded to seismic zone 1 in 2003 [7]. This notwithstanding the building stock in the historic centres of the two towns has substantial similarities in terms of building size and geometry, while quality of materials, workmanship and extent and type of retrofitting differed markedly as discussed in [7] and illustrated in section 3.

An important feature of this sequence was the sustained or increased magnitude of the subsequent events producing strong shaking and causing cumulative damage to masonry buildings. Research on effects of aftershocks has focused on steel, RC or timber frame structures ([9][10][11], respectively). The common assumptions are that the aftershocks will be of lesser magnitude of the main shock, thus implying that a Markovian approach can be used [12], and of shorter duration, thus becoming a function of the time elapsed after the mainshock [13]. Hence these models might not be directly applicable to the Central Italy 2016 sequence.

The effect of repeated event for masonry structure has been studied by [14]. For the Central Italy 2016 sequence, the empirical evidence of such effects has been reported by [1]. [15] and [16]. However, to the authors knowledge, such effect has not been quantified by using an analytical assessment approach for masonry buildings.
The main aim of this paper is to compare the damage suffered by the two historic centres Norcia and Amatrice under the two main events of the 2016 Central Italy earthquake sequence, by considering the increased seismic demand to structures experiencing repeated shakings. This paper presents in section 2 the methodology adopted and the steps followed to determine the damage-state dependent capacity curves; in section 3 a brief description of the two main events of the sequence considered in this study, the 24\textsuperscript{th} of August M\textsubscript{w} 6.2 and the 30\textsuperscript{th} of October M\textsubscript{w} 6.5 events. Section 4 is dedicated to the description of the two building stocks of Norcia and Amatrice. In section 5, the derived bilinear capacity curves for each of the buildings of the two datasets undergoing the spectra of the two shocks are presented in terms of performance points. A discussion on the results in terms of fragility functions concludes the paper.

2 PROPOSED METHODOLOGY

The proposed procedure to assess the cumulative damage of existing masonry structures exposed to repeated shaking can be summarized as shown in Figure 1.

Step (1) consists in the definition of a representative building sample, able to exhaustively represents the variety of building typologies of the case study under consideration. Section 3 explain how the buildings are chosen in the two cases presently analysed.

Step (2) is to choose an appropriate analytical assessment procedure able to evaluate the seismic vulnerability of the two samples. The procedure used in this work is the macro-element modelling approach FaMIVE [17], which allows to analyse a large number of buildings in a short time, producing simplified capacity curves based on ultimate lateral strength capacity and elastic and ultimate displacement. Having chosen suitable strong motion records, a capacity spectrum approach, such as N2 [18], can then be applied to determine the performance point for each capacity curve, representative of the response of each façade [19].

Figure 1: Proposed methodology to assess the cumulative damage to masonry structures
One of the main advantages of the FaMIVE method, is the possibility to tailor the routines to include specific characteristics of the building stocks considered which affect the mechanisms that might develop. A new module accounting for the effects of ring beams has been developed and included in the calculations. The work from [20] was used and further developed to correctly represent the current case studies. Specific assumptions on the way the ring beams are connected to the walls were implemented following the post-earthquake on site observations.

The procedure calculates the collapse load factor ($\lambda$) associated to all possible in plane (IP), out-of-plane (OOP), and combined (COM) failure mechanisms, then choses the one with the minimum $\lambda$, associated with the most damaging consequences for the analysed facade. The collapse load factor determines the lateral acceleration capacity of the structure.

The bilinear capacity curves are defined for each facade on the basis of three couple of points: $(0,0)$, $(a_u, k^*D_\zeta)$, $(a_u, D_\zeta)$, where $a_u$ represents the ultimate strength capacity and is defined as the ratio $\lambda/a$, namely the collapse load factor and the proportion of facade involved in the mechanism; the elastic limit displacement $D_\zeta$ is dependent on the natural period of the facade, which accounts for the presence of concrete ring beams, and the yielding strength $A_y$ which is the value of acceleration which will cause the stress distribution of the critical cross section to be triangular (elastic limit). The factor $k$ accounts for the difference between the initial elastic limit and the idealization of the curve, depending on each mechanism type. Finally, $D_\zeta$ represents the ultimate displacement of the facade considered. This is defined as a proportion of the displacement that determines the collapse of the facade ($D_c$) and is defined in relation to the occurring failure mechanism (i.e. IP, OOP or COM). More specifically, this is given as the ultimate displacement occurring at the resisting section of the wall (i.e. the effective proportion of the wall thickness in the case of OOP, the proportion of wall thickness and staggering ratio in the case of COM, the minimum value between staggering and wall thickness for IP).

The following step (3) is to compare each of the bilinear curve to the natural records of the events of interest in terms of spectral acceleration and displacement. Further information on the two main shocks considered in this study is given in Section 4. The outcome of this interpolation results is a cloud of representative performance points, Step (4), which, when compared with predetermined damage thresholds, defines the damage state of each facade of the sample undergoing the specific event. The final step (5) is to determine fragility functions for different limit states by using median and standard deviation values of the limit state displacement and deriving lognormal cumulative distributions. Three limit states are considered in agreement with the three representative points of the capacity curves, namely 1) damage limitation 2) structural damage and 3) collapse.

The novelty of the method lies in the way in which the cumulated damage due to consecutive strong shaking events is accounted for. The spectral displacement calculated to find the first set of performance points after the first event (i.e. interpolation with the first spectrum), is taken as permanent drift of each facade and used to determine its new stiffness and natural period. Moreover, the permanent drift is also accounted for as permanent leaning reducing the capacity of the structure to withstand further flexure caused by lateral acceleration, hence affecting the strength capacity determined by a new $\lambda$ factor. Each of the new capacity curves so constructed is interpolated with the representative record of the new event in the sequence, to obtain a new performance point which accounts for the previous damage condition.

This should be repeated for different building samples, for different spectra recorded at different locations for the same event. Depending on the number of iterations, which refer to the number of events considered, it is possible to observe a shift in both capacity and fragility functions, which clearly indicates the change in the overall sample' capacity due to repeated shocks.
3 NORCIA AND AMATRICE’S BUILDING STOCKS

During two consecutive EEFIT field missions the authors conducted two on-site investigations in Norcia and Amatrice and carried out a detailed post-event survey. Given the extensive level of damage observed in Amatrice and the very restricted time allowance within the red zone, it was preferred to collect very detailed chains of 360° images to document the damage condition of the building constructions along the main Corso Umberto I (East-West) and the perpendicular street (North-South) [8]. On the contrary, the damage condition of the residential buildings in Norcia was limited, allowing to conduct a detailed survey for a set of 82 buildings, corresponding to 110 facades (hereafter defined as N_sample) [6]. The sample was purposely chosen to evenly match the variety of buildings’ heights observed: more specifically, 74% of the sample is made of two-storeys buildings, with the remaining 26% distributed between three- storeys (18%) and one-storey (8%). Given the similarities in building size and structural systems between the two historic centres [7], a ‘simil’ sample of the Norcia dataset was modified to represent the Amatrice building stock in terms of material properties, number of storeys, horizontal structures (roof and flooring types) and traditional restraining elements applied (hereafter defined as A_sample), as obtained by the onsite ODC survey [8]. N_sample is characterized by 70% of buildings with a medium-high quality of masonry (M2), 17% of buildings made with a high quality of masonry (M1) and the remaining 13% of buildings characterized by a poor-quality masonry (M3). A_sample is characterized by 80% of the buildings with a poor-quality masonry (M3) and 20% of buildings with a medium-high quality (M2). Since no information is available to date on the mechanical properties test values for local masonry construction, relevant reference values are taken from Table C8A.2.2. of the NTC08 [22], summarised in Table 1, for the masonry fabrics identified on site.

<table>
<thead>
<tr>
<th>Masonry Typologies</th>
<th>f&lt;sub&gt;m&lt;/sub&gt; (N/mm&lt;sup&gt;2&lt;/sup&gt;)</th>
<th>τ&lt;sub&gt;0&lt;/sub&gt; (N/mm&lt;sup&gt;2&lt;/sup&gt;)</th>
<th>E (N/mm&lt;sup&gt;2&lt;/sup&gt;)</th>
<th>Weight (kg/m&lt;sup&gt;3&lt;/sup&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dressed rectangular stone masonry (M1)</td>
<td>6.0</td>
<td>0.090</td>
<td>2400</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.120</td>
<td>3200</td>
<td></td>
</tr>
<tr>
<td>Cut Stone with good bonding (M2)</td>
<td>2.6</td>
<td>0.056</td>
<td>1500</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>3.8</td>
<td>0.074</td>
<td>1980</td>
<td></td>
</tr>
<tr>
<td>Irregular stone masonry (pebbles) (M3)</td>
<td>1.0</td>
<td>0.020</td>
<td>690</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>0.032</td>
<td>1050</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Masonry types used in the case studies, retrieved from [22]

Given the general difference in fabric and mortar quality between the two towns’ buildings, the upper bound values of E (Young Modulus) were used to identify the characteristics of Norcia’s masonry typologies, while the lower bound were used to identify the ones of Amatrice. In a similar manner the values of cohesion τ<sub>0</sub> have been assigned. Figure 2 a) b) and c) show examples of masonry observed in Norcia, representative of the three typologies.

![Figure 2: Details of the M1, M2 and M3 masonry types in Norcia](Image)
Owing to the widespread level of damage in Amatrice, it was possible to observe the inner layers and the lack of connection in the thickness of the walls (Figure 3a). The majority of buildings were made of poorly dressed stones, mostly irregular and characterized by bad quality of mortar as shown in Figure 3 b). Very often, the walls were covered with thick layers of wired mesh and concrete jacketing [7], that would help bond the units together. Table 2 summarizes the key parameters characterizing the two building stocks.

![Figure 3: Details of the M2 and M3 masonry types in Amatrice](image)

Given the limited number of collapsed buildings in Norcia, the inspection of the horizontal structures was only partially possible, therefore the information regarding this specific aspect of the building stocks’ characterization refers to [23]. The three main flooring typologies encountered were wooden floors (WF) made of traditional joists, barrel vaults (VF) and reinforced wooden floors (RWF). In the case of Amatrice, additional presence of concrete floor slabs (CF) is added to the list, as these have been observed on site and reported in Figure 4 a) and b).

![Figure 4: Details of the concrete slabs observed in Corso Umberto I – Amatrice](image)

For the roof structures, the more traditional case of timber joists with screed and tiles (R1) was found only in 20% of the sample, while the remaining 80% showed to have roofs made with concrete slabs (R2) [23]. These typologies were implemented following the post-1979 building prescriptions listed within the Regional Law no.34 1981, enacted after the Irpinia earthquake [6]. While on site, it was possible to observe the lack of connection between these elements and the underneath masonry walls as shown in Figure 5 a), taken in Amatrice and
Figure b) taken in Norcia. With reference to the restraining elements, more traditional provisions such as buttresses (B) shown in Figure 5 a), ties (T) in Figure 5 b) were observed in Norcia, together with a number of buildings showing concrete ring beams (RB) (Figure 5 c)).

Conversely during the site investigation, no buttresses were observed in Amatrice. The main building stocks’ characteristics are summarized in Table 2.

<table>
<thead>
<tr>
<th>Building Stock</th>
<th>Masonry Type</th>
<th>Floor Type</th>
<th>Roof Type</th>
<th>Restraining elements %</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_sample</td>
<td>M1 17%</td>
<td>WF-VF 35%</td>
<td>R2 80%</td>
<td>T 25%</td>
</tr>
<tr>
<td></td>
<td>M2 70%</td>
<td>RWF 65%</td>
<td>R1 20%</td>
<td>B 33 %</td>
</tr>
<tr>
<td></td>
<td>M3 13%</td>
<td></td>
<td></td>
<td>RB 65%</td>
</tr>
<tr>
<td>A_sample</td>
<td>M2 30%</td>
<td>WF-VF 35%</td>
<td>R2 80%</td>
<td>T 25%</td>
</tr>
<tr>
<td></td>
<td>M3 70%</td>
<td>RWF 30%</td>
<td>R1 20%</td>
<td>RB 65%</td>
</tr>
</tbody>
</table>

Table 2: Parameters implemented in the FaMIVE procedure to reproduce Norcia and Amatrice’s building stocks.

In this section the characteristics of Norcia and Amatrice have been presented. The following section will describe the selected main shocks of the Central Italy sequence which have been considered relevant to describe the damage evolution of the two building stocks.

4 THE TWO MAIN EVENTS OF THE CENTRAL ITALY SEQUENCE

There are two different approaches to conduct seismic assessment at territorial scale that can be used to establish the seismic hazard of a region, namely probabilistic seismic hazard approach (PSHA) and deterministic scenario approach, directly correlated to the events occurred in the region. As discussed by [19], a deterministic approach might be more relevant for the assessment of historic centres, mainly because, if the seismicity of the area is well known, a credible and reliable earthquake scenario can be developed and be directly correlated to the building stock performance [24]. The 2016 Central Italy seismic swarm occurred in an area with the highest seismic hazard in Italy, where the PGA values expected with a probability of exceedance of 10% in 50 years are higher than 0.25 g [2][4]. The information on the records of the two events, extracted from the ESM database [20], are reported in Table 3:
<table>
<thead>
<tr>
<th>Station Name</th>
<th>Station Code</th>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Soil Type</th>
<th>Distance to city centre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amatrice</td>
<td>AMT</td>
<td>Free Field</td>
<td>42.63246</td>
<td>13.28618</td>
<td>B</td>
<td>0.9881 km</td>
</tr>
<tr>
<td>Norcia</td>
<td>NRC</td>
<td>Free Field</td>
<td>42.79254</td>
<td>13.09648</td>
<td>B</td>
<td>0.1727 km</td>
</tr>
</tbody>
</table>

Table 3: Details of the recording stations

Acceleration response spectra are extracted from the ESM database [25] and shown in Figure 6 a) and 6 b) respectively showing the 24\textsuperscript{th} of August and the 30\textsuperscript{th} of October events recorded at both AMT and NRC stations.

![Figure 6: Spectra of the 24/10/2016 event (a) and 30/10/2016 event (b) recorded at AMT and NRC stations](image)

The August event (a) has the largest ever peak ground acceleration recorded on Italian soil at the Amatrice instrument. It is also characterized by large amplifications in the 0.1-0.5 s period range, with a large drop for larger values. On the contrary, the event of the 30\textsuperscript{th} of October (b) shows amplification in the medium period range, with accelerations exceeding 1.0 g for periods around 1.0, which can help explaining the collapse of all the churches in Norcia [1][4][7].

Very relevant to the understanding of the building stock performance in the two towns is the demand in terms of spectral displacement imposed by the two seismic shocks considered as shown in Figure 7 a). For natural period lower than 0.5 s, the Amatrice record of the first event shows values of displacement demand up to 60 mm, while the demand recorded by the NRC instrument is in the range of 20 mm.

![Figure 7: Seismic demand in terms of spectral displacement for: a) the 24/08/2016 event with records from AMT and NRC station and the 30/10/2016 event with record from NRC station and b) close up of the same records in the range of 0 <T < 1](image)
A close up in the range of 0 <T1< 1 s [reported in Figure 7b)], which contains 97% of the two building samples natural periods, shows that for values of up to 0.5 s (85% of the building samples), the displacement demand of the 24th August event in Amatrice was substantially greater than the one in Norcia, while the 30/10/2016 event imposed on the Norcia building stock a demand twice as large as the one in August.

5 RESULTS AND DISCUSSION

The effect of the main shocks of the two events of 24th August 2016 and 30th October 2016th are represented in terms of performance points obtained by applying the N2 procedure [18] and computing their cumulative distribution to produce fragility functions with respect to three damage limit states: damage limitation, structural damage and collapse in accordance to EC8 part 3 [3]. For the present study, the EW direction of the earthquake ground motion record is used for both events, as this results in the highest demand on the two building samples.

Figure 8 a) shows the cloud of performance points representative of all the facades of the A_sample subjected to the 24th August 2016 event, and Figure 8 b) the corresponding fragility functions.

Figure 8: Performance points (a) and Fragility Functions (b) for Amatrice after the 24/08/2016 event

Figure 9 a) and b) show the performance of N_sample and related fragility functions for the same event, shows that the distribution of damage limitation performance points for A-sample is confined to very modest lateral displacement, when compared with the N_sample. This is due to the substantially greater strong ground motion experienced, but also to the stiffer and lower strength capacity of the A_sample with respect to the N_sample. This is confirmed by
the much narrower range of strength demand for the N_sample (0.08 g to 0.57 g) when compared to the A_sample (0.07 g to 0.92 g) for a similar range of displacement demand. It should be also noted that the performance points for collapse for A_sample are confined within a narrow range corresponding to the limit ultimate displacement of their capacity curves, which in turn is a function of the masonry units’ size and masonry quality. No collapse was computed for the N_sample.

Figure 10 a) shows the proportions of buildings in each damage state after the August event in each of the two town, providing a good correspondence to the empirical onsite observations. Figure 10 b) compares the corresponding fragility functions. While the structural damage condition is not substantially different for the two samples, the larger proportion of buildings with buttresses and ties, coupled with the lower level of acceleration and displacement demand for the lower period range, ensure a much better performance of the N_sample with respect to the damage limitation threshold.

![Figure 10: Comparison between the % of building stocks’ damage (a) and the two sets of fragility functions (b) for the 24/08/2016 event.](image)

<table>
<thead>
<tr>
<th>Building Stock Event</th>
<th>Damage state</th>
<th>Median (θ)</th>
<th>Standard deviation (β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_sample 24/08/2016</td>
<td>DL</td>
<td>0.011</td>
<td>1.023</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.912</td>
<td>0.915</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3.542</td>
<td>0.228</td>
</tr>
<tr>
<td>N_sample 24/08/2016</td>
<td>DL</td>
<td>0.049</td>
<td>1.032</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.839</td>
<td>1.182</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N_sample 30/10/2016</td>
<td>DL</td>
<td>0.038</td>
<td>1.086</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.422</td>
<td>1.198</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3.358</td>
<td>0.811</td>
</tr>
</tbody>
</table>

Table 4: Values of median and standard deviation characterizing the two samples under the 24/08/2016 and the 30/10/2016 event.

Table 4 shows the values of mean (θ) and standard deviation (β) of both samples after the August 2016 event, and for the N_sample after the 30th October event, for each damage state. Figure 11 a) and 11 b) show the condition of the N_sample after the 30th October event. The performance points and fragility curves presented are obtained by running a second iteration to the N_sample of Figure 9 a) as described in section 2.

Notwithstanding the clear increase in terms of lateral spectral acceleration demand Sa(T*), which now reaches 0.75 g, there is still a significant proportion of buildings remaining within
the DL state. However, the distribution of performance points in SD is much sparser, and almost 20% of the buildings enter the collapse state. The cloud of performance points has a general shift towards the right, indicating that the displacement demand has increased: some of the points reach collapse for 20 cm displacement against the 6.5 cm recorded after the August event.

![Figure 11: Performance points (a) and Fragility Functions (b) for Norcia after the 30/10/2016 event](image)

Figure 11: Performance points (a) and Fragility Functions (b) for Norcia after the 30/10/2016 event

Figure 12 a) and b) show the comparison of the status of N_sample after the first and second event, respectively. Beside the evident presence of collapses, the fragility functions for lesser damage states are shifted towards the left, as the residual values of the damage thresholds for each damage state is smaller than in the buildings exposed to the first event, and they have greater variability. Such shift is evident in the reduction of the median (θ) values and increase of the standard deviation (β) of the N_sample for the two events, summarised in Table 4.

![Figure 12: Comparison between percentages of Norcia building stock’s damage between the two seismic events (a) and the two sets of fragility functions (b) for the 24/08/2016 and the 30/10/2016 events](image)

Figure 12: Comparison between percentages of Norcia building stock’s damage between the two seismic events (a) and the two sets of fragility functions (b) for the 24/08/2016 and the 30/10/2016 events.

6 CONCLUSIONS

A limit state analysis of two samples representative of the two building stocks of Amatrice and Norcia, affected by the 2016/2017 Central Italy earthquake sequence has been conducted using the FaMIVE approach and N2 method, with the aim of assessing their seismic capacity and cumulated damage when exposed to sequences of high shaking, without being repaired. To this end a simple methodology has been developed whereby the masonry structures are characterized by bilinear curves and a capacity spectrum approach is adopted to compute performance point, using the natural spectra recorded at the sites for the various events.
Two samples with similar geometries and different mechanical characteristics are created on the basis of an accurate survey conducted in Norcia, and analysis of extensive omnidirectional camera records collected in Amatrice. The approach is successful in delivering buildings samples representative of the two diverse building stock. The extents of damage observed on site have been captured by the analytical method chosen and satisfactorily simulated via the determination of performance points and the derivation of fragility curves. The use of updated flexibility, reduced ductility and residual damage thresholds successfully capture the damage evolution through the two main events of the seismic sequence. The updated fragility functions show reduced median, higher standard deviation and an increase in damage probability of about 15% for the structural damage state.

Acknowledgements
The Authors wish to thank the Civil Protection Department of the Umbria Region and especially Nicola Berni and Paolo Putrino, the University of Perugia and Ferrara, Professor Antonio Borri, Professor Andrea Giannatoni, Dr Giulio Castori, Dr Romina Sisti and Dr Alessandro De Maria for their insights to the case study of Norcia. Miss Chen Huang is thankfully acknowledged for the support given during the Sept 2017 EEFIT Research mission.

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SEISMIC FRAGILITY OF RC STRUCTURES UNDER MAINSHOCK-AFTERSHOCK SEQUENCES RECORDED ON SOFT SOIL CONDITIONS

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Abstract

Past earthquake episodes of moderate-to-high magnitude have shown that strong mainshock trigger usually strong aftershocks forming, in such a way, the so-called mainshock-aftershock (MSAS) sequences. The latter can normally induce higher seismic losses to structures and infrastructures compared to the single mainshock event. High vulnerability has been also detected for structural systems being subjected to the soft soil earthquake strong ground motions due to the soft soils-driven amplification in structural demand. Along these lines, this study investigates the seismic fragility of an existing reinforced concrete (rc) structure subjected to MSAS sequences recorded on soft soil profiles. To this end, a preliminary selection of 28 and 52 soft-soil mainshocks and aftershocks strong GMs respectively served the basis to generate 36 artificial earthquake sequences. The latter were made of six mainshocks and six aftershocks seismic motions that are representative of the initially formed sets of records in terms of the predominant period, $T_g$, and the strong ground motion duration. An existing seven-story rc building located in Van Nuys, California, was modelled by using the OpenSees finite element code. The measured response of the building during the Northridge (1994) earthquake episode was used to validate the finite element model developed herein. Nonlinear time history analysis was performed and the soft-soil MSAS-induced structural response (i.e., roof drift, maximum interstory drift ratio and maximum residual interstory drift ratio) was comparatively assessed with the response induced by MSAS sequences recorded on firm soil conditions. Fragility analysis was also conducted and the relevant fragility curves were estimated accounting for varying intensity of the aftershocks. Based on the results derived herein, the firm-soil MSAS led to higher structural collapse capacity compared to the one that was calculated by using the soft-soil MSAS, the latter being more profound when the building was subjected to aftershocks with increased intensity.

Keywords: Earthquake Sequences, Strong Ground Motions, Reinforced Concrete Structure, Soft Soil Conditions, Nonlinear Response History Analysis, Drift Demand, Fragility Curves
1 INTRODUCTION

Structures and infrastructures being located in earthquake-prone areas of medium to high seismicity are usually subjected to intensive mainshocks that, in turn, may trigger strong aftershocks within a rather short time interval. The earthquake mainshock-aftershock (MSAS) sequences have been found to induce excessive damages to structural systems while the potential for collapse may increase since there is no sufficient time window to retrofit a damaged structure due to a preceding seismic episode (i.e., mainshock) in order, eventually, to withstand succeeding earthquake events (i.e., aftershocks). Plethora of earthquake sequences including the Michoacan (Mexico, 1985), Northridge (California, 1994), Chi-Chi (Taiwan, 1999), Wenchuan (2008, China), Chile (2010), Darfield (New Zealand, 2010), Christchurch (New Zealand, 2011), Tohoku (Japan, 2011), Nepal (2015) and Kumamoto (2016, Japan) have corroborated that structures experiencing strong mainshocks are more vulnerable to sustain increased damages during the successive aftershocks episodes. Moreover, the current state-of-practice (i.e., seismic codes and guidelines) prescribes the design of structural systems accounting only for the mainshocks; thus, the performance of the structural systems has not been tuned to resist the cumulative damage that the MSAS sequences may induce.

Several researchers have already focused on the effects of MSAS sequences on the performance of structures with various geometry, size and complexity. Especially, nonlinear single degree of freedom (SDOF) systems subjected to MSAS sequences were used in Refs. [1–6] and both global and local damage indices accounting for energy dissipation [7] were found to be highly sensitive in the MSAS seismic records. Contrarily, no sound influence was corroborated on deformations-based indices including the maximum and residual displacement as well as displacement ductility. Suitably chosen earthquake sequences ground motions were also used to excite multiple degree of freedom (MDOF) systems and their performance was thoroughly investigated [8–10]. For example, Ruiz-García and Negrete-Manriquez [8] assessed the peak and residual drift of steel frames subjected to both as-recorded and artificial MSAS sequences. The results found therein showed that the artificial MSAS sequences can overestimate the structural response compared to as-recorded MSAS sequences. The fragility analysis, being a probabilistic framework to account for the uncertainty, performance and reliability of any structural system exposed to the earthquake hazard, has been also utilized to quantify the effects of the MSAS earthquake sequences to structures [11–14]. To this end, a framework for fragility assessment of rc frame structures under MSAS sequences was developed by Wen et al. [14], who concluded that the aftershocks may increase by 15% the fragility of the frame structures. Increased vulnerability, quantified, though, without the use of the robust fragility analysis framework, has been also detected for structural systems located in soft-soil sites and subjected to aftershock seismic motions. Especially, Díaz-Martínez et al. [15] found that aftershock strong ground motions, recorded in soft soil sites, led steel moment resisting frames to experience higher inter-story drift demands compared to the ones caused by stiff soil earthquake aftershock motions.

Along these lines, the focal point of this paper is to quantify the performance and, eventually, estimate the fragility of an existing structure under real MSAS sequences that have been recorded in soft soil sites. Especially, a seven-story reinforced concrete (rc) building, located in Van Nuys, California, was adopted to constitute the numerical analysis testbed. The OpenSees finite element code [16] was used to develop a detailed, two-dimensional (2D) frame model. Attention was dedicated to select earthquake sequences consisting of several mainshock and aftershock ground motions recorded during past seismic episodes on soft soil sites. Nonlinear response history analysis was performed and the soft soil MSAS-induced structural response, quantified through drift demands, was comparatively assessed with the
response induced by MSAS sequences recorded on firm soil conditions. Fragility analysis was, finally, conducted and the collapse fragility curves were estimated accounting for varying intensity of the aftershocks.

2 STRUCTURAL MODEL

An existing, seven-story, rc hotel building located in Van Nuys, the North-West part of the Los Angeles (California, US) metropolitan area (Fig. 1, left) was used herein as the numerical analysis testbed. The building, seen by Fig. 1 (right), was designed in 1965 and its construction started one year later. The height of the typical story is 2.65 m while a 4.10 m high ground floor resulted in total height for the building equal to 20.0 m. Nine and four frame systems respectively (Fig. 2) were designed to resist the earthquake-induced, lateral forces along the two main horizontal directions of the building and its total size (in plan) is over 870,000 m² (19.05 m x 45.72 m). Concrete material of nominal strength, $f'_c$, equal to 5 ksi and 4 ksi were used for the columns of the first and second storeys, respectively, while the columns of the remaining stories were made of lower strength concrete ($f'_c=3$ ksi). Moreover, the nominal strength of the concrete material used both for beams and slabs of the first three floors (i.e., groundfloor and two first floors) is equal to 4 ksi and concrete of $f'_c=3$ ksi was used for the horizontal structural members placed from the third floor to the roof. Regarding the longitudinal and transverse reinforcing bars, steel members corresponding to A432-62T (Grade 60, yield strength, $f'_y$, equal to 60 ksi) and A305-56T (Grade 40, $f'_y=40$ ksi) were used therein. Detailed description of the specific building can be found elsewhere ([17,18]).

Figure 1. Seven-story reinforced concrete building in Van Nuys (California, US): location (left) and elevation (right).

Figure 2. Seven-story reinforced concrete building in Van Nuys (California, US): typical floor (left) and north frame presented in elevation (right).

The OpenSees finite element code [16] was used to develop the 2D representation of a typical frame of the building consisting of eight bays and seven storeys (Fig. 2). The columns and beams of the frames were modelled by the nonlinear beam elements, the latter enable ac-
counting for the concrete behavior under cyclic loading, residual strength and stiffness degradation through a fiber approach. The uniaxial Kent-Scott-Park material law with degraded linear unloading/reloading stiffness and no tensile strength was considered herein to model the concrete behavior while the reinforcing steel bars were modeled by using a uniaxial bilinear material relationship with kinematic hardening and optional isotropic hardening described by a non-linear evolution equation. The soil-structure interaction effects were neglected by the current study; hence, fixed-base conditions were considered at the base of the groundfloor columns. Additional details about the OpenSees model of the frame studied herein can be found elsewhere [19].

The Van Nuys hotel building, being instrumented with accelerometers mounted in different storeys, has experienced several earthquakes including, among others, the 1994 Northridge seismic event. Figure 3 presents the accelerations time history that was recorded during the Northridge mainshock at the groundfloor of the building. The nearly 0.45 g corresponding to the peak ground acceleration, PGA, reflects the increased intensity of this ground shaking. The groundfloor-recorded acceleration time history was used herein as the necessary base motion to excite the 2D frame model and perform response history analysis within the OpenSees finite element framework. The structural response in terms of roof accelerations, calculated by the numerical analysis, was compared with the acceleration time history recorded at the same location of the building during the Northridge mainshock (see Fig. 4). The rather satisfactory compliance between the recorded and simulated response serves a concrete validation of the specific finite element model and hence, allows its reliable use for conducting further analysis within the framework of this study.

![Figure 3. Acceleration time history of the Northridge earthquake episode (17/01/1994, moment magnitude, $M_w=6.7$, and epicentral distance, $R=7.2$ km).](image)

![Figure 4. Comparison between the recorded and numerically calculated roof acceleration response triggered by the 1994 Northridge seismic episode.](image)

### 3 EARTHQUAKE STRONG GROUND MOTIONS

The numerical study presented herein requires the selection of earthquake sequences including both mainshock and aftershock strong ground motions that have been recorded during past seismic events. Especially, the mainshock magnitude dependent time and distance window, originally proposed by Gardner and Knopoff [20], was adopted to select the necessary earthquake sequences while special effort was dedicated to consider mainshock and after-
shock records being associated with PGA higher than 0.05 g and 0.01 g respectively. Additionally, earthquake motions recorded on soft soil conditions, i.e., where the shear wave velocity, $v_{s,30}$, averaged over the upper 30 m of the soil profile is found lower than 180 m/s [21], were considered herein. The PEER-NGA earthquake strong ground motion database [22] was used to search for the necessary seismic records and the application of the aforementioned criteria led to 28 and 52 soft-soil mainshock and aftershock seismic records respectively. However, few soft soil mainshock and aftershock earthquake records were found to be recorded at the same station. Therefore, the already selected mainshock and aftershock seismic motions were randomly combined to create earthquake sequences that do not favor bias in the calculated response results. Such a random combination led to an extensive number of earthquake sequences that was reduced to facilitate the objectives of the current study by applying three additional criteria already discussed elsewhere [23-24]: (i) the duration of the mainshock ground motions, quantified by the using the index of the significant duration [25], should be longer than the one related to the aftershock ground motions, (ii) the predominant period, $T_g$, of the strong ground motions should be relatively long (i.e., $T_g>0.50$ s was considered herein), the latter being characteristic of soft soil recorded earthquake motions, and (iii) the mainshock seismic motions spectral acceleration values, $S_a$, calculated for the moderate-to-long periods range to include the first-mode period ($T=1.49$ s) of the frame, should be higher for the mainshock records compared to the aftershock ones.

<table>
<thead>
<tr>
<th>Earthquake name</th>
<th>Earthquake Magnitude, $M_w$</th>
<th>$T_g$ (s)</th>
<th>PGA (g)</th>
<th>Significant duration (s)</th>
<th>$S_a$ (g)</th>
</tr>
</thead>
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<tr>
<td>Loma Prieta (1989)</td>
<td>6.93</td>
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<td>0.27</td>
<td>8.41</td>
<td>0.38</td>
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<td>14.28</td>
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<tr>
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<td>11.23</td>
<td>0.58</td>
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<tr>
<td>Christchurch, New Zealand (2011)</td>
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<td>1.00</td>
<td>0.72</td>
<td>8.48</td>
<td>0.92</td>
</tr>
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</table>

Table 1. Details of the selected mainshock earthquake strong ground motions.

<table>
<thead>
<tr>
<th>Earthquake name</th>
<th>Earthquake Magnitude, $M_w$</th>
<th>$T_g$ (s)</th>
<th>PGA (g)</th>
<th>Significant duration (s)</th>
<th>$S_a$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperial Valley-06 (1979)</td>
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<td>0.25</td>
<td>0.24</td>
<td>11.89</td>
<td>0.36</td>
</tr>
<tr>
<td>Imperial Valley-06 (1979)</td>
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<td>0.15</td>
<td>0.22</td>
<td>14.15</td>
<td>0.32</td>
</tr>
<tr>
<td>Parkfield-02 (2004)</td>
<td>6.00</td>
<td>0.65</td>
<td>0.62</td>
<td>6.99</td>
<td>0.43</td>
</tr>
<tr>
<td>Parkfield-02 (2004)</td>
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<td>0.37</td>
<td>6.14</td>
<td>0.42</td>
</tr>
<tr>
<td>Parkfield-02 (2004)</td>
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<td>0.30</td>
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<td>0.34</td>
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<tr>
<td>Parkfield-02 (2004)</td>
<td>6.00</td>
<td>0.45</td>
<td>0.43</td>
<td>7.47</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 2. Details of the selected mainshock earthquake strong ground motions.

The application of the aforementioned strategy to select earthquake sequences resulted in six mainshock and aftershock records respectively (Tables 1 and 2) that were combined randomly forming, eventually, 36 MSAS sequences to be used for the subsequent time domain analysis and fragility estimations. A 10 s time window with zero acceleration was also added
between the two consecutive seismic motions. Moreover, four different levels of relative spectral intensity, $\Delta S_a$, were chosen herein to scale the aftershock ground motions:

$$\Delta S_a = S_{a,AS} / S_{a,MS}$$  (1)

where $S_{a,AS}$ and $S_{a,MS}$ are the spectral acceleration values of the mainshock and aftershock records respectively calculated at the fundamental period of the 2D frame. Such a scaling strategy enables investigating the effect that the aftershock motions with different intensity have on the seismic performance of the structure under study.

To favor the comparative assessment between soft-soil and firm-soil MSAS in terms of their effect on the structural response, 18 MSAS earthquake sequences (Table 3) recorded on firm soil sites (i.e., $v_{S,30} > 180$ m/s [21]) were chosen by following a strategy being almost identical to the one adopted for the selection of the soft-soil MSAS earthquake sequences records. It is only the criterion related to long predominant period, $T_g$, of the earthquake records that was not followed in order to select the firm soil records.

<table>
<thead>
<tr>
<th>Earthquake name</th>
<th>Earthquake magnitude, $M_w$</th>
<th>$T_g$ (s)</th>
<th>PGA (g)</th>
<th>Significant duration (s)</th>
<th>$S_a$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Chi (1999)</td>
<td>7.62/6.3$^\dagger$</td>
<td>0.48/0.30</td>
<td>0.13/0.12</td>
<td>16.63/10.28</td>
<td>0.14/0.02</td>
</tr>
<tr>
<td>L’Aquila, Italy (2009)</td>
<td>6.30/5.6</td>
<td>0.24/0.12</td>
<td>0.15/0.28</td>
<td>8.36/2.38</td>
<td>0.04/0.03</td>
</tr>
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<td>L’Aquila, Italy (2009)</td>
<td>6.30/5.6</td>
<td>0.10/0.16</td>
<td>0.14/0.25</td>
<td>8.86/3.47</td>
<td>0.03/0.04</td>
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<td>L’Aquila, Italy (2009)</td>
<td>6.30/5.6</td>
<td>0.66/0.13</td>
<td>7.55/5.33</td>
<td>0.19/0.02</td>
<td></td>
</tr>
<tr>
<td>L’Aquila, Italy (2009)</td>
<td>6.30/5.6</td>
<td>0.48/0.15</td>
<td>8.14/5.16</td>
<td>0.24/0.02</td>
<td></td>
</tr>
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<td>0.52/0.11</td>
<td>8.43/5.53</td>
<td>0.13/0.02</td>
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<tr>
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<td>0.25/0.19</td>
<td>12.55/14.22</td>
<td>0.11/0.07</td>
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<tr>
<td>Chalfant Valley-02 (1986)</td>
<td>6.19/5.4</td>
<td>0.18/0.12</td>
<td>16.77/16.19</td>
<td>0.11/0.06</td>
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<tr>
<td>Chalfant Valley-02 (1986)</td>
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<td>0.45/0.15</td>
<td>6.17/7.63</td>
<td>0.35/0.02</td>
<td></td>
</tr>
<tr>
<td>Whittier Narrows-01 (1987)</td>
<td>5.99/5.3</td>
<td>0.29/0.18</td>
<td>5.22/4.45</td>
<td>0.12/0.03</td>
<td></td>
</tr>
<tr>
<td>Whittier Narrows-01 (1987)</td>
<td>5.99/5.3</td>
<td>0.39/0.21</td>
<td>5.70/7.82</td>
<td>0.11/0.03</td>
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<tr>
<td>Whittier Narrows-01 (1987)</td>
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<td>7.95/5.17</td>
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<td>Whittier Narrows-01 (1987)</td>
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<td>0.32/0.16</td>
<td>7.13/6.15</td>
<td>0.10/0.07</td>
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<td>Whittier Narrows-01 (1987)</td>
<td>5.99/5.3</td>
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<td>0.10/0.04</td>
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<td>Whittier Narrows-01 (1987)</td>
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<td>4.49/2.14</td>
<td>0.04/0.03</td>
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<td>Whittier Narrows-01 (1987)</td>
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<td>0.23/0.13</td>
<td>11.17/10.26</td>
<td>0.05/0.03</td>
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<td>Whittier Narrows-01 (1987)</td>
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<td>0.27/0.15</td>
<td>9.53/10.45</td>
<td>0.05/0.03</td>
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<tr>
<td>Whittier Narrows-01 (1987)</td>
<td>5.99/5.3</td>
<td>0.40/0.13</td>
<td>6.58/10.84</td>
<td>0.07/0.04</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Details of the selected MSAS earthquake strong ground motions recorded on firm soil sites.

$^\dagger$ The nominator and denominator of the ratio represent the earthquake magnitude, $M_w$, of the mainschock, MS, and aftershock, AS, seismic motions respectively. An identical rationale is followed for rest of the parameters presented herein.

4 RESPONSE HISTORY ANALYSIS RESULTS

4.1 Effects of aftershock excitations on drift demand

The performance of response history analyses considering both the MSAS earthquake sequences ground motions and just the mainshock (MS) ones, enables scrutinizing the effects of the aftershocks on the building response. Especially, drift-related demand parameters including the roof drift (RD), maximum interstorey drift ($\text{IDR}_{\text{max}}$) and maximum residual interstorey...
drift ($\text{RDR}_{\text{max}}$) were provided herein as ratios between those values corresponding to the MSAS seismic records over the ones related to the MS records. By scaling the earthquake ground motions based on four levels of relative spectral intensity between the aftershock and the mainshock records (§3), the thorough investigation of the aftershocks effect on the drift demand is made feasible for a broad range of seismic intensity. Moreover, such a comparative assessment between soft and firm soil sites and the induced structural response allows investigating whether the soft soils-driven amplification in seismic demand, already observed for mainshock excitations [3,15,26–29], is valid for MSAS earthquake sequences.

Figure 5. Mean ratios of the roof drift induced by the MSAS earthquake sequences excitations and the mainshock ones: soft soil sites (left); firm soil sites (right).

Figure 6. Mean ratios of maximum interstorey drift induced by the MSAS earthquake sequences excitations and the mainshock ones: soft soil sites (left); firm soil sites (right).

Figure 7. Mean ratios of maximum residual interstorey drift ($\text{RDR}_{\text{max}}$) induced by the MSAS earthquake sequences excitations and the mainshock ones: soft soil sites (left); firm soil sites (right).
Along these lines, Fig. 5 plots the mean roof drift ratios in relation to the mainshock excitations spectral intensity, defined at the fundamental period, $T_1 = 1.49$ s, of the frame. As can be seen, the higher the intensity of the aftershock motions, the higher the roof drift caused by the MSAS compared to the single MS motions respectively. The latter is mainly valid for the soft soil seismic records while such a trend cannot be transparently seen for the firm soil MSAS earthquake excitations. For example, nearly 50% amplification (on average) was found for the soft soil-related roof drift demand ratio by increasing twice the relative intensity between the aftershocks and the mainshocks records, i.e., from $\Delta S_a = 0.5$ to $\Delta S_a = 1.0$. On the other hand, such an increase in the relative spectral intensity led to 1.09 times higher (on average) roof drift demand ratio caused by the firm-soil earthquake records. The soft-soil driven amplification of the roof drift demand can be further confirmed by almost 1.4 times higher average $RD_{MSAS}/RD_{MS}$ value that the $\Delta S_a = 1.0$-scaled soft-soil excitations induced in comparison with the firm-soil ones. Similar findings can be seen for the numerically calculated maximum interstorey drift (IDR$_{\text{max}}$, Fig. 6) and maximum residual interstorey drift (RIDR$_{\text{max}}$, Fig. 7).

### 4.2 Seismic fragility curves

The current study investigates the effect of the soft-soil MSAS earthquake sequences on the collapse of the specific rc building by estimating the relevant fragility. To this end, the fragility curve, $F$, of a structure subjected to MSAS earthquake sequences can be estimated as follows [14]:

$$F_{\text{MSAS}}(x_i, \Delta IM) = P\left[EDP_{\text{MSAS}} \geq EDP_{\text{LSi}} | IM_{MS} = x_i, \Delta IM \right]$$

(2)

where $EDP_{\text{MSAS}}$ is the engineering demand parameter of a structure under MSAS sequence, $IM_{MS}$ is the intensity measure of the mainshock earthquake excitation, $\Delta IM$ represents the ratio of the aftershock excitation intensity measure, $IM_{AS}$, over the one related to the mainshock strong ground motion, $IM_{MS}$, and $EDP_{\text{LSi}}$ is the threshold of the capacity corresponding to a specific limit state. Based on the lognormal distribution assumption of the structural response under the earthquake load, the probability that $EDP_{\text{MSAS}}$ exceeds $EDP_{\text{LSi}}$ is provided as:

$$P\left[EDP_{\text{MSAS}} \geq EDP_{\text{LSi}} | IM_{MS} = x_i, \Delta IM \right] = 1 - \Phi\left[\frac{\ln(EDP_{\text{LSi}}) - \ln(\lambda(EDP_{\text{MSAS}} | IM_{MS}, \Delta IM = i))}{\beta(EDP_{\text{MSAS}} | IM_{MS}, \Delta IM = i)} \right]$$

(3)

$$\lambda(EDP_{\text{MSAS}} | IM_{MS}, \Delta IM = i) = a IM_{MS}^{b}$$

(4)

$$\beta(EDP_{\text{MSAS}} | IM_{MS}, \Delta IM = i) = \frac{\sum \left[\ln(EDP_{\text{MSAS},i}) - \ln(\lambda(EDP_{\text{MSAS}} | IM_{MS}, \Delta IM = i))\right]^2}{N - 2}$$

(5)

where $\Phi(.)$ is the cumulative normal distribution function, $\lambda(.)$ is the median value of the $EDP_{\text{MSAS}}$ of a structure as a function of an IMMS, $a$ and $b$ are regression coefficients, $N$ is the number of numerical analyses, $EDP_{\text{MSAS},i}$ is engineering demand parameter of a structure under a MSAS sequence calculated by the ith numerical analysis. The $\beta$-term, denoted as dispersion, represents the uncertainty being related to the seismic demand. The robust method of incremental dynamic analysis [30] was used herein to facilitate the fragility estimates. Additionally, the maximum interstorey drift (IDR$_{\text{max}}$) is adopted as the engineering demand parameter while, based on FEMA P-356 [31], interstory drift, being equal to 4%, is considered herein as the corresponding limit state, $EDP_{\text{LSi}}$. 
Based on the fragility framework described above, Fig. 8 compares the collapse fragility curves obtained by using MSAS sequences excitations and mainshock ones recorded both on soft soil and firm soil conditions respectively. First, it can be seen that the MSAS earthquake sequences associated with the lowest intensity of aftershocks considered herein (i.e., $\Delta S_a=0.3$) was found to lead to nearly identical fragility with the one obtained by using only the mainshock excitations. The latter is valid irrespectively of the soil conditions, on which the seismic records have been recorded. It is also interesting to note that the firm soil-related fragility estimates were found marginally sensitive in increasing the aftershocks intensity while, on the other hand, higher relative spectral intensities (i.e., $\Delta S_a=[0.5, 0.8, 1.0]$) for the MSAS earthquake sequences excitations led to increased fragility estimates for the soft soil records. For example, the fragility was found equal to 0.5 for the soft soil recorded mainshock excitations associated with spectral acceleration of $S_{a,MS}=1.05$ g while 1.35 times less spectral intensity (i.e., $S_{a,MS}=0.78$ g) of the $\Delta S_a=1.0$-scaled MSAS excitations led to 50% probability of collapse for the building studied herein. Figure 8 facilitates also the quantitative comparison in terms of fragility estimations derived from the firm and soft soil sites respectively. Especially, lower vulnerability of the building studied herein was found to be associated with the firm soil seismic excitations compared to the soft soil ones since the former led to fragility estimate equal to 0.5 for 1.35 times higher mainshock-related spectral intensity, $S_{a,MS}$ compared to the soft soil seismic excitations. The latter observation is valid for the MSAS earthquake sequences with the highest aftershock motions intensity (i.e., $\Delta S_a=1.0$); however, the increased collapse fragility of the specific structure when subjected to soft-soil MSAS earthquake sequences can be further corroborated for the additional relative spectral intensities investigated herein.

![Figure 8. Collapse fragility curves for the building under MSAS sequences (left): soft soil sites; (right): firm soil sites.](image)

5 CONCLUSIONS

The current study elaborates the effect of MSAS earthquake sequences, recorded on soft soil sites, on the structural response and eventually, the fragility of an existing, seven-storey rc building located in Van Nuys, California, US. A refined finite element model was developed and its validation was based on response data recorded during the Northridge (1994) mainshock event. 36 MSAS sequences were formed on a random basis combination among six mainshock and six aftershock earthquake ground motions recorded on soft soil sites. Four mainshock-aftershock relative spectral intensity levels (i.e., $\Delta S_a=[0.3, 0.5, 0.8, 1.0]$) were also used to scale the selected seismic motions. Additionally, 18 MSAS earthquake sequences, recorded on firm soil sites, were chosen allowing the comparative assessment of the effect that seismic records from different soil conditions have on the structural response, the latter
being quantified herein by drift demand parameters. The numerical analysis results showed that the $\Delta S_a=1.0$-scaled soft soil MSAS excitations induced nearly 1.4 times higher average roof drift ratio ($\text{RD}_{\text{MSAS}}/\text{RD}_{\text{MS}}$) compared to the firm soil excitations. The increased susceptibility of the building studied herein to the soft soil earthquake sequences was further corroborated by the fragility analysis results. Lower vulnerability of the structure was found for the firm soil seismic excitations compared to the soft soil ones since, for $\Delta S_a=1.0$, the firm soil excitations led to fragility estimate equal to 0.5 for 1.35 times higher mainshock-related spectral intensity, $S_{a,\text{MS}}$ compared to the soft soil seismic excitations.

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PERFORMANCE ASSESSMENT OF BRIDGES UNDER A SEQUENCE OF SEISMIC EXCITATIONS
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Abstract
This study utilizes the current Pacific Earthquake Engineering Research (PEER) Performance-based Earthquake Engineering (PBEE) methodology to address the significance of considering a sequence of ground motions as the representation of seismic hazard in PBEE design and assessment frameworks. The current state of the art in design and assessment of structures, via PBEE, is founded on the conditional relationships between Intensity Measure (IM), Engineering Demand Parameter (EDP), Damage Measures (DM) and Decision Variable (DV) stringed together in a triple integral representing total probability theorem. Aside from presenting theoretical background, this study is conducted on two reinforced concrete (RC) ordinary highway bridges in California which are subjected to a sequence of earthquakes determined randomly using ARIMA model fitted to the history of events in Southern California. In contrast, the traditional PBEE approach, the non-linear analysis of the bridges is conducted without resetting the damage to the bridge models due to earlier ground motions. Methodologic risk assessment is conducted using Column Drift Ratio (CDR) as the primary EDP, to generate analytical functions that can assist engineers in the decision making of whether to repair the bridge after a ground motion or otherwise. This is achieved by providing a functional form to predict the amplification of EDP in a pre-damaged bridge due to a future unobserved ground motion given the last observed ground motions. Conclusions are based on this amplification factor which is described as the ratio of the EDP of a pre-damaged bridge to that of a repaired bridge under the influence of a certain history of ground motion intensity measures.

Keywords: Sequence of Earthquakes, Ground Motions, ARIMA, Ordinary Highway Bridges.

1. INTRODUCTION
Highway bridges are part of the lifeline infrastructure of urban environments and are designed with the expectation that they do not sustain significant damage and maintain their functionality even after major disasters such as major earthquakes \cite{8}. During the last three decades, however, bridges designed according to seismic design codes were observed to occasionally show poor performance during major earthquakes \cite{4}. Current methods of bridge analysis \cite{8} are based on a single ground motion excitation which neglects the effect of the pre-damaged state of the bridge during sequential earthquake events which can be due to a series
of mainshocks after mainshocks or combination of mainshocks and aftershocks, or both. Bridge structures are mainly designed to function for a life-span of 50 to 75 years [8]; in seismically active regions, bridges may experience more than one earthquake event during their lifetime. Based on these observations, not only the fragility curves need to be updated after the structure is exposed to a significantly intense earthquake, but decisions such as inspection, reevaluation, and repair need to be made to ensure and maintain the serviceability of the structure in future events.

Structures are likely to undergo multiple seismic excitations; this could be from a sequence of mainshocks, or mainshock and aftershocks. FEMA 352 [2] is among the first studies that investigated the effects of aftershocks on buildings; it focuses on the inspection of the post-earthquake damage by generating a hazard curve for damaged buildings subject to aftershocks. The main assumption in FEMA 352, however, is the independence between the mainshock shock and aftershock ground motion intensities. Yeo et al. [3] proposed a method of Aftershock Probabilistic Seismic Hazard Analysis (APSHA) in a similar manner of the traditional Probabilistic Seismic Hazard Analysis (PSHA) but taking the elapsed time between two events into consideration and assuming a non-homogeneous Poisson process. Furthermore, Amadio et al. [1] compared the effect of a single seismic event on intact single-degree-of-freedom (SDOF) system with the effect of repeated seismic events. They concluded that exposure to multiple seismic events results in a reduction in the collapse capacity as well as the capacity indicator of other damage limit states. Ruiz-Garcia et al. [4] investigated nine bridge models with 28 mainshock-aftershock excitations and concluded that the lateral drift demands are increased under scaled seismic sequences but do not significantly affect the highway bridges due to the inherent overstrength. Ghosh et al. [5] studied single column box girder bridge located near the San Andres Fault and assessed the damage accumulation under two scenarios: repeated mainshocks and mainshock-aftershock sequence. They used the Park and Ang damage index [19] which accounts for both ductility demand and dissipated hysteretic energy, as the damage measurement. They showed that for both scenarios, a significant increase in the probability of exceedance of damage index is plausible. Though there have been several studies on the effect of earthquakes sequences on structures, none of them took into account the randomness of the events along with the correlation between the consecutive events. Most importantly, in these studies, the earthquake processes are represented by distributions such as Poisson which inherently considers independence between the amplitude of the consecutive ground motions which is not necessarily true.

This research focuses on the evaluation of the performance of two highway-bridges of Southern California under a sequence of up to three seismic events considering the seismicity of the region. In this study, site-based simulated ground motions are used to conduct the Non-Linear Time-History Analysis (NLTHA) of bridges. The simulated ground motions are generated through a data-driven model ([14], [15]), denoted as the DRD model. Using Cybershake Forecast Model (UCERF2) database [17], the ground motions for a time-span of 100,000 years are generated for three sites in Southern California through the DRD model. Finite Element models of two real RC bridges located in southern California are adopted for the study. The seismicity of Southern California is modeled using time-series Auto-Regressive Integrated Moving Average (ARIMA) model. The data to model the seismicity of the region is obtained from NGAWEST2 database [12]. The model is fitted to the data of RotD50 Spectral Acceleration (Sa) at the first mode period of the bridges used in this study and observed during the past 75 years (i.e., 1945 - 2019). An ARIMA model is fitted to the data and 100 realizations are representing Sa sequences of the past 1000 years are simulated; three consecutive points of each ARIMA are randomly selected as the target Sa for the ground motions to be generated.
From the ground motions generated as per Cybershake scenarios for the 100,000 years using the DRD model, ground motions having the similar $Sa$ as determined from the simulated ARIMA process are used for the statistical analysis. A maximum of three consecutive ground motions are applied to the bridge models, and the obtained EDPs are used to deduce conclusions on the effect of the sequence of ground motions on the performance of bridge structures.

2. BRIDGE MODELS

Two California representative R.C. highway bridge structures are selected for the statistical analysis. Table 1 includes the details of the two ordinary bridges with seat-type abutments. The modeling of the bridges is conducted in OpenSees [6]. The models comprise of seat-type abutments, shear keys, column bents, elastomeric bearing pads, backfill soil, and superstructure. The concrete and steel used in modeling have a compressive strength $f_{c'} = 5.0$ ksi with a modulus of elasticity $E_s = 4030.5$ ksi and tensile strength $= 68$ ksi with a modulus of elasticity $E_s = 29000$ ksi, respectively.

The superstructure is modeled with elasticbeamcolumn element using uncracked section properties. To capture the dynamic response accurately, the mass of the superstructure is distributed throughout the length of the deck with each span’s mass being distributed in ten intervals. The bridge columns are modeled using beamwithhinges element (two gauss integration points) with fiber-discretized cross sections to model 1) confined concrete for the core, 2) unconfined concrete for the cover and 3) steel rebars. The plasticity of columns is concentrated at two plastic hinges at the opposite ends connected through a linear elastic element. The cap beam is modeled as a rigid bent using elasticbeamcolumn element with high torsional, in-plane and out-of-plane stiffnesses. The concrete and steel are modeled using concrete01 and steel02 materials of OpenSees, respectively. The base of bridge $A$ and bridge $B$ are simulated as fixed and pinned connections, respectively, with the stiffness of connections arising from piles beneath. The piles under the bridge columns are modeled using elastic springs with the horizontal stiffnesses [7].

Shear keys are designed and modeled in a brittle/isolated manner using the hysteretic spring model available in OpenSees [6]. The model is defined with a trilinear backbone curve. To determine the area of vertical reinforcement ($A_{vsk}$), the shear key is designed as per Caltrans SDC 1.7 [8]. As detailed in the experimental observations of [9], the strengths and stiffnesses of the initial, hardening and softening parts of the trilinear backbone curve are determined using the two states of isolated shear keys: 1) shear resistance at first sliding ($V_{slid}$) and 2) ultimate sliding shear resistance ($V_u$) right before the rupture of the dowel bars with an assumption of a smooth construction joint.

The model of abutment comprises of 1) abutment piles, 2) backfill soil and 3) elastomeric bearing pads. Piles of the abutments are modeled through a trilinear hysteretic spring model in OpenSees with the backbone curve defined as per [7]. The backfill soil is modeled using the hyperbolicgapmaterial material of OpenSees [6] with a Generalized Hyperbolic Force-Deformation (GHFD) backbone [10]. Hence, the active resistance of the abutment is provided by the piles while the passive action includes resistance due to the piles and backfill soil. The parameters described by [11] are used to model the elastomeric bearing pads using the steel01 material. The longitudinal behavior of the abutment is modeled using five springs in parallel connected by a rigid link while the transverse behavior is modeled using one spring on both ends of the abutment.
3. METHODOLOGY

Recurrence of earthquakes and their magnitude depends on the accumulation and release of seismic strain energy processes generated in the source. This process generally follows the energy balance principle; that is, the energy is stored and continues to accumulate in the seismogenic zone until it radiates around the fault and eventually gets dissipated. Consequently, the intensity of the sequence of earthquake events, which has different Intensity Measures (IM) such as Peak Ground Acceleration, Peak Ground Displacement, Spectral Acceleration, etc. should exhibit a certain time series pattern that includes random error terms as well as a dependency on the intensity of the previous events. The Intensity Measure (IM) used in this research is as per the current state of the art, RotD50 Spectral Acceleration at the first mode period of the bridges, which is termed as $Sa$ in this study.

To capture the sequential intensity pattern of earthquake events in Southern California, an Autoregressive Integrated Moving Average (ARIMA) time-series model is adopted. The ARIMA process can be split into three parts: 1) Autoregressive (AR) part, 2) Moving Average (MA) part, and 3) Integration (I) part. Autoregressive (AR) model of order $p$, abbreviated as AR($p$), is based on the idea that the current value of the series, $x_t$, can be explained as a function of $p$ past values $x_{t-1}, x_{t-2}, \ldots, x_{t-p}$, where $p$ determines the number of steps into the past needed to forecast the current value [13]. While, Moving Average (MA) model of order $q$, abbreviated as MA($q$), assumes the white noise components $w_t, w_{t-1}, w_{t-2}, \ldots, w_{t-p}$, for past $q$ observations can be combined linearly to form the current value of the series, $x_t$ [13]. In many situations, time series can be thought of as being composed of two components, a nonstationary trend component, and a zero-mean stationary component. Differencing such a process will lead to a stationary process. The differencing or integrating of order $d$ is represented as I($d$), where $d$ represents the number of times the series is differenced. Hence, it is understandable that the first difference ($d = 1$) is an example of a linear filter and eliminates linear trend, the second difference ($d = 2$) eliminates quadratic trend and so on. An ARIMA of orders $p, d, q$ for AR, I, and MA parts, respectively, is abbreviated as ARIMA ($p,d,q$). The process can be concisely expressed as Eq. (1), wherein $x_t$ is the current observation of the series, $w_t$ is white noise with zero mean and variance $\sigma_w^2$, and $B, \nabla, \delta, \phi(B)$ and $\theta(B)$, are the Backshift Operator, Difference Operator, Mean expression used when the mean of the stationary series is not zero,
Autoregressive Operator, and Moving Average Operator, respectively as expressed in Eq (2) to (6) [13]. In these equations $\phi_1$, $\phi_2$, \ldots, $\phi_p$, and $\theta_1$, $\theta_2$, \ldots, $\theta_q$ are constants for AR and MA parts, respectively.

$$
\phi(B)(1-B)^d x_t = \delta + \theta(B)w_t
$$

(1)

$$
B^k x_t = x_{t-k}
$$

(2)

$$
\nu^d = (1-B)^d
$$

(3)

$$
\delta = \mu(1 - \phi_1 - \ldots - \phi_p), \text{ if } E(\nu^d x_t) = \mu
$$

(4)

$$
\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p
$$

(5)

$$
\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q
$$

(6)

Using the NGAWEST2 database [12], ground motion records of past 75 years of the area are obtained for the seismic region centered at Downtown Los Angeles within a radius of 40 kilometers. The data consists of both mainshock-mainshock and mainshock-aftershock earthquake sequences, with a total number of 127 ground motions arising from 43 Earthquake events. If the ground motions are arising from the same mainshock or aftershock, and are recorded at multiple stations, only the record of the station closest to Downtown Los Angeles is used to train the ARIMA model. For each ground motion, the value of $Sa$ at bridge’s first mode period is obtained, hence a series of 127 distinct $Sa$ values is used as the representation of seismic hazard for the Southern California region and a time-series analysis is conducted on this data. Three CyberShake sites including LADT, PAS, and CCP, are selected for this study, for which the logarithmic $Sa$ series is normalized using the RotD50 Spectral Acceleration of Uniform Hazard Spectrum (UHS) of the site at bridge’s natural period; each combination of bridge and site is treated separately. The logarithmic normalized $Sa$ series is hereby denoted as $ln(Sa_{\text{norm}})$ and RotD50 Spectral Acceleration of Uniform Hazard Spectrum (UHS) of the site at bridge’s natural period is denoted as $Sa_{\text{hazard}}$. The normalization is conducted by multiplying each value of the $Sa$ series by a Normalizing Factor ($NF$) equal to the ratio of Maximum $Sa$ value of the series ($Sa_{\text{max}}$) and $Sa_{\text{hazard}}$. This is done to achieve the maximum value of the $Sa$ series equal to the $Sa_{\text{hazard}}$, while the other $Sa$ values in the series are adjusted proportionally. Various ARIMA models are fitted to the normalized $ln(Sa_{\text{norm}})$ series and evaluated by the average of the model selection parameters Akaike Information Criterion ($AIC$) and Bayesian Information Criterion ($BIC$). These measures are carefully chosen to test the goodness-of-fit as they penalize the addition of each predictor (complexity) that is included in the fitted model. Hence, unless the addition of a predictor/feature produces a better predictive equation, these measures tend to support the models of a lower order. $AIC$ [13] and $BIC$ [13] are estimators of the relative quality of statistical models for a given set of data and provide a means for model selection. Unlike other statistical measures, $AIC$ and $BIC$ are founded on information theory and models that minimize the information loss are considered to be of higher quality. Given the number of data points ($n$), the number of the estimated predictor ($k$) and the maximum value of the likelihood function ($\hat{\theta}$) for the model, the $AIC$ and $BIC$ values of a model are calculated using Eq. 7 [13] and Eq. 8 [13], respectively. Among all the candidate models, the model that minimizes the average of the $AIC$ and $BIC$ values, is selected as the true model. Based on this, ARIMA (2,1,0), i.e. Integrated AR (2), model is observed to be the true model for the normalized $ln(Sa_{\text{norm}})$ series. The model is further assessed by evaluating the residuals of the fitted model using the Autocorrelation Functions (ACF), Q-Q plot [13] and Ljung Box Test [13], results of which are given in the Figure 1. It can be observed that the ACF of the residuals lie well below the bands of white noise stating that the residuals are in fact not further modellable. High $p$-
values of the Ljung Box Test on the residuals also validate the results. Ljung Box Test conducts hypothesis testing on each value of the residual where in the Null Hypothesis states that the residual values are white noise and not further modellable. Higher \( p \)-values, as can be seen in Figure 1, intends to accept the Null Hypothesis. Lastly, the Q-Q plot also tends to follow a linear relationship thereby accepting the assumption of normality.

\[
AIC = \log \hat{\sigma}^2 + \frac{n + 2k}{n} \tag{7}
\]

\[
BIC = \log \hat{\sigma}^2 + \frac{k \log n}{n} \tag{8}
\]

Figure 1: Evaluation of residuals of ARIMA fit: a) Residuals of ARIMA (2,1,0), b) Autocorrelation function of the residuals, c) Q-Q plot of residuals, d) \( p \)-values of the Ljung-Box Test on the residuals

Using Cybershake Earthquake Rupture Forecast scenarios and the site-based DRD simulation model [14][15], around 10,000 ground motions are simulated for each of the three sites for a time-span of 100,000 years. The DRD model is a parameterized stochastic model
which generates ground motion in the two orthogonal horizontal directions. The model is developed by matching the major characteristics of recorded ground motions of California which include, near-fault effects of directivity and fling step; temporal and spectral non-stationarity; intensity, duration, and frequency content characteristics; directionality of components; and the natural variability of ground motions. The model also accounts for both pulse-like and non-pulse-like cases, especially in near-field stations. Thus, the DRD model generates an ‘observed’ set of model parameters for different earthquake source and site characteristics. The model is bifurcated into two parts by $R_{rup}$. For generating near-field ground motions using [14], the input parameters include: type of faulting ($F$) i.e. Strike-slip faults or Reverse and Oblique faults, moment magnitude ($M_w$), depth to the top of rupture plane ($z_{tor}$), closest distance between site and the fault rupture ($R_{rup}$), shear wave velocity of the top 30 m of soil at the site ($V_{s30}$), directivity parameter ($s$ or $d$) and directivity angle parameter ($\theta$ or $\phi$). And for simulating far-field ground motions using [15], the input parameters include: type of faulting ($F$) i.e. Strike-slip faults or Reverse and Oblique faults, moment magnitude ($M_w$), closest distance between site and the fault rupture ($R_{rup}$) and shear wave velocity of the top 30 m of soil at the site ($V_{s30}$). All these required input parameters along with the probabilities of ruptures are acquired from the Cybershake Earthquake Rupture Forecast scenarios for each site for 100,000 years’ time-span, thereby simulating around 10,000 ground motions for each site.

Using the fitted ARIMA (2,1,0) model, for each selected site, 100 realizations of the ARIMA processes representing 1000 years are simulated, which represent the normalized $\ln(S_{anorm})$ for 1000 years’ time-span. Then using the inverse exponential operation and de-normalizing by multiplying $1/\pi$, the simulated ARIMA series is transformed into linear $S_a$ series. This is done to account for the logarithmic nature (lognormal distribution) of the intensity measures ($S_a$) as well as to avoid generation of negative values for $S_a$ due to the random nature of the white noise in the ARIMA model. The transformed ARIMA series is hence a random simulation of the $S_a$ based seismicity of the site. The example ARIMA processes are shown in Figure 2. Then for each simulated ARIMA series, 1 data point out of the 1000 points is randomly selected along with the following next two consecutive data points. In other words, a consecutive three-point series is extracted from the simulated ARIMA series.
of $Sa$. This is assumed to represent a sequence of three ground motions with $Sa$ values same as that of the selected values of $Sa$ from the ARIMA series. Once the linear-scaled de-normalized three-points $Sa$ series is obtained, the three $Sa$ values are compared with the $Sa$ values of the ground motions simulated via DRD model for 100,000 years for the same site. Three ground motions with the identical $Sa$ values are selected and are arranged in the same order as the three-point $Sa$ series. The three consecutive ground motions are then applied to the two R.C highway bridge models in OpenSees software in the following four sequences decided based on the decision tree illustrated in Figure 3.

I. First sequence of ground motions, denoted as GM1-2-3, consists of time histories of all three selected ground motions concatenated together with an $n$ number of zeros added for the padding after each time history, where $n$ is equal to the number of time steps representing a time length equal to 10 times the period of the bridge.

II. Second sequence of ground motions, denoted as GM1-2, consists of time histories of first and second selected ground motions concatenated together with an $n$ number of zeros added for the padding after each time history as described in the previous step.

III. Third sequence of ground motions, denoted as GM3, consists of the time history of only third selected ground motion.

IV. Fourth sequence of ground motions, denoted as GM2, consists of the time history of only second selected ground motion.

The decision tree in Figure 3 is built based on the decision of repair or non-repair after experiencing a ground motion. After the occurrence of GM1, a decision is made to either repair the bridge or otherwise (i.e., not repair). This leads to the application of GM2 in two scenarios: GM1-2 and GM2. Similarly, after the application of ground motions 1 and 2 in sequence, it is decided whether to repair the bridge or otherwise. This leads to sequence GM1-2-3, when not repaired, and GM3, when the bridge is repaired after GM2. Hence a total of four ground motion sequences are considered in this study: GM1-2-3, GM1-2, GM3, and GM2. The ground motion sequence is limited to three number of shocks as it is assumed that the bridge will be retrofitted after undergoing three consecutive ground motions.

---

The Non-Linear Time History Analysis (NLTHA) of the two bridges is conducted by rotating the two orthogonal components of the selected ground motions through 180 degrees in an interval of 9 degrees. The median of maximum response in each direction ($Rot_{50}CDR$) is obtained and used as EDP. In this study the EDP of Column Drift Ratio (CDR) is termed in the form of $Rot_{pp}EDP$, where $Rot$ indicates the rotation of ground motion components, $pp$ indicates the percentile value used for the measure (e.g. “00”, “50” and “100” correspond to minimum, median and maximum values, respectively; the median value will be the commonly used measure in this study), and $EDP$ indicates that the measure is an EDP (i.e., Column Drift Ratio...
The EDP used in this study is the median bridge column drift ratio, hence denoted as Rot50CDR. In each of the four sequences, comparison of EDP is made in the form of the ratio of EDP obtained after damage due to initial ground motions of the sequence (termed as Rot50CDRiD) and EDP obtained after no-damage (i.e. repaired) bridge (termed as Rot50CDRiND), where i represents the last ground motion applied to the bridge. This means, for example, Rot50CDR3D represents the Rot50CDR obtained due to the third ground motion after the first two ground motions are applied in sequence (i.e., GM1-2-3). This is comparable to Rot50CDR3ND which represents the Rot50CDR in the no-damage state which is obtained due to applying only the third ground motion (GM3) portraying that the bridge was repaired after the second ground motion. Similarly, Rot50CDR2D represents the Rot50CDR obtained due to the second ground motion applied in a sequence to the first ground motion (i.e., GM1-2). This is comparable to Rot50CDR2ND which represents the Rot50CDR obtained only due to the second ground motion (GM2). This is illustrated in Figure 4 where column displacement response (u) in two orthogonal directions (x and y) is combined to determine the EDP i.e. Column Drift Ratio (CDR) and compared due to two ground motion sequences GM1-2-3 (Sequence I), representing damaged state, and GM3 (Sequence III), representing the not-damaged state. It should be noted that this illustration is based on one ground motion intercept angle. Hence the term Rotpp is not used. The comparison of the EDPs from the four ground motion sequences is done to highlight the effect of repairing the bridge structure after a shock. (i.e., reduction of EDP after the repair). This whole process is repeated for the 100 simulated ARIMA processes for all the three sites.

![Figure 4- Illustration for determining EDPs for ground motion sequences](image-url)

Finally, the data obtained from the application of the four ground motion sequences for the 100 simulated ARIMA processes for the three sites are clubbed into two groups. The first group consists of the ratio of the damaged EDP with not-damaged EDP and the IMs for the sequences
I and III, while the second group consists of the ratio of the damaged EDP with non-damaged EDP and the IMs for the sequences II and IV. IM used in this study is the Cumulative sum of SRSS of the Arias Intensities, termed as $I_0$, of the two orthogonal components of ground motion, where $SRSS$ of Arias Intensity is the integral $\int_0^{t_{\text{max}}} \sqrt{a_1^2(t) + a_2^2(t)} dt$, $a_i(t)$ represents the ground acceleration at time $t$ in $i^{th}$ orthogonal direction, and $t_{\text{max}}$ represents the length of the accelerogram [18]. Since, the Arias Intensity represents the energy of the ground motion, for the sequences of more than one ground motion, it can be said that the total energy is the summation of the individual Arias Intensities of the ground motions. Hence, group 1 contains the data for the ratio of $\text{Rot50CDR}_{3D}/\text{Rot50CDR}_{3ND}$ and summation of $I_{01} + I_{02}$, and group 2 contains the data for the ratio of $\text{Rot50CDR}_{2D}/\text{Rot50CDR}_{2ND}$ and $I_{01}$.

Due to the random nature of selecting the ground motions, some of the ground motions are selected from a very low IM zone of the simulated ARIMA processes which consequently result in an insignificant RotD50CDR. Hence, to include only the significant RotD50CDR in the statistical study, the data corresponding to a RotD50CDR of greater than 0.1% in at least one of the four sequences is considered. A linear regression model is used to fit the EDP ratio vs. the IM plot, which can be used to predict the RotD50CDR ratio (hence the increase in the EDP) due to an unseen event, given the Arias Intensities (IM) of the previous one or two ground. The results of the regression functions are provided in terms of both mean value point estimate and the probability of exceeding a certain level of RotD50CDR ratio condition on the Arias Intensity of the causative ground motion sequence.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Bridge A</th>
<th>Bridge B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>0.81</td>
<td>0.616</td>
</tr>
<tr>
<td>$b_1$</td>
<td>3.18</td>
<td>26.15</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.28</td>
<td>0.48</td>
</tr>
<tr>
<td>$b_3$</td>
<td>8.27</td>
<td>32.04</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0.11</td>
<td>0.23</td>
</tr>
<tr>
<td>$b_5$</td>
<td>0.99</td>
<td>9.5</td>
</tr>
<tr>
<td>$b_6$</td>
<td>0.016</td>
<td>0.015</td>
</tr>
<tr>
<td>$b_7$</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>$b_8$</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 2–Coefficients statistics of the regression equations

4. RESULTS AND DISCUSSIONS

Figure 6 and Figure 7 demonstrate the results of Bridge A, where the predicted variable ratio of $\text{Rot50CDR}_{3D}/\text{Rot50CDR}_{3ND}$ is plotted versus the predictor, summation of $I_{01} + I_{02}$ and $\text{Rot50CDR}_{2D}/\text{Rot50CDR}_{2ND}$ versus $I_{01}$, respectively. Similarly, Figure 8 and Figure 9 show the results of Bridge B, where the predicted variable ratio of $\text{Rot50CDR}_{3D}/\text{Rot50CDR}_{3ND}$ is plotted against the predictor summation of $I_{01} + I_{02}$ and $\text{Rot50CDR}_{2D}/\text{Rot50CDR}_{2ND}$ versus $I_{01}$, respectively. To illustrate the effect of the numerator EDP (Rot50CDR for the damaged bridge) and the denominator EDP (Rot50CDR for the repaired bridge), the marker size and the color of the circles are tuned based on the magnitudes of the corresponding data; the marker size is based on the numerator EDP value (Rot50CDR for the damaged bridge) where larger size denotes larger EDP and marker color shade portrays denominator EDP (Rot50CDR for the repaired bridge), where darker shade represents larger EDP. Figure 5 shows the indices of the marker size and the color shade corresponding to the value of numerator EDP (Rot50CDR for the damaged bridge) and the denominator EDP (Rot50CDR for the repaired bridge). Interpretation of the figures requires investigation in both predicted variable vs. predictor and the numerator EDP vs. denominator EDP. In general, it can be noticed that as the IM of the observed ground
motions \((I_{01} + I_{02} \text{ in Figure 6 and 7 or } I_{01} \text{ in Figure 7 and 8})\) increase, the ratio of \(\text{Rot50CDR}\) increases with the marker size increasing which denotes an increase in the numerator \((\text{Rot50CDR} \text{ for the damaged bridge})\). This goes well with the intuition that as the intensity of the previously observed ground motions increase, it causes a high amplification of the EDP due to the next ground motion as compared to the EDP of a repaired bridge caused by the next ground motion. However, this is true until the intensities of the observed ground motions are well higher than the unobserved ground motion (i.e., the next ground motion). For example, the largest and darkest dot in Figure 6 indicates that both \(\text{Rot50CDR}_{3D}\) and \(\text{Rot50CDR}_{3ND}\) are large. Therefore the increase in EDP due to ground motion 3 in a damaged state as compared to a repaired state of the bridge, is small. This is a case where all three ground motions possess high values of IM, and it shows that a high \(I_{01} + I_{02}\) does not necessarily lead to considerable amount of increase in EDP since the third (last) ground motion dominates the overall response of the bridge in the sequence. An opposite case can be analyzed via the two large size gray dots in Figure 6 and 7, in the case of Bridge A, which is indicative of large \(\text{Rot50CDR}_{3D}\) but a medium or small \(\text{Rot50CDR}_{3ND}\). In this case, the first two or one ground motions of the sequence have caused a significant impact on the bridge leading to a damaged state which furtherleads even a huge amplification (around ten times) of EDP due to even a smaller last ground motion. Similar results can be seen in Figures 8 and 9 for Bridge B. Since Bridge B has a two-column bent, it demonstrates is a much stiffer behavior compared to Bridge A, hence, the EDPs and the ratios are not as significant as Bridge A. The large gray markers in Figure 7 and 9 can be investigated in the same manner that an already damaged bridges will suffer an amplification of 10 in the EDP due to the second ground motion. The large and dark markers with a small value of \(I_{01}\) in Figure 7 and 9, are representative of another mechanism where the first ground motion is benign, and the second one governs the GM1-2 sequence. Hence, a small value of the predictor \(I_{01}\) results in a small EDP ratio, implying that the bridge remains intact after the end of the first ground motion and bridge response due to ground motion two is similar in the damaged and not-damaged states.

Statistical analysis is conducted by regressing the ratio of \(\text{RotD50CDR}\) of the damaged state and not-damaged state of the two groups \((\text{Rot50CDR}_{3D}/\text{Rot50CDR}_{3ND} \text{ vs. summation of } I_{01} + I_{02}, \text{ and Rot50CDR}_{2D}/\text{Rot50CDR}_{2ND} \text{ vs. } I_{01})\) on the cumulative Arias Intensity \((I_o)\) of the previous ground motions of the sequence. This aims to predict the increase in the EDP due to the next ground motions if the bridge is not repaired (i.e. \(\text{Rot50CDR}_{3D}\) and \(\text{Rot50CDR}_{2D}\)) as compared to if the bridge had been repaired (i.e., \(\text{Rot50CDR}_{3ND}\) and \(\text{Rot50CDR}_{2ND}\)), as a function of the IMs of the previous ground motions that the bridge has undergone (i.e., \(I_{01} + I_{02}\) and \(I_{01}\)). The regression of the two groups leads to an expression of the ratio of \(\text{Rot50CDR}_{3D}/\text{Rot50CDR}_{3ND}\) as a function of \(I_{01} + I_{02}\) and the ratio of \(\text{Rot50CDR}_{2D}/\text{Rot50CDR}_{2ND}\) expressed as a function of \(I_{01}\). As the ratio of \(\text{RotD50CDR}\) between the damaged and not-damaged cases is expected to be greater or equal to 1.0, the regression is conducted with the logarithm of the ratio of \(\text{RotD50CDR}\) as the predicted variable. Hence, the linear regression is adopted in this study by the traditional approach in the form of \(\text{EDP} = a \ IM^b\) [16]. The form of the regression equations is given in Eq (9) and Eq (10) along with the regression coefficients listed in Table 2. A monotonically increasing trend is captured by the regression model, and the corresponding 95% confidence interval is provided in Figure 2 and Figure 3. Also, the statistical significance of the model parameters is checked through Hypothesis testing and based on the Null Hypothesis that the parameters are not significant, \(p\)-values less than 0.05 suggest that the parameters of the regression model are significant. It is clear from the regression equation that when the previously observed ground motions have large \(I_0\), the \(\text{Rot50CDR}\) of the unrepaired bridge (i.e. \(\text{Rot50CDR}_{3D}\) and \(\text{Rot50CDR}_{2D}\)) will be
significantly higher than the Rot50CDR of the repaired bridge (i.e. Rot50CDR\text{3ND} and Rot50CDR\text{2ND}), even though the future unobserved ground motion possesses low \text{I}_0. However, there can be certain exceptions (e.g. the large darkest dot in Figure 2) to the increasing trend such as when the previous (observed) ground motions in the sequence are intense and possess high \text{I}_0, and they are followed by the last (unobserved) ground motion which is even more intense than the previous and control the overall response. This makes the ratio to fall on the right of the x-axis however, with the Rot50CDR ratio close to unity.

\[
\ln \left( \frac{\text{Rot50CDR}_{3\text{ND}}}{\text{Rot50CDR}_{2\text{ND}}} \right) = b_0 + b_1(\text{I}_{01} + \text{I}_{02}) \quad (9)
\]

\[
\ln \left( \frac{\text{Rot50CDR}_{2\text{ND}}}{\text{Rot50CDR}_{2\text{ND}}} \right) = b_0 + b_1(\text{I}_{01}) \quad (10)
\]

Figures 6 to 9, along with the regression equations, can be directly used by engineers in Southern California region to predict the amplification in the Rot50CDR that can occur if they decide to repair the bridge based on the IMs of previous seen one or two ground motions. This can be done by collecting the information about the last two (or one) ground motions observed under a given bridge. Summing the Arias Intensities (\text{I}_0) of the previously observed ground motions, the user can enter through the x-axis of Figure 6 (or 7) or Figure 8 (or 9) and obtain the ratio of EDP of the damaged state vs. not-damaged state, and if the amplification is above a certain threshold, consider fixing the bridge. This can also be simply done through the Eq 9 (or 10) using the suggested parameters for single-column and double-column two spanned bridges. A stepped example for the usage is given below:

Step 1: Consider a bridge located at Downtown Los Angeles Area. Collect the information of the last two (or one) ground motions observed by the bridge and compute the cumulative IM value of the observed ground motions, i.e., the sum of Arias Intensities of first and second ground motion (\text{I}_{01} + \text{I}_{02}) or Arias Intensity of first ground motion (\text{I}_{01}).

Step 2: Given the cumulative IM of the previously occurred ground motions and via the use of regression models, estimate the mean EDP ratio and the corresponding 95% confidence interval. This will give an estimate of how much amplification can the engineer expect to see in the bridge response if the bridge is decided not to be fixed.

Step 3: Engineer needs to decide if the repair of the current bridge will be economical through the loss estimation.

Figure 5- Indices of the marker size and the color shade corresponding to the values of Damaged Rot50CDR and Not-Damaged Rot50CDR
Figure 6- Amplification of Rot50CDR for damaged and not-damaged states of Bridge A for ground motion sequences I and III (GM1-2-3)

Figure 7- Amplification of Rot50CDR for damaged and not-damaged states of Bridge A for ground motion sequences II and IV (GM1-2)

Figure 8- Amplification of Rot50CDR for damaged and not-damaged states of Bridge B for ground motion sequences I and III (GM1-2-3)
Figure 9- Amplification of $Rot_{50CDR}$ for damaged and not-damaged states of Bridge B for ground motion sequences II and IV (GM1-2)

5. CONCLUSIONS

Bridge structures in California are designed and analyzed according to the Seismic Design Criteria [8]. The code specifications are based on a single event of ground motion under which the bridge structure is analyzed assuming not-damaged conditions. However, in practice, bridges undergo few substantial ground motions during their life span due to which the impact of otherwise benign ground motion can have a significant impact on their performance. In this study, two bridge structures are analyzed under a sequence of three ground motions which are randomly selected from 100 realizations of an ARIMA model fitted to the recorded data which represents the seismicity of Southern California. Four sequences are used to apply the selected ground motions. This is done based on a decision tree of repairing or not repairing the bridge after undergoing a ground motion. Through the selected sequences, the EDP of a damaged bridge is compared to that of a repaired bridge due to same ground motion. Using further statistical analysis, regression equations are proposed that can help engineers to predict the amplification of the EDP of bridges in the future event given the IMs of the previously observed ground motions if they decide not to repair the bridge. It is assumed in this study that a bridge’s lifespan is equal to the time that it undergoes at most three ground motions without repair.

Based on the results of this research, bridge structures are expected to experience a larger EDP value ($Rot_{50CDR}$) after undergoing consecutive ground motions compared to a repaired one which undergoes only one ground motion. A regression model is suggested to predict the EDP ratio of the unrepaired to the repaired bridge according to the IM (Arias Intensity) of the last two (or one) observed ground motions by the bridge. Conditioned that the observed ground motions were of high intensity, the bridge structure would experience large amplification (i.e., 10 or greater) of EDP after the following (i.e., unknown) ground motion if they are not repaired compared to the repaired case. The amplification of EDP in a previously damaged bridge is approximately equal to five in case the previously observed ground motions possess a moderate cumulative arias intensity. This ratio can be as high as 10 if the confidence interval of the prediction is considered. It should be noted that the models suggested in this study make predictions for the amplification ratio of the EDPs due to damaged state. Hence this needs to be used along with the updated techniques of predicting future IMs based on previously observed IMs of ground motions i.e. dependent IM hazard.
REFERENCES


STATE-DEPENDENT VULNERABILITY OF CASE-STUDY REINFORCED CONCRETE FRAMES

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Abstract

This study investigates the effect of mainshock-aftershock sequences on numerical fragility and vulnerability relationships of European reinforced concrete (RC) moment-resisting frames (MRFs). A four-story, four-bay nonductile RC MRF is selected for illustrative purposes. This index building is representative of a typical vulnerability class in the Mediterranean region. The influence of the masonry infills on seismic performance is also investigated. An advanced numerical nonlinear model is developed for the case-study frame and then assessed through nonlinear dynamic analysis using both real and artificial mainshock-aftershock sequences, via a ‘sequential cloud’ approach. The obtained seismic demand estimates allow to generate fragility functions for the undamaged frame when subjected to mainshocks only. Moreover, state-dependent fragility functions are derived for the mainshock-damaged frame when subsequently subjected to aftershocks. Damage-to-loss models, specifically calibrated on Italian post-earthquake data, are used to derive vulnerability functions for this case-study structure. Preliminary results from the study show that the frame experiences severe damages states and high losses for a range of ground-motion shaking intensities, with a clear damage increase due to aftershocks. An attempt to generate vector-valued mainshock-aftershock vulnerability relationships is finally presented. The proposed vulnerability surfaces can be more easily implemented into a time-dependent risk assessment framework.

Keywords: Mainshock-aftershocks sequences; Seismic performance; Fragility curves; Vulnerability curves; Reinforced concrete frames; Masonry infills.
1 INTRODUCTION

Earthquakes typically occur in sequences. Most often, each sequence is dominated by a seismic event with a larger magnitude than all others in the sequence. Such major seismic events constitute mainshocks, generally followed by several aftershocks clustered in both space and time. Earthquake sequences can cause huge losses due to repair costs, business interruption and casualties, especially if affected structures are left unrepaired after experiencing an initial damage due to the mainshock.

Recent events worldwide have demonstrated the extent to which the effect of such sequential earthquake-induced ground shaking can be devastating. On September 2010, the city of Christchurch, in New Zealand, was hit by a mainshock with a moment magnitude ($M_w$) of 7.1 which was followed, on February 2011, by a $M_w$ 6.3 aftershock. The seismic sequence resulted in approximately 185 casualties and $15 billion financial losses [1]. After the September mainshock, 90% of the reinforced concrete (RC) frames were classified as safe to re-occupy with none or minor damage in the central business district in Christchurch [2]. Following the aftershock of February 2011, only 53% were classified as safe to re-occupy, whilst the portion of unsafe buildings raised significantly to 18% and approximately 29% were either demolished or had limited accessibility [2].

A similar situation was observed during the recent 2016-17 Central Italy earthquake sequence. This sequence consisted of several moderate-to-high magnitude earthquakes, each centered in a different location and with its own sequences of aftershocks spanning several months (e.g., [3]). The first event of August 2016 with a $M_w$ of 6.1 was followed by two seismic events in October 2016 with $M_w$ of 5.9 and 6.5 respectively. The sequence hit a large sector of the Central Apennines of Italy, particularly affecting the Marche and Umbria regions, and significantly damaging several towns. Amongst others, the towns of Amatrice, Norcia and Accumoli were heavily damaged (e.g., [4]).

Many past studies have investigated the effect of the earthquake-induced damage accumulation on several types of structural systems. Generally, damage accumulation can be related to ground-motion duration (e.g., [5, 6] among others) or to the effect of seismic sequences. The seismic performance and vulnerability of structures under seismic sequences has been largely studied in the last decade (e.g., [7, 8, 9, 10]); a detailed review of these past studies is outside the scope of this short paper. However, some general findings can be highlighted. For instance, fairly limited research is available on the effects of aftershocks on advanced computational models, particularly in Europe, using nonlinear time history analyses (NLTHAs). Moreover, most of past studies focused mainly on collapse assessment and/or developing risk assessment frameworks. Also, those past studies mainly used artificial mainshock-aftershock (MS-AS) sequences rather than real (i.e., as recorded) sequences, with limited attention to the development of vulnerability (i.e., loss vs intensity) relationship for mainshock-damaged buildings.

The present study aims to develop fragility curves accounting for both the undamaged and mainshock-damaged states with an attempt of evaluating the effects of real as-recorded MS-AS sequences on the performance and fragility of the case-study structure. In addition, this study considers generating vulnerability relationships considering undamaged and mainshock-damaged states. Such vulnerability relationships express the likelihood that assets at risk will sustain varying degrees of loss (e.g., in terms of direct damage) over a range of aftershock intensity measures (IMs).

The paper is organized as follows. Section 2 discusses the geometric and material properties of the case-study structure; the non-linear modelling strategies used to capture different behavioral patterns of the structure; the definition of damage states; and the selection of ground-motion sequences. Section 3 deals with the seismic performance assessment and derivation of
fragility curves for the undamaged frames (both in the bare and infilled configurations, under the effects of mainshocks only), followed by a detailed assessment of the infilled frame under the effect of the entire MS-AS sequences, in order to derive conditional fragility relationships taking into account the initial mainshock-induced damage. Vulnerability relationships for the infilled frame are finally derived with a first attempt to also generate vector-valued mainshock-aftershock vulnerability relationships for this case-study structure. Section 4 summarizes the main findings from this study.

2 METHODOLOGY

2.1 Case-study structures

A generic four-story, four-by-four bays RC moment-resisting frame located in Torre del Greco (Naples, Italy) is considered as a case-study index building in this paper. The total height is 13.5 m with a first story of 4.5 m, upper stories of 3 m and a bay-width of 4.5 m in both directions [11]. The frame is designed for gravity loads only and do not conform to modern seismic code requirements. Figure 1 provides the layout and cross-sectional dimensions of the bare frame. Typical average values for the compressive strength of the concrete ($f_{cm}$) and yielding strength of the reinforcements ($f_{ym}$) are assumed; these are equal to 19 MPa and 360 MPa respectively. Infill walls typically used in Southern Italy in the 1970s (e.g., [12]) are considered with the following mechanical properties: compressive strength $\sigma_{m0} = 2.5$ MPa; shear strength $\tau_{m0} = 0.23$ MPa; Young modulus $E_m = 1495$ MPa and sliding resistance $\tau_0 = 0.28$ MPa.

![Figure 1: Layouts of the case-study bare frame (cross-sectional dimensions in cm).](image)

2.2 Non-linear modelling

The seismic response of the case-study structure is simulated by 2D numerical models in OpenSees [4]. The nonlinear behavior of the structural components is modelled through a lumped plasticity approach for both beams and columns (e.g., [13]). Zero-length rotational springs are assigned at the elements ends and the Ibarra-Medina-Krawinkler model [14], as implemented in OpenSees, is used to describe their moment-rotation nonlinear hysteretic behavior. Model parameters are defined according to [15]. Moreover, in order to capture the possible shear failure, additional shear springs are added in series to the flexural springs. The Setzler and Sezen model [16] is used to define their force-deformation relationship while the initial shear stiffness and the onset peak shear strength are determined accounting for the effects of diagonal cracks as recommended by [17]. Beam-column joints are modelled as rigid while the gravity load and the masses are respectively uniformly distributed on the beams and concentrated at the nodes.
Masonry infills have been modelled by idealized equivalent struts. The pinching factors for reloading strain and reloading stress and the degradation power of unloading stiffness have been defined according to [18]. The tensile response of the strut was assumed as 5% of the compressive response as the struts are meant to act in compression only. The force-deformation relationship developed and calibrated by [19] is used to simulate the effect of infills on RC frames. The infills-frame interaction can increase the shear demand on columns leading to brittle shear failure and hence, the equivalent compression struts are modelled by a dual compression struts as suggested by [20]. One strut is diagonal and connects the nodes at the beam-column intersections while the other is an off-diagonal strut connected to the shear springs of the columns. This allows to capture the increase in the columns shear demand due to interaction between the infill walls and RC frame. According to [20], 75% of the total strut strength and stiffness is assigned to the diagonal strut, whilst 25% is assigned to the off-diagonal one.

2.3 Damage State definition and thresholds mapping by pushover analysis.

Damage States (DSs) describe the damage conditions of a structure under the seismic input. Conventionally, the onset of different DSs can be identified by using thresholds of specific measurable global and/or local Engineering Demand Parameters (EDPs), such as maximum interstory drift ratio (MIDR), maximum top story drift, chord rotation, strength of cross-sections, material strains and others (e.g., [21]).

In this study, MIDR is selected as EDP to represent the global structural performance; MIDR has shown good correlation to both structural and non-structural damage. MIDR thresholds for three different DSs are calibrated based on pushover analyses according to multiple measurable criteria as summarized in Table 1. The parameters $\theta_y$ and $\theta_u$ of Table 1 denote respectively the yield and ultimate chord rotations and are defined according to [22] and to the Eurocode 8 Part 3 (EC8-3) [23].

The MIDR thresholds mapping for the DSs is based on [24] and on the EC8-3. The Moderate Damage State (DS1) is characterized by moderate structural and non-structural damages with no significant yielding and members preserve their stiffness and strength. The building in this damage state is occupiable but minor repairs may be required. The Extensive Damage State (DS2) incorporates severe damages in both structural and non-structural components. Buildings retain some residual strength and stiffness to remain stable, require major repairs, which might not be feasible in many cases. The Near-Collapse Damage State (DS3) represents a full exploitation of a building strength and ductility, very low residual strength and stiffness remain after the earthquake and the building is about to collapse. In addition, the Slight Damage State (DS0) is introduced to account for the non-structural damage, which initiate in infill walls due to lateral deformation. This damage state is achieved when the majority of infill walls reach the displacement of first crack.

<table>
<thead>
<tr>
<th>Damage Level</th>
<th>Moderate DS1</th>
<th>Extensive DS2</th>
<th>Near-Collapse DS3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section Level</td>
<td>Reaching yield bending strength in a column</td>
<td>Max. bending strength of a column is reached</td>
<td>Reaching shear failure in any element</td>
</tr>
<tr>
<td>Component Level</td>
<td>Reaching the value of $\theta_y$ in a supporting column</td>
<td>Reaching 75% of the $\theta_u$ in any component</td>
<td>Reaching the $\theta_u$ in any component</td>
</tr>
<tr>
<td>Global Level</td>
<td>Reaching global yield</td>
<td>Reaching the maximum strength</td>
<td>About 20% drop in maximum strength</td>
</tr>
</tbody>
</table>

Table 1. DSs thresholds mapping.
Structural models are developed for both bare and infilled frames and are characterized respectively by fundamental structural periods of 0.759 and 0.329 sec. Pushover analysis is performed to derive DS-thresholds in terms of MIDR based on the damage criteria in Table 1. The pushover incremental load patterns are defined according to the first mode as indicated in the Eurocode 8 (EC8) [25]. Figure 2 reports the results of the pushover analyses by showing the base shear coefficient (i.e., ratio of the total base shear to the weight of the structure) versus the roof drift ratio (i.e., roof displacement normalized by the total building height).

The pushover analysis shows that the story drifts are concentrated in the ground and first stories. The presence of infills worsens the situation and leads to the concentration of deformations in the ground story only. The infills provide a significant contribution in terms of strength and stiffness as reported by Figure 2; however, a significant drop of resistance can be observed due to the concentration of deformation in the ground story. The MIDR thresholds for the DSs are reported in Figure 2 and summarized in Table 2.

![Figure 2. Pushover curves and damage state thresholds for the bare and infilled frames.](image)

<table>
<thead>
<tr>
<th>Maximum interstory drift (MIDR) thresholds</th>
<th>DS0</th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS0*</td>
<td>0.16%</td>
<td>0.38%</td>
<td>1.61%</td>
<td>2.70%</td>
</tr>
</tbody>
</table>

Table 2. MIDR thresholds for the considered DSs.

### 2.4 Ground-motion sequence selection

As discussed earlier, most of the research studies to date have investigated the effects of aftershocks employing artificial MS-AS sequences and using mainshock records within back-to-back analyses (e.g., [1]). This procedure assumes that features of aftershocks such as duration, spectral shape and frequency content are similar to the one of the mainshocks, which might yield biased results due to the effects of these features on the seismic performance.

This paper attempts to tackle this issue by considering real (i.e., as-recorded) unscaled MS-AS sequences selected from two databases. The first one is the 2012 **KKiSK** **Ground-Motion Database** [26] and is based on the ground motions recorded by the national networks of strong ground-motions **K-NET/KiK-net**\(^1\). The other source of MS-AS sequences is the **NGA-WEST2 Ground Motion Database**\(^2\) developed by the Pacific Earthquake Engineering Research Center.

\(^1\) [http://www.kyoshin.bosai.go.jp/](http://www.kyoshin.bosai.go.jp/)
\(^2\) [https://ngawest2.berkeley.edu/](https://ngawest2.berkeley.edu/)
The total number of MS-AS sequences from the two databases is of 703; they have been identified using the criteria discussed in [26]. In the present study, each sequence consists of a mainshock followed by a single aftershock (i.e., the one with the largest magnitude) and the number of sequences was downsized in order to minimize the computational effort required for the analyses. Sequences considered not strong enough to cause structural damages by mainshocks or aftershocks have been disregarded based on the following criteria:

- Only the crustal earthquakes are considered from the 2012 KKiKSK database in order to match the seismogenic nature of the location under consideration (i.e., Italy).
- Sequences with mainshock spectral acceleration at the fundamental period, $S_a(T_1)_{MS}$, less than 10% of the value from the elastic spectrum of EC8 are disregarded as shown in Eq. (1).
  \[ S_a(T_1)_{MS} \geq 0.10 S_a(T_1)_{EC8} \]  
- All MS-AS sequences having $S_a(T_1)_{MS}$ greater than 10 times $S_a(T_1)_{EC8}$ are disregarded as shown in Eq. (2).
  \[ S_a(T_1)_{MS} \leq 10 S_a(T_1)_{EC8} \]  
- Aftershocks that are not strong enough might not result in damage increase in a mainshock-damaged structure, meaning that the structure will likely remain in the same initial damage state caused by a mainshock. Accordingly, all MS-AS sequences with, $S_a(T_1)_{AS}$ less than 50% of $S_a(T_1)_{MS}$ are disregarded.
  \[ \frac{S_a(T_1)_{AS}}{S_a(T_1)_{MS}} \geq 50\% \]  

According to the described criteria to the 2012 KKiKSK and NGA-WEST2 databases reduced the number of MS-AS sequences from 703 to: 255 for the bare frame and 358 for the infilled frame (due to their different fundamental periods). Figure 3 reports the scatter of the $S_a(T_1)_{MS}$ versus $S_a(T_1)_{AS}$ values for the selected sequences.

![Figure 3](image-url)

Figure 3. Scatter of $S_a(T_1)_{MS}$ and $S_a(T_1)_{AS}$ for the (a) bare frame and (b) infilled frame.

### 2.5 Cloud analysis and fragility curves derivation.

NLTHAs are performed on the case-study structures by using the set of unscaled ground-motion records discussed above. This produces a cloud of points in terms of ground motion IM and the corresponding EDP values (i.e., a cloud of IM vs EDP pairs) [27]. Probabilistic Seismic Demand Model (PSDM) are successively obtained by fitting the results of the Cloud analysis...
with a power law model \(i.e.,\) a linear regression in the bilogarithmic space allowing the derivation of fragility curves by a closed form solution \(e.g., [21]\). Fragility curves represent the conditional probability of exceeding a specific damage state given the level of IM \([27]\). As previously discussed, MIDR is assumed as EDP to represent the structural response while the 5%-damped (pseudo-)spectral acceleration at the fundamental structural period, \(S_a(T_1)\), is the assumed IM.

2.6 Vulnerability functions.

Vulnerability functions are widely used in seismic risk assessment of building portfolios. They can be effectively used to compute seismic losses (economic losses, downtime and casualties) for various earthquake scenarios and ground-motion IM values \([28]\). The vulnerability functions developed in this study address only direct economic losses; in fact, they are expressed in terms of loss ratio (LR), \(i.e.,\) the ratio of cost of repair to cost of replacement for a give asset, conditional on the level of ground-shaking intensity. Such functions can be derived by combining fragility curves, which provide the probability of attaining specific DSs given the IM, with consequence functions (or damage-to-loss ratios, DF) using the total probability theorem \([28]\), as reported in Eq. (4).

\[
LR(IM) = \sum_{i=1}^{9} DF_i P(DS_i | IM) \tag{4}
\]

\(DF_i\) is the damage-to-loss ratio for a given damage level \(ds_i\) \(e.g., [29]\). These ratios are region and building-type-specific and must be carefully selected \(e.g., [30]\).

The model proposed by \([31]\) is used in this paper in order to develop vulnerability functions. This model provides the damage-to-loss ratios for Italy and for different DSs based on the MSK-76 intensity scale \(e.g., [32]\). Table 3 shows the damage-to-loss ratios estimated by the author for six different DSs.

<table>
<thead>
<tr>
<th>Damage State</th>
<th>DS0</th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
<th>DS4</th>
<th>DS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage-to-loss ratio</td>
<td>0</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.75</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 3. Damage-to-loss ratios for five DSs \([31]\).

3 PERFORMANCE-BASED ASSESSMENT RESULTS

3.1 Performance assessment of undamaged structures

The case-study building is first subjected to the selected mainshocks to assess its performance considering the undamaged state. Figure 4 shows the results of NLTHA \(i.e., S_a(T_1)_{MS} \) vs. MIDRs and the PSDMs for the bare and the infilled case-study frames. It can be observed that both structures remain undamaged in several analysis cases, which constitute 46% of the cases for the bare frame and 30% for the infilled frame \(i.e.,\) the ND-cases in Figure 4). Similarly, in several instances, the structures experience either DS1 or DS2; specifically, 52% for the bare frame and 30% for the infilled. The near-collapse damage state \(i.e.,\) DS3 is also attained in 2% of the analysis cases for the bare frame and 3% for the infilled frame. The positive contribution of infills can be observed as the same levels of damage are observed for higher values of IM compared to the bare frame.
Figure 4. Undamaged state cloud analysis and Probabilistic Seismic Demand Models (i.e., $S_a(T_1)^{MS}$ vs. MIDR) for (a) bare and (b) infilled frames.

Fragility curves are successively derived and are reported in Figure 5. The comparison shows that the infills are damaged (i.e., DS0) for relatively low values of IM; on the other side, the infills contribute to the overall resistance of the frame and, the median IM values for DS1, DS2 and DS3 are significantly higher with respect to the ones of the bare frame. The median of IM values ($\theta_{IM}$) and the standard deviation ($\sigma_{IM}$) for each DS are also given in Table 4.

Table 4. Undamaged state fragility functions. Median and standard deviation values.

### Frame type

<table>
<thead>
<tr>
<th>Damage State</th>
<th>Bare frame $\theta_{IM}$ [g]</th>
<th>Bare frame $\sigma_{IM}$ [g]</th>
<th>Infilled frame $\theta_{IM}$ [g]</th>
<th>Infilled frame $\sigma_{IM}$ [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS0</td>
<td>-</td>
<td>-</td>
<td>0.136</td>
<td>0.396</td>
</tr>
<tr>
<td>DS1</td>
<td>0.136</td>
<td>-</td>
<td>0.136</td>
<td>0.396</td>
</tr>
<tr>
<td>DS2</td>
<td>0.698</td>
<td>0.3064</td>
<td>1.392</td>
<td>0.3906</td>
</tr>
<tr>
<td>DS3</td>
<td>1.254</td>
<td>2.184</td>
<td>1.254</td>
<td>2.184</td>
</tr>
</tbody>
</table>

3.2 Performance assessment of mainshock-damaged structures

For the sake of brevity, the following part of the study is limited to the assessment of the MS-AS performance of the infilled frame only. This model is more representative of actual building structures and allows a more realistic evaluation of their vulnerability. To simulate the
seismic performance of the mainshock-damaged structure during the aftershocks, the infilled frame is subjected to the entire MS-AS sequences. It should be noted that 40 seconds of free vibration are added to all MS-AS sequence after the mainshocks to allow the structure to reach the rest conditions prior to subjecting it to the aftershocks. Figure 6(a) shows the MIDR values attained due to the mainshocks versus the MIDR values obtained during the aftershocks for all analysis cases. The points located above the dashed diagonal line represent the cases in which the MIDR values due to the aftershock are higher than the corresponding values obtained due to the mainshocks only. Figure 6(a) shows that in several cases the frame experienced higher damage states as consequence of the MS-AS sequence with respect to the mainshock only. For instance, several observations in which the frame remains undamaged after the mainshocks, showed that it attains DS0 after the aftershocks. Similar situation can be observed with the frame passing from DS0 to DS1. For a few cases Figure 6(a) shows an increase in the damage state from DS1 to DS2, with only one observation for which the frame reached the collapse (DS3) after being initially in the DS2 range. However, the number of damage state increase observations is not statistically significant to derive fragility curves.

In order to increase the number of damage state increase observations, a back-to-back analysis (e.g., [33]) is also performed. For each of the three DSs, the most ‘critical’ five mainshocks causing MIDR values immediately below each DS-threshold were selected and then combined with the other mainshocks generating 2005 artificial sequences. Only the ground motions coming from the same database and sharing the same processing are combined together to perform back-to-back analyses.

This process allowed the definition of a statistically significant number of DS increases due to sequential ground motions, allowing the definition of state-dependent fragility functions. PSDMs are developed by filtering the points in the cloud and considering only the cases in which the MIDR of the aftershocks is higher than the MIDR of the mainshocks. Accordingly, Figure 6(b) shows the fragility curves for the undamaged structure together with the state-dependent fragility functions considering the DS3 conditioned to both DS1 and DS2 and DS2 conditioned to DS1.

![Figure 6. (a) MIDR values due to the mainshocks vs. aftershocks for the bare frame considering real sequences. (b) State-dependent fragility functions for the infilled frame.](image)

Figure 6(b) shows the increased fragility of the frame as consequence of the mainshocks-induced damage. $\theta_{m}$ of DS2 and DS3 are significantly reduced by 38.4% and 33.7% respectively when the structure has an initial damage of DS1 due to mainshocks. The reduction in $\theta_{m}$ for DS3 when the structure has an initial damage of DS2 reach values of 56.6%.

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3.3 Vulnerability assessment of undamaged and mainshock-damaged structure.

Vulnerability functions are finally developed, allowing the evaluation of expected seismic losses conditioned to the mainshock and aftershock IM values. As discussed above, the damage-to-loss ratios defined by [31] are used in this study. The original study [31] considered five DSs plus the undamaged state while the present study considers three DSs only plus the undamaged state. In order to couple the damage-to-loss ratios with the developed fragility curves, a matching of the different DSs was performed and is reported in Table 5.

<table>
<thead>
<tr>
<th>Di Pasquale et al. (2005)</th>
<th>DSs</th>
<th>DS0</th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
<th>DS4</th>
<th>DS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infilled frame</td>
<td>Damage Factor</td>
<td>0.00</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.75</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 5. Matching Damage Factors from [31] and the present study for infilled frames.

The probabilities of each DSs conditioned to the IM values are combined with the damage-to-loss ratios in order to produce conditional vulnerability functions as specified in Eq. (4). Figure 7(a) shows the vulnerability functions for the infilled frame in the undamaged state and with mainshock-induced DS1 and DS2. The undamaged structure has a loss ratio of about 80% with an $S_a(T_1)_{AS}$ equal to 2.5g. Differently, the structure experiences similar losses, and even higher, at significantly lower $S_a(T_1)_{AS}$ intensities when the mainshock-induced damage is already in place. For instance, the infilled frame has a loss ratio of 90% with values of $S_a(T_1)_{AS}$ equal to 2.11g and 0.69g respectively under the initial mainshock-induced DS1 and DS2.

State-dependent vulnerability functions can be also represented by plotting vulnerability surfaces, as discussed in [33]. The IMs of the mainshocks and aftershocks are plotted on the horizontal axes, while the vertical axis shows the resulting loss ratio. It should be noted that the surface passes through the state-dependent vulnerability functions, which are fixed at their intersection with the undamaged-state vulnerability function. The intermediate points between these functions are estimated using linear interpolation of 3D scatter of points. The vulnerability surface reported in Figure 7(b) allows to easily estimate the final expected losses based on $S_a(T_1)_{MS}$ and $S_a(T_1)_{AS}$. However, as per the initial assumption, only one single aftershock is
considered at this stage. The proposed vulnerability surfaces can be more easily implemented into a time-dependent risk assessment framework.

4 CONCLUSIONS

This study investigated the performance of a four-story nonductile RC frame, in both bare and infilled configurations, against MS-AS ground-motion sequences. 2D advanced nonlinear models were created using OpenSees, accounting for flexural and shear hysteretic behaviors in addition to the presence of masonry infill walls. Real MS-AS sequences as well as artificial sequences were adopted to carry out nonlinear time history analysis via a cloud-based approach. Fragility curves for the undamaged frames were first developed from mainshock analysis, highlighting the positive impact of the infills on the seismic performance of the case-study structure. Subsequently, state-dependent fragility functions were derived for the mainshock-damaged infilled frame based on the analysis results of the full MS-AS sequences. Results demonstrated that the mainshock-induced damage leads to significantly higher fragility compared to the structure in the undamaged state. It was also shown that higher initial damage caused by the mainshocks leads to larger fragility of the structure against aftershocks.

Vulnerability functions were finally developed for the infilled frame based on the results of fragility analysis in conjunction with damage-to-loss models calibrated on Italian post-earthquake data. These functions illustrated how the expected losses for the frame in the mainshock-damaged configuration are considerably higher than those for the undamaged-state frame across the entire range of ground-motion intensities. Vector-valued mainshock-aftershock vulnerability relationships were also generated in the form of a three-dimensional vulnerability surface that can be adopted to quantify the expected seismic losses based on the level of intensity of both mainshocks and aftershocks.

ACKNOWLEDGEMENTS

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REFERENCES


FORECASTING TIME-VARIABLE EARTHQUAKE RISK FOR REINFORCED CONCRETE BUILDINGS DURING AFTERSHOCK SEQUENCES BASED ON OPERATIONAL EARTHQUAKE FORECASTING AND RESONANCE PERIOD ELONGATION

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Abstract

The probability of a reinforced concrete building to sustain cumulative damage during aftershock sequences is a key issue for crisis management. Assessments of time-variant probabilities of damage states are made herein for reinforced concrete building models the city of Thessaloniki, Greece, in the case of a mainshock scenario. The elongation of the first eigenperiod of the building is used as the engineering demand parameter in the computation of the fragility curves for the building models. Period elongation is selected, as it is readily available in the case of buildings with permanent monitoring systems, which are part of a critical infrastructure, and it may be used complementary to post-earthquake building inspection in order to assess earthquake damage. The increase of the vulnerability of the buildings with time due to potential earthquake damage accumulation is modeled with a probabilistic approach, which is based on the Markov chain. During the aftershock sequences the probability to exceed period elongation thresholds accumulates. Additional risk due to aftershocks as a function of time is estimated and the developed framework is based on the traffic-light concept (red-orange-green) for building-tagging with respect to seismic risk to support earthquake risk management in the course of aftershock sequences.

Keywords: Aftershock Sequence, Time-variant Damage State Probability; Building Tagging.
1 INTRODUCTION

Immediately after a large earthquake, seismic risk management and protection of human life and infrastructure is more efficient, not only when the structural condition of buildings is rapidly and automatically evaluated, but also when near-real-time evaluations and short term forecasts of time-variable seismic risk are available [1]. On the scale of the city or in the case of critical structures, structural performance levels related to immediate occupancy or damage grades are ingredients to be considered in the decision-making strategy for short-term forecasting. Specifically, rapid risk assessment must be extended over the entire duration of the after-shock sequence to take into account potential damage accumulation due to aftershocks [2]. Operative procedures are then required to combine aftershock occurrence forecasts, modeling of damage accumulation, and building health characterization with respect to targeted levels of damage.

Decision-making in seismic emergen-cies has to consider the increased vulnerability due to mainshock damage and the time-variant vulnerability of buildings in view of probable after-shock damage accumulation. Underlying probabilistic frameworks have been developed to assess the contribution of aftershock damage accumulation with respect to damage caused by the mainshock and applied to specific building typologies [3–5]. Furthermore, the Markov-Chain approach has been used to assess seismic performance and to develop a closed-form reliability model of accumulated damage in structures [6–8], including probabilistic frameworks of the aftershock occurrence process. Most studies report the large contribution of aftershocks to consequence and loss forecasting. Short-term variability of buildings vulnerability related to the damage accumulated over the entire seismic sequence is a key element to be assessed for short-term decision-making.

Among the solutions available, monitoring the elongation of the fundamental period of buildings is a practical, efficient way to assess earthquake damage. Fundamental period (or frequency) is assumed to be a proxy for apparent structural stiffness and structural health [9–13]. For instance, the residual stiffness of masonry buildings from period measurements has been used to study the effect of seismic damage accumulation on macroseismic intensity assessment [9]. Michel et al. [10] quantified the fundamental frequency shift for unreinforced masonry specimens as a function of structural drift using dynamic tests in the laboratory. For reinforced concrete (RC) buildings, experimental and numerical approaches have produced empirical relationships between frequency shift and damage index [11] and Katsanos et al. [12] showed that transient period elongation during seismic response does not exceed 1.2 and 1.7 for the design earthquake and twice the design earthquake, respectively. Katsanos and Sextos [13] also showed the sufficiency (i.e., no magnitude-distance criteria required) of the structural period and the force-reduction factor (i.e. the ratio of the maximum seismic force to yield force) for damage prediction, using single-degree-of-freedom (SDOF) oscillators. Reuland et al. [14] predict fragility curves for subsequent earthquakes based on measured post-earthquake structural frequency and visual inspection. For seriously damaged buildings, Masi et al. [15] reported 40-50% period elongation with laboratory scaled specimens of RC without masonry infills under strong motion, and 15-30% for site-monitored RC buildings with masonry infills subjected to moderate seismic ground motion.

Clinton et al. [16] and Astorga et al. [17] showed the accumulated effect of successive earthquakes on frequency drop, in relation to the amplitude of seismic loading, for long-term structural monitoring. Furthermore, Gallipoli et al. [18] reported the co-seismic frequency drop of permanent monitored buildings during main and aftershocks. Post-earthquake experimental surveys for building damage classification have also highlighted residual elongations. For in-
stance, permanent frequency (or period) shift was first associated with the level of seismic damage after the 2003 Boumerdes earthquake in Algeria [19]. The damage was assessed by visual screening and based on the red-orange-green traffic light classification. The post-earthquake period elongation observed in classified buildings was <30% (green), between 10 and 70% (orange) and 50-70% (red). These values were confirmed by Vidal et al. [20] after the 2011 Lorca earthquake: period elongation was equal to 20%, 43% and 65% for EMS98 damage grades 1, 2 and 3-4, respectively.

This article presents a framework for predictive modelling of the time-variant seismic risk of reinforced concrete buildings during an aftershock sequence immediately after a large earthquake with Monte Carlo experiments. It uses a red-orange-green traffic light scheme to classify building models related to the immediate occupancy performance level. This tagging scheme refers to the seismic risk at a given moment in an aftershock sequence and should not be confused with other traffic light schemes referring to the degree of sustained seismic damage. This framework employs operational earthquake forecasting with the Epidemic Type Aftershock Sequence model (ETAS). The ETAS model is a statistical model, whose parameters are computed by analyzing earthquake catalogs. Moreover, this framework describes structural health of reinforced concrete buildings with the elongation of their natural period, a damage metric, which is continuously tracked in the case of instrumented critical buildings. Having such information available at almost real time allows short-term seismic risk forecasts based on updated structural conditions. The framework is applied in the case of a mainshock scenario in the vicinity of Thessaloniki, Greece.

2 EARTHQUAKE SCENARIO, AFTERSHOCK SEQUENCES AND BUILDING MODELS

2.1 Synthetic aftershock sequences for the earthquake scenario

The synthetic aftershock sequences in this framework are generated with the ETAS model. This model is based on empirical laws describing the distribution of earthquakes in space, time and magnitude. It predicts the rate of future earthquakes based on previous seismicity; it can also be used for OEF [21] and to generate synthetic catalogs. In this study, we use the ETAS model to generate synthetic catalogs of aftershocks caused by the mainshock scenario and its aftershocks.

The selected scenario corresponds to the Thessaloniki (Greece) case study: mainshock (depth: 10 km) magnitude 7.0, with epicenters located 30 km north-east of the city. The scenario is selected on the basis of seismic hazard deaggregation results for Thessaloniki using the OpenQuake Engine [22] and the computation input [23] from the 2013 Euro-Mediterranean Seismic Hazard Model (ESHM13) [24]. The ETAS model takes into account for the uncertainty of the strike and dip of the seismic fault. These parameter values are sampled by a Monte-Carlo simulation as random independent values from normal distributions, with a standard deviation of 10° and median values equal to 120° and 60° for strike and dip, respectively. Such fault geometries are found in the region of the scenario epicenter [25,26]. In the synthetic output catalogs, earthquakes with a magnitude in excess of 2.0 are simulated, but only earthquakes with a magnitude of 4.0 or more are included in the damage model. Moreover, the Guttenberg-Richter law is capped at magnitude 8 for the generation of synthetic earthquake sequences, as a conservative assumption compatible with the estimated maximum possible magnitude (M 7.1) in 100 years for the Thessaloniki region [27,28].

In this study, the parameters required by the ETAS model [29] are estimated based on the earthquake catalogs of the seismological station of Aristotle University of Thessaloniki (the 550BC-2010 catalog [30,31] and the 1995-2014 catalog [32]). A sub-catalog, which excludes
earthquakes outside a 200km radius around Thessaloniki, is used to consider events of similar tectonic settings. Fig. 1a gives the cumulative distribution of magnitudes in the catalogs and sub-catalog.

Figure 1: a) Cumulative magnitude distribution in the two historical earthquake catalogs for Greece (550BC-2010, 1995-2014) and in the catalog including earthquakes up to 200 km from Thessaloniki; b) number of aftershocks as a function of the magnitude of the triggering earthquake in the catalog of Greece (1995-2014 earthquakes with M>2 and distance less than 200 km from Thessaloniki). The solid line is a linear fit with an exponent of $\alpha=0.8$; c) Aftershock decay as a function of time a mainshock magnitude with 0.5 intervals (color scale). The straight lines represent Omori’s law with an exponent of $p=0.95$.

The ETAS model assumes that the seismicity rate is the sum of a background rate and the rate of the aftershocks triggered by each previous event. This work is concerned with seismicity following a large mainshock; the background rate is therefore ignored. The seismicity rate $\varphi$ at time $t$ following an earthquake of magnitude $M$ located at a distance $r$ is given by:

$$\varphi(t, r, M) = K \cdot 10^{\alpha M} \cdot \frac{1}{(t+c)^p} \cdot \frac{1}{(r+L(M))^{1+q}}$$  \hspace{1cm} (1)$$

where $L(M)$ is the associated rupture length of the earthquake, and $K$, $c$, $p$, $\alpha$ and $q$ the model parameters. The rate of aftershocks increases exponentially with the magnitude $M$ of the triggering event. In time, the seismicity rate follows Omori’s law: the rate of aftershocks decreases as a power law as a function of the time after the triggering event. In space, the density is constant for distances smaller than the rupture length $L(M)=0.01 \cdot 10^{M/2}$ km and decreases according to a power law at large distances $r >> L(M)$. The magnitude distribution is usually modeled by a Gutenberg-Richter law. The total seismicity rate $R(t, r)$ is the sum of the aftershock sequences of each previous earthquake and is given by:

$$R(t, r) = \sum_{t_i \leq t} \varphi(t - t_i, |r-r_i|, M_i)$$  \hspace{1cm} (2)$$

Where $|r-r_i|$ is the norm of the vector $r-r_i$, i.e. the 3D distance between the point corresponding to $r$ (where the seismicity rate is estimated) and the point corresponding to $r_i$ (location of earthquake $i$). Due to this cascading effect, the global rate of aftershocks is much larger than the rate of direct aftershocks modeled by Eq. 1. It also obeys Omori’s law, but with a smaller $p$ exponent and with much larger fluctuations than the rate of direct aftershocks given by Eq. 1 [33]. The cascade of secondary aftershocks also slightly modifies the spatial distribution of aftershocks, which decays as a power law with an apparent exponent smaller than $l+q$, and induces a slow diffusion of aftershocks as a function of time [34].

The model parameters are usually estimated using a maximum likelihood method [35]. However, this method often yields biased and unrealistic values for some parameters. For instance,
the exponent $\alpha$ describing the increase in aftershock productivity with the mainshock magnitude can be underestimated if the spatial distribution of aftershocks is not isotropic [36]. In this study, we use a different approach. We start by extracting aftershocks from the catalogs using the solution proposed by Helms et al. [37]. We then estimate the model parameters of Eq. 1 ($p=1.1$, $c=0.001$ day, $\alpha=0.8$, $K=0.008$ and $q=2.0$) by comparing the synthetic aftershock sequences generated with ETAS and the aftershock sequences extracted from the regional subcatalog including events within 200 km from the site. Specifically, $\alpha$ is estimated by fitting aftershock productivity as a function of mainshock magnitude (Fig. 1b). The parameters $K$, $p$ and $c$ of Omori’s law are found by fitting aftershock decay as a function of time since the mainshock (Fig. 1c). The total aftershock rate is characterized by an Omori exponent $p=0.95$, smaller than the value $p=1.1$ that characterizes the rate of direct aftershocks defined by Eq. 1. In space, the exponent, $q$ of Eq. 1 is obtained by analyzing the distribution of distances between the mainshock and aftershock hypocenters. Finally, for seismic risk analysis, the duration of the simulated aftershock activity should ideally enable return to a safe situation, i.e. after all cumulative damage to buildings. In our case, the maximum simulation time estimated by the earthquake catalog analysis is equal to 1,000 days.

1.1 Building models

A low-rise and a mid-rise reinforced concrete building model are used in this study (Fig. 2). The models are considered representative of Greek building typology, and are used to estimate losses [38]. The HAZUS [39] model naming convention is used and model characteristics are given in Table 1. The low-code design level refers to seismic design according to the first Greek seismic code in 1959. Numerical modeling is performed with OpenSees [40]. The inelastic behavior of the structural elements being considered with a distributed plasticity model of hysteretic response with stiffness degradation. The bilinear moment-curvature hysteretic models for the columns and beams are calibrated according to fiber section analysis results.

![Building models](image)

**Figure 2:** Building models representing reinforced concrete buildings in Greece (for details see Table 1).

<table>
<thead>
<tr>
<th>Type</th>
<th>Structural System</th>
<th>Height</th>
<th>Design Level</th>
<th>$T_{\text{orig}}$ (s)</th>
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<td>Moment resisting frames</td>
<td>Low-rise</td>
<td>Low code</td>
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<tr>
<td>C1M</td>
<td>Moment resisting frames</td>
<td>Mid-rise</td>
<td>Low code</td>
<td>0.61</td>
</tr>
</tbody>
</table>

**Table 1:** Description of the building models.
Figure 3: Flowchart of the proposed framework. Steps 1-4 are executed "offline", i.e. before the mainshock, steps 5-7 are executed immediately after the mainshock.

2 FRAMEWORK FOR TIME-VARIANT VULNERABILITY AND CUMULATIVE DAMAGE-STATE PROBABILITY

We consider two period elongation thresholds (40\% < $\Delta T_1 \leq 60\%$ and $60\% < \Delta T_1$) as the most critical values related to building state and according to the post-earthquake observations [19,20]. These values represent moderate and substantial to very heavy damage, respectively, used as performance levels for short-term decision-making in earthquake emergencies, i.e. immediate occupancy. At the end of the process, evaluations are made for potential decisions on the probability of damage states during the mainshock-aftershock sequence, computed by the
cumulative probabilities of exceeding the period elongation thresholds at a given time t
(P[40%<ΔT≤60%](t), P[60%<ΔT](t)) and, subsequently, the time-variant probability of damage states computed as the probability increment during a pre-defined time-window. Fig.3 shows the flowchart of the framework developed in this study and summarized below.

**Step 1.** A Probabilistic Seismic Demand Analysis (PSDA) of the undamaged building model is performed using inelastic finite element time-history analyses. Herein, the probabilistic model is arbitrarily formulated with the PEER ground motion bin approach [41]. Each bin consists of 15 physically-based synthetic ground motions generated with SeismoArtif [42], considered to be hazard-consistent. For each bin, ground motions are simulated using the specific barrier model [43]. Subsequently, they are adjusted in the frequency domain to be compatible with the median spectrum for each scenario and given by a Ground Motion Prediction Equation (GMPE), i.e. the Ambraseys et al. [44] model previously used to assess the seismic vulnerability of reinforced concrete buildings in Greece [45]. Periods in the [0.02 s - 2.0 s] range are selected for spectrum-adjustment, i.e. more than 2 times the longest eigen-period of the models herein.

**Step 2.** The stiffness degradation of the structural elements is modeled as a function of inter-story drift. Firstly, the stiffness degradation ratio is defined as the ratio of the secant modulus (K_{sec}) of the moment-curvature loops resulting at each section along the length of the structural elements, to the elastic stiffness (K_0 = EI) of these sections. The stiffness degradation ratio used here is equivalent to the stiffness reduction factors for concentrated plasticity hinges [46,47], which are based on the secant modulus in moment-chord rotation loops. In specific, hinge stiffness reduction factors are estimated as a function of inter-story drift, normalized to the yield chord rotation [48]. Fig. 4 shows the numerical results of the stiffness degradation ratio obtained at the bottom section of an outer column on the ground floor of the C1M low-code model. Several assumptions are made: (1) power functions are computed for sections of the model’s beam-column finite elements to estimate the stiffness variation along the structural elements; (2) the location of the sections along the length of the elements is defined by the Gauss-Legendre quadrature rule; (3) prismatic beam elements are used to model the damaged buildings, due to the fact that stiffness degradation varies along the length of each element (e.g., more damage is expected to occur at the top and bottom of a column than in the middle); (4) the cross-section dimensions remain constant from one section to another for each element, regardless of variations in bending stiffness; (5) the degraded stiffness in each section is computed with analytical relationships estimated from the moment-curvature results (as given for C1M in Fig. 4).

**Figure 4:** Example of the degradation of stiffness using the moment-curvature hysteretic loops (left insert) at the cross-section at the bottom of the outer columns of the C1M low-code model for each input motion represented by crosses. The analytical equation is obtained by considering a bi-linear degradation model (continuous line) to fit the data.
Numerical models of the building with increasing stiffness degradation are built to develop an operational method. The elongation of the first eigenperiod ($\Delta T_1$) of the degraded building relatively to the period of the original building ($T_{1,\text{orig}}$) is computed as follows:

$$\Delta T_1 = \frac{T_{1,\text{deg}} - T_{1,\text{orig}}}{T_{1,\text{orig}}}.$$  (3)

Fig. 5 shows $\Delta T_1$ with respect to maximum inter-story drift for building models configured according to the procedure described. Values of the relative elongation of the first eigen-period ($\Delta T_1 = 40\%$ and $60\%$) are shown for the damage state thresholds considered herein. The corresponding maximum inter-story drift corresponding to $\Delta T_1$ equal to $20\%, 40\%$ and $60\%$ is then obtained from Fig. 5 and used as the damage state thresholds. By applying the analytical relationships for the stiffness degradation of the element sections (Fig. 4), the stiffness of the element sections is computed for $\Delta T_1$ equal to $20\%$ and $40\%$. The buildings in the new damage state are then modeled using the degraded stiffness values corresponding to $\Delta T_1$ equal to $20\%$ and $40\%$. Fig. 6 shows the degraded stiffness ratio in the case of the C1M low-code model with $20\%$ period elongation. For the sake of simplicity, the uncertainty in the stiffness reduction functions is not propagated analytically in the $\Delta T_1$ threshold estimation or in the process of estimating the stiffness of the structural elements for each damage state. However, this uncertainty, as well as the damage state threshold uncertainty, are taken into account by adding empirical components to the dispersion of the estimated fragility curves.

Figure 5: Period elongation ($\Delta T_1$) of the building models as a function of maximum inter-story drift.

Figure 6: Example of the ratio of the stiffness of the structural elements with varying cross-section properties to the stiffness of the corresponding elements in the initial C1M low-code building model ($K/K_0$) for $\Delta T_1$ equal to $20\%$.
Step 3. A PSDA (Probabilistic Seismic Demand Analysis) is then performed on the degraded building models for $\Delta T_1$ equal to 20% and 40%. The seismic response analyses for the PSDA for each degraded model are run successively and in single-thread mode: on an Intel i7-4710HQ, this takes approximately 30 minutes, which is roughly equal to the time required for every undamaged model. The fragility curves are then computed, giving the probability of the degraded building model exceeding the selected period elongation threshold as a function of an Intensity Measure (IM), i.e. Peak Ground Acceleration (PGA), in our case. The fragility curves are evaluated with the “Cornell Method”, i.e. by fitting a linear function of PGA in log-scale to the inter-story drifts computed in Step 2. In Fig. 7, the fragility curves used in this study express the probability for the undamaged building models and the degraded building models in each damage state to exceed the $\Delta T_1$ thresholds. Although other intensity measurements may correlate better with structural response [49,50], PGA has been used in operational post-event response systems for structural evaluation [1] and is therefore adopted in this study.

![Figure 7: Examples of fragility curves in terms of period elongation according to [26].](image)

(a) Probability of $\Delta T_1 > 60\%$ at $t_i$ given that $\Delta T_1 = 0\%$ at $t_{i-1}$ (for undamaged building models); (b) Probability of $40\% < \Delta T_1 \leq 60\%$ at $t_i$ given that $\Delta T_1 = 20\%$ at $t_{i-1}$ (for damaged building models).

Step 4. For each seismic event (magnitude-distance pair) in the mainshock-aftershock sequence, a random IM value is computed based on the GMPE. In specific, the IM value is a sample from a random variable with median and standard deviation given by the GMPE for the seismic event. The fragility curves of the damaged and undamaged building models are then used to compute the elements of the Markov chain transition matrices [51,52]. The probability of exceedance of each damage threshold is computed using the fragility curves of the building models in the considered damaged states (numbered 1 to $m$) and for IM values corresponding to the events of the aftershock sequence. Here, time-variant vulnerability is obtained by considering that a building during an aftershock sequence is a system going through a chain of events and states, with the hypothesis that every subsequent state depends only on the current state, expressed as follows:

$$
T_{(t_i)} = \begin{bmatrix}
P_{11}(t_i) & P_{12}(t_i) & \cdots & P_{1m}(t_i) \\
0 & P_{22}(t_i) & \cdots & P_{2m}(t_i) \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
$$

(4)

where $T_{(t_i)}$ are the transition matrices, $P$ is the probability of transition from the original state (first index) to the next state (second index). The elements in the last column are computed by:

$$
P_{jm}(t_i) = P[DS(t_i) = m \mid DS(t_{i-1}) = j, IM(t_i) = a_i]
$$

(5)
where $P_{jm}(t_i)$ is the probability of exceeding the threshold of the highest considered damage state (DS = m) at the time of a seismic event ($t_i$), given that the building is in a lower damage state (DS = j, j < m) at the previous time-step ($t_{i-1}$). This value is equal to the value of the fragility curve of the building model in damage state j for damage state m, considering the intensity measurement ($a_i$). In other words, $P_{jm}$ is equal to the probability of transition from damage state j to damage state m for a given $a_i$. Subsequently, the elements of each matrix between the diagonal and the last column are computed by:

$$P_{jk}(t_i) = P[DS(t_i) = k | DS(t_{i-1}) = j, IM(t_i) = a_i] - P[DS(t_i) = k + 1 | DS(t_{i-1}) = j, IM(t_i) = a_i]$$  \hspace{1cm} (6)$$

where $P_{jk}(t_i)$ is the probability of transition from damage state j to damage state k, i.e. the probability of exceeding the threshold of damage state k but not the threshold of state k+1, given that the building is in damage state j (j < k). Both terms on the right hand side of Eq. 6 are the fragilities for a given intensity measurement ($a_i$), i.e. equal to the value of the fragility curves of the building model in damage state j for damage states k and k+1, respectively. Finally, the diagonal elements are computed by:

$$P_{jj}(t_i) = 1 - P[DS(t_i) = j + 1 | DS(t_{i-1}) = j, IM(t_i) = a_i]$$  \hspace{1cm} (7)$$

where $P_{jj}(t_i)$ is the probability of remaining in damage state j and not exceeding the threshold of damage state j+1 for a given intensity measurement ($a_i$) and given that the building is in state j. As with Eq. 6, the second term of the right hand side is the fragility. Elements below the diagonal of the transition matrices are assumed to be equal to zero, i.e. it is assumed that it is impossible to obtain a lower damage state associated with a decrease in structural period after stiffness degradation.

One transition matrix is computed for each seismic event of the mainshock-aftershock sequence and one series of matrices is computed for each building model. Since the value of the IM is given for each successive event, the Markov chain is time-inhomogeneous, i.e. the values of the elements of the transition matrix are time-dependent. The probability of the building being in a damage state at the time of the aftershock is then computed with a Markov chain with discrete time and space using the computed transition matrices as follows:

$$[P[DS(t_n) = 1] \ldots P[DS(t_n) = m]] = [1 \hspace{0.5cm} 0 \hspace{0.5cm} \ldots \hspace{0.5cm} 0] \Pi T(t_i)$$  \hspace{1cm} (8)$$

Zero values are assigned to all the elements of the initial vector on the right hand side of Eq. 8 with the exception of the first element of the vector, which is equal to 1.0. It is thus assumed that before the first seismic event of the mainshock-aftershock sequence, the building is in its original, undamaged state. In an underlying operational framework, it would be possible to update the initial vector using actual building measurements, i.e. considering actual structural health before the sequence.

**Step 5.** The synthetic sequences generated for the scenario in this study are used. The median and the uncertainties of the IM are computed using the GMPE given by Chiou and Youngs [53] for shallow crustal earthquakes. This model includes a flag-parameter to distinguish between main- and aftershocks. In this study, the uncertainty related to the IM is considered by sampling a random variable with the computed median and standard deviation for every seismic event in the synthetic catalogs.

**Step 6.** For each IM distribution, and based on the fragility curves of the building models (Step 4), $P[40%<\Delta T_1\leq60%](t)$ and $P[60%<\Delta T_1](t)$ are computed at the time of the events, with a variable time step. The time step of the computed $P[60%<\Delta T_1](t)$ for each synthetic sequence is not constant and actually depends on the time of the considered earthquakes within the sequence. Furthermore, the number and time of the seismic events varies from catalog to catalog.
according to the ETAS algorithm. The computed probabilities are then re-sampled with a constant time-step of 0.1 day, allowing computation of percentiles as a function of time. Fig. 8 shows the 5th, 50th and 95th percentiles of $P[60\%<\Delta T_1](t)$. The time of the mainshock is $t = 1.0$ day. As expected, the highest accumulation of $P[\Delta T_1>60\%]$ is during the first day, given that the seismicity rate decreases constantly with time (Fig. 1).

![Figure 8: Examples of cumulative probability of period elongation ($\Delta T_1$) exceeding 60% after the time of the mainshock ($t = 1$) for synthetic aftershock sequences generated in the case of the C1L and C1M building models. Dashed lines correspond to the 5th and 95th percentiles of $P[60\%<\Delta T_1](t)$, continuous lines correspond to its 50th percentile.](image)

The cumulative probability of exceeding $\Delta T_1=60\%$ provides information on the greater probability of observing the damage level (based on the period elongation threshold) at a given time after the mainshock, compared with the damage expected due to the mainshock alone. The ratio of the 50th percentile of $P[60\%<\Delta T_1](t)$ at the end of the scenario divided by the value of the 50th percentile due to the mainshock is given in Table 2. This ratio varies according to building model and we observe that the aftershock sequence contributes significantly to the resulting damage state.

<table>
<thead>
<tr>
<th>Type</th>
<th>Design Level</th>
<th>50th percentile of $P<a href="t">60%&lt;\Delta T_1</a>$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1L</td>
<td>Low code</td>
<td>0.0854 $t_{\text{main}}$ 0.1796 $t_{\text{end}}$</td>
<td>2.10</td>
</tr>
<tr>
<td>C1M</td>
<td>Low code</td>
<td>0.0028 $t_{\text{main}}$ 0.0064 $t_{\text{end}}$</td>
<td>2.26</td>
</tr>
</tbody>
</table>

Table 2: Ratio of the 50th percentile of the cumulative probability of period elongation 60% ($P[60\%<\Delta T_1](t)$) at the end of the aftershock sequences to the 50th percentile of $P[60\%<\Delta T_1]$ due to the mainshock.

3 TIME-VARIANT DAMAGE-STATE PROBABILITY AND TRAFFIC-LIGHT CONCEPT FOR DECISION MAKING

The so-called “traffic light” model is often introduced for decision-making. It evaluates the safety of structures according to criteria related to their operability and occupancy, assigning each building to one of three categories: the tolerable zone (green), the intermediate zone (orange) and the intolerable zone (red). This strategy has been already applied for post-earthquake crisis management, which consists in visually inspecting the buildings and classifying them...
according to occupancy safety based on an empirical expert appraisal [19,20]. It has also been used for continuous and online condition-based solutions for structural aging [54]. Typically, the “traffic light” defines a “green” level, indicating that the buildings are safe, with no apparent damage and can be occupied again, an “orange” level indicating that the buildings present all the characteristics of damaged buildings but with a high level of uncertainty, and a “red” level indicating that occupancy should be suspended immediately and the building demolished. Previous studies [19,20] have shown the benefits of experiment-based frequency assessment to reduce classification uncertainties, particularly for the “orange” level.

In our study, a frequency-based red/orange/green “traffic light” scheme is proposed to classify buildings according to the estimated time-variant probabilities of damage states. The probability increment between t and t + 7 days [1,5] \( P[60\%<\Delta T_1](t,t+7) \) is used to tag buildings based on tolerable damage state probabilities. The 5th, 50th and 95th percentiles of the computed probability are considered for two tolerable annual rates \( \lambda_{\text{tol}} \) of damage state probabilities (Table 3), based on the performance levels selected for seismic risk analysis. Silva et al. [55] propose \( 5\cdot10^{-5} \) and \( 10^{-4} \) as tolerable annual rates of collapse leading to loss of human life for low and moderate earthquake-prone countries, such as Greece \( 10^{-4} \). These values are adopted herein (\( \lambda_{\text{tol.1}} \) and \( \lambda_{\text{tol.2}} \)) for low and moderate seismicity regions, respectively, as “human life safety” performance levels.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Tolerable annual rate ( \lambda_{\text{tol}} )</th>
<th>Tolerable ( P(T=475y) )</th>
<th>Element in risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \lambda_{\text{tol.1}} = 1\cdot10^{-4} )</td>
<td>( 4.6\cdot10^{-2} )</td>
<td>Structure and human life (moderate seismicity region)</td>
</tr>
<tr>
<td>2</td>
<td>( \lambda_{\text{tol.2}} = 5\cdot10^{-5} )</td>
<td>( 2.4\cdot10^{-2} )</td>
<td>Structure and human life (low seismicity region)</td>
</tr>
</tbody>
</table>

Table 3: Tolerable risk thresholds used in the building tagging framework.

The thresholds of tolerable risk according to the standard values for earthquake design and performance based assessment are computed using a Poisson model as follows:

\[
P(T) = 1 - e^{-\lambda T}
\]  

with \( \lambda = \lambda_{\text{tol}} \) and arbitrarily assuming a time window T of 475 years. This arbitrary selection is similar to the arbitrary selection of a tolerable level for the probability of collapse in 7 days in [5].

Given the uncertainty of the estimated damage-state probabilities, tagging is possible after defining a desired level of confidence, i.e. the red, orange and green tag periods are determined based on two tolerable failure probability thresholds. On the one hand, if the 5th percentile of \( P[60\%<\Delta T_1](t,t+7) \) is higher than the corresponding tolerable threshold, the building is tagged red. On the other hand, if the 95th percentile of \( P[60\%<\Delta T_1](t,t+7) \) is lower than the corresponding tolerable threshold, the building is tagged green. Finally, the building is tagged orange throughout the period between the red and green tag periods, as proposed in [1].

Fig. 9 shows the results of the risk assessment and operational framework for risk management during aftershock sequences. As expected, the highest probability of the two damage states is observed at the time of the mainshock. Subsequently, a rapid decrease of the 5th, 50th and 95th percentiles of the computed probability is observed on the first day after the mainshock, followed by a period of slower decrease. The end of the risk period corresponds to the moment when the percentiles of the computed probability drop to very small values (e.g. less than \( 10^{-5} \)).
With these figures, decisions can be made based on the structure’s health estimated on the basis of the resonance frequency shift, measured using experimental data. For example:

- For a red-tagged building based on λ<sub><i>tol</i>1</sub> and P[60%<ΔT<sub>1</sub>](t,t+7), there is a time-period during which the probability of human life loss is intolerable due to extensive structural damage and based on a period elongation of more than 60%.

- All building models are tagged green regardless of the tolerable damage-state probability thresholds once a certain amount of time has elapsed since the mainshock.

Finally, Table 4 summarizes the time after the mainshock for buildings to switch from red to orange (t<sub>red</sub>) and orange to green (t<sub>orange</sub>) for the tolerable risk thresholds.

![Figure 9: Examples of cumulative probability of period elongation corresponding to 60% < ΔT<sub>1</sub> days after the main shock for the synthetic aftershock sequences considering the a) C1L and b) C1M building models. Black lines correspond to percentiles.](image)

<table>
<thead>
<tr>
<th>Tolerable Risk Threshold</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building type</td>
<td>Seismic design level</td>
<td>t&lt;sub&gt;red&lt;/sub&gt; (days)</td>
</tr>
<tr>
<td>C1L</td>
<td>low code</td>
<td>0.0</td>
</tr>
<tr>
<td>C1M</td>
<td>low code</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4: Estimated time in days after the mainshock until the red (t<sub>red</sub>) and orange (t<sub>orange</sub>) tag periods end, according to the tolerable risk thresholds considered in this study.

4 CONCLUSION

Here, we present a framework for predictive modelling of the time-variant risk of reinforced concrete buildings to exceed damage states, which tags buildings with a red-orange-green scheme as support to decision making for seismic risk management. The originality of this underlying framework lies in the reduction of resonance frequencies (or the period elongation) as a proxy for damage, based on the knowledge acquired during seismic sequences [19,20].

An underlying probabilistic framework for risk assessment completes the vulnerability study by simulating aftershock sequences for a mainshock scenario in the vicinity of Thessaloniki. The spatio-temporal variation of the aftershocks (M, R) is provided by an ETAS model, and the ground motion for each aftershock is produced by a GMPE model compatible with the context of our study. The risk assessment is then completed by associating the spatio-temporal variation
of the sequence’s ground motion with the probable structural damage and level of performance considered in this study, i.e. human life safety. These performance levels ultimately depend on the classification of the structures. This study confirms the major contribution of aftershocks to the risk associated with the scenario.

Having the resonance frequencies of buildings before damage, or the more expensive option of installing permanent instrumentation in critical structures, is indispensable for rapid damage assessment. For example, a real-time online algorithm could be developed using a solution like that proposed by Guéguen and Tiganescu [54], based on the traffic light concept. Additionally, false alarms would have to be taken into account by integrating natural frequency variations and the slow recovery of the elastic properties after a seismic event, as reported by Guéguen et al. [56] and Astorga et al. [17].

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LONG-TERM SEISMIC RISK ASSESSMENT CONSIDERING THE TRIGGERED AFTERSHOCKS

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Abstract

Assessment of the long-term risk profile considering the triggered seismic sequence is rendered complicated by the short-term clustering of aftershock events following the main event and the increased potential for damage accumulation. The assessment of long-term seismic risk profile for the main seismic events is based on the renewal of the structure to a prescribed state after each seismic event. In the case of a triggered aftershock sequence, it is very unlikely that the structure is repaired back to its intact condition during early phases of an ongoing sequence. Therefore, considering the (magnifying) effect of cumulative damage due to the triggered aftershock sequence in long-term seismic risk profile is not a trivial task. One viable strategy is to evaluate the increase in the (time-invariant) limit state excursion probability due to the combined effect of short-term aftershock clustering and the resulting cumulative damage for a prescribed post-event short-term time interval. As expected, in case the structure is assumed to be renewed to a given hypothetic damage state (such as main-shock damaged) after each event occurs, the formulation for limit state probability simplifies to a closed-form Poissonian formulation. The methodology is applied to an existing 3-story RC moment-resisting frame in central Italy. The Poissonian closed-form solution, with the hypothesis of renewal back to the main-shock damaged configuration, is shown to successfully capture the increase in the limit state probability due to the effect of the triggered sequence.

Keywords: Performance-based seismic assessment, Time-dependent reliability, Aftershock sequence, Cumulative damage, Non-linear dynamic analysis, Cloud Analysis.
1 INTRODUCTION

The damage-inducing potential of the aftershocks (AS) following the main event (mainshock, MS) is not considered explicitly in standard seismic risk assessment procedures. The guideline of advanced seismic assessment [1] provides a step-by-step procedure for estimating the fragility curves for a MS-damaged structure based on performance levels ranging from onset of damage to collapse. The seismic behavior of the MS-damaged structure is estimated by performing non-linear static analysis (pushover) on the damaged structure. The residual capacity of the MS-damaged structure is also estimated dynamically by subjecting the structure to a suite of back-to-back ground motion time histories (basically mimicking the effect of a mainshock and one severe aftershock) [2-4]. The use of back-to-back MS events (or their amplitude-scaled versions) as a proxy for the seismic sequence gives way to discussions on its adequacy for representing the frequency content and other ground motion characteristics of the aftershocks triggered by a strong earthquake [5-8]. One important message is that the back-to-back method may overestimate the damaging potential of the earthquakes with respect to when real seismic sequences are used. The demands induced by a sequence of as-recorded seismic events are investigated in [8-11]. Generating artificial MS-AS sequences are also devised in [12-13] paying attention to the frequency content of aftershocks.

Another issue that has received attention from the research community with regard to the aftershocks is the choice of the nonlinear dynamic analysis method for estimating the aftershock vulnerability. Most of the methods available in literature rely on incremental dynamic analysis (IDA, [14]) for calculating the residual capacity of the MS-damaged structure (see e.g., [2-4, 6, 15]). There are also methods in literature that use the so-called Cloud Analysis CA [16-18] for the prediction of the cumulative damage due to the aftershocks ([5, 10, 11, 19, 20]). CA employs linear logarithmic regression in order to predict the structural response versus the ground motion intensity measure based on as-recorded ground motions.

The evaluation of damaging potential of an AS sequence is a time- and history-dependent problem: the time-dependent decay in short- to medium-term seismicity and the history-dependent nature of structural behaviour in the nonlinear range. Quite a few works focus on tackling the problem of aftershock risk assessment considering this time- and history-dependence (see e.g., [4, 10, 11, 19]). Jalayer et al. [19] propose a method for calculating the limit state probability due to an AS sequence in an interval of time by using the total probability theorem. The methodology considers the uncertainty in the number of events of interest and the progressive damage induced by the occurrence of a sequence of aftershocks assuming independence between limit state exceedance in successive events. The method proposed in [19] has been refined in [10, 11] by considering the memory-dependence in calculating the limit state first-excursion due to successive AS records. The current work presents formal procedure for limit state probability assessment considering the MS and the triggered sequence of AS (MS+AS sequence) in contrast to risk calculated by only considering the strong motion (MS). This procedure, referred to as “best-estimate”, considers explicitly both the time-dependent rate of occurrence for AS and the cumulative damage caused by the triggered seismic sequence (MS+AS sequence). Moreover, it is shown that, the implementation of an (time- and event-) invariant fragility curve in the formulation employed for calculating the limit state probabilities due to the triggered sequence [10], leads to the same simple Poisson-type functional form adopted when the cumulative damage is not considered (assuming that the structure’s state is always “renewed” to the same initial state right after the event occurs) [11]. This, apart from verifying the formulation in a special and limiting case, provides the possibility of exploring alternative approximate and simplified solutions by adopting fragility curves corresponding to various initial conditions; such as, fragility curve for the intact struc-
ture, fragility curve for the MS-damaged structure, fragility curve for MS-plus-one-AS-damaged structure, and so on. It is particularly interesting to study the case of fragility curve for the MS-damaged structure since it (with some variations) is often used as a proxy for modeling the damage accumulation due to the MS+AS sequence.

As a numerical example, time-dependent risk related to MS+AS sequence is calculated through both the outlined “best-estimate” and “approximate” procedures. The numerical example herein uses a modified version of the sequential Cloud Analysis (CA) method [10, 11] considering explicitly the “collapse” cases, based on both back-to-back strong-motion (MS) records and also AS ground-motion records. Moreover, the sequential CA presented herein includes an embedded strategy in order to ensure that CA-based predictions do not involve extrapolations. Risk is evaluated in terms of the probability of exceeding the near-collapse limit state for a typical 3-story reinforced concrete (RC) frame building with infill panels located in L’Aquila, central Italy.

2 METHODOLOGY

To calculate the limit state exceedance probability due to a MS and the triggered AS sequence (MS+AS sequence), let \( P(\tau) = P(\text{LS} | \text{I}) \) be the probability of the first-excursion of a desired limit state \( \text{LS} \) given the information \( \text{I} \) about MS+AS sequence as shown in Figure 1. The MS with magnitude \( M_l \leq M_{ms} \leq M_u \) (\( M_l \) and \( M_u \) are the site-specific lower- and upper-bound magnitudes) is followed by an AS sequence with magnitudes \( M_{as} \) in the range \( M_{as, l} \leq M_{as} \leq M_u \) (\( M_{as, l} \) is the lower bound magnitude for AS) in the time interval \([t_{ms}, t_{ms} + \tau]\); \( t_{ms} \) is the time of occurrence of the MS and \( \tau \) is a given forecasting interval for the triggered aftershocks.

The probability \( P(\tau) \) can be further broken down as follows: \( \text{LS} \) is exceeded due to the MS with the probability \( P_{ms} \), or it is exceeded with probability \( P_{as}(\tau) \) during the AS sequence given that it is not exceeded due to the MS [11]:

\[
P(\tau) = P_{ms} + P_{as}(\tau) \cdot P_{ms}(\tau)
\]

(1)

The probability of exceeding the limit state \( \text{LS} \) in time interval \([0, \tau]\), denoted as \( P(\text{LS}) \) herein, can be expressed as (assuming a homogeneous Poisson stochastic model):

\[
P(\text{LS}) = 1 - \exp \left( - \nu_{ms} \cdot t \right) = 1 - \exp \left[ - \nu_{ms} \cdot \left( P_{ms} \cdot (1 - P_{ms}) \cdot P_{as}(\tau) \right) \right]
\]

(2)

where \( \nu_{ms} \) is the rate of occurrence of a MS with \( M_l \leq M_{ms} \leq M_u \). It is assumed that each time a MS occurs, it hits the intact structure. Fitting a (filtered) homogeneous Poisson model herein implies that limit state first-excursion due to MS+AS sequence is lumped at the time of occurrence of the MS, \( t_{ms} \); this is a reasonable assumption when \( \tau \) is more than an order of magnitude smaller than \( t \).

Adopting the first-mode spectral acceleration at period \( T \), \( Sa(T) \), as the desired ground motion intensity measure, \( P_{ms} \) in Eq. (1) or Eq. (2) can be calculated as:
where $sa_{ms}$ is $Sa(T)$ associated with the MS event; $m_0 = P(LS_{ms}|sa_{ms})$ is the conditional probability of LS first-excursion for the MS event (a.k.a. the fragility of the intact structure); $\lambda_{ms}$ is the site-specific hazard defined as the mean rate of exceeding $Sa(T)$ (note that the rate $\lambda_{ms}$ and time $t$ should have consistent units).

The probability $P_{as}$ (the dependence on $\tau$ is dropped for brevity) is the limit state first-excursion probability associated with the AS sequence following the MS event given that the MS has not caused the limit state excursion. This term can be expanded based on Total Probability Theorem over all possible MS wave forms [11]:

$$P_{as} = \frac{1}{N} \sum_{MS_i} P(LS_{as}|MS) p(MS) dMS \approx \frac{1}{N} \sum_{MS_i} P(LS_{as}|MS_i)$$

where $P(LS_{as}|MS)$ is the probability of LS first-excursion for an arbitrary AS event belonging to the AS sequence, given a MS (note that knowing the MS means that its waveform and other characteristics such as intensity are known); $p(MS)$ is the probability that a given MS takes place. It should be noted that $p(MS)$ is characterized for stochastic ground motions (see e.g., [21]); nevertheless, it has been assumed simplistically that different records in the set of MS ground motion are equally likely to take place (see also [21, 22] for similar applications). Therefore, Eq. (4) can be approximated by the average of $P(LS_{as}|MS)$ values over the set of $N$ MS ground-motion records which have not caused LS excursion for the intact structure. The term $P(LS_{as}|MS)$ can be estimated as follows [11] (see also the aftershock risk assessment procedure derived in [10, 19]):

$$P(LS_{as}|MS_i) = \sum_{n_{as}=1}^{N_{as}} P(LS_{as}|MS_i, n_{as}) P(n_{as}|MS_i)$$

where $N_{as}$ be the maximum number of AS events that may take place in the time interval $[0, \tau]$; $P(LS_{as}|MS_i, n_{as})$ is the probability of exceeding the limit state LS for the first time given that exactly $n_{as}$ aftershock events take place, and given the $i$th mainshock $MS_i$; the term $P(n_{as}|MS_i)$ is the conditional probability that exactly $n_{as}$ aftershock events take place, and is expressed herein by a non-homogenous Poisson probability distribution:

$$P(n_{as}|MS_i) = \left(\frac{v_{as,i}}{n_{as}}\right)^{n_{as}} e^{-v_{as,i}}$$

where $v_{as,i}$ is a time-decaying rate based on the Modified Omori (MO) aftershock occurrence model [23], and is equal to the number of aftershocks with magnitude between $M_{as} \leq M_{as} \leq M_{us}$ taken place in time interval $[0, \tau]$ measured with respect to the time of occurrence of the triggering MS, $t_{ms}$, with magnitude $m_i$ ($v_{as,i}$ is both time- and MS magnitude-dependent). This aftershock occurrence rate is calculated based on MO law (see [24] for more details):

$$v_{as,i} = \left(10^{a+b(m_i-M_{as})} - 10^{a+b(m_i-M_{us})}\right) I_0 \cdot I_0 = \left\{\begin{array}{ll}
\frac{(\tau + c)^{1-p} - (c)^{1-p}}{(1-p)} & p \neq 1 \\
\ln\left[(\tau + c)/c\right] & p = 1
\end{array}\right.$$

The parameters $a$, $b$, $c$ and $p$ of the MO model can be derived from a generic territorial model (e.g., [25]) or can be tuned-in to a specific sequence (see e.g., [19], [24], and [25]). The probability $P(LS_{as}|MS_i, n_{as})$ in Eq. (5) can be calculated by taking into account the set of mutu-
ally exclusive and collectively exhaustive (MECE) events that $LS$ first-excursion happens after one and just one of the $n_{as}$ AS events [10, 11]:

$$P(LS_{as} \mid MS, n_{as}) = \sum_{k=1}^{n_{as}} \Pi_{k,j} \cdot \prod_{j=1}^{k-1} (1 - \Pi_{j,k})$$

(8)

where $\Pi_{k,j}$ denotes the probability of $LS$ first-excursion due to the occurrence of the $k$th AS event in the sequence given that the limit state has not been exceeded in the previous $(k-1)$ events, and given the MS wave-form $MS_i$. The probability term $\Pi_{k,j}$ can be calculated as follows [10]:

$$\Pi_{k,j} = \int_{s_{as}} \pi_{k,i}(s_{as}) \cdot d\lambda_{as,i}(s_{as})$$

(9)

where $s_{as}$ is $Sa(T)$ associated with the $k$th aftershock event; $\pi_{k,i} = P(LS_k \mid s_{as}, MS_i)$ is an event-dependent fragility for the $k$th aftershock event (defined as the probability of exceeding the limit state $LS$ due to the $k$th AS event given that it has not been exceeded due to the previous $(k-1)$ AS events given $MS_i$; $\lambda_{as,i}$ is the mean (in units of time $t$) rate of exceeding $Sa(T)$ equal to $s_{as}$ in time interval $[t_{MS}, t_{MS} + \tau]$ given MS wave-form $MS_i$ (for detailed description of aftershock hazard assessment, see [24]).

The term $P(LS_{as} \mid MS, n_{as})$ in Eq. (5) can also be approximated with a closed-form analytic expression (see [11] for the complete derivation of the closed-form expression). A preliminary version of this closed-form expression was proposed in [27-29]. Assume that the set of probability terms $\{\Pi_{k,j} \mid j = 1 : n_{as}\}$ for a given MS wave-form $MS_i$ are identical and equal to the time-invariant function $\Pi$. Thus, the closed-form approximation to the time-dependent limit state probability $P(LS_{as} \mid MS)$ is derived as:

$$P(LS_{as} \mid MS) = 1 - \exp(-\Pi_{k,j}) = 1 - \exp\left(-\int_{s_{as}} \pi(s) \cdot d\lambda_{as,i}(s)\right)$$

(10)

The closed-form expression in Eq. (10) has the same functional form as the expression for limit state probability exceedance in Eq. (2). However, in this case, the prediction time interval for limit state first excursion is equal to the aftershock forecasting interval $[t_{MS}, t_{MS} + \tau]$ and it is implicitly considered in the aftershock hazard term $\lambda_{as,i}$. Arguably, based on the choice of fragility term $\pi(s)$ corresponding to $MS_i$, the closed-form derived in Eq. (10) can lead to different approximate solutions. The expression in Eq. (10) is derived by assuming that the structure is “renewed” to some invariant “average” state right after each aftershock takes place, represented by the fragility term $\pi(s)$. The most obvious choice for $\pi(s)$ is perhaps the fragility of the MS-damaged structure (denoted herein by $\pi_{1}(s_{as})$, i.e., $k=1$) since it is used (with some variations) by several researchers as a proxy to the aftershock vulnerability of a given structure. It is expected that the closed-form estimate to $P(LS_{as} \mid MS)$ in Eq. (10) and based on the MS-damaged fragility would capture to some extent (but not entirely) the damage accumulation potential of the sequence. Another choice for $\pi(s)$ is the fragility of intact structure denoted by $\pi_{0}(s_{as})$. In this case, the closed-from approximation cannot capture the damage accumulation potential of the aftershocks. Nevertheless, and compared to conventional risk estimation considering only the strong motion, Eq. (10) has the advantage of incorporating the time-dependent aftershock hazard.

Substituting $P(LS_{as} \mid MS)$ in Eq. (10) into Eq. (4), the closed-form approximation to $P_{as}$ can be written as:
\[
P_{as} \approx \frac{1}{N} \sum_{M_{si}} \left[ 1 - \exp \left( -\Pi_{i} \nu_{as,i} \right) \right]
\]  

(11)

This term can then be substituted in Eq. (2) in order to derive the first-excursion limit state exceedance probability in time interval \([0, T]\).

2.1 The (time- and history-dependent) structural performance variable

The structural performance variable denoted as \(Y_{LS}^{(k)}\) is adopted ([10, 11, 25]):

\[
Y_{LS}^{(k)} = \frac{D_{\text{max}}^{(k)} - D_{r}^{(k-1)}}{C_{LS} - D_{r}^{(k-1)}}
\]  

(12)

where \(Y_{LS}^{(k)}\) is the critical demand to capacity ratio due to the \(k\)th event for the structure that has already been subjected to the sequence of \((k-1)\) events; \(D_{\text{max}}^{(k)}\) is the maximum demand parameter due to the \(k\)th event; \(D_{r}^{(k-1)}\) is the associated residual demand corresponding to the sequence of \((k-1)\) events; \(C_{LS}\) is the limit state capacity of the (intact) structure. The performance variable \(Y_{LS}^{(k)}\) tends to reflect and to conditionally isolate the effect of the \(k\)th event on the structure.

2.2 Event-dependent fragility assessment

In order to estimate the event-dependent fragility term(s) \(\pi_{0}\) (Eq. 3) and \(\pi_{k,i}, k=1:N_{as}\) (see Eq. 9), a non-linear dynamic analysis procedure known as the Modified Cloud Analysis (MCA, [17]) is adopted (see also [11]). Herein, a sequential version of the MCA (described in a step-by-step manner in [11]) has been adopted in order to mimic the effect of back-to-back ground motion records on the structure. However, the sequential analysis procedure described and implemented in this work is an improved and version of the procedure reported in [10]: it is rendered more robust against extrapolations and it explicitly accounts for the “collapse” cases.

The event-dependent fragility assessment in [10] is performed by simple Cloud Analysis (CA, [16, 17, 30]). Although CA is a simple method to implement, it is subjected to a series of simplifying assumptions. One of these assumptions is that the conditional distribution of the structural performance variable \(Y_{LS}^{(k)}\) given the intensity measure \(s_{ak}\) is described by a Lognormal distribution whose parameters (median and logarithmic standard deviation) are estimated by a linear regression in the logarithmic space of CA pairs \((s_{ak}, Y_{LS}^{(k)})\) for the \(k\)th event. Moreover, it is assumed that the conditional standard deviation in the structural performance variable given intensity is a constant (i.e., does not depend on the intensity level).

Such assumption may lead to inaccurate estimates; especially in cases where the structure experiences global dynamic instability (denoted hereafter as \(C\), manifesting itself as very high global displacement-based demands) due to a certain record or records belonging to the suite of records used for CA. This problem can be addressed by applying the standard CA only to that portion of the suite of records which does not lead to dynamic instability (denoted hereafter as \(NoC\)). The rest of the records can be treated by employing alternative methods, such as, the logistic regression (see [17] for more details on MCA). In such an approach, the event-dependent fragility term \(\pi_{r}\) (note that subscript \(i\) is dropped both for the simplicity of the formulation and also because the treatment of collapse cases is applicable to both intact and damaged structures) can be expanded as follows using MCA procedure (see [17] for the derivation of this expression):
\[ \pi_k = \Phi\left( \frac{\ln (\eta_{Y_{LS|Sa, NoC}}^{(k)})}{\beta_{Y_{LS|Sa, NoC}}^{(k)}} \right) \frac{e^{-(\beta_0 + \beta_1 \ln s_{a_k})}}{1 + e^{-(\beta_0 + \beta_1 \ln s_{a_k})}} + \frac{1}{1 + e^{-(\beta_0 + \beta_1 \ln s_{a_k})}} \]  

(13)

where \( \Phi \) is the standardized Gaussian Cumulative Density Function (CDF); \( \eta_{Y_{LS|Sa, NoC}}^{(k)} \) and \( \beta_{Y_{LS|Sa, NoC}}^{(k)} \) are conditional median of \( Y_{LS}^{(k)} \) and standard deviation (dispersion) of the natural logarithm of \( Y_{LS}^{(k)} \) for the portion of the CA suite of records that does not lead to dynamic instability cases; \( \beta_0 \) and \( \beta_1 \) are the parameters of the logistic regression model for expressing the probability of global dynamic instability. With reference to Eq. (13): (see [10, 11])

\[ \ln (\eta_{Y_{LS|Sa, NoC}}^{(k)} (s_{a_k})) = \ln a_k + b_k \ln (s_{a_k}), \quad \beta_{Y_{LS|Sa, NoC}}^{(k)} = \sqrt{\frac{\sum_{n=1}^{N(k)} \left( \ln Y_{LS,m}^{(k)} - \ln a_k - b_k \ln (s_{a_k,m}) \right)^2}{N(k) - 2}} \]  

(14)

where \( N(k) \) is the number of records for CA and is equal to \( N(k) = N_{\text{cloud}}(k) - N_{\text{C}}(k) \); \( N_{\text{cloud}}(k) \) is the number of ground motion records employed in the CA in order to represent the record-to-record variability in the \( k \)th event (the sample size for different \( k \) levels can be variable); \( N_{\text{C}}(k) \) is the number of records that lead to the global dynamic instability (C or “collapse-cases”). The logistic regression model described in Eq. (13) is applied to all \( N_{\text{cloud}}(k) \) records; they are going to be distinguished by 1 or 0 depending on whether they lead to C or NoC.

3 NUMERICAL APPLICATION

3.1 Case-study structure

The methodology described in Section 2 is applied herein in order to perform risk assessment for MS+AS sequence for a typical partially-infilled moment-resisting RC building located in L’Aquila, central Italy. The case-study structure is a shear building model representing a two-dimensional, 3-story, and 2-bay RC frame with smooth rebars. It is representative of a prevalent class of RC building structures widely constructed between 1940 and 1970 in Italy. The frame is characterized with one-bay infill panel (see Figure 2(a) for the illustration). A nonlinear shear-type building model (assuming that the building mass is lumped at the floor levels and the floor beams are rigid) is constructed in OpenSees (http://opensees.berkeley.edu). Since this model was defined previously by the authors [10, 11], important issues about the modeling and underlying assumptions are briefly described herein. Preliminary analyses on the frame revealed that the damage is mainly accumulated in the first story; hence, in order to simplify the representative mathematical model, the nonlinear behavior is attributed only to the elements located in the first level (considering a lumped plasticity model), while the upper stories are considered to remain elastic. The displacement of the first story is taken to be the corresponding engineering demand parameter herein. Figure 2(b, c) also shows the cyclic response of columns as well as infill in the first story (see also [10] for comprehensive details). For columns, the cyclic response is calibrated against the full scale test on a specimen with dimension and reinforcement details matched with the columns in the case-study structure. The parameters of the Pinching4 Material in OpenSees are chosen to closely match the real hysteretic behavior, as illustrated in Figure 2(c). On the other hand, a simple hysteretic rule with no stiffness degradation is considered for infill panel using the Hysteretic Material in OpenSees library (see [10] for more discussions).

The performance objective for post-earthquake assessment of the aforementioned structure is defined in terms of the discrete limit state of Near Collapse (based on the European standard EC8 [31]). The near-collapse limit state threshold is defined with respect to the pushover...
analysis of the one-bay infill frame. It is conservatively set herein to 10% drop in the ultimate strength of the columns (a maximum of 20% is recommended by EC8 [30]). Figure 2(d) shows the onset of the limit state (referred to as the limit state capacity \( C_{LS} \) in Eq. 12) marked on the pushover curve by red star. The first-mode period of the building is equal to 0.27 sec. To perform nonlinear dynamic analysis, a Rayleigh damping is employed with a critical damping coefficient of 5% for the first two modes of vibration. The cyclic pushover of the structure is also shown in Fig. 2(e).

It is also important to define a threshold defining the Collaps (associated with the global instability, \( C \)) of the structures. A drift ratio of 10% is taken as the threshold of indicating the “collapse” cases (as shown in Figure 2d), which corresponds herein to around 60% reduction in ultimate capacity. It is noteworthy that for concrete frame structures, a 4% drift ratio for severe damage is proposed in [32], which is consistent with the near-collapse limit state threshold herein. Consequently, adopting a 10% drift ratio for the global instability, denoted as \( C \), somehow implies that drift demands beyond this threshold correspond to large deformation ranges.

![Figure 2](image-url)

Figure 2: (a) General configuration of the case-study; (b, c) monotonic and cyclic response of columns and infill panel, (d) pushover curve in terms of the base shear versus displacement of the first story level and the points corresponding to the NC and Collapse limit states, (e) cyclic pushover of the building.

### 3.2 Seismic hazard assessment and ground motion selection

The reference structure is located within the central Italy, Abruzzo region, the town of L’Aquila in the seismic zone 923 based on the ZS9 Italian Seismogenetic Zonation [33]. Fig. 3(a) shows the seismogenic zonation ZS9 with different zones identified on it. The seismic hazard is estimated for the designated site (red triangle in Figure 3b) according to the surrounding seismic zones 918, 919, 920, and 923, separately indicated also in Fig. 3(b). The seismicity rate \( \gamma_{ms} \) used in Eq. (2), is calculated as the sum of the seismicity of individual zones; the lower-bound magnitude \( M_l \) varies from 4.76 (Z920) up to 7.06 (Z923). Based on the seismicity data of each zone, a simplified site-specific probabilistic seismic hazard analysis (PSHA) is performed on the
desired site. The Sabetta and Pugliese attenuation relation (SP96, [34]) is chosen because of its wide use in Italy.

The aftershock hazard is calculated by integrating the adopted ground-motion prediction equation over all possible aftershock magnitudes and distances within the desired aftershock zone (see [19, 24, 35] for more details). The aftershock zone considered herein is the one presented in [10, 24], which is indicated as ZAS with cyan color in Figure 3(b). The aftershock magnitudes are within the range of $4.0 \leq M_{as} \leq 7.06$. The occurrence of aftershocks is modeled by the MO model with the parameter estimated for the Italian generic aftershock sequence based on aftershock events occurring from 1981 to 1996 [26]. Figures 3(c) shows the comparison between seismic hazard considering only MS (long-term seismic hazard $\lambda_{ms}$) and the short-term seismic hazard associated with triggered aftershock events ($\lambda_{as,i}$ for $\tau=1$ day, and MS number $i=14$).

Figure 3: (a) The Italian seismogenic zonation and the site of interest indicated by a pentagram, (b) the four seismogenic zones surrounding the case-study site (defined by red triangle), and the aftershock zone (ZAS) with cyan color, (c) site-specific seismic hazard

A set of 50 European (especially Italian) strong ground motion (MS) records are selected from the NGA-West2 database [36], and listed in [11]. This suite of records covers a wide range of magnitudes between 5.50 and 7.50, and closest distance to ruptured area ($R_{\text{RUP}}$) up to around 80 km. AS record set consists of a set of 43 European (especially Italian) aftershock ground-motion records selected based on the classification of the NGA-West2 database for aftershock records [36], and listed in [11]. This suite of selected records covers the range of low magnitudes between 4.20 and 6.20, and $R_{\text{RUP}}$ up to around 40 km. Both set of ground motions are chosen without emphasizing on detailed record selection.
3.3 Step-by-step procedure for time-dependent risk calculation

To estimate the event-dependent fragilities and the time-dependent limit state probabilities, the sequential MCA is applied herein. While the procedure described in [10] was specifically targeted to MS-damaged structures (conditioned on knowing the MS wave-form), the modified version described herein can be applied to risk assessment in general considering the effect of aftershocks. A step-by-step procedure is described as follows:

**Step (1):** A suite of ground-motion records is selected from the pool of MS strong-motion recordings which represent the MS records. The fragility of the intact structure, $\pi_0$, (Level 0) is calculated by adopting the MCA. In the next step, $P_{ms}$ is calculated from Eq. (3). Figure 4(a) illustrates the Cloud regressions and the associated MS-induced fragility curves, $\pi_0$, for the case-study frame using the set of strong motion (MS) records outlined in [11]. For each scatter Cloud data (colored squares), the corresponding record ID is shown. The red-colored squares indicate the collapse-cases, $C$. The line $Y_{LS}=1$ showing the onset of near-collapse limit state excursion is also indicated on this figure. The fragility curve is calculated by using the expression in Eq. (13) considering the collapse-cases explicitly and plotted as thick black lines. They are compared with the fragility curves calculated from CA considering only the non-collapse cases (dashed lines, see [10, 11]). It can be seen that the explicit consideration of collapse-cases in MCA leads to a slight difference (the fragility shifts to the left which means that the structure becomes more vulnerable).

**Step (2):** The selected suite of MS ground motions is partitioned into two mutually exclusive subsets: those which cause the structure to exceed the limit state (herein, Near Collapse), and the rest of the records that do not lead to limit state excursion. It should be noted that the ground motion records that cause C (collapse-cases) are a subset of records that cause limit state excursion.

**Step (3):** Each one of the records that does not lead to limit state excursion is followed (in a back-to-back manner) with a suite $N_{cloud}(1)$ of records (i.e., $N_{cloud}(1)$ sequences of MS-AS) consisted of AS recordings (selected from the pool of aftershock records) plus those mainshock records that have led to limit state excursion in Step 2. The latter set of records has been included in order to ensure that the Cloud response has few data points with ($Y>1$), so that no extrapolation is necessary. The MS records that have led to limit state excursion are only used for ensuring that the Cloud procedure has enough data points associated with $Y>1$. Note that these records are not going to be used for constructing an (ongoing) AS sequence. This is because any seismic sequence that has one of this records in Level 1 or higher is going to be automatically interrupted (i.e., a record that has caused the intact structure to exceed the limit state is definitely going to lead the damaged structure to exceed the same limit state). This is an effective way of making sure that the CA results are reliable (avoid extrapolations). By adopting MCA, the fragility of MS-damaged structure conditioned on a given MS waveform $MS_i$ (a.k.a. the event-dependent fragility $\pi_{1,i}$, Level 1) is calculated (see Fig. 4b; note that the CA and the associated event-dependent fragilities in Fig. 4 correspond to $MS_{20}$ presented in [11]). The procedure leads to estimating $\Pi_{1,i}$ from Eq. (9) by integrating the MS-damaged fragility and aftershock hazard.

**Step (4):** In the next step, the MS-AS sequences that have not led to limit state exceedance are going to be expanded into $N_{cloud}(2)$ MS-AS-AS sequences (by appending a ground motion record in a back-to-back manner). The AS records are in part permuted from the
suite of AS recordings described in Step (3), and in part from the MS records that have led to limit state excursion defined in Step (2). The event-dependent fragility $\pi_{2,i}$, (Level 2) conditioned on a given MS waveform $MS_i$, is calculated by adopting the MCA (see Fig. 4c). As a result, $\Pi_{2,i}$ can be again calculated from Eq. (9).

**Step (5):** Step 4 is going to be repeated until there is at least one $Y$-value smaller than 1. Following this step, the event-dependent fragility $\pi_{k,i}$, conditioned on a given MS waveform $MS_i$, is calculated by adopting the MCA (see e.g., Fig. 4d for $\pi_{3,20}$, Level 3).

![Cloud Regression for MS Events, Level 0](image1)

![Cloud Regression, Level 1, MS20](image2)

![Cloud Regression, Level 2, MS20](image3)

![Cloud Regression, Level 3, MS20](image4)

Figure 4: The schematic diagram of the sequential MCA procedure, (a) Calculation of the fragility of intact structure, $\pi_0$; (b, c, d) the event-dependent fragilities $\pi_{1,20}$ up to $\pi_{3,20}$ (given $MS_{20}$) for the case-study structure; the AS sequence is constructed based on back-to-back records from the MS record set.
Step (6): The event-dependent limit state probabilities $\Pi_{k,i}$, $k=1:N_{as}$, (estimated through Steps 1-5) are going to be employed in order to calculate the probability $P(L_{Sas}|MS_{i},n_{as})$ from Eq. (8).

Note: The limit state probability $P(L_{Sas}|MS_{i},n_{as})$ (due to aftershocks) given the $MS_{i}$ waveform and $n_{as}$ can be calculated from the recursive formulation in Eq. (8) as a function of $\Pi_{k,i}$ for $k=1:N_{as}$. Calculating $\Pi_{k,i}$ by using Eq. (9) leads to the “best-estimate” limit state excursion probability due to the aftershocks $P_{as}(\tau)$. However, we propose three alternative approximations based on the closed-form approximation derived in Eq. (11) by setting:

(a) $\Pi_{i} = \Pi_{0}$ (the fragility of the intact structure)
(b) $\Pi_{i} = \Pi_{1,i}$ (the fragility of the MS-damaged structure given $MS_{i}$)
(c) $\Pi_{i} = \Pi_{2,i}$ (the fragility of the MS-plus-one-AS-damaged structure given $MS_{i}$)

Step (7): The limit state probability $P(L_{Sas}|MS_{i})$ denoting the probability of exceeding the limit state $LS$ for the first time given $MS_{i}$ is calculated from Eq. (5). With reference to Eq. (5), the conditional probability of having exactly $n_{as}$ aftershock events, $P(n_{as}|MS_{i})$, is estimated by the Poisson probability distribution from Eq. (6).

Step (8): The $P(L_{Sas}|MS_{i})$ values, calculated for each of the MS waveforms that did not lead to limit state excursion in Step (2), are averaged according to Eq. (4) in order to approximate the probability that the first limit state excursion is due AS sequence, $P_{as}$. Figure 5(a) and 5(b) depict the quantities $P_{as}(\tau)$ and $P_{ms}$ as a function of the aftershock forecasting time window $\tau$ elapsed after the MS, where the AS sequence is assembled from the pool of MS records and from the pool of AS records (see [11] for the set of MS and AS records).

Step (9): The $P_{as}$ (from Step 8) and $P_{ms}$ (from Step 1) are substituted in Eq. (2) in order to calculate the probability of exceeding the limit state in time interval $[0,t]$ taking into account also the effect of aftershocks. Figures 5(c) and 5(d) plot the limit state excursion probabilities $P(L_{S})$ in Eq. (2) in time interval $[0,t]$ for a fixed value of aftershock forecasting interval $\tau=7$ day, where the AS sequence is constructed based on the two distinct MS and AS sets of records, respectively.

3.4 Discussion on the Results

In Figure 5(a) and 5(b), slight difference can be noticed in the results obtained based on the two different pools of records. This confirms that construction of AS sequence by back-to-back positioning of strong-motion records might cause overestimation in $P_{as}$ (this is also observed in previous research efforts, see e.g. [5-8]). Nevertheless, this overestimation is not highly significant herein. Next, it can be observed that taking into account the effect of triggered aftershocks (the “best-estimate” represented by the thick black line) changes the limit state probability significantly, as compared with a “classical” risk assessment due to mainshocks only (i.e., $P_{ms}$ represented by dotted red line). In addition, the approximate solution based on $\Pi_{i} = \Pi_{0}$ (plotted as dashed-dot cyan line) leads to significantly higher estimates for $P_{as}(\tau)$ with respect to $P_{ms}$, although it does not consider the damage accumulation due to the aftershock sequence. The reason is that this approximate solution considers the increased short-term seismicity due to the aftershock sequence. The approximate solution based on $\Pi_{i} = \Pi_{1,j}$ and plotted as dashed blue line, that partially manages to capture the damage accumulation due to aftershocks, leads to $P_{as}(\tau)$ estimates closer to those obtained based on the “best-estimate” procedure. Finally, the approximate solution based on $\Pi_{i} = \Pi_{2,j}$ and plotted as grey
thick line, manages to provide estimates for $P_{as}(\tau)$ still closer to those provided by the “best-estimate” procedure.

Figure 5: Comparison between different risk-related metrics by considering only MS and the MS+AS sequence when the AS sequence is generated from

Figures 5(c) and 5(d) plot the limit state excursion probabilities $P(LS)$ in Eq. (2) in time interval $[0,t]$ for a fixed value of aftershock forecasting interval $\tau=7$ day, where the AS sequence is constructed based on the two distinct MS and AS sets of records, respectively. It can be observed that the approximation based on $\Pi=\Pi_1$ (MS-damaged structural fragilities, dashed blue line) does not manage to fully capture the cumulative damage due to the aftershock sequence, although it provides a very good partial estimate. To a much lesser extent, the approximation based on $\Pi=\Pi_0$ (i.e., intact structural fragility, dashed-dot cyan) manages to follow the trend in the limit state probability due to the MS and the triggered AS sequence. Although this approximation does not capture the damage accumulation, it manages to take into account the (time-dependent) increase in short-term seismicity due to the triggered aftershock sequence. The solution based on $\Pi=\Pi_2$ (MS-plus-one-AS-damaged structural fragility, thick grey line) seems to provide an excellent balance between reduced analysis effort (a maximum of three back-to-back events are considered) and accuracy (close match with “best-
estimate” results). These plots also feature the admissible limit state probability level (plotted as a thick green line) taken to be equal to $1-e^{(-0.0021t)}$ (associated with 10% exceedance probability in 50 years). It can be seen that the limit state probability calculated by considering the mainshocks only (plotted as a dotted red-line) is below the admissible level; however, considering the triggered AS sequence (in the “best-estimate” procedure and the three alternative approximations) leads to risk levels that exceed the admissible level. As before, it is observed that the results are sensitive to the selected pool of ground motion records for construction of AS sequence.

4 CONCLUSIONS

- The classical risk assessment based on strong ground-motion (only MS) cannot consider the effect of triggered aftershocks and—under certain conditions—may lead to significant underestimation. In order to consider the sequence of aftershocks in seismic risk assessment, both the increase in short-term seismicity and the potential for damage accumulation should be considered.
- The simplified (closed-form) solution proposed herein with the fragility of intact structure is somehow equivalent to considering the short-term increase in seismicity, without considering the effect of cumulative damage.
- Using the closed-form solution with the MS-damaged fragility manages to consider the short-term increase in seismicity as well as the effect of cumulative damage as some sort of a first-order approximation. The risk estimate improves with respect to MS-only and closed-form based on intact fragility results. However, it still underestimates the risk when compared to the best-estimate results.
- The proposed methodology is implemented using a non-linear dynamic analysis routine known as the Modified Cloud Analysis (MCA). The sequential MCA methodology presented herein, compared to the earlier version presented in [10], has two significant advantages. First, it adopts a simple but effective technique in order to make sure that performance assessment based on the Cloud Analysis procedure avoids extrapolations; second, it can explicitly account for the “collapse” cases where the structure experiences global dynamic instability (i.e., very large global displacement-based demands).

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REFERENCES


RETROSPECTIVE OPERATIONAL AFTERSHOCK FORECASTING FOR 2016 AMATRICE-NORCIA SEISMIC SEQUENCE IN CENTRAL ITALY

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Abstract

The first days elapsed after the occurrence of an earthquake and its triggered aftershocks are crucial in terms of emergency decision-making. To this end, the adopted novel and fully-probabilistic procedure succeeds in providing spatio-temporal predictions of aftershock occurrence in a prescribed forecasting time interval (in the order of hours or days). The procedure aims at exploiting the information provided by the ongoing seismic sequence in quasi-real time. The versatility of the Bayesian inference is exploited to adaptively update the forecasts based on the incoming information as it becomes available. The aftershock clustering in space and time is modelled based on an Epidemic Type Aftershock Sequence (ETAS) model. One of the main novelties of the proposed procedure is that it considers the uncertainties in the aftershock occurrence model and its model parameters. This is done by pairing up the Bayesian robust reliability framework and the suitable simulation schemes (Markov Chain Monte Carlo Simulation) provides the possibility of performing the whole forecasting procedure with minimum (or no) need of human interference. This procedure is demonstrated through a retrospective spatio-temporal early forecasting of seismicity associated with the 2016 Amatrice-Norcia seismic sequence in central Italy. Seismicity forecasts are issued with various time intervals in the first few days after the main events within the sequence.

Keywords: Time-dependent reliability, Aftershock sequence, ETAS, Markov Chain Monte Carlo Simulation, Operational aftershock forecasting, Central Italy seismic sequence.
1 INTRODUCTION

Short-term operational seismicity forecasts (in the order of days to months), in the presence of a vast number of aftershocks following a large earthquake, are of utmost importance for emergency decision-making and risk mitigation in the disaster area [[1]-4]. The Epidemic Type Aftershock Sequence (ETAS) model [5, 6] is the stochastic model most frequently used to describe earthquake occurrence within a seismic sequence. It is an epidemic stochastic point process in which every earthquake within the sequence is a potential triggering event for subsequent earthquakes, and therefore generates its own well-defined Modified Omori [7] (MO) aftershock decay. Hence, it is capable of accounting for the triggering effect of all the events that have taken place before a desired time. The ETAS model performed quite well in operational seismic forecasting during the L’Aquila 2009 (central Italy) seismic sequence [8]. The model parameters are usually calibrated a priori based on the maximum likelihood criterion. The first effort on the calibration of temporal model parameters has been carried in [5], and extended later [6, 9, 10] to estimate the spatio-temporal model parameters. In addition, several attempts are made for developing improved algorithms to attain maximum likelihood estimates of ETAS parameters [11-13]. Adaptive model parameter estimation based on the events in the ongoing sequence (e.g., calibrating the parameters of MO and ETAS models based on the ongoing catalogue by employing Bayesian parameter estimation [14-18]) has the advantage of both tuning a sequence-specific model, and also capturing possible time variations of the model parameters. As the original purpose of the present paper, we propose a fully simulation-based method to provide a robust estimate [16, 18] for the spatial distribution of the events in a prescribed forecasting time interval after the main event. In the context of this robust estimate, the uncertainty in the ETAS model parameters is taken into account as the posterior joint probability distribution for the model parameters conditioned on the events that have already occurred (i.e., registered events in the ongoing seismic sequence before the beginning of the forecasting interval). The Markov Chain Monte Carlo (MCMC) simulation scheme [16-19] is used to sample directly from the posterior probability distribution for ETAS model parameters (i.e., conditioned on the registered events in the ongoing sequence). Moreover, this robust estimate also considers the sequence of events that is going to occur during the forecasting interval (and hence affect the seismicity in an epidemic type model like ETAS). Although this sequence is unknown at the time of forecasting, we propose a stochastic procedure to generate it. The procedure leads to the stochastic spatio-temporal distribution of the forecasted events and consequently to the uncertainty in the estimated number of events, corresponding to a given forecasting interval. The resulting robust forecasts are directly applicable in adaptive daily aftershock hazard and risk assessment procedures [15, 20-22].

The proposed methodology is applied to provide retrospective early forecasting of seismicity associated with the 2016 Amatrice seismic sequence activities in central Italy. Robust spatio-temporal short-term seismicity forecasts are provided with various time intervals in the first few days elapsed after main events within the sequence, which can predict the seismicity within plus/minus two standard deviations from the mean estimate within the few hours elapsed after the main event.

2 METHODOLOGY

The aftershock occurrence is described herein by a non-homogenous Poisson point process over the two-dimensional space and time. Hence, the aftershock zone can be described as the set \( A \) in the Cartesian space discretized into mutually exclusive and collectively exhaustive (MECE) subsets or spatial cell units centered at \((x,y)\in A\). In this manner, \( \lambda(t,x,y,m|\text{seq},Ml) \) represents the rate of occurrence of events in the forecasting interval \([T_{\text{start}}, T_{\text{end}}]\) at time \( t\).
elapsed after the main event (a.k.a. main-shock) occurred at time of origin $t_0$ with magnitude greater than or equal to $m$ and in the cell unit centered at $(x,y) \in A$, given (a) the observation history $\text{seq}$ which is the sequence of $N_o$ events (including main-shock and the sequence of aftershocks) taken place before the forecasting interval (i.e. in the interval $[T_{start} , T_{end})$), and (b) the lower cut-off magnitude $M_l$. Hence, $\text{seq}$ can be expressed as $\text{seq} = \{(t_i,x_i,y_i,m_i), t_i < T_{start}, m_i \geq M_l, i=1:N_o\}$, where $t_i$ is the arrival time for the $i$th event with magnitude $M_i$ and location $(x_i,y_i) \in A$. The average number of events in the spatial cell unit centered at $(x,y)$ with magnitude greater than or equal to $m$ in the forecasting interval $[T_{start},T_{end}]$ can then be calculated as:

$$N(x,y,m|\text{seq},M_i) = N_h(x,y,m|M_i) + \int_{T_{start}}^{T_{end}} \lambda(t,x,y,m|\text{seq},M_i) \, dt$$  \hspace{1cm} (1)$$

where $N_h(x,y,m|M_i)$ is a constant representing the background seismicity of the area. Let $\theta$ denote the vector of model parameters for $\lambda(t,x,y,m|\text{seq},M_i)$. Given a particular space-time model and a realization of the vector of model parameters $\theta$, one can calculate a plausible value for the rate of occurrence denoted as $\lambda(t,x,y,m|\theta,\text{seq},M_i)$. A robust estimate $[16, 18, 19, 23]$ of the average number of events in the spatial cell unit centered at $(x,y)$ with magnitude greater than or equal to $m$ in the forecasting interval $[T_{start},T_{end}]$, and over the domain of the model parameters $\Omega_\theta$ can be calculated as:

$$\mathbb{E}[N(x,y,m|\text{seq},M_i)] = N_h(x,y,m|M_i) + \int_{\Omega_\theta} \int_{T_{start}}^{T_{end}} \lambda(t,x,y,m|\theta,\text{seq},M_i) \cdot p(\theta|\text{seq},M_i) \, dt \, d\theta$$  \hspace{1cm} (2)$$

where $p(\theta|\text{seq},M_i)$ is the conditional probability distribution function (PDF) for $\theta$ given the $\text{seq}$ and the lower cut-off magnitude $M_i$.

As mentioned above, $\text{seq}$ denotes the sequence of events taking place before the beginning of the forecasting interval (i.e., in the interval $[T_{start} , T_{end})$). However, the triggering effect of the events taking place during the forecasting interval $[T_{start},T_{end}]$ is expected to play a major role. The sequence of events taking place during the forecasting interval denoted as $\text{seqg}$, which is unknown at the time of forecasts, is simulated/generated herein. Let us assume that a plausible $\text{seqg}$ is defined as the events within the forecasting interval defined as $\text{seqg}=\{(IAT_i,x_i,y_i,m_i), T_{start}\leq t_i \leq T_{end}, m_i \geq M_i\}$, where $IAT_i=t_i-t_{i-1}$ stands for the inter-arrival time. The robust estimate for the number of aftershock events in Eq. (2) should also consider all the plausible sequences of events $\text{seqg}$ (i.e., the domain $\Omega_{\text{seqg}}$) that can happen during the forecasting time interval:

$$N(x,y,m|\text{seqg},M_i) = N_h(x,y,m|M_i) + \int_{\Omega_{\text{seqg}}} \int_{T_{start}}^{T_{end}} \lambda(t,x,y,m|\text{seqg},\theta,\text{seqg},M_i) \, dt \cdot p(\text{seqg}|\theta,\text{seqg},M_i) \, d\text{seqg} \int_{\Omega_\theta} \int_{T_{start}}^{T_{end}} \lambda(t,x,y,m|\theta,\text{seqg},\text{seqg},M_i) \, dt \cdot p(\theta|\text{seqg},M_i) \, d\theta$$  \hspace{1cm} (3)$$

where $p(\text{seqg}|\theta,\text{seqg},M_i)$ is the PDF for the generated sequence $\text{seqg}$ given that $\theta$ and $\text{seq}$ are known and $\lambda(t,x,y,m|\text{seqg},\theta,\text{seqg},M_i)$ is the space-time clustering model. Herein, we have employed the space-time clustering ETAS model considering also the sequence of events taking place within the forecasting interval.

The ETAS model is an epidemiological stochastic point process in which every earthquake is a potential triggering event for subsequent earthquakes $[5, 6, 9]$. Errore. L'origine riferi-
According to the general ETAS model, we adopt the spatio-temporal triggering effect of a given sequence on the seismicity rate, denoted as $\lambda_{\text{ETAS}}$, as:

$$
\lambda_{\text{ETAS}}(t, x, y, m|\theta, \text{seq}, M_j) = e^{-\beta(m-M_{j})} \sum_{j<t} K e^{\beta(M_{j}-M_{l})} \frac{K_j}{(t-t_j + c)^p} \frac{K_R}{(r_j^2 + d^2)^q}
$$

where $\text{seq} = \{(t_j, x_j, y_j, M_j), t_j < t, M_j \geq M_l\}$ is the observation history up to the time $t$; parameter $\beta$ is related to Gutenberg-Richter seismicity; parameters $c$ and $p$ are similar to those of the Modified Omori’s Law [7] defining the decay in time of short-term triggering effect; $d$ and $q$ characterize the spatial distribution of the triggered events; $r_j$ is the distance between the location $(x, y) \in A$ and the epicenter of the $j$th event $(x_j, y_j)$; parameters $K$, $K_t$, and $K_R$ satisfy the achievement of asymptotic compatibility between ETAS predictions and the long-term seismicity (see [18] for a comprehensive discussion on satisfying the compatibility condition for the parameters $K$, $K_t$, and $K_R$).

Thus, the vector of model parameters $\theta$ can be defined as $\theta = [\beta, K, K_t, K_R, c, p, d, q]$. It is to note that parameters $K$, $K_t$, and $K_R$ are derived as a function of other model parameters in $\theta$; therefore, the main parameters of the ETAS model include $[\beta, c, p, d, q]$. The rate of events in the ETAS model with magnitude (exactly) equal to $m$, denoted herein as $\mu_{\text{ETAS}}$, is calculated by taking the derivative of Eq. (4) with respect to magnitude $m$:

$$
\mu_{\text{ETAS}}(t, x, y, m|\theta, \text{seq}, M_j) = \beta e^{-\beta(m-M_{j})} \lambda_{\text{ETAS}}(t, x, y, M_j|\theta, \text{seq}, M_j)
$$

The integral with respect to time in Eq. (3) cannot be calculated analytically over the entire interval $[T_{\text{start}}, T_{\text{end}}]$, and is approximated by summing over the sub-intervals $[t_{i-1}, t_i]$ within $\text{seq}$:

$$
\int_{T_{\text{start}}}^{T_{\text{end}}} \lambda(t, x, y, m|\text{seq}, \theta, \text{seq}, M_j) dt = \sum_{\Omega_{\text{seq}}} \int_{t_{i-1}}^{t_i} \lambda_{\text{ETAS}}(t, x, y, m|\text{seq}, \theta, \text{seq}, M_j) dt
$$

$$
= \sum_{\Omega_{\text{seq}}} \left( KK_t K_R e^{-\beta(m-M_{j})} \sum_{j<t_i} \frac{e^{\beta(M_{j}-M_{l})} I_0(t_i, t_{i-1}, t_j)}{(r_j^2 + d^2)^q} \right)
$$

where $\lambda_{\text{ETAS}}$ has the functional form presented in Eq. (4), and $\text{seq}_{i-1}$ is the previous $(i-1)$ events within the generated sequence. In the following sections, it is described first how sequence of events $\text{seq}$ for the forecasting interval is sampled based on $p(\text{seq}|\theta, \text{seq}, M_i)$. Later on, the method for sampling $\theta$ from the distribution $p(\theta|\text{seq}, M_i)$ is discussed.

### 2.1 Generating sequences according to $p(\text{seq}|\theta, \text{seq}, M_i)$

The probability distribution $p(\text{seq}|\theta, \text{seq}, M_i)$ in Eq. (3) can be written as follows [18]:

$$
p(\text{seq}|\theta, \text{seq}, M_i) = \prod_i p(IAT_i, x_i, y_i, M_i|\text{seq}_{i-1}, \theta, \text{seq}, M_i)
$$

where $\text{seq}_i$ is the generated sequence up to the $i$th event, where $\text{seq} = \{\text{seq}_{i-1}, (IAT_i, x_i, y_i, m_i)\}$, and the sequence of events that precede the $i$th generated event is $\{\text{seq}_{i-1}\}$. The probability distribution $p(IAT_i, x_i, y_i, m_i|\text{seq}_{i-1}, \theta, \text{seq}, M_i)$ can be further expanded (again using the probability product rule) as follows:
where \( p(m_i | \text{seq}_i^{-1}, \theta, \text{seq}, M) \) is the marginal PDF for the magnitude \( m_i \) given the sequence of events that precede it, \( \theta \), and \( M \); \( p(IAT | m_i, \text{seq}_i^{-1}, \theta, \text{seq}, M) \) is the (conditional) marginal PDF for inter-arrival time \( IAT \) given that the value of magnitude is equal to \( m_i \); finally, the term \( p(x, y | IAT, m_i, \text{seq}_i^{-1}, \theta, \text{seq}, M) \) is the conditional joint PDF for the spatial position \((x, y)\in A\) given that \( IAT \) and \( m_i \) are known. It should be noted that the break-down into the product of several conditional PDFs is necessary during the sequence generation process.

To generate a plausible sequence of events during the forecasting interval, the procedure, illustrated by the flowchart in Fig. 1 is adopted (for a comprehensive discussion on generating the \( i \)th event within the sequence \( \text{seq}_g \), see the steps discussed in [18]).

### 2.2 Sampling \( \theta \) from the distribution \( p(\theta | \text{seq}, M) \)

The probability distribution \( p(\theta | \text{seq}, M) \) in Eq. (3) can be calculated using Bayesian parameter estimation:

\[
p(\theta | \text{seq}, M) = \pi(\theta | \text{seq}, M)
\]
where \( p(\text{seq}|\theta, M_l) \) denotes the likelihood of the observed sequence given the vector of model parameters \( \theta \) and lower cut-off magnitude \( M_l \), \( p(\theta|M_l) \) is the prior distribution for the vector \( \theta \), and \( C^{-1} \) is a normalizing constant. In lieu of additional information (e.g., statistics of regional model parameters), the prior joint distribution \( p(\theta|M_l) \) can be estimated as the product of marginal uniform probability distributions for each model parameter. In order to sample from \( p(\theta|\text{seq}, M_l) \), Markov Chain Monte Carlo (MCMC) simulation routine is employed which is particularly useful for cases where the sampling needs to be done from a probability distribution that is known up to a constant value [19] (herein, \( C^{-1} \)). The MCMC routine uses the Metropolis-Hastings (MH) algorithm [24, 25] in order to generate samples as a Markov Chain sequence used first to sample from the target probability distribution \( p(\theta|\text{seq}, M_l) \), and later to estimate the robust reliability in equation (3). The MH routine is shown in Fig. 2 (see [18] for complete discussion on the MH algorithm).

The likelihood for the observed sequence \( p(\text{seq}|\theta, M_l) \), where \( \text{seq} = \{(t_i, x_i, y_i, m_i), t_i < T_{\text{start}}, m_i \geq M_l, i = 1, N_o \} \), with \( N_o \) events, including the main-shock (with \( i = 1 \)) and the sequence of aftershocks is derived in [18].
3 CASE-STUDY APPLICATION

3.1 Amatrice-Norcia 2016 seismic sequence

The proposed methodology presented in Section 2 is applied to provide retrospective forecasting for seismic activities of the 2016 Amatrice-Norcia sequence by analysing the registered data of quasi real-time catalogues from INGV (Istituto Nazionale di Geofisica e Vulcanologia). The corresponding aftershock zone, as shown in Fig. 3a by the gray-colored area, is located mostly within the seismic zone 923 based on the ZS9 Italian Seismogenetic Zonation [26]. Fig. 3a shows also the seismogenic zonations surrounding the aftershock zone. It is to note that based on the Italian seismic zonations, the upper-bound magnitude for seismic zone Z923 is $M_{\text{max}}=7.06$. On the 24th of August 2016 at 01:36 UTC, a Mw 6.0 earthquake struck the Central Italy between towns of Norcia and Amatrice, devastating Amatrice, Accumoli and several surrounding small towns and villages, causing almost 300 fatalities and leaving almost 30,000 homeless. The seismic sequence, including a Mw 5.4 aftershock (occurred almost one hour after the main shock at 02:33 UTC), triggered hundreds of earthquakes per day until the mid-September. Two months after, on the 26th of October, a Mw 5.4 followed within a two-hour delay by a Mw 5.9 earthquake (at 17:10 and 19:18 UTC, respectively) took place in the east of town Visso, and preceded the largest event of the sequence, a Mw 6.5 on October 30 at 06:40 UTC, North of Norcia. This one is the largest earthquake recorded in Italy since the Mw 6.9 1980 Irpinia event. Fig. 3b and Fig. 3c illustrate the seismic activities within the aftershock zone during the first two months highlighting the key events taken place. The present study strives to perform robust forecasts for the spatio-temporal evolution of the events in specific time intervals within the very complex sequence described above that is distinguished by three main events (“main-shocks”) of moment magnitudes 6.0, 5.9, and 6.5, respectively (as illustrated in Fig. 3b). We divide the sequence into three parts: (a) from 24-August to 25-October, (b) from 26-October to 29-October, and finally (c) from 30-October to 1-November. We have used two different catalogues herein in order to gather data backwards in time (see [18]).

3.2 Daily forecasts of seismicity from August 24 up to October 25

Figure 4 shows the forecasted seismicity maps in terms of the mean plus two logarithmic standard deviation (98% confidence interval) for the number of events with $M\geq3.0$ within each spatial cell unit issued for the 24-hour time forecasting intervals. The prediction time window $[T_{\text{start}}, T_{\text{end}}]$ indicates a 24-hour interval where $T_{\text{start}}$ is 6:00 UTC of the following day. The sequence $\text{seq}$ comprised of events registered in the Catalogue including the main event up to the time $T_{\text{start}}$. To issue the first forecast in Fig. 4a, the observation history, $\text{seq}$, comprises the main event with Mw 6.0 at 01:36 UTC and the triggered events up to 6:00 UTC of 24 August 2016, where the lower cut-off magnitude, $M_l$, of Catalogue is equal to 3.0 based on the two methods discussed in [15] (the procedure adopted for evaluating the completeness magnitude $M_c$ throughout the various phases of this multiple seismic sequence is described in detail in [18]). In any case, $M_l=3.0$ is considered as the cut-off threshold for the computation of the aftershock rates for the upcoming days.

The first step towards providing seismicity forecasts (with reference to Eq. 3) is sampling from the distribution of modal parameters $\theta$ based on posterior (target) probability distribution $p(\theta|\text{seq}, M_l)$. The vector $\theta=[\beta, K, K_r, c, p, d, q]$ is updated on a daily basis by applying the Bayesian updating routine illustrated in Eq. (9) and considering that parameters $K, K_r, K_e$ are derived as function of other parameters within vector $\theta$ (see [18]). Samples for $\theta$ are generated as a Markov Chain sequence directly from $p(\theta|\text{seq}, M_l)$, as noted in Section 2.2.
Figure 3: Amatrice-Norcia 2016 seismic sequence: (a) The aftershock zone indicated by the grey-coloured box in perspective with the surrounding Italian seismogenic zonation; (b) The spatial distribution of aftershock events based on Catalogue from August 24, 2016 (01:36 UTC) up to November 2, 2016 (10:32 UTC) bordering four neighbouring provinces in Italy. The grey-coloured box defines the considered aftershock zone and the most damaged towns are highlighted with green boxes. The main seismic events are illustrated as follows: M6.0 and M6.5 with red stars; M5.4 (24/08/16), M5.4 (26/10/16), M5.9 (26/10/16) with magenta triangles; aftershocks M≥3.0 with grey circles. (c) The number of events (with M≥3.0) in Catalogue occurred within a 24-hour interval starting from 6:00 UTC of the desired day.

The MCMC procedure for updating the model parameters is carried out adaptively (the evolution in the statistics (mean and COV) of model parameters $\theta=[\beta, K, K_R, c, p, d, q]$ are illustrated and discussed in [18]). The earthquakes of interest occurred within the correspond-$
The two main events of the sequence with $M \geq 5.0$ (see also Figure 3) are identified with coloured stars (these events are shown for reference only and they did not necessarily take place in the illustrated map’s corresponding forecasting interval). We also report the forecasted daily probabilities of having earthquakes of magnitude equal to or larger than $m=4, 5$ and $6$ in the whole aftershock zone (see [18]).

At the right-hand side of each sub-figure, the observed (shown as a red star) vs. forecasted number of events (shown in an error-bar format) is illustrated for events with $M=3.0$ for the entire aftershock zone. The error-bar for the forecasted number of events features: the median value (the 50th percentile, equivalent of the logarithmic mean in the arithmetic scale) inside a grey-filled square, the (logarithmic) mean plus/minus one (logarithmic) standard deviation indicating the interval between 16th and 84th percentiles (marked with blue horizontal lines), and the (logarithmic) mean plus/minus two (logarithmic) standard deviations indicating the interval between 2nd and 98th percentiles (marked with black horizontal lines). This is done to help in locating the observed number of events within plus or minus certain number of standard deviations from the mean estimate. It can be seen that the observed number of events lies within plus/minus one standard deviation of the mean estimate.

Figure 4: Forecasted vs. observed seismicity distribution in the aftershock zone after the event with Mw 6.0 at 01:36 UTC of 24/08/2016; the maps report the mean + 2 standard deviation confidence interval for the number of events per [km$^2$] (latitude/longitude cells of a 0.01° × 0.01° grid) equal to or greater than magnitude $M=3$ in the indicated 24-hour forecasting time window. The sub-figures illustrate the observed (plotted in red star) vs. the error-bar for the forecasted number of events with $m \geq M$ corresponding to the forecasting time interval: the median value (the 50th percentile, equivalent of the logarithmic mean in the arithmetic scale) inside a grey-filled square, the (logarithmic) mean plus/minus one (logarithmic) standard deviation indicating the interval between 16th and 84th percentiles (marked with blue horizontal lines), and the (logarithmic) mean plus/minus two (logarithmic) standard deviations indicating the interval between 2nd and 98th percentiles (marked with black horizontal lines).
3.3 Daily forecasts of seismicity from October 26 up to October 29

As mentioned before, on 26th of October, a Mw 5.4, followed within a two-hour delay by a Mw 5.9 earthquake (at 17:10 and 19:18 UTC, respectively), took place in the east of town Visso (located in the north-western part of the aftershock zone, see Figure 1a). This triggered a new aftershock sequence within the ongoing one. At this stage, given the time elapsed from the occurrence of the mainshock (i.e., around two months), it seemed quite tedious to consider all the events of interest up to the time of origin (i.e., 24th of August) for each forecasting interval. To achieve this (i.e., avoid considering all the events back to 24th of August), we performed a shift in the time of origin $T_o$ from August 24th to 17:10 UTC of October 26th (time of occurrence of the Mw 5.4 earthquake, see Figure 3). After the occurrence of the event with Mw 5.4 at 17:10 UTC of 26/10/2016, we provide a 6-hour prediction of seismicity for the forecasting interval starting from $T_{start}$ set to 18:00 UTC of 26/10/2016 (i.e., 50 minutes after the occurrence of Mw 5.4 event). At this point, we performed a shift in the time of origin by setting $T_o$ to 17:10 UTC of 26th of October (see Figure 4).

The sequence seq includes all the triggered events with $M \geq 3$ occurred after 17:10 UTC of 26/10/2016 (including the main Mw 5.4 event). It should be noted that the event Mw 5.4 was not preceded by any foreshocks (i.e., no $M \geq 3$ events took place between 06:00 UTC and 17:10 UTC of 26 of October). Given the very low seismic activity prior to the major event and given the presence of very few events in seq, we did not perform Bayesian updating on the model parameter $\theta$ and used the statistics issued for 26/10/2016 (see [18]). It is important to note that the forecasted seismicity for the 24-hour interval elapsed after 06:00 UTC of October 26 is used herein (after proportioning it to a 6-hour forecasting interval) as the background seismicity $N_b(x,y,m|M_l)$. The background seismicity usually considers the long-term seismicity in the calculations and was assumed to be equal to zero in our previous calculations for the first part of the sequence staring from 24th of August. Herein, we use this background seismicity to conservatively approximate the triggering effect of the events occurred in the first part of the sequence (from August 24th to October 26th). The background seismicity is added as a constant term to the contribution of the triggering events (see Section 2). The forecasted seismicity map in terms of the mean plus two standard deviation for the number of events with $M \geq 3.0$ is shown in Fig. 6a. Observed events with $M \geq 3.0$ (coloured dots) occurred within the corresponding 6-hour forecasting interval are also highlighted on the map. The main two events with Mw 5.4 at 17:10 UTC assigned as the main-shock and the Mw 5.9 event at 19:18 UTC (which lies within the 6-hour forecasting interval) are shown with magenta stars. According to the right-hand side error-bar plot of Fig. 6a, the total number of registered events within the 6-hour forecasting interval (red star) is significantly higher than the forecasted values. This can be attributed to very few number of observed input data in seq for preforming the robust estimation and to the fact that model parameters were not tuned to the
a newly triggered sequence. Although less successful in predicting the number of events, the model predicts exceedance probabilities $P(M \geq 5)$ and $P(M \geq 6)$ to be very high.

After the occurrence of the event with $Mw$ 5.9 at 19:18 UTC, the seismicity forecast is provided in Fig. 6b again for the interval starting from 20:00 UTC (42 minutes after the $Mw$ 5.9 event) up to 24:00 UTC of 26/10/2016 (i.e., a 4-hour time interval). The corresponding $\text{seq}$ includes all the events with $M \geq 3.0$ which occurred (including the main event of $Mw$ 5.4 at 17:10 UTC and $Mw$ 5.9 at 19:18 UTC) after 17:10 UTC up to the starting time (20:00 UTC) of October 26. The model parameters $\theta$ are updated based on the information provided by the sequence $\text{seq}$ with $M_l$ set to 2.5 (see [18]). Note that the cut-off magnitude lower than 3.0 is assigned only for model updating purposes to gain more data and the seismicity rate is later calculated with $M_l=3.0$. Fig. 6b illustrates the forecasted seismicity map in terms of the mean plus two standard deviation for the number of events with $M \geq 3.0$ within the 4-hour forecasting interval. Note that for the 4-hour time interval, the exceedance probabilities $P(M \geq 5)$ and $P(M \geq 6)$ increase in Fig. 6b after the occurrence of the event with $Mw$ 5.9 at 19:18 UTC (compared to Fig. 6a). In addition, the observed number of events within the 4-hour time interval (sub-figure) lies within the plus/minus one standard deviation confidence interval.

Figure 6: Forecasted vs. observed seismicity distribution in the aftershock zone after the event with $Mw$ 5.4 at 17:10 UTC of 26/10/2016

3.4 Daily forecasts of seismicity from October 30 up to November 1

As mentioned before, on 30th of October, a $Mw$ 6.5 event occurred in the North of Norcia at 6:40 UTC (located in the north-western part of the aftershock zone, see Figure 3). The first forecasting is performed for the same day of 30/10/2016 with $T_{\text{start}}$ set to 12:00 UTC and $T_{\text{end}}$ set to 06:00 UTC of 31/10/2016 (i.e., 18-hour interval). At this stage, we performed a shift in the time of origin $T_o$ from 17:10 UTC of 26th of October to 6:40 UTC of 30th of October. The background seismicity $N_b(x,y,m|M_l)$ is set (and proportioned to an 18-hour interval) to that of 30th of October for a 24-hour interval with starting time set to 6:00 UTC (see [18]). Fig. 7a
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shows the map of forecasted seismicity with the back-drop of events occurred in this interval. The error-bar plot for the forecasted number of events manages to capture the observed number of events within one standard deviation confidence interval.

To measure the effect of the shift in the time of origin, the same forecasting presented in Fig. 7a is performed with time of origin set to 17:40 UTC of 26th of October. Fig. 7b shows the forecasted map of seismicity and the error-bar for the predicted number of events. The forecasted number of events are slightly lower than those predicted in the previous step in Figure 5c (after shifting the time of origin). This is to be expected since the latter forecast employs a time-invariant background seismicity to consider the events of interest occurred in the time interval between 17:10 UTC of 26th of October and 6.40 UTC of 30th of October. This is while the former forecast explicitly considers the triggering contribution of these events and the associated time-decay. Overall, it is reassuring to note that the two forecasts provide essentially the same information.

3.5 Discussion on the results

It is observed that after an initial transition time (in the order of few hours, enough to accumulate enough events for updating the model parameters), the model quickly tunes into the sequence and provides forecasting that is reliable in most cases up to plus/minus one standard deviations. As expected, the procedure falls short of predicting the “main-shock” of 24th of October (17:10 UTC) as it happened when the sequence had decayed. The procedure, however, did a better job for forecasting the events occurred at 19:18 UTC of 26th of October and on 6.40 UTC of 30th October. This relative success can be attributed to the fact that these events took place at the initial stages of the newly triggered sequence of 26th of October when the seismic activity was still very high. The estimated model parameters present some time-dependent fluctuations but after a certain number of days elapsed after the main event, they

Figure 7: Forecasted vs. observed seismicity distribution in the aftershock zone after the event with Mw 6.5 at 6:40 UTC of 30/10/2016
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... seem to stabilize. In general, the first sub-sequence (Mw 6 “mainshock” occurred at 1:36 UTC 24th of August) seems to be the mildest one in terms of the time decay in seismicity and is the least active in terms of sequence’s productivity. The second sub-sequence (Mw 5.4 “mainshock” occurred at 17:10 UTC 26th of October) is intermediate both in terms of the rate of time-decay and the productivity. The third sub-sequence (Mw 6.5 “mainshock” occurred at 6:40 UTC 30th of October) has the steepest time-decay of seismicity and is the most active in terms of the productivity of the sequence.

4 CONCLUSIONS

- A fully simulation-based procedure is proposed for (a) Bayesian model updating of an epidemiological-type aftershock spatio-temporal clustering model; (b) robust operational forecasting of the number of events of interest expected to happen in each time frame.
- The robust forecasting considers the uncertainty (i.e., the joint probability distribution) in the model parameters.
- The model updating and forecasting procedure is carried on without human interference and use of expert judgement. The model is simply “tuning-in” automatically into the sequence of observed events.
- The choice of the recent Central Italy sequence of events as a demonstration of this procedure proved to be very challenging. This is because the sequence embedded three “sub-sequences” with different productive and decaying properties. We used the peculiarities of this sequence to test several different strategies for forecasting.
- To perform early forecast within an ongoing seismic sequence, a shift in the time of origin of the sequence is proposed by conservatively introducing a constant background seismicity (calculated by the procedure). This shift proved to be quite useful as it relieved us from the burden of summing up the triggering properties of all the events that took place in the previous “sub-sequence” (or the previous part of the sequence as we may wish to call it) at the small price of neglecting the time-decay in their triggering contribution.
- The proposed procedure for robust forecasting is conditioned on the available catalogue of events and the epidemiological model adopted for capturing the spatio-temporal aftershock clustering.

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REFERENCES


EMPIRICAL FRAGILITY CURVES BASED ON RANDOM GROUND SHAKING FIELDS: EMPLOYING COPERNICUS-EMS DAMAGE GRADING MAPS FOR 2016 AMATRICE SEQUENCE

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Abstract

Significant structural and non- structural damage suffered by residential buildings due to recent earthquakes world-wide provide a sad testimony to the vulnerability of the existing building stock. This evidence stresses the need to develop accurate seismic vulnerability assessment tools in the service of strategic planning and applicable at a territorial scale. The empirical fragility curves have been used quite often in order to characterize the vulnerability of prescribed buildings classes as opposed to individual buildings. In this approach, the fragility curve represents both the building-to-building variability within the class and the uncertainty in the building class response to future earthquakes. Development of empirical fragility curves for classes of buildings depends to a large extent on the pairs of seismic ground shaking intensity and observed damage data gathered from spatially distributed buildings. The seismic intensity experienced at the site of each building is usually estimated by employing maps of ground shaking for the earthquake of interest (a.k.a., shakeMaps). The accuracy in the estimation of the ground shaking intensity is arguably as important as that of the evaluation of the incurred damage. The former involves explicit characterization of the uncertainties in the ground-shaking intensity estimation. This can be achieved through application of ground motion prediction equations and the corresponding spatial correlation structure in the residuals (of the predictive equation), resulting in generation of random fields of ground shaking. Each field represents a plausible distribution of ground motion intensity for a given earthquake scenario. Then, the random ground shaking field can be updated based on available observations of ground shaking intensity from nearby stations. The generated random fields can be further modified to consider the local stratigraphic and topographic amplification. For a given damage pattern observed for a prescribed building class, the generation of random ground motion fields leads to a set of plausible empirical fragility curves (corresponding to exceedance of different damage states). The fragility curves and their 16th, 50th and 84th percentiles, obtained based on the generated random ground shaking fields and observed damage in the aftermath of Amatrice 2016 Earthquake (based on Copernicus EMS...
damage maps) show reasonable agreement with the empirical fragility curves available in the literature for similar buildings classes.

**Keywords:** Vulnerability assessment, empirical fragility curves, masonry buildings remote sensing, visual survey.

## 1 INTRODUCTION

Accurate assessment of seismic risk for buildings at a territorial scale depends to a large extent on the availability of reliable and accurate fragility curves. The seismic fragility curves can be distinguished, based on the type of data used for deriving them, into four categories [1]; namely, analytic [2-3], empirical, based on expert opinion, and hybrid [4]. The empirical fragility curves can provide a realistic picture of post-earthquake damage. The shortcomings related to empirical fragility assessment are associated to the difficulties in creating a homogenous class of buildings, inaccurate estimation of seismic ground shaking, site effects and the observed damage [5-6]. One essential requirement for derivation of useful empirical fragility curves is to refer to a standardized damage scale. The European Microseismic Scale (EMS-98, [7]) is a refined example of a standard damage scale; very often used to report the empirical fragility curves in Europe. With respect to Italian masonry buildings, Lagomarsino and Giovinazzi [8] have derived vulnerability curves reporting the mean damage ratio versus seismic intensity expressed in the EMS-98 scale. Zuccaro and Cacace [9] proposed a methodology to reduce the variability in the vulnerability classification of EMS-98 through the application of vulnerability modifiers. Rota et al. [10] derived empirical fragility curves based on damage survey data for 150,000 buildings from past Italian earthquakes, occurred in the period spanning from Irpinia (1980) to Molise (2002) earthquakes. It is to note that this work follows the masonry buildings classes definition presented in [10]. De Luca et al. [11] derived empirical fragility curves based on a database of 131 post-earthquake building surveys conducted on RC buildings located in Pettino town after the 2009 L’Aquila Earthquake. Del Gaudio et al. [12] have derived empirical fragility curves based on the large database of post L’Aquila Earthquake (2009) damage survey conducted by the Italian Civil Protection [13] on RC buildings. This work presents a methodology for the generation of conditional GMPE-based ground shaking fields for derivation of empirical fragility curves. The application of this methodology is demonstrated in deriving empirical fragility curves based on the Copernicus-EMS damage data for masonry buildings available right after the Amatrice Earthquake of 24th of August 2016. The basic underlying idea (like the method described in [14] for portfolio loss assessment) consists of the following considerations: (a) The ground shaking levels recorded at adjacent buildings are going to reveal significant spatial correlation. This calls for adopting a full probabilistic model based on the ground motion prediction equation (GMPE), where the inter-event and intra-event correlations between the GMPE residuals are characterized (e.g., [15-19]). The GMPE’s considering a spatial correlation structure are usually expressed in terms of multi-variate Lognormal joint probability distributions; (b) The ground shaking propagated to the bed rock level using the GMPE can be modified (or propagated to the surface) based on site-specific stratigraphic and topographic considerations; (c) a “complete” GMPE representation through the joint Lognormal probability distribution can be updated both based on the recorded registrations of the earthquake event of interest at the surrounding stations (e.g, [5, 20-21]) and the observed damage pattern; (d) an alternative shake map can be generated as stochastic realizations of the ground shaking field according to the updated GMPE description at the ground surface; (e) structural fragility can be interpreted
as the expected value and standard deviation of the fragility curves calculated based on numerous plausible ground shaking field realizations.

2 METHODOLOGY

2.1 Basic assumptions for empirical fragility assessment for a class of buildings

Strictly speaking, the concept of fragility curve is applicable to Poisson-type limit state excursion [22-23]. In seismic risk applications, the uncertainty related to earthquake occurrence can be classified as leading to Poisson-type limit state excursion. Using the damage survey data from spatially-distributed surveyed buildings belonging to the same class for fragility modelling implies that the building-to-building variability within the same class is inevitably going to be considered. It can be argued that using a single class fragility curve is equivalent to the assumption that each of the surveyed buildings is replaced by a nominal building for that class. Then, the building-to-building variability is considered as contribution to record-to-record variability. However, the ground motions registered at each building location cannot be independent as they correspond to the same earthquake event and they can exhibit correlation based on the mutual distance between the buildings. Excluding such correlation may lead to unrealistic and un-conservative fragility curves [24]. Unconservative because the real information content in the data is going to be less than the information content represented by those fragility curves that do not consider the possible correlations. Herein, it is assumed that the building class definition is going to capture all the common building features (and hence correlations) that can be captured by the survey data. Moreover, the spatial correlation in the ground motion is modelled by considering the spatial correlation structure in the residuals of the ground motion prediction equation (GMPE). Although assigning a damage grade to buildings is certainly subjected to errors, herein, the uncertainty in assigning the damage grades to surveyed buildings and the fragility model parameter uncertainties are not considered.

2.2 Empirical fragility assessment for a class of buildings

The empirical fragility herein is calculated within an updated robust reliability assessment framework [25-26]. The concept of robust reliability is used for calculating “robust” empirical fragility curves; where the term robust implies that the uncertainty in the evaluation of the ground shaking is considered (see [2-3] for examples of robust analytical fragility assessment). Let the survey dataset \( D_{cl} \) define the set of \( N_{cl} \) damage states \( D_i \) (e.g., the EMS-98 damage grade, \( i \) can vary between 1 and 5) for the buildings surveyed for a certain class \( CL \). Let \( PGA = \{PGA_i, i=1:N_{cl}\} \) denote the vector of ground shaking values (in terms of PGA) estimated at the position of each surveyed building belonging to class \( CL \). The robust empirical fragility is calculated by considering the uncertainty in generating a plausible field of PGA values conditioned on both the vector of PGA values registered at various accelerometric stations, \( D_{PGA} \), and the observed damage pattern \( D_{cl} \). For a given building class \( CL \), the updated empirical fragility conditioned on available data \( D_{cl} \) and \( D_{PGA} \) is written as:

\[
P(D > D_i | pga = x, D_{cl}, D_{PGA}) = \int_{\Omega_{PGA}} P(D > D_i | x, D_{cl}, PGA) \cdot f(PGA | D_{PGA}, D_{cl}) \cdot dPGA
\]

where \( D_i \) denotes a given damage grade; \( pga = x \) is a given level of ground-shaking; \( D_{cl} \) (bold indicates that it is a vector of damage survey data) is the survey data acquired for buildings class \( CL \); \( f(PGA | D_{PGA}, D_{cl}) \) denotes the joint probability density function (PDF) for the ground-shaking field; \( \Omega(D_{cl}) \) denotes the domain of all plausible PGA fields; the joint PDF herein represents the joint probability of observing the ground shaking levels for a given building class survey locations \( f(PGA | D_{PGA}, D_{cl}) \) given the damage survey data \( D_{cl} \) and the
registered PGA data $D_{PGA}$ where $PGA = \{PGA_i, i=1:N_{cl}\}$. In general, it is quite complicated to calculate the integral in Eq. (1) analytically. Herein, it is estimated using standard Monte Carlo Simulation (MCS). In the adopted MCS approach, $N_{sim}$ realizations of the ground shaking field PGA are generated based on the joint PDF $f(PGA|D_{PGA}, D_{cl})$. The simulation procedure yields estimates of the updated empirical fragility and its standard deviation (see [27-28]):

$$P(D > D_i | x, D_{cl}, D_{PGA}) \approx \frac{\sum_{i=1}^{N_{sim}} P(D > D_i | x, D_{cl}, PGA_i) }{N_{sim}}$$

$$\sigma^2[P(D > D_i | x, D_{PGA}, D_{cl})] \approx \frac{\sum_{i=1}^{N_{sim}} P(D > D_i | x, D_{cl}, PGA_i)^2}{N_{sim}} - P(D > D_i | x, D_{cl}, D_{PGA})^2$$

where $P(D > D_i | x, D_{PGA}, D_{cl})$ is the updated fragility curve and $\sigma^2[P(D > D_i | x, D_{PGA}, D_{cl})]$ is the standard deviation of the updated fragility curve based on empirical damage data $D_{cl}$ and registered PGA data $D_{PGA}$.

2.3 Generation of scenario- and GMPE-based ground shaking fields based on $f(PGA|D_{PGA}, D_{cl})$

The joint probability density function $f(PGA)$ for the vector of $PGA = \{PGA_i, i=1:N_{cl}\}$ values at the location of $N_{cl}$ buildings of interest belonging to a specific class $CL$, for a given earthquake scenario, can be evaluated by employing a ground motion prediction equation (GMPE). Assuming that the PGA values at the location of each surveyed building are distributed as a joint multi-variate (log) Normal distribution, a full probabilistic representation of GMPE, which is identified by its expected value vector $M$ and covariance matrix $\Sigma$, can be constructed. Once the first two moments are known, several realizations of the ground shaking field can be generated and their median can be calculated. Referring to the GMPE developed by Bindi et al. [29] for the PGA (the geometric mean of two horizontal components) and based on a collection of Italian seismic events, the median predicted PGA can be written as:

$$E[\log_{10}PGA] = e_1 + F_D(R_{jb}, M) + F_M(M) + F_S + F_{sof}$$

where $E[\log_{10}PGA]$ is the expected value for the (base 10) logarithm of $PGA$; $e_1$ is a constant term, $F_D(R_{jb}, M)$, $F_M(M)$, $F_S$ and $F_{sof}$ represent the distance function, the magnitude scaling, the site amplification and the style of faulting correction, respectively. $M$ is the moment magnitude, $R_{jb}$ is the Joyner–Boore distance. The values $E[\log_{10}PGA]$ $(i=1:N_{cl})$ constitute the components of the mean vector $M$ for $N_{cl}$ buildings belonging to a specific class $CL$. The covariance matrix, $\Sigma$, is defined as the sum of two inter-event and intra-event components:

$$\Sigma = \sigma_{\text{INTER}}^2 \cdot e + \sigma_{\text{INTRA}}^2 \cdot R$$

where $\sigma_{\text{intra}}$ represents the intra-event variability and $\sigma_{\text{inter}}$ represents the inter-event variability (both parameters are tabulated in [29]); $e$ is the all ones matrix and $R$ is the matrix of correlation coefficients. $R$ is composed of unit diagonal terms and off-diagonals equal to $\rho_{jk}$, $j\neq k$ (varying from 1 to $N_{cl}$). The covariance matrix is obtained according to $\rho_{jk}$ formulation [30]:

$$\rho_{jk} = \exp\left[-3 \cdot \frac{h_{jk}}{b(T)}\right]$$

where $h_{jk}$ is the distance between sites $j$ and $k$ and $b(T)$ is a coefficient equal to 10.8km.

The GMPE adopted herein [29] considers the site effects as a function of the shear wave velocity (Vs) dependent European Code soil classifications. Nevertheless, it is important to incorporate the results of more sophisticated seismic microzonation studies or soil class map (i.e. [31]) for the surveyed buildings sites. Landolfi et al. [32] and later Tropeano et al. [33]
propose site-specific stratigraphic coefficients that consider non-linear soil column propagation effects. Then, the stratigraphic amplification factors from [32], that consider non-linear soil column propagation effects, to ground shaking fields propagated to bedrock are applied [34]. In particular, the site effects are evaluated through the stratigraphic amplification factor, directly multiplied by the reference (i.e., propagated to bed-rock) peak ground acceleration from the GMPE by [29] to obtain the peak acceleration at surface. In a similar manner, it is also possible to apply topographic factors (ST) to the GMPE (directly multiplying ST to the reference PGA from GMPE). ST depends on the shape of slopes; since irregular surface geometry affects the focusing, defocusing, diffraction and scattering of seismic waves. A geometrical parameter more suitable for small scale studies is the slope curvature, which can be obtained from the Digital Elevation Model (DEM) of the area. This index permits to mark the concave and the convex features of a landscape, with negative and positive values accounting for attenuation in valleys and the seismic waves focusing on ridges. The effectiveness of this parameter was also adopted in seismic slope stability analyses by Silvestri et al. [35].

One interesting feature of the joint Normal distribution attributed herein to the PGA values at the buildings’ sites for a specific class is that the distribution parameters can be updated based on the registered accelerometric values. With reference to the vector of log10\(PGA\) values denoted as \(D_{PGA}\), let the vector of mean values \(M\) and the covariance matrix \(\Sigma\) be partitioned as follows [20]:

\[
M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}
\]

where \(M_1\) is the mean (of the base 10 logarithm) vector of \(PGA=[PGA_i, i=1:N_{cl}]\) values according to the adopted GMPE; \(M_2\) is the mean vector of calculated log10\(PGA\) at the stations within the area of interest (according to the adopted GMPE); \(\Sigma_{11}\) is the covariance matrix for the calculated log10\(PGA\) for the surveyed buildings of class \(CL\); \(\Sigma_{12} = \Sigma_{21}\) is the cross-covariance matrix for the log10\(PGA\) values calculated at the location of the surveyed buildings and those calculated at the location of the stations; \(\Sigma_{22}\) is the covariance matrix for the log10\(PGA\) values calculated at the stations.

The conditional distribution of the calculated log10\(PGA\) values given the registered log10\(PGA\) values at the stations is a joint Normal distribution with mean vector \(M_{12}\) and covariance matrix \(\Sigma_{1122}\):

\[
\begin{align*}
M_{12} &= M_1 + \Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot (D_{PGA} - M_2) \\
\Sigma_{1122} &= \Sigma_{11} - \Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot \Sigma_{21}
\end{align*}
\]

where \(D_{PGA}\) is the vector of the registered log10\(PGA\) values for the stations.

Once the first two moments of the updated ground shaking field corresponding to the Lognormal PDF \(f(PGA|D_{PGA})\) are obtained, various plausible realizations of the ground-shaking field can be generated. It is to note that the ground-shaking field is also conditioned on the damage survey data for a given building class. This conditioning is implemented based on the premise that only those plausible ground-shaking fields that lead to physically meaningful fragility data are considered. The not-meaningful realizations of ground-shaking fields. This is achieved by checking whether the gradient of resulting empirical fragility curve for all damage levels is always non-negative.

2.4 The empirical fragility curves \(P(D>D_i|x,PGA_k,D_{cl})\)

This section describes how the fragility curve \(P(D>D_i|x,PGA_k,D_{cl})\) can be obtained for a given realization of vector of ground shaking values at the location of surveyed buildings.
generated based on pdf $f(PGA|D,PGA, D)$ and given the damage survey $D$ for a given class. The empirical fragility curves $P(D>D_i|x,PGA_k,D)$ ($k=1:N_{sim}$) have been calculated according to a logistic regression probability model [28, 36-37]. This type of regression is suitable for cases where the dependent variable is binary (i.e., either 0 or 1). Thus, it is especially suitable for estimating the probability of exceeding a damage state $D_i$. That is, with regard to damage state $D_i$, each surveyed building can either: a) exceed the designated damage state denoted as 1 or b) NOT exceed the designated damage state, denoted as 0. Denoting probability of exceeding damage state $D_i$ as a function of the ground-shaking level $PGA=x$ as $\pi(x)$, the likelihood of having $r_i$ buildings that exceed damage state $D_i$ for the $N_{cl}$ buildings surveyed for class $CL$ can be expressed as follows (assuming a Binomial Distribution):

$$p(r_i | N_{cl}, \pi_i(x)) = \binom{N_{cl}}{r_i} \cdot \pi_i(x)^r_i \cdot (1 - \pi_i(x))^{N_{cl}-r_i}$$

(9)

where $\pi_i(x)$ is expressed through the following relation:

$$\pi_i(x) = P(D > D_i | x, PGA_k, D) = \frac{1}{1 + e^{-\left(\beta_0 + \beta_1 x\right)}}$$

(10)

3 CASE STUDY: MASONRY BUILDING DAMAGED BY AMATRICE EARTHQUAKE

3.1 The area and the buildings portfolio considered in this study

Between August and October 2016, Central Italy was stricken by three damaging earthquakes. The vulnerability study carried out in this paper refers to the first event, with Mw 6.0 event and occurred on August 24th at 01:36 UTC close to Accumoli village (herein referred to as Amatrice Earthquake, see [38] for more details about the Central Italy seismic sequence). The European Macroseismic Scale EMS 1998 [7] classification has been used herein in order to identify the damage to the portfolio of masonry buildings considered. The grades of damage are described as follows: Grade 1 (D1): Negligible to slight damage. There is no structural damage and slight non-structural damage; Grade 2 (D2): Moderate damage. There is slight structural damage and moderate non-structural damage; Grade 3 (D3): Substantial to heavy damage. There is moderate structural damage and heavy non-structural damage; Grade 4 (D4): Very heavy damage; Grade 5 (D5): Destruction. Identification of the damage level for different buildings has been carried out through satellite imagery through Copernicus-EMS damage grading map. The Copernicus (https://www.copernicus.eu/en, last access 31/01/2019) EMS provides rapid assessment of the damages through generation of “damage grading” maps, made possible by comparing pre- and post-event satellite images. It turns out that satellite imagery is more reliable for damage grades higher than or equal to Grade 4 and less reliable for damage grades lower than Grade 4 [39]. Therefore, although it is quite efficient for performing damage survey for vast areas, its use for a complete damage classification should be ideally accompanied by other means of damage surveying such as field surveys and areal photos.

We have limited our portfolio of surveyed buildings to residential masonry buildings. The buildings have been selected and divided in different classes through a visual survey. The visual survey [40] was based on photography available from field trips (courtesy of G. Forte and A. Santo), videos provided by drone [41] for areas that were difficult to access and google street view (https://www.google.it/streetview, last access 01/06/2017). Further breaking-down was based on the recommendations in [10]. In [10], the masonry buildings are classified according to four parameters: number of floors, the presence of tie rods or tie beams, the type of horizontal structure (flexible or rigid floors), regular or irregular masonry layout. However,
due to limitations posed by visual survey, the breaking down into more detailed classifications within the portfolio of residential masonry buildings in this work is limited to two factors: the number of storeys and the presence of tie rods or beams. As far as it regards the type of masonry, most surveyed buildings offered visual evidence of being made of irregular masonry. As a result, four distinct classes of masonry buildings have been defined:

- a) Masonry buildings without tie rods or tie beams with number of stories ≤2 (Masonry Buildings Class 1, MBC1);
- b) Masonry buildings without tie rods or tie beams with number of stories >2 (Masonry Buildings Class 2, MBC2);
- c) Masonry buildings with tie rods or tie beams with number of stories ≤2 (Masonry Buildings Class 3, MBC3);
- d) Masonry buildings with retrofit operations with number of stories >2 (Masonry Buildings Class 4, MBC4).

Assuming that the surveyed portfolio is prevalently consisted of irregular masonry buildings, MBC1 can be associated to the union of classes IMA2 and IMA4 of [10]; MBC2 to the union of classes IMA6 and IMA8; MBC3 to the union of classes IMA1 and IMA3; and MBC4 to union of classes IMA5 and IMA7. This is due to fact that distinction between masonry buildings with and without rigid floor system was not rendered by the visual survey.

3.2 Site effects for the surveyed buildings

On 13th – 14th September 2016, post-earthquake field recognition identified the valley of Tronto river as the most affected area in terms of site-specific effects. The Tronto valley hosts several municipalities. The villages have developed either close to the river (e.g., Trisungo) or on top of the cliffs overlooking it. With the latter being located usually on the top of small ridges and ancient erosional terraces (e.g. Amatrice, Accumoli, Arquata del Tronto) or located on the slopes (e.g. Pescara del Tronto, Illica, Tino). The villages located on cliff-type morphology are bordered by steep slopes (25°-35°) with heights varying from 20 to 80 m. For these areas, buildings labelled with higher levels of damage are widespread and localized near the steep escarpments and in the narrower part of the ridges. These buildings are affected by seismic waves’ focalization due to topographic shape effects (e.g., [42]). These topographic effects are not present in lowland areas of the valley which suffered less damage (see Trisungo). On the other hand, other towns (Amatrice, Pescara del Tronto, Illica and Tino), suffered widespread damage due to both topographic and stratigraphic effects. Some of these towns lie on slopes characterized by few meters of soft soils resting on a stiffer material (Accumuli); where stiff arenaceous formation of the Laga Flysch is buried by few meters of weathered deposits and colluvium mainly made of silty sands. Pescara del Tronto hamlet lies on debris and travertine sands resting above a limestone bedrock. Vs associated to these deposits were based on the lithological association reported in [31]. For instance, D1 and D2 are more frequent on stiff rock such as the Arenaceous flysch and limestones; while D4 and D5 are more frequent on coarse alluvial deposits, as they are constituted of soft soil.

3.3 Generation of GMPE-based ground-shaking fields for the Central Italy Earthquake (24 August 2016) scenario

According to the procedure outlined in Section 2, N_{sim}=25000 realizations of the GMPE-based ground shaking fields are generated for the Amatrice Earthquake scenario (M=6.0) providing the PGA for engineering bedrock site conditions. The realizations are generated for each building class separately. In the next step, the generated ground-shaking fields on bedrock are multiplied by the stratigraphic and topographic amplification/deamplification factors.
Since the uncertainty in the evaluation of these amplification factors is not considered, their application affects only the median $M$ (multiplied linearly by amplification factors) and leaves the covariance matrix $\Sigma$ unaffected. Finally, in order to obtain the conditional GMPE-based fields, PGA registrations for eighty one accelerometric stations are employed in order to update the ground motion fields according to the procedure described in Section 2. Figure 1a maps the median PGA values rendered by a mesh-grid of 500mx500m resolution. Figure 1b, demonstrates the median of the conditional GMPE-based ground-shaking fields for the surveyed buildings in Amatrice (the largest of the seven towns considered).

Figure 1: (a) Median of the conditional GMPE-based ground-shaking fields for the Amatrice Earthquake scenario; (b) Median of conditional GMPE-based ground-shaking fields for the surveyed buildings in Amatrice.

3.4 The Empirical Fragility Curves

The updated empirical fragility curves and the corresponding plus/minus one standard deviation interval for each class of masonry buildings defined in Section 3 (MBC1-4) and for the different damage grades defined in Section 3 (EMS-98, D2-D5) have been calculated following the procedure described in Sections 2 and plotted in Figures 2. In each sub-figure of Figure 2, the fragility curve is plotted in thick solid lines (using established colour codes, i.e., green for D2, yellow for D3, orange for D4 and red for D5). This curve is calculated based on the conditional ground-shaking fields generated as described in Section 2, corresponding to a total set of 25000 simulations. Moreover, the expected value plus/minus one standard deviation fragility curves are plotted in dashed lines; the observed damage fractions are plotted as circles. The observed damage fractions, which are reported only for the sake of comparison with the resulting fragility curves, are obtained by dividing the PGA domain into equidistant bins—making sure that each bin contains at least one surveyed building. The 50th damage fraction percentile (shown in full circles) for damage grade $D_i$ and a given bin of PGA is calculated as the 50th percentile of the ratio of number of buildings in that bin whose damage grade is equal to or exceeds $D_i$ to the total number of buildings in the same bin over all the realizations (posterior to conditioning on $D_i$). The damage fraction percentiles for each bin are positioned at the center of each bin. It can be observed that the fragility curves obtained through the logistic regression procedure show a very good agreement with the observed damage fractions percentiles. It is to mention that only those ground-shaking fields leading to plausible fragility curves are considered. Table 1 summarizes the equivalent lognormal median and logarithmic standard deviation (dispersion) for the fragility curves.
Figure 2: Empirical fragility curves and their plus/minus one standard deviation confidence intervals for classes MBC 1-4 and damage levels D2-D5 (Damage Survey: Copernicus EMS).

Table 1: Equivalent lognormal median and logarithmic standard deviation (dispersion) for the fragility curves.

<table>
<thead>
<tr>
<th>D2 Damage level</th>
<th>( \eta )</th>
<th>( \beta )</th>
<th>D3 Damage level</th>
<th>( \eta )</th>
<th>( \beta )</th>
<th>D4 Damage level</th>
<th>( \eta )</th>
<th>( \beta )</th>
<th>D5 Damage level</th>
<th>( \eta )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBC1 COP</td>
<td>0.08</td>
<td>2.76</td>
<td>MBC1 COP</td>
<td>0.19</td>
<td>2.51</td>
<td>MBC1 COP</td>
<td>0.41</td>
<td>2.32</td>
<td>MBC1 COP</td>
<td>0.74</td>
<td>1.90</td>
</tr>
<tr>
<td>MBC1 COP+</td>
<td>0.12</td>
<td>1.67</td>
<td>MBC1 COP+</td>
<td>0.21</td>
<td>1.82</td>
<td>MBC1 COP+</td>
<td>0.53</td>
<td>1.91</td>
<td>MBC1 COP+</td>
<td>0.94</td>
<td>1.69</td>
</tr>
<tr>
<td>MBC1 COP-</td>
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<td>MBC1 COP-</td>
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<td>3.60</td>
<td>MBC1 COP-</td>
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<td>3.01</td>
<td>MBC1 COP-</td>
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<td>2.10</td>
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<tr>
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<td>MBC2 COP</td>
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<td>1.29</td>
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<td>1.28</td>
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<td>MBC4 COP+</td>
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<tr>
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<td>MBC4 COP-</td>
<td>0.80</td>
<td>1.45</td>
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Figure 3 shows a comparison with empirical fragility curves proposed in [10] for analogous classes of masonry buildings:

- The fragility curves (solid lines with the standard colour code for various damage levels) together with the corresponding one-standard deviation confidence intervals (dashed lines), calculated according to the methodology proposed in Section 2 based on Copernicus-EMS damage grading maps;
The fragility curves reported in [10] (solid black line) are plotted for analogous irregular masonry building classes. In particular, MBC1 is defined as the union of classes IMA2 and IMA4; MBC2 is defined as the union of classes IMA6 and IMA8; MBC3 is defined as the union of IMA1 and IMA3; MBC4 is the union of classes IMA5 and IMA7.

The fragility curves obtained based on the INGV shakeMap and on Copernicus-EMS damage grading maps;

The hazard curve (dotted red curve) for the town of Amatrice ([http://esse1.mi.ingv.it/d2.html](http://esse1.mi.ingv.it/d2.html), last access 15/01/2019) is also plotted in Figure 3. It is to note that, the Bindi et al. [29] GMPE employs the geometrical mean of the two horizontal components of the PGA; this is while the hazard curve is reported for an arbitrary component. A correction factor (see [43]) has been applied to the attenuation law in order to make it suitable for one arbitrary component.

Table 2 reports the annual probabilities of exceeding the different damage levels (risk $R$). $R_F$ denotes the risk calculated based on fragility curve $F$; $R_{F\pm \sigma}$ denote the risk calculated based on fragility plus/minus one standard deviation curves, respectively. Except for MBC3 (D4-D5) and MBC4 D5, the agreement observed between the fragility curves is good. This is particularly interesting if we consider the inevitable differences in the portfolio of buildings (the upgraded buildings in Amatrice did not exactly behave as expected), building class definition, and the estimation of ground shaking level. The comparison in Table 2 reveals that there is agreement in terms of (order of magnitude of) risk between the fragility curves of [10] and the ones proposed herein. The fragility curves obtained based on shakeMap and those obtained by following the procedure in Section 2 demonstrate differences for all the classes (to the unconservative side). This provides evidence for the remarkable sensitivity of the empirical fragility curves to the methods used for the estimation of ground shaking level.
Figure 3: Comparison between the empirical fragility curves obtained based on damage data from Copernicus-EMS and visual survey with analogous empirical fragility curves in Rota et al. (2008).

Table 2: Annual probabilities of exceeding the damage levels for the different categories of masonry buildings.

<table>
<thead>
<tr>
<th>Fragility curve</th>
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<th>MBC1 and D3</th>
<th>MBC1 and D4</th>
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<td>-</td>
<td>1,00</td>
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4 CONCLUSIONS

Empirical fragility assessment can reveal significant sensitivity to the definition of the building class, building-to-building variability within a given class, and observed damage and ground shaking levels for the considered portfolio of buildings for a given class. In such context, accurate estimation of the ground shaking level at the location of buildings and considering the possible correlations between these ground shaking levels is arguably as important as accurate estimation of the damage. The (Bayesian) concept of robust updated fragilities can be applied in order to consider the uncertainty in the estimation of the ground shaking level at the position of the buildings of the interest. The joint probability distribution of the ground shaking levels can be modelled through simulation of plausible realizations of a random ground motion field. This random field is defined based on the (updated) statistics of the GMPE considering the ground shaking levels recorded at the nearby accelerometric stations. The set of plausible ground motion fields can be further filtered as those fields which lead to physically-meaningful fragility curves for all damage levels. This work uses damage data obtained based on Copernicus-EMS damage grading maps due to the Amatrice Earthquake of 24th of August 2016 and the aftershocks immediately following it. The paper explores several factors that may severely affect empirical fragility assessment. Moreover, a comparison with the fragility curves reported in [10] for analogous classes of masonry buildings, shows general agreement for all classes and damage levels except for MBC3 D4-D5 and MBC4. This agreement is particularly interesting in the light of objective differences in class definition, portfolio of buildings considered, and the procedures employed for the estimation of ground-shaking level. The differences for ultimate damage levels related to upgraded buildings classes (MBC3 and MBC4) might be somewhat justified by the worse-than-expected performance of irregular masonry buildings due to Amatrice August 24th Earthquake [44]. The results demonstrated herein are subjected to the following limitations: a) it is assumed that the building class definition is going to adequately capture all the common building features (and hence correlations); b) there were very few accelerometric stations inside the polygon defining the portfolio of buildings considered; c) newer generations of GMPE might have allowed for more detailed modelling of site effects and an integrated characterization of spatial correlation; d) it is assumed that the damage survey is not subjected to error. However, if there is information available for calibrating the accuracy of damage results, the predictive fragilities can be extended to consider the error in damage survey.

ACKNOWLEDGEMENTS

This work is based on visual damage survey results described in the B.Sc. thesis of Filomena Castagna. This support is gratefully acknowledged.

REFERENCES


ENGINEERING DEMAND PARAMETERS FOR THE DEFINITION OF COLLAPSE IN CODE CONFORMING RC BUILDINGS

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Keywords: Engineering Demand Parameters (EDPs), Collapse, Reinforced concrete.

Abstract. The focus of the paper is on the choice of Engineering Demand Parameters (EDPs) for the definition of collapse limit state in code-conforming Reinforced Concrete (RC) buildings. Different definitions of collapse for buildings are found in building codes and the published literature. The differences derive from the EDPs used to measure collapse and from the EDPs’ thresholds to assess if collapse is reaches. Definition of collapse in new, code-conforming buildings is somehow simpler than for older buildings, since the application of capacity-design principles avoids all brittle mechanisms. Still, the selection of the EDP is not unique. There are local EDPs, such as rotation with respect to the chord as suggested by Eurocode 8, or global EDPs, such as inter-story drift or top floor displacement. The study presented in this paper compares different EDP definitions and different limit values of the selected EDPs through the analyses of two case studies, a six-story and a nine-story Moment Resisting Frame (MRF). Both buildings were designed according to the Italian seismic code. Multiple-stripe, nonlinear dynamic analyses are carried out on the two frames modeled with lumped plasticity elements. The results indicate that different collapse definitions lead to different safety assessments, thus pointing to the need for a single definition of collapse, measured through a single EDP value.
1 INTRODUCTION

The Authors of this work took part of the RINTC project [1], an initiative that spanned over multiple years with the aim of assessing the collapse risk of buildings designed according to the NTC2008 Italian Building code. More specifically, this work is focused on assessing the collapse risk of cast-in-situ reinforced concrete (RC) residential buildings [2, 3].

Generally speaking, building collapse can be defined as the inability of the structures to guarantee a given performance for a given earthquake. At collapse, a structure loses its capability to carry lateral loads. Collapse can be assessed through either local or global Engineering Demand Parameters (EPDs) at the local (element) or global level [4]. However, the definition of buildings collapse is not unique [5, 6] and different definitions can be found in different building codes (EC8, NTC2008), design guidelines (FEMA356,SEAOC Vision 2000,NZS 1170-5) and published literature (Ricci at al. [2] and Camata et al. [3], Ghobarah (2004) [7],Park and Ang [8]).

In this work, some of the aforementioned collapse definitions are compared by analyzing two different code-conforming RC residential buildings of different heights in three different configurations: Bare Frame (BF), Infill Frame (IF), Pilotis Frame (PF).

2 CASE STUDY DEFINITION

The RC buildings analyzed in this work are all characterized by regularity both in plan and in elevation. The ground level is 3.4 [m] high while all the upper stories are 3.05 [m] high. Two building types are considered, as shown in Figure 1: one six-story high and the other nine-story high. In both cases, the floor plan is identical (with the exception of the columns’ sizes) and the roof is flat.

Figure 1: Case study buildings: floor plan, fixed reference grid, and one-way slab orientation.
2.1 Building design

The buildings' structural design is carried out according to the Italian NTC2008 building code. A low ductility class is assumed for the Response Spectrum Analysis. The six-story building is assumed to be located in Naples, while the nine-story building is assumed to be located at L'Aquila. The seismic hazard is evaluated for both sites using the NTC2008 guidelines and assuming soil type C. The six-story and the nine-story building both have a moment-resisting frame (MRF) structure type assumed fixed at the base. A linear elastic frame model is considered for the design of beams and columns that are assumed cracked with a reduced stiffness of 50%\(I_{tr}\) and 75%\(I_{col}\), respectively. The stairs were designed and modeled with knee-beams. The minimum column size is 35 [cm], the outer beams are all deep, while all internal beams are flat (for this reason the building automatically falls into the low ductility category of NTC 2008). Three different infill configurations are considered. In the Bare Frame (BF) configuration, infills are considered only as dead load; in the Infill Frame (IF) configuration, infills are uniformly distributed in elevation and in the Pilotis Frame (PF) configuration, infills are considered in the same way as the IF configuration but are absent at the ground level. The MRF design according to the BF and IF configurations are identical, because at the design level the infills are not modeled as structural elements but are only included as dead loads. For the PF configurations all actions are increased by 40% for the vertical elements at the ground level, following the NTC2008-7.2.3 prescriptions for buildings with an infill distribution irregular in elevation.

The following materials are used: concrete C28/35 (\(f_{ck} = 28 \text{[MPa]}\), \(f_{cd} = 15.9 \text{[MPa]}\), \(\varepsilon_{cu} = 3.5\%\), \(E_{cm} = 32000 \text{[MPa]}\)) and steel B450C (\(f_{yk} = 450 \text{[MPa]}\), \(f_{yd} = 391.3 \text{[MPa]}\), \(E_s = 206000 \text{[MPa]}\)).

Table 1 reports the buildings' main design data, with \(\Sigma A_{pls}/A_{floor}\) the total column area at the ground floor/total floor area, \(\rho_{tr,de,1L}\) the first level average deep beams' longitudinal steel ratio, \(\rho_{tr,fl,1L}\) the first level average flat beams' longitudinal steel ratio and \(\rho_{pls,gr}\) the ground floor average base floor columns' steel ratio.

<table>
<thead>
<tr>
<th></th>
<th>(\Sigma A_{pls}/A_{floor} \text{[%]})</th>
<th>(\rho_{tr,de,1L} \text{[%]})</th>
<th>(\rho_{tr,fl,1L} \text{[%]})</th>
<th>(\rho_{pls,gr} \text{[%]})</th>
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<td>0.84</td>
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<tr>
<td>L'Aquila-9s</td>
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<td>0.75±1.80</td>
<td>0.9±1.26</td>
<td>1.45±3.19</td>
</tr>
</tbody>
</table>

Table 1: Summary of buildings design data

2.2 Non-linear Modeling

The OpenSees [9] software was used to run nonlinear analyses. Beams and columns are modeled with lumped plasticity elements placed at the ends of the elastic members through zeroLength elements with flexural hinges. The assumed hinge model is the well-tested model by Ibarra et al. (2005) [10], that is implemented in OpenSees as "ModIMKpeakOriented" [11]. The model parameters are computed using the predictive equations by Haselton et al. [12], that describes the moment-rotation relationship in a single plane of flexure and for a fixed axial load. The columns' parameters were calibrated by defining two distinct and independent relationships in the two orthogonal bending planes, evaluated for the axial force produced by the gravity
loads of the seismic load combination. The beams’ parameters were then calibrated by defining non-symmetrical relationships, given the sections’ different top and bottom reinforcements. The stairwell elements modelling requires particular care due to the large shear forces in the inclined beams that induce large shear forces in the adjoining members. For this reason, in the nonlinear model the axial stiffness of the elasticBeamColumn staircase elements was set to zero and an inelastic bar element was added in parallel, with a non-symmetric elasto-plastic constitutive law for compression and tension (uniaxialMaterialHysteretic). The following materials are used in the non-linear model: $f_c = 35 \, [MPa]$, $E_c = 32000 \, [MPa]$ and for steel $f_s = 488 \, [MPa]$, $E_s = 206000 \, [MPa]$. The contribution of the masonry infills was modelled with a diagonal strut element (compression only) using the models proposed by Decanini et al. [13, 14, 15, 16]. These models describe the monotonic and cyclic behavior of the infill as a function of its mechanical and geometrical characteristics. The infills are made single-leaf 30 [cm] thick hollow bricks with the following mechanical characteristics: vertical stress $\sigma_0 = 0 \, [MPa]$, compression strength $\sigma_{mo} = 6 \, [MPa]$, shear strength $\tau_{mo} = 0.77 \, [MPa]$ and sliding strength $u = 0.54 \, [MPa]$. The Decanini et al. models provide only the peak strength, while the other parameters of the phenomenological law are determined based on simple rules that provide stiffness and drift values. The masonry infill drift values was updated based on recent works [17, 18] and the influence of openings was taken into account by using reduction factors for the panels’ stiffness and ultimate strength, based on the work by Decanini et al. (2014 [16]. The infills were modeled in OpenSees with the Concrete01 material by fitting initial stiffness, peak strength, and ultimate deformation. All other details of non-linear modelling can be found in [1, 2, 3].

2.3 Results of nonlinear analyses

Nonlinear static and dynamic analyses were carried out using the nonlinear models. Figure 2 shows the pushover (PO) curves for two different force distributions (Uniform and Modal) for the 6-story and 9-story buildings. In the Y-direction two different PO curves are shown that refer to the positive (solid) and negative (dotted) direction, respectively, due to the different staircases response. In the present work, the collapse EDPs are computed from "multi-stripe analyses" (MSA) [19, 20], carried out for both the six and nine-story building. Ten stripes were considered, each corresponding to 20 couples of ground motion records. The ground motions records have increasing return periods for each stripe, ranging from 10 to $10^5$. The record selection was carried out in [21, 22]. Rayleigh damping is assumed proportional to mass and tangent stiffness with a damping ratio set equal to 5% for two period corresponding to first period and the period corresponding to the second mode of the second group of modes. The damping was modeled only in the elastic beam/column elements between the 2 zero-length elements.

The MSA results were computed and shown in terms of roof displacement of a control node (Figure 3) or roof drift ratio (RDR) and the maximum inter-story drift ratio (IDR) of all floors in both the X and Y directions (Figure 4).
Figure 2: Pushover Curve

Figure 3: Maximum roof control node displacement
Figures 3 and 4 show significant top displacement and IDR for the nine-story buildings located at L’Aquila. This is due to the high ductility of the structures, that were designed following earthquake resistant design principles.

3 DEFINITION OF COLLAPSE EDPs

The EDPs are structural response values that can be used to assess the performance of structural and nonstructural components [4, 6]. They can be distinguished in Local EDPs (chord rotation, Local Damage Index) or Global EDPs (Inter-Story Drift Ratio (IDR), Roof Drift Ratio (RDR), Global Damage Index). In this work, the collapse EDPs are divided into those defined in design codes and guidelines and those proposed in the published literature.

3.1 Collapse EDPs by design code and guidelines

In building codes there is no unique definition of the collapse EDPs. Italian NTC2008 building code (and similarly Eurocode 8) do not specifically define collapse EPDs for code-conforming buildings, but limit the EDP definitions to existing buildings. In any case, the EDPs in EC8 and NTC2008 are all local and refer to the response of the single structural members. For brittle mechanisms (typically shear failures) these can be checked by simply monitoring whether the shear force demand exceeds the corresponding capacity. Performance for ductile is checked by monitoring that the chord rotation does not exceed given capacities. Strictly speaking, the collapse limit state is reached when the first brittle or ductile mechanism is reached in any structural element. Brittle mechanisms are not present in code-conforming buildings, as they are designed following capacity design principles. In this work it is assumed that beam collapse does not lead to structural collapse. Collapse is assumed to be reached when the first column
reaches the chord rotation limits given by NTC2008.

On the other hand, other building design codes such as SEAOC Vision 2000, FEMA356 and New Zealand code NZS 1170-5 monitor a global EDP, i.e. the Inter-Story Drift Ratio (IDR). Limit values are provided in SEAOC Vision 2000 \((IDR \geq 2.5\%\)) , FEMA356 \((IDR \geq 4.0\%\)) , NZS 1170-5 \((IDR \geq 3.0\%\)) . These codes do not distinguish between building models with and without infills.

### 3.2 Other collapse EDPs

The published literature provides reports a number of studies that deal with this topic [6]. Among the most recent studies, Ricci at al. [2] and Camata et al. [3] use the Roof Drift Ratio (RDR) as global EPD to assess the implicit risk of code-conforming Italian buildings (RINTC) [1, 2, 3]. The RDR capacity of a building is defined in each direction as the RDR measured in pushover analyses at 50% capacity drop in base shear past the peak base shear RDR demand is measured from Nonlinear History Analyses. Table 2 shows the RDR capacities for the two buildings considered in this study.

<table>
<thead>
<tr>
<th>Typology</th>
<th>Naple-6s</th>
<th>L’Aquila-9s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X [m]</td>
<td>Y [m]</td>
</tr>
<tr>
<td>BF</td>
<td>1.04</td>
<td>0.98</td>
</tr>
<tr>
<td>IF</td>
<td>0.86</td>
<td>0.71</td>
</tr>
<tr>
<td>PF</td>
<td>1.04</td>
<td>0.89</td>
</tr>
</tbody>
</table>

An alternative global EDP is the Inter-Story Drift Ratio (IDR). Ghobarah (2004) [7] defines different IDR limit values for different RC frames, mainly Ductile Moment Resisting Frames \((IDR > 3.0\%\)) and Moment Resisting Frames with Infills \((IDR > 0.8\%\)) for BF and IF buildings, respectively. For PF buildings, the two approaches are combined (ground floor assessed with 3.0% IDR limit and remaining floors with 0.8%).

Another EDP collapse measure derives from the calculation of a local damage index and a global damage index. The most popular local damage index is proposed by Park and Ang [8] and consists of a simple linear combination of normalized deformation and hysteretic energy dissipated:

\[
DI = \frac{\delta_M}{\theta_u} + \frac{\beta}{P_y \delta_u} \int dE_h
\]

where \(\delta_M\) maximum strain for earthquake, \(\theta_u\) ultimate strain, \(\beta\) strength degrading parameter equal to 0.15 [23], \(P_y\) yield force and \(\int dE_h\) hysteretic energy dissipated during the earthquake.

The local damage index for the structural elements is computed as:

\[
DI = \frac{\theta_M}{\theta_u} + \frac{\beta}{M_y \theta_u} E_h
\]

where \(\theta_M\) maximum chord rotation for earthquake, \(\theta_u\) ultimate rotation capacity of section, \(\beta\) strength degrading parameter equal to 0.15 [23], \(M_y\) yield moment and \(E_h\) hysteretic energy dissipated during the earthquake. In the column, the damage index was calculated for each element by combining the two directions step by step and taking the maximum value during the time history between node i and node j as the element local damage index. In analogy with the
columns for beams the local damage index of the element is the maximum value during the time history between node i and node j. The infill panels were treated using the Eq. 1, taking as data those deriving from the behavior of the equivalent concentric single-strut. More specifically, the local damage index is the maximum between the two concentric single-strut. The local damage index can be correlated to the global damage index by attributing to each local damage index a weight. Some of these correlations are collected in [24]. Similarly to the local criterion, for the global one the approach proposed in [8] was used, where the global damage index is calculated by weighing the local damage index \((\lambda_i)\) of each structural element (beams and column), according to the hysteretic energy dissipated compared to the total hysteretic energy of all the elements.

\[
D_T = \sum_{i=1}^{N} \lambda_i D_i
\]  
\[
\lambda_i = \frac{E_i}{\sum_{i=1}^{N} E_i}
\]

when \(N\) is number of elements, \(E_i\) hysteretic energy dissipated by the i-th element and \(D_i\) local damage index of the i-th element. Being new structures designed according to the capacity design when the column collapse after the beam, furthermore, damage to the column is difficult to repair, it is proposed to weigh the local damage index of column \((D_{pls})\) and beams \((D_{trv})\) differently:

\[
D_{global,structures} = 1.25 \sum_{i=1}^{N_{pls}} \lambda_{pls} D_{pls} + 0.75 \sum_{i=1}^{N_{trv}} \lambda_{trv} D_{trv}
\]

The global damage index (Eq.6) takes into account exclusively damage to the structural elements and neglects damage to non-structural elements such as infills. The same approach for calculating the damage index was extended for including the infills. The global damage index is therefore the sum of the global structural damage and the global damage of the infills, suitably combined and weighed differently:

\[
D_{global,structures+infills} = \frac{2}{3} D_{global,structures} + \frac{1}{3} D_{global,infills}
\]

4 COLLAPSE RATES FOR DIFFERENT EDPs

Table 3 summarizes the previously defined EDPs, while Figures 5 - 6 reports the D/C ratios computed for the different nonlinear history analyses at all 10 Intensity Measure Levels (IML). Collapse is reached when \(D/C \geq 1\).
Table 3: Collapse EDPs used in this work

<table>
<thead>
<tr>
<th>Name</th>
<th>collapse EDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTC2008 and EC8</td>
<td>Achievement of chord rotation in the first structural element (column)</td>
</tr>
<tr>
<td>FEMA 356</td>
<td>IDR ≥ 4% (Transient or Permanent)</td>
</tr>
<tr>
<td>NZS 1170-5</td>
<td>IDR ≥ 3.0% (Transient)</td>
</tr>
<tr>
<td>Vision 2000</td>
<td>IDR ≥ 2.5% (Transient or Permanent)</td>
</tr>
<tr>
<td>RINTC [2, 3]</td>
<td>RDR. Collapse is reached when demand RDR reaches RDR capacity, assessed as RDR corresponding to 50% drop in base shear capacity in Pushover analysis</td>
</tr>
<tr>
<td>Ghobarah (2004) [7]</td>
<td>IDR: BF ≥ 3% , IF ≥ 0.8% , PF Ground floor ≥ 3% and remaining floor ≥ 0.8%</td>
</tr>
<tr>
<td>Park and Ang [8]</td>
<td>Global Damage Index</td>
</tr>
</tbody>
</table>

Figure 5: $D/C$ results for different EDPs for the Naples 6-story building
Figure 5 shows that for the six-story building the RINTC criterion is the least conservative among all the criteria selected for the 3 configurations (BF, IF, PF), followed by the element EDP (NTC2008, EC8) and the Global Damage Index. The FEMA356 EDP having the highest value in terms of IDR is the most conservative criterion followed by NZS 1170-5, in the BF typology followed by the Ghobarah EDP and the SEAOC Vision2000 EDP, in the IF and PF the latter two criteria are reversed. As for the buildings with infills (IF, PF), the Ghobarah EDP turns out to be overly conservative since the $D/C$ value is much larger than in the other criteria.

![Figure 6: D/C results for different EDPs for the L’Aquila 9-story building](image)

In the nine-story building the results in terms of $D/C$ (Figure 6) show greater homogeneity for the different collapse EDP. Global Damage Index is always the one that supplies the lowest number of collapses in each single strip, followed by the RINTC EDP and the element EDP (NTC2008, EC8). The criteria in terms of IDR have the same trend as the 6-story building except for stripe number 10 where they are all the same. The Ghobarah EDP appears to be overly conservative for the buildings with infills (IF, PF) for the IM strips with lower intensity.

5 CONCLUSIONS

This paper discusses the use of different collapse definitions from building codes and the published literature. For the six-story buildings presented in this paper, the results obtained with the different definitions considered are highly scattered. It is difficult to assess the best collapse criteria, as they are all based on reasonable grounds. All EDPs considered in this work show limitations. The chord rotation used in Eurocode 8 and NTC 2008 is a local parameter, the first section that reaches the collapse chord rotation indicates global collapse. The link between local and global collapse remains questionable. On the other hand, IDR and RDR are global parameters that consider the overall building behavior: on the other hand, different documents...
propose different capacity values, more specifically for the IDR. In the BF typology the IDR limit varies from 4% (FEMA 356) to 2.5% (Vision 2000), in the case of infills (IF, PF) the differences are even greater because the Ghobarah criterion is the only one that differentiates the infilled typology with a limit IDR of the 0.8%. In case of RDR used in the RINTC project, the capacity was defined from preliminary PO analyses, but the limit value of the capacity seems arbitrary and could be conventionally chosen at different points of the post peak softening PO curve. The damage criterion by Park and Ang is a local damage index, which can be correlated to the global damage index. The limit of the application lies in the difficulty to correlate local to global damage. The approach proposed in this work is one of many in the literature reported in [24].

It is difficult to identify with of the above collapse criteria is most accurate in assessing collapse. All criteria discussed in this work indicate that a code-conforming building has a low collapse rate. There is a clear need for a clear, unified definition of collapse, based on the definition of an easy to monitor EDP.

Acknowledgments

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REFERENCES


ANALYTICAL MODEL FOR CONCRETE CONFINED BY STEEL STIRRUPS AND/OR FRP JACKETS IN RECTANGULAR SECTIONS

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Abstract

This paper illustrates the extension to the rectangular section of an analytical model for confined concrete proposed for square sections confined by steel stirrups and/or FRP jackets. The confined concrete model is developed starting from the elasticity theory and under the key assumption that the confinement acts under the plane-strain conditions. The confining pressure within the section core is obtained by an analogy formulated in polar coordinates regarding internal co-axial cylinders for obtaining the equivalent confining pressure within the section core. The model is suitable to be implemented by using any active confinement triaxial model and for describing the also the contribution offered by external FRP wraps and/or steel jackets. The paper concludes with some experimental results demonstrating the accuracy of the new analytical model proposed in predicting the confined concrete stress-strain relationship of rectangular section cores.

Keywords: confined concrete, fiber section, FRP wraps, stress-strain relationship, steel jackets.
1 INTRODUCTION

As known, the confinement increases the ductility and strength of compressed concrete, since it contrasts the internal cracks and volume increase near and beyond the strength peak. In practice, it is provided by transverse and longitudinal reinforcements acting in a state of "passive confinement", that is the confining action is depending on the axial strain of the confined concrete.

Many research groups in the last decades have addressed their efforts in estimating the uniaxial stress-strain curve of confined concrete [1, 2, 3, 4, 5, 6, 7]. Among these models, it has been also recently proposed an analytical model for confined concrete developed on the key assumption of plane-strain conditions with the elasticity theory [8]. The analytical formulation at the base of this analytical model has been developed by referring to a square section. It permits, by referring to an iterative and incremental approach [9], of evaluating the action of different internal transverse arrangements, as well as additional external wrapping in any material (such as, for instance, fiber reinforced polymers, FRP) or steel jackets. In addition, by applying this model in previous works, stress-blocks parameters for confined concrete [10], and design equations linking the transverse reinforcement amount with the actual curvature ductility of RC sections [11] have been proposed.

This paper presents the extension to the rectangular section case of the BGL model [8], formerly proposed for square cores. It is briefly shown the new analytical formulation together with some numerical applications for validating the extended model [12]. The new model for confined concrete, indicated briefly as BDGL model, has a generalized mathematical formulation providing, if the two dimensions of the confined core are equal, the same analytical relationships obtained in the case of the BGL model. The confined uniaxial stress-strain law to be assigned to each core fiber is derived by supposing the Reinforced Concrete (RC) member as a column subjected to an increasing concentric axial load up to the failure. Then, in the numerical simulations any additional bending moment may be taken into account through section equilibrium equations referred, however, to a non-uniform axial strain profile that respects the plane-sections assumption. The paper concludes showing some comparisons with experimental results for validating the new generalized formulation of rectangular confined cores.

2 PASSIVE CONFINEMENT: PROBLEM DEFINITION

The analytical formulation of the confined concrete is based on the key assumption that the increment of axial stress within the core due to the transverse confinement arises without any out-of-plane axial strain. This assumption yields to:

$$\sigma_z(\varepsilon_z) = \sigma_{z0}(\varepsilon_z) + \Delta\sigma_z(\varepsilon_z)$$

where $\Delta\sigma_z$ represents the increment of axial strength due to the concrete triaxial state, $\sigma_z$ and $\sigma_{z0}$ are the stresses of confined and unconfined core, respectively.

By assuming that the section is contained in the $xy$ plane, the plane strain condition also implies that:

$$\Delta \varepsilon_x = \gamma_{zx} = \gamma_{zy} = 0$$

where $\Delta \varepsilon_x$ is the strains field, $\gamma_{zx}$ and $\gamma_{zy}$ are the shear strains in the $zx$ and $zy$ plane, respectively; and $z$ is the column longitudinal axis.

The solution of the problem is given by the direct integration of the equilibrium equations together with compatibility equation, and respecting the boundary conditions. In particular, as
for the square core also in the case of rectangular one the analytical formulation is rewritten by introducing the Airy’s stress functions [13] so that:

$$\sigma_x = \frac{\partial^2 f}{\partial x^2}, \quad \sigma_y = \frac{\partial^2 f}{\partial y^2}, \quad \tau_{xy} = -\frac{\partial^2 f}{\partial x \partial y}$$

(3)

These functions identically satisfy the equilibrium equations. By substituting the Eq. (3) into compatibility equations defining the problem we obtain the Eq. (4) expressing the elastic problem of a confined rectangular core section.

$$\frac{\partial^4 f}{\partial x^4} + 2 \frac{\partial^4 f}{\partial x^2 \partial y^2} + \frac{\partial^4 f}{\partial y^4} = \nabla^4 f = 0$$

(4)

With the Eq. (4) the problem is formally expressed through an unique equation, whose solution \(f(x,y)\) identically must satisfy the equilibrium, respecting the compatibility with the boundary conditions. Therefore, the solution \(f(x,y)\) plays a central role in the problem of a rectangular confined core. In the model here proposed it assumed the existence of a function \(f(x,y)\) given as superposition of second-order \(f_2(x,y)\) and fourth-order \(f_4(x,y)\) polynomial functions, as follows:

$$f(x,y) = f_2(x,y) + f_4(x,y)$$

(5)

$$f_2(x,y) = \frac{C_{20}}{2} x^2 + \frac{C_{02}}{2} y^2, \quad f_4(x,y) = \frac{C_{40}}{4 \cdot 3} x^4 + \frac{C_{22}}{2} x^2 y^2 + \frac{C_{04}}{4 \cdot 3} y^4$$

(6)

where \(C_{ij}\) represent the polynomial constants, with subscripts \(i\) and \(j\) corresponding to the \(x\) and \(y\) variables power, precisely. The assumed \(f_2(x,y)\) and \(f_4(x,y)\) functions are illustrated in Figure 1 and Figure 2, respectively, where the polynomial constants respect the following relationships:

$$C_{20} = -B_x l_x^2, \quad C_{02} = -B_y l_y^2, \quad C_{40} = -A_x, \quad C_{04} = -A_y, \quad C_{22} = +\frac{1}{2}(A_x + A_y)$$

(7)

Therefore, under the assumptions made the confining state within the section core is completely defined once the four Airy’s constant \(A_x, A_y, B_x\) and \(B_y\) are known.

Figure 1 – \(f_2(x,y)\): lateral confining stresses.
2.1 Analytical solution of the rectangular confined core

For solving the problem of a rectangular confined core, it is necessary to determine the four unknown Airy’s constants $A_x$, $A_y$, $B_x$ and $B_y$. In the proposed model they are calculated by minimizing, along the two principal directions $x$ and $y$, the scatter between the total displacement of concrete core and the one of the transverse hoop, both derived taking into account the actual confinement stress state.

It is demonstrated that the Airy’s constants have the following expressions [12]:

- $A_x$ coefficient

\[
A_x = \frac{21SE^2cE_xA_yN_{A,y} - 6E_xA_y(1+\nu)N_{A,y} - 2SEcE_xA_yN_{A,y} - 6E_xA_y(1+\nu)N_{A,y} - 2SEcE_xA_yN_{A,y} - 6E_xA_y(1+\nu)N_{A,y}}{D_{A,y}} \cdot \varepsilon_z
\]

where

\[
N_{A,y} = SEcE_xA_yN_{A,y}, \quad N_{A,y} = 105SEcE_xA_yN_{A,y}
\]

\[
D_{A,y} = 2\left\{54E^2cE_xA_y(1+\nu)\left[S^2E^2cE_xA_yN_{A,y} - 35SEcE_xA_yN_{A,y} - 2SEcE_xA_yN_{A,y} - 6E_xA_y(1+\nu)N_{A,y} - 2SEcE_xA_yN_{A,y} - 6E_xA_y(1+\nu)N_{A,y}ight]ight\}
\]

\[
D_{A,y} = 7vI_x + 2l_y^2(10-3\nu) + 7vI_y
\]

\[
D_{A,y} = l_y^2(9\nu^2 + 3\nu - 19) - 7vI_x^2 - 7vI_y^2 - 7vI_y^2(9\nu^2 + 3\nu - 19)
\]

\[
D_{A,y} = 22050E^2cE_xA_yN_{A,y} - 2SEcE_xA_yN_{A,y} - 6E_xA_y(1+\nu)N_{A,y} - 2SEcE_xA_yN_{A,y} - 6E_xA_y(1+\nu)N_{A,y}
\]

\[
D_{A,y} = 63I_x + 13l_y^2(4 + 7\nu) + 13l_y^2(4 + 7\nu) + 63l_y^2
\]

\[
D_{A,y} = 4l_y^2(4 + 3\nu) + 3l_y^2(38 + 39\nu) + 2l_y^2(2 + 3\nu) + 2l_y^2(38 + 39\nu) + 4l_y^2(4 + 3\nu)
\]
\[
D_{A_6} = 396900 E_s I_x^2 I_y^2 (l_x + l_y)(1 + \nu)
\]
\[
D_{A_7} = 2 S^2 E_s I_x^2 I_y^2 \left[ 2 l_x^4 + 9 l_y^2 l_x^2 + 21 l_y^4 \right] + 315 S E_s E_s I_x \left[ 4 l_x^3 (4 + 3 \nu) + 21 l_x^3 l_y + 21 l_x l_y^3 + 4 l_y^5 (4 + 3 \nu) \right] +
\]
\[
+ 1190700 E_s^2 I_x^2 I_y^2 (1 + \nu)
\]

- **A_y coefficient**

\[
A_y = \frac{21 S E_s^2 E_s A_y \left[ E_s I_x I_y N_{A_y,1} - 6 E_s A_y \left( 1 + \nu \right) N_{A_y,2} \right]}{D_{A_y}} \cdot \varepsilon_x, \quad D_{A_y} = D_{A_x}
\]

The numerator of \( A_y \) may be obtained by changing in the numerator of \( A_x l_x \) with \( l_y \) and viceversa, while the denominator is the same of \( A_x \) coefficient.

\[
N_{A_y,1} = SE_s I_x^2 I_y \left[ \left( 8 l_x^2 - 21 l_x l_y + 21 l_y^2 \right) + 630 E_s I_x \left[ -l_x^3 \left( 2 + 3 \nu \right) + l_y^3 \left( 4 + 3 \nu \right) \right] \right]
\]
\[
N_{A_y,2} = 105 E_s I_x \left[ l_x^3 (2 + 3 \nu) - l_y^3 (4 + 3 \nu) \right] + SE_s I_x^2 I_y \left[ -13 l_x^2 + 7 l_y^2 \right]
\]

- **B_x coefficient**

\[
B_x = \frac{3 E_s E_s A_y \left[ 6 E_s A_y \left( 1 + \nu \right) S^3 E_s I_x I_y N_{B_x,1} + S^3 E_s I_x I_y N_{B_x,2} + 210 S^2 E_s^2 E_s I_x I_y N_{B_x,3} + N_{B_x,4} \right]}{D_{B_x}} \cdot \varepsilon_x, \quad D_{B_x} = l_x^2 D_{A_x}
\]

\[
N_{B_x,1} = S^2 E_s^2 I_x^2 I_y^2 \left[ 21 l_x^4 + 9 l_y^2 l_x^2 + 7 l_y^4 \right] + 105 S E_s E_s I_x \left[ 9 l_x^5 (5 + 2 \nu) + 21 l_x l_y^3 + 7 l_y^5 (4 + 3 \nu) \right] +
\]
\[
+ 132300 E_s^2 I_x^2 I_y \left( 1 + \nu \right)
\]
\[
N_{B_x,2} = 28 l_x^4 - 8 l_x^2 l_y^2 + 49 l_y^4 - 21 l_y^4
\]
\[
N_{B_x,3} = 4 l_x^5 (4 + 3 \nu) + 21 l_x l_y^3 + 7 l_y^5 (5 + 3 \nu) - 3 l_y^5 (4 + 3 \nu)
\]
\[
N_{B_x,4} = 793800 S E_s E_s I_x^2 I_y^2 I_x (1 + \nu)
\]

- **B_y coefficient**

\[
B_y = \frac{3 E_s E_s A_y \left[ 6 E_s A_y \left( 1 + \nu \right) S^3 E_s I_x I_y N_{B_y,1} + S^3 E_s I_x I_y N_{B_y,2} + 210 S^2 E_s^2 E_s I_x I_y N_{B_y,3} + N_{B_y,4} \right]}{D_{B_y}} \cdot \varepsilon_x, \quad D_{B_y} = l_y^2 D_{A_y}
\]

The numerator of \( B_y \) may be obtained from the \( B_x \) numerator [Eq. (12)] by changing \( l_x \) with \( l_y \) and viceversa, while the denominator is the same of \( B_x \) coefficient.

\[
N_{B_y,1} = S^2 E_s^2 I_x^2 I_y^2 \left[ 7 l_x^4 + 8 l_y^2 l_x^2 + 21 l_y^4 \right] + 105 S E_s E_s I_x \left[ -l_x^5 (4 + 3 \nu) + 7 l_x l_y^3 + 21 l_y^3 l_x^2 + 9 l_y^5 (5 + 2 \nu) \right] +
\]
\[
+ 132300 E_s^2 I_x^2 I_y \left( 1 + \nu \right)
\]
\[
N_{B_{1.2}} = -21l_x^2 + 49l_x^2l_y - 8l_x^2l_y^2 + 28l_y^4 \\
N_{B_{1.3}} = -3l_x^2 (4 + 3\nu) + 7l_x^2l_y (5 + 3\nu) + 21l_x^2l_y^2 + 4l_y^4 (4 + 3\nu) \\
N_{B_{1.4}} = 793800SE_c E_s^2 l_x^2 l_y^2 (1 + \nu)
\]

(15)

2.2 Confining pressure within the section core

It results convenient to consider the confining pressure acting on cylinders enclosed within the concrete section core. To this scope, the confining action is expressed in polar coordinates by the means of radial \(\sigma_n\) and tangential \(\tau_{nm}\) stresses in each point along any internal circumference having radius \(r\) and a center distant \(x_0\) from the axes origin, as plotted in Figure 3.

\[
\sigma_n(r, \varphi, x_0) = \frac{1}{2} (A_x + A_y) r^2 \cos 4\varphi - B_x l_x^2 \cos^2 \varphi - B_y l_y^2 \sin^2 \varphi + D_1(x_0)
\]

\[
\tau_{nm}(r, \varphi, x_0) = \frac{3}{2} (A_x + A_y) r^2 \cos 2\varphi + r^2 \left( A_x \cos^2 \varphi - A_y \sin^2 \varphi \right) - B_x l_x^2 + B_y l_y^2 \right] \sin \varphi \cos \varphi + D_2(x_0)
\]

(16)

where

\[
D_1(x_0) = x_0 \left\{ \frac{1}{2} (A_x + A_y) x_0 \cos^2 \varphi + A_x \left[ (-2 + 3\cos^2 \varphi) r \cos \varphi - x_0 \sin^2 \varphi \right] + A_y \cos \varphi - 2 (A_x + A_y) r \sin \varphi \right\}
\]

\[
D_2(x_0) = x_0 \left[ \frac{1}{2} (3A_x + A_y) \left(x_0 + 2r \cos \varphi \right) \sin \varphi \cos \varphi + \left( A_x + A_y \right) r \sin \varphi \cos 2\varphi \right]
\]

(17)

![Figure 3](image)

**Figure 3** – Confining pressures along internal circumferences and variation of the mean value (left), tangential stresses in correspondence of the circumference having the center at the axes origin.

It is useful, for obtaining a global parameter of the confinement state of the entire section core, to refer to the mean confining pressure \(f_{in}\) acting on an internal cylinder having radius \(r\) and \(x_0\) center. By referring for convenience only to a section quarter we obtain:
\[
\frac{\pi/2}{\pi/2} \int \sigma_n(\varphi) d\varphi = \frac{1}{2} \left( B_i l_i^2 + B_j l_j^2 \right) + \frac{x_0^2}{4} \left( A_y - A_x \right) \quad \forall r
\] (18)

One can observe that \( f_{rm} \) does not depend on the cylinder radius \( r \), varying with the distance \( x_0 \) ranging from zero to the value \( l_c-l_y \). In order to quantify the confinement action within the entire section core is possible, for instance, to determine the pressure \( \bar{f}_{rm} \) calculated as mean value of all the mean values \( f_{rm} \):

\[
\bar{f}_{rm} = \frac{1}{l_c-l_y} \int f_{rm}(x_0) dx_0 = -\frac{1}{2} \left( B_i l_i^2 + B_j l_j^2 \right) + \frac{\left( l_c-l_y \right)^2}{12} \left( A_y - A_x \right)
\] (19)

The so-derived mean value \( \bar{f}_{rm} \) may be intended as “equivalent confining pressure”, because of it measures the confinement of the entire core. In this way one can use, as in the case of square core, the triaxial compressive tests results for obtaining the confined uniaxial law of the rectangular core. However, it should be remarked that in the case of rectangular sections with high aspect ratio (very lengthened sections), as well as eccentrically loaded with biaxial bending, \( \bar{f}_{rm} \) could be excessively approximated. In these cases, it is suggested of using the Eq. (19) with a section partitioning in accordance with the adopted multiple transverse reinforcements. Accordingly, different confined stress-strain relationships may be assigned to the section core.

2.3 Confinement along the column

In order to take into account the confinement along the column a reduction factor \( k \) of \( \bar{f}_{rm} \) is introduced. It accounts for two different tridimensional transferring mechanisms: the arching action, developing between two adjacent hoops (\( k_{al} \), and the transfer of the corner forces exerting by the hoops through the longitudinal bars, depending on their bending stiffness before the buckling (\( k_c \)). However, it is considered that \( k_{al} \geq k_c \). More details about these two reduction factors are reported in [12].

2.4 Confinement provided by external steel jackets/FRP wraps

The proposed model is also suitable for accounting the external confinement offered by steel jackets/FRP wraps, representing typical interventions on existing RC columns inadequately confined. In these cases the confining pressure due to the external strengthening may be still calculated with the previous formulations, by assigning a zero value to the inertia moduli of the external reinforcements, since their bending inertia modulus is negligible. Thus, the formulas of the Airy’s constants \( A_s, A_c, B_s, B_c \) may be derived with the following assumptions:

\[
I_s = I_m = 0, \quad E_s = E_m, \quad S = S_m, \quad A_m = b_m t_m
\] (20)

where the subscript \( m \) is referred to the external reinforcement (FRP wraps or steel jackets) having inertia modulus \( I_m \), elastic modulus \( E_m \), wrapping depth \( t_m \), wrapping width \( b_m \), and wrapping spacing \( S_m \). When continuous wrapping is used \( b_m/S_m = 1 \).
Figure 4 – Different type of external strengthening: a) FRP continuous wraps, b) FRP spaced strips, and c) steel jackets.

The resulting confining pressure $f_{cm}$, due to the combined action of external strengthenings and internal hoops and longitudinal bars, may be calculated through the weighted average (with respect to the confined areas) of the confining pressures:

$$f_{cm} = \frac{A_f}{A_e} f_{ri} + f_{re}$$

(21)

where $f_{re}$ and $f_{ri}$ are the confining pressures related to external strengthening (such as FRP wraps or steel jackets), and to the internal reinforcements, respectively; $A_e$ and $A_f$ the areas enclosed within the external strengthening and internal reinforcements, precisely.

3 VALIDATION OF THE PROPOSED MODEL

In this paragraph the proposed model it is applied for obtaining uniaxial stress-strain relationships of confined concrete. The confined concrete constitutive law for rectangular core is derived with the iterative and incremental procedure proposed in [9] for square section. Accordingly to this procedure, it is still possible to use any active confinement model for evaluating the confined stress-strain state at any axial strain increment of the compressed column. Definitively, the resultant confined concrete curve, obtained in passive confinement conditions, matches all the active curves related to the increasing confining pressures. In the case of simple rectangular hoop, the highest value of confining pressure is reached at the transverse reinforcement yielding, and it remains constant until the concrete failure. In addition, in this study the active model presented in [14] is employed.

Figure 5 – Confined concrete stress-strain and confining pressures obtained with the proposed model.

At first, in Figure 5 it is considered an existing RC column section poorly confined with internal hoops, and externally strengthened with FRP wraps. The uniaxial stress-strain relationship singularly refers to the single confinement action offered by hoops + longitudinal bars (HL), by FRP wraps (W) and, then, by both acting simultaneously (HL+W). In the numerical simulations shown the ultimate compressive concrete strain is set equal to 0.03 and, for com-
pleteness, also the unconfined concrete law is plotted. In the same figure the related confining pressure $f_c$ is plotted, too.

In the chosen case study it is very easy to observe that the confining action is almost totally offered by the external FRP wraps, while the contribution of the internal reinforcements (hoops + longitudinal bars) in terms of confinement is almost negligible. Moreover, the heavy confinement of FRP wraps implies a significant improvement of the concrete ductility rather than of the confined concrete strength.

In Figure 6 a comparison with experimental results in terms of stress-strain relationship is also illustrated. In particular, the specimen named Wall n. 12 detailed in [15] is considered. The experimental stress-strain of concrete core was derived by subtracting the load sustained by cover concrete and longitudinal reinforcements from the total measured vertical load, and by dividing by the core area (measured to the center line of the perimeter hoop reinforcement). Whereas, the longitudinal strain represents the average value of the strains measured by four potentiometers installed within the wall specimens. In the graphs they are illustrated the confining pressures with the contributions separately offered by the two transverse reinforcements (perimeter hoop and square internal one). The resultant confining pressure $f_{r,tot}$ is obtained by applying the weighted average (with respect to the confined areas) of the confining pressures, as follows:

$$f_{r,tot} = f_{re} + 4 \frac{A_i}{A_e} f_{ri}$$  \hspace{1cm} (22)

where $f_{re}$ and $f_{ri}$ are the confining pressures due, respectively, to perimeter hoop and to square internal one, calculated with the Eq. (19); $A_e$ and $A_i$ the related confined core areas. In addition, two simplified relationships for confining pressures are also considered and compared with the ones analytically predicted with the model. They represent the usual values chosen by several design codes for indicating the global confining pressure within the rectangular core, such as for example in [16, 17] and are calculated starting from $f'_{lc}$ and $f'_{ly}$ (including the confinement effectiveness coefficient $k_c$) corresponding to the hoop yielding calculated with the free body diagram approach along the two section directions. The comparison shows a good agreement among the experimental and numerical results obtained, validating the BDGL model proposed.

![Confined stress-strain relationships and confining pressures](image)

**Figure 6** - Confined stress-strain relationships and confining pressures by considering the Wall n. 12 [15].

A further validation of the proposed model is demonstrated with the scatter diagram of Figure 7, where $f_{cc,exp}$ and $f_{cc,thor}$ are compared, corresponding to the experimental and theoretical values of the strength peak of confined concrete. In this comparison a data set of experimental results is considered, whose details are reported in [12]. The scatter diagram clearly shows that the predicted values of the confined peak strength result in good agreement with the experimental ones. In this case the $R^2$ coefficient is higher than 0.8, and the regression line
has an angular coefficient very close to unity. These results confirm the accuracy of the proposed model, validating it also in the case of applications with external strengthenings, such as FRP wraps.

![Trendline-R^2=0.817 y=0.7928x+8.0429](image)

**Figure 7** – Scatter diagram of confined stress peak related to the considered database of externally wrapped specimens.

4 CONCLUSIONS

In this paper it is presented the generalization of an analytical confined concrete model, named BGL model, formerly developed for square concrete core. The new extended model, named BDGL model permits of obtaining the uniaxial stress-strain relationship through the calculation of the “equivalent confining pressure”, measuring the actual confinement state within the rectangular core. The new model is versatile for both newly-designed and existing RC columns, also accounting for external strengthenings such as FRP wraps and/or steel jackets.

The model may be easily implemented, due to its iterative and incremental approach, within a general software framework aided to develop design equations linking the amount of the transverse reinforcement with the actual ductility of RC members. In this way different transverse reinforcement ratios and additional external strengthenings may be considered. In the case of existing RC columns it will be also possible to take into account the pronounced bond slips of longitudinal bars, by referring to recent published simplified models [18, 19, 20]. In addition, the influence of concrete confinement on anchorage condition may be investigated, too.

Finally, in future simplified formulations of the Airy’s constants should be obtained starting from the proposed ones, for allowing an easier application and a more straightforward implementation in a designing procedure.

REFERENCES


INFLUENCE OF NONLINEAR MODELING ON CAPACITY ASSESSMENT OF RC FRAMED STRUCTURES

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Abstract

Many existing buildings in the world present serious seismic deficiencies and need to be retrofitted. However, the basis for an effective seismic retrofit intervention is a reliable assessment of the structure. To this end, nowadays structural engineers can simulate the response of structures subjected to earthquake excitation by nonlinear numerical models. These models consider explicitly the mechanical nonlinearities of the structural members, identify the parts of the structure where yielding takes place, quantify the demand of plastic deformation and force. Furthermore, a number of models is available to carry out the nonlinear analysis of structures. All these models are able to provide a detailed representation of the seismic response of the structure. However, they are controlled by many parameters that need to be properly set to obtain an accurate prediction of the response.

Based on the framework depicted above, the target of the Reinforced Concrete Work Package 2 of the ReLUIS 2018 project was to examine and compare different nonlinear modelling techniques used to evaluate the response of structures by pushover analysis. To this end, a case study building is analysed by the eight research units involved in the project by different nonlinear numerical models. The building presents very different lateral stiffness and strength in the longitudinal and transverse directions. Each numerical model is run two times including and not including the masonry infills. Furthermore, pushover analysis is run two times with forces in the longitudinal and transverse directions. Finally, the results are compared to illustrate advantages and limitations of each nonlinear modelling technique.

Keywords: Existing buildings, RC framed structure, infills, nonlinear modeling, pushover analysis, seismic assessment.

1 INTRODUCTION

In Italy, as well as in other earthquake prone countries, buildings were erected in the past without considering the effects of seismic excitation or according to obsolete seismic design provisions. Furthermore, many existing buildings suffer from significant structural degradation because of the original use of materials with low mechanical characteristics or the natural decay of their features. The vulnerability of the existing building stock is a serious economic and social concern in many countries and the need for retrofitting or rebuilding grows as time progresses and existing structures become older and degrade further. The precondition of any effective seismic retrofit intervention is a reliable assessment of the structure safety. A correct seismic vulnerability analysis should accurately detect the seismic deficiencies and quantify the seismic capacity of the structure. This target can be achieved by means of nonlinear numerical models and nonlinear methods of analysis, which explicitly consider the inelastic response experienced by the structural members. A great variety of nonlinear numerical models is available, each one presenting advantages and limitations. Furthermore, nonlinear numerical models require many parameters to be set. An improper choice of the numerical model or the incorrect selection of the model parameters may undermine the accuracy of the analysis.

Using the case study of an existing reinforced concrete frame structure, this paper analyses different types of nonlinear numerical models, their behavior and their response predictions. Both lumped and distributed plasticity models are considered. Phenomenological and fiber section models are used to simulate the nonlinear response of the cross-sections. The analyses are run by means of different computer programs by different research teams of the Rein-
forced Concrete Work Package 2 of the ReLUIS 2018 project. A FE model with shell elements is considered too. Two numerical models are built by each research team, with and without masonry infills. Preliminarily, the parameters that control the numerical models are set based on shared assumptions made to obtain results as homogeneous and comparable as possible (in terms of Base shear – Roof displacement relationship).

The results are used to detect the seismic deficiencies of the structure, to determine the collapse mechanisms and to evaluate the capacity of the structure (maximum base shear and roof displacement) the structure can sustain. Finally, the results are compared to illustrate advantages and limitations of each numerical model and to analyze the effects of the infills on the result of the seismic assessment of the structure.

2 CASE STUDY BUILDING

The case study building is derived from the De Gasperi-Battaglia school building in Norcia (Italy) shown in Figure 1. The construction of the building dates back to the early sixties. In 1997, the building was stricken by the Umbria–Marche earthquake and suffered significant damage. In 1999, a comprehensive structural survey was executed to serve as basis for the design of seismic upgrading interventions. The survey included the analysis of constructive details of non-structural elements, the collection and study of the design drawings and reports of the structure, the verification of the geometry of the structure, and the characterization of the mechanical features of materials by experimental investigation.

The building is approximately rectangular in plan, four storeys high, endowed with unidirectional hollow clay block-cement mix slabs, and protected by a pitched roof. Two Gerber joints separate the building in three independent blocks with RC framed structure. The left block (Fig. 1), which is the one analysed in this paper, is rectangular shaped in plan with maximum and minimum dimensions equal to \( L = 24.5 \) m and \( B = 12.2 \) m. The inter-storey height is equal to 3.5 m at the 1st storey, 3.3 m at 2nd, 3rd and 4th storey, and 2.1 m at the ridge of the roof. The structure of the analysed block is constituted by three seven-bay frames and seven two-bay frames arranged along the longitudinal and transversal directions, respectively. The unidirectional floor slabs rest on the beams of the frames arranged along the transversal direction and on the beam sustained by the adjacent block by means of the Gerber joint. The external frames are endowed with masonry infills constituted by two layers of clay bricks. The external layer of the infill panels is made with clay solid bricks of 12 cm thick, while the internal one is 8 cm thick and is made with clay hollow bricks. The masonry infills encased in the longitudinal frames are partial height because they are surmounted by windows that extend from column to column. Out of the transversal frames, only the one located on the left side of the block is infilled. In this case, the infills are full height.

![Figure 1.](image_url) De Gasperi-Battaglia school building: (a) plan layout of 2nd and 3rd floor, (b) north front view
The analysis of the technical drawings provides the description of the structural and non-structural elements and in turns the data for the evaluation of the gravity loads. The characteristic values of the dead \((g_k)\) and live \((q_k)\) loads are listed in Table 1. The design drawings show also that the beams of the frames arranged along the longitudinal direction are provided with flat beams. Instead, deep beams are used for the transversal frames. Longitudinal reinforcement of the beams is made by bent-up bars in compliance with the design practice of the time. Rectangular cross-section oriented with their short side orthogonal to the plane of the transversal frames is used for all the columns. The size of the column cross-sections reduces along the height of the frame. The rebars are mainly placed along the short sides of the cross-section. The experimental investigation conducted in 1999 includes uniaxial compression tests on concrete and uniaxial tensile tests on steel rebars. The compressive strength of concrete was determined on 11 cylinder samples extracted from the structure. The minimum and maximum values are discarded and the collection of data thus obtained provides an average value of 25.2 MPa. The elastic modulus of concrete is equal to 22000 MPa. The yield strength of rebars was determined for 4 specimens and the average value is equal to 374 MPa.

<table>
<thead>
<tr>
<th>Type of load</th>
<th>(g_k) (kN/m²)</th>
<th>(q_k) (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard floor</td>
<td>5.10</td>
<td>3.00</td>
</tr>
<tr>
<td>Top floor</td>
<td>4.10</td>
<td>1.00</td>
</tr>
<tr>
<td>Pitched roof</td>
<td>4.22</td>
<td>1.75</td>
</tr>
<tr>
<td>Infill</td>
<td>3.00</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 1: Loads per square meter.

3 NONLINEAR NUMERICAL MODELS

3.1 Common features of the numerical models

Two three-dimensional numerical models are built by each research unit to predict the seismic response of the case study building. The difference is in the treatment of the masonry infills, whose contribution to the lateral stiffness and strength of the structure is neglected in the first numerical model (bare frame model) while it is considered in the second one (infilled frame model). Beams and columns are modelled by one-dimensional beam-column elements, while infills (if modelled) are simulated by a pair of equivalent diagonal trusses. Figure 2 shows a schematic view of the structural model. The columns of the first storey are clamped at the base. The in-plan position of each column is coincident with the centroid of its cross-section at the first storey. The model describes the four floor decks and the pitched roof. The part of the deck sustained by the transversal frame located in the right side and by the Gerber joint is not explicitly modelled; in particular, it is included in the numerical model only considering its contribution in terms of gravity load and mass. Since the floor decks are endowed with a concrete slab, their in-plan stiffness is assumed very large, even though this is achieved by means of different modelling strategies by the research units. The mass of each floor is determined from the gravity loads in the seismic design combination. The mass of the pitched roof was added to that of the fourth floor. The floor masses, which are resumed in Figure 2, are distributed among the nodes of the floor on the basis of their tributary areas. The mass of the part of the deck not explicitly modelled is added to the mass of the right side nodes.

Gravity loads are introduced into the numerical model in the form of loads distributed on the beams and forces applied in the nodes of the upper ends of the columns. The nodes of the right side of the deck are loaded also with the forces transmitted by the part of the deck that has not been explicitly modelled.
The same material properties of concrete and rebars are assumed for the development of all the numerical models. Compression strength $f_c$ and elastic modulus $E_c$ of concrete are assumed equal to the value determined in Section 2, i.e. 25.2 MPa and 22000 MPa, respectively. The strain corresponding to the peak compressive stress is equal to $2 \times 10^{-3}$. The tension strength of the concrete is neglected. The yield strength $f_y$ of the rebars is assumed equal to the value determined in Section 2 (374 MPa) and the elastic modulus is equal to 200000 MPa.

The columns of the perimeter longitudinal frames are modelled by two finite elements joined in an intermediate node. In the infilled model, these nodes allow the insertion of the equivalent trusses used to simulate the partial height infills. No tension strength is assigned to the equivalent trusses, while their response in compression is simulated by the trilinear force-displacement relationship based on the proposal of Decanini et al. [1-3]. Openings in the infills are taken into account reducing lateral stiffness and peak strength according the suggestions of Decanini et al. [2]. The parameters that control the model are set based on the data collected by Liberatore et al. [4] and elaborated within the RINTC project [5].

3.2 Model of UniRM3 with distributed plasticity finite elements

The numerical model of the research unit UniRM3 (University of Roma Tre) is developed in OpenSEES [6] environment and simulates beams and columns by finite elements with distributed plasticity based on the iterative force-based formulation ($forceBeamColumn$ element). The beams are simulated with three or one element depending on the variation of the rebar configuration along the longitudinal axis. The columns are simulated by one or two elements depending on the presence of the infills. The number of integration points is three or five depending on the length of the finite element. The concrete part of the cross-sections is discretized in fibres with area of about $2 \times 2 \text{ cm}^2$. The longitudinal rebars are simulated by single fibres placed on the cross-section in their actual position. The uniaxial model Concrete04 without tensile resistance is adopted to simulate the stress-strain relationship of concrete according to Mander et al. [7], while the response of the steel rebars is simulated by means of the Giuffrè-Menegotto-Pinto model [8] and implemented in OpenSEES as Steel02. The isotropic hardening is neglected, while the parameter $b$ responsible for the kinematic hardening is assumed equal to 0.005. Finally, the Hysteretic material model is adopted to simulate the infill behaviour. Each material parameter is calibrated on the basis of the material characteristics defined in Section 3.1 and the same concrete properties are assumed for the confined and unconfined parts in accordance with the models of the others research units. The rigidDiaphragm command of OpenSEES is used to create in-plan rigid constraints between the nodes of the floors.
3.3 Models with fibre discretization and plastic hinges of finite length

The numerical models developed by the research units of UniCT (University of Catania) and UniSA (University of Salerno) simulate beams and columns by finite elements with plastic hinges at their ends based on force formulation. The model is developed in the OpesSEES environment and the beamWithHinges element is used. The length of the plastic hinges of the beams is assumed equal to the depth of the cross-section while the one of the columns is equal to the average of the two dimensions of the cross-section. The central part of the elements is elastic and the elastic modulus is reduced to account for the cracking of the concrete. In particular, it is assumed equal to 50% and 90% of the elastic modulus assigned to the concrete (22000 MPa) for beams and columns, respectively. For all the analysed elements, the cross-section is discretized in 2x2 cm fibres. The longitudinal rebars are simulated by single fibres placed in their actual position. The steel reinforcement of the beams considered in the model is that effectively anchored, as deduced from the technical drawings. The uniaxial models Concrete04 without tensile resistance, Steel02 and Hysteretic material are adopted to simulate the cyclic response of concrete, rebars and infills (in the infilled model), respectively. The main difference between the numerical models of the two research units is the strategy adopted to simulate the presence of the floor concrete slabs. This is simulated in the model of UniCT by rigid diaphragms that mutually connect the nodes of the floor decks. Beams are connected to the rigid diaphragms by axial buffer elements to avoid fictitious axial forces caused by the interaction with the rigid diaphragm [9]. Instead, the model of UniSA replicates the stiffness of the floors by elastic concrete trusses connecting opposite corners of the floor decks. Elastic modulus of the trusses is equal to 22000 MPa and their cross-section is assumed to be equal to 4 cm deep and 70 cm wide (about 1/10 of the truss length).

3.4 Models with phenomenological lumped plasticity elements

The research units of PoliBA (Politecnico di Bari) and UniNA (University of Naples Federico II) reproduce beams and columns by using a phenomenological lumped plasticity approach. In particular, elastic elements with nonlinear rotational springs (plastic hinges) at the two ends are used. Plastic hinges are located at the critical zones: end cross-sections of beams and columns for the bare frame model and additionally, for the model of the infilled frame, at the end cross-sections of the captive columns. The computer codes used to run the models of PoliBA and UniNA are SAP 2000 [10] and OpenSEES, respectively. Preliminarily, a sectional analysis is performed to define the backbone of the moment-rotation $M$-$T$. In the case of columns, the sectional analysis is performed assuming an axial load value equal to that due to gravity loads. Internal diaphragm constraints are assigned to each floor, consistently with the assumed condition of rigid floor.

The research unit of PoliBA assumes a moment-rotation relationship with plastic hardening and post-peak softening. The yielding moment $M_y$, the ultimate moment $M_u$, the yield rotation $\theta_y$ and the ultimate rotation $\theta_u$ are determined by the sectional analysis and the formulas suggested by the Italian Seismic Code [11]. The sectional analysis is performed considering the constitutive laws stipulated in the Italian Building Code: parabola-rectangle for concrete and elastic-plastic for steel. The residual moment resistance is assumed equal to 0.2 $M_y$. No reduction is applied to the concrete stiffness.

The moment-rotation relationship of the research unit of UniNA reproduces the first cracking, yielding, maximum, ultimate (20% strength drop) and zero resistance point of the cross-section. First cracking and yielding moments are calculated by a sectional analysis. The maximum moment and the chord rotation values defining the response backbone are calculated by
the empirical expressions proposed by Verderame and Ricci [12] and calibrated for RC elements with plain bars.

### 3.5 Models with phenomenological beams and fibre discretization of columns

The research units of UniCH (University of Chieti-Pescara) and UniAQ (University of L’Aquila) adopted a phenomenological lumped-plasticity approach for the modelling of beams, and finite elements with distributed plasticity and fibre discretization of the cross-sections to simulate the columns. Internal diaphragm constraints have been assigned to the nodes of each floor to simulate the presence of the concrete slab. The numerical models of the research units of UniCH and UniAQ are developed in OpenSEES and SAP 2000 environment, respectively.

The research unit of UniCH modelled the beams of the analysed structure by elastic members with concentrated plastic hinges at their ends. The plastic hinges are simulated by nonlinear zero-length elements whose moment-rotation relationship is reproduced by the phenomenological model of Ibarra et al. [13]. In particular, the ModIMKPeakOriented uniaxial material of OpenSEES, which adopts a deterioration model with peak-oriented hysteretic response [14], is used. The parameters that control the model are determined by the equations proposed by Haselton et al. [15]. The forceBeamColumn element of OpenSEES with three integration points is used for each column segment. The uniaxial models Concrete04 without tensile resistance and Steel02 are used for concrete and rebars, respectively. The isotropic hardening is neglected, while the parameter $b$ responsible for the kinematic hardening is assumed equal to 0.005. The uniaxial material Concrete01 is used to replicate the cyclic response of the infills in the infilled model.

The zero-length plastic hinges of the beams of the model developed by the research unit of UniAQ are characterized by a moment-rotation law defined in accordance with FEMA356 [16]. Instead, the columns are modelled by means of elastic beam-column elements provided by fibre plastic hinges of finite length at the two ends. In particular, the cross-section of the plastic hinges is discretized adopting a mesh of 15x15 fibres. The length of the plastic hinges is set equal to the average size of the columns cross-sections along the two directions. Neither for beams, nor for columns the stiffness reduction due to concrete cracking is considered. Concerning the infilled numerical model, non-linear link elements characterized by “multilinear plastic” properties in accordance to Decanini law have been introduced to model the masonry infills.

### 3.6 Numerical model displacement-based finite elements

The research unit of UniPR (University of Parma) developed a numerical model using Abaqus 2018 software [17]. Displacement-based finite elements B31 and B32 are adopted depending on the number of elements used for the beam and column discretisation and depending on assumed plastic hinge length. For beams, the nonlinear response is considered only for bending moment about the local 1-axis of the cross-section and is defined specifying the moment-curvature relationship $M_1-\chi_1$. For columns, the nonlinearity is assigned to the responses to the bending moments about the local axes 1- and 2-. It is assumed that these nonlinear responses are uncoupled. The moment-curvature relationships are obtained by means of the Biaxial software considering a parabola-rectangle stress-strain relationship for concrete (compressive strength equal to 25.2 MPa, peak and ultimate strains equal to $2\times10^{-3}$ and $3.5\times10^{-3}$) and a plastic hardening behaviour for rebars (yield strength equal to 374 MPa, elastic modulus equal to 200000 MPa, post-yield modulus equal to 500 MPa, and ultimate strain equal to $4\times10^{-3}$). The $M_1-\chi_1$ and $M_2-\chi_2$ relationships assigned to the columns are determined.
considering the axial force caused by gravity loads. The moment-curvature relationship is enriched with a linear descending part that connects the peak point of the curve to a zero moment in correspondence of a curvature equal to 4.5 time the peak curvature value. The effect of the floor concrete slab is simulated connecting the floor nodes by diagonal rigid trusses.

4 NONLINEAR ANALYSIS AND RESULTS

The numerical models are analysed by pushover analysis. The gravity loads in the seismic design combination has been applied in load step 1. Hence, horizontal forces are applied alternatively in the longitudinal ($x$-) and transverse ($y$-) direction. The distribution of applied forces is proportional to the floor masses. The results of the analyses are used to predict both local and global response, and the collapse mechanism of the building.

4.1 Analysis of the local response and collapse mechanism

The local response of beams and columns to the incremental loading is represented in terms of moment-rotation relationship. In the case of members modelled by finite elements with lumped plasticity, the plastic rotation is plotted against the bending moment. Instead, when distributed plasticity models are used, the curvature of the end cross-section is obtained from the output returned by computer code. Hence, the rotation is determined multiplying the curvature by the assumed length of plastic hinge. The response of the end cross-sections of all the members is analysed to detect where the yielding localises and identify the collapse mechanism of the structure.

Figure 3 shows the results of the middle column of the interior frame arranged along the $x$-direction and the beams framing into this column. The column is marked with a red circle in Figure 3. The results plotted herein are obtained by means of the pushover analysis of the bare frame models subjected to forces in $x$-direction. The moment-rotation relationship is plotted only for the cross-sections that have yielded under the incremental loading. All the numerical models predict the formation of the plastic hinges in the same cross-sections, i.e. the bottom cross-section of the first storey column and the end cross-sections of the beams of the four floors. All the models, with the exception of those developed by the research group of UniRM3 and UniSA, predict the same peak moment of the beam. Instead, the beams of the numerical model of UniRM3 exhibit much larger bending moment due to the effect of the compressive axial force induced in the beams by the rigid diaphragm [9]. This effect is mitigated in the model of UniSA modelling the floor concrete slab by deformable trusses rather than a rigid constraint. Nevertheless, the peak moments returned by this model are generally larger than those returned by the models with lumped plasticity beams and by the model of UniCT, which adopts buffer elements on the beams to eliminate the fictitious axial force. The considered numerical models provide similar results also for the columns. Indeed, all the numerical models basically provide the same peak moment response of columns, with differences only in the post-peak phase. In fact, finite elements with distributed plasticity and fibre discretization of the cross-section (UniAQ, UniCH, UniCT, UniRM3 and UniSA) have exhibited strong degradation of the moment resistance. The resistance degradation is less significant in the finite element of UniPR or is even missing in the elements of PoliBA and UniNA. Note that the results illustrated in Figure 3 are qualitatively the same for the other columns and beams. Hence the considerations can be extended to the whole structure and the yield pattern illustrated in Figure 3 for one column reflects the collapse mechanism of the bare framed structure.
Figure 3. Collapse mechanism of the bare frame models under forces in \(x\)-direction and moment-rotation relationships of the yielded cross-sections.
Figure 4. Collapse mechanism of the bare frame models under forces in $y$-direction and moment-rotation relationships of the yielded cross-sections.
Figure 5. Collapse mechanism of the infilled frame models under forces in x-direction and moment-rotation relationships of the yielded cross-sections.
Figure 6. Collapse mechanism of the infilled frame models under forces in \( y \)-direction and moment-rotation relationships of the yielded cross-sections.

Figure 4 reports the results of the bare frame structure loaded with forces in \( y \)-direction. The results refer to the middle column of the frame arranged along the \( y \)-direction and located on the right side of the building. This frame has been selected because it is the one that sustains the largest displacement demand. Indeed, the mass is not symmetrically distributed with
respect to the rigidity centre of the structure and the centre of mass is located at its right. The moment resistance of the beams of the frames arranged along the $y$-direction is comparable to that of the columns. Consequently, all the numerical models detect a two-storey collapse mechanism; the beams yield at first and second and floor, while remain in the elastic range of behaviour at third and fourth floors, and the column yields at the bottom cross-section of the first storey and at other location as well. The considerations on the agreement/disagreement between the local responses predicted by the considered numerical models apply also for this analysis.

Finally, Figures 5 and 6 summarize local response of members and collapse mechanism of the infilled frame numerical models. Regardless of the direction of loading, the yielding pattern of the analysed columns is the same as that found in the case of the bare frame structure. Furthermore, the relation between the local responses predicted by the considered numerical models replicate those found in the case of the bare frame models.

### 4.2 Analysis of the global response

The global response of the structure is represented in Figure 7 in terms of base shear – roof displacement relationship. The results evidence that the structure is significantly stiffer and stronger when loaded in $y$-direction. When the structure is loaded in $x$-direction, the bare frame models with lumped plasticity elements (PoliBa, UniAQ, UniNA and UniPR) are less flexible than those with finite element with distributed plasticity or plastic hinges of finite length and fibre discretization of the cross-section (UniCH, UniCT, UniRM3 and UniSA). The good agreement between the lateral stiffness of the latter models is the result of a proper reduction of the elastic modulus of the elastic segments of the members modelled with plastic hinges of finite length. These observations are confirmed also in the case of the infilled frame models. Instead, in the case of the analysis executed with forces in $y$-direction, the initial lateral stiffness is similar for all the models. The only exception is the infilled frame model of PoliBA, whose lateral stiffness is significantly larger than that of its counterparts.

The considered numerical models generally provide similar lateral resistance. The numerical model of UniRM3 exhibits a significantly larger lateral resistance, which reflects the differences already observed in the local responses analysed in Section 4.1. If this case is
excluded, the maximum percentage difference is recorded in the case of pushover analysis with forces in x-direction of the bare frame models; the minimum lateral resistance, obtained by the model of UniCT, is 25% smaller than that determined by the model of UniPR.

A good agreement between the numerical models is generally achieved on the prediction of the roof displacement corresponding to the peak lateral strength. The only two exceptions are the model of UniNA, whose displacement is much larger than those obtained by the other models for pushover analysis with forces in y-direction, and the infilled frame model of PoliBA loaded in y-direction, that conversely exhibits the smallest displacement corresponding to peak resistance.

The lateral response evaluated by the pushover analysis with forces in y-direction is characterised for some models by a significant strength degradation. Indeed, the strength degradation in the post-peak response is the most important difference between the responses predicted by the considered numerical models. It ranges from the virtually null value exhibited by the models of PoliBA and UniNA to the 60% reduction of lateral strength recorded for the model of UniSA at the roof displacement of 250 mm.

5 CONCLUSIONS

The paper presents the results of a cooperative research project devoted to compare different nonlinear modelling techniques for the prediction of seismic response of buildings. A real building, one of the blocks of the De Gasperi-Battaglia school building in Norcia, is used as case study and is analysed by pushover analysis in two directions. Eight research units shared the data on geometrical, dynamic and mechanical features of the building and developed their own numerical model. Each research unit carried out the analysis two times, including and not including the infills in the numerical model.

A good agreement between the results obtained by the considered numerical models is generally observed in the four cases analysed: pushover analysis in x- and y-direction including and not including infills. All the models have basically predicted similar moment response of the members and have detected the same collapse mechanism. With few exceptions, the considered models return similar base shear – roof displacement relationship until the attainment of the peak lateral resistance. Instead, in the post-peak phase the response in terms of base shear may be quite scattered. This is evident when the building is pushed in y-direction. In this case, the lateral strength of some models exhibits significant degradation while it remains virtually constant and close to the peak value for others.

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RINTC-E PROJECT: TOWARDS THE SEISMIC RISK OF LOW AND PRE-CODE SINGLE-STORY RC PRECAST BUILDINGS IN ITALY

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Abstract

This paper reports the preliminary results of a research project (RINTC-E) aimed at computing the risk of collapse in RC precast industrial buildings designed according to the codes in force in Italy in Seventies and Nineties. Companion papers describe the overall research project, funded by the Italian Civil Protection Department, its different areas of application (reinforced concrete, masonry, steel buildings, etc), and the overall seismic risk calculation procedure. This paper describes the design, modelling and pushover nonlinear analyses of one-story precast RC building, designed according to codes in force in Italy in Seventies. The structural nonlinear behavior is modeled using a lumped plasticity approach and the beam-to-column connection is based on friction forces. Collapse of the building is evaluated considering two failure conditions: i) 50\% degradation of the maximum base shear recorded on the pushover curve; ii) beam-to-column connection failure.

Keywords: precast concrete, existing buildings, nonlinear analyses, lumped plasticity models, pushover analysis, friction connection.
1 INTRODUCTION

Great consideration is given to the seismic vulnerability of Italian buildings, since severe consequences have occurred after earthquakes in different areas of the country. Seismic safety of precast structures was an underestimated topic in last century, even if damage and collapse of these constructions cause huge economic losses; for example, productive activity stops in industrial facilities, because of damage. Furthermore, this kind of structures is frequently subjected to heavy interventions and intended use changes, which can produce an increase in seismic vulnerability, if not adequately designed.

Recent seismic events, in particular Emilia Romagna earthquakes in 2012 [1], highlighted the importance of taking into account the seismic behavior of RC precast buildings, pointing out a significant design inadequacy for existing structures under seismic loads. These events have given to the research community the possibility to improve their knowledge about precast structures, providing a lot of data and practical experiences. Ercolino et al [2] handled the seismic assessment of a precast structure in Emilia Romagna that showed severe damage after last earthquakes, consisting in significant rotation at the base of the columns along one direction and connection failure. The developed model is able to reproduce the seismic behavior of the assessed structure and it can be very useful for modelling exiting precast buildings with friction connections.

Structural response and failure mode of existing precast constructions under seismic forces denote several weaknesses, mainly due to the fact that a high percentage of precast RC buildings in Italy were erected in areas only lately declared as high seismic zones; for this reason, the adopted design procedure meets the requirements of a non-seismic (or low seismic) code, providing buildings with insufficient resistance and ductility. Column slenderness, effects of seismic input asynchrony, floor deformations, influence of seismic vertical components, beam-to-column connection based on friction and eccentricity between columns and beams represent the main problems, as also demonstrated in Magliulo et al [3]. Despite all these problems, few studies on the seismic vulnerability of precast structures were conducted. In 2015 Casotto et al [4] developed a seismic fragility model for Italian existing RC precast buildings, with variable geometry and materials; the damage state was defined performing non-linear analyses and comparing the maximum demand for each state to the structural capacity. In the same year Palanci et al [4] developed a similar study considering a set of 98 one-story precast buildings in Turkey; the resulting fragility curves allow the classification of precast building in three groups, according to the strength and the ductility capacity, in order to make risk assessment and loss estimation easier and faster. Some studies focused the attention on the frictional beam-to-column connection behavior, whose failure causes the instantaneous collapse of the structure, because of the beam loss of support. Therefore, the neoprene-concrete frictional coefficient plays a very important role in the seismic assessment of existing precast structures, in which dowel connections were often not provided. Demartino et al [6] analyzed two different models, the first elastic and the second rigid non-linear, in order to evaluate the influence of different parameters on the minimum frictional coefficient necessary to avoid sliding. Magnitude, epicenter distance and soil type, besides dynamic characteristics of the structure, are the most conditioning factors. Magliulo et
al [7] carried out a wide experimental campaign aimed to develop formulas for the neoprene-concrete frictional coefficient evaluation; pulling tests gave the most relevant outcomes, relating friction to the axial load acting on the neoprene pad.

In the present paper a seismic assessment of single-story RC precast buildings, assembled in 1970s, is carried out through static nonlinear analyses. Six different structures are designed according to the Italian DM 30/5/1974 [8] and CNR 10012/1967 [9], for three different sites (Milano, Napoli, Catania) and for two different heights of the columns (6 or 9 m), considering the same soil type. The development of a three-dimensional nonlinear model is needed; the base of the columns is intended to act as a plastic hinge, according to a lumped plasticity approach. Nonlinear pushover static analyses are performed along both the orthogonal directions, in order to validate the model and identify the building capacity.

2 CASE STUDIES: DESIGN

The assessed buildings are designed according to Italian codes DM 30/5/1974 [8] and CNR 10012/1967 [9]. Such regulations do not take into account seismic loads and the design follows a deterministic approach according to the allowable stress design.

Three sites (Milano, Napoli and Catania) are considered. Figure 1 shows the layout of the case studies, in particular plan and section views are illustrated. The global geometrical features are the same for all cases: buildings present one bay in transversal direction and four bays in longitudinal direction; principal beam span is 15 m, secondary beam span is 6 m. For each site, two cases with different values of column height (6 and 9 m) are analyzed, highlighting the influence of column slenderness on the global structural behavior. The presence of a crane is modelled only in terms of mass and vertical force; brackets, supporting the crane, are located at 1.5 m from the top of the columns.

According to the structural typology, roof elements and beams are designed only for vertical loads (permanent and variable actions) whereas the design of columns takes into account the wind load and temperature variation also, which are the only horizontal forces acting on the structure in the design phase.

Roof covering is made up of double T prestressed elements, disposed one close to the other and joined through a concrete slab with a thickness of 5 cm. This system allows to consider a rigid behavior for the floor in its own plane. Double T elements are linked to the principal beams through steel pins and plates, bolted to the jointed elements, ensuring in this way a hinged connection. They are designed for permanent loads (slab, screed and waterproofing), live loads (1 kN/m² for accessible roof) and snow, evaluated according to the site altitude and the roof geometric characteristics. The selected sizing is reported in Table 1.
The principal beams present two kinds of variability: the first is a height variability, since the beam is higher in the mid-span than in the ends, and the second is a section variability, since the section shape changes gradually, from a T-section at the lateral sides to an I-section in the center part. This peculiar configuration aims to improve the beam structural behavior, increasing flexural and shear strengths where they are required. Because of all this variability, mean value of the height and a particular base dimension have to be considered in the nonlinear model (Table 1). This assumption does not affect the reliability of the results, since the assessed structural typology contemplates a nonlinear response under seismic forces only for columns.

Design of secondary beams is neglected, and pre-determined section dimensions are assigned to these elements (Table 1).

For both the covering elements and the beams the amount of steel reinforcement is not calculated because it does not influence nonlinear analyses.

Columns are monolithic precast square-shaped elements, assumed all equal in terms of section dimension and reinforcement quantity. According to DM 30/5/1974 [8], preliminary sizing is performed considering the columns subjected to a compression load and reducing the allowable stress by 30%.

<table>
<thead>
<tr>
<th>Element</th>
<th>B [m]</th>
<th>H [m]</th>
<th>Site</th>
<th>H&lt;sub&gt;col&lt;/sub&gt; [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof element</td>
<td>1.60</td>
<td>0.40</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td>Principal beams</td>
<td>0.25</td>
<td>1.14</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td>Secondary beams</td>
<td>0.30</td>
<td>0.50</td>
<td>All</td>
<td>All</td>
</tr>
</tbody>
</table>

Table 1 - Dimensions of the roof elements, the principal beams and the secondary beams

The acting axial load is given by the sum of the element self-weight, live loads, snow and the forces related to the crane (supporting beam and hook). Nevertheless, the preliminary sizing does not satisfy the verifications taking into account the second order effects; so the
adopted section and longitudinal reinforcement ratios have to be increased for all the case studies. Figure 2 illustrates the final choices.

Socket foundations host the base of the columns allowing to consider a fixed restrain at the base.

![Figure 2 Column sections for different cases](image)

Figure 2 Column sections for a) the case of Milano with 9 m-height columns, b) the case of Catania and Napoli with 9 m-height columns, c) the case of Milano with 6 m-height columns and d) the case of Catania and Napoli with 6 m-height columns

The principal beams are simply supported on the columns through a friction connection, made with neoprene bearings. Bearing dimensions are chosen to be equal to 25x25x1 cm. Secondary beams, instead, are hinged to the columns with their connections through steel plates and bolts; mechanical connections are necessary due to wind actions. A neoprene pad is also placed at secondary beam-column interface, with dimensions equal to 10x10x1 cm, in order to allow a better stress distribution between the connected elements.

Such building configuration produces horizontal and vertical eccentricities between columns and beams, in both the principal horizontal directions. The value for these eccentricities are evaluated with the following formulas:

\[
\begin{align*}
\varepsilon_{h,pr} & = \frac{B_{col}}{2} - \frac{b}{2} \\
\varepsilon_{v,pr} & = \frac{H_{bpr}}{2} \\
\varepsilon_{h,sec} & = \frac{B_{col}}{2} - 0.05m \\
\varepsilon_{v,sec} & = \frac{H_{bsec}}{2}
\end{align*}
\]  

(1)
where $B_{col}$ is the base of the column, $b$ is the side length of the neoprene pad, $H_{p_p}$ is the section height of principal beams and $H_{sec}$ is the section height of secondary beams. Final geometric configuration is illustrated in Figure 3.

![Figure 3 - Elastic model of the assessed structures: a) transversal direction and b) longitudinal direction](image)

3 NONLINEAR MODEL

Nonlinear analyses require a modification of the elastic model illustrated in Figure 3. Plastic hinges are added at the base of the columns, in order to simulate the nonlinear behavior of the structure, according to a lumped plasticity approach. In these plastic zones, representing all the ductility resource of the structure, dissipation of seismic energy occurs. Hinges are modelled through a trilinear moment-rotation curve, fixing a yielding point, a capping point and an ultimate point. Values of bending moments and curvatures corresponding to the abovementioned points are estimated according to Fischinger et al [10]. Yielding moment is evaluated through an elastic fiber analysis of the column transversal section, considering three different materials: concrete core, concrete cover and steel. Unconfined concrete cylindrical mean strength is assumed equal to 42.96 MPa. This value is derived from the study [2], where the same type of structure with the same concrete characteristic strength was analyzed, and where some information were directly taken onsite. Indeed, a lack of knowledge can be denoted in the field of onsite testing of concrete belonging
to RC precast structures built in the Seventies. For the material steel more data are available, based on extensive experimental campaigns on existing RC building steel bars, and the yielding mean strength results to be equal to 448 MPa. Yielding rotation is calculated according to Fardis, whose formulation allows to take into account the dependence on the shear span, and all the other parameters are estimated according to Haselton, since they do not depend on the shear span. The case of Milano with 9 m-height columns represents an exception to this validated procedure, demonstrating the lack of specific knowledge for existing precast RC structures modelling. Fischinger model [10] is inadequate for very high columns with a low amount of reinforcement, since it provides a very large value of yielding rotation and at the same time a too small value for the capping rotation. This produces a post-yielding stiffness larger than the elastic one, providing an unrealistic and unacceptable model. For this reason, only for this case, the monotonic behavior of plastic hinges is modelled completely according to Fardis [2]. Furthermore, as it can be found in the next paragraph, nonlinear static analyses show that collapses for all the assessed structures occur because of friction connection failure in the elastic field. Therefore, choosing Fardis approach seems to be reasonable since the model variation does not affect the elastic branch of the trilinear curve. Plastic hinges are different for corner columns and lateral columns, since the value of axial load acting on them changes according to the area loading the column. Furthermore, due to the design depending on vertical loads, column sections show different reinforcement along the two orthogonal directions; for this reason, two different plastic hinges are needed along x and z directions.

Concerning the hysteretic behavior, the Ibarra et al peak-oriented model [11], based on energy dissipation deterioration increasing with the number of cycles, is adopted. A normalized energy dissipation capacity represents the fundamental parameter; it depends on the axial load ratio, the ratio of stirrups spacing to column section dimension, the effective ratio of transverse reinforcement and the ratio between the value of the shear force in equilibrium with the maximum flexural strength and the shear strength. Figure 4 and Figure 5 show the resulting moment-rotation curves for the cases with 9 m-height columns. For the sake of brevity, the case of buildings with 6 m-height columns is not reported, but the trend of the curves is the same of the case of Napoli and Catania illustrated in Figure 4. Obviously, since for the sites of Napoli and Catania the buildings are identical, same curves are provided for both cases.
Figure 4 Plastic hinge moment-rotation curves for Napoli and Catania
Modelling is carried out by means of the software OpenSees, implementing columns and beams as elastic elements and plastic hinges with the Ibarra deterioration model with a peak-oriented hysteretic response. Considering that columns and plastic hinges are connected in series, in order to provide columns with elastic behavior and to avoid numerical problems for plastic hinges, a small amount of elasticity is to be provided to plastic hinge, slightly modifying the column stiffness [12, 13]. Friction connections are implemented by means of flat slider bearing elements of OpenSees, which allow the translation in both the principal horizontal directions (Figure 6), at the reaching of a friction force, computed according to the Coulomb formula. Friction coefficient is found to be equal to 0.125, according to [7].
4 ANALYSES OUTCOMES

Nonlinear static analyses are performed in order to evaluate the seismic capacity of the structures. This represents the first step for the vulnerability assessment, since the calculated capacity values are then compared to the seismic demand, provided by multi-stripe analyses through the application of 20 ground motions for each of the 10 intensity level, chosen for each site. Nonlinear multi-stripe analyses results are not included in this paper.

Nonlinear static analyses show that the collapse state is reached for all the structures in their elastic field, because of friction connection failure. Figure 7 and Figure 8 illustrate the resulting pushover curves, providing the building top center of mass displacement (assumed at the height of the mean axis of the main beams) vs the total base shear. Increasing the displacement, the force increases following the elastic branch of the capacity curve, up to the value of the friction force. At this point, beams start sliding upon the neoprene pad on column head, and a brittle failure occurs because of the loss of support of the beams, when the sliding displacement exceeds a certain value of displacement, depending on geometric features of the structures. The collapse displacement can be evaluated as the sum of two contributions: i) the column top elastic displacement under the friction force; ii) the sliding displacement before the loss of the support.
Figure 7 Pushover curves for Catania and Napoli (top) and Milano (bottom) in the case of columns 6 m high.
Figure 8 Pushover curves for Catania and Napoli (top) and Milano (bottom) in the case of columns 9 m high

5 CONCLUSIONS

The main conclusions of the research study shown in this paper are summarized in the following.

- Design according to Italian code in force in Seventies, in zones that were not classified as seismic zones, provides slender structures with a small amount of transversal reinforcement.
- Design according to Italian code in force in Seventies, in zones that were not classified as seismic zones, provides friction beam-to-column main connections.
- Further research is needed in order to model the nonlinear behavior of columns of existing single-story RC precast buildings.
- Seismic capacity of existing single-story RC precast buildings designed in Italy in Seventies, in zones that were not classified as seismic zones, is provided.

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STRUCTURAL ANALYSIS OF THE WALLS SUPPORTING THE RESURRECTION OF CHRIST BY PIERO DELLA FRANCESCA MURAL PAINTING AT SANSEPOLCRO, ITALY

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Abstract

The Resurrection of Christ by Piero della Francesca (about 1460) at Sansepolcro (Italy) is a masterpiece in the development of painting. This mural painting was realized on a brick wall (15 cm thick) placed in a different place as it is today. Shortly later, the painting was moved by means of a transport to solid-wall and placed against a pre-existing wall (60 cm thick), where it is today, erecting against this wall a counter wall (15 cm thick) where the mural painting wall-panel (225x200 cm) had been inserted. Sansepolcro area presents a strong historical seismicity (six earthquakes > VII MCS in the last 1000 years), which caused several damages to buildings; moreover in the IIWW the Nazi retiring blasted some edifices causing local vertical rebound up to 1g. For safeguarding the painting from seismic hazard, the Sansepolcro Municipality supported studies regarding the structural behaviour of the hall where the Resurrection of Christ mural painting is. Type and characteristics of the masonries constituting the museum hall and the wall where the mural painting is placed were defined by means of GPR, ultrasonic and micro-endoscopies. These studies outlined a crack pattern subdividing the wall into more vertical panels, the mural painting resulting inserted in the middle of the central panel, but free to move independently thanks to soft link between the mural painting solid-wall-panel and the surrounding masonry. This wall also has a soft link with the lateral walls. A 3D FEM of the museum was built using the data obtained from the investigations on the masonries. Imposing to the FEM the constraints deriving from in situ dynamic tests, we obtained a fine tuning of the 3D FEM and a reliable reproduction of the 3D dynamic behaviour of the building suitable to describe the real motu proprio of the structure and the contribute by the single walls. In detail, the main movement of the wall supporting the mural painting is oscillatory on the wall plane. Wall soft connections and dynamic behaviour explains because the Piero della Francesca’s Resurrection of Christ mural painting remained unharmed, despite the numerous seismic events occurred in the area in the past centuries.

Keywords: Seismic hazard, Resurrection of Christ, Piero della Francesca, Sansepolcro, masonry, no destructive tests, FEM.
1 INTRODUCTION

Hurley [1] in 1925 defined the Resurrection of Christ by Piero della Francesca (Fig. 1) at Sansepolcro (Italy) "the greatest picture in the world". This mural painting was realized on a brick wall (15 cm thick) placed in a different place as it is today. Shortly later, the painting was moved by means of a transport to solid-wall and placed against a pre-existing wall (60 cm thick), where it is today, erecting against this wall a counter wall (15 cm thick) where the mural painting wall-panel (225x200 cm) had been inserted. The Sansepolcro area presents a strong historical seismicity (six earthquakes > VII MCS in the last 1000 years), which caused several damages to buildings; moreover in the WWII the Nazi retiring blasted some edifices causing local vertical rebound up to 1g.
In the last years, the Sansepolcro Municipality supported a series of studies regarding the structural behaviour of the hall where the Resurrection of Christ is, in order to safeguard it from seismic hazard.

The researches we developed regarded the type and the characteristics of the masonries constituting the museum hall and the wall where the mural painting is placed. For this target, we studied the history of the building and its present assemblages and performed a series of no destructive tests (NDT): GPR, sonic and ultrasonic, micro-endoscopies and thermic.

We also defined the dynamic behaviour of the hall by means of a FEM analysis and matched that to the results by the monitoring system at work since 5 years.

2 GEOLOGICAL BACKGROUND

Sansepolcro is settled in an inner Neogene extensional basin of the Northern Apennines orogenic belt [2]. The basin presents master faults of the NE side and a thick clastic sedimentary infilling [3] that can reach a few hundreds of meters in the depocenter of the basin.

In the museum area, the bedrock is at about 40 m below the ground datum and the clastic sequence presents silt, sand and clay of low plasticity. According to the rule in force (NTC2008/2018, by Eurocode 7) this soil is of the C Type.

Sansepolcro is a seismic area with a strong historical seismicity with six earthquakes > VII MCS in the last 1000 years (Tab. 1), which caused several damages to buildings because MCS VII is the threshold level for building damages.

<table>
<thead>
<tr>
<th>Date</th>
<th>Local magnitude</th>
<th>MCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1352/12/25</td>
<td>Me 5,6</td>
<td>VIII</td>
</tr>
<tr>
<td>1353/01/01</td>
<td>Me 5,8</td>
<td>IX</td>
</tr>
<tr>
<td>1458/04/26</td>
<td>Me 5,8</td>
<td>VIII-IX</td>
</tr>
<tr>
<td>1789/09/30</td>
<td>Me 5,8</td>
<td>IX-X</td>
</tr>
<tr>
<td>1897/12/18</td>
<td>Me 5,2</td>
<td>VII-VIII</td>
</tr>
<tr>
<td>1917/04/26</td>
<td>Me 6,0</td>
<td>IX-X</td>
</tr>
<tr>
<td>1948/06/13</td>
<td>Me 4,9</td>
<td>VII</td>
</tr>
<tr>
<td>2001/11/28</td>
<td>Me 4,3</td>
<td>VI</td>
</tr>
</tbody>
</table>

Table 1. Main historical seismic events occurred at Sansepolcro.

3 BUILDING HISTORY AND MASONRY

The building containing the Resurrection of Christ mural painting was built in the XIV century as a large hall for public assemblies; in the XV century the hall was divided into two smaller halls by the construction of a wall in false, this wall had a fireplace and a chimney.

A little bit later, on the side opposite the fireplace was placed the Resurrection of Christ mural painting by means of transport to solid-wall. For this purpose against the preexisting wall (60 cm thick), was erected a counter wall (15 cm thick) where the mural painting wall-panel (225x200 cm) had been inserted (Fig. 2).

During centuries, two doors were opened/closed/reopened laterally in this wall and the fireplace closed by means a thin wall presumably in the XVIII century.

In the XX century, the vaulted ceiling partially renewed and the roof rebuilt with the insertion of several still rods.

Actually, the Resurrection of Christ mural panting is on a wall-panel 15 cm thick that is insert into a counter-wall 15 cm thick, built all around it, and placed against a wall 60 cm thick not jointed to the two main lateral walls (Fig. 3).
3.1 Masonries

The masonry structure and assemblages had been defined by means of direct observations in the cellar and attic and no destructive tests (NDT): GPR, sonic and ultrasonic velocity, micro-endoscopy, thermography.

The result of these analysis (Fig. 4) outlines the lateral walls were built by means of mortared sandstone stone slabs and boulder, the intermediate wall by means of mortared bricks and sandstone boulder with the chimney buffered by bricks.

The counter-wall supporting the mural painting wall-panel is in mortared sandstone cobbles and bricks laterally and bricks all around the wall-panel; mortared bricks make the wall-panel.
3.2 Fissures pattern

All around the mural painting there is a fissures pattern (Fig. 5) monitored in the last fifteen years. The instruments record seasonal movements of the order of 0.01 mm, in the events of 2016 remote earthquakes the movements reached the order of 0.2 mm in both the events with no drift towards enlargement of the opening.
In the original body of this wall there is a large fracture (up to 5 cm large) parallel to the wall itself.

4 STRUCTURAL DYNAMIC ANALYSIS

The surveys and the tests carried out are the cognitive framework necessary for a dynamic analysis of the building.

The objective of the analysis is the dynamic identification of the structures of “Piero della Francesca” and “Matteo di Giovanni” rooms and the evaluation of the behaviours due to the very small displacements. To achieve this goal, a linear elastic analysis was carried out.

Because the target of our analysis was the first small movements that can damage the mural painting, we are not interested in the further development of the crack pattern; therefore, it is correct to develop a linear elastic dynamic analysis.

In our analysis we used the software SAP2000 (®Computers and Structures, Inc.) to build a complete 3D finite element model of the building.

For this purpose, we modelled and discretised the main walls of the halls with the Resurrection of Christ mural painting in 3D FEM as an elastic and isotropic continuum (Fig. 6).

Whereas for correctly simulating the package of the wall supporting the mural painting wall-panel, the intermediate wall had been modelled by using “shell” and “layered” elements (Fig. 7).
The physical-mechanical properties of the masonries, in lack of direct destructive tests, had been assigned on the base of abacus developed by national and regional boards [4] [5] [6].

4.1 Dynamic behaviour

The FEM model developed and the physical-mechanical properties assigned to the masonries permitted to obtain a dynamic behaviour of the building. This behaviour was matched with the in situ dynamic measures developed on the base of seismic passive monitoring [7], the results are fully comparable (Tab. 2). This has proved the correctness of the developed model.

<table>
<thead>
<tr>
<th>Modal</th>
<th>Form</th>
<th>In situ experimental frequency [Hz]</th>
<th>FEM model frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Flexural</td>
<td>3.766</td>
<td>3.765</td>
</tr>
<tr>
<td>2</td>
<td>Flexural</td>
<td>4.405</td>
<td>4.876</td>
</tr>
<tr>
<td>3</td>
<td>Flexural</td>
<td>7.217</td>
<td>7.318</td>
</tr>
<tr>
<td>4</td>
<td>Rotational</td>
<td>8.204</td>
<td>8.264</td>
</tr>
</tbody>
</table>

Table 2. In situ and FEM frequencies and behaviour of the modal forms.

The main modal forms and movements are oscillatory. In detail, the first mode of deformation for the whole structure (Fig. 9) is oscillatory along the short sides and for the intermediate wall supporting the mural painting wall-panel the movement results oscillatory in the plane of the wall (Fig. 10).
Figure 9. Left: first modal forma at the frequency of 3.765 Hz is oscillatory along the short sides (N-S), right: second modal form at the frequency of 4.876 Hz is mainly up and down.

Figure 10. Dynamic behaviour of the first modal form for the intermediate wall supporting the mural painting wall-panel.

4.2 FEM analysis results

The numerical model gives an idea of the actual situations found on the building at the level of injuries and shows how the structural strengthening implemented over time have increased the overall strength of the structure.

Specifically for the wall of the Resurrection, it appears to be stressed with about one order of magnitude smaller than the side and front walls, both for its position around the middle, and for its loose connection to the external walls. In fact, the wall that contains the wall panel with the painting seems to be little tightened to the walls, but at the same time confined by these. The position of the Resurrection therefore appears to be optimal, as it is in the portion of structure, less stressed in the event of a seismic event.
However, despite this optimal location of the Resurrection painting within the structure, the dynamic local behaviour does not appear to exclude any effect on the painting. In fact, if on the one hand the partially "released" behaviour of the wall-panel of the painting reduces the seismic actions, at the same time this can lead to reform the lateral cracks resulting from the different behaviour between the two masonry units.

The numerical model implemented is therefore an important tool for assessing the dynamic behaviour of the structure and for any future comparison monitoring.

5 CONCLUSIONS

The FEM simulation executed on the dynamic behaviour of the walls of the halls, which contains the Piero della Francesca mural painting wall-panel, is able to fully describe the movements to which the wall-panel is subject. In particular, the movements result in the order of 0.01 mm, 0.2 mm during the events of a remote earthquake and the main modal form is oscillatory in the plane of the intermediate wall containing the wall-panel.

The intermediate wall supporting the mural painting has a soft connection with the two main lateral walls. The cracking pattern subdivide the intermediate wall into three vertical panels that can accomplish the oscillatory movements with a “bookshelf” behaviour. On the other hand, the mural painting wall-panel, due to its soft connection to the encasing counter-wall, is free to move as a solid block in respect to the rest of the walls.

These movements and their entity is fully compatible with the fissure pattern and the monitoring results and fit to the in situ dynamic passive survey.

The outlined dynamic behaviour and the soft connections and cracking pattern can explains because the Piero della Francesca’s Resurrection of Christ mural painting remained unharmed, despite the numerous seismic events occurred in the area in the past centuries.

In our opinion, the large fracture presents in the original body of the intermediate wall parallel to the wall itself can be related to the blasting of the civic tower made by Nazi retiring in the July 1944, more than to an earthquake.

REFERENCES


RESIMUS: A RESEARCH PROJECT ON THE SEISMIC VULNERABILITY OF MUSEUMS’ COLLECTIONS

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Abstract

The art goods represent a priceless asset of the cultural and artistic identity of communities. Nevertheless, the seismic performance of museums’ content is only rarely checked, despite the several losses occurred in the last earthquakes (Emilia Region 2012, Aquila 2009, Central Italy 2016). As a consequence, risk mitigation and management have been paying an increasing attention to artistic and cultural goods. Such artifacts, despite presenting an intrinsic vulnerability - due to their irregular shape, slenderness, fragility - rarely are adequately protected from earthquakes. The research project “RESIMUS” has developed in this scenario, and it focuses on the seismic vulnerability of the collections exposed in the Museum of Bargello in Florence. A special attention is paid to the role played by the staging on the seismic performance of the artifacts. The Museum of Bargello represents a privileged case-study, both for the importance and the value of its collections and for its staging variety. The staging adopted in the Museum, indeed, presents different types, materials, ages and dynamic properties, representing a special example of site-specific outfitting. The research project has involved different scientists and disciplines, providing results which concern the laser-scanner survey of the objects, their classification according to the main mechanisms of collapse, the site-specific seismic input and the finite element modeling of some case-studies.

Keywords: Artefacts, Sculptures, FEM models, Seismic performance of artifacts, Ammannati’s sculptures
1 INTRODUCTION

The research project focuses on the issues of conservation and seismic protection of artifacts on exhibit within museums and historical buildings, i.e. site-specific museums [1] The recent seismic events occurred in Italy (Emilia Region 2012, Aquila 2009, Central Italy 2016), such as in the rest of the world (Sendai 2011), have drastically pointed out the seismic vulnerability of cultural property and their contests [2]. As a consequence, risk mitigation and management have been paying an increasing attention to artistic and cultural goods. Such artifacts, indeed, present an intrinsic vulnerability, due to their irregular shape, slenderness and fragility; nevertheless, rarely they are adequately protected from earthquakes.

In these years, many studies have been devoted to the analysis of the seismic performance of art goods. Most of them focused on the seismic response of specific case-studies, which are investigated by performing numerical analyses having different levels of detail and computational effort. Whichever method is selected for the seismic analysis, a complete check of the entire content of an art museum is difficult to achieve. There are only few general contributions referring to art collections that differ from each other for material, shape and seismic behavior. The Getty Museum (Malibù, California) has been the first one to deal with a systematic classification of artifacts shown in museums on the basis of their seismic vulnerability [3-7]. Even in Italy there have been interesting experiences concerning the classification of art and archeological collections [8].

The research project RESIMUS faces the assessment of the seismic vulnerability of the art collections exhibited at the Museum of Bargello in Florence (Italy), which represents a precious case-study, since it contains a large variety of collections, each of which of a great artistic value. The project, still in progress, aims to achieve a general overview of the art collections. A special attention has been paid to the role of the staging in such vulnerability. In the Museum, indeed, there are windows and staging devices very different from each other (see Figure 1); some of them have become essential components of the Museum asset; therefore, they cannot be considered as simple containers, which can be replaced, but they need to be assumed as artifacts to protect.

Figure 1. Some windows for collections staging at the Bargello Museum.

The results achieved by now can be divided into two main sections, respectively focused on theoretical and experimental work. The theoretical activity is presented in Sections 2 and 3, respectively. Section 2 concerns the general overview of the collections’ vulnerability, and it has been performed by using a form-filled approach, based on specific vulnerability-sensitive descriptors. Section 3, consists of numerical analyses, both rigid-blocks and Finite Element procedures, performed on three case-studies opportunely selected among the Museum’s goods. The experimental activity is presented in the Sections 4 and 5, respectively focused on the mechanical properties (strength, friction coefficient) of the artifacts materials and on the characterization of the foundation soil, since the expected seismic intensity is not exactly known.
so far. In each section, a general overview of the research is provided, together with the main achievements and proposals.

2 THE FORM-FILLED QUALITATIVE APPROACH

One of the main problems experienced by administrations in charge of wide artistic heritages is the systematic inadequacy of the economic resources that are needed to face seismic risk mitigation campaigns [9]. The need to monitor a large number of artifacts with the availability of limited economic resources has encouraged the development of qualitative evaluation methods. In Italy this approach has been extensively adopted to check the seismic vulnerability of buildings, both for preventive surveys (GNDT/CNR) and for post-earthquake interventions (AeDES), with reference to “homogeneous” categories of buildings, such as housing, schools, hospitals, churches, etc..

In these last years, this qualitative approach to the seismic vulnerability assessment of large groups of items has been extensively adopted also with reference to cultural goods [8, 10,11]. Just like what happens in the assessment of immovable property, the quantitative evaluation of artistic heritage enables to monitor many samples of artifacts [12]. Although such evaluation does not provide for exhaustive information on the seismic performance of the sample, it allows to evaluate the most vulnerable items which can consequently become the object of a more specific and analytical investigation on their seismic performance [13]. Moreover, the introduction of proper numerical coefficients to “weight” each descriptor allows to change the evaluation of the artifact vulnerability from a qualitative index to a quantitative - although not exact - one, with the consequent possibility to compare the items’ vulnerability within each collection.

The adoption of a qualitative assessing method for artifacts requires a preliminary classification of the art collections that present a substantially homogeneous seismic response.

In this research the classification proposed by Ciampoli and Augusti [8] has been assumed as starting point. It has been implemented by introducing the relationship between each artifact and the other vulnerability sources within the room (architectural and non-structural components, further artifacts and so on), to take into account the possible occurrence of secondary vulnerabilities. Even the information suggested by Podany [7] in 2017 for the damage mitigation has been integrated in the classification. The work, still going on, started with the creation of a computer database of the artifacts belonging of the Museum, since there was not any available information. Three levels of database have been set for each item, referred, respectively, to the exhibition room where the item is located, to the staging and to the item itself. A special attention has been paid to the mutual distance between the artifacts exhibited in the same room, and to the possibility of non-structural elements to induce damages to the collections. At the current time, only the forms referred to the collections exhibited in “Sala Donatello” have been filled. The outcome achieved on this subject right now have been published in the papers made by Mattoni and Tanganelli [14,15] in 2018.

3 THE NUMERICAL ANALYSES

The rupture mechanism more likely to occur to art goods can be related to their type and geometrical features. According to the assumed classification [14], marble and stone statues resulted to be the most vulnerable art goods of the Museum, due to their high mass and slenderness. The adoption of a pedestal can further worsen the seismic vulnerability of the art works [16], increasing their slenderness and changing their dynamic response. The main numerical approaches for the seismic analysis of marble sculptures consist of the rigid block and the Finite Element Method (FEM) procedures. The rigid block analysis has been introduced in
the 60s [17], and – thanks to its simplicity - is widely used even nowadays [18-22], even though it can provide information regarding only the motion of the system. The adoption of FEM to artifacts is a recent achievement [23-25] allowed by the digital scanner 3D technology and by the increased computational effectiveness of computing devices. FE analysis is effective in providing information both on the motion of the system and on its stress level, and it can describe different collapse mechanisms, depending on the assumptions made for the material behavior and the interface conditions. In this research project both the rigid blocks and the FEM analyses have been performed with reference to three case-studies. The first one is Cerere, a marble statue made by Bartolomeo Ammannati in the XVI century as the central figure of the Juno’s Fountain. The second case-study is the statue Marzocco, made by Donatello in 1420, and representing the symbol of the town. It stands over a marble pedestal, which is an art good itself; it was made by Benedetto da Maiano in 1480 as pedestal of the Lion in the main square of Florence (Piazza della Signoria), and it was successively adopted as pedestal for the Marzocco. The third case-study is the marble statue Oceano, made by Giambologna between 1572 and 1576 for a sculptures group to place in the Boboli Gardens, and currently located in the courtyard of the Museum.

The geometrical representation of the case-studies, has been made on the basis of laser-scanner and photogrammetric surveys [15, 26-28], which provided a very detailed surface model for each object. The FE models have been found after some changes made on the geometrical models provided by the geometrical survey. Each geometrical model was simplified through the software MeshLab [29] in order to reduce the number of polygons describing its surface; then it was further changed, by introducing a set of nodes in the volume inscribed by the surface, in order to obtain a volume model. Figures 1, 2 and 3 show, for the three case-studies, the main information regarding the rigid block and the FEM models, whilst in [15, 28,30] a detailed descriptions of the models can be found.

<table>
<thead>
<tr>
<th>Material</th>
<th>statue</th>
<th>pedestal</th>
</tr>
</thead>
<tbody>
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<td>616</td>
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<tr>
<td>Height (cm)</td>
<td>230</td>
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</tr>
<tr>
<td>Number of final 4-nodes elements</td>
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<td>9337</td>
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Figure 1. Case-study #1: Cerere

<table>
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Figure 2. Case-study #2: Marzocco.
In the rigid blocks analysis, the collapse condition is assumed at the overturning of the sculpture occurred as a consequence of the seismic input. Each sculpture has been modeled as a single rigid block, neglecting the possible interface between the pedestal and the upper sculpture, such as the vertical component of the earthquake. The maximum seismic acceleration which each sculpture can sustain is found by imposing the equilibrium between the overturning moment demand and the corresponding capacity [2]. The equilibrium between demand and capacity provides a limit value for the horizontal acceleration, which has been compared to the spectra provided for the site by the Italian Technical Code [31, 32]. As a result of the simplified analysis, the amplification factor, $f_a$, has been found for each considered Return Period, according to the NTC 2018 instructions [31, 32]. In Figure 4 the values found for the Vulnerability Index, defined as $1/f_a$, have been shown. Further and more detailed results can be found in [15, 29, 32].

The Finite Element analysis has been performed by using different analytical procedures, differing from each other for the assumed description of materials (elastic and inelastic) and the interface behavior between the statues and their supports. The seismic input has been described through an ensemble of seven ground motions, spectrum-compatible to the elastic spectrum provided by NTC 2018 for a soil-type B, and a seismic intensity corresponding to a Return Period of 1950 years, i.e. the Collapse Prevention limit state, with a class of use ($c_u$) equal to 2 (strategic buildings). Several analyses have been performed [30, 33], which pointed out the importance of the different assumptions made in the analysis. Figure 5 shows the comparison between the seismic response of Cerere, expressed in terms of its top displacement, found by assuming a fixed restraint between the statue and the pedestal and by introducing an interface friction between the contact surfaces. Further results regarding the

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**Table 1: Material Properties**

<table>
<thead>
<tr>
<th>Material</th>
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<th>pedestal</th>
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<tr>
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</tr>
</tbody>
</table>

**Figure 3. Case-study #3: Oceano**

**Figure 4. Vulnerability Index found for the three case-studies (soil-type B)**

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effects connected to the modeling of the interface surface between the statue and its support are presented in [27] and [33].

![Figure 5. Top displacements of Cerere in the X-Y plane (from [27])](image)

Figure 5. Top displacements of Cerere in the X-Y plane (from [27])

![Figure 6. Inelastic strains in the Cerere’s pedestal (from [33])](image)

Figure 6. Inelastic strains in the Cerere’s pedestal (from [33])

The inelastic behavior of material has resulted to scarcely affect the seismic response of the case-studies. As regards the case-study #1, only the pedestal, consisting of masonry, overcomes the elastic limit, achieving inelastic deformations (see Figure 6). Further analyses of the case-studies are in progress, which consider the three components of the seismic input, and check the effects of different values of the friction coefficient between the pedestal and the statue.

4 EXPERIMENTAL TESTS ON MECHANICAL PROPERTIES OF MATERIALS

The assessment of the seismic safety of artifacts requires the knowledge of the material behavior, as regards the strength and the sliding attitude of the items.

The limit strength of the material is the most crucial information about its structural capacity; indeed, only when the limit and the yield stress are known, it is possible to evaluate the quality (elastic vs plastic) of its dynamic response and to predict its damage level.

At the current time, the compressive test has been performed on the marble only [34]. The tested marble has been extracted by a Carrara quarry, and it is consistent to the material used for the case-studies #1 (Cerere) and 3 (Oceano). Indeed, in the XVI Century, under the Medici’s dynasty, many marble sculptures were made in the Florentine area were made with marble taken by the Carrara quarry. The Uniaxial Compressive Strength (UCS) test (see Figure 7a) has been performed on six cylindric samples, through a hydraulic press INSTRON MODEL 5592 with a maximum stress equal to 600 kN, at a constant velocity of 1±0.5 MPa/s, according to UNI EN 1926 [35] and ASTM [36] standards. The UCS test was even enhanced by ultrasound lectures. The experimental investigation provided a compressive mean strength equal to 50.8 MPa and a standard deviation equal to 10 MPa.

Another quantity which largely affects the results provided by the numerical analysis is the friction coefficient (FC), whose amount can change the type of collapse mechanism (sliding vs rocking or overturning) of the artifact. The quantity FC is checked both through static and dynamic tests. Right now only part of the static tests has been performed. The test has been made through an inclined plane, having an adjustable slope (see Figure 7b). Three samples have been tested. Each sample had a cubic shape, with a different finishing for each of the six sides. Each sample has been tested over three different support planes made by as much types of mortar, respectively made with concrete, lime and hybrid. The test should be repeated for other materials, such as different types of stones; moreover, further materials (glass, metal, plexiglas) should be considered for the support plane. When the static test will be competed,
all the samples, with the respective support planes, will be tested over a shaking-table, in order to determine the dynamic values of $FC$.

5 EXPERIMENTAL TESTS ON THE FOUNDATION SOIL

The Museum area, such as the rest of the Florence basin, consists of plio-pleistocene palustrine and alluvial deposits, followed by two sedimentary cycles related to the paleo-Arno river and the holocene geomorphic evolution [37]. The seismicity of the area has been carefully investigated in these years [38,39]; the Peak Ground Acceleration of the town – defined for a rigid soil and an exceeding probability of 10% in 50 years - is equal to 0.13g. Specific investigations, consisting of 36 down-hole tests and over 1800 drillings [38-40] provided the stratigraphy of the urban soil and the main properties of each layer and the distribution of the amplification factor within the urban area. As a result of this survey, a grid has been obtained, with the main information (Fundamental Period of the soil, bedrock depth and main stratigraphy) defined at each intersection. The mechanical properties of the most superficial layers – the alluvional deposits – depend on the consistency of the soil, which largely varies within same areas. As a consequence, a reliable classification of the foundation soil requires to check the consistence – and the exact depth - of the uppermost layers of the soil; according to the general information currently collected, the soil could belong to the classes B, C or D, depending of the depth and density of the single layers. In some of the performed analyses [32] different soil classes have been assumed for the foundation soil, whilst in the most part of the investigations [25,27,33] the soil-type B, which has the higher probability to correspond to the foundation soil, has been assumed.
Some investigations have been planned to check the foundation soil, in order to define the mechanical properties of the most superficial layers. Since the Museum is located inside the historical center of Florence, only non-invasive investigations can be made. Right now, a capacitative electric tomography (CET) has been performed [41-43], whilst further investigations, like the 3d electric tomography and the georadar, should be performed in the next months. The CET has been performed along the perimeter of the Museum. The measures have been accomplished through a OhmMapper device, by adopting 5 bipolar receivers 5m long, with a distance to the transmitter ranging between 5m and 10m. The data have been processed through the software MagMap 2000 (Geometrics Inc, 2001), by adopting the despiking technique to reduce the anomalies. A detailed description of the investigation can be found in [44], while Figure 8 shows the map where the survey has been made, and a picture of the lecture.

6 CONCLUSIONS

In this work a comprehensive description of the research project RESIMUS has been presented. The project, still in progress, is focused on the seismic safety of the art collections exhibited in the Museum of Bargello (Firenze), and it has collected several contributions belonging to different fields. A special attention has been paid to the staging adopted for the exhibition, which can largely affect the seismic behavior of each item.

The assessment of the seismic performance of the collections, indeed, has been faced through two different approaches, respectively based on form-filled qualitative procedures and numerical analyses. The qualitative approach is aimed at achieving a comprehensive view of all the art collections of the Museum and their vulnerability level. It can indicate the most vulnerable artifacts of the collections, in order to proceed with more detailed analyses.

The numerical analyses have been performed with reference to three case-studies, i.e. three marble or stone sculptures. Both the rigid block simplified analysis and the Finite Element Method have been performed. In the Finite Element analysis, different hypotheses have been assumed regarding the material behavior and the interface between the statues and their pedestals. Finally, some experimental tests have been performed to investigate the mechanical properties of the materials, and those of the foundation soil of the Museum, in order to define the seismic input to assume for the analyses. The results, which have been mentioned in the paper, suggested further research to perform. Namely, the form-filled classification of the art collections of the Museum should be completed in the next months, and a larger number of case-studies, belonging to different types, should be analyzed. Further FEM analyses will be done, in order to better evaluate the role of each single assumptions on the obtained results. An experimental campaign has been planned, including dynamic tests to do on shaking table, to validate the numerical models. Finally, the experimental tests made on the soil should be integrated with electric tomography and geo-electric 3D, in order to achieve deeper layers of the foundation soil, to improve its seismic classification.

ACKNOWLEDGMENTS

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DYNAMIC IDENTIFICATION OF THE SANSEPOLCRO (ITALY) MUSEUM AND THE WALL SUPPORTING THE RESURRECTION OF CHRIST BY PIERO DELLA FRANCESCA

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Keywords: Dynamic Identification, Operational Modal analysis, Structural Health Monitoring

Abstract. Dynamic identification is considered a powerful technique for testing the conservation status of buildings as their natural frequencies, damping and modal shapes are directly related to their rigidity and structural integrity. This issue is of crucial interest when historical structures are in area of high seismic level, as SanSepolcro city which has experienced earthquakes up to a maximum intensity of \( I_{\text{max}} = X \) MCS scale. The dynamic response is suitable for ancient structures because is performed by measuring ambient vibrations instead of actively shaking the building. The results obtained from dynamic tests might be used to validate the assumptions used in numerical modeling in order to reduce the uncertainties related to the soil-structure interaction and to the structural parameters. In this framework, we show how an automatic identification procedure can track in real-time the dynamic response of the museum in Sansepolcro (Italy) using operational modal analysis (OMA). The dynamic characteristics, of the SanSepolcro museum and the wall supporting the mural painting of Resurrection of Christ, were identified by installing a temporary network of 10 seismic sensors. The OMA technique was performed using enhanced frequency domain decomposition (EFDD) which has allowed to detect the first four modal shapes, damping and structural frequencies. The dynamic characteristics were used to calibrate a more realistic and reliable 3D-FEM numerical model. Integrating, the results of the dynamic indetification with 3D-FEM model analysis, has provided important information to understand the complicated system of connections between the existing walls.
1 INTRODUCTION

In recent few decades, Operational Modal Analysis (OMA), also known as Output Only or Ambient Vibration Test, has become a powerful tool for a wide range of applications in the field of civil engineering [1, 2, 3, 4]. The Output Only modal test represents an extraordinary alternative when the structures are too important to be artificially excited. This method takes advantages from natural sources of vibration (wind, traffic, sea waves etc.) instead of having to shake the structure artificially [5, 6]. The difficulty associated with output-only test is that the measured response is often noisy and contains the characteristics of the structure as well as the characteristics of the unknown excitation force. So, the power of Output Only modal identification method is to distinguishing the structural modes from the ambient vibration [7, 8].

Dynamic characterization is a method for testing the health status of structure because its natural frequencies and mode shapes are directly related to its integrity [9, 10]. The dynamic response of the buildings is increasingly growing when the functionality and the health status of the edifices have to be estimated. This issue is of crucial interest when historical structures are in area of high-medium seismic risk, as the SanSepolcro museum which is located in area with a medium seismic hazard (PGA 0.22g), i.e. the highest seismicity in Tuscany. In this framework, we here present an application of the automatic Enhanced Frequency domain decomposition method [11, 12, 13, 14] to define the dynamic behaviour of the Museum in SanSepolcro and the wall supporting the Resurrection of Christ. The Resurrection is a mural painting painted during the Italian renaissance (1460 A.D) by Piero della Francesca and it is considered a masterpiece in the development of painting. The wall supporting the mural painting is located in the middle of room embedded by lateral walls (Figure 2a). The dynamic identification was used to derive a more accurate 3D FEM numerical model [15 - this volume] able to reproduce the real dynamic behaviour of the building. Integrating, the results of the dynamic identification with 3D FEM model analysis, has provided important information to understand the complicated system of connections between the existing walls of this historical building.

2 DYNAMIC IDENTIFICATIIO TEST

The main aim of the seismic survey was to investigate the dynamic behaviour of the wall supporting the famous mural painting of Resurrection of Christ. Therefore, the ten seismic stations were installed in south – side area of the museum that contains the mural painting (Figure 1, 2).

The seismic survey presented in this work was carried out for 17 hours, from 9 to 10 November 2017, and with ten seismic stations (Figure 1, 2). Each seismic station was equipped with 3-component seismometers: six stations with Lennartz 3D/5s seismometers (sensitivity 400 V/m/s and with a 5 s eigenperiod); three stations with Guralp CMG-6T seismometer (sensitivity 2400 V/m/s with a 10 s eigenperiod) and one station with a Guralp CMG-40T seismometer (sensitivity 800 V/m/s with a 30 s eigenperiod). Seismic data were digitized using a 24 bits Guralp CMG24 Digitizer at 100 Hz, and the time synchronization between stations was achieved using GPS. The three stations were installed at ground level (Figure 1), the others seven stations were located on the roof (Figure 2).
Figure 1. Position of 3 seismic stations installed at the ground level.

Figure 2. Position of 7 seismic stations installed at the roof of the museum.
3 AUTOMATIC OPERATIONAL MODAL ANALYSIS

The enhanced frequency domain decomposition (EFDD) is a frequency domain technique for operational modal analysis of structures for which the theoretical background is described in numerous papers [11, 12, 13, 14] and therefore we here only summarize the fundamental theoretical steps. The EFDD is an extension of the Basic Frequency Domain technique (BFD), often called Peak-picking method [16], and it is based on the Singular Value Decomposition (SVD) of the Power Spectral Density (PSD) matrix $G_{yy}(\omega)$, where $\omega$ are the single angular frequency.

Here we used the automatic modal identification method first introduced by Brincker et al. (2007) [12] and implemented by Rainieri and Fabrocino (2010) [13] for computing automatically the modal parameters of the buildings. The main steps of automatic EFDD method applied in the present study can be summarized as follows.

3.1 Spectral density matrix

For all the signals in each 300 s long data set, the spectral density matrix $G_{yy}(\omega)$ is calculated using the Welch’s technique with Hanning weighting function of 60 s, an overlap of 66 % and frequency resolution of 0.0122 Hz. The Hanning function ensures that all data are equally weighted in the averaging process, minimizing leakage and ringing effects. The size of the matrix is in our case 30x30 for each frequency $\omega$ and where 30 are the total number of channels (3 components x 10 stations) used for the analysis of the seismic response of the museum.

3.2 Single Value Decomposition (SVD)

The SVD of the spectral density matrix $G_{yy}(\omega)$ is calculated to identify the modal shape by looking at which frequency the first singular value (Figure 3c) has a peak ($\omega_{\text{peak}}$). At this frequency, the first singular vector is used to estimate the modal shape of the structure [1, 7 8].

3.3 Automatic Identification of Modal Domain

The modal shapes are automatically searched within the modal domain (or modal bandwidth) using the modal coherence [13, 14]. The latter is evaluated between each frequency line of two first singular vectors calculated for two consecutive records 300 s length. The modal coherence will be 1 for stationary signals and will allow to define the modal domain due to the structural modes (Figure 3ab). The mean of >0.96 (Figure 3a) and the standard deviation of <0.01 (Figure 3b), calculated for a set of 10 consecutive modal coherences are used to reduce the noise and to identify the modal domain [23, 30]. This procedure is repeated every 300 s and for the entire record.

3.4 Automatic Identification of Modal Shape.

Within each modal bandwidth (Figure 3c), the maximum amplitude of the first singular value will correspond to the modal frequency. For this frequency peak, the corresponding first singular vector $u_1$ provides the modal shape $\varphi_{r,\text{mode}} = \{u_1(\omega_{\text{peak},\text{mode}})\}$ every 300 s long data set.
3.5 Natural Frequency and Damping

The natural frequency and damping of each mode is then evaluated interpreting each amplification region in the first singular value line as the spectral bell of the power spectral density [17] of a Single Degree of Freedom (SDOF) system. The spectral bell is calculated using a MAC (Modal Assurance Criterion) rejection level of 0.9 of the modal shape $\Phi_{rj\text{mode}}$ and then it is converted in the time domain to estimate the so-called normalized correlation function. The resonance frequency of the structure is then simply obtained by counting the number of times the normalized correlation function crosses the zero axis per second [17]. Whereas, the damping is calculated by using the logarithmic decrement technique of the normalized correlation function (Figure 6c).

Figure 3. The automatic EFDD technique is based on detecting the modal bandwidth using the mean a) and the standard deviation b) of the modal coherence calculated during the seismic survey on 10 consecutive records, the white lines on the both colour bars represent the thresholds $>0.96$ and $<0.01$ used for the identification of mode bandwidth for mean a) and standard deviation of modal coherence b), respectively. c) first singular values estimated for all the observation time (17 hours).
4 MODAL PARAMETERS OF THE MUSEUM

The EFDD method has automatically identify the first four modes of the Sansepolcro Museum with structural frequency at 3.76 Hz, 4.40 Hz, 7.21 Hz and 8.20 Hz (Figure 3). Figure 4 shows the three-dimensional representation of the four modes. The Museum is characterized by a first and second transversal modes with a movement trend in NE-SW direction, with a frequency of 3.76 Hz and 4.40 Hz, respectively (Figure 4). The third modal shape, at frequency of 7.21 Hz, presents a transversal movement in out of phase of the external walls in NW-SE direction (Figure 4). The fourth mode is torsional with a frequency of 8.20 Hz (Figure 4). The wall, supporting the mural painting the Resurrection of Christ, follows the movement of the building with amplitude lower than the lateral and external walls, for all the four modal shapes.

The overall dynamic behavior of the Museum can be then represented in terms of structural frequencies (Figure 5a), damping ratios (Figure 5b) and absolute displacements (Figure 5c) for the four modal shapes. The building show larger values of damping ratios of the first and second modes than the third and fourth modes which are, for the whole 17 hours long period of observation, less than 1% (Figure 5b).

To assess the absolute value of the movement of each modal shape of the Museum as induced by the very low level ambient noise excitation, we calculate the displacement in µm by i) integrating the ground seismic velocity and ii) filtering in a 1 Hz bandwidth around the natural frequency of each mode (Figure 3c). The maximum absolute displacement of the single modal oscillation is then calculated in a 20 s time window. Displacement here represents (Figure 5c) the absolute movement in µm relative to the equilibrium position and allows to assess the influence of each modal shape on the dynamics of the structure. The first mode (black line Figure 5c) induce a displacement of ~0.3 µm, 3 time larger than those produced by the 4th mode.

Figure 4. Three-dimensional representation of the four modes.
Besides, displacements for each mode show a remarkable decrease during night-time (from 23:00 to 05:00 GMT) ten times smaller than during the day-time, indicating the high sensitivity of the seismic sensors and EFDD technique to detect the dynamic response of the building by the very low amplitude of the ambient vibration.

Figure 5. a) Trend over time of the structural frequencies, b) trend over time of the Damping ratio for each modal shape, c) trend over time of the displacement for each modal shape of the museum of Sansepolcro.
5 CONCLUSIONS

An experimental seismic survey with ambient vibrations has been carried out on the Museum of Sansepolcro containing the famous mural painting Resurrection of Christ by Piero della Francesca. Ten seismic stations were installed along the building recording the ambient vibrations due to anthropic activity. The seismic signals were analyzed in frequency domain using the EFDD method. Four main frequencies were identified with the associated modal shapes, damping and displacement. The modal parameters were used to calibrate a numerical 3D finite element SAP2000 to identify the elastic modulus of the masonry [15 – this volume], by an iterative procedure that minimizes the difference between experimental and numerical modal results (Table 1).

Integrating, the results of the dynamic identification with 3D FEM model analysis, has extended the knowledge of the structural behavior of entire building and consequently of the wall supporting the Resurrection of Christ.

The wall supporting the fresco is embedded in the middle of central wall-panel and confined by the lateral walls with soft links [15], this system connections between the walls explains its limited movement respect to the lateral and external walls of the building, as detected by dynamic identification and confirmed by the numerical model.

Our results indicate clearly that the operational Modal analysis using ambient vibrations provides detail information on dynamic response of the building, useful to derive reliable model with respect to their real dynamic behavior for structural analysis.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Experimental Frequency (Hz)</th>
<th>Numerical Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.766</td>
<td>3.765</td>
</tr>
<tr>
<td>2</td>
<td>4.405</td>
<td>4.876</td>
</tr>
<tr>
<td>3</td>
<td>7.217</td>
<td>7.318</td>
</tr>
<tr>
<td>4</td>
<td>8.204</td>
<td>8.264</td>
</tr>
</tbody>
</table>

Table 1. Comparison between numerical and experimental modal frequencies.

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IF SAFETY IS NOT ENOUGH. A MULTIDISCIPLINARY RESEARCH ON SEISMIC PREVENTION OF MUSEUM COLLECTIONS: THE MUSEOGRAPHICAL ANALYSIS.

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Abstract

The paper presents part of a work-in-progress research on earthquake damage mitigation for museum collections, focusing on the museographical aspects. The essay is part of the research project RESIMUS - Resilience Museums, developed by a group of researchers of DIDA of the University of Florence. We present two fundamental museographical tools: the drawing as an instrument of study, and the study of coherent museographical references. The paper tries to merge the literature on proper devices for the seismic reliability of artifacts and the updated museographical literature, in order to develop a framework where the seismic prevention comprehends the entire exhibition design.

Keywords: Museography; Exhibition design; Seismic assessment; Museum collection; seismic vulnerability
1. INTRODUCTION

As beauty as fragile, Italy is continuously chasing urgent situations and fixing damages provoked by natural disasters. The almost perpetual state of emergency had at least one “positive” effect: building a strong know-know in the management of the emergencies. Unfortunately, this did not activated strategic intervention programs to prevent such damages [1]. The aftermaths of the recent Italian earthquakes (Amatrice, Emilia, L’Aquila) have as a common denominator the huge damages occurred to the cultural heritage. Besides the destruction and harms to the historical buildings, impressive damages to collections and works of art were registered; losses that, according to ICCROM [2], may have consequences on communities. Other seismic countries and regions around the world present similarities, like Turkey, Greece, New Zealand, Japan, and California, to mention some. Museum institutions, research centers, and academics started studying mechanism of prevention to be applied to museum collections (on show and in the storages), and the sensibility about the fragility of the cultural heritage raised [3,4,5,6]. Unfortunately, there is a lack of quantitative and qualitative instruments allowing the seismic assessment of museum collections.

The paper presents part of a work-in-progress research on earthquake damage mitigation for museum collections, focusing on some museographical aspects. In section 2, it introduces the general knowledge about the seismic assessment of the movable museum collections. In section 3, we present the research project RESIMUS, RESIlience MUSeums, of DIDA, Department of Architecture of the University of Florence. After a panoramic introduction about the RESIMUS methodology and the case study of the National Museum of Bargello of Florence, the article focuses on one of the employed tools: the drawing. We illustrate as the act of sketching is a primary resource to understand the architectural space and the museum setups, and how the drawing is used inside the RESIMUS research. In section 3.5, we underline the necessity of studying and comparing existing setups before proceeding with the design of new museum settings. We present some examples of museum exhibitions that have been built inside historical museum architectures, showing the importance of the study of the references. The article concludes with general remarks and outlines the future developments of the research.

2. SEISMIC MITIGATION OF MUSEUM COLLECTIONS: AN OVERVIEW

In 1983, the J. Paul Getty Museum in Los Angeles starts investing on the seismic mitigation of its collections. After the building analysis commissioned to Lindvall, Richter, and Associates [7], Aghhabian, Masri, and Nigbor [8] begin testing the behavior of art objects in case of seismic events. They inaugurate a field of research, continued both internally at the Getty [9] and externally by other scholars [10]. In these years of testings, the dialogue between academic research and museum practice produces interesting results. Augusti and Ciampoli develop a method to index and rank the vulnerability of museum objects in case of seismic events. They inaugurate a field of research, continued both internally at the Getty [9] and externally by other scholars [10]. In these years of testings, the dialogue between academic research and museum practice produces interesting results. Augusti and Ciampoli develop a method to index and rank the vulnerability of museum objects in case of seismic events. So far, the limit of this significant research is in its application and in the lack of quantitative instruments [11]. The studies concentrate on the mitigation of damages on singular artifacts (i.e. Prigioni by Michelangelo - [12, 13], Riace Bronzes, [14]) without considering neither the presence of close elements (works of art, objects, lighting, etc.) nor the whole setting. In his book [15], Podany sums up twenty years of findings and experiences on this matter. He draws the attention on the lack of a general analysis about the whole museum set-
ting, on the need of sharing knowledge among institutions worldwide, and on the necessity of close collaborations among academia, institutions, and professionals. Since the 90’s, the Getty Museum starts applying upstanding anti-seismic procedures with remarkable results. The museum team successfully realizes some safety devices then installed in the museum galleries. The safety solutions change from object to object: big statues, for example, are arranged on isolator unit [16], medium vases are anchored to the bases with clips (small points of contact that constrain the movement without support), interfaces are used in case an object has an uneven base or a small point of contact (it requires a custom interface to distribute the load and a fastener to be secured to the base), and contour mounts are applied to objects with specific shapes. Other museums follow the Getty example starting installed anti-seismic devices, for example: the Archaeological Museum of Olympia installs a four friction pendulum system for the statues of Hermes by Praxiteles, the Tokyo National Museum displays ancient pottery on a large isolator base, while, in Italy the Michelangelo’s Prigioni and the Riace Bronzes have isolators units. In New Zealand, the Christchurch Art Gallery Te Pan o Waiwhetu operates directly isolating the entire museum. The MUNDA - Museo Nazionale d’Abruzzo (National Museum of Abruzzo) collects the works of art recovered after the quake of 2009. This renovated building embeds the most recent and sophisticated anti-seismic solutions and the permanent setup is realized using anti-seismic devices.

The safety of collections concerns both temporary and permeant exhibitions. According to Podany, temporary exhibitions are the most exposed setups. With the exception of big exhibition events, temporary exhibitions struggle with low budgets and few resources, forgetting about the seismic safety. In general, it might be sufficient applying simple precautions [17], for example by inserting weights inside the vases and fixing the objects to the bases. Little but effective actions would prevent significant damages. On the other side, the awareness about the mitigation of the seismic risk of museum collection is lacking, although it should be considered as important as the anti-theft systems.

3. THE RESIMUS RESEARCH PROJECT: THE MUSEOGRAPHICAL ANALYSIS

3.1. The RESIMUS research project

In general, the systematic application of anti-seismic devices on exhibitions come after the seismic event, highlighting that, as said, a shared culture on protection of the movable heritage is missing. RESIMUS - (RESIlience MUSEums) is a multidisciplinary ongoing research developed by scholars at DIDA - Department of Architecture of the University of Florence. The research efforts focus on the preservation of the cultural heritage and on raising awareness about the seismic hazard for museum collections and museums. To do so, we provide innovative responses to vulnerability. We try to understand the fragilities of museum collections and exhibitions, and last, we propose solutions that integrate both coherent museographical design and anti-seismic devices. One of the RESIMUS’ goals is the progressive improvement of the museum display settings (updating the existing setups or replacing them), just like anti-seismic solutions are applied to buildings [18].

Although the presence of anti-seismic devices is not new in museums, their wide and wise application is missing. In the book When Galleries Shake, Podany highlights the lack of a document of analysis that allows a univocal interpretation of the various museum settings. He also reports the low level of coordination among institutions and research centers about seis-
mic assessment in museum. The research tries to fulfill these lacks and contributes to the general topic on seismic assessment and resilience of the ‘museum system’. The RESIMUS key strengths are the multidisciplinary approach and the close collaboration with museum institutions, one for all the National Museum of Bargello in Florence, Italy.

3.2. The RESIMUS approach

Within RESIMUS, we are developing a multidisciplinary methodology to analyze and rate the seismic vulnerability of museum collections and their setups. Such methodology is composed by several steps. First, we built up a form: a reasoned combination of pre-existing classifications, Augusti and Ciampoli, Ertürk and Sungay [19], Podany, and the one developed by RESIMUS. The goal is to test a universal simple tool allowing every institution to analyze its museum setting and to rank the seismic vulnerability of its exhibitions. The form is composed by three parts: 1) General report of the building and of the museum context; 2) Room form: a qualitative report for one room of the museum; 3) Object form: a qualitative report for one object and its display kept in the previous analyzed room (Room form). The sum of the Object forms of each room determine the level of vulnerability of that room.

In each museum, the spaces, the displays, and objects on view are different. Given the infinitive variables we might encounter, we are customizing the form-filled qualitative approach in order to release a single universal form of analyses. At the National Museum of Bargello, we tested this system of analyses (form) on the Donatello room [20]. Following the principle of “learning by doing,” such application was a trial test and subsequent upgradings of the original form has been made.

What do the form tell us? Likely, the results will show the safest and the riskiest objects on show and the vulnerability of both the exhibition typologies and the rooms. Such results should make aware the museum of the level of vulnerability of its collections. The museum might use these results in a strategic way. For example, if it plans a refurbishment of one wing of the museum or to proceed with some restorations, it might start from the most vulnerable room or exhibition display. Lastly, we are improving a tool (form) that can be used to classify the level of safety of the museum collections and that can help in planning future decisions. Assuming the correct use of the form by a consistent group of museums, from Florence for example, with the sum of these results we can trace and rank the vulnerability of the museums of Florentine area.

The second step is the numerical analyses applied to significant artworks [21]. We started from the most vulnerable, or the most significant from geometrical or masses point of view. The seismic analyses bases on the rigid block with Finite Element (FEM) procedures and are applied on 3D models acquired during 3D scanning campaigns [22]. This phase comprehends also mechanical tests on materials [23].

Given the data and the findings acquired in these phases, the third step consists in improving existing exhibition setups or in designing from scratch new museum settings. The design is a combination of both anti-seismic solutions and museographical coherent design.

3.3. The preliminary phase: the drawing as an instrument of study

As anticipated, a fundamental aspect of the research is the study of the museum spaces and their setups. Following the analysis pattern, from general (the containers: the museum space)
to detail (the object), each passage is characterized by the use of a fundamental instrument of study: the drawing. The act of sketching, indeed, is both an instrument of research and project. The in situ fast representation of the spaces and of the settings is useful to freeze concepts, understand the articulation of the spaces, and analyses the volumes. The transfer of the signs on paper forces a specific kind of reasoning that is not comparable to the act of taking pictures. Furthermore, the drawings might synthesize and illustrate some morphological aspects that a text or a table might not be able to efficiently communicate. The drawing is intended as a communication tool, has to be clear and correct, in this case the esthetics does not matter.

Through the sketches, we begin localizing the room we are examining inside the museum plans. This helps investigating the relations between the room and the other spaces. We draw plans and sections of the current situation of the rooms. Each room has specific proportions and dimensions. The sketch can immediately communicates how one can move inside the spaces, where the exhibition display and the works of art are located, the relation among museum setting, objects, and architecture, the composition of the architectural space (windows, doors, passages, etc.), the set up, the position of objects, and the prevalent visitors’ path. Further elements illustrate the kind of lightings and their location, the presence of safety and security devices, of the emergency exit routes, and of other movable elements (chairs, benches, barriers, etc.). All these information are ordered into the form and integrated by a photographic campaign and further data information.

The sketches are fundamental to understand the complexity of the museum setting, from the general characters (how the spaces are connected and works among each other, how one moves inside the spaces), throughout the level of the room (how the collections are exhibited, what kinds of display are used and the relations among each other), to the single object and its own exhibition device (intrinsic elements of the object, how is exhibited, the kind of display is used).

The process of learning by sketching is not a novelty. Such methodology is the same that artists and architect used since the *Grand Tour d'Italie* and *d'Europe*, the novelty is the aim: a preliminary study of the museum settings with the purpose of understanding the level of vulnerability of the movable collections.

Figure 1: Sketches of the National Museum of Bargello.
3.4. The designing phase

The results coming from the analysis are functional to the third phase of the project: the redesign of an exhibition rooms (or the design of temporary exhibition). The multidisciplinary work tries to address further questions: can museum display be both safe and coherent, according to updated exhibiting criteria? Can museography integrate with seismic prevention and museum policies? How? We can answer to these questions through the design and realization of exhibition projects holding together: consistent curatorial program, anti-seismic devices, and coherent museographical project.

About the anti-seismic devices, in section 2 we saw some successful experience adopted by some museums (isolator base, clips, interfaces, contour mounts, etc.). We already stressed the fact that every object, collection, and museum is different and therefore, even if we use the same safety device, this has to be declined according to its site specific context. Moreover, our goal is to consider the whole collection of the rooms instead of one single object or exhibition device. It is necessary developing a museographical projects holding together all the aspects of an exhibition design.

Curators, architects and museographers, restorers, engineers, etc. are all involved and work as a team. Prior the designing phase, there is a further preparation step consisting in a review of the literature, prepared by each discipline and then shared among the group. Even when academics and professionals speak the same idiom, they could not “understand” each other because they use their specific vocabulary. Therefore, it is necessary building up a shared basic knowledge and establishing a common vocabulary.

3.5. Architecture and museography: the relevance of the references

As architects and museographers, our contribution is presenting outstanding examples to share with colleagues. In learning from the past and looking up to the future, we seek for some remarkable case studies, provide critical analyses of museum references, and study anti-seismic museum solutions.

Speaking of exhibitions, we start from the principle of displaying. The act of showing is apparently simple, but, as stated by Newhouse [24], is a powerful gesture. A display project is, or should be, made by carefully and pondered decisions. The art of displaying starts before the birth of museums. Let think about the rise to the Acropolis with the position of statues and of the architectures [25], the debate about the arrangement of the Michelangelo’s David into the Signoria square [26], the interior layout of the Renaissance Studioli, like the one of Francesco I de Medici in Florence, or, still in Florence, the setup of the Uffizi Tribuna. Both public and private spaces contains messages to be indirectly communicate to an audience. The way of arranging statues and paintings in the spaces changes during the centuries, according to the transformation of taste and social context; following this path we follow also the history of museum. Although before the birth of museums the position of the objects (we do not speak of works of art yet) followed political or personal basis, with the opening of the public museums the display starts following curatorial principles. Therefore, positioning the artifacts in the space requires a careful study and can not be an improvised act.
In Italy, architects confront themselves with pre-existences and operate often inside historical architectures. The acknowledged capacity of some of them is link, beyond their specific skills, to the capacity of reading the sites, proposing measured solutions, and working closely with the curators. It is not by chance that the most successful display projects of the half of the XX century come from the collaborations between curators and architects [27]. Scarpa and Magagnato at Castelvecchio in Verona, Albini and Marcenaro at Palazzo Bianco and Palazzo Rosso in Genoa, BBPR and Baroni at Castello Sforzesco in Milan, Scarpa and Vigni at Palazzo Abatellis in Palermo are some famous examples. The refurbishment of the mentioned museums coincides with the Italian economic boom. In those years, temporary exhibitions became places of experimentation. New technological devices are tested and then commercialized, thanks also to the support of enlighten businessmen. Some prototypes become shops and home furniture; it is the case of the bookcase LB7 designed by Albini for the temporary exhibition Scipione e il bianco e il nero in 1940. The modular bookcase is used inside the Olivetti stores and then produced by Poggi and Cassina. Scarpa’s projects are tailored examples. They enhance the hand-crafted abilities of the workers, express the respect to both site and art-pieces, and state his “obsession” for the details. If several Scarpa’s remarkable solutions might be taken as references, other audacious outcomes are not suitable for anti-seismic setups. As an example, the sequence of the rooms that in Palazzo Abatellis conducts to the bust of Eleonora d’Aragona. Such sequence is a little lesson of museography: the position of the pieces in the space, the combination and choice of materials and colors, the arrangement of the objects in relation to the light sources, etc. Scarpa places the statues on tall and thin bases (concrete pillars, wooden columns, metallic wire) that, excluding the bust, are the extrusion of the bottom of the statues. Speaking of seismic vulnerability, by an external analysis, these configurations are weak from the geometric point of view: high center of the mass and slender forms. Obviously, to define the solidity of the display system we should investigate the kind of connection between object and base and the intrinsic features of the latter. To be precise, we should apply the study we pursue on the Ammannati’s fountain at the Bargello [28].

Figure 2: C. Scarpa, Palazzo Abatellis, Palermo.
The outstanding setup of the Pietà Rondanini in the Castello Sforzesco in Milan by BBPR took into consideration the relationship of one piece with the collection, the room, and the site. The visit of the room was a journey inside the museum tour. The pieces along the room worked as introduction to the view of the iconic Michelangelo’s work, the climax of the path. Once reached the top of the hall, the visitor walked along the semi-circular wall until discovering the marble: a kinetic approach. From 2015, the statue is in an other area of the castle, the Ospedale Spagnolo. The architect De Lucchi designed the new set up. The statue is placed at the center of an almost empty room, the base level of the marble has ben lowered, and the entire room is dedicated to it. We passed from the discovery of the Pietà in an intimate space to the so-called one-man show setting. The new setup embeds all the recent exhibition technologies, from lighting to anti-seismic [29]. The case of the Pietà exemplifies as the display of a museum object is the resulting combination of curatorial programs, managerial choices, museographical approaches, and applied technological solutions. The inauguration of the renovated spaces of the Castello Sforzesco, including the relocation of the Pietà, was programmed jointly with the most important event of the year: the Milan Expo 2015.
Although the fortunate season of exhibitions and experimentations come to an end, today we can profit of the possibilities that the agreements between academia and museum institution can offer. If practical experimentations are limited, in other words the possibility of using temporary exhibition as an experimentation field, we can use digital tools and technological devices to simulate and test possible solutions. Furthermore, we can profit of the presence of international networks composed by academics, professionals, and museums to easily share and compare results and experiences.

The presented examples are useful references for the Bargello’s case because they exemplify a successful dialogue between the display and the historical architectures, and the ability in enhancing the collections. Though these references are mostly not anti-seismic, they are useful to understand the museographical approaches. In the next months, jointly with the National Museum of Bargello, we will decide which room of the museum will be redesign. At that point, and before the effective design, other suitable references will be deeply examined and other safety devices tested. It is going without saying that each collections need specific references. For example the setting up for big statues follows different exhibit principles and solutions than an armory room, or a design dedicated to textiles is not applicable to ceramics. Once defined the room, and with the support of the analysis form, we will start the designing phase.

4. CONCLUSION

In designing a museum setup, whether if permanent or temporary, the simple introduction of anti-seismic devices is not enough. It is a multidisciplinary team work that has to take into consideration several variables. A non recent document, edited by the French Ministry of Culture in 1986, suggests some useful guidelines for new museums. About the museographical aspects, it expresses few basic principles, still valid. For examples, “a wrong codification of the display could lead to serious dangers. In particular, one risk is the standardization of museums in a rigid way and the trivialization of the museographical display. A mistaken approach would destroy one of the richest quality of museums, their uniqueness”[30]. The text enhances the importance of the object and its preservation: “in general, everything should start from the object and everything should put at the service of the object and its display. The general program of the exhibition has to be integrated with the evaluation of the “spatial potential” of each piece and its presence in the general scenography.” From these pages we have the confirmation of the importance of the specificity of the site, of the link between object and place, of the respect of diversity, but also the necessity of a scientific program and of prior detailed study of the site and of the object.

The museographical analysis composes by two main tools: the drawing, as an instrument of study, and the study of the references. These two tools are necessary to develop a coherent and correct museographical planning that integrates with both the curatorial project and the technological anti-seismic solutions (together with all the other safety elements). The choice and the study of the references relate to the site, to the kind of collection, and to the curatorial program.

The next phases of the research RESIMUS foresees the development of an integrated project at the National Museum of Bargello. The project will follow the curatorial direction
and the results of the analysis coming from the compilation of the RESIMUS forms. The group will collaborate in designing the best and coherent display setting. The final outcomes of the research will be a detailed design for a room of the museum.

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REFERENCES


RESPONSE SPECTRA OF RIGID BLOCKS WITH UNCERTAIN BEHAVIOR

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Abstract

The dynamic behavior of rigid blocks can effectively describe a wide variety of structural problems. The seismic behavior of art objects, the out-of-plane response of masonry façades, the foundation uplift of stiff structures, the rocking response of bridges’ piers (often adopted as seismic isolation strategy), represent only few examples of how the concepts behind dynamic rigid body motion govern problems with very different features. There is a wide literature on the topic aiming at understanding the dynamic behavior of rigid blocks and at proposing strategies to employ the basic principles of rocking motion to assess and seismically protect buildings and objects. In this paper an experimental survey on the seismic response of rigid blocks, characterized by different aspect ratios and scale factors, is presented. The results are reported in terms of response spectra considering the maximum rotation of the blocks. The experimental research is focused on small rigid blocks. However, the experimental results are also numerically simulated showing that widely used numerical models can fail in the prediction of the actual dynamic response of rigid blocks leading to wrong safety considerations. A marked uncertainty in the response of the blocks is identified under specific conditions that the author tried to identify and discuss. The conclusive remarks could be of help to assess the risk with respect to the overturning of rigid blocks under seismic excitation and to provide some considerations with respect to seismic protection strategies.

Keywords: Rigid blocks, Shaking table, Response spectra, Size effect, Aspect ratio.

1 INTRODUCTION

A wide variety of physical systems can be assimilated to the dynamic behavior of rigid blocks. In spite of its simplicity, it can provide useful results for different structural problems, which attracted the attention of many researchers. Among the other applications, the rigid block model was employed to describe the out-of-plane response of masonry structures [1]-[3], as well as the seismic vulnerability of art objects [4]. The principles on which the theory
of rigid block motion relies on is also the basis for isolation strategies of bridges with rocking piers [5]. It is evident how this simple system can be used to study blocks with very different size factors and aspect ratios, which can be investigated with different purposes.

The mentioned aspects led many researches, interested in very different fields, to address their attention to the rigid block model. In this context, the first pioneering studies on the dynamic behavior of rigid blocks were devoted to numerical investigations [6]. The first experimental studies were presented about 20 years later [7]. A wide literature on this topic was developed in the last 30 years, including alternative models to interpret the rigid block physical phenomenon, such as the ‘roll and rock’ model [8]; furthermore, additional features to enrich the basic model such as the presence of an elastic foundation [9]-[12], considering the occurrence of sliding [13] or adapting this approach to flexible structures [14] were also developed.

The theoretical studies found practical engineering applications mainly devoted to assess the seismic behavior of structural systems. The rigid block model was extensively employed to describe the out-of-plane behavior of masonry walls [15] considering different boundary and load conditions, or the seismic vulnerability of ancient Greek columns [16]. In the field of the seismic assessment other studies were devoted to the response of structures with foundation uplift allowed [17] and the earthquake protection of art objects [18]-[20].

The rigid block model was also employed with reference to the seismic protection of structures. In particular, a strategy which attracted the attention of the academic community consists of the application of rocking walls to existing structures [21] which allows controlling the drifts in multi-story buildings. Based on the same principles, another interesting retrofitting strategy for frame structures is the employment of controlled concrete rocking columns [22]. More recently, the concept of rocking isolation emerged also with reference to other structural typologies such as bridges [23].

The brief overview on the development of researches on rigid block models shows the great versatility of this model which can be successfully employed for very different studies. For this reason, further investigations decontextualized from specific engineering applications may be of help for a better understanding of the physical phenomenon on which the rigid block model is based. In this paper, a single degree of freedom shaking table is utilized to conduct an experimental campaign to investigate the seismic response of several specimens with different aspect ratios and size factors. The results are provided in terms of response spectra and compared with well-known numerical models. The presented results show a significant response variability of the experimental outcome in specific conditions, and highlight the limits of several numerical assumptions commonly accepted. In addition, the results obtained in the experimental campaign led to some more general remarks on the seismic vulnerability and on the possible seismic protection strategies of structures whose behavior can be adequately described by the dynamics of rigid blocks.

2 RIGID BLOCK MODEL

The dynamics of a can be investigated by integrating the differential equations, from a numerical point of view, or performing experimental campaigns.

In the following the hypotheses that the block is always in contact with the base and that no sliding can occur are assumed. From the numerical point of view, under these assumptions the block can be associated to two possible phases:
- full contact with the base;
- rocking.
The condition associated to the first of the two phases is simple since the block follows the ground motion and no rotation occurs. The rocking phase is governed by the following governing equation

\[
\left( m d^2 + I \right) \ddot{\theta} + m (h \cos \theta \pm b \sin \theta) \ddot{u}_g - m (h \sin \theta \mp b \cos \theta) (g + \dot{v}_g) = 0
\]

(1)

according to the scheme reported in Figure 1.

The reported signs refer to the possibility of clockwise (positive \( \theta \)) or anticlockwise (negative \( \theta \)) rotation of the block. \( \ddot{u}_g \) and \( \dot{v}_g \) represent the horizontal and vertical ground motions, respectively. In the following, according to the shaking table features, it will be assumed \( \dot{v}_g = 0 \). Eq. (1) can be easily conveniently integrated by means of the well-known Runge-Kutta algorithm.

Two transitions conditions have to be identified according to this model, namely incipient rocking (from full contact to rocking) and impact (from rocking to full contact).

The uplift of the block is associated to a value of the moment \( M_o \) applied to the block larger than the stabilizing moment \( M_s \), that is

\[
M_o > M_s
\]

(2)

being

\[
M_o = |\ddot{u}_g| mh
\]

\[
M_s = \alpha_s mgb
\]

(3)

where \( 0 \leq \alpha_s \leq 1 \) is a coefficient to reduce the stabilizing moment which accounts for block imperfections and can be evaluated experimentally.

Combining Eqs. (2) and (3) the incipient rocking conditions can be expressed as

\[
|\ddot{u}_g| > \alpha_s g \frac{b}{h}
\]

(4)

that is the theoretical uplift acceleration is given by \( \ddot{u}_{g,up} = \alpha_s gb/h \). Regarding the definition of the impact conditions, several models can be considered. Here the impact and the subsequent re-uplift implies a loss of energy associated to an empirical coefficient \( e \). In particular, given an impact velocity \( \dot{\theta}^- \) the outcome of the impact is a velocity \( \dot{\theta}^+ \) which can be defined as follows

\[
\dot{\theta}^+ = e \dot{\theta}^-
\]

(5)

The coefficient \( e \) can be evaluated according to theoretical assumptions or experimentally. Under the hypothesis of rigid block and rigid ground, some authors assume that when the rotation reverses, it continues smoothly from point A to B, there is no bouncing of the block and the assumption of conservation of the angular momentum is made [24]. Equating the momentum about B before and immediately after the impact, it's possible to obtain, \( e_{\theta} = 1-3/2 \cdot \text{sen}^2 \delta \). Other authors assume the foundation in contact at the instant of impact in a single point and the vertical component of the momentum vanishes during the impact [14]. These assumptions lead to \( e_c = h/d \). Here this coefficient has been also evaluated empirically by means of a free oscillations test. If the block shows an asymmetric behavior, the coefficient \( e \) can be also differentiated according to the impact side.
By employing a single degree shaking table and making use of a contactless acquisition data strategy, some aspects of the dynamic behavior of rigid blocks are here investigated. The experimental setup is briefly described in section 3.1.

Nine wooden specimens have been tested, Figure 2, considering three different aspect ratios and three size factors, as better specified in Table 1. The Scale factor (Sc.f.) represents length $d_i$ relative to the $i$-th block normalized by the corresponding value $d_1$ of the reference largest block.

In Table 1 three further parameters have been introduced, that is the critical angle $\delta = \arctg \left( \frac{b}{h} \right)$, the minimum friction coefficient $\mu = \frac{b}{h}$ to prevent sliding when no vertical component of the ground motion is considered, and the frequency parameter $p$.
\[ p = \sqrt{mgd/(I_o + md^2)} \]. It was experimentally verified that for all the considered tests the available friction resistance was sufficient to prevent the occurrence of sliding.

Three sets of coefficients of restitution have also been adopted in the numerical simulations, that is \( e_H \) and \( e_C \) evaluated according the theories reported in [6] and [14], respectively, as well as \( e_e \) which was assessed experimentally. Then, for a given ground motion, response spectra associated to an increasing value of the PGA have been derived, highlighting the influence of the aspect ratio and of the size factor. In addition, behavior zones are identified considering the outcome of the experimental campaign.

3.1 The experimental setup

The experimental setup includes a single degree of freedom shaking table and a camera for contactless displacement data acquisition, which has been chosen in view of several crucial aspects:

i. it does not require the use of devices to be constrained to the specimens, which could interfere in the test changing the dynamic properties of the system, particularly in the cases of smaller size specimens;

ii. since the range of investigated frequencies is not very high, displacement acquisition appears to be appropriate and accurate.

Figure 2: Tested specimens.
The block response is determined employing markers whose position can be traced during test by means of a high-resolution and frequency-acquisition camera located on a tripod in front of the shaking table, as shown in Figure 3.

![Figure 3: Experimental setup.](image)

Once the video acquisition has been completed, the frames are processed using the Tracker Video Analysis and Modeling Tool software ver. 5.0.7" [25]. In the following a brief description of the experimental devices and of the data acquisition procedure are reported.

### 3.1.1. Shaking table

The earthquake was simulated through the use of a single degree of freedom shaking table designed to perform seismic simulations or vibration tests. The vibrating table, from the LO.F.H.I.S. series ND13014 (Low Frequency High Stroke and Velocity Shaker) distributed by CENTROTECNICA SRL, consists of an aluminum plate forced to move in one direction using the linear motor technology that defines the position of the plate over time. The shaking table has a total weight of 650 kg and a working area of 650x660 mm (expandable by introducing a top plate bolted to the table supports) to accommodate a maximum payload of 100
The vibrating table, with displacement control, allows a maximum displacement capacity of 28.6 mm (± 14.3 mm), maximum speed of 2.5 m / s and works in a frequency range between 0 ÷ 100 Hz.

3.1.2. Camera

The Sony DS-RX100 M5 camera used for video records is equipped with a 20.1-megapixel Exmor RS® CMOS sensor with the ability to record in 4K up to 120 fps, and 1,920 x 1,080 to 240, 480 and 960 fps in HFR (High Frame Rate) mode. The camera is located on a VCT-R640 stand and activated by means of the RMT-VP1K remote control, which allows not to disturb the initial frames. The huge amount of data is managed through the use of SF-UX2 series SD memory card (Speed Class 10) with high acquisition speed. To improve the lighting conditions, the block was lit with three 120W halogen lamps.

3.1.3. Tracking software and data processing procedure

Through the adopted tracking motion software, it is possible to track the location of some reflecting markers of very little weight and size placed on the object and on the vibrating table and reconstruct the trajectory with respect to a fixed reference system. The free software Tracker [25] is based on the Java Open Source Physics (OSP) framework.

The data processing procedure is first calibrated in order to reduce the errors connected to the camera optical distortion. In particular, a simple test pushing the table to the maximum positive and negative displacement capacities is performed. The displacement data obtained as output of the shaking table are then compared with those acquired through the processing of the frames considering a marker constrained to the table. It turned out that the correction to be applied to the data obtained through visual acquisition is proportional to the distance from the central point of the shaking table; thus, a corrective coefficient can be calibrated to get unbiased the data.

Once the procedure is calibrated, the rotation of the block at each step is computed considering the horizontal displacements of two markers placed on the specimen on the same vertical alignment. For convenience, these two points have been chosen at the center of gravity of the block (point G, Figure 4) and at the center top of the block (point P, Figure 4), respectively. Then, the rotation can be simply assessed as:

\[ \theta = \arcsen \left( \frac{\Delta x}{h} \right) \]  

being \( \Delta x = x_p - x_G \), the relative displacement between top and center of gravity for a generic frame.
3.2 The experimental campaign

3.2.1. Assessment of the coefficient of restitution $e_E$

It was necessary to reproduce in laboratory the ideal conditions that allow the applicability of the model assumed, for these reasons the blocks were placed on a support equipped with a sheet of sandpaper on the surface, so as to ensure contact with the base support and avoid slipping. In this way the coefficient of friction $\mu$ is sufficiently large to exclude sliding.

The calibration of the restitution coefficient was carried out starting from experimental tests in free vibrations in which the upper end of the block was displaced by a nylon float which was ignited. Once the free vibration test was processed and response in terms of rotation of the block was computed the coefficient of restitution was evaluated by employing the strategy proposed in [15]. In particular, by considering the peak rotations $\theta_i$ and $\theta_j$ at two generic rocking phases $i$ and $j$, being $i > j$, the coefficient of restitution can be computed as

$$e_E = 2(i-j) \frac{\cos(\delta - \theta_j) - \cos\delta}{\cos(\delta - \theta_i) - \cos\delta}$$

In order to reduce the measurement errors, here it was assumed $i=6$ and $j=0$. The obtained results are reported in Table 1.

3.2.1. Response spectra tests

The nine wooden specimens were subjected to the ground motion associated to the Friuli earthquake registered in Tolmezzo (Italy) in 1976 (EW component), which is associated to a Peak Ground Acceleration (PGA) equal to 0.3513 g, (see Figure 5). The signal was then scaled and 60 tests were performed for each specimen considering a variable PGA ranging between 0 and 0.60 g with a step size equal to 0.01 g. The 540 tests were then processed according to the procedure described in the previous section, and the maximum observed rotation normalized by the critical angle $\delta$ was computed.
In Figure 6 for the nine wooden specimens, the observed maximum normalized angular response is reported as a function of the assigned PGA. It is worth to mention that three behaviors of the block can be identified, namely full contact, rocking, overturning. Such behaviors can be better described as follows:

- full contact means that for all the duration of the ground motion the incipient rocking condition is not achieved;
- rocking means that the block tilts but doesn’t overturn;
- the overturning area is associated to a band that starts with the first overturning PGA irrespective of the fact that there could be cases for higher PGAs which do not lead necessarily to the overturning of the blocks (e.g. blocks A1).

In the same Figures the theoretical uplift acceleration ($\ddot{u}_{up}$), numerically computed according to Eq. (4), is reported by a vertical dashdot line. It is worth to mention that the first rocking is not easy to be identified experimentally because of the measurement limits and because the blocks possess an intrinsic deformability. For this reason, conventionally, here the first rocking was associated to a measured normalized maximum angle equal to the 0.5% of the critical angle $\delta$. The transition PGAs are then reported in Table 2. The experimental PGA at rocking outset is in all the considered cases slightly lower than the theoretical one; this is probably due to the imperfections of the contact surface of the block and to the block deformability. In terms of first overturning PGA slender blocks overturn at lower value of the PGA than thicker blocks, and smaller blocks are more keen to overturning than larger blocks.
Figure 6: Maximum normalized angular response vs PGA for the nine tested specimens.

<table>
<thead>
<tr>
<th>Block</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>First rocking</td>
<td>0.15</td>
<td>0.17</td>
<td>0.16</td>
<td>0.24</td>
<td>0.27</td>
<td>0.23</td>
<td>0.33</td>
<td>0.40</td>
<td>0.38</td>
</tr>
<tr>
<td>First overturning</td>
<td>0.24</td>
<td>0.22</td>
<td>0.22</td>
<td>0.31</td>
<td>0.37</td>
<td>0.26</td>
<td>0.45</td>
<td>0.48</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 2: Transition PGAs

For each specimen corresponding numerical simulations were performed considering the set of three coefficients of restitution $e_H$, $e_c$ and $e_E$ reported in Table 1. It is worth to note that the uplift coefficient $\alpha_s$ was assumed equal to 1 in all the numerical simulations. The numerical results usually underestimate the actual response of the blocks in terms of rotation amplitude: this happens in all the considered cases but B2. Another remark to be brought to
light is that even considering an experimental evaluation of the coefficient of restitution, the results provided by the numerical simulations appear to underestimate the actual safety of the block, as shown in Figure 7 where experimental and numerical results are compared.

3.2.1. Variability of the response spectra

During the experimental campaign, when some tests were repeated (claiming to consider the same conditions in terms of ground motion, specimen and initial conditions), it was observed that when the object tilts the reproducibility of the tests was almost impossible. The chaotic character of the dynamic response of rigid blocks and the consequent deterministic unpredictability was already observed in [26] for the case of sinusoidal excitation. Here, a specific campaign to assess the response variability for a specific block (B2) was performed. In particular, for each value of the considered PGAs ten tests were performed and processed for a total amount of 600 additional tests. In Figure 8 the results obtained from this additional...
set of experiments is depicted identifying the upper and lower bound of the maximum normalized angle achieved. In this case four bands have been highlighted. The first band (up to 0.2645 g) is related to full contact and correspond to that identified in Figure 6. The second (up to 0.33 g) and third (up to 0.41 g) bands correspond to the rocking behaviors with a distinction: for lower values of the PGA the response variability is limited and the block behavior can be considered (almost) deterministic; then, as the PGA and the number of impacts increase, a higher response variability is encountered. Finally, the fourth band starts with the lowest value of the PGA which leads the block to overturning at least in one test.

Figure 8: Response variability of the tests for Block B2

4 CONCLUSIONS

In this paper the first results of a larger experimental campaign on the dynamic behavior of rigid blocks is presented. Small rigid blocks have been tested on a single degree shaking table. A contactless measurement strategy was adopted to assess the response of the blocks. Nine wooden specimens were tested considering different aspect ratios and size factors. The specimens were subjected to free rocking motion aiming at the identification of the coefficient of restitution and to ground motions with increasing PGAs. The experimental results were compared with numerical simulations which showed the limit of numerical models to simulate the dynamic behavior of rigid blocks, especially when the response is characterized by rocking with many impacts. In addition, it is shown how the reproducibility of the experimental tests is difficult when the tilting behavior is relevant, and the variability of the response is shown. The presented results, to be confirmed with other ground motions and to be extended to other blocks with different aspect ratios and size factors, represent a first step aiming at a reliable identification of behavior maps useful for seismic vulnerability assessments and protection design purposes.

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DYNAMIC ANALYSIS OF ARTIFACTS: EXPERIMENTAL TESTS FOR THE VALIDATION OF NUMERICAL MODELS

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Abstract

The protection of art goods is an important issue of the seismic engineering. Artifacts are often made by fragile and ancient material, and they can easily present irregular shapes and high slenderness. In these years many contributions have been devoted to their analysis, based on numerical models having different complexity and computational effort. The Finite Element Models represent the most common and versatile approach for the representation of artifacts. Nevertheless, their reliability depends on the numerical assumptions made for the analysis, which requires a wide number of information regarding the dynamic behavior of the artifacts, such as the friction between the analyzed object and its support, the effective damping, the inelastic involvement of material, etc. In this work an experimental campaign has been started aimed at determining the main factors which affect the dynamic representation of artifacts through FEM analysis and simplified models. A large number of experimental tests, both static and dynamic, have been performed, by adopting both real and reduced scale objects.

Keywords: Validation of numerical models, shaking table test, experimental campaign
1 INTRODUCTION

The art goods play a crucial role in defining and understanding the identity of communities, and therefore they should be carefully protected against possible dangers and hazards. The seismic events occurred in the last decades (L’Aquila 2009, Emilia 2012, Centro Italia 2016), however, have induced serious injuries to many monuments and artifacts [1], evidencing their seismic vulnerability and pointing out the importance of increasing the current prevention policy [2,3].

The assessment of the seismic safety level of an art good is achieved by comparing its expected seismic response to the corresponding limit value. The most advanced approach for determining the seismic response of artifact is the Finite Element Method. The adoption of FEM, however, requires many assumptions regarding the mechanical properties of the materials and the boundary conditions to assume for the object and for its restraint. Previous studies developed by some of the Authors [4,5] pointed out the importance played by some quantities, such as the strength and the friction coefficient of the materials, in the achieved collapse mechanism and the expected dynamic response of sculptures. The reliable calibration of the numerical model to use for seismic analysis requires the availability of experimental results. In these years several researchers provided precious contributions to the calibration of numerical models for seismic analyses of artifacts [6-9]. However, the available experimental data regarding art goods are still few, and the calibration of numerical models is an open issue.

In this paper, the first results of an experimental campaign focused on the calibration of FE models to use for representing the seismic behavior of art goods are presented. The campaign, still in progress, consists of two main phases.

The first phase, presented in the Sections 2 and 3, refers to the mechanical properties of some of the materials most used for ancient and current artifacts. The materials have been investigated with reference to their strength (Section 2) and to their attitude to slide over a standing surface (Section 3). The strength of the material has been checked through uniaxial compressive strength (UCS), combined to ultrasound lectures, according to the current Code [10-12] provisions, whilst the friction coefficient has been checked both through static and dynamic tests. Right now, only two materials, i.e. the Carrara marble and the sandstone (pietra serena), have been checked. Both the materials are taken from quarries located in the Florentine area, and they can be representative of the sculptures made in the Tuscan Renaissance. Further materials, such as ceramic and glass, should be included in the experimental campaign. At the current time, the friction coefficient has been checked with reference to the marble only, and to standing surfaces made of different materials, such as masonry and mortar, glass, Plexiglas, timber and steel.

The second phase of the survey, whose beginning is planned for September 2019, is focused on real artifacts. The tests will be performed through shaking table, checking the dynamic response of the artifacts to sinusoid acceleration histories and real ground motions. In Section 4 the experimental program is described. The experimental results obtained for the dynamic response of artifacts will be compared to those provided by the FE models. Two different FE models will be adopted in the simulation, and compared to the experimental results, in order to evaluate the role of the single assumptions as a function of the peculiarity of each model.
2 EXPERIMENTAL EVALUATION OF THE MATERIALS STRENGTH

The compressive strength is usually assumed as reference quantity for the assumptions of all the main mechanical properties, such as the Young modulus. The test should be made on all the main materials used for art goods, such as stones, ceramics, etc. Right now, however, two stones only have been tested, i.e. the Carrara marble and the sandstone (“pietra serena”). The strength of the material has been found through a uniaxial compressive strength (UCS), combined to ultrasound lectures, whilst the friction coefficient has been found through an inclined plane devise.

2.1 Marble

Most part of the marble sculptures made in the Florence area in the XVI Century, were made with Carrara marble. Therefore, the compressive test has been made on six samples taken by a Carrara quarry. The samples have a cylindrical shape, with a diameter of 5.4 cm. The strength was found through a Uniaxial Compressive Strength (UCS) test made through a hydraulic press INSTRON MODEL 5592 with a maximum stress equal to 600 kN, at a constant velocity of 1±0.5 MPa/s, according to UNI EN 1926:2007 and ASTM standards (ASTM 1985). The UCS test was even enhanced by ultrasound lectures. The ultrasound velocity was measured by DSP – UTD 1004 model N034 Boviar, by applying two transducers on the opposite sides of each sample. Figure 1 shows the images of the samples at the beginning of the crushing, whilst in Figure 2 shows the strength values provided by the tests, together with the main data on the samples.

![Figure 1. Samples under the UCS test.](image1)

<table>
<thead>
<tr>
<th>density</th>
<th>Ultrasound lectures</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[KN/m³]</td>
<td>[μs]</td>
</tr>
<tr>
<td>M1</td>
<td>26.66</td>
<td>19.5</td>
</tr>
<tr>
<td>M2</td>
<td>26.71</td>
<td>19.6</td>
</tr>
<tr>
<td>M3</td>
<td>26.50</td>
<td>15.4</td>
</tr>
<tr>
<td>M4</td>
<td>26.78</td>
<td>18.2</td>
</tr>
<tr>
<td>M5</td>
<td>26.84</td>
<td>16.2</td>
</tr>
<tr>
<td>M6</td>
<td>26.84</td>
<td>16.9</td>
</tr>
<tr>
<td>mean</td>
<td>26.72</td>
<td>17.6</td>
</tr>
<tr>
<td>C.o.V.</td>
<td>0.5%</td>
<td>9.2%</td>
</tr>
</tbody>
</table>

![Figure 2. Test on the marble: data and results](image2)
2.2 Sandstone

The sandstone has been used in Tuscany since the Etruscan age. In the Renaissance it has been widely used for palaces and art goods. Indeed, it is the most valuable (strong and durable) type of the “macigno” stone, that is one of the most common stone of the Florentine area. The color is gray at the extraction, and it becomes ochre with the time, due to the oxidation process. Due to its large diffusions, there have been, along the centuries, several quarries, spread in the country. Each of them has provided material with different colors and mechanical properties. In this survey, four types of sandstone, made by six samples each, have been tested. In Figure 3 the location of the quarries of the checked stone has been shown. The samples have a cubic shape, with a side of 5 cm. Some further tests have been performed to check density, absorption and porosity of the material; in this case, smaller cubic samples, with sides equal to 2 cm, have been adopted.

An extensive presentation of the results can be found in [13], while Figure 4 and Table 1 show, respectively, the values obtained for the compressive strength of each sample and the main data provided by the test. As can be noted, the strength of the sandstone is much more variable than the marble one; the Coefficient of Variation (CoV) of the sandstone, indeed, varies between 34% and 52% in the considered samples. Moreover, the strength of the material results to be very sensitive to the extraction quarry: the sandstone coming from the “Bigi” quarry has a strength three times lower that the one of the samples coming from the other
quarries. Even the other mechanical properties, such as porosity and imbibition coefficient, result to be sensitive to the extraction quarry.

<table>
<thead>
<tr>
<th>sample</th>
<th>density [g/cm³]</th>
<th>imbibition coeff. [%]</th>
<th>porosity [%]</th>
<th>Ultrasound lectures [μs]</th>
<th>strength [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trassina</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>2.62</td>
<td>1.76</td>
<td>4.56</td>
<td>22.6</td>
<td>26.2</td>
</tr>
<tr>
<td>T2</td>
<td>2.62</td>
<td>1.86</td>
<td>4.78</td>
<td>20.6</td>
<td>25.7</td>
</tr>
<tr>
<td>T3</td>
<td>2.63</td>
<td>1.79</td>
<td>4.60</td>
<td>25.0</td>
<td>21.7</td>
</tr>
<tr>
<td>T4</td>
<td>2.63</td>
<td>1.88</td>
<td>4.85</td>
<td>26.3</td>
<td>21.4</td>
</tr>
<tr>
<td>T5</td>
<td>2.62</td>
<td>1.76</td>
<td>4.54</td>
<td>21.2</td>
<td>21.8</td>
</tr>
<tr>
<td>T6</td>
<td>2.62</td>
<td>1.74</td>
<td>4.50</td>
<td>18.3</td>
<td>21.0</td>
</tr>
<tr>
<td>mean</td>
<td>2.62</td>
<td>1.80</td>
<td>4.6</td>
<td>13.9</td>
<td>20.0</td>
</tr>
<tr>
<td>CoV</td>
<td>0.2%</td>
<td>3.2%</td>
<td>3.1%</td>
<td>65%</td>
<td>39%</td>
</tr>
</tbody>
</table>

| Carrara |              |                        |              |                          |               |
| C1     | 2.62          | 1.73                   | 4.46         | 20.5                     | 14.7          | 14.6         | 27         |
| C2     | 2.61          | 1.78                   | 4.57         | 10.4                     | 10.5          | 10.2         | 67         |
| C3     | 2.61          | 1.92                   | 4.93         | 24.7                     | 14.3          | 14.5         | 85         |
| C4     | 2.61          | 2.00                   | 5.13         | 10.2                     | 10.1          | 10.3         | 141        |
| C5     | 2.61          | 1.94                   | 4.98         | 10.0                     | 10.3          | 10.2         | 84         |
| C6     | 2.61          | 1.81                   | 4.66         | 9.8                      | 10.7          | 11.4         | 47         |
| mean   | 2.61          | 1.86                   | 4.8%         | 12.6                     |               | 75.2         |            |
| CoV    | 0.2%          | 5.6%                   | 5.5%         | 71.2%                    | 39%           |

| Villa "I Tatti" |              |                        |              |                          |               |
| VT1     | 2.61          | 1.78                   | 4.59         | 16.2                     | 14.9          | 15.9         | 61         |
| VT2     | 2.61          | 1.78                   | 4.59         | 16.6                     | 15.3          | 16.4         | 82         |
| VT3     | 2.61          | 1.74                   | 4.48         | 15.6                     | 16.6          | 16.3         | 46         |
| VT4     | 2.61          | 1.90                   | 4.87         | 13.0                     | 13.2          | 13.8         | 63         |
| VT5     | 2.61          | 1.81                   | 4.67         | 16.0                     | 14.2          | 14.3         | 70         |
| VT6     | 2.62          | 1.68                   | 4.34         | 16.8                     | 14.2          | 14.3         | 26         |
| mean    | 2.61          | 1.78                   | 4.6%         | 15.2                     |               | 58.0         |            |
| CoV     | 0.2%          | 0.07%                  | 3.9%         | 69.3                     | 34.0%         |

| Bigi      |              |                        |              |                          |               |
| B1       | 2.47          | 3.60                   | 8.60         | 22.6                     | 26.2          | 29.4         | 20         |
| B2       | 2.48          | 3.32                   | 7.98         | 20.6                     | 25.7          | 19.4         | 21         |
| B3       | 2.48          | 3.52                   | 8.43         | 25.0                     | 21.7          | 22.1         | 40         |
| B4       | 2.48          | 3.44                   | 8.26         | 26.3                     | 21.4          | 21.5         | 16         |
| B5       | 2.47          | 3.70                   | 8.83         | 21.2                     | 21.8          | 24.5         | 12         |
| B6       | 2.47          | 3.38                   | 8.10         | 18.3                     | 21.0          | 17.2         | 22         |
| mean     | 2.5           | 3.49                   | 8.4          | 22.2                     | 21.8          |               |            |
| CoV      | 0.2%          | 4.06%                  | 3.8%         | 39.9                     | 44%           |

Table 1. Test on the sandstone: data and results

3 EXPERIMENTAL EVALUATION OF THE FRICTION COEFFICIENT

The friction coefficient plays a fundamental role in the numerical simulation of the dynamic response of artifacts, since the amount of friction determines the collapse mechanism experienced by the artifact under seismic excitation. At the current time, the friction coefficient has been determined only for the marble samples, i.e. cubes having 10 cm sides. Several materials, instead, have been considered for the standing plane. The experimental campaign, still going on, consists both of static and dynamic tests.

3.1 Static test

The angle between the sample and the standing plane corresponding to the sliding activation has been checked through the device represented in Figure 5. The device has a horizontal fixed plane, and an inclined, adjustable plane where the standing plane is fixed, whose angle

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can be read through a goniometer. The test has been performed by checking the sliding angle which activate the sliding of the samples, which are marble cubes of 10 cm sides. Each of the six sides of the samples has a different finishing (shown in Figure 5), i.e. respectively: diamond sawcut, polished, fine chiselled, rough chiselled, gradined and bush hammered.

The test has been performed by fixing the standing plane to the device, placing a cube of marble over it, and gradually increasing the slope until achieving the sliding of the specimen.

<table>
<thead>
<tr>
<th>Device for testing the sliding slope</th>
<th>Textures of the six sides of the marble samples</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.jpg" alt="Image" /></td>
<td><img src="image2.jpg" alt="Image" /></td>
</tr>
<tr>
<td><img src="image3.jpg" alt="Image" /></td>
<td><img src="image4.jpg" alt="Image" /></td>
</tr>
<tr>
<td><img src="image5.jpg" alt="Image" /></td>
<td><img src="image6.jpg" alt="Image" /></td>
</tr>
<tr>
<td><img src="image7.jpg" alt="Image" /></td>
<td><img src="image8.jpg" alt="Image" /></td>
</tr>
<tr>
<td><img src="image9.jpg" alt="Image" /></td>
<td><img src="image10.jpg" alt="Image" /></td>
</tr>
<tr>
<td><img src="image11.jpg" alt="Image" /></td>
<td><img src="image12.jpg" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure 5. Device for determining the friction coefficient and finishing of the sample sides.

The test is aimed to provide data regarding both big sculptures which stand over pedestals or floors, and smaller art goods, exhibited over showcases’ shelves. As a consequence, the different materials have been considered for the standing plane. Three planes consisting of masonry covered by mortar, have been considered to represent the interface of sculptures with their pedestals, and four further planes, made respectively of glass, Plexiglas, timber and steel have been considered to represent the interface of small artifacts with shelves.

The masonry planes differ from each other for the covering mortar, that is, respectively, cement, lime and mixed one. In Figure 6 the values provided for the Friction Coefficient (FC) by the performed tests have been shown. Each result shown in Figure 6 represent the mean values of six results, referring to as much samples.
3.2 Dynamic test

A dynamic test through the shaking table has been planned in order to check the effects of the friction coefficient on the dynamic response of the checked systems (samples over standing planes). Previous experimental campaigns [14] pointed out interesting relationships between the friction coefficient and the type of dynamic response (rest, sliding and rocking), as much as the fundamental frequency of the system, and the amplification in the acceleration.

The test will be performed on the same samples and standing planes considered in the static test, by assuming the equipment described in Section 4.1.

4 EXPERIMENTAL BEHAVIOUR OF ARTIFACTS

4.1 Dynamic response of artifact

The dynamic test on handcraft objects will be performed through the bidirectional shaking table at the Disaster Resilience Simulation Laboratory at the Politecnico di Torino, Italy. The structure of the shaking table consists of steel profiles, whereas the upper platform, where specimens can be fixed, is made of aluminium. Two parallel tracks are located side by side and connected through transversal rectangular sections. Tracks’ profiles are 3 meters long and
the section’s size is 40x100x4 mm. Upon the steel profiles there are aluminium guides allowing the motion, along the longitudinal direction, of sliders that support two 600x500x10 mm aluminium platforms. Each track has its own platform, which is moved by a linear electric actuator anchored under it. On the small platforms, other two tracks and platforms are fixed. Type and section of the steel profiles are the same of the bottom ones, while the length is shorter (600 mm). For the transversal motion other two linear electric actuators are anchored under the aluminium platforms. If necessary, a bigger platform (1500x1500x10 mm) can be installed (Figure 8). The linear electric actuators adopted are manufactured by the company LinMot and each is made of a stator, a slider and a motor. The longitudinal ones have a slider’s length of 800 mm and a maximal stroke of 510 mm, whereas the transversal ones have a slider’s length of 500 mm and a maximal stroke of 330 mm. The power supply, the two transformers and the four drivers to control the motors are provided by LinMot as well (Figure 9). The drivers are fundamental for the tuning of the motors (i.e. the initial configuration of all the control parameters) to have a response coherent with the input data. This operation is done through the software LinMot-Talk that is also used to switch on the actuators and to bring them in the home position. The software used for the activation and control of the shaking table is LabView. The seismic input is sent to the shaking table through a myRIO device manufactured by National Instruments. This device is physically connected to the motors’ drivers and also to an accelerometer, which is located on the platform and allows catching the actual response of the system. Simply, it is possible to connect a USB pen drive containing the seismic signal in terms of displacements to the myRIO device. The LabView code is used to set the input and output sampling rates, to generate a sinusoidal seismic signal or to load a real one, to scale it, to start and stop the motion and finally to compare the data obtained from the accelerometer with the theoretical ones.

Figure 8. Shaking table at Politecnico di Torino.
Two different loading conditions will be considered in the test. The first one consists of mono-dimensional periodic acceleration histories. The choice of the frequency content to assume will be made on the basis of a preliminary experimental modal analysis [7], which will be performed on the assumed case-studies, by considering different standing surfaces. Different seismic intensities will be considered ranging between 0.10 g and 0.35 g, i.e. the range of interest of most part of the art goods exhibited in the Italian Museums.

The second loading case consists of real ground motions. The ground motions will be selected in order to be spectrum-compatible to the elastic spectrum provided by the Italian Code for different Italian location. The first considered location is Florence, where is placed the Museum of Bargello, related to the research project (RESIMUS) which promoted the experimental campaign. The dynamic tests will be performed with the main horizontal component of the ground motion, with both the horizontal components and with all the 3 effective components, including the vertical one, in order to check the effects of these assumptions in the numerical analysis. Different ground motions will be considered as seismic input, in order to simulate the effect of the foundation soil on the seismic response of the artifact, according to the current soil classification provided by the Technical Code (NTC 2018).

4.2 Validation of numerical models

It is well known that the reliability of the numerical analysis depends on the assumptions made in the analysis.

Some numerical analyses were performed within the research project “Resimus” [4,5,15-17] on the statue “Cerere” by Bartolomeo Ammannati. Two advanced Finite Element models have been adopted for representing the artifact. The two models differ from each other both for the material behavior and for the restraint condition: the first model [18] assumes a non-linear elastic behavior of materials and a perfect continuity between the statue and the pedestal. The second model [19], instead, assumes a linear elastic behavior for the materials, and introduces a proper contact surface between the statue and the pedestal. The results provided by performing a time-history analysis through the two models evidenced the crucial role played by the assumed friction coefficient when the detachment between statue and pedestal is taken into account. In the above mentioned analysis, the considered Friction Coefficient has been assumed on the basis of a static test made on the same materials of the artifact.

The planned experimental test through shaking table requires a preliminary geometrical survey of the artifact, in order to obtain the geometrical model to use for the analysis. The geometrical model provided by a photogrammetric or laser-scanner survey is usually very detailed and limited to the object external surface. Therefore, it needs to be simplified and
changed in a 3-d model, in order to maintain a satisfactory precision, reducing the computational effort required by the structural analysis. The dynamic test should be performed on artifacts made of different materials, shape and dimensions, in order to check the effectiveness of the numerical simulation at the varying of the conditions.

A third simplified model will be validated through the experimental tests. It consists of a 3-d representation of the dynamic behavior of the statues under the earthquake. Two rigid perpendicular beams have been considered to model the contact surface, and another beam element is used to identify the position of the center of gravity above the contact surface (Figure 10). By defining different lengths of rigid beams, the model is able to take into account the eccentricity as a variable. To consider the effect of friction, a Friction-Pendulum Isolator element has been used to model the contact surface. This element is able to model combination of different conditions varying from at-rest to slide, or from uplift to slam-down for the cases of friction and rocking, respectively. The pendulum radius of the slipping surface was set to zero to consider the flat surface friction. The element models the coupled biaxial friction at contact surface considering the post-slip stiffness. The friction forces are proportional to both external normal force and friction coefficient. The axial force \( P \) is modeled with a compression-only gap element that does not carry the tension force in the case of uplift and it is given by:

\[
P = \begin{cases} 
K_z d_z & \text{if } d_z < 0 \\
0 & \text{otherwise}
\end{cases}
\]  

where the \( K_z \) is the vertical stiffness in negative axial direction (-Z) and \( d_z \) is the vertical displacement of the rigid body base at the contact surface. \( K_z \) is set to some large value in order to consider the rigidity of the contact surface. The nonlinear behavior is considered for each shear (friction) degree of freedom in \( x \) and \( y \) directions. The friction force-deformation relationship is given by:

\[
f_x = -P \mu_x z_x, \quad f_y = -P \mu_y z_y
\]

where \( f_x \) and \( f_y \) are the friction forces in \( x \) and \( y \) directions, \( \mu_x \) and \( \mu_y \) are velocity-dependent friction coefficients, and \( z_x \) and \( z_y \) are internal hysteretic variables. In order to accurately model the problem, the fast and slow friction coefficients are considered as a function of velocity. The initial values of \( z_x \) and \( z_y \) are zero and they evolve according to following differential equation:

\[
\begin{bmatrix}
\dot{z}_x \\
\dot{z}_y
\end{bmatrix} = \begin{bmatrix}
1-a_x z_x^2 \\
-a_x z_x z_y
\end{bmatrix} \begin{bmatrix}
\frac{K_x}{P \mu_x} \dot{d}_x \\
\frac{K_y}{P \mu_y} \dot{d}_y
\end{bmatrix}
\]

for \( \sqrt{z_x^2 + z_y^2} \leq 1 \)  

where \( K_x \) and \( K_y \) are the elastic shear stiffness constants in the absence of sliding, and \( a_x \) and \( a_y \) are binaries parameters deepening on velocity in \( x \) and \( y \) direction:

\[
a_x = \begin{cases} 
1 & \text{if } \dot{d}_x z_x > 0 \\
0 & \text{otherwise}
\end{cases}, \quad a_y = \begin{cases} 
1 & \text{if } \dot{d}_y z_y > 0 \\
0 & \text{otherwise}
\end{cases}
\]

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5 CONCLUSIVE REMARKS

In this paper, an experimental campaign focused on the calibration of FE models to use for representing the seismic behavior of art goods is presented.

The experimental campaign, still in progress, consists of both static and dynamic tests. In the paper, only the results provided by the static tests have been shown. Such results refer to the mechanical properties of some of the materials most used for ancient and current artifacts (marble and sandstone) and to their attitude to slide over a standing surface, respectively consisting of masonry and mortar, timber, glass, steel and Plexiglas. The strength of the material has been checked through uniaxial compressive strength (UCS), combined to ultrasound lectures, according to the current Code provisions, whilst the friction coefficient has been checked both through static and dynamic tests.

Several dynamic tests will be performed through a shaking table, using both the same samples of the static tests and handcraft objects, compatible for material, shape and scale to common art goods. The results obtained on the handcraft objects will be used for the validation of the presented 3-d numerical model.

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REFERENCES


AN INTEGRATED COMPUTATIONAL APPROACH FOR HERITAGE MONUMENTAL MUSEUMS

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Abstract

This paper discusses and presents a workflow for the processing of the geometrical information obtained from point clouds data, through BIM (Building Information Model) implementation and geometric object-oriented modelling, for FE (Finite Element) structural analysis. The methodology includes two important phases for the construction of a full HBIM (Heritage BIM), namely: i) the transformation of the point cloud data into a BIM, and ii) the transition from the geometric model to a FE Model. The proposed semi-automatic process allows to explicitly export all the geometric elements with structural relevance; a software chain is proposed between the Rhinoceros 3D geometric editor, able to handle the complex geometries of historical buildings (such as double curvature surfaces, non-perpendicular faces, etc.), and the Salome Meca open source platform for structural analysis. The workflow, the software chain and the BIM related procedures here proposed as a general approach are discussed with reference to a specific and emblematic monumental masonry building: the Museum of the Galleria dell’Accademia di Firenze. This case study is particularly relevant due to its intrinsic geometrical complexities which cover almost all the issues existing in a historic structure.

Keywords: Museum; Heritage-BIM; Terrestrial Laser Scanning (TLS); FE dynamic analysis; Uncertainty; Metamodel.
1. INTRODUCTION

Existing masonry structures like the historic Museums which populate the city centres of many Italian towns are often characterized by very complex spatial organisations due to the changes and transformations occurred over the centuries. Daily use of such buildings, where structural maintenance and preservation needs are strictly correlated, requires modern technologies and effective instruments able to connect and manage different field of expertise.

Building Information Modelling (BIM) is an appealing methodology, already consolidated in the field of construction industry ([1]), which allows, through a single 3D digital model, to include the different technologies and processes needed for the management of a whole wide-range project. In the field of existing buildings, BIM may represent a new way of coordinating the different needs required for preservation, seismic risk assessment, management and daily use by providing the digital support that connects knowledge (documentation), technologies and skills. However, the application of BIM in the field of existing structures, and particularly of Heritage structures, still poses important challenges ([2], [3]).

In addition, thanks to the growing development of new analysis techniques and new technologies for rapid survey, such as terrestrial laser scanning (TLS), one interesting issue is to investigate the possibility to directly - or semi-directly - correlate the geometric information derived from point cloud data to those needed for the structural management of the existing structure.

This paper, in order to identify a best practice for employment of BIM in the field of Heritage structures, presents a full workflow aimed at processing the geometric information obtained from point cloud data, through BIM implementation and geometric object-oriented modelling, for finite element (FE) structural analysis. The proposed workflow is aimed to include two of the main steps needed for the set-up of a full Heritage BIM (HBIM), namely: i) the transformation of the cloud point data into a BIM, and ii) the transition from the geometric model to a FE Model (FEM). The proposed process, which is semi-automatic, allows to explicitly export all the geometric elements with structural relevance from the BIM to the FEM. A software chain, in particular, is developed starting from the BIM to the Salome_Meca open source platform for FE structural analysis. The software Rhinoceros, able to handle the complex geometries typical of historical buildings (such as double curvature surfaces, non-perpendicular faces, etc.), is employed as intermediate element of the chain.

The workflow, the software chain and the BIM related procedures here proposed as a general approach, are discussed with reference to a specific and relevant monumental masonry building: the Museum of the Galleria dell’Accademia di Firenze (Florence, Italy). The case study is particularly interesting due to its intrinsic geometrical complexities which cover almost all the problematics existing in a historic structure. The origins of the Galleria dell’Accademia di Firenze date back to the eighteenth century, when Grand Duke Pietro Leopoldo of Tuscany founded a modern Academy of Fine Arts (along with other schools) incorporating the ancient Ospedale di San Matteo and the convent of San Nicola di Cafaggio. The most significant transformation took place in 1783 when a new building (the Tribuna), designed by the Italian architect De Fabris, was built inside the complex for the relocation of Michelangelo’s David from Piazza della Signoria. The Galleria dell’Accademia di Firenze shares a large part of an urban block with the most prestigious institutions for arts, education and restoration in Florence: the classrooms, auditorium and libraries of the Accademia di Belle Arti, the Conservatorio Musicale Cherubini, and the laboratories and museum of the Opificio delle Pietre Dure. The whole building as a result is a very complex historical fabrica whose structural behaviour is strongly characterised by the interaction between different units. The various elements which today constitute the historical fabrica are articulated in an intri-
icated and complex spatial organisation, as a result of the transformations occurred over the centuries without any unitary project.

In the following, after a short introduction of the context, the workflow proposed to build the finite element model starting from point cloud data within the HBIM framework is presented. Then, the FE model of a structural unit is employed as illustrative application aimed to discuss pros and cons of the proposed workflow. Several parametric dynamic analyses performed to demonstrate the feasibility of the procedure, are the basis for a future seismic assessment of the Museum.

2. THE CONTEXT

2.1 From BIM to HBIM

The creation of information management systems for the built environment in a spatial form using BIM or GIS systems is one of the main issues of contemporary research and therefore the bibliography on the subject is very extensive and rapidly growing. Building Information Modelling is the new paradigm of digital design and management, and its use is already mandatory in some countries, so it is applied for the design of all kind of buildings including, of course, new museums. In combination with Geomatics techniques for 3D data acquisition, it is also increasingly applied for the documentation and management of historical buildings ([4], [5]). In this case, its use is still episodic, also because, unlike new constructions, their material and constructive technology must be largely reconstructed only knowing their exterior characteristics. However, it’s evident the advantage that can be derived from collecting all the information of a building in a single database, which can be gradually integrated and updated with further investigations (especially for large historic complexes whose knowledge is usually divided between many specialists).

Given the challenge of applying a methodology designed for new buildings to existing ones, the focus of the research was mainly on the problem of the representation of forms and irregularities of historical architecture. For the realisation of a BIM model of an existing building surveyed with geomatic techniques (as-found BIM), there are different approaches ([6]). A possible method is to adapt the components available in the BIM software (walls, floors, etc.) to an existing 2D CAD drawing (CAD-to-BIM) or to construct simplified parametric geometries ([7]). When point clouds from a survey performed with geomatic techniques are available, whose use is not natively included among the modelling tools of the BIM software, different workflows (Scan-to-BIM) have been proposed for extracting automatically or semi-automatically the desired features ([8]), in particular for modelling the irregular geometry of historical buildings. Many BIM softwares can display point cloud data for visual reference, other workflows involve the importation of mesh ([9]) or NURBS geometries generated in other software ([10], [11]).

A smaller number of studies concern the use of BIM models of existing and historical buildings, in particular for building conservation and management activities ([12], [13]). Most of them focused the BIM models use for structural analysis and in particular for the realisation of FE models ([14], [15], [16]), others tackled the issue of energy retrofitting of historical buildings. While the digitisation of museum objects is a well-known topic, especially for making models for the web or the creation of virtual museums, as far as we know, no previous research has investigated the use of BIM for the integrated management of the museum building and the collections. A recent study ([17]) is focused on the creation of semantically enriched 3D models (called Collection Information Models - CIM) and mentions the possibility to operate on the system of relationships between objects (contents) and museum (container) within the BIM environment.
2.2 The Salome_Meca platform

The Finite Element Method is amongst the most diffused technique used to assess the structural behaviour of existing buildings. It includes, shortly, the following steps: i) formulation of the system of equations and its boundary and initial conditions, ii) selection of the correct finite element typology to define the approximation functions for the solution, iii) a mesh, properly representing the discretized domain of the problem, iv) the solver algorithm implemented for the problem, and v) the calculation of the approximation error. Finite Element Procedures are still widely used to solve the majority of the problems involving structural design and optimization with respect to performance and cost, but their importance has also increased in the assessment of the behaviour of existing structures, with respect to the reliability of non-linear implementations. It is important to notice that, while the solution of the problem can be easily calculated with the help of a reliable solver algorithm, the most delicate phase is the creation of a consistent model, with a dataset consisting at least in i) the mesh or geometrical domain, ii) the properties of the elements, iii) the boundary conditions and iv) the loads.

The Code_Aster FE solver, developed by EDF (Électricité de France, [20]) and used internally since the late ‘80s, has been surrounded by a complete graphical interface since the beginning of 2000s, when it was decided to make the code available under the open source LGPL license. This interface includes, at the present state of development, four basic modules: i) one for the management of the geometry (GEOMETRY), ii) one for the generation of the discrete domain (MESH), iii) one for the analysis data setting and the consequent run (ASTER STUDY) and iv) one for the visualization and post-postprocessing of the results (PARAVIS). Other more advanced modules are available for parametric calculations (YACS and ADAO), for the statistic and probabilistic approach (OPENTURNS, [21]) related to computational mechanics and even for mesh manipulation and adaptation (HOMARD).

The current state of development, however, isn’t a fixed parameter: its open nature can be exploited fundamentally for two aspects, the transparency of the implemented algorithms and the possibility to access and modify the code for specific purposes without the need of re-implementing the Finite Element Method, but rather relying on the extensive validation of the code inside its quality assurance development process.

3. FE MODEL FROM BIM

The proposed workflow, here presented as a best practice (Figure 1), aims to build a full procedure in order to link the construction of a numerical (simulation model) derived from TLS survey with HBIM procedures.

![Workflow diagram](image)

Figure 1: Workflow description, from the BIM model, realized in Revit, to the FEM model, inside the salome_meca platform, passing through the automated Python scripts in the Rhinoceros environment.

Two main inputs are needed to consistently build a FE Model: i) the mesh and ii) the additional information to be implemented within the finite elements (material properties, loads, boundary conditions, etc.). The mesh can be obtained from a geometrical input of topological
content, such as a combination of primitive forms, or producing this content from other sources, such as lidar or laser scanner surveys, based on cloud point outputs. The former procedure is more compatible inside a BIM or HBIM context, where the availability of data, or data storing procedures, can assist in improving the interoperability and easily retrieving information from the same databases.

Actually, the Salome_Meca platform allows to handle complex geometrical inputs from external sources, such as neutral format files (BREP, STEP, IGES, STL etc.), and consequently generate a finite element compatible mesh as the starting geometry for the model. Information on the model properties can also be loaded from external sources and implemented in the analysis command script or directly on the geometry (for instance the identification of coherent geometrical groups). All the three steps (geometry importing and manipulation, mesh generation and the analysis command file creation) can be wrapped in Python compatible language and operated to build a full-run finite element analysis.

To build the geometry and define the objects dedicated to structural purposes, a sufficiently flexible environment - Rhinoceros - was chosen: starting from the original HBIM container (a Revit model in this case, ideally common for all purposes (architectural, structural, management of the exhibitions or collections, etc.) semantic translation were performed to Rhinoceros. The choice was motivated by: i) the need for the implementation of complex and non-regular shapes, such as structural vaults, out-of-plane deformed walls, ii) the need to simplify the structural elements representation (1D frames and 2D shell elements instead of three-dimensional finite elements), which results in an object-oriented programming approach, iii) the possibility to build additional, or modify existing, complex geometrical entities, and iv) the possibility to manage internal or external databases. All these operations are possible in the Rhinoceros environment thanks to the Python programming language-based interface, which is employed as a trait d’union with the Salome_Meca platform.

![Figure 2: Superposition of the DWG 2D layer, derived from the HBIM, and plan view of the model.](image)

With the implemented workflow it is possible to:
• manage the geometry of the object to be built with the Rhinoceros modeler and ideally derived from the external HBIM model (Figure 2);
• add information on the geometrical structural objects and ideally derive it from the HBIM database;
• export the enriched geometry in a Salome_Meca compatible form;
• import the enriched geometry in the Salome_Meca environment (Figure 3);
• build the computational finite element model and perform the analyses (Figure 3).

Figure 3. Geometry environment and FEM mesh environment, with inherited assignments for the elements.

Additional steps, not yet implemented, could involve the direct integration of the HBIM model management by using the Rhinoceros plug-ins VisualARQ which maintains the original environment flexibility.

4. APPLICATION: PARAMETRIC ASSESSMENT OF THE DYNAMIC BEHAVIOR

For illustrative purpose, with the aim to discuss pros and cons of the proposed workflow, the FE application has been focused on a portion of the whole Galleria dell’Accademia di Firenze, the so called Sala dei Gessi (Figure 4). It is an extended rectangular unit, confining with the structures of the Accademia delle Belle Arti on the western side and by corridors on the NE and SE side. Several parametric modal analyses have been performed in order to assess the stiffness effect of the non-modelled confining structures. The sparsity of the mode shapes, or a high number of the mode shapes needed to obtain an acceptable cumulated mass participation factor, is an index which shows the main presence of local rather than global effects. This is particularly true for masonry and complex historical structures, with flexible diaphragms and located in an aggregate built context ([18], [19]).

A suitable solution was investigated in order to build a reliable and robust model of the Sala dei Gessi (Figure 5). Ideal restraints, to represent the walls intersecting from the outside the Sala dei Gessi unit, were adopted in the form of 1000 mm extended ideal walls, with 100 mm of thickness and a full horizontal constraint on the free vertical edge. These fictitious walls are
oriented as the original one and their stiffness must be properly calibrated in order to reliably represent the restraint they apply to the structure in relationship to its dynamic behaviour.

A preliminary set of simple and idealized wall test cases was performed in order to calibrate the restraint offered by aggregate walls (which are not included in the model of the building) with different extensions. The ideal restraint is modelled as a 100 mm thick wall with 1000 mm of extension, aligned as the original wall with horizontal constraints imposed at the vertical free edge. The interaction is reported in the following and the characteristics of the fictitious material (with ideal elastic modulus) to be used in a more general context, are parametrically calibrated with respect to the first mode shape frequency. The following list of test cases has been performed to correctly capture and qualify the procedure:

- In-plane behaviour of a simple wall without openings;
- In-plane behaviour of a wall with typical openings;
- Out-of-plane behaviour of a simple wall.
4.1 In-plane behavior of a typical wall

A typical masonry wall (10 m high and extended for 20 m, with a 0.30 m thickness and with or without openings; Figure 6) has been parametrically tested to assess its numerical in-plane dynamic behaviour. A density of 1.8 t/m³ has been considered for the wall material.

Figure 6. Parametric analysis of the in-plane interaction between adjacent/aggregate continuous walls: typical wall, 10 m high, 20 m long and 0.30 m thick, without opening (left) and with openings (right), for different adjacent element extension ratios (L/H = 0.5/1.0/2.0) and a fictitious element, in red.

Figure 7. Parametric analysis of the in-plane interaction between adjacent/aggregate continuous walls. From left: first in-plane mode shape of the wall with fictitious restraint and with different adjacent wall extensions (L/H=0.5, 1.0 and 2.0).
From the results (summarized in Figure 7 and Figure 8), it is clear that the calibration is loosely guided by the extension of the adjacent walls. Other parameters, such as concentrated masses at floor levels, could be considered for a refined calibration.

4.2 Out-of-plane behavior of a typical wall

The out-of-plane behaviour of the wall has been analysed with a varying set of parameters for the restraint level (Figure 9). A fixed lower boundary has been assumed and out-of-plane rotations have imposed on the sides, as for symmetric conditions. On the other hand, the influence of the fictitious appendix has been tested for the in-plane behaviour (Figure 10).

From the results, it can be summarized that i) regarding the out-of-plane behaviour of the wall, the first modal shape is heavily influenced even by low elastic modulus values (meaning that only the detached or not connected condition is really influent) and ii) regarding the influence of the fictitious appendix on the in-plane behaviour, not a crucial variability can be observed even for thick appendixes (t = 0.30 m).
4.3 Testing the Sala dei Gessi

A complex structure such as the Sala dei Gessi in the Galleria dell’Accademia di Firenze exhibits a very sparse behaviour in terms of dynamic response. This response depends, among others, on several input such as: i) the masonry mechanical properties (elastic modulus), ii) the expected masses involved in the vibration on each floor and iii) the restraint offered by aggregate buildings. Since none of the above parameters can be evaluated with a deterministic approach, uncertainties qualification of the results (frequency value of the mode shapes) based on the uncertain values of the input parameters has been performed.

Based on a Latin Hypercube Sampling (LHS, [22]) with a reliable number of samples (168), the parameter space was explored exploiting the Salome-Meca FEM model inside the OpenTURNS environment and considering an initial hypothesis for the probabilistic distributions of the parameters themselves. Then a surrogate analytical model, the so-called metamodel, was created and validated upon the simulation sample, with the polynomial chaos expansion method ([23]), in order to have the possibility to variate the initial distributions and evaluate the output (the first frequency) under different conditions.

Figure 9. Parametric analysis of the out-of-plane interaction between adjacent/aggregate walls, considering a fictitious material (E_f) for the equivalent restraint offered by the adjacent wall.

Figure 10. Parametric analysis of the in-plane behaviour of a typical wall restrained for out-of-plane effects, considering a fictitious material (E_f) for the equivalent restraint offered by the adjacent wall.
Figure 11. First 4 mode shapes of the *Sala dei Gessi* for different restraint values offered by the buildings surrounding the structure. From top: $E_f = 0$ (no restraint), $E_f = 200$, and $E_f = 10^6$ MPa (full restraint).

<table>
<thead>
<tr>
<th>N.</th>
<th>$E_{wf} = 0$</th>
<th>$E_{wf} = 50$</th>
<th>$E_{wf} = 100$</th>
<th>$E_{wf} = 200$</th>
<th>$E_{wf} = 300$</th>
<th>$E_{wf} = 10^6$</th>
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<td>7.277</td>
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<td>6.202</td>
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<tr>
<td>11</td>
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<td>12</td>
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</tr>
<tr>
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<td>6.941</td>
<td>7.120</td>
<td>7.271</td>
<td>8.035</td>
</tr>
</tbody>
</table>

Table 1: Frequencies [Hz] of the first 15 mode shapes of the *Sala dei Gessi*. In red the frequencies whose mode shapes are reported in Figure 11.
Table 2: Description of the input parameters and the hypothetical distributions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_m</td>
<td># Masonry elastic modulus [MPa]</td>
<td>LogNormal (muLog = 7.29361, sigmaLog = 0.198042, gamma = 0)</td>
</tr>
<tr>
<td>E_wf</td>
<td># Elastic modulus of the fictitious material for boundary conditions [MPa]</td>
<td>Uniform (a = 0.01, b = 300)</td>
</tr>
<tr>
<td>q_sl2</td>
<td># Mass per unit of surface on level 2 [kg/m³]</td>
<td>Normal (mu = 1000, sigma = 100)</td>
</tr>
<tr>
<td>q_sl3</td>
<td># Mass per unit of surface on slab 3 [kg/m³]</td>
<td>Normal (mu = 300, sigma = 30)</td>
</tr>
</tbody>
</table>

Figure 12. Parameters model space exploration, realized with a 168 simulations LHS (above) and the derived Chaos Expansion Polynomial metamodel with a much larger population (below), for the Sala dei Gessi unit. Masonry elastic modulus E_m, floor loads/masses q_sl2 and q_sl3 and the aggregate restraint elasticity E_wf are taken into account. A clear predominance of the restraint factor is evidenced.

The results of the parametric analysis (shown in Figure 11 and Table 1) confirm a great variability of the first modal shape, which involves the participation of the 3rd level of the building (the roof), or at least a part of it. Around a certain value of the restraint offered by the surrounding buildings (300 MPa of equivalent stiffness) a shift of the first modal shape could be observed. Fitting the surrogate model on these initial parametric results, the distribution obtained with the metamodel of the output (the first modal shape frequency) is presented with
a much bigger sample (Figure 12), and its sensitivity is then tested with Sobol indices ([24]). What can be derived from the illustrated results (Figure 13) is a clear predominance of the restraint factor among the others, which means that the behaviour of the local vibrating mode depends strongly on the boundary conditions, with a non-trivial input-output relationship.

Figure 13. First frequency results distribution based upon the metamodel simulations, and Sobol sensitivity indices for the input parameters - masonry elasticity $E_m$ [MPa], aggregate restraint $E_{wf}$ [MPa], mass density on the floors $q_{sl2}$ and $q_{sl2}$ [kg/m$^2$].

5. CONCLUSIONS

The workflow presented aims to finalize an effective and efficient semi-automatic procedure able to link the construction of a numerical model with TLS survey data interpreted by BIM procedures. The proposed process allows, in particular, to explicitly export all the geometric elements with structural relevance from the BIM to a FE Model. It has been shown that the information included in a BIM container could be used to efficiently produce a numerical model to be employed for structural analysis and, in addition, uncertainty quantification/assessment of the results. With respect to the various (unknown) input parameters which characterize existing buildings, the workflow can help to avoid misleading interpretation of deterministic results and can support in defining a correct decision-making procedure. Focusing on the preservation needs, this procedure could possibly be generalized and adopted for various purposes related to: i) the number of input parameters that could be investigated (soil-structure interaction, diaphragm stiffness, wall-wall and slab interlocking, etc.) and the output type (stress or damage criteria, etc.), ii) the order of analysis that could be used (non-linear statics, non-linear time-histories, etc.), and iii) the possibility to optimize the transformation of survey data, starting from the cloud point data, into a BIM compatible with the FEM.
REFERENCES


SEISMIC PROTECTION OF STATUES. A CASE STUDY

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Abstract

The analysis of rocking behaviour is a key aspect of safety assessment the maintenance of artifacts, like statues vases, since the rocking itself can be the cause of damage at the base of the object, due to impact with the support. By seismic excitation, a freestanding object may in fact enter a rocking state which would cause overturning, so that often the sliding is desirable, given the possibility of evaluating relative displacements to avoid collisions. In the majority of cases, museum exhibitions are not equipped with isolation devices for every single contained object. This paper presents the theoretical numerical bases of a simple low cost isolation system to protect small art objects. Smaller artifacts are in fact the more prone to overturning since the rocking response is highly influenced by the size of the object. The insertion of a further rigid body between the moving base the statue can reduce the effect of base motion due to earthquakes. The case study is a real marble statue by Donatello, placed in the Museo dell’Opera del Duomo in Florence.

Keywords: Rigid body, isolation, statues, friction, rocking dynamics.
1 INTRODUCTION

The first complete study on the rocking behavior of a rigid block supported on a base undergoing horizontal motion is the seminal paper by Housner on the inverted pendulum [1]. After his study the behaviour of tall, slender structures subjected to ground motion was extensively analyzed with that model [2, 3]. The motion of rigid bodies under base excitation includes in fact the behaviour of piece of equipments, hospital devices, statues, storage tanks, or even that of tall buildings [4, 5]. Only in the following decades the model has been used to represent the behaviour of particular rigid bodies, i.e. the art objects in the museums in seismic areas. In the first studies performed in Japan [6], the computer simulation showed that the horizontal velocity as well as the acceleration must be taken into account as criteria for overturning. Nevertheless the rocking behavior is a feature to be avoided in case of artifacts, to prevent damage at the base of the object, and greater attention should be given to sliding motion to avoid collisions [7]. In every case the response under earthquake of the aforementioned artefacts significantly depends on both the dynamic characteristics of the objects themselves and those of the building in which they are arranged [8, 9], as well as the input ground motion [10, 11]. Other aspects to take into account are the friction coefficient between the block and the base [12] and the geometry of the object [13, 14]. The problem becomes more complex when the behaviour of two stacked rigid bodies is examined: the highly nonlinear formulation needs some simplifying assumptions [15, 16]. In the case of two superimposed blocks examined by Spanos [17] a large friction coefficient is considered to prevent sliding. This last hypothesis should be removed in the case of analysis of real structures [18, 19]. Like the protection of the buildings containing such artifacts has been for a long time [20, 21], the protection of the contents is a prominent issue, so that several protection devices have been proposed [22]. For small objects, for which classical isolation systems could involve expensive efforts, simple, low cost protection systems are required.

Based on the above considerations, this paper present a simple isolation system developed for statues and artifacts freestanding on a pedestal simply supported on the museum floor. The problem examined is the dynamic behaviour of two stacked rigid bodies. The case study is a real marble statue, the Zuccone by Donatello, that represents the Biblical prophet Habakuk, whose geometrical and inertia characteristics have been reported in a recent paper by Wittich [23]. The two rigid bodies can represent respectively the statue and its pedestal. The results can be useful in the determination of the optimal geometry of the pedestal, together with the friction coefficient between the statue and the moving floor. The geometry of the pedestal induces the only sliding motion, while rocking is the only possible motion for the statue, due to the high friction coefficient with the base. The differential equations governing the problem have been derived and included in the numerical procedure developed with Mathematica®.

2 THE MODEL

The analysis of two stacked blocks needs the study of the single block motion. In the following the rocking motion of one symmetric rigid block with aspect ratio $B/H$, simply supported on a moving plane with acceleration $\ddot{x}_g(t)$ as in Figure 1 is examined. The static friction coefficient $\mu_s$ takes into account the amount of force needed to start the sliding motion, while the force necessary to keep the object sliding is proportional to the kinetic friction coefficient $\mu_k$, with $\mu_s > \mu_k$. The block can rotate alternatively around the two base corners $O$ and $O'$ with rotation angle $\theta$, clockwise positive. Impact, when the angle of rotation reverses is the only dissipative event.
According the classical inverted pendulum theory [1], the velocity field after a perfect plastic and centered impact is related to the pre-impact field by means of a reduction coefficient $e$

In particular $e = \sqrt{r}$ being $r$ the restitution coefficient in the Housner sense. The coefficient $e$ can be assumed constant during the motion, so that the angular velocity $\dot{\theta}^+ (t)$ of the block after the impact can be given as function of the angular velocity $\dot{\theta}^- (t)$ before it (Figure 1):

$$\dot{\theta}^+ (t) = e \dot{\theta}^- (t).$$  \hspace{1cm} (1)

![Figure 1: The single rocking block at rest (left) and in rocking mode for $\theta(t) < 0$](image)

In these hypotheses the conservation of angular momentum about point $O'$ just before the impact and right after the impact is:

$$\left(I_o - 2mRb \sin \alpha \right) \dot{\theta}^- (t) = I_o \dot{\theta}^+ (t).$$  \hspace{1cm} (2)

where $I_o$ is the moment of inertia (defined with respect to $O$ or $O'$), $R$ is the distance of the center of mass $G$ from the corners $O$ or $O'$ and $\alpha$ is the angle between $R$ and the vertical edge of the block (see Figure 1). The combination of (1) and (2), gives, for a rectangular block:

$$e = 1 - \frac{3}{2} \sin^2 \alpha \quad \text{with} \quad 0 < e < 1.$$  \hspace{1cm} (3)

Energy dissipation is involved for $e < 1$. Rocking motion is present when the static friction with the base plane prevents sliding. Adopting the notation by Shenton [11], let $f_x$ and $f_y$ be the horizontal and vertical reactions at the tip $O'$ of the block, at all times rocking motion holds true if:

$$|f_x| \leq \mu f_y.$$  \hspace{1cm} (4)

In other words, starting from an equilibrium configuration of the system and given the condition (4), the angular momentum of inertia forces is greater than that due to gravity force. The rocking motion, according to the D’Alembert principle, is governed by the following set of differential algebraic equations (DAEs):

$$I_o \ddot{\theta} (t) + mg R \sin (-\alpha - \theta (t)) = -m \ddot{x}_g (t) R \cos (-\alpha - \theta (t)) \quad , \quad \theta (t) < 0$$

$$I_o \ddot{\theta} (t) + mg R \sin (\alpha - \theta (t)) = -m \ddot{x}_g (t) R \cos (\alpha - \theta (t)) \quad , \quad \theta (t) > 0$$

$$\dot{\theta}^+ (t) = r \dot{\theta}^- (t) \quad , \quad \theta (t) = 0$$  \hspace{1cm} (5)
where $\ddot{x}_g(t)$ is the horizontal base acceleration and $I_o = I_{o'}$ is the polar inertia moment with respect to the two points $O$ and $O'$. The rocking motion starts when $|\ddot{x}_g(t)| > g \frac{b}{h}$, being $g$ the gravity acceleration. The first two ordinary nonlinear differential equations are relative to the rotation motion around $O$ and $O'$ and the third algebraic equation relates the two angular velocities in $O$ and $O'$ and holds true at the impact instant only. The angle $\alpha = \arctan \frac{b}{h}$ takes into account the slenderness of the block. The system (5) can assume the following form:

\[
\frac{I_o}{mR} \ddot{\theta}(t) + g \text{sgn}(\theta(t)) \sin(\alpha - \text{sgn}(\theta(t)) \theta(t)) = -\ddot{x}_g(t) \cos(\alpha - \text{sgn}(\theta(t)) \theta(t)) \quad , \quad \theta(t) \neq 0
\]

\[
\theta^+(t) = r \theta^-(t) \quad , \quad \theta(t) = 0.
\]

where $\text{sgn}(\cdot)$ is the signum function. The numerical solution of the DAEs (6) may be put more conveniently in terms of a key point displacement, considering two reference systems with origin in the two rotation points $O$ and $O'$, namely $R_1 = \{O, x, y\}$ for $\theta(t) > 0$ and $R_2 = \{O', x', y'\}$ for $\theta(t) < 0$. Let $\theta(t)$ be the rotation function, the position of the point $P$ at time $t$ in the two frame systems above described is related to the position vector at the starting time:

\[
r_p^{(1)} = \begin{bmatrix} x_p^{(1)} \\ y_p^{(1)} \end{bmatrix}, \quad \theta(t) < 0 ; \quad r_p^{(2)} = \begin{bmatrix} x_p^{(2)} \\ y_p^{(2)} \end{bmatrix}, \quad \theta(t) > 0
\]

so that the actual position of the point $P$ is given by the rotation matrix $R \circ \theta(t)$ applied on $r_p^{(1)}$ and $r_p^{(2)}$:

\[
OP(t) = R \circ \theta(t) r_p^{(1)} \quad , \quad \theta(t) > 0
\]

\[
O'P(t) = R \circ \theta(t) r_p^{(2)} \quad , \quad \theta(t) < 0
\]

where the rotation matrix $R \in SO(2)$, being $SO(2)$ the orthogonal group of matrices with unit determinant, is:

\[
R \circ (\cdot) = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) \\ -\sin(\cdot) & \cos(\cdot) \end{bmatrix}.
\]

from (8) the acceleration is derived as:

\[
\frac{\partial^2}{\partial t^2} OP(t) = \frac{\partial^2}{\partial t^2} [R \circ \theta(t)] r_p^{(1)} \quad , \quad \theta(t) > 0
\]

\[
\frac{\partial^2}{\partial t^2} O'P(t) = \frac{\partial^2}{\partial t^2} [R \circ \theta(t)] r_p^{(2)} \quad , \quad \theta(t) < 0
\]

after some manipulations the (10) can be rewritten as follows:

\[
\frac{\partial^2}{\partial t^2} OP = \left[ \ddot{\theta}(t) \partial R \circ \theta(t) - \partial^2 \circ \theta(t) R \circ \theta(t) \right] r_p^{(1)} \quad , \quad \theta(t) > 0
\]

\[
\frac{\partial^2}{\partial t^2} O'P = \left[ \ddot{\theta}(t) \partial R \circ \theta(t) - \partial^2 \circ \theta(t) R \circ \theta(t) \right] r_p^{(2)} \quad , \quad \theta(t) < 0
\]
where the first derivative of the rotation matrix $R$ belongs to the orthogonal group of matrices with $\det(R) = 1$:

$$\partial R(\cdot) = \begin{bmatrix} -\sin(\cdot) & \cos(\cdot) \\ -\cos(\cdot) & -\sin(\cdot) \end{bmatrix} \in SO(2).$$

The horizontal component of relative acceleration can be deduced by (11):

$$\ddot{x}(t) = \begin{cases} \frac{\partial^2}{\partial t^2} OP \cdot \hat{i}, & \theta(t) > 0 \\ \frac{\partial^2}{\partial t^2} OP' \cdot \hat{i}, & \theta(t) < 0 \end{cases}$$

with $\hat{i}$ unit vector of $x$ axis. The horizontal acceleration $\ddot{x}(t)$ can be put in the explicit form:

$$\ddot{x}(t) = \begin{cases} -[x_1 \cos(\theta(t)) + y_1 \sin(\theta(t))] \dot{\theta}(t) + [-x_1 \sin(\theta(t)) + y_1 \cos(\theta(t))] \dot{\theta}(t), & \theta(t) > 0 \\ -[x_2 \cos(\theta(t)) + y_2 \sin(\theta(t))] \dot{\theta}(t) + [-x_2 \sin(\theta(t)) + y_2 \cos(\theta(t))] \dot{\theta}(t), & \theta(t) < 0 \end{cases}.$$ (12)

The absolute acceleration:

$$\ddot{x}_a(t) = \ddot{x}_g(t) + \ddot{x}(t).$$ (13)

is the sum of the base acceleration and the block one.

The configuration of the block in case of sliding motion can be characterized by the translation of a generic point of the block with respect to the base. The friction force is function of the vertical forces applied on the block and is opposite to the motion. Starting from an equilibrium configuration, sliding motion begins when the maximum horizontal force due to the static friction coefficient is attained:

$$m|\ddot{x}_g(t)| > m \mu_s (\ddot{y}_g(t) + g).$$ (14)

The differential equation governing the sliding motion problem is:

$$m(\ddot{x}_g(t) + \ddot{x}(t)) = -\text{sgn}(\ddot{x}(t)) \mu_s m (\ddot{y}_g(t) + g).$$ (15)

The numerical procedure has been developed in the general case of nonzero vertical acceleration of the base whose equations are deducted by (14) and (15) with $\ddot{y}_g(t) = 0$:

![Figure 2: The single sliding block](image)
The differential equation of sliding (15) is integrated until the relative velocity $\dot{x}(t)$ is non-zero from the instant in which the friction contact force is exceeded by the inertial forces related to (13).

$$\begin{align*}
m |\dot{x}_g(t)| &> m g \mu_i, \\
m (\dot{x}_g(t) + \ddot{x}(t)) &= -\text{sgn}(\dot{x}(t)) m g \mu_k. \\
\end{align*}$$

(16)

When the velocity becomes null the block is in relative equilibrium with the base (rest) until the external force attains a value able to reactivate the sliding motion.

The geometrical characteristics of the statue-pedestal problem [24] are reported in Figure 3. The system has two degrees of freedom, namely the rotation $\theta$ (clockwise positive) of block 2 (statue) and the centroid position $x_G$ of block 1 (pedestal). Assuming $m_1$ and $m_2$ masses of the two blocks whose center of masses are $G_1$ and $G_2$, $M = m_1 + m_2$ is the total mass of the system and $(x_g(t), y_g(t))$ are the components of the base motion. Let $r_{OG_1}$ and $r_{OG_2}$ be position vectors of $G_2$ relative to $O'$ and $O$ in the initial. Their components in the two Cartesian reference systems $R_1$ and $R_2$ pictured in Figure 3 are given by:

$$
r_{OG_1} = \begin{bmatrix} R \sin \theta \\ R \cos \theta \end{bmatrix}, \quad r_{OG_2} = \begin{bmatrix} R \sin \theta \\ -R \cos \theta \end{bmatrix}. \tag{17}
$$

The hypothesis of sliding motion for the statue lead to null vertical component of its relative motion with respect to the base. The position of the center of mass $G_1$ is:

$$\begin{bmatrix}
x_{G_1}(t)
\end{bmatrix} = \begin{bmatrix} x_g(t) + x_{G_1}(t) \\
y_g(t) \end{bmatrix}$$

and that of $G_2$ is:

$$\begin{align*}
x'_{G_2}(t) &= x_{G_2}(t) + R \cos \theta(t) r_{OG_2}, \quad \theta(t) < 0 \\
x_{G_2}(t) &= x_{G_2}(t) + R \cos \theta(t) r_{OG_2}, \quad \theta(t) > 0
\end{align*}$$

so that the actual position $P(t)$ of the keypoint $P$ is:
\[ O'P(t) = R \odot \theta(t) r_p^{(1)} \quad , \quad \theta(t) < 0 \]
\[ OP(t) = R \odot \theta(t) r_p^{(2)} \quad , \quad \theta(t) > 0 \]

where \( R \odot \theta(t) \) is the rotation matrix (9) that takes into account the rocking motion of block 2 and the position vectors \( r_p^{(1)} \) and \( r_p^{(2)} \) are represented in Figure 5.

The total kinetic energy of the system is:
\[ T(t) = T_1(t) + T_2(t) \]

with \( T_1(t) \) and \( T_2(t) \) the kinetic energies of pedestal and statue:
\[ T_1(t) = \frac{1}{2} m_1 \dot{x}_{G_1} \cdot \dot{x}_{G_1} \quad , \quad T_2(t) = \frac{1}{2} [ J_{G_2} \ddot{\theta}^2(t) + m_2 \dot{x}_{G_2}^\prime (t) \cdot \dot{x}_{G_2}^\prime (t) ] \quad , \quad \theta(t) < 0 \]
\[ T_1(t) = \frac{1}{2} m_1 \dot{x}_{G_1} \cdot \dot{x}_{G_1} \quad , \quad T_2(t) = \frac{1}{2} [ J_{G_2} \ddot{\theta}^2(t) + m_2 \dot{x}_{G_2}^\prime (t) \cdot \dot{x}_{G_2}^\prime (t) ] \quad , \quad \theta(t) > 0 \]

being \( J_{G_2} \) the centroid moment of inertia of the statue. The potential energy of the system is given by:
\[ V(t) = V_1(t) + V_2(t) \]

with \( V_1(t) \) and \( V_2(t) \) the potential energies of pedestal and statue:
\[ V_1(t) = m_1 g x_{G_1} \cdot \vec{j} \quad , \quad V_2(t) = m_2 g x_{G_2}^\prime \cdot \vec{j} \quad , \quad \theta(t) < 0 \]
\[ V_1(t) = m_1 g x_{G_1} \cdot \vec{j} \quad , \quad V_2(t) = m_2 g x_{G_2}^\prime \cdot \vec{j} \quad , \quad \theta(t) > 0 \]

being \( \vec{j} \) the unit vector of \( y \) axis. The friction force at the base of pedestal during the sliding movement is given by:
\[ F_{x,\text{friction}} = -\mu_k M \left( g + \ddot{y}_x (t) \right) \dot{x}_x (t) \text{sgn}(\dot{x}_x (t)) \]

so that the Lagrangian formulation of the problem states:
\[ L(t) = T(t) - V(t) \]

with the two Lagrangian parameters:
\[ q_1(t) = x_{G_1} (t) \quad , \quad q_2(t) = \theta(t) \]

The motion is governed by two differential equations derived by the Euler-Lagrange relation:
\[ \frac{\partial^2 L(t)}{\partial t \partial q_k} - \frac{\partial L(t)}{\partial q_k} = Q_k (t) \quad , \quad k = 1, 2 \]

and \( Q_k (t) \) is the generalized non conservative force dual to \( q_k (t) \). The system assumes two different expressions according to the sign of \( \theta(t) \). In view of equations (18)-(22), the DAEs can be expressed as:
\[
\begin{align*}
J_o \ddot{\theta}(t) - m_2 R \cos(\alpha - |\theta|) \left(\ddot{x}_g(t) + \ddot{x}_{\hat{g}}(t)\right) + m_2 R g \text{sgn}(\theta(t)) \sin(\alpha - |\theta|) &= 0, \quad \theta(t) \neq 0 \\
M(\ddot{x}_g(t) + \ddot{x}_{\hat{g}}(t)) + \text{sgn}(\theta(t)) \left( -m_2 R \sin(\alpha - |\theta|) \dot{\theta}(t) + \right. \\
&\left. - \cos(\alpha - |\theta|) \dot{\theta}(t) + M \mu_k g \right) = 0, \quad \theta(t) \neq 0 \\
\dot{\theta}'(t) &= r \dot{\theta}^{-}(t) \\
\end{align*}
\]  

(23)

The motion problem (23) is composed by two ordinary nonlinear differential equations and a single algebraic one, that involves the pre-and-post-impact angular velocity of the statue during the rocking motion. Uncoupling of the two differential equations is not possible. The DAEs (23) govern the motions of the two blocks, while the rocking of the top block when the flat block is at rest with respect to the ground is governed by the following relation [25]:

\[
J_o \ddot{\theta}(t) - m_2 R \cos(\alpha - |\theta|) \ddot{x}_g(t) + m_2 R g \text{sgn}(\theta(t)) \sin(\alpha - |\theta|) = 0.
\]  

(24)

derived from (23$_1$) with the condition \(q(t) = x_{\hat{g}}(t) = 0\). Equation (23$_2$), analogous of (16) with the condition \(\theta(t) = 0\), describes the sliding motion of the two blocks:

\[
M(\ddot{x}_g(t) + \ddot{x}_{\hat{g}}(t)) = -\text{sgn}(\dot{x}_{\hat{g}}(t)) M \mu_k g.
\]  

(25)

The numerical procedure implemented in Mathematica$^\copyright$ takes into account the only sliding motion for the pedestal, so that its mass is the only necessary mechanical parameter. The Zuccone statue can undergo rocking motion only and the procedure involves the aspect ratio \(B/H\). The analysis has been performed considering the minimum inertia plane for the statue (Figure 4).

![Figure 4: The Zuccone by Donatello](image)

Given the geometrical conditions, at the starting point statue and pedestal are at rest with respect to the moving base, until (23) or (24) is activated according to the mechanical parameters involved, reported in Table 1 and derived from [23] for the statue and by the authors for the pedestal:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mass [kg]</th>
<th>Footprint X [m]</th>
<th>Z [m]</th>
<th>Height [m]</th>
<th>Center of mass G$_x$ [m]</th>
<th>G$_y$ [m]</th>
<th>G$_z$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statue</td>
<td>576</td>
<td>0.41</td>
<td>0.55</td>
<td>1.99</td>
<td>0.19</td>
<td>0.91</td>
<td>0.27</td>
</tr>
<tr>
<td>Pedestal 1</td>
<td>200</td>
<td>0.60</td>
<td>0.75</td>
<td>0.30</td>
<td>0.30</td>
<td>0.15</td>
<td>0.37</td>
</tr>
<tr>
<td>Pedestal 2</td>
<td>340</td>
<td>0.60</td>
<td>0.75</td>
<td>0.30</td>
<td>0.30</td>
<td>0.15</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 1: Mechanical parameters of the statues
The analysis of the dynamic system begins with the evaluation of the type of motion related to the base acceleration. Three possible patterns of motion have been examined (Figure 5):

- a) rocking of the top block with pedestal at rest with respect to the ground
- b) sliding motion of the complex statue-pedestal as one rigid body
- c) combined motion patterns: rocking of the statue and sliding of the pedestal.

The conditions for the activation of the motion a) are given by:

\[
\begin{cases}
    M \ddot{x}_g(t) < \text{sgn}(\dot{x}_g(t)) M \mu_s g \\
    \ddot{x}_{c_2}(t) h > g b.
\end{cases}
\]  

(26)

The time \(t_a\) is the instant at which:

\(\ddot{x}_{c_2}(t_a) h = g b\).  

(27)

It corresponds to the change of motion to range b), whose conditions for the activation are given by:

\[
\begin{cases}
    M \ddot{x}_g(t) > \text{sgn}(\dot{x}_g(t)) M \mu_s g \\
    \ddot{x}_{c_2}(t) h < g b.
\end{cases}
\]  

(28)

The balance of friction force with inertia forces corresponds to the time \(t_b\) in which:

\[M \ddot{x}_g(t_b) = \text{sgn}(\dot{x}_g(t_b)) M \mu_s g.\]  

(29)

The conditions for the activation of the motion c) are given by:

\[
\begin{cases}
    M \ddot{x}_g(t) > \text{sgn}(\dot{x}_g(t)) M \mu_s g \\
    \ddot{x}_{c_2}(t) h > g b.
\end{cases}
\]  

(30)

In general, the motion a) lasts until the ground acceleration does not allow the overcoming of the static friction force at base interface. The transition of motion from a) to c) occurs when:

\[M \ddot{x}_g(t) - m_a R \text{sgn}(\dot{\theta}(t)) \{\sin(\alpha - |\theta(t)|) \dot{\theta}(t) - \cos(\alpha - |\theta(t)|) \ddot{\theta}(t)\} \geq \text{sgn}(\dot{\theta}(t)) M \mu_s g \]  

(31)
Equation (28) corresponds to the (23) substituting the kinematic friction coefficient with the static one and \( x_c(t) = 0 \rightarrow \dot{x}_c(t) = 0 \). The inertial forces due to the ground acceleration \( \ddot{x}_g(t) \) and those due to the rocking of statue are involved in (29) with a term containing \( \ddot{\theta}^2(t) \) and another containing \( \dot{\theta}(t) \) respectively projections on the x-axis of the centripetal and tangential interactions through the contact point \( O \) or \( O' \). The sliding motion of pedestal stops when \( \dot{q}_l(t) = \dot{x}_c(t) = 0 \). A frequent circumstance involves the activation of the rocking of statue: the differential equation of rocking (24) is integrated until conditions for sliding (29) are attained, so that the system (23) is integrated until the velocity of the pedestal with respect to the ground is null. From this point on the program loop can restart and the procedure is able to choose the right motion according the dynamic conditions involved. In the examined cases equation (24) is activated before than the static friction force between the pedestal and the ground is exceeded. Only in this last case the flat block initiates sliding.

3 RESULTS AND DISCUSSION

In the following pictures the results of preliminary analyses have been displayed. In particular, the influence of the restitution coefficient has been analyzed. The graphs of Figure 6 and 7 represent the time histories obtained for the statue placed on the two different Pedestals reported in Table 1 and compared with the time of the statue directly placed on the floor. Reference has been made to marble pedestals in both cases. The following mechanical parameters have been considered in all the analyses:

\[
\mu_s = 0.6 \quad \mu_k = 0.45 \quad f = 1.5\,Hz
\]

![Fig. 6 Rotation time histories for the statue placed on Pedestal 1 (red line) and the statue freestanding on the floor (blue line). Displacement time history for the pedestal only (gray line)](image-url)
Fig. 7. Rotation time histories for the statue placed on Pedestal 2 (red line) and the statue freestanding on the floor (blue line). Displacement time history for the pedestal only (gray line).

In Figure 7 the time histories corresponding Pedestal 2 and to two different values of the restitution coefficient are considered [26]. In both cases the statue overturning is possible for the examined frequency. A lower value of the restitution coefficient is the cause of the overturning with the statue placed on pedestal, while in the higher value case the overturning happens in presence of the pedestal. Reverse considerations can be applied in the case of a pedestal with lower mass (Pedestal 1), where overturning is caused by the lower restitution coefficient in presence of the pedestal.

4 CONCLUSIONS

The first analyses of the complex composed of a statue and its pedestal considered as stacked rigid blocks have been presented in this paper. The case study is a real marble statue, the Zuccone by Donatello, that represents the Biblical prophet Habakuk. It has been shown that a small variability of the restitution coefficient strongly changes the response of the system, so that in real cases (exhibition in the museums) the geometry of the exposition devices should be accurately designed, given that a large set of numerical results is available. Future developments include larger numerical analyses and a broad set of experimental data.

5 ACKNOWLEDGEMENTS

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ROCKING RESPONSE AND OVERTURNOING OF MUSEUM ARTEFACTS DUE TO BLAST LOADING

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Abstract. The dynamics of museum artefacts and statues under fast-dynamic excitations arising from explosions is investigated here. The study takes root in the existing knowledge and theory of inverted pendulum structures subjected to earthquake loadings and extends them to non-symmetrical pulses due to blast waves. New analytical, closed-form solutions for the rocking response are derived and the overturning domain of slender blocks is determined.

The analytical findings and assumptions are validated through detailed three-dimensional numerical simulations, which consider the full interaction between the blast waves and the structure (e.g. diffraction, rarefaction, multiple reflections, no-normal incidence, up-lifting etc.). We prove that unilateral rocking response and overturning are predominant mechanisms compared to sliding and up-lifting under explosive loadings.

We develop design charts to be used as a straightforward decision-making tool for determining the critical stand-off distance between the explosive source and the target to prevent overturning, which can be helpful in different applications.

Finally, the assessment of the minimum perimeter around museum artefacts for protection against explosions is discussed for some emblematic statues.
1 INTRODUCTION

The rocking dynamics of museum artefacts and statues under fast-dynamic excitations arising from explosions is studied herein. Our modelling approach is analytical and focuses on providing closed-form solutions. The most important physics of the problem are considered in order to describe the dominant features of the rocking motion due to blast waves. Museum artefacts are modelled herein as inverted pendulum structures. Their modelling involves several difficulties. In particular, the inherent non-linearity and the unilateral contact conditions at the base of the inverted pendulum (rocking) make the dynamics of the system much different from the classical single- or multi-degree-of-freedom harmonic oscillators [1].

The problem of rocking attracts significant scientific research, mostly in the domain of earthquake engineering (construction of bridges, seismic isolation, masonry structures, historical monuments, etc.). We refer, for instance, to the seminal works of Omori [2, 3] and especially to the investigations of Housner [4], who was the first to study the response of a rigid, free-standing block subjected to constant and square pulse seismic (ground) accelerations. Extensive research has provided useful insights on the dynamic response of a rocking block, see e.g. [5, 6, 7, 8, 9].

Our developments take root in the existing knowledge and theory of inverted pendulum structures subjected to earthquake loadings and extend them to non-symmetrical pulses provoked by blast waves. We model blast actions using the modified Friedlander equation. Moment balance equations and overturning conditions are presented and used to determine the critical (minimum) stand-off distance between the source and the target to prevent toppling. This is accomplished by deriving new analytical, closed-form solutions, which lead to the identification of the central and dominant dimensionless parameters that govern the dynamical behaviour of the system. These dimensionless parameters improve and extend the current understanding of rocking due to blasts (cf. [10, 11, 12, 13, 14, 15]) and lead to fundamental insights for design.

Contrary to earthquake actions, the excitation due to explosions presents additional complexity. This is not only because of the ultra-high rates involved—the characteristic time of a blast is of several milliseconds instead of \( \approx 1 \div 10 \) s for earthquakes—but also due to several phenomena related to wave propagation and fluid-structure interaction (e.g. diffraction, rarefaction, reflections, damage etc.). The effect of these phenomena on our modelling assumptions is extensively discussed as follows. More specifically, the predictions of our analytical model are compared to detailed numerical analyses that consider the above mentioned phenomena, a combined sliding/rocking behaviour, and the possibility of uplifting (flight mode).

Engineering applications of the present study can be found in several domains. Of interest here is the preservation of un-anchored equipment and museum (slender) artefacts from overturning, e.g. statues. Such objects belong to the world cultural heritage and their protection has raised important issues throughout history. We refer e.g. to the lost and/or destroyed artefacts of Athena Parthenos, Colossus of Rhodes, the statue of Zeus at Olympia, and more recently the Buddhas statue of Bamiyan.

The proposed analytical model can further be used for securing historical buildings made of monolithic columns from collapse (e.g. classical Greek and Roman temples [16, 18]).
2 STATEMENT OF THE PROBLEM

The problem of a rigid block resting on a horizontal plane is studied based on the following assumptions (Fig. 1):

i. A rectangular slender, rigid block is assumed with a uniformly distributed mass \( m \). The dimensions of the block are \( 2b \times 2h \times 2w \) and the radial distance from the rocking pivot point \( O \) to the centre of gravity is \( r = b \sec \alpha \), where \( \alpha \) is the slenderness angle.

ii. The contact with the horizontal plane is assumed punctual at point \( O \) (no contact moment). Contact is considered to be unilateral. The angle of friction, \( \varphi \), is assumed to be sufficiently large to prevent sliding.

iii. The pressure load due to the explosion is exclusively applied on the front surface \( S \) (incident surface, see Fig. 1) and the blast wave is assumed to impinge all points of \( S \) at the same time (simultaneously) and with the same magnitude (uniformly). We consider the resulting load to act always horizontally and at the block’s centroid as the loading pulse duration is extremely short (i.e., small inclination angle within the duration of the loading). Diffraction phenomena are neglected. The effects of induced ground shocks are also omitted [11, 13].

These simplifying assumptions are helpful for reducing the complexity of the problem and for deriving analytical, closed-form solutions. Their adequacy is explored in Section 4, where it is shown that the analytical solution represents quite well the dynamic behaviour of the system and the overturning condition. In particular, it is shown that the minimum distance that has to be assured between the explosive source and the target, such that toppling is avoided, is in good agreement and on the safety side with the one determined by the full numerical model and experimental results presented in Section 4.

Figure 1: Configuration considered for the rocking problem: a rectangular slender, rigid block resting on a horizontal plane with uniformly distributed mass, subjected to uniform pressure load due to an explosion.
2.1 Blast actions

Explosion produces a blast wave of high-pressure accompanying high-temperature and supersonic expansion of gases. The abrupt increase of the pressure carried by a blast wave can produce severe structural damage. When the primary shock meets a target, it generates on it the so-called reflected overpressure, $P_r$, which is the difference between the pressure determined by the explosion increased by the reflection at target’s surface and the ambient one, $P_o$. Figure 2 shows the schematic time variation of $P_r$, which is determined by the arrival time of the shock wave, $t_A$, the overpressure peak, $P_{ro}$, the positive phase duration, $t_o$, negative phase duration, $t_o^-$, and the underpressure peak, $P_{ro^-}$. These parameters are functions of the distance $R$ and the explosive weight (conventionally expressed in TNT equivalent). Herein we consider only the positive phase of the blast wave (safety approach).

The pressure acting on a target due to blast loading is the algebraic sum of the hydrostatic overpressure and the dynamic pressure

$$ C_D q := \frac{1}{2} \rho u |u| $$

with $C_D$ the drag coefficient (function of the target shape and Mach and Reynold numbers), $\rho$ the density, and $u$ the velocity of gas particles.

![Overpressure graph](image1)

Figure 2: Time evolution of overpressure (i.e. the pressure measured relatively to the atmospheric one) due to an explosion acting on a target. The proposed analytical model only considers the positive phase of the overpressure (safe estimate of the rocking response). The negative pressure may have stabilising effects (increase the block resistance to overturning).

The simulation of a blast can be conducted by using different approaches [20, 21], i.e., empirical or physics-based ones. These models are briefly presented below.

2.1.1 Empirical models

Empirical models rely on best-fit interpolations of experimental results and mainly on those of Kingery and Bulmash [22], which allow to determine the blast parameters and pressure loading from the knowledge of the trinitrotoluene (TNT) equivalent explosive weight, $W$, and the Hopkinson-Cranz scaled distance, $Z = R/\sqrt{W}$ (see [28]). The time evolution of the positive phase of the reflected pressure is modelled with the well
established modified Friedlander equation,

\[ P_r(t) = P_{ro} \left[ \left(1 - \frac{t}{t_o}\right) \left(1 - H[t - t_o]\right) \right] \exp \left(-d \frac{t}{t_o}\right), \quad (1) \]

where \( H[\cdot] \) denotes the Heaviside (step) function, \( d \) is the exponential decay coefficient, and \( t_A \) is taken as the origin of the time axis. The impulse \( i_r \) associated to the positive phase, which represents the area beneath the pressure curve, reads

\[ i_r = \int_0^{t_o} P_r dt = \left[e^{-d} + d - 1\right] \frac{P_{ro} t_o}{d^2}, \quad (2) \]

The above equation allows to determine the exponential decay coefficient, \( d \), by equating it with the best-fit interpolation of \( i_r \) from experiments (as presented in [28]).

2.1.2 Numerical physics-based models

Physics-based, numerical approaches allow a rather detailed description of the main features of the blast phenomenon with, of course, an increased calculation cost. They rely on the definition of two domains: the explosive charge and the surrounding air. Through numerical simulations, detonation, propagation of shock waves and their interaction with deformable structures can be efficiently modelled with a Coupled Eulerian-Lagrangian (CEL) scheme. For more, see [23, 16, 24].

3 ROCKING RESPONSE TO AN EXPLOSION

3.1 Equation of motion

The moment balance around the rocking pivot point gives the equation of motion

\[
\begin{align*}
\mathcal{I}_o \ddot{\theta} + m g r \sin(\alpha - \theta) = S r P_r \cos(\alpha - \theta), & \quad \theta(t) > 0, \\
\mathcal{I}_o \ddot{\theta} + m g r \sin(-\alpha - \theta) = S r P_r \cos(-\alpha - \theta), & \quad \theta(t) < 0,
\end{align*}
\]

where \( \mathcal{I}_o = (4/3)m r^2 \) is the moment of inertia with respect to the pivot point, \( \theta = \theta(t) \) is the inclination angle, and \( P_r = P_r(t) \) is the loading which is given by the Friedlander equation as mentioned above.

For tall, slender blocks, angles \( \theta \) and \( \alpha \) are small and the equation of motion \((3)\) can be linearised using the first-order approximations \( \sin(\cdot) \cong \cdot \) and \( \cos(\cdot) \cong 1 \). Furthermore, as the load is acting always on the same direction (positive \( \theta \), Fig. 1), a unilateral response mechanism is expected. Equation \((3)\) can be hence restricted to positive angles only, and considering its dimensionless form we obtain

\[ \ddot{\phi} = \phi + \chi p - 1, \quad (4) \]

where \( \phi = \theta/\alpha \) is the dimensionless angle of rotation; \( \chi = \frac{1}{2 \log \alpha} \frac{P_{ro}}{\alpha} \) is the normalized rocking moment, i.e., the ratio between the moment due to the blast load and the restoring moment due to gravity; \( p = \left[\left(1 - \frac{\tau}{\tau_o}\right) \left(1 - H[\tau - \tau_o]\right)\right] e^{-d \tau_o} \) the normalized Friedlander time-history; and \( \tau_o \) the ratio between the characteristic time of the load and the time parameter, \( T = \sqrt{\frac{\mathcal{I}_o}{m g r}} \), related to the response of the rigid block.
Initially the block is at rest ($\dot{\phi}(0) = 0$, $\phi(0) = 0$). Equation (4) admits a close form solution, whose complete mathematical expression is given in the Appendix.

It is worth noticing that the characteristic time parameter, $T$, of blocks of centimetric scale or larger, is much higher than the characteristic time of blast loads $t_o$ and, therefore, $\tau_o \ll 1$.

3.2 Overturning domain and minimum stand-off distance

For unilateral excitations, overturning happens when the rocking angle $\theta \geq \alpha$ or, equivalently, when $\phi \geq 1$. The overturning condition can be found by equating the total work done by the blast load to the difference in potential energy between positions $\theta = \alpha$ and $\theta = 0$ (see also Housner, [4]):

$$\int_0^\infty rSP_r \dot{\theta} \cos(\alpha - \theta) \, dt \geq mgr(1 - \cos \alpha).$$

(5)

Noticing that $P_r(t \geq t_o) = 0$ and rearranging the inequality in terms of the non-dimensional rocking angle and normalized time, one obtains

$$\alpha rSP_{ro} \int_0^{t_o} p\dot{\phi} \cos[\alpha(1 - \phi)] \, d\tau \geq mgr(1 - \cos \alpha).$$

(6)

For slender blocks, the power series expansion at the first order gives $(1 - \cos \alpha) \cong \alpha^2/2$ and $\cos[\alpha(1 - \phi)] \cong 1$, hence the overturning condition simply becomes

$$2I \chi \geq 1,$$

(7)

with $I = \int_0^{t_o} p\dot{\phi} \, d\tau$. The left-hand side term in inequality (6) represents the non-dimensional overturning moment.

From the overturning condition ($2I \chi = 1$) we can determine the minimum required distance between the explosive source and the target, $R^*$, in order to avoid toppling. Figure 3 shows the contours of the critical distance $R^*$ for different slenderness angles, block heights, densities, and explosive quantities.

4 VALIDATION OF THE OVERTURNING DOMAIN

Our analytical approach allows to determine the minimum stand-off distance to prevent toppling. The calculations lead to closed-form solutions, which are useful for identifying the main factors that influence the dynamic response of the system under explosive loads. However, the aforementioned approach is based on some simplifying assumptions (see Sect. 3), whose validity is here explored.

Assumptions (i) and (ii) of Section 3 are first investigated. In other words, we quantify a) the linearisation of the equations of motion, and b) the effect of combined sliding, rocking, and uplift (flight mode). We consider Coulomb friction at the interface of the block with the rigid base, with an angle of friction equal to $\phi = 35^\circ$, which is common for many geomaterials (concrete, marble, stone etc.). Blast loadings are applied as in Section 3, relying on the best-fit interpolations in [28]. ABAQUS commercial software is used for the computations. A hard contact formulation is used, i.e., no penetration is allowed at the contact of the rocking block with the base [25]. The rigid base is fixed and the rigid
block is free to translate along \( y \)- and \( x \)-axes, rotate around \( z \), and uplift, see Figure 3. The results of this comparison are presented in paragraph 4.1.

Next, we focus on assumption (iii) which concerns the simplifications related to the blast loads and their approximation by empirical models. In particular, we investigate the effects due to the interaction between blast waves and the rocking block. The analyses are performed again using ABAQUS software. The same modelling approach is used for the interaction of the block and the base as before. The results of this comparison are presented in paragraph 4.2.

4.1 Sliding and uplifting effect

We investigate the minimum stand-off distance \( (R_{\text{num}}^*) \) for several combinations of slenderness angles and explosive weights. Table 4.1 displays the comparison between the threshold range \( (R^*) \), derived in Section 3, and the one obtained from the numerical simulations \( (R_{\text{num}}^*) \).

The numerical analyses show that rocking and sliding happen together. However, for slender structures sliding is limited and rocking prevails. This justifies the no-sliding assumption. Moreover, uplifting, even for combinations of stand-off distance and explosive weight close to the critical ones, is found to be usually negligible. For blocks of high slenderness, sliding becomes even smaller and practically only rocking is observed. For blocks...
of lower slenderness, sliding has a more important effect and becomes more pronounced for increasing explosive weights. In the worst case studied here \((W = 50 \text{ kg} \text{ and } \alpha = 20^\circ)\) the analytical approximation overestimates the minimum stand-off distance by approximately 9%. Consequently, our analytical estimation provides a close upper bound for the critical distance and, therefore, can be safely used for the design of protective measures.

Table 1: Comparison of the overturning domain between the analytical solution, \(R^*\), and the numerical one, \(R_{\text{num}}^*\). The rocking block has \(h = 1 \text{ m} \), \(\rho = 2000 \text{ kg/m}^3\), and variable slenderness angle \(\alpha\). Different weights of TNT, \(W\), are considered. Good agreement is found, being always on the safety side.

<table>
<thead>
<tr>
<th>(W) [kg]</th>
<th>(\alpha = 20^\circ)</th>
<th>(\alpha = 15^\circ)</th>
<th>(\alpha = 10^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^*) [m]</td>
<td>2.18 3.24 5.51</td>
<td>3.40 5.10 8.78</td>
<td>6.53 9.96 17.51</td>
</tr>
<tr>
<td>(R_{\text{num}}^*) [m]</td>
<td>2.0 3.0 5.0</td>
<td>3.16 4.75 8.05</td>
<td>6.15 9.3 16.3</td>
</tr>
<tr>
<td>(R^<em>/R_{\text{num}}^</em>)</td>
<td>1.09 1.08 1.102</td>
<td>1.077 1.073 1.09</td>
<td>1.053 1.071 1.074</td>
</tr>
</tbody>
</table>

4.2 Fluid-structure interaction effect

We account for three-dimensional Fluid-Structure Interactions (FSI) with a CEL approach (cf. [23]): the balloon analogue models the explosive source and air is assumed as an ideal gas. The material parameters for the constitutive laws of the balloon are those detailed in [16] (p. 645, model #6). To ensure mesh convergence, the elements size of the Eulerian domain is fixed to 1.0 cm.

The numerical analyses account for the blast negative phase, diffraction and rarefaction phenomena, multiple reflections, no-normal incident angle of the blast waves with all the faces of the rocking block, and the three-dimensionality of the shock front. Notice that wave diffraction phenomena as well as the negative blast phase result in an overall reduction of the blast impulse with respect to the analytical model.

Table 4.2 presents the critical distance, as obtained from the numerical. The analytical model provides a safe estimate of the critical stand-off distance (upper bond). We notice that an intrinsic factor of safety slightly higher than 2 (ordinary value in any engineering design) is obtained with respect to the detailed numerical simulations.

Table 2: Comparison of the overturning domain between the analytical solution, \(R^*\), and the numerical one, \(R^*_{\text{FSI}}\). The rocking block has \(h = 1 \text{ m} \), \(\rho = 2000 \text{ kg/m}^3\), and slenderness angle \(\alpha = 15^\circ\). Different weights of TNT, \(W\), are considered. The analytical model gives a factor of safety \(\approx 2.5\) with respect to the numerical solution.

<table>
<thead>
<tr>
<th>(W) [kg]</th>
<th>(\alpha = 15^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^*) [m]</td>
<td>3.40 5.10 8.78</td>
</tr>
<tr>
<td>(R^*_{\text{FSI}}) [m]</td>
<td>1.50 2.25 3.35</td>
</tr>
<tr>
<td>(R^<em>/R^</em>_{\text{FSI}})</td>
<td>2.26 2.26 2.62</td>
</tr>
</tbody>
</table>
5 OVERTURNING OF MUSEUM ARTEFACTS

In this section we consider the overturning of museum objects due to blast loading. We are interested in the identification of the minimum perimeter around artefacts for protection against explosions.

The artefact is modelled as a rigid block, with front surface \( S \) equal to the front surface of the artefact and moment of inertia around the pivot point \( O \) equal to the one of the artefact, \( \mathcal{J}_o \). With reference to Figure 4, the centre of gravity is located at distance \( r \) from the pivot point, at a height \( h_g = r \cos \alpha \) from the ground and width \( b = r \sin \alpha \). The centroid of the front surface, impinged by the blast wave (simultaneously and uniformly), is at height \( h_c \) from the ground (see Fig. 4).

Blast loads are modelled as in Section 3 (cf. Sect. 4 (iii)). The drag coefficient \( C_D \) is supposed to be equal to 2 (\( C_D \) of a rectangular target) for front surfaces of any shape. We use the empirical predictions of \( P_{ro} \) [28], which are valid for rectangular objects. This assumption is on the safety side. For instance, a human body-like shaped target has a drag coefficient \( C_D \approx 0.97 - 1.43 \) [29].

Assuming small slenderness angles \( \alpha \) and a unilateral rocking response, the dimensionless equation of motion still holds. However, the dimensionless rocking moment and normalized time are corrected to consider the real geometry of the artefact as follows

\[
\chi \to \chi (1 + \delta), \quad \tau \to \frac{\tau}{\sqrt{\varsigma}}, \quad \tau_o \to \frac{\tau_o}{\sqrt{\varsigma}},
\]

with \( \varsigma = \frac{\mathcal{J}_o^2}{\mathcal{J}_o^2}, \quad \delta = \frac{h_c - h_g}{r} \) (11)

where \( \varsigma \) is the ratio of the moment of inertia of the artefact \( \mathcal{J}_o^2 \) and of the rectangular block.
\( \mathcal{J}_o \) and \( \delta \) is the dimensionless contribution to the rocking moment due to the misalignment of the surface centroid and the centre of gravity.

Accordingly, the linearised overturning condition (7) remains the same, i.e., \( 2\chi I \geq 1 \). As discussed above, the minimum stand-off distance \( R^* \), between a given artefact and a selected explosive quantity to avoid overturning is computed from inequality (7). The knowledge of \( R^* \) can be used to design barriers to prevent visitors from getting closer than the critical overturning distance.

We consider herein some emblematic museum statues belonging to the world cultural heritage as case studies for the assessment of protective barriers, see Figure 5. For each statue, we consider the worst case scenario: a blast wave with a direction such that the statue rocking resistance is the smallest one (minimum slenderness angle \( \alpha \) and moment of inertia \( I_o \)). Table 5 shows the overturning domain for each artefact as function of the explosive weight, \( W \). The case of Michelangelo’s David is particularly interesting. A large height and a high slenderness angle confer to the statue an excellent resistance to rocking, hence overturning. Notice that the protective barrier around the statue of Michelangelo’s David at the Gallery of the Academy of Florence is such that it is impossible to approach the statue closer than \( \approx 1.50 \) m, meaning that the artefact is safe for explosive weights as high as 30 kg and greater. The same holds for Aphrodite of Milos. On the contrary, the statue of Laocoön and His Sons is found to be highly vulnerable to explosive threats.

![Figure 5: Different museum artefacts considered. From left to right: Michelangelo’s David (Gallery of the Academy of Florence, Florence), Laocoön and His Sons (Vatican Museums, Vatican City), and Aphrodite of Milos (Louvre Museum, Paris). The three-dimensional models are recovered from the platform Scan The World [30].](image)

6 CONCLUDING REMARKS

We investigated the dynamics of inverted pendulum structures under fast-dynamic excitations arising from an explosion with particular interest to museum artefacts. The purpose of our analysis is to derive reliable decision making tools in the design of protective devices to preserve the historical heritage.

By virtue of a simplified expression of blast actions and based on established empirical
Table 3: Rocking and overturning parameters for the considered artefacts and corresponding critical stand-off distances for different explosive weights.

<table>
<thead>
<tr>
<th>Museum artefact</th>
<th>$m$</th>
<th>$J^2_o$</th>
<th>$h_y$</th>
<th>$h_c$</th>
<th>$\alpha$</th>
<th>$b$</th>
<th>$S$</th>
<th>$R^*$ (10 kg)</th>
<th>$R^*$ (20 kg)</th>
<th>$R^*$ (30 kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Michelangelo's David</td>
<td>5800</td>
<td>$1.65 \times 10^3$</td>
<td>2.28</td>
<td>2.35</td>
<td>17.6</td>
<td>0.70</td>
<td>5.02</td>
<td>0.25</td>
<td>0.32</td>
<td>0.37</td>
</tr>
<tr>
<td>Laocoon and His Sons</td>
<td>1328</td>
<td>13.6</td>
<td>0.61</td>
<td>0.79</td>
<td>27.1</td>
<td>0.32</td>
<td>1.83</td>
<td>1.13</td>
<td>1.64</td>
<td>2.06</td>
</tr>
<tr>
<td>Aphrodite of Milos</td>
<td>565</td>
<td>16.4</td>
<td>0.86</td>
<td>0.87</td>
<td>18.4</td>
<td>0.28</td>
<td>0.83</td>
<td>0.7</td>
<td>1.0</td>
<td>1.25</td>
</tr>
</tbody>
</table>

models, we derived analytical, closed form solutions for the rocking of inverted pendulum structures (rectangular block). Relying on an energetic approach, we obtained the overturning condition in terms of the minimum distance (critical stand-off distance) that has to be assured between the explosive source and the target, such that toppling is avoided. The proposed model was then compared and validated with detailed numerical simulations that consider the fluid-structure interaction phenomena, a combined rocking/sliding behaviour, and the possibility of uplifting (flight mode). Finally, we considered the overturning of un-anchored museum artefacts. The previously derived equation of motion and overturning conditions still holds. Correction factors were introduced to consider the actual geometry and mass distribution of artefacts. We investigated the safety of some emblematic statues against explosions. By means of the proposed analytical model we highlighted the criticality of the preservation state of some emblematic statues.

We derived the critical stand-off distance in the form of design charts, which can be helpful in many applications. Of interest here was the use of the analytical estimations in preserving museum objects, for determining the minimum perimeter around statues of high historical and aesthetic value. The proposed analytical model can be further used in the protection of existing buildings and assets as well for devising energy absorbing systems based on rocking.
APPENDIX. ANALYTICAL SOLUTIONS OF THE EQUATION OF MOTION

Analytical, closed-form solution of Equation (4): angular displacement, $\phi$, and angular velocity, $\dot{\phi}$ (with $\zeta = d/\tau_o$).

$$\phi(\tau) = \begin{cases} 
\frac{1}{(\zeta^2-1)^2} \left\{ \left[ 1 - \left( 1 - \frac{2}{\tau_o} \right) \zeta \right] \cosh \tau + \\
\left( \zeta + \frac{1}{\tau_o} \right) \left( \zeta^2 - 1 \right) - \frac{2}{\tau_o} \cosh \tau - \\
\left( \zeta^2 - 1 \right) (\zeta - 1) - 2 \frac{\zeta}{\tau_o} \right \} \sinh \tau + \\
\frac{\tau_o}{\tau} (\zeta^2 - 1) - \left( \zeta + 2 \frac{2}{\tau_o} \right) \zeta + 1 \right \} \sinh \frac{\tau_o}{\tau} \right \} & \text{if } \tau \leq \tau_o, \\
\frac{1}{(\zeta^2-1)^2} \left\{ \left[ (\zeta^2 + 1) e^{-\zeta \tau_o} \right] \chi \sinh \tau - \\
\zeta (\tau_o + \zeta - \zeta^2 \tau_o) + 1 \right \} \chi \sinh(\tau + \tau_o) - \\
\left( \zeta^2 - 1 \right) \tau_o \left( \zeta^2 + \chi - 1 \right) - 2 \zeta \chi \right \} \cosh(\tau + \tau_o) - \\
2 \zeta \chi e^{-\zeta \tau_o} \cosh \tau \right \} + 1 & \text{if } \tau > \tau_o.
\end{cases}$$

$$\dot{\phi}(\tau) = \begin{cases} 
\frac{1}{(\zeta^2-1)^2} \left\{ \left[ 1 + \left( 1 - \frac{2}{\tau_o} \right) \zeta \right] \cosh \tau + \\
\left( \zeta + \frac{1}{\tau_o} \right) \left( \zeta^2 - 1 \right) - \frac{2}{\tau_o} \cosh \tau - \\
\left( \zeta^2 - 1 \right) (\zeta - 1) + 2 \frac{\zeta}{\tau_o} \right \} \sinh \tau + \\
\frac{\tau_o}{\tau} (\zeta^2 - 1) - \left( \zeta - 2 \frac{2}{\tau_o} \right) \zeta + 1 \right \} \sinh \frac{\tau_o}{\tau} \right \} & \text{if } \tau \leq \tau_o, \\
\frac{1}{(\zeta^2-1)^2} \left\{ \left[ (\zeta^2 + 1) e^{-\zeta \tau_o} \cosh \tau + 2 \zeta^2 e^{-\zeta \tau_o} \cosh \tau \right] \chi - \\
\zeta (\tau_o + \zeta - \tau_o \zeta^2) + 1 \right \} \chi \cosh(\tau + \tau_o) - \\
\left( \zeta^2 - 1 \right) \tau_o \left( \zeta^2 + \chi - 1 \right) - 2 \zeta \chi \right \} \sinh(\tau + \tau_o) - \\
2 \zeta \chi e^{-\zeta \tau_o} \sinh \tau \right \} & \text{if } \tau > \tau_o.
\end{cases}$$

REFERENCES


FRAGILITY ASSESSMENT OF BASE ISOLATED FREE STANDING MUSEUM ARTIFACTS

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Abstract

The seismic response evaluation of free standing building contents, has become a challenging task for the structural engineers. Artistic assets, which are placed on museums, are subjected to the floors’ acceleration signal. In this view, it is obvious that the seismic action that affect the response of the floors non-structural contents is affected by the seismic response of the host building. The building contents, such as museum artifacts, are usually standing unanchored on a rigid base, and as such, they demonstrate rocking behavior under ground motion excitation. In the present study, the seismic response of rigid rocking blocks, with dimensions corresponding to museum assets, placed on the first floor of a two story reinforced concrete building is investigated. Base isolation is currently applied in contemporary museum buildings in order to reduce the ground motions’ intensity. However, especially on existing museum buildings, base isolation can be easily implemented on the base of a specific cultural heritage object or a certain floor area in which artifacts of great importance are placed. These two different alternatives of base isolation configurations, mentioned above, are considered in the present study and fragility analysis is performed in order to evaluate each seismic mitigation method.

Keywords: Rocking Artifacts, Rocking Blocks, Seismic Isolation, Fragility Analysis.
1 INTRODUCTION

The seismic response evaluation of building contents which present rocking behavior, has become a challenging task for the structural engineers. At museums, laboratories or nuclear power stations, the damage of their contents, which usually stand free unanchored at the floors, are of great importance. As such, their seismic performance is affected by the response of the host building and especially by the acceleration signal of the specific floor. The performance of both free standing contents and nonstructural components placed on building structures, subjected to ground motion acceleration has been examined the last years [1-6]. In the present study a seismic fragility analysis of a free standing rocking rigid block, without considering sliding, with dimensions corresponding to a museum asset, such as a marble statue, hosted in the second floor of a two storey reinforced concrete frame building is performed.

The seismic protection of rocking rigid blocks using base isolation devices has been thoroughly examined [7-10]. Moreover, for acceleration sensitive building contents, the application of floor isolation systems has been studied [11, 12]. In this scope, two alternatives of base isolation implementations, using friction pendulum bearings, are examined. In the first one the host building is considered isolated, while in the second one the seismic isolation is applied at the base of the examined content. The latter, could be considered as a seismic mitigation strategy in order to reduce the vulnerability of artifact hosted into conventional museum buildings.

2 NUMERICAL ANALYSIS

2.1 RC frame building

For the scope of that study a two storey building is considered. That choice is justified due to the fact that, museum buildings are in common low rise and stiffer than the conventional ones. The building is designed based on the EC2 and EC8. The building dimensions, as well as, the structural elements cross sections are depicted in Figure 1. The selected concrete quality is C25/30, while the reinforcing steel quality is B500C. Concentrated plasticity elements are used in order to simulate the structures nonlinear response. The dynamic analyses are performed without considering the simultaneous rocking response of the examined block as it mass is lesser than the 5% of the floors mass. Due to that fact, the seismic response of the rocking block does not affects the seismic response of the hosting structure. The nonlinear dynamic analyses are performed in the software SAP2000.

![Figure 1: RC Frame Structure.](image-url)
## 2.2 Free standing rigid block seismic response

The unanchored content of a building can be modeled as a rigid block (Figure 2). A rigid block standing free on a rigid base, with slenderness $\alpha$, semi-diagonal $R$ and frequency parameter $p$, oscillates about the centers of rotation $O$ and $O'$ when base acceleration exceeds $\ddot{u}/g \geq \tan(\alpha)$. The problem of a rigid rocking block motion under a seismic excitation, without considering sliding, can be described by the following equation \[13\]:

$$\ddot{\Theta} = -p^2 \cdot \left\{ \sin[\alpha \cdot \text{sgn}(\Theta) - \Theta] + \frac{\ddot{u}}{g} \cdot \cos[\alpha \cdot \text{sgn}(\Theta) - \Theta] \right\}. \tag{1}$$

where $\text{sgn}(\cdot)$ is the sign function and $p = \sqrt{3g/4R}$ is the frequency parameter of the rigid block.

During the rocking motion, energy is lost only during impact (when the rotation changes sign at $\Theta = 0$) which causes a reduction of the rotational velocity after it:

$$\left(\dot{\Theta}_{n+1}\right)^2 = r \cdot \left(\dot{\Theta}_n\right)^2 \tag{2}$$

where $r$ is the restitution coefficient, $\dot{\Theta}_n$ is the velocity before the impact and $\dot{\Theta}_{n+1}$ is the velocity after the impact.

Considering that the angular momentum remains constant about point $O$ exactly before the impact and right after it, the coefficient of restitution for a rigid rectangular block is given by the following equation \[14\]:

$$r = \left[1 - \frac{3}{2} (\sin \alpha)^2\right]^2 \tag{3}$$

It has to be noticed that the mass of the base in which the rocking block stands, has considered as infinite. As such, the above relationships are compatible with the problem \[7, 8\].

![Figure 2: Rigid rocking block.](image)

The considering block has height $2h = 2.0$ m and width $2b = 0.5$ m, dimension that correspond to a semi-diagonal $R = 1.03$ m and slenderness $\alpha = 0.245$ rad. The rocking block is examined as it is placed on the second floor of the examined structure. Thus, it is subjected to the floors acceleration signal. The floors acceleration signals are derived from the results of the nonlinear time history dynamic analyses of the RC structure.
2.3 Isolation Implementation

Friction pendulum systems (FPS) [15] are studied as isolation devices. In Figure 3 the two isolated methods are pictured. In the first one, where the whole building is isolated, FPS devices with radius $R_{\text{eff}} = 2.24\, \text{m}$ and friction coefficient $\mu = 0.1$ is considered. Regarding the rocking block isolation, FPS devices with radius $R_{\text{eff}} = 2.24\, \text{m}$ and friction coefficient $\mu = 0.03$ are used. In both cases the isolation period is $T_{\text{is}} = 2\, \text{s}$. When base isolation is implemented on the rocking block, the acceleration signal that subjects the rocking block is measured on the isolated base instead of the floor. In that case the floors acceleration is equal to the fixed based structure.

![Rocking Content](a)

![FP Isolator](b)

Figure 3: Base isolation (a) of the whole building and (b) of a specific artifact.

2.4 Ground Motion Records

A set of 35 natural ground motion record has been used for the dynamic analyses. The initiation of rocking on the fixed base structure due to all the seismic excitations are the main criteria of the specific records selection. Both near-fault and far-fault records are employed, in order to present a wide range of intensities and frequency contents A rigorous search on the PEER [16] and the European strong motion database [17] are carried out concluding to the used records. In Table 1 the range of the PGA, PGV and PGV/PGA values are listed.

<table>
<thead>
<tr>
<th>IM</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA (g)</td>
<td>0.17 – 0.72</td>
</tr>
<tr>
<td>PGV (cm / s)</td>
<td>5.80 – 106.91</td>
</tr>
<tr>
<td>PGV/PGA (s)</td>
<td>0.02 – 0.44</td>
</tr>
</tbody>
</table>

Table 1: Intensity Measures Values Range

2.5 Fragility Analysis

In the present study the effect of the seismic protection alternatives are evaluated by generating fragility curves of the rocking block. To estimate the probability of overturning the
problem should be considered as a categorical one, by grouping the data into non-collapse and collapse ones. So to estimate the parameters of the fragility function (mean $\mu$ and standard deviation $\beta$) that provides the probability of collapse, the maximum likelihood approach is adopted [18].

The introduced engineering demand parameter (EDP), adopted herein, is the absolute maximum developed rotation $|\theta_{\text{max}}|$ normalized to the critical overturning rotation (Equation 4).

$$EDP = \frac{|\theta_{\text{max}}|}{\alpha}$$  \hspace{1cm} (4)

As intensity measures, PGA, PGV and the ration PGV/PGA is used for the vulnerability analysis. The velocity based parameters and the frequency content ones has been approved to be better damage indicators for rocking blocks of large size [19,20]

3 RESULTS

3.1 Seismic excitation properties at the base of the artifact

In order to estimate the efficiency of the base isolation alternatives, their affection on the floors seismic acceleration signal is studied. The three main excitation parameters presented above, are the Peak Floor Acceleration (PFA) and the Peak Floor Velocity (PFV) and the ratio PFV/PFA. These parameters are presented normalized to PGA, PGV and PGV/PGA values of the initial record respectively.

Regarding the PFA values (Figure 4), the statue isolation is the most efficient. Examining the fixed base structure, the PFA values are on average higher than the PGA ones. Only in cases that the structure respond inelastically, the PFA seems to be reduced. By performing seismic isolation on the base of the building, the PFA took lower values than the corresponding PGA ones, while in the case of the artifact isolation the PFA values are even more reduced. When the structure behaves elastically, the increase of the PFA values exceeds 150% of the PGA values, whereas when severe structural damage occurs, the PFA are reduced.

![Figure 4: PFA/PGA values.](image-url)
Due to the fact that the peak acceleration is an optimal IM regarding the prediction of only small in size rocking response, the PFV values are also presented (Figure 5). In the fixed base structure, the PFV values are increased under all the seismic excitations. Examining the base isolated structure the PFV values are in common slightly increased. Reduction of the floors velocities are presented only when base isolation is performed under the monument base.

![Figure 5: PFV/PGV values.](image)

In Figure 6 the PFV/PFA values are depicted. The main feature of the base isolation technique except of decreasing the acceleration values of the seismic signal, is that elongates the period of the excitation wave. That can be easily observed in Figure 6, in which the normalized PFV/PFA values are depicted. Increasing of the period parameter is occurred in base isolation cases, while for the fixed base structure the values are altered slightly.

![Figure 6: (PFV/PFA)/(PGV/PGA) values.](image)
3.2 Rigid rocking block response

In Figure 7 the rocking rotations, normalized over the blocks slenderness, in respect with PGA, PGV and PGV/PGA are presented. As expected, the base isolated free standing block developed less rocking rotations. Specifically, when the structure is considered fixed base the block overturns under 9 seismic events, while it overturns under 5 excitations when the host structure is base isolated. The most efficient solution seems to be the isolation of the specific artifact, or the specific floor, in which the block overturns only under one excitation.

3.3 Fragility curves

In Figure 8 the overturn fragilities are displayed in respect of the PGA, PGV and PGV/PGA values. The standard deviation values, using the PGA as an IM, are 0.47, 0.39 and 0.27 for the fixed base structure, the base isolated structure and the statue isolation case respectively. Using the PGV these values took the values of 0.37, 0.37 and 0.14. Finally using PGV/PGA as ground motion parameter, the standard deviation values are 1.21, 1.71 and 2.26, respectively. Comparing the standard deviation values, a factor that indicates the efficiency of an IM to predict the seismic response, the PGV/PGA seems to have the worst performance. It can be remarked that the standard deviation values of the fragility curves in terms of PGV are much lower than those in terms of PGA. By the fragility curves, the enhanced seismic response of the isolated block is obvious. It has to be noticed that seismic isolation, using bearings at their base, has beneficial effect of the seismic response of small rocking blocks [8].
4 CONCLUSIONS

In the present study the seismic response of rocking building contents is examined. Specifically, the seismic performance of a museum rocking content hosted on a two storey RC building is studied. Moreover, two base isolation alternatives are investigated for the protection of the building contents. In the first the host building is considered base isolated, while in the second one only the rocking content at the level of the floor is isolated. The base isolation is performed using friction pendulum bearings. In both alternatives the examined, small in size, rocking block presents enhanced response compared with the fixed base structure. The results indicated that the floor isolation seems to be more beneficial for the seismic protection of the free standing contents hosted on a fixed base reinforced concrete structural system, than the base isolation of the whole structure.

REFERENCES


UNCERTAINTY QUANTIFICATION IN ULTRASONIC GUIDED-WAVES BASED DAMAGE LOCALIZATION

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Keywords: Bayesian inverse problem, SHM, ultrasound, guided waves, damage localization, hyper-robust model, uncertainty quantification.

Abstract. Bayesian methods for inverse problems offer higher robustness to noise and uncertainty than deterministic, yet accurate, inference methods. Both types of techniques typically focus on finding optimal model parameters that minimize an objective function, which compares model output with some acquired data. However, uncertainties coming from different sources, such as: (1) the material manufacturing process, (2) material’s mechanical properties, (3) measurement errors, or (4) the model and its parameters, may cause inference errors and loss of information should they are not properly taken into account. These uncertainties might have important safety and economic consequences in damage-related applications, such as in structural health monitoring of aerospace structures. This paper aims at illustrating the benefits of using probability based methods instead of deterministic approaches. A case study is presented, which illustrates the use of a hyper-robust Bayesian damage localization method when compared to a deterministic one. The results show that Bayesian inverse problem is more robust to data noise and uncertainties stemming from the model parameters than deterministic methods.
1 INTRODUCTION

Damage localization and reconstruction in plate-like structures using guided-wave based structural health monitoring (SHM) have been mainly addressed using post-processing techniques applied to ultrasonic signals [1]. The exploration of large areas with a small attenuation [2] is one of the most remarkable characteristics that has led industries, such as the aerospace industry, to focus on guided-waves. Sparse or phased-array sensors’ layouts are typically placed so that the structure is actively interrogated on demand, which confers higher accuracy and reliability [3]. Potential safety and economical implications in condition-based maintenance are extra-motivations for the use of this SHM technique. The required damage-related information is extracted from the data by using inverse problems (IPs) that minimize the distance between model predictions and observations [4].

IPs can be solved using different methods such as the deterministic or the Bayesian ones. Deterministic approaches seek the best value of the model parameters that provides the closest prediction to the available observation. Alternatively, Bayesian inverse problems (BIPs) provide a set of parameters values associated to their posterior plausibilities, namely the posterior probability density function (PDF). To this end, the prior degree of belief of the parameters is updated by using observations (data) and the Bayes’ Theorem [5, 6]. In the particular case of ultrasound-based damage detection, two general approaches are typically adopted: (1) model-based IPs, whereby detailed damage information (e.g. the severity of damage as residual strength) [7] can be obtained from the measured signal at a considerable computational cost; and (2) efficient IPs based on signal features, whereby other relevant information, e.g. the damage position or severity, can be obtained [2, 8–10]. Among them, the time-of-flight (ToF) has been extensively used as a signal feature for its efficiency in obtaining information about material properties along with damage localization using post-processing scattered signals. It is worth mentioning that ultrasound-based damage localization conveys sources of uncertainty that are mostly related to the measurement system and physical properties of the material.

To partially address this modeling issue, a number of researchers have proposed the use of BIP applied to ultrasound based damage localization [11, 12]. More recently, a Bayesian framework, which combines the information coming from different post-processing techniques, i.e., time-frequency models [13–16], using their posterior plausibilities to obtain a more robust damage localization, has been proposed in [17]. In this paper, a comparison between BIPs and deterministic IPs in the context of ultrasonic guided-wave based damage localization is provided. In the context of BIP, probability is interpreted as a multi-valued logic that expresses the degree of belief of a proposition conditioned on the given information [5, 18]. Both deterministic and Bayesian approaches are applied to an aluminum plate with one and two damaged areas. The asymptotic independent Markov sampling (AIMS) [19, 20] algorithm is adopted to solve the resulting BIP, while genetic algorithms (GA) are used to solve the deterministic IP. In general, the results show that BIP is superior in terms of robustness of the damage reconstruction while the deterministic problem provide very efficient solutions but at risk of obtaining incomplete and biased information.

The remainder of the paper is organized as follows: Section 2 comprises the probabilistic methodology used to obtain the robust estimate of the ToF for each sensor. In Section 3, the proposed framework is applied in two case studies and the results are discussed. Finally, Section 4 provides concluding remarks.
2 METHODOLOGY
2.1 Bayesian damage localization

In this section, the ultrasonic guided-wave based damage localization is addressed by using a model-based BIP using an ellipse-based ToF model [21]. To this end, \( N_p \) actuator-sensor paths are considered in a plate-like structure to excite and receive guided-waves for damage detection by screening changes of their ToF. To this end, the ToF information of the scattered signals can be theoretically obtained as follows [22]:

\[
\text{ToF}^{(a-s)} = \frac{\sqrt{(X_d - X_a)^2 + (Y_d - Y_a)^2}}{V_{a-d}} + \frac{\sqrt{(X_d - X_s)^2 + (Y_d - Y_s)^2}}{V_{d-s}}
\]

(1)

where \((X_d, Y_d)\) are the coordinates of the damage, \((X_a, Y_a)\) are the actuator transducer coordinates, \((X_s, Y_s)\) are the coordinates of one arbitrary sensor transducer, and \(V_{a-d}\) and \(V_{d-s}\) are the wave propagation velocities of the actuator-damage and damage-sensor paths respectively. These velocities are the same under the assumption of isotropic materials and a concentrated damage within a bounded region, i.e. \(V = V_{a-d} = V_{d-s}\).

To probabilistically describe the ToF model given by Equation (1), a set of uncertain model parameters \(m = \{X_d, Y_d, V\}\) are considered in this problem to describe the uncertainty about the damage coordinates as well as the wave propagation velocity. The set \(m\) of model parameters is augmented with a model error term \(e \in \mathbb{R}\), resulting in a set of model parameters defined as \(\theta = \{m, \sigma_e\} = \{X_d, Y_d, V, \sigma_e\} \in \Theta\), where \(\sigma_e\) is the standard deviation of the error term \(e\) and \(\Theta\) is the model parameter space. The referred model error term \(e \in \mathbb{R}\) is considered to account for the non-existence of a theoretical ToF model that fully represent the reality, so that [17]:

\[
\text{ToF}^{(a-s)}_M = \text{ToF}^{(a-s)}_D (m) + e = \text{ToF}^{(a-s)}_M (\theta)
\]

(2)

where subscripts \(M\) and \(D\) from \(\text{ToF}^{(a-s)}_M\) and \(\text{ToF}^{(a-s)}_D\) refer to modeled and measured ToF, respectively. Note in Equation (2) that \(e\) provides the discrepancy between \(\text{ToF}^{(a-s)}_M\) and \(\text{ToF}^{(a-s)}_D\) values. By the PMIE, this error term can be conservatively described as a zero-mean Gaussian distribution with covariance \(\sigma_e\) as \(\mathcal{N}(0, \sigma_e)\).

Next, the posterior PDF of the model parameters given the ToF data \(D = \{D^{(1)}, \ldots, D^{(N)}\}\), where \(N\) is the total number of active sensors, by applying the well-known Bayes’ Theorem as:

\[
p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}
\]

(3)

where \(p(\theta)\) is the prior PDF of the model parameters, and \(p(D|\theta)\) is the likelihood function for the set of data \(D\). Given the stochastic independence of the measurements, the likelihood can be expressed as \(p(D|\theta) = \prod_{k=1}^{N} p(D^{(k)}|\theta)\). Finally, \(p(D)\) is the evidence of the data under the model specified by \(\theta\). This term, which acts as a normalizing factor within the Bayes’ theorem, can be bypassed through sampling, e.g. using Markov Chain Monte Carlo (MCMC) methods [23]. In this paper, the AIMS algorithm [19][20] is used as MCMC method due to its efficiency in addressing multimodality of the posterior PDF. Thus, Equation (3) can be rewritten as:

\[
p(\theta|D) \propto \left\{ \prod_{k=1}^{N} p\left(D^{(k)}|\theta\right) \right\} p(\theta)
\]

(4)
2.2 Deterministic damage localization

Similarly, the deterministic approach pursues the minimization of the discrepancy between predictions and observations. In this case, however, the error term that measures the aforementioned distance $r$ is considered as a deterministic variable as follows:

$$\text{ToF_D}^{(a-s)}(m) = \text{ToF_M}^{(a-s)}(m) + r$$

where $m$ denotes the unknown parameters. The minimization problem can be formulated using the $L_2$-norm between prediction and observation, as follows:

$$m_{opt} = \arg \min_m \|\text{ToF_D}^{(a-s)} - \text{ToF_M}^{(a-s)}\|_2^2$$

where $m_{opt}$ is the set of optimal parameters that minimize the discrepancy between model and data. In addition, the logarithm of the objective function is considered to boost the convergence of the minimization problem, as follows:

$$m_{opt} = \arg \min_m \log \left( \|\text{ToF_D}^{(a-s)} - \text{ToF_M}^{(a-s)}\|_2^2 + \Delta \right)$$

where $\Delta$ is a small value that avoids the trivial solution when the objective functions tends to zero [24]. Equation (6) is then addressed by using GA [25], given that the objective function is non-convex, and hence more than one local minimum may be found. The GA are configured so that the convergence of the objective function is achieved.

Figure 1: General framework used to compare deterministic and Bayesian IPs.

Figure 1 depicts the workflow for the comparison of both the BIP and the deterministic IP using a GA-based approach. Note that both IPs use the same ToF data to address the damage localization so that the outputs are comparable. Note also that the BIP provides the values of the model parameters associated to their posterior probabilities by the posterior PDF, while the deterministic approach only provides the best individual that minimizes the corresponding objective function (recall Eq. (7)). Thus, the BIP is able to quantify the uncertainty associated to the model parameters stemming from different sources.
3 CASE STUDIES

3.1 Synthetic signal generation

For the purpose of signal generation, a 300mm × 300mm aluminium plate of 2mm thickness has been modeled in Abaqus. The material is considered to be an aluminum alloy 2024-T351 with Young’s modulus $E = 73.1$ GPa, density $\rho = 2780$ kg/m$^3$, and a Poisson’s ratio of 0.33. Here, the damage is modeled as a rectangular hole of dimensions $2\text{mm} \times 4\text{mm}$. S4R (4-node, quadrilateral, stress-displacement elements with reduced integration and large-strain formulation) shell elements \cite{26} are used with 1mm mesh size to avoid spatial aliasing. The excitation is modeled as a perpendicular action with a 5 cycle sine tone-burst centered at $f = 100$ kHz. In addition, the sensors are arbitrarily placed as can be observed in Figure 2.

![Figure 2: Layout of the aluminum plate with both one and two damaged areas. The red area correspond to a single damage location in case study 1, while the two blue damaged areas correspond to the case study 2.](image)

The acquired signals are then post-processed by applying different time-frequency models, which are then ranked based on their posterior plausibilities. Consequently, a robust time-frequency model is built by using such probabilities \cite{17}. The robust ToF values are then used as data to reconstruct the damage location by using the ToF model-based approaches, i.e., the deterministic and the Bayesian ones.

3.2 One damaged area

Figure 3a depicts the damage reconstruction when one damaged area is simulated. In this case study, both the BIP, whose posterior PDF is represented by iso-probability lines, and the deterministic IP, whose best individual is depicted by a gray point, provide similar damage localization. Here, the deterministic IP stands out computationally due to its efficiency in providing the minimum. However, the superior robustness of the posterior PDF provided by the Bayesian approach, which accounts for uncertainties from several sources, is manifested in its dispersion, while the deterministic solution provides one unique value. As can be observed in Figure 3a, the deterministic solution is located at the tail of the posterior PDF, which is the result of a different wave propagation velocity inference. In particular, the velocity obtained by the deterministic approach is 1962 m/s, while the mean of the marginal posterior PDF obtained by the BIP is 2000.4 m/s. This small variation is the cause of the misplacement of the damage position in the deterministic IP with respect to the BIP. These results emphasize the strengths of BIP when dealing with uncertain data, models and parameters.
3.3 Two damaged areas

In case of two or more damaged areas, the BIP shows its clear superiority in comparison with the deterministic IP. As can be observed in Figure 3b, the deterministic solution of the damage location, depicted by the gray point, is relatively accurate with respect to one damaged area. This result points out one of the limitation of the GA, which is only able to provide with one minimum that corresponds to the damage location of one hole. In contrast, the Bayesian approach is able to provide with a relatively accurate damage location of both holes due to the ability of the AIMS algorithm to address multimodality. Note that the posterior PDF is depicted by iso-probability lines in Figure 3b. It is also worth mentioning that both the high probability areas coincide with the closer corner holes to the actuator located at the center of the plate. This behavior agrees with the wave scattering phenomenon when the guided-waves reach the square holes.

4 CONCLUSIONS

A comprehensive comparison between deterministic and Bayesian approaches to address ultrasonic guided-wave based damage localization has been illustrated in this paper. To this end, a state-of-the-art robust Bayesian damage localization approach that uses the AIMS as MCMC algorithm to draw samples from the posterior PDF has been compared with the classical approach of GA-based IP. Two case studies have been selected to illustrate the strengths of each approach, using one structure with one and two damaged areas, respectively. The results show the superiority of the BIP in comparison with the deterministic IP in more complex cases such as in the presence of multiple damage areas, while the GA-based approach stands out due to its efficiency in providing local minimum.

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References


IMPACT DAMAGE IDENTIFICATION IN A COMPOSITE STRUCTURE BY SURROGATE MODELLING AND MARKOV-CHAIN MONTE-CARLO METHOD

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Abstract

Bayesian approaches have proven to be successful in structural health monitoring, especially for inverse problem solutions, such as the identification of damage parameters from some indirect observations. Operatively, they aim at generating random samples of the damage parameters based on some prior distributions and then assigning each sample a weight according to its likelihood, i.e. a probability which, in the Bayesian context, quantifies the degree of agreement between the observations and the predicted sample. However, when fast analytical expressions for likelihood assessment do not exist, numerical simulations might be required for the evaluation of the trajectory likelihoods, thus making the whole procedure for the posterior distribution estimation computationally unfeasible. In this work, the computational problem is addressed by leveraging on surrogate modeling, while the Markov-Chain Monte-Carlo method provides the Bayesian framework for approximating the posterior probability distribution of the damage parameters. Specifically, the method is applied to the impact damage identification, including damage position and extent, on a rectangular composite panel, based on observations of the strain field pattern acquired at some specific nodes. A surrogate model consisting of an artificial neural network, trained on a set of experiments virtually generated by numerical analyses, is used to predict the strain as a function of the damage position and extent, thus allowing a fast calculation of the strain observation likelihood. The algorithm is successfully tested with respect to a numerical case study of an impact damage occurring on a composite plate.

Keywords: SHM, diagnosis, damage identification, delamination, Monte Carlo Markov Chain, surrogate model, Metropolis-Hastings, Bayesian.
1 INTRODUCTION

Fiber reinforced composite structures have become increasingly popular in recent decades, mainly due to their performance, offering several distinct advantages over more conventional engineering materials, and are nowadays extensively employed in the transport industry. However, composite materials tend to exhibit high vulnerability to localized impact-like loading, e.g. a dropped tool or a runaway debris strike. Indeed, there are at least two main reasons that make impact damage a serious issue, especially when considering aircrafts. First, it is often very difficult to detect and diagnose impact damage, unless extensive and costly Non-Destructive Testing (NDT) campaigns are regularly undertaken, mainly because impact damage – unlikely for metallic structures – is often barely visible. Second, even a low-velocity low-energy impact causing no (or barely) visible damage can lead to a serious decrease in the component strength, e.g. component compressive strength might halve in presence of a barely visible damage [1]. As a consequence, the scientific and industrial communities have put considerable effort into the development of frameworks able to autonomously assess the integrity level of the structures of interest, generally known as Structural Health Monitoring (SHM) [2], [3]. According to SHM general prescriptions, the automatic evaluation of the health state of the structure is based on a network of permanently installed sensors retrieving data from the structure response. Data are then integrated within statistically based models providing knowledge about the actual health state of the system (diagnosis), focusing on the identification, localization and quantification of the potential damage [4]. The intrinsic advantage of statistical frameworks over deterministic ones is that they take into consideration the impact of uncertainty that is congenital on measurements and modelling assumptions.

The continuous on-line real-time monitoring of composite structures in the context of SHM has reached increasing attention in the past decades, and there is an extensive body of scientific literature on the topic; many SHM methods exist in literature, either based on data [5] or on models [6], [7]. The former rely on real world data (measurements) and methods for identifying patterns within the data in order to infer the health state of the system; these are generally limited by data availability. The latter leverage on models which provide simulated data of the healthy and damaged system, from which it is possible to extract damage dependent features, enabling assumptions on the health state of the system under study.

Model based SHM methods take advantage of the capability of modern computers in order to provide data out of numerical simulations; thus, they avoid the need of prohibitive test campaigns to generate the necessary amount of data. However, model based methods, despite being very effective and manageable in most situations, are typically structured as inverse problems, targeting the identification of some parameters of interest for model updating [8]. This means that given some relation – i.e. a function – mapping any parameters space (e.g. damage parameters) onto the space of observations (e.g. measurements of variables of interest), one desires finding the parameters conditioned on some measured quantities (observations). Within this groundwork, Bayesian inference lays the foundations for a statistical framework for damage identification [9], [10]. According to Bayesian Model Updating (BMU) methods, the joint posterior probability density function (pdf) is computed as a function of the observation from indirect measurements [7], [9], [11]. Despite the apparent simplicity of such methods it is almost always impossible to find a closed form solution for these problems, especially when dealing with real world applications. A solution to this problem comes from the Monte Carlo Sampling (MCS) techniques [7], [12]. Among the MCS, the Monte Carlo Markov Chain (MCMC) with Metropolis-Hastings (M-H) algorithm has been extensively used, due to its power and ease of application. Despite its advantages, the MCMC algorithms involve the computation of the sample likelihood for the posterior pdf approximation at each algorithm
iteration, which entails running a model which might be rather complex, e.g. a Finite Element (FE) model, at each iteration of the algorithm. That is usually not effective nor practical, since it would take considerable time, given that MCMC algorithms usually requires thousands of iterations. A way to address this issue lies in the surrogate modelling, e.g. based on response surfaces, Gaussian processes or on ANNs [13], [14] just to mention few approaches, which allow mapping the space of parameters (e.g. damage parameters) onto the space of the observed variables (e.g. measurements) via manageable and efficient functions, thus enabling fast likelihood assessment [15]. An interesting example of ANN surrogate modelling applied in a Bayesian framework for damage estimation can be found in [13], where, given a limited set of strain field measurements, MCMC algorithm is used to estimate the probability distribution of damage parameters.

In the present work, a Bayesian statistical framework for the impact damage assessment of a Carbon Fiber Reinforced Polymer (CFRP) based on a net of strain sensors is illustrated and verified on a CFRP panel. The curse of dimensionality is addressed by simplifying the impact damage description; the parameters space characterizing the impact damage is thus reduced to a level which achieves a reasonable compromise between the data set dimensions for the training of the surrogate model and the appropriate number of damage parameters to mimic with sufficient fidelity real world data. Then, a MCMC M-H algorithm estimates the impact damage location and extent, based on a series of observations collected from the sensor network. Several Artificial Neural Network (ANN) surrogate models trained off-line with numerical (FE) strain field simulations in presence of various impact damages are embedded into the MCMC M-H algorithm routine. These surrogate models draw the functions between damage parameters and the strain field at precise locations (sensor positions), enabling fast sample likelihood calculation in the MCMC M-H algorithm iterations. The algorithm is verified in a simulated framework, providing good agreement with test case data.

The paper is structured as follows: Section 2 provides an overview of the MCMC algorithms and an inside in surrogate modelling; Section 3 discloses the application framework of both the surrogate models and MCMC algorithm, with their respective results. A conclusive section is provided in the end.

2 METHODS

In this section the probabilistic framework for the model-based damage diagnosis is illustrated. The Bayesian approach is used to express the pdf of damage parameters conditional on the available observation drawn from sensors measurements; Monte Carlo Markov Chain (MCMC) algorithm is then employed to estimate that conditional pdf. Finally, surrogate modelling is introduced to enhance a fast MCMC execution.

2.1 Bayesian inference and Monte Carlo Markov Chain

The Bayesian approach to model-based SHM allows to solve the inverse problems which challenge the damage identification deterministic approaches by expressing the pdf of the damage parameters $\theta$ conditional on the observed measurement data $z$. According to Bayes’ Theorem, this pdf is called the posterior distribution, and can be formulated in this way:

$$ p(\theta | z) = p(z | \theta) p(\theta) / p(z) $$

where $\theta \in D \subseteq \mathbb{R}^{M \times 1}$ (bold variables denote vectors) is a $M \times 1$ vector, with $M$ the number of model parameters and $z \in \mathbb{R}^{K \times 1}$ is a $K \times 1$ vector, with $K$ the number of observed features. $D$ is the physical domain of the parameters, represented by a partition of the set $\mathbb{R}^{M \times 1}$. The posterior probability $p(\theta | z)$, or the probability of the parameters vector being equal to $\theta$
conditional on the observations vector being \( z \), is thus expressed combining the prior distribution \( p(\theta) \) with the likelihood \( p(z|\theta) \). The prior distribution \( p(\theta) \) represents any available knowledge on the damage parameters before the measurement \( z \) is known; the prior distribution should integrate the user expertise on the model parameters features. The likelihood \( p(z|\theta) \) models the level of agreement between the observations \( z \) and the response \( z^* = f(\theta^*) \) predicted by the model \( f \) when model parameters are equal to \( \theta^* \).

It should also be noticed that, for real applications, observations in \( z \) are typically corrupted with noise, thus considering \( \tilde{\theta} \) as the target exact parameter to be identified, \( z = f(\tilde{\theta}) + n \), where the noise vector components \( n_i \) can be assumed to be i.i.d. and zero-mean Normally distributed with variance \( \sigma \) in most applications, thus \( n_i \sim N(0, \sigma^2) \). Furthermore, if the model \( f \) is inadequate – i.e. not being capable of appropriately replicate the real-world data – selection of \( n_i \) should consider modelling errors. This can be done, as in [13], by adjusting (i.e. increasing) \( \sigma \) to include additional uncertainties associated with the model simplifications.

Based on these assumptions, the following expression for the likelihood is provided:

\[
p(z|\theta^*) = \prod_i p(z_i|\theta^*_i) = \prod_i \frac{1}{\sqrt{2\pi\sigma}} \exp \left(-\frac{(z_i - z_i^*)^2}{2\sigma^2}\right) \tag{2}
\]

It is clear that the closer \( z \) and \( z^* \) are and the higher the likelihood (and the posterior probability) will be.

The denominator in (1), \( p(z) \), is the evidence, which plays the role of a normalization factor and ensures that the posterior probabilities sum to unit. Thus, the following relation holds:

\[
p(\theta|z) \propto p(z|\theta) p(\theta) \tag{3}
\]

The problem with the Bayesian approach is that the posterior pdf can rarely be analytically computed, especially when considering real applications. In this respect Monte Carlo (MC) algorithms are a powerful method for obtaining information about distributions, allowing the characterization of a probability distribution by randomly sampling values out of it. Specifically, a Monte Carlo Markov Chain (MCMC) method consists in a sequential drawing process that guarantees to draw random samples out of a target unknown probability distribution independently from the starting point of the chain, hence the ergodicity of the chain.

If a Markovian process is considered, each sample is dependent only on the previous one, that is to say the probability to draw a sample at discrete time \( j \) only depends on the state attained at \( j-1 \) (this property is also referred to as “memory lessness”). The latter hypothesis is often not completely met in real applications, as one typically resort to methods that adapt the proposal variance as a function of the samples previously stored within the chain, in order to provide a more efficient scan of the parameters space. This would include some memory within the sampling process, thus making the sampling at discrete time \( j \) dependent on previous samples, although maintaining the ergodicity of the chain.

### 2.2 The adaptive proposal Metropolis-Hastings algorithm

Among all the MCMC algorithms, the M-H is possibly the most popular, mainly because of its simplicity and ease of application. The M-H is a random walk algorithm in which subsequent samples are linked by a transition kernel, which is a pdf satisfying the reversibility and ergodicity of the chain, so that an unbiased estimation of the posterior pdf \( p(\theta|z) \) is guaranteed. A proper transition kernel can be assembled via a proposal distribution \( q(\theta^*|\theta^{j-1}) \) and an acceptance probability \( \alpha(\theta^*, \theta^{j-1}) \); both terms are further defined. After initialization, the M-H algorithm draws a trial sample, \( \theta^* \), of the parameters vector \( \theta \) at each iteration (see Table 1 for the complete algorithm procedure). This trial sample \( \theta^* \) is drawn from the proposal...
distribution \( q(\theta^* | \theta^{j-1}) \), which is assumed to be symmetric (Gaussian) and centered on the previous sample value \( \theta^{j-1} \):

\[
q(\theta^* | \theta^{j-1}) = N(\theta^{j-1}, \Sigma_q)
\]

(4)

Being the proposal symmetric the following equality holds: \( q(\theta^* | \theta^{j-1}) = q(\theta^{j-1} | \theta^*) \). The covariance matrix \( \Sigma_q \) has to be specified by the user, based on prior knowledge on the parameters \( \theta \). Then, the algorithm decides whether to keep or reject the sampled \( \theta^* \) based on the acceptance probability \( \alpha(\theta^*, \theta^{j-1}) \). The acceptance probability, \( \alpha(\theta^*, \theta^{j-1}) \), depends on the sample likelihood \( p(z|\theta^*) \), on the proposal distribution \( q(\theta^* | \theta^{j-1}) \) and on the prior pdf \( p(\theta^*) \).

One of the main difficulties in the algorithm implementation is the tuning of the proposal covariance matrix \( \Sigma_q \). If the proposal variance is too large, the number of accepted samples will be too low, and the convergence of the chain can’t be assured. Vice versa, if the variance is too low, the chain will require too many steps to reach convergence. To effectively addresses the issue, an adaptive proposal M-H algorithm can be used [16]. The method herein employed recursively adapts the covariance matrix of the proposal distribution using the residuals of the chain, introducing two additional parameters \( U \) and \( H \), which are called the update frequency and the memory respectively. The former establishes the interval between the matrix updates; the latter defines the number of necessary samples used to compute the residuals and the proposal covariance matrix.

M-H Algorithm with adaptive proposal:

Initialize the algorithm:
\( \theta_0 \)

\( U = U_0, H = H_0 \)

for \( j = 1: N \)
- draw a candidate in the parameters space:
  \( \theta^* \sim q(\theta^* | \theta^{j-1}) = N(\theta^{j-1}, \Sigma_q) \)
- evaluate the likelihood:
  \( p(z|\theta^*) \)
- accept and store \( \theta^* \) with probability:
  \[
  \alpha(\theta^*, \theta^{j-1}) = \min \left( 1, \frac{\frac{p(z|\theta^*)}{q(\theta^{j-1}|\theta^*)} \frac{q(\theta^{j-1}|\theta^*)}{p(\theta^*)}} \right)
  \]

If the remainder of \( (j/U) \) is null
- Generate the residual of the chain:
  \( R(\theta_H) = \theta_H - E(\theta_H) \)
  where \( \theta_H \) is the matrix of the last \( H \) samples of the Markov chain and \( E(\cdot) \) is the expected value
- Update the covariance matrix of the proposal:
  \[
  \sigma^2 = \sigma^2_{\theta_H} = \frac{C_d^2}{H-1} R(\theta_H)^T R(\theta_H)
  \]
  where \( C_d^2 = \frac{2d}{\sqrt{d}} \) and \( d \) is the number of parameters to be estimated

### 2.3 Surrogate models

The focus is now on the relation \( z^* = f(\theta^*) \). In this respect one should find an efficient way to unambiguously relate the parameters space - to which \( \theta^* \) belongs - to the observations (measures) space, to which \( z^* \) belongs. Despite advances in computational power of modern
computers, running complex simulations (e.g. FE analysis) is far from being practical or efficient. That conundrum can be solved via surrogate models, also known as metamodels [14]. Surrogate models are intended to mimic the underlying High-Fidelity (HF) model as closely as needed over the parameters space domain, thus consisting of a computationally “lighter” approximated version, fitted on a dataset collecting the output of the HF model for a limited number of input conditions and parameters. Replacing the HF model with its surrogate model one should benefit of: a) reduced computation time, b) easier implementation into the Bayesian framework. However, one should also be aware that surrogate modelling has some drawbacks:

a) accuracy issues. A compromise between the data set dimensions – i.e. the number of HF model simulations – and the surrogate model accuracy must be established;
b) the surrogate models are often unreliable out of the parameters support used for training; their work frame is thus bounded.

Let $\mathbf{\theta}^{DS}, \mathbf{Z}^{DS}$ be the input-output dataset generated by the HF model. $\mathbf{\theta}^{DS} = \begin{bmatrix} \theta_{11} & \cdots & \theta_{1M} \\ \vdots & \ddots & \vdots \\ \theta_{1P} & \cdots & \theta_{PM} \end{bmatrix}$ is a $P \times M$ matrix, with $P$ the number of HF simulated scenarios for $P$ different input parameters scenarios, selected in order to guarantee a sufficiently informative scan of the parameters support $D$. $\mathbf{Z}^{DS} = \begin{bmatrix} Z_{11} & \cdots & Z_{1K} \\ \vdots & \ddots & \vdots \\ Z_{1P} & \cdots & Z_{PK} \end{bmatrix}$ is a $P \times K$ matrix, collecting the $K$ simulated observed features for each simulated scenario. Therefore, the surrogate model can be simply thought of a function $f$ which establish a relation between the parameters and the observations spaces ($f = \mathbb{R}^{M \times 1} \rightarrow \mathbb{R}^{K \times 1}$).

$$\mathbf{z} = f(\mathbf{\theta})$$

The function $f$ is approximated based on the dataset $\mathbf{\theta}^{DS}, \mathbf{Z}^{DS}$, specifically minimizing the error quantity $\mathbf{Z}^{DS} - f(\mathbf{\theta}^{DS})$.

It is worth noting that the surrogate model parameters, depending on the framework, should also include boundaries and loading conditions of the system, unless these remain the same. However, it is sometime possible to take advantage of the system linearity, e.g. in case of varying load in linear elastic models, thus avoiding the need of increasing the parameters space.

3 APPLICATION TO IMPACT DAMAGE IDENTIFICATION IN CFRP STRUCTURES

The case study which is here presented consists of a CFRP rectangular panel affected by low velocity impact damage and in-plane loaded. The layup is symmetric $[45, -45, 0, 45, -45, -45, 45, 90, 0]$s, and consists of 18 plies. The main features of the panel are reported in Table 2 and Table 3.

The panel HF model shall establish a relationship between spaces, namely the parameters space $\mathbf{\theta}$ and the observable features space $\mathbf{z}$, specifically at some discrete $\mathbf{\theta} \in D$. Once the relationship is identified, one should be able – via the M-H MCMC algorithm integrated with a surrogate version of the HF model – to trace back estimates of the parameters posterior pdfs when a measure of the observed features is given. Naturally, the chosen features have to exhibit some sensitivity to the parameters variation. Ergo, while the HF/surrogate models allow one to relate the parameters space to the observable features space, M-H MCMC enables the inverse solution of finding parameters conditioned on observations.

In this scenario, the parameters in $\mathbf{\theta}$ are those which univocally characterize the impact damage [17]. The observed features are represented by a pattern of strain field components measured at the panel outermost plies, always in correspondence of the same boundaries and loading case.
In real-world applications observable data are limited, thus one should try to minimize the number of sensors maximizing their perceived information, choosing appropriate observed features and sensor positions. Preliminary sensitivity analyses showed that the external plies strain component which exhibits the highest sensitivity to the damage parameters is $\varepsilon_{22}$, i.e. the strain along the panel longitudinal direction, also corresponding to the load direction. Based on the above considerations, equally spaced sensors are placed in the vicinity of the vertical edges of the plate, externally with respect to the area subject to potential impact damage (Figure 1). The number of selected sensors is set to 18 and from now on the observations vector will be referring to the $\varepsilon_{22}$ variable ($z$ is thus characterized by 18 entries). This has to be intended as a first tentative sensor network layout, based on logistic limitations, then leveraging on numerical tests of the diagnostic system, an optimization of the network is possible as a compromise between costs and benefits.

When the low velocity impact takes place, it gives rise to delaminations (inter-laminar damage) and matrix cracks in the surroundings of the impact location, potentially also inducing failure of the reinforced fibers on a limited region near the impact (intra-laminar damage), however dependent on the impact energy. Experiments show that the delamination extent and shape vary across the specimen thickness [17], depending mainly on the specimen layup. In the present framework, a simplified model of the impact damage is considered, and the following assumptions are made: a) the delaminations involve all the plies at the same extent; b) delaminations have a circular shape; c) according to continuum damage mechanics, a straightforward way to model intra-laminar damage is to introduce some damage indexes which work as stiffness penalty factors. Hence, the following stiffness degradation scheme is used:

$$\tilde{E}_{11} = (1 - d_f)E_{11}$$  \hspace{1cm} (6)

$$\tilde{E}_{22} = (1 - d_f)(1 - d_m)E_{22}$$  \hspace{1cm} (7)

$$\tilde{G}_{12} = (1 - d_f)(1 - d_m)G_{12}$$  \hspace{1cm} (8)

Where $[\tilde{E}_{11}, \tilde{E}_{22}, \tilde{G}_{12}]$ are the modified longitudinal ($E$) and shear ($G$) stiffness of the ply and $d_f$ and $d_m$ can be interpreted as penalty factors for including fiber and matrix damages, respectively.

Therefore, the damaged zone consists of:

1. a circular shaped delamination area, having radius equal to $R_{DEL}$, in which the damage indexes assume the values disclosed in Table 4;
2. a circular shaped area, concentric to the delamination area, having radius $R_0$ equal to $R_{DEL}/3$, in which the damage indexes are the highest.

Finally, three parameters are collected into $\vartheta$, the x- and y-coordinates of the impact damage and the radius $R_{DEL}$, thus $\vartheta = [x, y, R_{DEL}]$.

<table>
<thead>
<tr>
<th>Panel length [mm]</th>
<th>Panel width [mm]</th>
<th>Panel thickness [mm]</th>
<th>Ply thickness [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>270</td>
<td>3,6</td>
<td>0,2</td>
</tr>
</tbody>
</table>

Table 2: CFRP panel design.

<table>
<thead>
<tr>
<th>$E_{11}$ [MPa]</th>
<th>$E_{22}$ [MPa]</th>
<th>$G_{12}$ [MPa]</th>
<th>$G_{13}$ [MPa]</th>
<th>$G_{23}$ [MPa]</th>
<th>$\nu_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>157487</td>
<td>9946</td>
<td>4950</td>
<td>4950</td>
<td>3209</td>
<td>0,24</td>
</tr>
</tbody>
</table>

Table 3: Unidirectional ply elastic properties.
3.1 Finite Element model

The HF finite element model has been realized via Abaqus software (Figure 2). In order to effectively represent the post-impact delamination damage, continuum shells (SC8R element) have been used for the modelling of each ply. Between adjacent plies a narrow-band of vanishing thickness (0.001mm), termed the cohesive zone, is inserted (Figure 2). Adjacent plies are connected to these cohesive layers via tie constraints, except for the zones in which delamination has occurred, which are tie-free. Here, the cohesive zone modelling is exploited for the delamination modelling only [18]. The panel is loaded longitudinally; a 10kN force is applied to one of the free ends, while the remaining free end is constrained (encastre constraint). The loading of the panel is kept constant independently from the damage scenario; thus, it is not included in $\theta$.

As previously anticipated, delamination is here assumed to involve in equal measure all plies and has a circular shape; a cylindrical delaminated zone is thus observable in 3-D space. Impact damage is taken into consideration by adjusting the plies stiffness in the impact area, according to matrix and fiber damage indexes and considering the stiffness degradation scheme described above. The mesh is incrementally refined in the surroundings of the delaminated area (Figure 3). The resulting strain field on the external plies is shown in Figure 3.

For each damaged scenario a pattern of virtual strain observations has been extracted in correspondence of the sensor locations shown in Figure 1. Being the layup symmetric and the loading and boundary conditions of the panel symmetric too with respect to midplane, the two outermost surfaces are being characterized by the same strain field. Thus sensors have been placed on one surface only.

<table>
<thead>
<tr>
<th>$E_{11}$ [MPa]</th>
<th>$E_{22}$ [Mpa]</th>
<th>$G_{12}$ [Mpa]</th>
<th>$G_{13}$ [Mpa]</th>
<th>$G_{23}$ [MPa]</th>
<th>$\nu_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>157487</td>
<td>9946</td>
<td>4950</td>
<td>4950</td>
<td>3209</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 5: Lamina elastic properties.
Figure 2: Panel Finite Element model (a) and cohesive layer modelling (b).

Figure 3: FE mesh (a), external ply resulting strain field $\varepsilon_{11}$ and $\varepsilon_{22}$ (b) and (c).

3.2 Surrogate model

Aiming at reducing the computational burden of the FE model described in the above section, a surrogate model of the same has been implemented through Artificial Neural Networks (ANNs). The data set for the surrogate model realization has been generated performing several FE analyses varying discretely the damage parameters, namely the delamination center location $(x, y)$ and extent $(R_{DEL})$, in order to span the parameters space with a reasonable number of cases. The coordinates of the delamination center $(x, y)$ relate to a cartesian coordinate system which is centered in the panel centroid, with the $y$-axis parallel to the panel longitudinal direction. The data set is composed of:

1. a $P \times M$ matrix $\theta^{FEA}$ in which $P$ combinations of the $M$ damage parameters are listed: each row provides a different case scenario, covering the bounded parameters space
(9). The parameters space domain is characterized by the bounds and step sizes listed in Table 6. Therefore, for each row a HF model simulation is performed.

2. $P$ $K$-dimensional ($K$ is the number of sensors) column vectors $z_{p=1:P}^{FEA} = [\epsilon_p^{S_1}, ..., \epsilon_p^{S_K}, ..., \epsilon_p^{S_K}]^T$ representative of the $\epsilon_{22}$ strain values at each sensor location $(S_1, ..., S_K, ..., S_K)$ for the $P$ damage scenarios. These vectors can be assembled in a unique $P \times K$ matrix $Z^{FEA}$ (9).

<table>
<thead>
<tr>
<th>$R_{DEL}$ [mm]</th>
<th>$x$ [mm]</th>
<th>$y$ [mm]</th>
<th>Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 : 2 : 30</td>
<td>-84 : 21 : 84</td>
<td>-150 : 25 : 150</td>
<td>1507</td>
</tr>
</tbody>
</table>

(9)

Table 6: Bounds and steps size on the discretized parameters space.

Given the data set, the aim of the surrogate model is to establish a relationship between the damage parameters matrix $\theta^{FEA}$ and the response matrix $Z^{FEA}$. To facilitate input-output regression by the surrogate model, $K$ surrogate models are built, each one relating the $P$ scenarios in $\theta^{FEA}$ with the $P$ responses of a single sensor, the latter consisting in a column of matrix $Z^{FEA}$ in (9). The $k$-th surrogate model is thus represented by the function $f_k$:

$$z_{k}^{FEA} = z_{1:P,k}^{FEA} \approx z_{1:P,k}^{FEA} = f_k(\theta^{FEA})$$

(10)

Where $z_{1:P,k}^{FEA}$ are the observations predicted by the surrogate model and correspondent to the targets $z_{1:P,k}^{FEA}$. The underlying method behind the surrogate model here exploited leverages on Artificial Neural Networks (ANN) theory. $K$ multilayer perceptron ANNs (one for each sensor) are fed with the FE generated dataset; input to the $k$-th ANN is the ($\theta^{FEA}$) matrix, while the output is represented by the $k$-th column vector of $Z^{FEA}$, $z_{k}^{FEA}$. The $k$-th ANN returns then a function which takes as input the damage parameters (three-dimensional column vector) and provides a scalar estimate of the strain at the $k$-th sensor location. The ANN is constituted by a 3-neurons input layer, a 40-neurons hidden layer and a single neuron output layer; the hidden layer transfer function (tf) is a sigmoid ($f^1$) and the output layer tf is linear ($f^2$). The representative equation of the $k$-th ANN is shown in (11), where $W$ are the weights matrices and $b$ the bias vectors.

$$\epsilon_{S_k} = f^2(W_{2,1}f^1(W_{1,1}\theta + b_1) + b_2)$$

(11)

Levenberg-Marquardt algorithm is selected for the ANN training, coupled with early stopping to avoid data overfitting, specifically dividing the training, validation and test subsets with a 70%-15%-15% proportions respectively. The fitting performance of the surrogate model for the strain prediction at sensor ID 2, 3, 4 and 5 (with reference to Figure 1) can be appreciated in Figure 4, where symbols represents data used for training. Then, the performance of the surrogate models is tested considering a randomly drawn test set (Figure 5a) which was intentionally removed from the training set $\theta^{FEA}$. The performance of sensor ID1 is shown in Figure 5b for example. The level of agreement is excellent, provided that the sensor is located in a remote area of the panel, where the sensitivity to the damage parameters is by far worsened.
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Figure 4: Strain sensitivity for sensors ID 2, 3, 4 and 5 to a varying size delamination.

Figure 5: Test case ANN performance for sensor ID 1. Randomly selected test case parameters (a); ANN strain prediction (b).

3.3 Metropolis-Hastings Markov Chain Monte Carlo (M-H MCMC) with adaptive proposal

The M-H MCMC algorithm described in Section 2 is here exploited to sample the posterior pdf $p(\theta | z)$ of the impact damage parameters $\theta$ given a set of virtual (simulated) observations $z$ consisting of a collection of strain measurements gathered according to the sensors positions shown in Figure 6. The observation data set consists of $K$ strain measures ($K$ being the number of sensors) retrieved from a subset of FE simulations which were not employed for the surrogate models training; the same test set was exploited for the ANNs assessment. A uniform distribution is selected for the prior pdf $p(\theta)$ since there is no available knowledge on the damage parameters; prior pdf bounds are listed in Table 6. The variance of the likelihood $\sigma^2$ includes all the un-modelled features and is estimated here as the sum of two contributions: a) a Gaussian sensor noise associated to electrical/optical disturbances and b) the uncertainty of the surrogate model with respect to HF simulations on the test set, for which the resulting error histogram is also fitted with a Gaussian distribution. These result in a pdf with zero mean and variance $\sigma^2 = 100 \mu\varepsilon$. 
For each of the test case, the length of the trace plots (Markov chain length) is fixed to 10000. The first 2000 samples are discarded for the burn-in period (20%) and a thinning interval of 10 sample is used to reduce autocorrelation.

Three damage parameters are simultaneously estimated via the M-H algorithm: the impact damage location (x and y) and extent (delamination radius $R_{DEL}$). The initial guess for initialization $\theta_0 = [X_0 Y_0 R_{DEL,0}]$ is randomly selected, as shown in Figure 6; the covariance matrix $\Sigma_q$ for the proposal distribution is initialized with variance values which are 30% of the variables range (with reference to variables bounds in Table 6). Covariance values are initialized to zero, since there is no evidence of correlation between damage parameters variables. The updating frequency $U$ of the covariance matrix $\Sigma_q$ and the memory parameter $H$ are set to 1000, meaning that the covariance matrix $\Sigma_q$ is being updated every 1000 iterations and the residuals and the proposal covariance matrix are computed based on the last 1000 samples. Figure 6 shows the test case damage location with respect to the panel borders and sensors arrangement (further details are listed in Table 7).

Figure 7 is showing the trace plots of the proposal sampled parameters values. Initially, the selection of a too large proposal variance in $\Sigma_q$ manifests as a stepped Markov Chain, then, after proposal variance updating based on the accepted samples, the algorithm focuses on the high probability density region, thus showing a good behavior of the sampling procedure with rejection rate around 30%. Figure 8 shows the damage parameters pdf considering the complete observation set (18 sensors). It can be seen that the provided estimates of the damage parameters are almost coincident with their true values. Then, the algorithm is tested with reduced number of sensors and while adding a certain amount of noise to the observed simulated strains, sampling a zero mean Gaussian distribution with variance $\sigma^2$, equal to the one defined above for likelihood assessment. Results are shown in Figure 9. As expected, the performance decreases as the observations set reduces; the sensors IDs of the tested sets are collected in Table 8, and are referring to the sensors IDs shown in Figure 6. The decrease in performance is expressed by the increase in the parameters distribution variance, while the mode exhibits only little deviation with respect to the full sensors set. The robustness of the method when observations are polluted with noise is also demonstrated.

<table>
<thead>
<tr>
<th>$X_{true}$</th>
<th>$Y_{true}$</th>
<th>$R_{DEL true}$</th>
<th>$X_0$</th>
<th>$Y_0$</th>
<th>$R_{DEL,0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>-25</td>
<td>24</td>
<td>100</td>
<td>100</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 7: Test case parameters true values and initial guess.

Figure 6: Test case position with respect to sensors and panel perimeter.
Figure 7: Trace plot of sampled damage parameters.

Figure 8: Pdf estimates of the damage parameters considering the full sensors set. Vertical dashed lines indicate targets.

<table>
<thead>
<tr>
<th>6S</th>
<th>10S</th>
<th>18S</th>
</tr>
</thead>
<tbody>
<tr>
<td>S3, S5, S7,</td>
<td>S1, S3, S5, S7,</td>
<td>S1:S18</td>
</tr>
<tr>
<td>S12, S14,</td>
<td>S9, S10, S12,</td>
<td></td>
</tr>
<tr>
<td>S16</td>
<td>S14, S16, S18</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Sensors sets with sensors IDs.

Figure 9: Pdf estimates of the damage parameters with varying the number of observations. Vertical dashed lines indicate targets.
4 CONCLUSIONS

In this study, a Bayesian probabilistic model-based framework for the impact damage identification on a CFRP panel is illustrated. Based on a collection of strain observations, simulated with a damaged CFRP panel FE model which is in plane loaded, the probability distributions of the damage-characterizing parameters are estimated, using Bayesian inference and the Metropolis-Hastings Monte Carlo Markov Chain algorithm. Subset of data are collected via the FE model at locations that simulate a realistic application; specifically, sensors are equally spaced and placed line wise along the panel borders. A preliminary sensitivity analysis demonstrated that the longitudinal strain component $\varepsilon_{22}$ (in the direction of the load) exhibits the highest sensitivity to the damage parameters variations, and is thus selected as observed feature. The M-H MCMC algorithm efficiency is substantially increased by replacing the panel high fidelity FE model with a surrogate model of the same, consisting of a series of Artificial Neural Networks mimicking the input-output relations between the observed variable at a specific panel location (sensors positions) and the damage parameters. By doing this, the likelihood assessment, which takes place at each algorithm iteration, is notably sped up. The ANNs are trained offline with a data set which has been assembled via several FE simulations. In order to limit the training data set dimensions, the number of damage parameters has been reduced, being set equal to three, i.e. damage location ($x$ and $y$ coordinates of the delamination center) and extent (delamination radius). The M-H MCMC algorithm is furthermore refined by making the proposal covariance matrix adaptive.

The capability of the method to estimate the impact damage parameters on a CFRP plate based on a set of observations is demonstrated. The method accuracy and robustness are tested polluting the observed variables with a Gaussian-distributed zero-mean noise and reducing the availability of measurements, i.e. limiting the set of sensors, proving to be able to correctly estimate the damage parameters even with a limited amount of data, although an increase in the variance of the parameters probability distribution estimates is registered. The results which are herein shown are significant, and the method, apart being efficient and adequately accurate, has also proven robust. Nevertheless, as in most Monte-Carlo based Bayesian model updating strategies, the results strongly depend on the selection of the proposal variance. Thus, a method for selecting a more informative initial guess on the proposal covariance matrix should be studied. Future researches should also extend the present work to more realistic and complex damage features, increasing the number of parameters featuring the damage, and thus increasing the parameters space, possibly leveraging on improved version of the MCMC algorithm, e.g. taking advantage of parallel computing capabilities of modern computers, in order to tackle the curse of dimensionality.

REFERENCES


BAYESIAN DAMAGE CHARACTERIZATION BASED ON PROBABILISTIC MODEL OF SCATTERING COEFFICIENTS AND HYBRID WAVE FINITE ELEMENT MODEL SCHEME

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Abstract

Ultrasonic Guided Wave (GW) has been proven to be sensitive to small damage. Motivated by the fact that the quantitative relationship between wave scattering and damage intensity can be described by scattering properties, this study aims at proposing a new probabilistic damage characterization method based on the scattering coefficients in tandem with hybrid wave finite element model (WFEM) scheme. The probabilistic distribution properties of the scattering coefficients estimated using measured ultrasonic guided waves in the frequency domain are inferred based on absolute complex ratio statistics. The theoretical scattering coefficients can be efficiently calculated using WFEM which combines conventional finite element analysis with periodic structure theory. Based on the probabilistic distribution of reflection/transmission coefficients, the likelihood function connecting the theoretical model responses containing the parameters to be updated and the measured responses are formulated within a unified Bayesian system identification framework to account for various uncertainties. The transitional Monte Carlo Markov Chain (TMCMC) is used to sample the posterior probability density function of the updated parameters. A numerical example is utilized to verify the accuracy of the proposed algorithm. Results indicate that the strategies proposed in this study can quantify the uncertainties of damage characterization.

Keywords: Wave Finite Elements; Ultrasonic Guided Waves; Damage Identification; Uncertainty Quantification; Bayesian Analysis.
1 INTRODUCTION

Structural Health Monitoring (SHM) involving the observation of a structure using response measurements, the extraction of damage-sensitive features and the analysis of these features to assess structural health condition has attracted widespread attention [1, 2]. Low-frequency damage detection methods utilizing dynamic responses have been applied extensively in engineering structures [3]. However, the damage detection approaches based on the low-frequency characteristics are usually insensitive to small damage.

Nowadays, ultrasonic GW-based SHM methodologies have been widely reported to be sensitive to small damage, convenient and efficient in detecting structural damage [3], such as fatigue cracks in metallic structures, debonding and delamination in composite structures. The scattered waves need to be analyzed using certain damage identification algorithms to extract various characteristics containing essential information about the damage [4, 5]. Over the past decades, tremendous efforts have been directed to extract structural conditions using GWs. In the campaign of structural damage characterization, one critical issue that has been widely accepted is that uncertainties due to endogenous factors should be appropriately considered [6].

Bayesian statistics has been widely considered an excellent candidate for uncertainty quantification in GW-based damage detection [6, 7] since it considers probability as a multi-valued propositional logic for plausible reasoning [8, 9].

By making full use of the Bayesian system identification framework for accommodating measurement noise and modeling errors properly, this study aims at formulating a generic methodology for probabilistic damage characterization based on wave scattering characteristics [4, 5]. The scattering coefficients are probabilistically modelled by using absolute complex ratio random variables [10]. A Bayesian scheme makes inferences about the damage characterization parameters directly by processing the statistical information contained in the experimentally measured scattering properties. The TMCMC [11] is finally used to sample the posterior Probability Density Function (PDF) of the updated parameters. The hybrid WFE methodology is hereby employed in order to efficiently simulate wave scattering when ultrasonic GW impinge a damaged segment within a structure of arbitrary layering. Furthermore, a cheap and fast Kriging surrogate model will be employed in tandem with the WFE scheme in order to approximate the output in function of model parameters. The procedure is verified using numerical data in different damage configurations.

2 THEORETICAL BACKGROUND

The piezoelectric transducer excites propagating waves within the structure. The incoming GWs (+) impinge on the damaged structural segment and generate a set of outgoing (-) reflected and transmitted waves. The propagation of waves is often described in terms of “wave modes”. The reflection and transmission coefficients denoted by \( \{ R_k, T_k \} \) at \( \omega_k \) are determined by dividing the frequency spectra of the reflected/transmitted signal by that of the incident wave signal [4]. The reflection/transmission coefficients predicted by the structural damage model are denoted as \( \{ R_{\theta}, T_{\theta} \} \), defined by a set of damage parameters \( \theta \), which are to be identified. Each implemented damage scenario can be FE-modelled and the associated scattering coefficients can be numerically computed.

In the field of wave propagation, the scattering coefficients can be estimated by taking the ratio of the FFT of reflected/transmissive wave and the FFT of the incident wave as:
\begin{align}
\mathcal{R}_k &= \left[ \begin{array}{c} X_r(o_k) \\ X_i(o_k) \end{array} \right] \\
\mathcal{J}_k &= \left[ \begin{array}{c} X_r(o_k) \\ X_i(o_k) \end{array} \right]
\end{align}

The statistical inference can be executed by embedding the “deterministic” structural models within a class of probability models so that the structural models give a predictable (“systematic”) part and the prediction error is modeled as an uncertain (“random”) part [8]. In the context of Bayesian inference with scattering coefficients, the measured outputs and the numerical model outputs are connected as follows [8]:

\begin{align}
\mathcal{R}_k &= R_k(\theta) + \mu_{re} \\
\mathcal{J}_k &= T_k(\theta) + \mu_{tr}
\end{align}

In Eq. (2), the error term \( \mu_{re} \) and \( \mu_{tr} \) are usually modeled as white noise with constant variances.

The variances of the FFT coefficient of the reflected wave signal and the transmitted wave can be approximated by:

\begin{align}
\sigma^2_{re} &= \sigma^2_{in} \left( R_k^2(\theta) + \gamma_{re} \right) \\
\sigma^2_{tr} &= \sigma^2_{in} \left( T_k^2(\theta) + \gamma_{tr} \right)
\end{align}

where \( \sigma^2_{re} = \text{var}(X_{re}) \), \( \sigma^2_{in} = \text{var}(X_{in}) \) and \( \sigma^2_{tr} = \text{var}(X_{tr}) \) denote the variations of the incident wave, the reflected wave and the transmitted wave, respectively; \( \gamma_{re} \) and \( \gamma_{tr} \) denote the variances of the prediction errors of reflection and transmission coefficients. \( R_k(\theta) = \eta^{(k)}(\theta) \) and \( T_k(\theta) = \eta^{(k)}(\theta) \) denote the reflection and transmission coefficients predicted at \( \theta \), which are achieved by using the Kriging model [12, 13]. To formulate a Kriging predictor model, it requires initial Design of Experiments (DoE). These samples are frequently referenced as the training set or support points. Appropriate DoE plays a vital role in constructing a high-fidelity Kriging model because DoE influences the creation of the most informative training data. A common choice for the training design is the Latin Hypercube Design (LHD), which guarantees to spread design points evenly across each input parameter dimension. With the training set at hand, one can then calculate the predicted values of the surrogate model at various sample points in the parameter space by performing an “experiment” at each of those samples based on the hybrid WFE scheme [14-16].

The PDF of the scattering coefficients \( \mathcal{R}_k \) and \( \mathcal{J}_k \) are given by [10]

\begin{align}
p_{\mathcal{R}_k}(r_k | \gamma_{re}) &= \frac{2\gamma_{re}}{\left( R_k^2(\theta) + \gamma_{re} + r_k^2 \right)^2} \\
p_{\mathcal{J}_k}(e_k | \gamma_{tr}) &= \frac{2\gamma_{tr}}{\left( T_k^2(\theta) + \gamma_{tr} + e_k^2 \right)^2}
\end{align}

Conditioned on the set of measurements \( D = \{ \mathcal{R}_k, \mathcal{J}_k \}_{k=k_1,\cdots,k_2} \) formed over \( \omega \in [k_1 \Delta \omega, k_2 \Delta \omega] \), the likelihood function is given by
According to the Bayes’ theorem, we can condition the prior on the training data and integrate over the prior distribution of the coefficients to obtain the posterior uncertainties of $\lambda = \{\theta, \gamma_r, \gamma_v\}$ [8,9]:

$$p(\lambda | \mathcal{M}, D) = p(\lambda | \mathcal{M}) \exp(-L(\lambda))$$ \hspace{1cm} (6)

With $L(\lambda)$ denoting the negative-log likelihood function given by

$$L(\lambda) = \sum_{k=1}^{k_2} \ln \left( \frac{2r_k (R_k^2(\theta) + \gamma_{r_k})}{(R_k^2(\theta) + \gamma_{r_k} + r_k^2)^2} \right) + \sum_{k=1}^{k_2} \ln \left( \frac{2\varepsilon_k (T_k^2(\theta) + \gamma_v)}{(T_k^2(\theta) + \gamma_v + \varepsilon_k^2)^2} \right)$$ \hspace{1cm} (7)

As a result, the posterior distribution $p(\lambda | \mathcal{M}, D)$ of the damage identification parameters and prediction-error parameters can be achieved using TMCMC algorithm [11].

3 STEP-BY-STEP DESCRIPTION

The procedures of the proposed methodology are outlined below:

(a) Determine the scattering coefficients for the structure under investigation by GW measurements;

(b) Construct Kriging surrogate model to numerically compute the relationship between the scattering coefficients and the damage characterization parameters $\theta$;

(c) Formulate the likelihood function with the scattering coefficient estimates and those predicted by surrogate model in tandem with WFE;

(d) Calculate the posterior uncertainty of $\theta$ with TMCMC.

4 CASE STUDY

The accuracy of the proposed algorithm is demonstrated by a fundamental spring-mass system shown in Figure 1. The model is parameterized through the stiffness of the spring $k_0 = 10^4 N/m$ and the mass of each block $m_0 = 10^{-3} kg$. Assume that damage occurs in the 200th spring, and the damage extent is assumed to be 80%, i.e. $k_d = \alpha k_0$ with $\alpha = 0.2$. The system is assumed to be excited by a 9-cycle Hanning-windowed sinusoidal tone burst. The reference pseudo-experimental damage signature is provided by an explicit solution of the full system comprising 400 masses. Contamination is added to the explicit solution by Gaussian noise. Reflection and transmission coefficients for the GW are therefore acquired.

![Figure 1: Schematic diagram of the spring-mass system](image-url)
The frequency band $f_r = [10, 30]$ kHz is selected for identification, which is symmetrically around the central frequency of the excitation. A thousand training points are generated for the damage identification parameter $\alpha$ using LHD. For each sampling point, the scattering coefficients corresponding to frequencies falling in $f_r = [10, 30]$ kHz are calculated as training outputs using hybrid WFE shown in Figure 2. The training inputs and outputs are then used for constructing Kriging model between scattering coefficients and the damage identification parameter. The parameter vector set to be identified includes $\theta = \{\alpha, \gamma_r, \gamma_l\}$. The lower bound and upper bound of the parameters are set to be $\theta_{\text{lower}} = \{0.5\alpha, 0.001, 0.001\}$ and $\theta_{\text{upper}} = \{1.5\alpha, 1, 1\}$. Then the Bayesian inference is performed by TMCMC, resulting in 10 stages in total. Figure 3 presents the convergence diagram of the TMCMC algorithm at different stages, which demonstrates that the proposed algorithm is rather efficient. The histogram of the stochastic samples of the final stage is shown in Figure 4 accompanied by the kernel density estimation. The MPV of the damage identification parameter is approximately 0.201, which is only 5% away from the actual value. An insignificant c.o.v. (less than 0.3%) is estimated for the extracted damage identification parameter. A relatively large uncertainty of the order of 7% is observed for the prediction errors.

The posterior marginal distribution of the identified parameters using TMCMC and Laplace approximation are compared in Figure 4. Results from Figure 4 indicate that discrepancy is found for the PDFs using two different approaches, especially regarding the results of the prediction-error parameters. The mean values of all parameters identified through TMCMC is less than those acquired through Laplace approximation. The results identified through Laplace approximation are much more dependent on the initial guesses. Given that the initial values deviate the true values significantly, the results can be poor, and it can cause significant divergence. Therefore, the TMCMC algorithm has an important advantage over Laplace approximation as it avoids the need for estimating the initial values of the parameters, which is non-trivial in a number of real cases.
5 CONCLUSIONS

Ultrasonic GWs have played an important role in modern SHM technologies due to their high sensitivity to small damage. We hereby investigate the possibility of using scattering coefficients for probabilistic damage identification, through the uniqueness of GW interactions with each damage scenario. In the context of damage detection with GWs, modelling error as well as measurement noise will inevitably affects the results. This emphasizes the importance of using a comprehensive statistical framework to account for the uncertainties in the parameters and their propagation when in need for robust predictions consistent with experimental data. By making full use of the Bayesian system identification framework to account for measurement noise and modeling errors, this study formulates a new, generic framework for probabilistic damage identification by integrating a hybrid WFEM scheme employed for scattering coefficient estimates, a Kriging predictor model as well a TMCMC stochastic simulation technique. A numerical study has been used to verify the algorithm properly.

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REFERENCES


DAMAGE DETECTION IN COMPOSITE CARBON FIBER TUBES BASED ON EXPERIMENTAL MEASUREMENTS AND FINITE ELEMENT MODEL UPDATING TECHNIQUES

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Abstract

Reliable damage detection is critical for decision making on optimum design solutions and maintenance actions of structural systems. Therefore, there is the need to develop efficient methodologies for damage identification and prognosis of damage accumulation. The present study focuses on the implementation of a methodology that combines analysis with experimental measurements to provide reliable damage predictions. The methodology is applied in composite carbon fiber tubes, which have been used widely in industry in the last years. The composite cylindrical parts are produced on a spinning axis by winded carbon fibers, cascaded on specified number of plies, in various angles and directions. 3D FE models of the examined cylindrical parts are developed in robust finite element analysis software simulating each carbon fiber ply and resin matrix and analyzed against static, thermal and dynamic loading to investigate their linear and nonlinear response. In addition, experimental tests on composite cylindrical parts are conducted based on the corresponding analysis tests. The specimens are equipped with sensors to measure the strains induced and the output vibrations. The number of the sensors that are required is considered as a study parameter. To develop a high fidelity FE model for reliable damage prediction, modal residuals and mode shapes are combined with response residuals and time-histories of strains and accelerations by using the appropriate updating algorithm. Finally, potential damages and remaining lifetime are estimated with the use of full stress time histories of the composite parts at critical locations. Fatigue is evaluated using the Palmgren- Miner damage rule, S-N curves, and rainflow cycle counting. Based on the conclusions derived, the applied methodology is found to be a reliable tool for detecting structural damages.

Keywords: Model updating, Large Scale Structures, Structural Dynamics, Damage Detection
1 INTRODUCTION

Aerospace and automotive industry to building reinforcement and retrofit methodologies, as well as cryogenic fuel storage tanks technology, have paid much attention in recent years to the industrialized implementation and use of carbon fiber reinforced polymer (CFRP) composites as a structural material for static and dynamic load bearing as well as resistance to accidental excitations and actions [1-6]. Compressing pre-stressed carbon fibers against a liquid resin polymer matrix produces CFRP composites. As the carbon fibers are weaved on a revolving axial rod, in certain orientations and angles, a number of plies are cascaded in a specific pattern and in a certain volumetric fraction and thickness, before heated for a specific amount of time in order to obtain full strength and hardening characteristics [7-9].

Although plain woven CFRPs present tension-compression asymmetric characteristics and strong anisotropic mechanical behavior, they have gained popularity, attributed mainly to the cheap manufacturing process and to their low-density, low thermal expansion and high strength, stiffness and corrosion resistance. However, their non-ductile brittle failure behavior under extreme loading conditions has not yet been thoroughly investigated despite recent studies in failure and damage mechanisms [10-14] and impact performance [15].

Automated damage detection and identification methods have surfaced as tools for ensuring structural integrity. A great challenge that vibration-based methods tackle, is that a limited number of sensors are distributed along the examined structure, used to quantify the response to natural excitation, extracting information regarding structural condition. The main goal of this work is to use the measured responses of a healthy CFRP structural system under operational vibrations, in order to tune an initially parameterized Finite Element (FE) model and perform damage identification and prognosis of damage accumulation.

Tuning of the initial FE model representing the healthy reference structure is performed using a state-of-the-art optimization algorithm, namely, covariance matrix adaptation evolution strategy (CMA-ES) [16-19], applied in parallel computing, to solve the single-objective optimization problem, arising from time domain residuals that compare numerical time-histories of strains and accelerations to experimental data [20-25]. The updated FE model can then be interrogated in order to estimate strain and stress time histories of critical locations of the structural system. Rainflow-counting algorithm follows to produce all simple stress reversals from a spectrum of varying stress, obtaining the equivalent stress amplitude cycles. The results of the rainflow-counting are fed into a Palmgren-Miner rule, a deterministic fatigue damage accumulation method in order to predict the remaining life of the structural system [3, 17, 24-28].

The presentation in this work is organized as follows. The theoretical formulation of finite element model updating based on modal characteristics, frequency response functions is briefly presented in section 2. Section 3 presents the adopted residual in time domain using measured acceleration and strain time histories. Section 4 presents the theoretical basis of fatigue damage accumulation using Palmgren-Miner rule. Section 5 presents the experimental application, the development of the FE model of CFRP pin-joined frame, its modal identification along with the FE model updating parameterization and results for orthotropic material characterization. Finally, the effectiveness of the fatigue methodology is demonstrated in section 6. Conclusions are summarized in section 7.

2 MODAL AND FREQUENCY RESPONSE RESIDUALS – LINEAR MODELS

Let a parameterized class of linear structural models used to model the dynamic behavior of the structure and let \( \theta \in \mathbb{R}^n \) be the set of free structural model parameters to be identified using the measured modal data. The overall measure of fit of the linear model, between the measured and
the model predicted characteristics is formed in the following expression, combining modal and frequency response residuals [26, 27]:

\[
J(\mathcal{O}, \mathcal{W}) = w_1 J_1(\mathcal{O}) + w_2 J_2(\mathcal{O}) + w_3 J_3(\mathcal{O}) + w_4 J_4(\mathcal{O})
\]

using equally weighting factors \(w_i \geq 0\), \(i = 1, 2, 3, 4\), with \(w_1 + w_2 + w_3 + w_4 = 1\).

For the first group, the measure of fit \(J_1(\mathcal{O})\) is selected to represent the difference between the measured and the model predicted frequencies for all modes. For the second group, the measure of fit \(J_2(\mathcal{O})\) is selected to represent the difference between the measured and the model predicted mode shape components for all modes, given by:

\[
J_1(\mathcal{O}) = \sum_{r=1}^{m} \varepsilon^2_{\omega_r}(\mathcal{O}) \quad \text{and} \quad J_2(\mathcal{O}) = \sum_{r=1}^{m} \varepsilon^2_{\phi_r}(\mathcal{O})
\]

where the modal data are used \(\{\omega_r(\mathcal{O}), \phi_r(\mathcal{O})\} \in \mathbb{R}^n, r = 1, \ldots, m\) to formulate the following residuals:

\[
\varepsilon_{\omega_r}(\mathcal{O}) = \frac{\omega^2_r(\mathcal{O}) - \hat{\omega}^2_r}{\hat{\omega}^2_r} \quad \text{and} \quad \varepsilon_{\phi_r}(\mathcal{O}) = \left\| \beta_r(\mathcal{O}) \phi_r(\mathcal{O}) - \hat{\phi}_r \right\|_2
\]

and for the second group the measure of fit \(J_3(\mathcal{O})\) and \(J_4(\mathcal{O})\) represent the frequency response measures of fit as follows:

\[
J_3(\mathcal{O}) = \sum_{r=1}^{m} \left[ 1 - x_a(\omega_r, \mathcal{O})^2 \right] \quad \text{and} \quad J_4(\mathcal{O}) = \sum_{r=1}^{m} \left[ 1 - x_s(\omega_r, \mathcal{O})^2 \right]
\]

where

\[
x_a(\omega_k) = \frac{\left\| \{H_X(\omega_k)^H \{H_A(\omega_k)\}\}^2 \right\|}{\left( \{H_X(\omega_k)^H \{H_A(\omega_k)\}\} \right) \left( \{H_A(\omega_k)^H \{H_A(\omega_k)\}\} \right)^2}
\]

and

\[
x_s(\omega_k) = \frac{2 \left\| \{H_X(\omega_k)^H \{H_A(\omega_k)\}\} \right\|}{\left( \{H_X(\omega_k)^H \{H_A(\omega_k)\}\} \right) + \left( \{H_A(\omega_k)^H \{H_A(\omega_k)\}\} \right)^2}
\]

constitute the global and amplitude correlation coefficients [28], where \(\{H_X(\omega_k)\}\) and \(\{H_A(\omega_k)\}\) are the experimental (measured) and the numerical (predicted) response vectors at matching excitation - response locations, for any measured frequency point, \(\omega_k\).

### 3 TIME DOMAIN RESPONSE RESIDUAL

Additionally, parameter estimation of nonlinear model is based on response time history measurements such as acceleration and displacements. This formulation has the advantage of applicability over both linear and non-linear systems; it compares the measured raw data of the experimental arrangement to the equivalent predictions of the numerical model. In this way, all available information is preserved and systematic errors of the identification procedure are alleviated.

The measure of fit consisted of both accelerations and strains equally weighted, expanding to the following equation, where \(a(\mathcal{O})\) and \(s(\mathcal{O})\) are the analytical time-histories of accelerations and strains of the introduced FE model and \(\hat{a}_i\) and \(\hat{s}_i\) are the respective experimental acceleration and strain signals.
4 ESTIMATION OF FATIGUE DAMAGE ACCUMULATION USING STRESS RECONSTRUCTION

The process of inception and spread of cracks through a structural member due to action of fluctuating stress is also known as fatigue [29]. When dealing with a uni-axial stress state with axial stress time history $\sigma_k$, $k = 1, \ldots, N$, the Palmgren-Miner rule [30, 31] is commonly used to predict damage accumulation due to fatigue. According to this rule, fatigue damage at a point in the structure subjected to variable amplitude stress time history $\sigma_k$, is defined as the ratio of the number of cycles of operation to the number of cycles to failure at a given stress level. When dealing with multiple stress levels, according to the Palmgren-Miner rule the sum of fatigue cycles at various levels yields the total damage

$$D = \sum_{i=1}^{\kappa} \frac{n_i}{N_i}$$

where $n_i$ denotes the number of cycles at a stress level $\sigma(i)$ of the stress time history $\sigma_k$, $N_i$ stands for the number of cycles required for failure at a stress level $\sigma(i)$, and $\kappa$ is the number of stress levels identified in a stress time history at the corresponding structural point. When dealing with arbitrary stress time histories at a point of interest on a structure, the number of cycles at a stress level is usually obtained by applying cycle counting methods, such as the rainflow cycle counting [32-34]. Design codes determine the stress levels for fatigue of commonly used engineering materials; furthermore, the so-called S-N curves are included in the design codes to specify the number of cycles required for failure in terms of the stress level $\sigma(i)$ [29, 34]. The S-N curves are obtained via experimental tests on real size specimens. In doing so, uniaxial, constant amplitude cyclic stresses applied to various structural members with different shapes. In design codes, the S-N curves are expressed by log-log curves, in which plots the number of cycles for fatigue failure versus its associated stress range. Each curve is designated with a number that specifies the function, shape and built of the considered structural member.

From what precedes, it is evident that accurate estimate of the fatigue damage accumulation at a point is contingent upon accurate prediction or measurement of stress time histories, as well as accurate cycle counting procedures for determining the stress range spectrum. It is noteworthy that, the number of cycles to fail depends also on the mean stress; as the mean stress increases for a given level of alternating stress the fatigue life decreases [35]. Therefore, the fatigue accumulation model must be revised to account for a non-zero mean stress according to the Goodman relationship [36, 37]

$$\Delta\sigma_m = \Delta\sigma_r \left(1 - \frac{\sigma_r}{\sigma_u}\right)$$

where $\Delta\sigma_m$ stands for the modified stress cycle range, $\Delta\sigma_r$ signifies the original stress cycle range, $\sigma_r$ denotes the mean stress, and is calculated by the cycle counting algorithm; and $\sigma_u$ is the ultimate tensile strength of the material.

Once the stress range spectrum for a structural member is obtained, and the relevant detail category is determined, S-N curves are used for estimating fatigue strength. In this regard,
Miner summation is employed, and the fatigue damages pertinent to the stress ranges are linearly summed. The parametric representation of fatigue damage reads [29]

\[
D = \sum_{i=1}^{\kappa_1} \frac{n_i}{N_{D_i}} (\Delta\sigma_i)_{n_i} + \sum_{j=1}^{\kappa_2} \frac{n_j}{N_{L_j}} (\Delta\sigma_j)_{n_j}
\]

where \(\Delta\sigma_D\) denotes fatigue limit for constant amplitude stress ranges at \(N_D = 5 \times 10^6\) cycles; \(\Delta\sigma_L\) stands for the cut-off limit; \(\Delta\sigma_i\) and \(\Delta\sigma_j\) are the \(i^{th}\) and \(j^{th}\) stress ranges; \(n_i\) and \(n_j\) are the number of cycles in each \(\Delta\sigma_i\) and \(\Delta\sigma_j\) block; \(\kappa_1\) and \(\kappa_2\) represent the number of different stress range blocks above or below the constant amplitude fatigue limit \(\Delta\sigma_D\), respectively.

5 EXPERIMENTAL APPLICATION

In order to examine the orthotropic material mechanical behavior of the CFRP used, the experimental arrangement presented in Figure 1 was set up. Four (4) tri-axial accelerometers and four (4) strain gauges with three bridges at 120° angle rosette each, were placed at characteristic points of the frame of the pin-joined CFRP structure, which was anchored on flat plate parallel to the ground, on a vertical concrete column. An electromagnetic shaker was mounted on a free end of the frame where a load cell sensor was placed to record imposed forces under dynamic excitation load.

![Figure 1](image1)

Figure 1 Experimental setup of examined CFRP pin-joined structure.

Furthermore, an impact hummer was also was in impulse excitation experimental tests in order to apply modal identification techniques and compute the elements of the frequency response functions (FRFs) matrix.

The CFRP is consisted of a stack of nine (9) plies with equal thickness and orientation angles apart from one ply. Specifically, plies 1 to 6 and 8 to 9 have a nominal thickness of \(t = 0.175\, \text{mm}\) at \(\theta = 55^\circ\) and \(\theta = -55^\circ\) nominal orientation angles consecutively. Ply 7 has a nominal thickness
of \( t = 0.16 \text{mm} \) at \( q = 86^\circ \). The nominal material parameters of the 2D orthotropic material used to model the CFRP were \( E_1 = 146.45 \text{GPa} \) and \( E_2 = 7.73 \text{GPa} \) for the modulus of elasticity in X and Y direction respectively, \( v_{xy} = v_{yx} = 0.12 \) is the Poisson’s ration for in-plane bi-axial loading, and \( G_{12} = 3.54 \text{GPa}, G_{xz} = 3.95 \text{GPa} \) and \( G_{yz} = 2.80 \text{GPa} \) are the in-plane, transverse for shear in XZ plane and transverse for shear in YZ plane shear moduli and \( \rho = 1600 \text{kg/m}^3 \) is the density.

### 5.1 Development of the FE model and modal analysis

The geometry of the CFRP pin-joined frame is discretized with composite shell elements and tetrahedral solid elements for the aluminum connectors using appropriate pre-processing commercial software [38]. The total number of DOFs was 2,500,000 [39].

![Figure 2 FE model of examined pin-joined frame and indicative eigenmode.](image)

The detailed FE model is presented in Figure 2, along with an indicative mode shape, predicted by the nominal FE model colored by spectrum colors of the normalized deformations.

### 5.2 Experimental modal analysis

After developing the nominal finite element model, an experimental modal analysis procedure of the CFRP cantilever beam was performed in order to quantify the dynamic characteristics of the examined structure.

![Figure 3 Typical FRFs for modal identification.](image)
First, all the necessary elements of the FRF matrix, required for determining the response of the structure were determined by imposing impulsive loading \([20-23, 25]\). The measured frequency range of the test was 0-200 Hz. An initial investigation indicated that the beam has seven (7) natural frequencies in this frequency range. Figure 3 presents typical Frequency Response Functions (FRFs) at three components X, Y and Z for all measuring points under a specific impulse location and direction.

5.3 FE model parameterization and updating results

The parameterization of the finite element model is introduced in order to facilitate the applicability of the updating framework. The parameterized model is consisted of six (6) parts, as shown in Figure 4.

Part 1 is modeled with composite shell elements and orthotropic material properties while parts 2 to 6 are modeled with solid elements and isotropic material properties. Specifically, parts 2 and 3 represent the aluminum connection joints, part 5 is the glue between the CFRP and the aluminum joints and part 6 is the foam that fills the inner of the CFRPs at the joint location. All orthotropic material properties along with the nine ply thicknesses \(t\) and orientation angles were used as design variables of part 1. Additionally, Young’s moduli and the material densities of isotropic material parts were also used as design variables. Apart from material properties parameters, the Rayleigh modal damping ratios are used as design variables. Specifically, modal damping ratios \(\xi_1\) to \(\xi_7\) pertaining to the first seven (7) eigenmodes are included in the design variables, so as to enhance fitting of compared time histories and FRFs, using nominal damping ratio of 3\%, as the most common for a composite and steel structures. The total number of design variables for the FE model is thirty-six (36).

![Figure 4 Parts of the parameterized FE model. Detail of CFRP tube and aluminum drop-out.](image)

The CMA-ES framework is applied at \(\pm 10\%\) from the nominal values as design bounds, in order to update the developed FE model combining modal residuals that include the lowest identified modal frequencies with mode shapes and response residuals that include shape and amplitude correlation coefficients considering measured and numerical frequency response functions including components at all sensor locations, along with time domain acceleration.
and strain time-histories. Finally, the results of the FE model-updating framework are presented in Table 1. A comparison between identified, nominal and updated FE predicted modal frequencies is also presented.

**Table 1** Comparison between identified, nominal and updated FE predicted modal frequencies.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Identified Frequency (Hz)</th>
<th>Damping (%)</th>
<th>Numerical (before updating) Frequency (Hz)</th>
<th>Error (%)</th>
<th>Numerical (after updating) Frequency (Hz)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>78.456</td>
<td>0.91</td>
<td>83.379</td>
<td>5.90</td>
<td>79.126</td>
<td>0.85</td>
</tr>
<tr>
<td>2</td>
<td>84.932</td>
<td>0.87</td>
<td>89.935</td>
<td>5.56</td>
<td>85.251</td>
<td>0.37</td>
</tr>
<tr>
<td>3</td>
<td>87.651</td>
<td>0.92</td>
<td>97.991</td>
<td>10.55</td>
<td>88.325</td>
<td>0.76</td>
</tr>
<tr>
<td>4</td>
<td>113.542</td>
<td>0.68</td>
<td>123.381</td>
<td>7.97</td>
<td>114.206</td>
<td>0.58</td>
</tr>
<tr>
<td>5</td>
<td>185.584</td>
<td>1.19</td>
<td>203.573</td>
<td>8.84</td>
<td>188.562</td>
<td>1.58</td>
</tr>
<tr>
<td>6</td>
<td>190.736</td>
<td>1.21</td>
<td>206.131</td>
<td>7.47</td>
<td>190.09</td>
<td>0.34</td>
</tr>
<tr>
<td>7</td>
<td>195.502</td>
<td>1.05</td>
<td>208.214</td>
<td>6.11</td>
<td>196.147</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The time-histories of normal, shear and maximum Von Mises criterion stresses predicted by the optimal FE numerical model (red continuous line) are compared, in Figure 5 to the equivalent time-histories of stresses coming directly from the measured data (black continuous line) at two indicative measurement locations SG1 and SG4. The experimentally obtained stress time histories under real operating conditions, result very close to those computed numerically resulting in a high fidelity FE model that could be used for damage accumulation and fatigue prognosis.

![Figure 5](image_url) Comparison between numerical – FE model results and experimental measurements at reference locations SG1 and SG7.

6 **FATIGUE MONITORING USING OPERATIONAL VIBRATIONS**

Using the updated FE models of the CFRP pin-joined frame, the stresses under dynamic load conditions are calculated. The ultimate aim is the identification of those points in the frame
where the larger stresses appear. Figure 6 presents major principal composite stresses (max of all layers) of the updated FE model loaded by random force time history.

Figure 6 Predicted maximum stress locations using the updated FE model.

Incorporating representative dynamic excitation for a relatively short time interval, the stress time histories at the most critical points ST1 – ST4 are computed for the same time duration. The stress pattern during this short time interval is assumed to repeat, thus extrapolating the stress time histories to the fatigue time. Using the calculated stress time histories at locations (ST1-ST4) and utilizing the available S-N fatigue curves from the manufacturer, the Miner’s rule is applied to estimate the fatigue damage accumulation. Using equation (10) and the repeatable pattern of stress time histories measured during a short monitoring period of minutes. The calculated fatigue life for the four locations is presented in Table 2.

<table>
<thead>
<tr>
<th>Location</th>
<th>Calculated Fatigue Life (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST1</td>
<td>10822.51</td>
</tr>
<tr>
<td>ST2</td>
<td>56338.33</td>
</tr>
<tr>
<td>ST3</td>
<td>97557.56</td>
</tr>
<tr>
<td>ST4</td>
<td>14522.96</td>
</tr>
</tbody>
</table>

7 CONCLUSIONS

A computational framework is proposed for estimating fatigue damage accumulation in a CFRP pin-joined frame. A FE model of the frame is developed and updated to match the dynamic characteristics measured. This is achieved through coupled use of numerical and experimental methods for identifying, updating and optimizing a high fidelity FE model. The full acceleration and stress time histories of the CFRP frame are estimated, at critical locations, by imposing operational vibration measurements from a limited number of sensors in the updated FE model. Fatigue damage and remaining lifetime is subsequently estimated via commonly adopted
engineering approaches, such as the Palmgren-Miner damage rule, S-N curves, and rainflow cycle counting. Incorporation of a numerical model of the structure in the response estimation procedure permits stress estimation at unmeasured locations, thereby enabling the drawing of a complete and substantially dense fatigue map consistent with the vibration measurements. Based on these conclusions, the applied methodology is found to be a reliable tool for detecting structural damages.

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SEISMIC ASSESSMENT AND STRENGTHENING OF URM AND MIXED MASONRY-RC BUILDINGS IN LISBON, PORTUGAL

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Abstract

Vulnerability analysis and seismic assessment of old URM residential buildings is a crucial measure for the preservation and safeguarding of the built heritage, particularly in seismic prone areas as Lisbon, Portugal. In this study, the procedures adopted are briefly presented with reference to the old masonry and mixed masonry-reinforced concrete (RC) residential buildings in Lisbon. Reference is made to the contribution of the historical information, the material and the geometric characterization of the building structures (the “knowledge path”), and to the procedure to assess the seismic performance and to identify potential vulnerabilities, in view of proposing adequate retrofit solutions. For the seismic assessment of built heritage constructions in Lisbon, first capacity curves and then fragility functions are derived, using the Performance Based Analysis (PBA) and considering a Displacement Based Approach and are presented. Based on the main results achieved, some retrofitted solutions are identified and discussed.

Keywords: Seismic Assessment, Seismic Strengthening Techniques, URM buildings, mixed masonry-RC buildings.
1 INTRODUCTION

The seismic vulnerability assessment of unreinforced masonry (URM) residential buildings is a complicated task because it refers to a wide variety of constructions, characterized by different types of masonry. Masonry is a heterogeneous material defined by units (stone, clay brick, adobe, etc.) and joints (filled or not by mortar). The mechanical properties of the material are thus related to the properties of the constituents and to the dimension, shape and interlocking of the units and, in the case of joints filled by mortar, to the quality of mortar. The traditional masonry buildings are constructed based on local materials, traditional construction processes and empirical rules. These are usually characterized by unreinforced masonry (URM) walls and timber floors and roof structures. In some cases, the systematic introduction of earthquake-resistant elements (e.g. steel tie-rods, timber frames reinforcing the masonry walls) is observed. Despite this, the seismic vulnerability of URM buildings has been showed worldwide.

The development of detailed vulnerability models at territorial scale requires the identification of different building classes/typologies. This is supported on the idea that buildings with similar architectural and structural features and located in similar geotechnical conditions are expected to have a similar seismic performance.

Focusing on the residential buildings from Lisbon, one with the highest seismic risk in Portugal, different types can be defined. In Figure 1 it is represented the evolution of construction typologies and seismic design codes. It is shown different categories of URM buildings, until 1930, mixed masonry-RC buildings and finally the reinforced concrete (RC) ones. As identify in Figure 1, four main types of masonry buildings are usually recognizable in the Lisbon District: 1) “Pre-Pombalino” buildings – erected before the 1755 earthquake –; 2) “Pombalino” buildings – the new design used in the post 1755 reconstruction –; 3) URM buildings constructed between the 19th and 20th centuries (known as “Gaioleiro”) and 4) mixed masonry-reinforced concrete (RC) apartment buildings built between 1930 and 1960 (preceding the RC buildings and commonly known as “Placa” buildings).

In Figure 1, the red in the horizontal arrow indicates the periods in which there was concern regarding the seismic resistance of the buildings. This corresponds mainly to two different periods: (i) the reconstruction of the Lisbon downtown, in the sequence of the earthquake of 1755;
(ii) and to the introduction of the seismic-design codes in Portugal [1, 2], crucial for the ade-
quate seismic performance of our constructions.

All these masonry residential buildings (including the mixed masonry-RC) are vulnerable
and poorly resilient to earthquakes, not only due to their intrinsic fragility but also given their
long exposure to all sorts of aging effects and all the structural alterations they have been sub-
jected over the years. Thus, vulnerability analysis and seismic assessment of these residential
buildings is a crucial measure for their preservation and safeguarding. In this study, the proce-
dures adopted are briefly presented about the old masonry and mixed masonry-RC residential
buildings in Lisbon. For the seismic assessment of built heritage constructions in Lisbon, first
capacity curves and then fragility functions are derived, using the Performance Based Analysis
(PBA) and considering a Displacement Based Approach. Based on the main results achieved,
some retrofitted solutions will be recognized and discussed.

2 BRIEF DESCRIPTION OF MASONRY RESIDENTIAL BUILDINGS

As illustrated in Figure 1, Lisbon masonry buildings were erected for many centuries. They
were built based on the available materials and empirical provisions, justifying the strong un-
certainties about their structural behavior. The first typology, identified in Figure 1, corresponds
to the “Pre-Pombalino” buildings, and is quite heterogeneous as it results from different periods
of construction (and reconstruction after the 1755 earthquake).

In this work focus is given to the three typologies enclosed by the black rectangle in Figure
1: the “Pombalino”, the “Gaioleiro” and the “Placa” buildings. In Figure 2 the map of Lisbon
shows the main areas where the most representative masonry and mixed masonry-RC buildings
are located in the city. The “Pombalino” buildings, in yellow, are in Lisbon downtown, close
to the Tagus River. Then, further north, and followed by growth of the city at that time, one
have the “Gaioleiro” buildings in an area called “Avenidas Novas” (New Avenues). And finally,
the mixed masonry-RC buildings, “Placa” buildings, despite are spread in town, they are con-
centrated in two main areas: Alvalade and Arieiro. For each of these three main typologies it is
displayed, in Figure 2, a recent photography (on the left side) and the 3D GIS model (on the
right side) created with a rule-based approach by using the CityEngine software. These build-
ings are generally mid-rise structures with three to six storeys (Figure 2). The façade walls are
made of rubble stone masonry with decreasing thickness up to the height.

The 1755 earthquake turned out to be the momentous which made engineers become more
aware of seismic behavior and of the use of new and standardized construction techniques such
as uniform configuration of the blocks, producing orthogonal grids, and development of struc-
tural systems consisting of three-dimensional mixed timber-masonry structures (the “Pomba-
lina” cage) [3]. The cage is composed of timber floors and mixed timber-masonry shear walls
(“frontal” walls). The foundation system is composed of timber piles linked at the top by hori-
zontal timber cross-members placed on the base of the walls to stiffen the alluvium layers of
the Lisbon downtown area. The ground floor of the building consists of solid stone walls and
piers linked by a system of masonry arches.

The “Gaioleiro” buildings represents a period of downgrade of the construction quality in
Lisbon. There are, in fact, records of collapsed buildings during the construction phase which
originated a public demonstration against the “Gaioleiro”, the name given to the contractors
responsible for these buildings. These buildings, mainly located in “Avenidas Novas” (Figure
2), were built during a period of real estate speculation which ended up affecting the structural
reliability and general quality of the buildings. The timber structure from the “frontal” walls
(presented in the “Pombalino” buildings) was replaced by clay brick masonry walls, usually
solid bricks on the bottom storeys and hollow on the top storeys. The thickness of the walls
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decreases by changing the orientation of the bricks and, on the top floors, these were often replaced by light timber partition walls. Architectonical and structural features of these masonry buildings are described in detail in [4].

Figure 2: Map of Lisbon - Masonry Buildings in Lisbon.

The proliferation in Lisbon of the use of reinforced concrete in the construction was developed throughout the first half of the 20th century. After 1930, the use of RC became more common, particularly in kitchens and bathrooms. Throughout the decade of 1930s this material was increasingly being used in more elements of the construction: on the separation of commercial/ground floors and the floors above, on balconies and terraces and finally on most of the floors. However, the insertion of RC elements is not conceived with a well-defined role with respect to the seismic action: the RC slabs are slender and lightly reinforced and occasionally without continuity between spans, the concrete has low to moderate resistance and the transverse reinforcement is insufficient. The characterization of structural elements of this mixed typology is presented in detail in [5].

3 SEISMIC STRUCTURAL PERFORMANCE ASSESSMENT

The first step of the procedure for the vulnerability analysis and seismic assessment of these old residential buildings corresponds to the “knowledge path”. The characterization of all masonry residential buildings in Lisbon started with the analysis of the historical background of the construction period. Subsequently, the main architectural and structural features were examined. Then the structural characterization of the main elements was defined in terms of materials characterization, geometry and behaviour. The most common modifications made on the buildings along time were also identified to highlight the main structural singularities and weaknesses. Finally, structural analyses were performed leading to the definition of fragility functions.

The fragility functions were defined, addressing the global response of the structure, mainly governed by the in-plane capacity of the walls, and the local response, related to the activation of out-of-plane collapse mechanisms of parts of the buildings. As it schematically exemplified
in the left flowchart of Figure 3 [6], to derive fragility functions it is proposed non-linear analyses but considering the in-plane and out-of-plane response of the buildings and the propagation of uncertainties. The capacity curves are obtained, in-plane, through non-linear static (pushover) analyses, and, out-of-plane, through non-linear kinematic analyses.

Fragility curves are defined by lognormal functions and they define the probability that a generic Performance Level (PL) is reached given a value of Intensity Measure (IM) in Eq. (1):

\[ P_{PL} = p(d > D_{PL}|\text{im}) = p(\text{IM}_{PL} < \text{im}) = \Phi \left( \frac{\text{im}}{\beta_{PL}} \log \left( \frac{\text{im}}{\text{IM}_{PL}} \right) \right) \]

where d is a displacement representative of the building seismic behavior, \( D_{PL} \) is the displacement corresponding to the attainment of PL, \( \text{IM}_{PL} \) is the median value of the lognormal distribution of the intensity measure IM that induces the attainment of PL and \( \beta_{PL} \) its dispersion. Fragility functions are thus defined by two parameters: median \( \text{IM}_{PL} \) and dispersion \( \beta_{PL} \). The performance limit states were defined in terms of performance levels (PLk, with k=1,…,4). The three limit states recommended by Part 3 of Eurocode 8 [8], were assumed to correspond to the performance levels PL2, PL3 and PL4, while PL1 was assumed to correspond to the operational limit state. In addition, a direct correspondence between performance levels (PLk, with k=1,…,4) and damage levels in the structure (DLk, with k=1,…,4) was considered.

For the global seismic assessment, mainly governed by the in-plane capacity of the walls, procedures based on nonlinear static analyses were followed and comparisons were made between the displacement capacity of the structures, identified for different PLs, and the seismic demand, expressed by a properly reduced acceleration-displacement response spectrum.

For the “Gaioleiro” and “Placa” buildings detailed procedures were established for the definition of the fragility curves. For both typologies, similar processes were followed by making use of well-known tools/analysis methods whose use was standardized in a systematic and efficient way. The correspondent flowchart is presented in the right of Figure 3 [7]. Any of the two procedure could be also adopted for the “Pombalino” buildings.

In these procedures (Figure 3, right), the definition of the rational range of variation of the materials mechanical properties followed the same routine: (i) firstly, given the absence of a database for Lisbon, the values proposed in the Italian Code [8] were used as a starting point; (ii) then, the results obtained by the in situ and laboratory experimental campaigns, addressed to characterize the materials in Lisbon old buildings, were collected; (iii) finally, a Bayesian approach was followed and an interval of possible values was proposed for different types of materials [4][5].

The standard deviation \( \beta_{DL} \) (assumed equal to \( \beta_{DL} \)), which describes the total variability associated with each fragility curve, was defined according:

\[ \beta_{DL} = \sqrt{\beta_{C,DL}^2 + \beta_{D,DL}^2} \]

where \( \beta_{C} \) is the uncertainty of the capacity of buildings and \( \beta_{D} \) is the uncertainty in the seismic demand. For the “Gaioleiro” and “Placa” buildings profounder analyses were performed to define properly the parameters \( \beta_{C} \) and \( \beta_{D} \). The main differences between the procedure followed for these two typologies is related to the tools chosen to estimate \( \beta_{C} \). Further details can be found in [9] for the “Gaioleiro” buildings and in [10] for the “Placa” buildings. Similar procedures is intended to be developed for “Pombalino” buildings.

For the “Placa” buildings the seismic behavior analysis did not address the local response, related to the occurrence of out-of-plane mechanisms, due to the presence of such quite systematic RC beams at the height of the window lintels. In the other hand, the “Pombalino” and
"Gaioleiro" buildings are vulnerable to the activation of local mechanisms and thus the local response, needs to be addressed. For the “Gaioleiro” buildings typology both the local and global seismic behavior were already taken into account and the final fragility curves were derived from the combination between the global and the local seismic [6, 9].

The fragility functions and the probability distribution of Damage States reached in previous works [5, 6, 11] put in evidence the seismic vulnerability of these three typologies of residential buildings and the need for the design of retrofitting measures and for structural interventions.

4 SEISMIC STRENGTHENING TECHNIQUES

The strengthening of the masonry residential buildings is an important issue since all of them have significant cultural value and establish a heritage that certainly deserves to be preserved. Reinforcing a building with cultural value, to meet new seismic codes requirements, can destroy much of a building’s appearance and integrity. The interventions should not be too intrusive, should be introduced sensitively and the following three important preservation principles should be kept in mind:

- Existent (historic) materials should be preserved and retained to the greatest extent possible and not replaced indiscriminately in the process of seismic strengthening;
- New seismic retrofit systems, whether unseen or exposed, should respect the character and integrity of the building and be visually compatible;
- Seismic work should be "reversible" as much as possible.
For the adequate definition of strengthening techniques, one should first assessing seismic performance of the building. As previously referred, the results achieved until now with “Pombalino”, “Gaioleiro” and “Placa” buildings have shown their seismic vulnerabilities and clear indicate the need to strengthen.

The pathologies and vulnerabilities identified, after the field survey and the seismic assessment, should then be tackled. The different deficiencies could be due: (i) the ageing of the materials (masonry, wood, iron and concrete); (ii) their inherent vulnerabilities, as they all were built before the introduction of the first seismic-design code in Portugal; (iii) vulnerabilities resultant of the changing they have been subjected all over the years, by altering their use or merely an adaptation to nowadays usages and facilities (e.g. introduction of toilets, introduction of elevators, air conditioning, widen up of rooms, etc.).

The essential approach to improving the seismic performance of these three typologies of residential buildings is to:

1. secure all unconstrained parts that means falling hazards to the public (e.g. chimneys, parapets, etc.);
2. strengthen specific structural elements;
3. improve the wall-to-wall and wall-to-diaphragm connections;
4. increase the in-plane stiffness and strength of floor and roof diaphragms;
5. consider, if necessary, adding new structural components.

In this paper is tackling issues 2. to 4., and for topic 2. only masonry walls are referred.

4.1 Strengthen of masonry walls

In the following different retrofit interventions of masonry walls are listed:

a. improving the masonry quality (e.g. injections of grout (Figure 4 [12]), insertion of transversal connections, etc.), with the twofold advantage of avoiding wall crumbling and increasing the wall capacity (i.e. the in-plane strength and stiffness);

b. Strengthening of masonry walls with reinforced cement coating (Figure 5 [13]; this solution is certainly not recommended for “Pombalino” and “Gaioleiro” buildings if it wants to preserve the existent materials), or with polypropylene meshing (Figure 6 [14]) or with composite materials (Carbon or Glass Fiber Reinforced Polymer - CFRP or GFRP).

For the injection of fluid grout in the wall (type a. intervention) it is worth noting that, being a passive technique, restores the integrity of the building and improves its strength. Nevertheless, the grout cannot always be penetrated in all the required parts of the walls due to the inexistence of voids. In the other hand, it is almost impossible to reach the desirable strength of the grout without adding cement. In [12] and figure 4, it is reported a study developed at Instituto Superior Técnico, Lisbon, in which a hydraulic lime-cement based grout was developed and injected in a damaged rubble stone masonry panel. As cement was added to the grout, it was clear refer that this grout is recommended for the “Placa” buildings, as this typology has already RC structural elements. The results obtained in [12] showed that this strengthening technique was effective in allowing the damaged specimens to successfully restore the initial characteristics and in improving their behavior under cyclic loads.

More information related to the strengthen of masonry walls could be found in [15].
4.2 Increase the wall-to-wall and wall-to-diaphragm connections

The improvement of the wall-to-wall and wall-to-diaphragm connections is essential to develop a global behaviour of the building controlled by the in-plane capacity of walls. This way it is possible to avoid the activation of local failure mechanisms. The improvement of the connections can be achieved with the insertion of tie-rods (Figure 7), steel beams angles or ring beams.
4.3 Increase the in-plane stiffness and strength of diaphragms

The stiffening of the in-plane stiffness of flexible timber floor and roof diaphragms may considerably improve the seismic behavior of old masonry buildings, modifying the building global response and, in particular, allowing a considerable redistribution of internal forces to the stiffer and stronger vertical resisting elements of the structure.

According to previous studies, there are different procedures to increase the in-plane stiffness and strength of diaphragms, for instance:

- application of different kind of strips on the timber floors. Corradi et al. [17] studied the application of FRP strips glued to the floor together with the extra layer of planks. This solution attained higher increases of stiffness than the solution with only the extra layer. In their study, Piazza et al. [18] used steel strips screwed to the floor and FRP strips glued to the floor with the same placement, Figure 8 Left. Both solutions show an increase of in-plane stiffness, but the FRP strips glued provide higher stiffness. Valluzi et al. [19] studied the addition of wood strips and the addition of metal strips. When comparing both proposals with the same placement, the proposal with metal strips provides higher values of stiffness.

- Brignola et al. [20] analysed the addition of steel elements around the diaphragm perimeter and verified that the steel elements increase the in-plane stiffness.

The proposals presented abovementioned refer to solutions in which the new elements were placed on top of the floor, itself. As for the old masonry buildings in Lisbon it is important to preserve the authenticity of the existing timber floors, Nunes et al. [21] developed and studied a strengthening solution to be placed underneath the floors: a light steel structure consisting of steel bars at 45° degrees to the bars of the wood pavement and beam angles at the contour of the pavement g (Figure 8, right). The proposed solution can be easily implemented. This solution increase its in-plane stiffness and the connections between the floors and the walls, and thus the floor contribution to obtain a better distribution and transference of forces to the lateral load resisting walls. In this study, numerical simulations and laboratory tests have been performed. The results allowed to identify the failure mode and reveal a significant increase of the in-plane stiffness (about 27 times).
5 CONCLUSIONS

In this paper, the procedures that have been adopted for the seismic assessment of built heritage constructions in Lisbon, are briefly presented and the adequate references given. In the adopted methodologies, first capacity curves and then fragility functions are derived, based on the Performance Based Analysis and considering a Displacement Based Approach. According to all existing uncertainties, for both the definition of the capacity curves and of the seismic demand, a probabilistic approach is recommended and has been followed in the studies reported. The main results achieved for the “Pombalino”, “Gaioleiro” and “Placa” buildings allowed the identification of their main vulnerabilities. The retrofitted solutions identified in this paper considered not only the main seismic vulnerabilities of the three buildings typologies analyzed but also the cultural value of the constructions.

REFERENCES


SEISMIC ASSESSMENT AND STRENGTHENING OF WALL-FRAME RC BUILDING THROUGH A CASE STUDY IN LISBON

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Abstract

Earthquake loss estimation is an effective tool to provide owners and stakeholders with useful information to support financial and social decisions related to risk mitigation programs. In the framework of the Performance Based Earthquake Engineering the probabilistic estimation of monetary loss can be used as meaningful indicators of the building’s performance. In this work, a seismic loss assessment approach is defined for old Reinforced Concrete buildings and applied to an old Reinforced Concrete wall-frame building in Lisbon. The building presents non-ductile detailing typical of buildings designed before the introduction of modern seismic codes (pre–1980). The analytical methodology, which uses multiple stripe analysis, is used for the building case study and different retrofitting strategies are identified and selected, in order to improve its seismic performance. Fragility and vulnerability functions are developed for this structure, before and after strengthening. The feasibility of different retrofitting strategies is investigated in terms of economic loss.

Keywords: Seismic assessment, Loss assessment, RC Wall-frame building, Fragility curves, Retrofitting strategies.
1 INTRODUCTION

The evaluation of the seismic structural behavior is an important task in order to verify the actual need of structural retrofitting, whereas seismic rehabilitation interventions may involve significant costs. In this respect, earthquake loss estimation is an effective tool to provide owners and stakeholders with useful information to support financial and social decisions related to risk mitigation programs.

Assessment methodologies have evolved significantly and methodologies based on deformation have been implemented in alternative to methodologies based on strengths [1] [2]. The definition of intervention methodologies depends on the deficiencies detected in the seismic structural evaluation: a less efficient evaluation may lead to inappropriate intervention solutions and/or high intervention costs.

There are few studies in the literature concerning the development of loss estimation models for old reinforced concrete (RC) frame buildings in Portugal [3] [4] [5]. On the other hand, according to the authors knowledge, there are no loss estimation models applied to old wall-frame RC buildings in Portugal. So, this work focuses on the vulnerability assessment of wall-frame building typology with smooth steel rebars, built between 1960 and 1980, which is currently missing. This typology represents a high percentage of the building cities’ stock in Lisbon. For this typology, old RC wall-frame with steel smooth rebars, it is discussed:

- a procedure for the seismic performance and the loss assessment;
- the effectiveness of two strengthening methodologies involving diagonal X steel braces and the application of Fibre Reinforced Polymers (FRPs).

It is worth noting that the applicability of the retrofitting solution follows the main literature in the field, e.g. [6], and regulation codes, e.g. [7] [8] [9] [10].

2 LOSS ASSESSMENT METHODOLOGY

In this work, a building specific loss estimation methodology, based on the Pacific Earthquake Engineering Research (PEER) approach, is proposed to estimate economic losses of old RC buildings. Due to the lack of data regarding post-earthquake damage for this type of buildings in Portugal, an analytical methodology is followed.

The procedure adopted for the loss analysis aims to balance both aspect of efficiency and accuracy of the component based-fragility function of FEMA P-58 [11] and the building generic loss function of HAZUS [12]. Component-based fragility functions are used for the damage assessment of the structural elements, while the fragility functions provided by the HAZUS methodology are used to estimate damage on non-structural components and the corresponding monetary loss. The expected level of damage for a given ground motion intensity is estimated through the employment of numerical modelling and nonlinear dynamic time-history analysis.

2.1 Hazard analysis

The ground motions for each intensity measure (IM) level should be selected using a conditional mean spectrum (CMS) as the target spectrum, which provides the expected (mean) response spectrum characterized by a certain variance, conditioned on the occurrence of a target spectral acceleration value at the period of interest [13].
2.2 Structural analysis

For the structural analysis, a multiple stripe analysis (MSA) is proposed. A MSA consists of performing nonlinear response-history analyses at discrete IM levels using different sets of ground motion records for a range of IM levels [13]. The ground motions for each IM level would be selected using a conditional mean spectrum (CMS) as the target spectrum, which provides the expected (mean) response spectrum conditioned on the occurrence of a target spectral acceleration value at the period of interest.

For old RC wall-frame building structures, different engineering demand parameters should be adopted, namely the inter-storey drift ratio (IDR) and the peak floor acceleration (PFA).

2.3 Damage analysis

The damage parameter proposed in this study makes use of the local element seismic response (RC columns and walls) to define the global structural damage. Four damage states are defined at the local level, based on the work of [14]. Based on a collection of experimental results, mean inter-storey drift ratios \( \overline{IDR} \) were derived, corresponding to different damage levels in non-ductile RC columns. These damage state (DS) correspond to (i) Light damage (DS1): drop of 30% or more in lateral stiffness, evaluated from the hysteresis loops; (ii) Moderate damage (DS2): inter-storey drift ratio approximately equal to the chord rotation in the column estimated when the longitudinal reinforcement is at yielding; (iii) Severe damage (DS3): the inter-storey drift ratio at which shear failure is reached; (iv) Collapse (DS4).

The median inter-storey drift ratios corresponding to light cracking and severe cracking are equal to 0.35% and 0.71%, respectively. The median inter-storey drift ratios at which DS3 and DS4 occur were derived using closed form analytical expressions [14], and are given, respectively, by Equations 1 and 2.

\[
\overline{IDR}_{DS3} = \frac{1}{0.26 \left( \frac{P}{A_g f'_c \rho''} \right) + 25.4} \geq \frac{1}{100}
\]

\[
\overline{IDR}_{DS4} = \frac{1}{0.2 \left( \frac{P}{A_g f'_c \rho''} \right) + 4.6} \leq \frac{1}{10}
\]

where \( P \) is the axial load at the instant of shear failure, \( A_g \) is the column gross section area, \( f'_c \) is the nominal concrete compressive strength, \( \rho'' \) is the transverse reinforcement ratio, \( b \) is the column width and \( s \) is the tie spacing.

To the best authors’ knowledge, appropriate fragility curves for RC wall fragility with light horizontal reinforcement and smooth rebars are currently missing from the state of the art; therefore, they are currently being developed by the authors.

2.4 Loss analysis

The total expected losses in a building as a function of a certain ground motion intensity can be considered as the sum on three components: losses resulting if the building collapses, \( E[Loss|C] \), losses associated with repairs given that the structure has not collapsed, \( E[Loss|NC \cap R, IM] \), and losses resulting from having to demolish the building due to excessive residual
drifts, \(E[Loss_{NC} \cap R, IM]\). The following equation is used to develop the building-specific relationship that relates ground motion intensity to economic monetary loss [15]:

\[
E[Loss_{NC} | IM] = E[Loss | C] \cdot P(C | IM) + E[Loss_{NC} \cap R, IM] \cdot \\
\left(1 - P(D | NC, IM)\right) \cdot \left\{1 - P(C | IM)\right\} + E[Loss_{NC} | D] \cdot P(D | NC, IM) \cdot \left\{1 - P(C | IM)\right\}
\]

(3)

where, \(P(C | IM)\) is the probability that the structure will collapse under a ground motion intensity \(IM = im\), \(P(D | NC, IM)\) is the probability that the structure will be demolished given that it did not collapse when subjected to an earthquake with intensity level \(IM = im\), \(P(NC | IM) = 1 - P(Coll | IM)\) is the probability that the structure will not collapse, given \(IM = im\). More details about how expression 3 was obtained can be found in the study of [15].

3 CASE STUDY BUILDING

For this work, a building belonging to the typology of RC wall-frame buildings built within 1960 and 1980 in Lisbon, is selected (Figure 1). The building is an eight-storey (ground floor plus seven storeys above ground) structure, characterized by an open ground storey and infills in the upper storeys. The structure is symmetric with respect to the Y direction and moderately asymmetric along the X direction. It features three main RC frames in the longitudinal direction (X) and two stiff RC cores that provide an acceptable lateral stiffness in the transverse (Y) direction. The building was designed according to the old Portuguese code for reinforced concrete and for earthquake resistant design, introduced in 1958 [16]. The building, which was designed to withstand low seismic forces, has non-ductile behaviour and insufficient seismic detailing, e.g., (i) smooth longitudinal reinforcing bars; (ii) columns and RC walls with low confinement and tie reinforcement (lower than 1%); (iii) beams framing eccentrically to the columns.
The site’s seismic hazard curve, $\lambda_{Sa}$, is defined by means of a probabilistic seismic hazard analysis. In this work, the 5% damped pseudo-spectral acceleration, $Sa(T_1)$, is used as the intensity measure. Figure 2 shows the hazard curve for Lisbon considering soil type B as defined in Part 1 of Eurocode 8 [17], corresponding to the average period $T_1$ of the structure equal to 0.9 seconds (s).

![Seismic hazard curves for Lisbon, for a return period $T_1 = 0.9$ seconds.](image)

The RC wall-frame building is modelled in OpenSees [18]. Force-based beam–column elements and a fibre modelling approach are employed for beams, columns and RC walls.

The main features of the building are replicated in the model, such as the infills and the smooth reinforcing bars, typical of RC structures built in the 60s. As for the smooth rebars, the approach described in [19] is used, involving the reduction of the Young modulus and the maximum strength of the reinforcing steel to simulate the increase of the member flexibility due to strain penetration effects. Based on [19], for the RC walls, the Young modulus is reduced by 40% and the maximum strength of the rebars of the RC walls by 30%, at the ground storey level.

### 3.1 Results of loss assessment

Figure 3 shows the variations of the expected losses normalized by the replacement value of the building. The total economic loss of the building at different levels of ground motion intensity have been computed as the sum of non-collapse losses due to repair, non-collapse losses due to demolition and collapse losses [15].

The information presented in Figure 3 can be used together with the seismic hazard curve of the site (Figure 2) to estimate the expected annual loss (EAL) [15], that is equal to 0.32% of the replacement value of the case study building. This value of EAL is slightly higher when compared to results of similar studies on non-ductile building in Portugal, e.g. [4], which reported values within 0.12% and 0.23%. This value can be justified by the non-ductile behavior of the RC walls with smooth bars, which have a critical influence on seismic performance of the building structure, and thus to its earlier collapse.
4 CRITERIA FOR STRENGTHENING

The effectiveness of two retrofitting strategies are evaluated with the aim of improving the performance of the RC walls in the X direction, which are more vulnerable to brittle shear failure and tend to cause earlier collapse of the building. This is evident from Figure 4 where the shear demand, represented by means of bi-linearized curves obtained through nonlinear static analysis, is compared with the shear strength. On the left of Figure 4 it is represented the pushover curve of shear wall in the X direction versus the top displacement ductility (top displacement divided by the yield displacement), whereas on the right the correspondent for Y walls. The shear strengths, represented in Figure 4, is obtained with the expressions provided by EC8-3 [10] and EC2-1 [20] which give very similar results. The shear failure of the wall in the X direction is reached for a very low value of the displacement ductility demand. As stated before, this result is due to the very low amount of horizontal reinforcement.

An improvement of behaviour may be achieved by adopting one of the following approaches or strategies, or even combining them: (i) by reducing the seismic demands on members, or/and (ii) by increasing the member capacities. The deformation capacity and shear strength of individual members may be significantly upgraded through FRP-wrapping, without modifying at all their stiffness. Reduction of seismic demand on the walls through retrofitting may be achieved by increasing the lateral stiffness. The lateral stiffness can be increased by adding a new lateral load resisting system to take almost all the effects of full seismic action, e.g., steel bracing or new concrete walls. In this work, the effectiveness of applying steel braces at the ground storey level to reduce the shear demand on the RC walls is investigated. This partial strengthening, as opposed to a global one, has the double aim to re-
duce the cost of intervention and allow the continued usage of the building during the retrofitting work.

Figure 5 shows the pushover curves, i.e. base shear versus top displacement at the centre of mass, for the X direction (the results in the two senses of loading are identical, as the structure is symmetric with respect to the Y axis) determined with a modal load pattern distribution. The strengthening solution which involves the application of the steel braces is depicted in Figure 5a, while the strengthening solutions which involves the application of FRP in the RC walls is showed in Figure 5b. It is evident that the use of steel bracing significantly reduces the potential for shear failures in the walls at the ground storey level. The shear demand is reduced as to keep the RC walls in the elastic region. The strengthening solution which involves the application of the FRP is depicted in Figure 5b. As stated before, the application of the FRP does not modify the stiffness of the structural elements but increases the shear strength of the walls, allowing them to reach their flexural capacity without developing a brittle mechanism. By comparing the two pushover curves (black solid lines) it is evident that the application of the steel braces at the ground storey did not result in a significant increase of the total lateral strength (Figure 5a).

![Figure 5: Pushover curves after retrofitting interventions: (a) with steel braces, (b) with FRP.](image)

4.1 Cost-benefit analysis

A preliminary evaluation of the cost and benefit of each strengthening intervention is herein made, in order to evaluate the actual convenience in choosing one of the examined strengthening strategies. As state in [21] cost of retrofitting that give a positive return should be no more than about 10 to 30% of the building replacement value. In this work, an effort is made to consider cost-effective retrofitting solutions, which should also allow continued usage of the building during the retrofitting work.

In order to consider reliable and realistic repair costs, the CYPE database [22], which contains detailed and up-to-date construction costs for the Portuguese building stock, has been utilized. The prices absent in this database refer to the prices applied by the construction companies that work with these techniques. The cost of retrofitting is showed in Table 1, as a percentage of the replacement value of the building. As for the benefits, they are considered over a building lifespan of 50 years ($T$) and assuming a value of zero for the discount rate ($r$). Benefit-cost ratios were evaluated as in Expression 4 [2].

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The results of this preliminary study are shown in (Table 1) as a percentage of the replacement value of the building, which amounts to 1 807 600 €. It is worth noting that the indirect loss were not considered in the estimation of the EAL, which leads to an underestimation of the benefit of retrofitting [23]. The results show benefit-cost ratios higher than one, therefore a positive effect of the retrofitting strategies.

Table 1: Benefit-cost ratios for the original building and for each one of the strengthening strategies.

<table>
<thead>
<tr>
<th>EAL (%)</th>
<th>Original</th>
<th>FRP</th>
<th>Braces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefit (%)</td>
<td>-</td>
<td>2.95</td>
<td>1.91</td>
</tr>
<tr>
<td>Cost of retrofit (%)</td>
<td>-</td>
<td>1.47</td>
<td>1.91</td>
</tr>
<tr>
<td>Benefit-Cost Ratio</td>
<td>-</td>
<td>2.01</td>
<td>1.02</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

In this study, a methodology for the seismic loss assessment of old RC frame building is presented. Then a case is addressed representative of old RC wall-frame building in Lisbon with the following main characteristics: (i) RC walls; (ii) smooth reinforcing bars; (iii) vertical irregularity; (iv) plan asymmetry in one direction. The feasibility of partial strengthening of such buildings was examined, with the ultimate aim to develop an efficient retrofitting plan for this typology. Two local methods of retrofitting were used, the first involving the partial strengthening at the open ground storey with steel braces, the second the FRP-wrapping of single elements (individual RC walls). It is worth noting that FRP composite materials have received increasing attention in the past few decades as a potential material for retrofitting of existing RC structures.

A preliminary cost-benefit analysis was performed, which showed that local methods of intervention, as opposed to a complete strengthening to comply with current standards for new buildings, are perhaps the only retrofitting possibility that might be acceptable by the owners of such buildings, for two important reasons: (i) low cost of intervention and (ii) continued usage of the building during the retrofitting work.

REFERENCES


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CYCLIC NONLINEAR MODELING OF SEVERELY DAMAGED AND RETROFITTED REINFORCED CONCRETE STRUCTURES

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Abstract

Advanced numerical methods for seismic assessment of existing substandard reinforced concrete (RC) structures have significantly evolved in the last two decades. Nonetheless, existing numerical tools have numerous limitations (e.g. numerical instabilities, applicable only in 1D or 2D problems, computationally demanding, etc.) and lack of objectivity and accuracy in providing robust, numerically accurate and objective solutions. As a result, new numerical approaches are necessary to solve some of these limitations.

This paper presents a detailed 3D modelling approach that solves some of the current modelling limitations. The approach is used to predict the experimental results from i) a substandard RC joint with inadequate detailing subjected to cyclic loading, and ii) the same RC joint rehabilitated and retrofitted with carbon fiber reinforced polymer (CFRP) sheets and subjected to cyclic loading. It is shown that the proposed numerical method reproduces the experimental results of both substandard and CFRP retrofitted specimens in a robust and computationally efficient manner. Current research is investigating the behavior of more RC components and full-scale retrofitted structures. This study contributes towards providing engineers and researchers with an advanced analytical tool to study the cyclic nonlinear behaviour of substandard RC structures.

Keywords: RC Joint, Nonlinear analysis, Finite Element Modelling, CFRP strengthening, deficient structures
1 INTRODUCTION

Many numerical models have been proposed during the last decades for simulating the mechanical behavior of reinforced concrete (RC) structures. Nevertheless, a numerical approach combined with a constitutive model for concrete behavior that produces accurate and computationally efficient results for any RC structural member is still an open research subject. An accurate, objective and computationally efficient approach will constitute a powerful tool for any professional Civil Engineer and scientist that study the seismic performance of existing RC structures or perform the design of retrofitting interventions of severely damaged RC structures.

Many models that are found in the international literature can only describe certain aspects of the concrete material behavior and their implementation is limited to cases of small practical interest since they place emphasis on post-peak material characteristics, which are described by the introduction of several material parameters that have no physical meaning. A detailed literature review in regards to the modeling of RC structures can be found in [1, 2]. The numerical approach which was proposed in [1], is the latest research work that emphasizes on the ability of realistically predicting the behavior of concrete structures in a wide range of problems ranging from static cyclic to dynamic loading conditions, performed in a computationally efficient way.

The model that was proposed by Mourlas et al. [1], presents a numerical procedure which is based on the brittle nature of the material of concrete. The constitutive model is based on the experimental findings of Kotsovos and Newman [3] and a mathematical approach described in Kotsovos and Pavlovic [4]. This model is also used for the analysis of large-scale structures as described in Markou et al. [5], where a 4-storey RC building specimen was modeled through the Hybrid Modeling (HYMOD) approach [6, 7]. The under study RC building [5] was retrofitted with RC infill walls and carbon fiber reinforced polymer (CFRP) jacketing at its base, where three different load histories were applied until the frame reached its ultimate capacity. According to the HYMOD approach, the shear-dominated areas (RC joints, shear walls) are simulated by a detailed model that foresees the use of 8-noded hexahedral elements for discretizing the concrete domains and embedded beam elements for discretizing the steel reinforcement. The rest of the structure (bending dominated structural members) are modeled through the use of the beam-column finite element. It has to be noted here that in their work [5], the building was also retrofitted with RC infill walls and CFRP jacketing. Even though the proposed modeling method in [5] managed to capture the experimentally derived curves with high accuracy, the pinching effect was not reproduced accurately given that the concrete material deterioration (due to the multiple imposed cycles) and the potential rebar slippage at the base of the building were physical phenomena that were not numerically accounted for based on the adopted constitutive material models [1].

In this research article, a parametric investigation of a newly proposed material factor is performed, where it takes into account the loss of bond between the steel reinforcement and the surrounding damaged concrete, thus reduces the energy dissipation of the RC specimen’s overall response based on the level of concrete damage. The methodology used in this research work foresees the validation of the proposed modeling method through the use of bare and retrofitted RC joints that were tested experimentally by Reyes et al. [8]. The numerically obtained P-δ curves are compared with both the experimental and numerical data in order to test the accuracy of the proposed method.
2 MATERIAL MODELING

2.1 Concrete Material Modeling and Damage Factor

The constitutive modelling of concrete has to describe a realistic behavior of concrete under generalized three dimensional states of stress as mathematically described in Kotsovos and Pavlovic [4]. The constitutive relations take the following form:

\[ \varepsilon_0 = \varepsilon_{0(h)} + \varepsilon_{0(d)} = (\sigma_0 + \sigma_{id}) / (3K_s) \]

(1)

\[ \gamma_0 = \gamma_{0(d)} = \tau_0 / (2G_s) \]

(2)

where \( K_s \) and \( G_s \) are the secant forms of bulk and shear moduli, respectively. The secant forms of bulk, shear modulus and \( \sigma_{id} \) are expressed as functions of the current state of stress which derived by regression analysis of the experimental data found in [3].

The model treats cracking with the smeared crack approach (see Fig. 1). In this way the models simulate the geometrical discontinuity by the assumption of displacement continuity. The crack opening strategy during each load increment foresees the use of the unified total crack approach (UTCA) proposed by Lykidis and Spiliopoulos [9], which foresees that the state of crack formation or closure is treated in a unified way within every Newton-Raphson internal iteration. When a crack is opened, the material properties normal to the crack plane are set to zero. The concrete material assumes that it loses all of its carrying capacity along the vertical direction of the crack, where it behaves in a brittle manner. The expression of the strength envelope of concrete is provided in Eq. 1 and it’s based on the Willam and Warkne [10] formulae.

\[ \tau_{0u} = \frac{2\tau_{0c}(\tau_{0c}^2 - \tau_{0e}^2)\cos\theta + \tau_{0c}(2\tau_{0e} - \tau_{0c})\sqrt{4(\tau_{0c}^2 - \tau_{0e}^2)\cos^2\theta + 5\tau_{0c}^2 - 4\tau_{0e}^2\tau_{0c}}}{4(\tau_{0c}^2 - \tau_{0e}^2)\cos^2\theta + (2\tau_{0e} - \tau_{0c})^2} \]

(3)

where the rotational variable \( \theta \) defines the deviatoric stress orientation on the octahedral plane. The \( \tau_{0c} (\theta=0^\circ) \) and \( \tau_{0e} (\theta=60^\circ) \) correspond to the state of \( \sigma_1=\sigma_2>\sigma_3 \) (triaxial extension) and \( \sigma_1>\sigma_2=\sigma_3 \) (triaxial compression), respectively and are expressed analytically by experimental data.

Figure 1: Local axes for the case of two cracks at a specific Gauss point. [5]
where $\varepsilon_i$ is the current strain in the i-direction that is normal to the crack plane and $\varepsilon_{cr}$ is the strain that causes cracking formation. Parameter $b$ is the number of the imposed displacement branch of the load history, while $n_{cr}$ and $n_{tot}$ are the numbers of increment that the crack is formed at and the total number of increments that an imposed displacement branch is divided into, respectively.

When the criterion of crack-closure is satisfied at a Gauss Point, which had prior to that only one crack formation, then a part of the stiffness is lost along the previously crack plane (material deterioration) that was assumed to form in an orthogonal direction to the maximum principle tensile stress. Therefore, the constitutive matrix takes the following form:

$$
C_i = \begin{bmatrix}
da_i(1-D_x)(2G_x+\mu) & a_i(1-D_x)\mu & a_i(1-D_x)\mu & 0 & 0 & 0 \\
a_i(1-D_x)\mu & a_i(1-D_x)(2G_x+\mu) & a_i(1-D_x)\mu & 0 & 0 & 0 \\
a_i(1-D_x)\mu & a_i(1-D_x)\mu & 2G_x+\mu & 0 & 0 & 0 \\
0 & 0 & 0 & a_i(1-D_x)\beta G_x & 0 & 0 \\
0 & 0 & 0 & 0 & a_i(1-D_x)\beta G_x & 0 \\
0 & 0 & 0 & 0 & 0 & a_i(1-D_x)\beta G_x
\end{bmatrix}
$$

(5)

where $\beta$ is a shear retention factor, $a_n$ and $a_s$ are constants with recommended values of 0.25 and 0.125, respectively, based on the parametric investigation presented in [11]. The proposed expressions of the constitutive matrix describes the anisotropic behavior of concrete at the local coordinate system, therefore, it has to be transformed to the global system by using the standard coordinate system transformation laws as follows:

$$
C_g = T^T C_i T
$$

(6)

where $T$ is the transformation matrix consisted by the direction cosines that define the relative orientation of the local to global axis. The factor $D_i$ is a damage factor proposed in [11], describing the accumulated energy loss due to the number of times a crack has opened and closed. As it was suggested in [11], the damage factor has the following form:

$$
D_i = e^{-(1-a)\psi f_{cc}} = e \left( \frac{1 - \frac{\varepsilon_{cr}}{\varepsilon_{tan}}} {f_{cc}} \right) = e \left( \frac{\varepsilon_{cr}}{\varepsilon_{tan}} \right)
$$

(7)

where $f_{cc}$ is the number of times where a crack has closed and is updated in every iteration at every Gauss Point. A schematic representation of Eq. 7 can be seen in Fig. 2.

During the nonlinear analysis, the constitutive matrix is calculate according to Eq. 5 when a crack is closed at a Gauss point, which had previously one or two cracks. After the crack closure, the stresses are corrected through the use of the following expression:

$$
\sigma_i = \sigma_i^{-1} + C_g \cdot \Delta \varepsilon_i
$$

(8)

Finally, when all the cracks have been closed (of a previously cracked Gauss Point) and the reduction factor $a$ (Eq. 4) of one of the previous cracks is larger than 0.5, then the constitutive matrix takes the following form:
2.2 Steel Material Modeling and Modified Damage Factor

According to any nonlinear behavior of cracked RC structural members, the level of damage that occurs due to the opening of cracks affects the respective contribution of the steel reinforcement to the surrounding concrete areas. Consequently, the cracks directly affect the internal force transfer mechanism from the rebars to the surrounding concrete domain, especially in the anchorage areas. Therefore, a modification of the steel stress-strain relation of Menegotto-Pinto [12] is proposed herein that is related to the level of concrete cracking surrounding the rebar element during the nonlinear cyclic analysis. The loss of bonding between steel reinforcement with the surrounding cracked concrete can indirectly be taken into account by reducing the stiffness contribution of steel reinforcement [11], thus simulate the pinching effect. Hence, the introduction of a bond-slip model, which will require the definition of new material parameters and an additional dof at each reinforcement bar, can be avoided.

Based on the proposed formulation presented in [11], the average of all parameters \( a \) (found in Eqs. 4 and 10, and expressed by Eq. 11) at the 8 Gauss Points of any concrete hexahedral element can be used to determine the level of damage of the concrete material as expressed through Eq. 10.

\[
D_s = \left[ 1 - a_{\text{Element}} \right] \tag{10}
\]

where,

\[
a_{\text{Element}} = \frac{\sum_{i=1}^{ncr} a_i}{ncr} \quad \text{, } ncr \text{ is the number of cracked Gauss Points} \tag{11}
\]

In the case of unloading, when the structure reaches its initial deformation, a material deterioration of the steel reinforcement is computed based on the following proposed formulae:

\[
E_s' = (1 - D_s)E_s \tag{12}
\]
The material deterioration is applied when the criterion $\sigma_s \cdot \varepsilon_s < 0$ is satisfied, which describes the situations when crack closures and re-openings occur, where the pinching phenomena are excessive.

![Figure 3: Menegotto-Pinto steel model that uses the modified formulation for parameters $E'$ (Eq. 12) and $R'$ (Eq. 14) with different values of the damage factor $D_R$ and $D_S$.](image)

Additionally, so as to capture the pinching effect at a material level, the reduction of parameter $R$ (see Eq. 13) of the Menegotto-Pinto [12] model is proposed.

$$R = R_0 - \frac{a_1 \xi}{a_2 + \xi}$$

(13)

It must be noted here that $R_0$, $a_1$ and $a_2$ are experimentally determined parameters and assumed to be 20, 18.5 and 0.15, respectively, in this study. By using the same concept de-
scribed above, the reduction factor $D_R$ is proposed in order to decrease the parameter $R$ into $R'$ which is calculated through the following expression:

$$ R = (1 - D_R)R, \text{ where } D_R = D_s $$

(14)

The material deterioration is once more applied when the criterion $\sigma_s \cdot \epsilon_s < 0$ is satisfied. The modified Menegotto-Pinto material model’s numerical response is illustrated in Fig. 3, according to the proposed formulation. Before moving to the next section, it is noteworthy to state at this point that the material model used to simulate the stress-strain relationship of the CFRP jacketing foresaw a linear behavior until complete failure for both tension and compression states, as shown in Fig. 4. As it can be seen, it was assumed that when the ultimate stress level at any Gauss Point of the CFRP hexahedral elements was reached, the model foresees for a complete loss of its capacity. According to the reported material properties [8], the ultimate CFRP tensile strength was set to 4,140 MPa and the Young Modulus of elasticity was set equal to 241 GPa. In addition to that, it was also assumed that the CFRP hexahedral elements that were used to discretize in detail the jacketing, had full bonding with the concrete finite elements.

3 NUMERICAL IMPLEMENTATION

The numerical investigation foresaw the use of finite element meshes that will be described in this section, followed by a discussion on the obtained numerical findings. All analyses were performed by using a displacement control algorithm that adopted an energy convergence criterion expressed in Eq. 15. The energy convergence tolerance was set equal to $10^{-5}$, whereas the CPU used to perform all the analyses presented in this section, had a 3.7 GHz computing power.

$$ e_{err} = \frac{\Delta u_i \left[ F_{i}^{\text{ref}} - R_{s}^{\text{ref}} \right]}{\Delta u_i \left[ R_{s}' - R_{s}' \right]} \leq \text{tolerance} $$

(15)

Figure 5: Geometric and reinforcement details of the beam-column joints tested in Garcia et al. [8].
The numerical implementation foresaw the study of an exterior joint that was tested by Garcia et al. [8] under extreme cyclic loading (see Fig. 5). The column had a cross-section of 260 x 260 mm and the longitudinal reinforcement were 16 mm in diameter. The beam was designed to have a cross section of 260x400 mm and was also reinforced with 16 mm rebars, as it can be depicted in Fig. 5.

Two types of bar anchorage detailing were examined for the top beam reinforcement as shown in Fig. 5. The beam reinforcement of type A and B (Fig. 5c) were anchored into the joint for a length of 220 mm (approximately 14\(d_b\)) without foreseeing for any hooks nor bends. This short anchorage was designed to be insufficient in order to develop slippage, thus extreme nonlinearities within the joint volume. Furthermore, the flexural strength of the RC column was designed to be the same as the beam’s, a design strategy that does not follow the modern design philosophy that foresees a strong column-weak beam approach. The specimens were designed to fail at the core, where there are no confining stirrups to provide the joint with the ability to behave in a ductile manner. In order to avoid the shear failure outside the core of the joint, 8 mm transverse stirrups were provided at both column and the beam members spaced every 150 mm. The mean concrete compressive strength was reported to be 32 MPa and 31.3 MPa for specimens JA2 and JB2, respectively, where the respective tensile splitting strength was 2.44 MPa and 2.41 MPa for the two specimens, respectively. The yield and tensile strength of the steel reinforcement were reported to be \(f_y = 612\) MPa and \(f_u = 726\) MPa for the 8 mm rebars and \(f_y = 551\) MPa and \(f_u = 683\) MPa for the 16 mm rebars. Additionally, the elastic modulus was found [8] to be equal to \(E_s = 209\) GPa.

The frame joints were subjected to different cyclic loading sets according to the experiment’s setup. The initial loading history applied on the bare specimens consisted of three push-pull cycles at drift ratios of ±0.25%, ±0.5%, ±0.75%, ±1.0%, ±1.5%, ±2.0%, ±3.0%, ±4.0% and ±5.0%. Furthermore, a second actuator applied a constant axial load equal to N = 150 kN on the column as illustrated in Fig. 6.

![Figure 6: Test setup (units: mm) and instrumentation of RC joints. [8]](image-url)
According to the experimental results reported in [8], the bare specimens suffered from severe damages, where excessive diagonal cracking and partial concrete spalling at the region of the core had been observed, as shown in Fig. 7.

Figure 7: Crack patterns after the failure of JB2 bare RC specimen. [8]

For the needs of this research work, specimen JB2 was studied through the use of the proposed numerical model that uses the damage factors for concrete and steel reinforcement, as presented in section 2.2. So as to study the mechanical behavior of specimen JB2, the concrete domain was discretized with 8-noded hexahedral finite elements and the steel rein-
forcement was discretized with Natural Beam-Column Flexibility-Based elements [14]. A total number of 174 concrete and 324 steel elements were used to discretize the RC joint, where 18 hexahedral steel elements (red color) were used at the areas of the boundary conditions in order to prevent local failure (see Fig. 8). Furthermore, for the retrofitted RC joint (JB2RF), 156 hexahedral elements were used for the CFRP jacking as shown in Fig. 9. The FE mesh and material details for both RC joints are given in Tables 1 and 2, respectively.

<table>
<thead>
<tr>
<th>a/a</th>
<th>Model</th>
<th>Total Number of Nodes</th>
<th>Hexahedral Elements</th>
<th>Embedded Rebar Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>JB2</td>
<td>396</td>
<td>192</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>JB2RF</td>
<td>620</td>
<td>348</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 1: FE mesh details of the two RC Joint models.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young Modulus (GPa)</th>
<th>Hardening Modulus (GPa)</th>
<th>Yielding Stress / Tensile Strength (MPa)</th>
<th>Compressive Strength (MPa)</th>
<th>Shear remaining strength β</th>
<th>Poisson Ratio</th>
<th>Ultimate Strain ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>20</td>
<td>-</td>
<td>2.411</td>
<td>31.3</td>
<td>0.03</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>High Strength Concrete</td>
<td>30</td>
<td>-</td>
<td>3.911</td>
<td>55.3</td>
<td>0.05</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Steel Rebars</td>
<td>190</td>
<td>1</td>
<td>551 (Ø16) / 612 (Ø8)</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>15%</td>
</tr>
<tr>
<td>CFRP</td>
<td>50</td>
<td>-</td>
<td>3001</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Table 2: Material details used in the FE model.

Figure 10: JB2 Joint. Comparison between the numerical and experimental curves.

The computed force-displacement curves for the two specimens (JB2 and JB2FR) are compared with the corresponding experimental data in Figs. 10 and 11, respectively. As it can be seen, the numerical results match very well with the experimental data, where the stiffness and the resulted load-carrying capacity of the specimen were predicted in an accurate manner. The numerical results show that the concrete damage factors managed to capture the stiffness
and load-capacity degradation, where the pinching characteristics for both specimens were described with an acceptable accuracy. It is evident that the proposed modeling approach manages to capture this highly complicated physical phenomenon through the use of the modified Menegotto-Pinto model and by adopting the proposed damage factors $D_S$ and $D_R$ given in section 2.2 by Eqs. 10 and 11, respectively.

![Diagram of joint B](image)

Figure 11: JB2FR Joint. Comparison between the numerical and experimental curves.

According to the numerical findings, the FE model developed for simulating the retrofitted specimen JB2FR not only managed to capture numerically the hysteretic behavior of the specimen and its strength’s corresponding degradation, but it was also able to predict the stiffness degradation of a structural joint that foresaw three different solid domains:

a. Old damaged concrete,
b. New high-strength concrete found in the joint and
c. CFRP jacketing.

It must be noted herein that the numerical analysis failed to converge during the final cycle of imposed displacements due to excessive cracking of the concrete domain within the joint domain. Finally, the proposed numerical model for the case of the retrofitted specimen was able to predict the overall strength enhancement with a deviation that was less than 5%.

4 CONCLUSIONS

In this research work, a 3D detailed modeling approach is used for the simulation of bare and CFRP retrofitted RC joints. The proposed numerical model has been integrated with a newly proposed damage factor for the steel material that is directly connected to the number of opening and closing of concrete cracks that occurs during the nonlinear analysis. This damage factor, which is an extension of the damage factor proposed in [11], was developed to account the loss of bonding between steel reinforcement and the cracked surrounding concrete within areas that are expected to undergo significant cracking.
According to the numerical investigation’s findings, the proposed model managed to capture the hysteretic behavior of both the bare and CFRP retrofitted RC joints, which were subjected to ultimate limit cyclic loading conditions. The modification of the steel reinforcement model was able to reproduce the severe pinching effects that occurred during the cyclic loading of the bare joint. Additionally, the corresponding extreme pinching effect of the retrofitted RC joint was also numerically reproduced with an acceptable accuracy, where the numerical and experimental curves were found to be in a good agreement. Furthermore, for the case of the CFRP retrofitted specimen, the proposed modeling approach managed to reproduce the overall strength enhancement of the joint with an accuracy that foresaw a deviation of less than 5% compared to the experimental data.

Future research work foresees the implementation of the proposed algorithm in predicting the mechanical behavior of additional specimens that are characterized by extreme pinching phenomena, while use the developed technology in the parametric investigation of the mechanical response of full-scale structures that undergo extreme cyclic loading. Finally, the proposed modeling approach will be used for the study of the effects that different interventions derive in-terms of the overall mechanical behavior and strength enhancement of RC structural framing systems.

REFERENCES


COMPUTATIONAL ISSUES OF HINGED WALLS USED AS RETROFITTING OF EXISTING RC FRAMES

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Abstract

In recent years hinged walls have been implemented as a retrofit technique for existing RC buildings. To investigate the effectiveness of the proposed solution on different frame typologies, non-linear 2D pushover analyses have been carried out. Two main configurations were adopted, representing an inner frame with weak beams and strong columns and a side frame with strong beams and weak columns, respectively. The study shows that some computational aspects are of fundamental importance in providing reliable results, namely: the dead load distribution on the beams and the moment-axial force interaction in the columns. The hinged wall technique proves to be an effective retrofit solution only if conceived properly for each structural typology; whilst in some cases it may be detrimental when applied in the traditional way. Some new configurations are herein proposed based on new connection layouts in order to be suitable for the different typologies of existing RC frames.

Keywords: Existing RC structures, retrofit techniques, hinged walls, computational issues.
1 INTRODUCTION

The poor state of conservation of the building heritage requires a deep renovation action, considering that 40% of it has already exhausted its nominal service life, and most of existing buildings are obsolete and vulnerable to seismic actions. In detail, the post Second World War reinforced concrete heritage represents the 60% of such building stock (Belleri & Marini, 2016 [1]; Marini et al., 2017 [2]; Labò et al., 2017 [3]; Belleri et al., 2016 [4]; Feroldi et al., 2013 [5]). Those buildings are typically multi-storey structures, featuring one-way reinforced concrete (RC) frames, designed for gravity loads, with inadequate structural details. These features highly contribute to increase their seismic vulnerability, which is typically associated with the possible onset of soft-storey mechanisms or brittle failure of short columns. Hinged walls represent a possible retrofit solution, aiming at linearizing the deformation of the frame along its height (Mac Rae et al., 2004 [6]). Such linearization entails a more uniform deformation demand in frame stories and thus a more predictable damage pattern. The proposed system can be used both in existing and new structures: it can be connected to the existing building from outside, or even some existing elements can be adapted to this function, for example the stairwell walls. In the last few years, application of hinged walls from outside as retrofit solution has increased (Wada et al., 2011 [7]; Gioiella et al., 2017 [8]), and it could be even more widely used in the future. The present paper investigates the suitability of hinged wall solution in the seismic retrofit of existing RC buildings. In particular, the role of the beam-column capacity ratio and the number of effective links are investigated by means of nonlinear static analyses.

2 APPLICATION TO AN EXISTING BUILDING

Since the application of hinged walls in existing buildings as a retrofit solution is getting attention worldwide, the weaknesses and the possible beneficial effects of their application need to be further investigated. On the whole, hinged walls are conceived as external walls connected to the existing building by means of links at each floor level; the wall is usually designed to remain in the elastic field during a target seismic event. The crucial difference between the proposed solution and the more widespread rocking wall is the presence of an explicit pinned constraint at the base, which allows to reduce the bending moment transferred to the foundation. In the following, the hinged wall is implemented in some typical configurations of RC existing buildings, represented by 2D frames.

2.1 Design of the existing building

For the sake of simplicity, the reference frame is assumed to be regular both in plant and elevation. The geometrical scheme of the frame is represented in Figure 1; geometrical and mechanical characteristics of beams and columns are assumed to be homogeneous along the height of the building. A concrete cylindrical strength of 30MPa and steel yielding stress of 450MPa are assumed. The frame has five floors and five bays, with a total dimension and height of 22.5 m and 15 m, respectively. In existing buildings, two main structural configurations are generally possible: a frame with weak beams and strong columns or a frame with strong beams and weak column. Here, the inner longitudinal frame (case A) and the side (case B) longitudinal frame of a building are analysed to describe the aforementioned typologies, respectively. The structural elements are designed according to the Italian building code enforced in the ‘60s, with reference to the “admissible stresses method”. Structural details such as main dimensions, and bottom (Aab) and top (Aa) rebar are reported in Table 1 and Table 2.

Once the two principal configurations have been defined, two more frames are modelled fostering their characteristics to obtain a frame with very weak beams and very strong col-
umns (case C) and the opposite (case D). This further investigation allows a better understanding of the influence of the frame configuration on the response of the retrofitted building. So, the four models used in the following analyses are:

- **CASE A**: Existing inner frame: weak beams, strong columns
- **CASE B**: Existing side frame: strong beams, weak columns
- **CASE C**: Modified inner frame: weak beams, strong columns
- **CASE D**: Modified side frame: strong beams, weak columns

![Figure 1: 2D Frame geometrical configuration (dimensions in cm)](image)

In order to understand the influence of the beams and columns capacity, the “nodal ratios” are computed for each frame type, as the ratio between the sum of the ultimate resisting moment of the beams and the sum of the ultimate resisting moment of the columns converging in a node (Table 3). Given the layout of the nodes, the mean nodal ratio is calculated for each floor.

**Table 1: Reinforcement of beams**

<table>
<thead>
<tr>
<th>CASE</th>
<th>b (cm)</th>
<th>h (cm)</th>
<th>c (cm)</th>
<th>A_{sb}</th>
<th>A_{su}</th>
<th>Stirrups</th>
<th>A_{sb}</th>
<th>A_{su}</th>
<th>Stirrups</th>
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<td>80</td>
<td>24</td>
<td>3</td>
<td>3φ16</td>
<td>9φ16</td>
<td>φ 10/10 cm</td>
<td>6φ16</td>
<td>3φ16</td>
<td>φ 10/250 cm</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>35</td>
<td>3</td>
<td>3φ16</td>
<td>4φ16</td>
<td>φ 10/10 cm</td>
<td>3φ16</td>
<td>3φ16</td>
<td>φ 10/250 cm</td>
</tr>
<tr>
<td>C</td>
<td>80</td>
<td>24</td>
<td>3</td>
<td>3φ16</td>
<td>6φ16</td>
<td>φ 10/10 cm</td>
<td>4φ16</td>
<td>3φ16</td>
<td>φ 10/250 cm</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
<td>50</td>
<td>3</td>
<td>3φ16</td>
<td>4φ16</td>
<td>φ 10/10 cm</td>
<td>3φ16</td>
<td>3φ16</td>
<td>φ 10/250 cm</td>
</tr>
</tbody>
</table>

**Table 2: Reinforcement of columns**

<table>
<thead>
<tr>
<th>CASE</th>
<th>b (cm)</th>
<th>h (cm)</th>
<th>c (cm)</th>
<th>A_{sb}</th>
<th>A_{su}</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>30</td>
<td>3</td>
<td>2φ16</td>
<td>2φ16</td>
<td>φ 8/30 cm</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>30</td>
<td>3</td>
<td>2φ16</td>
<td>2φ16</td>
<td>φ 8/30 cm</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>30</td>
<td>3</td>
<td>2φ20</td>
<td>2φ20</td>
<td>φ 8/30 cm</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
<td>30</td>
<td>3</td>
<td>2φ16</td>
<td>2φ16</td>
<td>φ 8/30 cm</td>
</tr>
</tbody>
</table>
Concerning the static loads on the structure, a permanent structural and non-structural load (4 kN/m² and 2 kN/m², respectively) and a live load of 2 kN/m² are considered. In addition, a linear load of 6 kN/m has been included in the side frame to account for perimetral infills. The actual stiffness of beams and columns has been reduced to 50% and 70% of the initial value respectively, to account for concrete cracking. The structure has been modelled through the software MidasGEN [9].

### 2.2 Design of the retrofit intervention

The analysis of the effects of hinged wall solution is the main objective of the present study. Typically, the main design parameter for hinged walls is the stiffness required to linearize the deformation of the frame along its height. A hinged wall to frame storey lateral stiffness ratio is identified as $\alpha$ (1):

$$\alpha = \frac{E_w J_w}{K_s H^3}$$  \hspace{1cm} (1)

where the $H$ is the structure total height, $K_s$ is the frame storey approximate lateral stiffness, $E_w$ and $J_w$ are the Young modulus and the moment of inertia of the hinged wall, respectively. Previous studies (Mac Rae et al., 2004 [6]) have demonstrated that $\alpha$ equal to 0.1 is the optimal value required to linearize the frame deformation; beyond such value, there is no beneficial effects from increasing the dimensions of the wall. Concerning the estimation of the frame storey lateral stiffness, the approximated method proposed by Schultz, 1992 [10] has been adopted. The method was derived for regular frames, fixed at the base, and accounting only for flexural deformations. The following simplified expression is used in the present study:

$$K_s = \left( \frac{24}{h_c^2} \right) \left( \frac{1}{2} \sum k_c + \frac{1}{2} \sum k_{ga} + \frac{1}{2} \sum k_{gb} \right)$$  \hspace{1cm} (2)

where $h_c$ is the storey height, $\sum k_c$ the sum of the stiffness of the column in a given storey, $\sum k_{ga}$ and $\sum k_{gb}$ the sum of the flexural stiffness of the girders framing into the joint above and below the columns, respectively. The stiffness of each member (column or girder) is assumed as $k=EJ/L$. So, the hinged wall moment of inertia can be obtained from the former definition of the ratio $\alpha$ (1); then, the ratio between the wall dimensions is assumed herein as $b=0.15h$ and the height of the wall section is calculated (3). The dimensions of the hinged walls for the four frames are summarized in Table 4.

$$h = \sqrt{\frac{12 J_w}{0.15}}$$  \hspace{1cm} (3)
Table 4: Hinged wall dimensions required to linearize the frame deformation

<table>
<thead>
<tr>
<th>CASE</th>
<th>b (cm)</th>
<th>H (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>27</td>
<td>180</td>
</tr>
<tr>
<td>B</td>
<td>28</td>
<td>190</td>
</tr>
<tr>
<td>C</td>
<td>27</td>
<td>180</td>
</tr>
<tr>
<td>D</td>
<td>32</td>
<td>210</td>
</tr>
</tbody>
</table>

3 COMPUTATIONAL ISSUES

The nonlinear static (pushover) analyses carried out on the frames have shown significant results concerning some computational aspects of the modelled frames. In particular, it is important to include the distributed loads on the beams and to account for the moment-axial forces interaction in the columns to capture the global system performance. In addition, connecting the hinged wall at each floor is not always beneficial.

3.1 Columns Design and Load distribution

The frame described in case A is here implemented to explain some relevant computational aspects: the analyses carried out show how the building modelling affects the response of the structure. In detail, it is important to take into account the actual distribution of the gravity loads on the beams: indeed, the distribution of gravity loads produces an initial rotation at the beam ends, which can significantly reduce the residual deformation capacity of the retrofitted building. The hinged wall, aiming at linearizing the deformation of the building along its height, produces an additional large rotation in the beams and may cause the premature beam failure. Besides, lumping the distributed loads into point loads applied at the nodes produces an overestimation of the system deformation capacity. Such results are summarized in Figure 2 and Figure 3, where the base shear-displacement graph for each modelled frame is reported.

The other crucial aspect to be considered is the appropriate definition of the axial-flexural interaction for RC columns. When considering such interaction (PM_int), a substantial increase in the capacity of the building is recorded; it is worth noting that the detrimental effect highlighted before (i.e. when distributing the gravity loads on the beams) is observed only when such interaction is considered.

The beneficial effect of the hinged wall application can be seen as an increase in both capacity and moreover ductility of the retrofitted system, while the opposite effects are considered to be detrimental. For instance, in Figure 2 it is clear that for the same structure, the hinged wall solution appears to be detrimental if considering the axial-flexural interaction (A_HW_PM_int) for columns and beneficial without considering it (A_HW_no_PM_int), with respect to the corresponding as-is case (corresponding dotted lines). At the same time, such detrimental effect is not detected when loads are modelled as point loads and lumped at the nodes (Figure 3): in this case, the hinged wall solution appears to be always beneficial, even though the only increase in ductility is detected by accounting for axial-flexural interaction.

So, in this specific case, it is possible to observe the detrimental effect for the system only when accounting for both the distributed loads on the beams and the axial-flexural interaction for the columns; beside the mentioned example, the results show that in any case the computational aspects need to be carefully defined in order to have a reliable estimation of the structural behaviour and so an optimal design of the retrofit solution.
Figure 2: Models comparison in the case of distributed dead loads on the beams: A=Frame in Case A, HW=Hinged Wall as a retrofit solution, PM_int/no_PM_int = accounting or not for the Moment-Axial forces interaction in the columns.

Figure 3: Models comparison in the case of concentrated dead loads at the nodes: A=Frame in Case A, HW=Hinged Wall as a retrofit solution, PM_int/no_PM_int = accounting or not for the Moment-Axial forces interaction in the columns.
3.2 Links Configuration

The previous results show that in some cases the high deformation demand due to the introduction of the hinged wall can be critical for the structural elements. Each link controls the displacement of the associated storey, forcing the elements deformation.

A possible solution is thus investigated by modifying the number and configuration of links along the building height. The results highlight the influence of the nodal ratios of the frames in the effectiveness of the retrofit solution. When dealing with a “weak beams – strong columns” frame type, the imposed displacement profile leads to a deformation demand for beams that is far larger than their capacity, leading to their premature failure. Such results are addressed herein. First, all the considered link configurations are shown in Figure 4, then the pushover curves are reported for each configuration and the behaviour of the frames before collapse highlighted (Figure 5 - Figure 12).

![Figure 4: Links configuration considered in the models](image)

The following graphs show the comparison between the retrofitted building response in the case of different configurations of the links for each frame type; the model “as is” represents the building before retrofit. The results show that the effectiveness of the traditional hinged wall solution (i.e. connecting the wall to the frame at each floor) is substantial in the case of “modified side frame”, while the adoption of hinged walls is detrimental in the case of the “modified inner frame”. This is associated with the linearization of the deformation along the height, which is particularly useful in the case of possible onset of soft storey mechanisms, and so in the case of strong beams and weak columns. Such linearization prevents the collapse of the columns in a soft storey mechanism. In the other cases, as explained before, the imposed linearization causes a rotation demand in some beams which exceeds their rotation capacity, and so a detrimental effect is observed.

The nodal ratios of the frame stories seem to be closely related to the collapse behaviour of the different frames: the larger are the nodal ratios and the more effective is the application of the hinged wall in the traditional way, i.e. connected at each floor. For this reason, nodal ratios are relevant parameters to be investigated for the retrofit solution with hinged walls.

To overcome the possible detrimental effects observed, different configurations of the links were investigated: for each frame type, the best solution was identified as the one that produced the highest deformation capacity of the system.
Figure 5: Comparison of pushover results for the frame C different configurations: M0 STEP 70 (7 cm) represents the hinge status result described in Figure 6. Optimal solution: Model 2.

Figure 6: Plastic hinge status result at the step before collapse of frame C in the case of Hinged Wall connected at each floor – Model 0 STEP 70 (7 cm) in Figure 5.
Figure 7: Comparison of pushover results for the frame A different configurations: M0 STEP 140 (14 cm) represents the hinge status result described in Figure 8. Optimal solution: Model 3.

Figure 8: Plastic hinge status result at the step before collapse of frame A in case of Hinged Wall connected at each floor – Model 0 STEP 140 (14 cm) in Figure 7.
Figure 9: Comparison of pushover results for the frame B different configurations: M0 STEP 310 (31 cm) represents the hinge status result described in Figure 10. Optimal solution: Model 4.

Figure 10: Plastic hinge status result at the step before collapse of frame B in case of Hinged Wall connected at each floor – Model 0 STEP 310 (31 cm) in Figure 9.
Figure 11: Comparison of pushover results for the frame D different configurations: M0 STEP 290 (29 cm) represents the hinge status result described in Figure 12. Optimal solution: Model 0.

Figure 12: Plastic hinge status result at the step before collapse of frame D in case of Hinged Wall connected at each floor – Model 0 STEP 290 (29 cm) in Figure 11.
4 CONCLUSIONS

The use of hinged walls as retrofit intervention for RC buildings is getting attention worldwide for its low invasiveness and the possibility of carrying out the retrofit intervention form outside. Furthermore, the solution contributes in linearizing the deformation of the building along the height and does not require heavy works to the foundations. In this paper the effectiveness of such solution has been investigated through nonlinear static analysis of 2D typical RC frames, considering various ratios of the beam-to-column capacity. Some computational aspects were emphasized:

- Modelling dead loads as distributed loads applied to the beams, rather than as point loads lumped at the structure nodes, is crucial to enable the correct estimation of the beam ultimate rotation demand.
- If the moment-axial load interaction is not considered in the RC columns, the effectiveness of the hinged wall application may be overestimated and possible detrimental effects may not be detected.
- The floor nodal ratio, that is the ratio between the beams and columns bending resistance at each floor, seems to be quite relevant in order to identify the collapse mechanism of the existing building and therefore the efficiency of the hinged wall solution.
- Results show that in some cases the adoption of hinged walls may be detrimental of the structural response, if applied in the traditional way (i.e. connecting the wall at each floor). To overcome this drawback, different links configurations have been studied and proposed, in order to identify the optimal solution for each frame typology.

REFERENCES


A COMPARATIVE STUDY ON TARGET DISPLACEMENT EVALUATION IN BUILDINGS WITH SOFTENING RESPONSE

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Abstract

Nowadays, the use of nonlinear static (Pushover) analysis has become widespread in structural engineering practice, as it provides a robust estimation of structural response with reduced computational cost. However, the accuracy of the analysis depends on the calculation of seismic demand, i.e. the maximum roof displacement during the design ground motion. Modern structural Codes like Eurocode 8-1, the Italian Code (NTC, 2018) and the Greek Code of Structural Interventions (KANEIE, 2017) propose a methodology where global nonlinear structural response is approximated by a bilinear SDOF system, while target displacement is derived with the use of the elastic response spectrum. However, this approach seems to be inadequate for structures with post-peak softening response, like masonry and infilled RC buildings, as it neglects the negative-stiffness branch of the Pushover curve. Consequently, different variants of the above methods have been proposed, where a four-linear SDOF behaviour is used instead. In addition, the next generation of Eurocode 8-1 (Annex E of the draft EN1998-1) proposes a novel approach to the problem, where a multilinear SDOF system is extracted from the Pushover curve and target displacement is calculated in terms of nonlinear time history analysis. The purpose of this paper is to test and compare all the aforementioned methods and to provide an insight on their applicability in real-life structures. For this reason, nonlinear static analysis is performed using FespaR software to a multistory infilled RC structure and a 2-story masonry building. After the derivation of the capacity curves, target displacements are calculated for each method separately and the results are compared in terms of accuracy and safety.

Keywords: Pushover Analysis, Target Displacement, Infilled RC Building, Masonry Structures, SDOF Nonlinear Time History Analysis, FespaR Software.
1 INTRODUCTION

Reinforced Concrete (RC) and masonry buildings constitute the vast majority of the built environment in seismically prone regions around the Mediterranean Sea. An outburst in the construction of multi-storey RC residential and office buildings occurred during the 1960s and 1970s, hence, nowadays these structures have already exceeded or approached their life expectancy. Consequently, there is an urgency for seismic assessment and retrofitting of these structures in order to adapt to modern code provisions and contemporary knowledge on seismic-resistant technology. Realistic seismic evaluation leads to the proper retrofitting and strengthening strategies, a decision-making procedure which has significant life-cost and economic consequences.

In this context, modern design codes such as the European Norm for the design of structures for earthquake resistance (EN 1998) [1], the Greek Code for the Seismic Retrofit of existing buildings (KAN.EΠE, 2017) [2], the ASCE standard for the Seismic Rehabilitation of Existing Buildings (ASCE 41-13, 2013) [3] and the Italian Building Code (NTC-18) [4] offer specific guidelines for the evaluation of the nonlinear properties of structural members and the estimation of the nonlinear structural response. The concept of the performance-based design, which demands a given structure to withstand various levels of loading intensity with adequate damage levels, is the basic core of modern codes [5].

Nonlinear Time History analysis remains, for the time being, impractical for every day engineering applications, as it requires a large number of input ground motions, resulting to huge computational time, in order to cover the variability and directivity of the seismic action. For this reason, nonlinear static (Pushover) analysis consists the standard procedure for the determination of nonlinear structural behaviour, as it provides a good estimation of nonlinear response with the minimum computational cost.

In addition, nonlinear analysis provides the capability to realistically describe the computational model of an RC structure in full detail with the addition of infill walls. However, infill panels are very brittle, hence, their effect on the structure’s overall response is the appearance of softening branches with negative stiffness at the early stages of the nonlinear path. This is also the case for masonry buildings, or dual mixed RC and masonry structures. Target displacement determination for these types of structural systems, should consider the loss of lateral loads bearing capacity in the softening region of the Pushover curve. Modern codes for the seismic evaluation of existing structures don’t consider any negative stiffness part on the Pushover curve, as they are based on bilinear approximations of the actual nonlinear base shear-displacement relation. Future codes, like the draft of the new Eurocode 8-1, or other methodologies in the literature try to address this issue and to provide a robust procedure for the determination of target displacement.

2 MODELLING THE NONLINEAR BEHAVIOUR OF STRUCTURAL MEMBERS

The accuracy of nonlinear analyses depends on the constitutive laws incorporated for the structural members. Herein, the FespaR software [6] is used to perform Pushover analyses, within the context of lumped plasticity beam - column elements. The nonlinear properties of different kind of structural members are presented in the following paragraphs.

2.1 RC columns and beams

Plastic hinge nonlinear properties at both RC column and beam ends are expressed in moment-chord rotation terms as presented in Figure 1(a) and Figure 1(b) respectively. Moment-chord rotation diagrams are deduced from cross sectional moment - curvature analysis, in-
cluding also shear and bond-slip phenomena. The yield point and the plastic branch on column diagrams are clearly affected from the axial load. Largest values of the dimensionless axial coefficient $\nu$ lead to increased yield moment, but on the contrary the plastic rotation capacity of the column is reduced.

![Diagram](image)

Figure 1: Moment - chord rotation diagrams in (a) RC columns and (b) beams

### 2.2 Infill masonry walls

Infill masonry walls and panels are modeled as hinged diagonal braces with zero axial capacity in tension (Figure 2). Infill walls lose all their load carrying capacity in compression when axial deformation exceed the ultimate failure point of Near Collapse (NC) limit state. Consequently, RC infilled frames with dense distribution of infill panels exhibit a sudden and significant loss of strength at initial stages of nonlinear response.

![Diagram](image)

Figure 2: Computational model and force-deformation diagram of infill walls

### 2.3 Masonry piers and sprandels

Masonry piers and sprandels are modeled as beam macro-elements in terms of shear and element chord rotation. Piers are modeled for both in plane and out of plane nonlinear response. The critical failure mode among flexural, shear sliding or diagonal cracking failure is first identified and then yield strength and ultimate deformation are calculated (Figure 3). After the ultimate point, masonry members are considered heavy damaged to carry any further lateral load and they lose half of their shear strength capacity. This local constitutive behaviour in the element level is depicted with a softening branch in the overall building’s Pushover curve.
3 TARGET DISPLACEMENT EVALUATION METHODS

Estimation of seismic demand in Pushover analysis is not directly provided. The nonlinear capacity curve of the MDOF system should first be modified to an equivalent curve of an idealized SDOF system. Then, this equivalent curve is linearized and the seismic demand is derived using the elastic response spectrum. Current seismic codes, like EC8-1 and the Italian Building Code (NTC-18) are based on a bilinear approximation of the Pushover curve, neglecting softening and residual strength segments of the nonlinear curve. On the other hand, alternative methodologies have appeared in the literature and are about to make their appearance in future seismic codes. In the following, both existing code and alternative methodologies for target displacement evaluation are briefly explained.

3.1 EC8-1 2004

In Eurocode 8-1 [7], the target displacement is derived according to the N2 method [8], which is based on two equivalent nonlinear models, one MDOF and one SDOF system. The MDOF is converted to the equivalent SDOF with the following coefficient Γ:

\[ \Gamma = \frac{m^*}{\sum m_i \Phi_i^2} \]  

(1)

where, \( m^* = \sum m_i \Phi_i \) is the equivalent mass of the SDOF system.

Then, the Pushover capacity curve is converted from base shear \( F_b \) versus control node displacement \( d \) form to spectral acceleration \( S_a \) versus spectral displacement \( d^* \) form (capacity spectrum) as follows:

\[ S_a = \frac{F_b}{\Gamma \cdot m^*} \]  

\[ d^* = \frac{d}{\Gamma} \]  

(2)

On this plot, the elastic spectrum is also drawn in ADRS format:

\[ d^* = S_e(T) \cdot \left[ \frac{T}{2\pi} \right]^{-2} \]  

(3)
Next, from the smooth nonlinear capacity curve, its bilinear representation is derived following the rule of equal areas below the two curves, while the post-yield line is considered horizontal without any hardening.

Target displacement is evaluated as the point in the bilinear curve, where the plastic deformation of the SDOF system is equal to the respective inelastic response spectrum, i.e. the modified elastic spectrum divided with plastic deformation index \( \mu \) (Figure 4). Then, target displacement is easily calculated from the following relations:

\[
\begin{align*}
  d'_{t} &= S_{y}(T'_{c}) \cdot \left[ \frac{T'_{c}^{2}}{2\pi} \right], & T'_{c} &\geq T_{c} \\
  d'_{t} &= S_{y}(T'_{c}) \cdot \left[ \frac{T'_{c}^{2}}{2\pi} \right] \cdot C_{I}, & T'_{c} &< T_{c}
\end{align*}
\]

(4)

Where, \( C_{I} \) is the inelastic to elastic deformation ratio.

![Figure 4: Target displacement calculation according to EC8-1 2004](image)

3.2 NTC-18

The main disadvantage of the EC8-1 method is that the second horizontal branch of the bilinear curve has the same ordinate with the performance point, rendering this procedure inefficient for Pushover curves with negative stiffness segments. To deal with this drawback, the Italian Building Code (NTC-18) [4] proposes a different method for the derivation of the bilinear curve. The yield point on the curve is calculated by the rule of equal areas, considering also the following two constraints (Figure 5):

- The first linear branch intersects with the smooth nonlinear curve at the 70% of the maximum base shear (~\( V_{max} \)).
- The second horizontal branch expands until displacement \( du \), which corresponds to the point where a drop of 20% in maximum base shear (~\( V_{max} \)) occurs.
3.3 N2 EXTENDED METHOD

To address the issue of target displacement evaluation in RC infilled frames, Dolsek and Fajfar [9], [10] proposed a four-linear approximation of the Pushover curve. The quadrilateral curve is the sum of two virtual parallel systems, the bare frame alone and the infill panels separately, that together represent the whole generalized SDOF system. Each sub-system can be simulated by a nonlinear spring describing the corresponding nonlinear force-displacement relation.

The idealized quadrilateral curve, similarly to the bilinear one, is deduced from the capacity curve equalizing the areas below the two curves up to the target displacement (Figure 6). The relations that represent the idealized curve and the subsequent derivation of the target displacement can be found in [9].
3.4 The new generation of EC8-1 (2020)

Although the N2 extended method deals with infilled RC frames, it can’t be generalized to masonry buildings, dual systems or other structural systems with softening response. The next generations of Eurocodes, and more specifically the draft of Eurocode 8-1 [11], tries to fill the gap in the contemporary seismic codes and proposes a general methodology, able to cover all aspects of nonlinear response.

Annex E describes the determination of target displacement by using a series of nonlinear time history analysis on the equivalent SDOF model. First, the MDOF system is converted to the corresponding SDOF by using the transformation factor $\Gamma$ of relation (1). Then, in the general case, a multilinear approximation of the nonlinear curve is determined in terms of best fitting, using for example least squares, while also imposing equality of the areas under the actual and the idealized force-displacement curve. As far as the hysteretic behaviour of the SDOF is concerned, the peak-oriented hysteretic model with degrading unloading stiffness is recommended as the most appropriate. In the present paper, the modified peak-oriented Ibarra-Medina-Krawinkler hysteretic model [12], presented in Figure 7, is utilized. This model consists of a four-linear backbone curve with strain hardening and residual strength. The model in its generalized form can describe asymmetrical hysteresis and cyclic stiffness and strength deterioration. However, for a monotonic Pushover curve these phenomena may be neglected as it is proposed in EC8-1.

![Figure 7: Peak-oriented Ibarra-Medina-Krawinkler hysteretic model](image)

In addition, viscous damping is considered for the SDOF system according to the following relation:

$$c = a_0 m^* + a_1 k^*$$  \hspace{1cm} (5)$$

where, $m^*$ and $k^*$ are the mass and elastic stiffness of the equivalent SDOF model and the $a_0$ and $a_1$ coefficients are calculated according to relations (6):
where, $\xi$ is the viscous damping ratio and $T_1$, $T_2$ are the fundamental and the second period of vibration of the structure.

A minimum of $N_a=15$ artificial ground motion accelerograms are generated [13] and are used in a series of $N_a$ nonlinear time history analyses. The final target displacement is determined as the median value of the set of the peak displacements calculated in each time history analysis:

$$d'_i = \exp\left(\frac{1}{N_a} \sum_{i=1}^{N_a} \ln(d'_{i,i})\right)$$

(7)

4 EXAMPLES

4.1 6-storey infilled RC structure

In the first example, the 6-storey, including the basement, RC infilled wall building shown in Figure 8 is examined. Concrete compressive strength is $f_{cm}=20.0$ MPa, reinforcement yield stress is $f_{ym}=280.0$ MPa, and masonry infill compressive strength is $f_{wc}=3.0$ MPa. Three different limit states (DL, SD, NC) with peak ground accelerations of $a_g=0.16$ g, $a_g=0.24$ g and $a_g=0.36$ g respectively, are arbitrarily considered and for each level of seismic action the target displacement is determined.
structural collapse. Heavy damage is present at the long shear walls of the first three floors. Also, the infill walls of the 1st and 2nd floor exceed ultimate plastic deformation and their lateral load bearing capacity drops to zero. This enables the onset of a soft storey collapse mechanism which is enhanced under the action of $P-\Delta$ effects.

First, the target displacement is determined in terms of the N2 extended method and the current EC8-1, as presented in Figure 10. The target displacement is determined equal to $d_t=11.10$ cm, for the four-linear curve, where plastic deformation demand for the whole structure is estimated $\mu=2.82$. When the EC8-1 2004 methodology is applied, the target displacement is calculated equal to $d_t=8.07$ cm with plastic deformation demand $\mu=2.05$.

The target displacement is alternatively evaluated with the new generation of EC8-1 2020 method. A total of 15 artificial ground motions are generated appropriately to fit the elastic response spectrum at each performance level (Figure 11). The check of the fitting criteria is performed according to the existing EC8-2.
The hysteretic behaviour of the SDOF which corresponds to the time history where target displacement is determined, is presented in Figure 12.

In Table 1, a comparison on the target displacement results of the different methods is listed. Differences are smaller when the intensity of the accelerogram is not strong enough to cause large inelastic deformations which threaten building’s structural integrity. On the contrary, results start to diverge at larger peak ground accelerations, where the softening branch of the Pushover curve influences the outcome a lot. If we consider a priori that the nonlinear time history analysis is more accurate, the N2 extended method seems to overestimate considerably the performance point.
Target Displacement Evaluation Method | \(a_g\) | \(0.16\) g | \(0.24\) g | \(0.36\) g
--- | --- | --- | --- | ---
EC8-1 2004, \(d_t\) (cm) | 3.55 | 5.95 | 8.07 |
N2 extended, \(d_t\) (cm) | 3.55 | 6.18 | 11.10 |
EC8-1 2020, \(d_t\) (cm) | 3.37 | 4.81 | 7.28 |

Table 1: Target displacement comparison for the RC infilled building

4.2 2-storey masonry building

In the second example, a 2-storey unreinforced masonry building with rigid horizontal diaphragms (Figure 13) is tested for the same 3 performance levels (DL, SD, NC) with peak ground accelerations of \(a_g=0.16\) g, \(a_g=0.24\) g and \(a_g=0.36\) g respectively. Compressive masonry strength is \(f_{wm}=5.0\) MPa.

Figure 13: Plan view of the 2-storey masonry building

First, a Pushover analysis with lateral load distribution according to the 1st eigenmode is performed along the longitudinal horizontal direction of the building and afterwards the target displacement for the NC limit state is determined using NTC-18 and the current version of EC8-1 (Figure 14).
Figure 14: Target displacement determination at NC limit state according to (a) NTC-18 and (b) EC8-1 2004

The performance point is alternatively evaluated with the new generation EC8-1 2020 method. A new set of 15 artificial ground motions are generated to fit the elastic response spectrum at each performance level (Figure 15).

Figure 15: Fitting of the generated artificial ground motions at the elastic response spectra

Figure 16 depicts the hysteretic loops of the SDOF and the determined target displacement which corresponds to the maximum displacement measured throughout the nonlinear analysis. The backbone of the hysteretic curve indicates the fitting of the multilinear model to the actual Pushover curve.
In Table 2, a comparison on the target displacement results of the different methods is listed. At lower seismic intensities, EC8-1 2004 and NTC-18 produce exactly the same results, as the inelastic deformation of the structure is not important. Small differences with the method provided by EC8-1 2020 are attributed to differences in the elastic stiffness of the SDOF system due to different linearization techniques. On the other hand, at the high seismic intensity level of $a_g=0.36 \, g$ the difference between the results of each method is larger. This time, the nonlinear time history analysis of the SDOF produces the largest target displacement of all methods.

<table>
<thead>
<tr>
<th>Target Displacement Evaluation Method</th>
<th>$a_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.16g</td>
</tr>
<tr>
<td>EC8-1 2004, $d_t$ (cm)</td>
<td>0.99</td>
</tr>
<tr>
<td>NTC-18, $d_t$ (cm)</td>
<td>0.99</td>
</tr>
<tr>
<td>EC8-1 2020, $d_t$ (cm)</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 2: Target displacement comparison for the masonry building

5 CONCLUSIONS

In this paper, different methods for the determination of target displacement, available on the literature and on both current and future seismic codes, are tested in buildings with severe softening response. Differences on the results of all methods are getting larger as the seismic intensity considered increases. This behaviour occurs due to the fact that high seismic intensity results to larger inelastic deformations at levels where softening response becomes significant. For the time being, this region of inelastic response is not addressed by contemporary codes, while other methodologies in the literature, like the N2 extended method, contain a lot of assumptions. On the other hand, the next generation of Eurocode 8-1 propose a novel method that combines Pushover analysis on the global MDOF system and nonlinear time history analysis on the SDOF level. This procedure seems to be very promising in terms of accu-
racy and robustness as it can deal with different kinds of structural systems with various sub-assemblages, provided the fact that the proper hysteretic laws are used. Another crucial point that affects the results of the method is the selection of the appropriate ground motions that represent the seismicity of the examined region. The two examples considered in this paper provide a preliminary study, however, they are not enough to draw solid conclusions on the limitations of each method and further research is deemed necessary.

REFERENCES
AN APPROXIMATE METHOD TO ASSESS THE SEISMIC CAPACITY OF EXISTING RC BUILDINGS

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Abstract

A new approximate method for the assessment of reinforced concrete buildings is demonstrated and validated through a realistic example via non-linear static analysis. This method aims to handle the problem when a building’s reinforcement information is unknown. In order to deal with such a problem, two reinforcement assumptions are made. The first one considers zero reinforcement, while the second one considers the minimum reinforcement amounts of Eurocodes 2 and 8. For both assumptions, the safety indices of an existing building are calculated, and the results are compared with the corresponding ones of a non-linear static analysis. It is shown that this approximate technique is able to predict with acceptable accuracy the safety indices and thus, can be successfully used for the preliminary analysis and assessment of a building with unknown reinforcement amounts. Moreover, both assumptions, i.e., ignoring or accounting for minimum reinforcement, produced quite similar results.

Keywords: Approximate method, Reinforced concrete, Preliminary seismic analysis, Building assessment, Non-linear static analysis
1 INTRODUCTION

During the last 20 years or so, the need for a strengthening assessment of reinforced concrete (RC) buildings has been increased rapidly, mostly because the majority of existing buildings were constructed in the years 1950-1970, which means that these buildings have exceeded their design working life with regard to present regulations. In addition, these buildings were designed according to old regulations and with simple calculation tools, which do not reflect today's higher standards. Since their number is quite large, it is difficult to simultaneously assess all of them with advanced analysis methods and thus, they should be prioritized on the basis of their seismic vulnerability. Moreover, difficulties are increased when their design and construction details are not available to engineers, which is the case for many existing buildings.

To solve these problems, recently, a working group set up by EPPO (Earthquake Planning and Protection Organization of Greece) has proposed a method for the preliminary assessment of RC buildings [1]. It is an approximate method for assessing the seismic capacity of existing RC buildings in accordance with the seismic requirements of current regulations. Since it is based on simple calculations, there is no need of a detailed model and time-consuming advanced analysis methods.

A major characteristic of this approximate method is its ability to estimate the capacity of structures, for which their reinforcement details are unknown, making it particularly important and original. The proposed method is applied for two assumptions for the amount of reinforcement. The first one considers zero longitudinal and transverse reinforcement, while the second one considers the minimum reinforcement amounts, for all the horizontal and vertical members.

The above method is demonstrated herein in detail through an application to an existing RC building with unknown reinforcement details. Its degree of accuracy is validated through comparisons with the more advanced and accurate non-linear static analysis method. The values of interest are the safety indices of the examined building as found by both methods, i.e. approximate and advanced, and according to these results, conclusions are drawn for the accuracy of the proposed method to predict the seismic capacity.

The proposed approximate method consists of two main parts: Informational and computational. The first part is based on 13 criteria from which a reduction factor $\beta$ is deduced, to be used in the computational part. These criteria defined by the proposed method are the following: Existing structural damage, reinforcement oxidation, normalized axial load, regularity in plan, stiffness distribution in plan – torsion, regularity in elevation, stiffness distribution in elevation, mass distribution in elevation, short columns, vertical discontinuities, forces route and transfer, neighboring buildings, and faulty workmanship or non-structural damage that has occurred either during or after construction [1]. The second part of the proposed method includes the following steps: i) Determination of the seismic demand, ii) determination of the seismic resistance and iii) determination of the safety indices [1].

The present work mainly focuses on the second part of the method, which consists of the computational process. In section 2, the building to be examined is presented and in section 3 it is evaluated by the proposed method, while in section 4 it is re-evaluated by the more accurate non-linear static analysis. In section 5 the results are compared and finally in section 6 all the important conclusions are drawn.
2 DESCRIPTION OF THE CASE STUDY RC BUILDING

The examined RC building was constructed in 1988. Its floor plan is square-shaped at all stories, with a total length and width equal to 15 m, as shown in Figure 1. The building consists of 5 floors, with the ground floor height being 5.50 m and the remaining floor heights being 3.50 m (Figure 1). The ground, 3rd and 4th floors are used as offices, while the 1st and 2nd floors contain machinery. The columns are 0.60x0.60 m in the ground floor, 0.50x0.50 m in the 1st and 2nd floors and 0.40x0.40 m in the 3rd and 4th floors. The Π-shaped shear wall of the elevator is 3x3x0.25 m, and the beams are 0.25x1.00 m. The material properties are C16/20 for concrete and S500 for the reinforcing steel.

![Figure 1: Story plan and section A-A.](image)

2.1 Loads

The dead loads (G) include the self-weight of the structure from material properties (25 kN/m$^3$ for the RC), the toppings (1 kN/m$^2$), the outer masonry walls (3.6 kN/m$^2$), and the roof insulations (2 kN/m$^2$). The live loads (Q) include the ones in the office floors (2 kN/m$^2$ with $\psi_2 = 0.3$, where $\psi_2$ is a combination coefficient for the quasi-permanent variable action), in the machinery floors (5 kN/m$^2$ with $\psi_2 = 0.9$), in the stairs and balconies (2 kN/m$^2$ with $\psi_2 = 0.3$), and in the roof (1 kN/m$^2$ with $\psi_2 = 0.3$). Seismic loads (E) were calculated in accordance with the EC8 [2] response spectrum with a ground acceleration equal to $a_g = 0.24g$ (where $g$ denotes the acceleration due to gravity, 9.81 m/sec$^2$), soil type B (medium dense sand or stiff clay), and seismic zone II.

The influence of the concrete slabs was taken into account by modeling all beams as T-beams with an effective width, $b_{eff}$, by defining a diaphragm on each floor and by considering a dead load distributed appropriately on beams. The total mass of the building was found to equal $M_{tot} = 1589$ tonnes.
2.2 Dynamic characteristics

The period, $T$, of the building was calculated in two different ways:

1. According to the approximate equation of EC8 [2], which is called empirical period and was obtained according to the following equation:

$$T = C_t H^{\frac{3}{4}}$$

where $C_t$ is equal to 0.05 and $H$ is the height of the building starting from the foundation.

2. By applying an elastic analysis, which is called the analysis period. The analysis period resulted from modal analysis using the secant-to-yield stiffness for all the members, which was determined by section analysis. The percentage of mass participation for the 1$^{\text{st}}$ mode resulted in 85.6% and 0.28% for the x and y directions, respectively, and for the 2$^{\text{nd}}$ mode in 0.95% and 78.7% for the x and y directions, respectively.

In Table 1 the empirical and analysis periods are presented for each direction.

<table>
<thead>
<tr>
<th>Seismic Direction</th>
<th>Empirical $T$ (sec)</th>
<th>Analysis $T$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0.46</td>
<td>1.82</td>
</tr>
<tr>
<td>y</td>
<td></td>
<td>1.27</td>
</tr>
</tbody>
</table>

Table 1: Empirical and analysis periods for each direction.

2.3 Reinforcement details

Since the reinforcement of the examined structure is unknown, two assumptions for the members’ reinforcement are made: i) zero longitudinal and transverse reinforcement values and ii) minimum reinforcement according to the EC8 and EC2 requirements [2,3].

The minimum longitudinal reinforcement, $\rho_{\text{min}}$, for the beams was taken as equal to $\rho_{\text{min}} = 0.5f_{\text{ctm}}/f_{\text{yk}}$ (where $f_{\text{ctm}}$ is the mean tensile strength of concrete and $f_{\text{yk}}$ is the characteristic yield strength of the reinforcement). For the columns, the minimum longitudinal reinforcement was taken as equal to 10$\phi$20, 10$\phi$16 and 12$\phi$12 for the 0.60x0.60, 0.50x0.50, and 0.40x0.40 columns, respectively, which corresponds to 8% of their cross-sectional area. For the walls, it was taken as equal to 4$\phi$12 at the corners and 7$\phi$8 in between per 30 cm, as shown in Figure 2. The transverse reinforcement of all the members was considered to be equal to $\phi$8/250.

![Figure 2: Wall reinforcement.](image)

3 APPLICATION OF THE PROPOSED APPROXIMATE METHOD

The determination of the existing reinforcement amounts of the structural elements in RC buildings is often a time-consuming procedure. In order to minimize the time of collecting all the reinforcement information, the proposed method can be used easily for alternative assumptions of the reinforcement amounts, as it avoids this time-consuming part.
This section describes in detail the steps of the computational process of the proposed approximate method, by applying them to the examined RC building. In particular, the approximate process for determining the safety indices, including seismic demand and resistance, is described. Both assumptions of members’ reinforcement amounts, i.e., zero and minimum, are used in the calculations.

### 3.1 Determination of the seismic demand

The first step is to calculate the seismic demand, $V_{req}$, in terms of base shear force, which is determined according to the design spectrum for each direction and is defined by the following equation [2]:

$$V_{req} = M S_d(T)$$

where, $M$ is the structure mass and $S_d$ is the design spectrum acceleration at period $T$. This study is performed for the Significant Damage performance level (or Level B). The behavior factor, $q$, is obtained by KANEPE [4], depending on the performance level and the direction for which the check is being conducted. It was taken to equal $q_{B,x} = 1.70$ and $q_{B,y} = 2.30$ due to unfavorable and favorable presence of the infills in the structure for the x and y directions, respectively. Table 2 summarizes the values of the seismic demand, $V_{req}$, for the empirical and analysis period of the structure.

<table>
<thead>
<tr>
<th>Seismic Direction</th>
<th>$V_{req}$ (kN)</th>
<th>Empirical $T$</th>
<th>Analysis $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>6602</td>
<td>1813</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>4878</td>
<td>1923</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Seismic demand, $V_{req}$, for each period.

### 3.2 Approximate determination of the seismic resistance

The next step is to calculate the basic seismic resistance, $V_{R0}$, of the members of the ground floor by the following equation [1]:

$$V_{R0} = \alpha_1 \sum V_{Ri}^{columns} + \alpha_2 \sum V_{Ri}^{walls} + \alpha_3 \sum V_{Ri}^{short columns}$$

where $V_{Ri}^{columns}$ is the seismic resistance of each column, $V_{Ri}^{walls}$ is the seismic resistance of each wall, $V_{Ri}^{short columns}$ is the seismic resistance of each short column and $\alpha_1$, $\alpha_2$ and $\alpha_3$ are values that can be taken according to the proposed method as [1]:

- $\alpha_1 = 0.5$, $\alpha_2 = 0.7$, $\alpha_3 = 0.9$ in structures with columns, walls and short columns
- $\alpha_1 = 0.7$, $\alpha_2 = 0.9$ in structures with columns and walls but without short columns
- $\alpha_1 = 0.7$, $\alpha_3 = 0.9$ in frame structures without walls, and with short columns
- $\alpha_1 = 0.8$ in frame structures without walls and short columns

The examined RC structure has columns and walls, but not short columns. Therefore, $\alpha_1 = 0.7$ and $\alpha_2 = 0.9$.

In the case that the amount of reinforcement of the examined building is equal to minimum values, the strength of the vertical members, $V_{Ri}$, is obtained by:
\[ V_{RI} = \min\{V_{Rd,s}, V_{R,max}\}, V_M \]  \hspace{1cm} (4)

where the shear strengths \( V_{Rd,s} \) and \( V_{R,max} \) are obtained by EC2 or KANEPE (or similar to EC8-3) [3,4,5] and \( V_M \) is the flexural strength and is equal to \( V_M = M_R/L_S \), where \( L_S \) is obtained by KANEPE [4].

In the case that the amount of longitudinal and transverse reinforcement is being ignored, i.e. equal to zero, the strength of the vertical members, \( V_{RI} \), is obtained by:

\[ V_{RI} = \min(V_{Rd,s}, V_{R,max}) \] \hspace{1cm} (5)

where calculations are made considering zero total longitudinal reinforcement ratio \( \rho_{tot} = 0 \), zero contribution of transverse reinforcement to shear resistance \( V_w = 0 \) and the plastic part of chord rotation ductility factor \( \mu_{pl} = 1 \). In this case, the bending contribution is not taken into account.

The final seismic resistance, \( V_R \), is defined by the following equation [1]:

\[ V_R = \beta V_{R0} \] \hspace{1cm} (6)

Among the 13 criteria mentioned in the introduction of this paper, the ones that most affected the reduction factor value were: i) the normalized axial load with the maximum value on the center column (K4) of the structure and with the criterion grade equal to \( \beta_3 = 4 \), ii) the stiffness distribution in plan – torsion with the criterion grade equal to \( \beta_5 = 1 \), and iii) the stiffness distribution in elevation with the criterion grade equal to \( \beta_7 = 3 \). The reduction factor resulted in \( \beta_x = 0.81 \) and \( \beta_y = 0.83 \) for the x and y directions, respectively. In Table 3, the basic \( (V_{R0}) \) and the final \( (V_R) \) seismic resistance of the vertical members of the ground floor are presented, for each assumption of the reinforcement amounts and seismic direction.

<table>
<thead>
<tr>
<th>Seismic Direction</th>
<th>Total strength</th>
<th>Ignoring Reinforcement</th>
<th>Minimum Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>Basic ( V_{R0} ) (kN)</td>
<td>1628</td>
<td>1513</td>
</tr>
<tr>
<td></td>
<td>Final ( V_R ) (kN)</td>
<td>1319</td>
<td>1226</td>
</tr>
<tr>
<td>y</td>
<td>Basic ( V_{R0} ) (kN)</td>
<td>1928</td>
<td>2006</td>
</tr>
<tr>
<td></td>
<td>Final ( V_R ) (kN)</td>
<td>1600</td>
<td>1665</td>
</tr>
</tbody>
</table>

Table 3: Seismic resistance \( V_{R0} \) and \( V_R \) of the vertical members according to the proposed method.

### 3.3 Determination of the safety index

Finally, the last step is to calculate the safety index, \( \lambda \), for each direction, without taking into account the effect of the transverse direction, using the following equation [1]:

\[ \lambda = \frac{V_{req}}{V_R} = \frac{V_{req}}{\beta V_{R0}} = \frac{\lambda_0}{\beta} \] \hspace{1cm} (7)

In Table 4, the final safety indices are presented, which were calculated for performance level B, using the empirical and analysis period, for the assumptions of ignoring and accounting for minimum reinforcement amounts.
<table>
<thead>
<tr>
<th>Seismic Direction</th>
<th>Ignoring Reinforcement</th>
<th>Minimum Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Empirical $T$</td>
<td>Analysis $T$</td>
</tr>
<tr>
<td>$x$</td>
<td>5.01</td>
<td>1.38</td>
</tr>
<tr>
<td>$y$</td>
<td>3.05</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Table 4: Safety indices of the proposed approximate method for ignoring and accounting for minimum reinforcement amounts.

From Table 4, the high influence of the period assumption on the calculated safety indices can be observed. The use of the empirical period in the calculations results in 3-4 times higher safety indices than the ones resulted from the use of the more accurate period (i.e. analysis period). Moreover, the examined assumptions for the existing reinforcement (ignoring or not) resulted in quite similar safety indices values. Differences between them did not exceed 10%.

In the next section, in order to check the above results use is made of the more accurate non-linear static analysis method, for the assessment of the same building.

4 APPLICATION OF THE NON-LINEAR STATIC ANALYSIS METHOD

In order to validate the proposed method, the non-linear static analysis method is employed. This method, also known as pushover analysis, is a widespread method used to evaluate and redesign old buildings, as well as to design new ones. A significant advantage of the method versus approximate methods is the ability to simulate with high accuracy the inelastic behavior of members through a stress-strain (or force-displacement) diagram.

The modeling of the building, i.e., beams, columns and walls, was made according to KANEPE [4] with frame elements with plastic hinges at their ends, using the finite element software SAP2000 [6]. All cross-sectional data, i.e. yield moment, axial force and moment interaction, curvature and chord rotation angle, were calculated using an appropriate cross-section analysis tool [7].

The mechanical behavior of the structural elements was described by a force-displacement diagram. This behavior is defined at the two ends of each member (plastic joints). In this work, all members were modeled to account for flexural failure based on the bending moment-chord rotation, $M - \theta$, relationship (Figure 3). Moreover, for columns and walls, the bending moment-axial load, $M - N$, interaction was taken into account, while for beams this axial force was assumed to equal zero. The Π-shaped shear wall (elevator) was modeled by three frame-elements with dimensions of $3 \times 0.25$ m that were appropriately connected to each other with rigid elements.

![Figure 3: Capacity curve in terms of $M - \theta$.](image-url)
The three performance levels identified by KANEPE and EC8-3 [4,5] are the following: Damage Limitation or Level A, Significant Damage or Level B and Near Collapse or Level C. The limits of each performance level are defined as a function of the deformation value, \( \delta_d = \theta \), of the members, where \( \theta \) is the chord rotation. These limits are obtained according to the following equations of KANEPE [4]:

\[
\begin{align*}
\delta_d &= \delta_y, \text{ for the performance level A} \\
\delta_d &= \frac{0.5(\delta_y + \delta_u)}{\gamma_{Rd}}, \text{ for the performance level B} \\
\delta_d &= \frac{\delta_u}{\gamma_{Rd}}, \text{ for the performance level C}
\end{align*}
\]

(8)

where \( \delta_y \) is the yield deformation, \( \delta_u \) is the ultimate deformation and \( \gamma_{Rd} \) is a partial safety factor.

In this paper, the results will be demonstrated only for the Significant Damage performance level (Level B) due to space limitations.

A representative model of the building is shown in Figure 4.

![Figure 4: Sections along the x and y directions and 3D model view.](image)

4.1 **Determination of seismic resistance and seismic demand**

In this paper, two alternative ways are used to determine the seismic resistance of the whole structure. In the first one (Local definitions), the maximum resistance of the structure is defined when one vertical element reached the maximum acceptable deformation for the examined performance level. In the second one (Global definitions), the maximum resistance of the structure is defined through Eqs. (8), where \( \delta_y \) and \( \delta_u \) are defined by linear approximation of the capacity curve of the building as shown in Figure 5.

Figure 5 shows the shear force-displacement curve resulting from the non-linear static analysis, for the x and y directions. This curve is appropriately linearized according to KANEPE [4], i.e. converted into two straight lines. The yield displacement, \( \delta_y \), is considered as the point in which the first failure in one vertical member occurs, i.e. the first exceedance of the performance level A for local and global values. The local ultimate displacement, \( \delta_u \), is considered as the point in which the maximum limit of the performance level C is exceeded, and the global ultimate displacement \( \delta_u \) is considered as the point in which the maximum base shear force is achieved. In Figure 5, only the local and global limits of the performance level B are shown.
In Table 5, the displacement and base shear force acceptable limits, $\delta_{lim,B}$, and, $V_{lim,B}$, respectively, of performance level B are presented.

<table>
<thead>
<tr>
<th>Damage</th>
<th>Seismic Direction</th>
<th>$\delta_{lim,B}$ (m)</th>
<th>$V_{lim,B}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>x</td>
<td>0.085</td>
<td>1210</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>0.103</td>
<td>1884</td>
</tr>
<tr>
<td>Global</td>
<td>x</td>
<td>0.114</td>
<td>1352</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>0.195</td>
<td>2310</td>
</tr>
</tbody>
</table>

Table 5: Seismic resistance for performance level B in terms of deformation and base shear force.

The seismic demand, $V_{req}$, is determined according to the design spectrum for each direction and is defined by Eq. (2). In this case, the behavior factor $q$ is obtained by the shear force-displacement curve, which resulted from the non-linear analysis, as the ratio of the ultimate shear force to the yield shear force and is equal to $q_x = 1.51$ and $q_y = 1.56$ for the x and y directions, respectively. Finally, the seismic demand for the non-linear analysis resulted in $V_{req,x} = 7483.3$ kN and $V_{req,y} = 7170.5$ kN using the empirical period and $V_{req,x} = 2042$ kN and $V_{req,y} = 2837$ kN using the analysis period, for the x and y directions, respectively.

4.2 Safety indices of the non-linear static analysis

The safety indices of the non-linear static analysis are defined in two different ways: a) based on the base shear force and b) based on the displacement. Both ways refer to local and global definitions.

For the 1st case, the safety index, $\lambda$, for each direction is defined as follows:

$$\lambda = \frac{V_{req}}{V_s}$$  \hspace{1cm} (9)

where $V_{req}$ is obtained using the behavior factor which was calculated as described in section 4.1 and $V_s = V_{lim,B}$ is the base shear force and is obtained from the shear force-displacement curve of
the non-linear analysis depending on the performance level being examined. For performance level B, the values of \( V_S \) are presented in Table 5. The safety indices are found using the local and global definitions and are indicated in this paper as Force Local Values (FLV) and Force Global Values (FGV), respectively.

For the 2\textsuperscript{nd} case, the safety index, \( \lambda \), for each direction is defined as follows:

\[
\lambda = \frac{\delta_t}{\delta_d}
\]  

(10)

where \( \delta_t \) is the target displacement calculated according to KANEPE [4] using only the analysis period of the building and \( \delta_d = \delta_{lim,B} \) is the acceptance limit of the examined performance level in accordance with KANEPE [4]. The safety indices are found using the local and global definitions and are indicated in this paper as Displacement Local Values (DLV) and Displacement Global Values (DGV), respectively.

In Table 6, the final local and global safety indices of the non-linear static analysis are presented, which were calculated for performance level B and based on the base shear force and displacement, using the empirical and analysis period.

<table>
<thead>
<tr>
<th>Damage</th>
<th>Seismic Direction</th>
<th>Base shear force</th>
<th>Displacement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Empirical ( T )</td>
<td>Analysis ( T )</td>
<td>Empirical ( T )</td>
</tr>
<tr>
<td>Local</td>
<td>x</td>
<td>6.18</td>
<td>1.69</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>3.81</td>
<td>1.51</td>
<td>0.60</td>
</tr>
<tr>
<td>Global</td>
<td>x</td>
<td>5.53</td>
<td>1.51</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>3.10</td>
<td>1.23</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 6: Local and global safety indices of the non-linear static analysis based on the base shear force and displacement.

From Table 6, large differences between the results from the empirical and the analysis period can be observed, as in Table 4. Here, the empirical period is used only for comparison purposes, because the use of non-linear analysis methods implies that the more accurate period (i.e. the analysis period) can be easily calculated and hence used for the analyses.

5 RESULTS COMPARISON

This section presents comparisons between the approximate and the more accurate non-linear analysis method, in the form of diagrams. Figures 6 to 8, present the ratios \( \rho = V_{approximate}/V_{analysis} \) and \( \varepsilon = \lambda_{approximate}/\lambda_{analysis} \), indicating the ratio of the approximate to the accurate method for the seismic resistance and the values of the safety indices, respectively. The cases near to unity indicate that the results of the approximate and the accurate methods are close. The seismic resistance of the members as resulting from the approximate method is calculated on the basis of the following assumptions: a) ignoring the existence of the reinforcement, i.e. zero amounts of longitudinal and transverse reinforcement and b) taking into account the minimum reinforcement amounts. On the other hand, the seismic resistance of the members for the accurate method is calculated by taking into account only the minimum reinforcement amounts because it is impossible to do the analyses with zero reinforcement.
The indicators of Figures 6 to 8, i.e. FLV, FGL, DLV and DGV represent the cases of the non-linear analysis as described in section 4.2 of the present study, which were compared to those of the proposed method. All the results refer to the use of the empirical and analysis period of the structure.

5.1 Seismic resistance

Figure 6 presents the ratio $\rho$ of the seismic resistance resulting from the approximate and the accurate methods, for the x and y directions. It can be observed that the ratio $\rho$ ranges from almost 0.7 to 1.1. Generally, the seismic resistance of the structure resulting from the proposed method appears to be in quite good agreement with the respective ones obtained from the non-linear analysis, as all cases are quite close to unity. It appears that higher accuracy is achieved when the local definition of the safety index is used (FLV case).

![Figure 6. Ratio $\rho$ for the seismic resistance for a) x direction and b) y direction.](image)

5.2 Safety indices

Figures 7 and 8 demonstrate the ratio $\varepsilon$ of the safety indices resulting from the approximate and the accurate methods, for the x and y directions and for the empirical and analysis period, respectively.

From Figure 7, it can be observed that the ratio $\varepsilon$ ranges from almost 0.8 to 1.0. Moreover, it can be seen that higher accuracy is achieved when comparing the proposed method with the global values of the non-linear analysis (FGV case). Using the empirical period of the structure, it is quite clear that the results of the proposed method, for both assumptions of the reinforcement amounts, are very close to the ones obtained by the base shear force of the non-linear analysis. This is because, in contrast to displacements, forces are not so sensitive to the stiffness (or period) assumption.
From Figure 8, it can be seen that the ratio $\epsilon$ ranges from almost 0.4 to 1.0. Moreover, it can be observed that higher accuracy is achieved when comparing the proposed method with the global values of the non-linear analysis based on the base shear force (FGV case). On the other hand, when comparing the proposed method with the values of the non-linear analysis based on the displacement (DLV and DGV cases), the two methods have great differences. Using the analysis period of the structure, it is quite clear that the results of the proposed method, for both assumptions of the reinforcement amounts, are very close to the ones obtained by the base shear force of the non-linear analysis.

6 CONCLUSIONS

In this study, a new approximate method for the assessment of RC buildings proposed by EPPO (Earthquake Planning and Protection Organization of Greece) has been demonstrated through a realistic application on a RC building and validated via non-linear static analysis. This method simplifies the assessment procedure, while it can also deal with the problem when a building’s reinforcement information is unknown. When comparing the results of both methods, the following conclusions can be drawn:
• The seismic resistance of the structure which resulted from the proposed method appears to be in quite good agreement with the respective ones obtained from the non-linear analysis, with the best results obtained when the local definition of the safety index was used.

• Examined assumptions for the existing reinforcement (ignoring or not) result in quite similar safety indices values. Differences between them did not exceed 10%. Furthermore, the results were found very close to those obtained from the non-linear analysis when considering the seismic capacity in terms of base shear, and a higher accuracy can be observed when the global definition of the safety index was used.

The same degree of accuracy was not found when indices were defined through the deformation capacity of the structure. However, it is worth noticing that also in this case, the global definition of the safety index results in better accuracy than when the local definition was used.

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ENGINEERING PRACTICE FOR SEISMIC REHABILITATION AND STRENGTHENING OF EXISTING BUILDINGS IN BULGARIA

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Abstract

The Rehabilitation and Strengthening of existing structures represent very significant part of the contemporary construction practice. One of the main problems in the case of Rehabilitation and Strengthening is related to the accepted degree of the interventions, the necessity to reach the safety of the new buildings and the evaluation of the risk, in case of lack of interventions for upgrading the performance of the building.

The position of the country in one of the most seismically active areas imposes on Bulgarian structural engineers to prove the seismic structural safety for both the newly designed and the existing structures. The existing national technical regulations in Bulgaria are mainly turned on new structures. Only particular articles in the codes give some general recommendations which are not enough for a detailed and correct design. The Eurocode 8-3 is the other possibility for the design of the Rehabilitation and Strengthening but the problems in the application of this standard create serious difficulties in the design practice.

The paper makes an overview of the common engineering practice in Bulgaria and presents the full process of seismic Rehabilitation and Strengthening, starting with the procedure of seismic assessment. The most popular strengthening techniques in the Bulgarian practice are presented as well.

The analysis is based on several realistic project situations that make possible to identify and present the main difficulties and problems in the engineering practice.

Using these specific examples, some key findings and recommendations are made.

Keywords: Assessment, Rehabilitation, Strengthening, Earthquake Engineering.
1 INTRODUCTION

The contemporary trends in the construction sector include the increasing rate of adaptation and modernization of existing structures compared with construction of new structures. The design approaches for Rehabilitation and Strengthening of existing buildings have an important contribution to guaranteeing the high quality of the life with respect of the contemporary requirements. The process of Rehabilitation and Strengthening of existing buildings must lead to a selection of realistic, practical and cost-effective techniques to mitigate the deficiency of the existing buildings.

Finding the potential for future development of existing buildings is an important task of the contemporary construction sector. It is necessary to clarify all the doubts concerning the actual reliability of the existing structures and to make an analysis of existing structures to identify their possibilities for meeting sustainability goals. The most significant parameters for evaluation of the quality of the buildings and the comfort of occupancy could be summarized in the following:

- possibilities for different operational changes and rehabilitation of the building;
- possibilities for increasing the comfort of occupancy and the extension of the remaining lifetime of the buildings;
- expected level of security and durability of the structure;
- thermal, waterproofing and soundproofing insulations;
- operational quality of the building installations and equipment- water supply system and sewerage system, heating system, lift, staircases, electric system, gas system etc.

The target of the design is to fulfill the following requirements according to article 169 of [1]: mechanical resistance and stability, safety in case of fire, hygiene, safety and accessibility in use, protection against noise, energy economy and heat retention, sustainable use of natural resources. To reach these requirements it is necessary to provide:

- the improvement of the structural behaviour;
- the upgrading of the technical systems in the buildings;
- the upgrading of the finishes of the floors and the walls with new materials;
- the improvement of the waterproofing and soundproofing;
- the renewal of the facades (new coverings, adequate thermal insulation for maximum energy efficiency) and the entrances (possibilities for handicapped people), etc.

The most important structural problems could be summarized in the following:

1) Low material characteristics of the original structure;
2) Poor detailing of the existing elements;
3) Lack of structural system for seismic actions;
4) Previous inappropriate interventions;
5) Insufficient information about the existing structure and obstacles for its acquiring;
6) Conflicts with the architectural design;
7) Technological issues, related to changes of the structural elements and changes of the loading;
8) Different types of damages: damages related to the structure, to the facades, to the roof, in the staircases, in the basement, in the technical systems, in the vertical planning around the building etc.;
9) Deterioration of the structural behavior and the material characteristics.

Concerning the rehabilitation and strengthening of existing buildings the following regulations in Bulgaria are available: Territory Planning Act [1], Ordinance N 5 on technical passports of buildings [2], Ordinance РД 02-20-2 for the design of buildings and facilities in seismic areas [3] and BDS EN 1998-3:2005 [4].
Article 137 of [1] gives 6 categories of the construction sites according to their importance factor. The most significant are the sites category 1. All these categories include not only the new sites but also the activities for Rehabilitation and Strengthening of the corresponding category. The Ordinance N 5 is the regulation giving instructions for the elaboration of the technical passports including the existing buildings. The content of the technical passport is: A – Technical characteristics of the site (building); B – Construction measures and deadlines; C – Instructions for safe operation. For the existing buildings the elaboration of the technical passport is related to article 169 of [1] and the special part for the mechanical resistance and stability of the structure. The procedure of the assessment is described in details in [2]: analysis of the available documentation, as-built documentation, damages, checks for the capacity of the structure. This procedure is very close to the requirements of [4] and [3]. Finally, the assessment should provide the prescriptions for rehabilitation and strengthening and the deadlines for these activities on the site.

2 PROCEDURE OF SEISMIC ASSESSMENT. MOST POPULAR STRENGTHENING TECHNIQUES IN BULGARIAN CONSTRUCTION PRACTICE

The most important cases for seismic assessment, rehabilitation and strengthening of the structures are related to: operational changes in the loading or the equipment of industrial buildings, problems in the structure, changes in the structure, construction of a new stories, convert in compliance with the contemporary requirements, etc.

The process of assessment and design, the possible solutions and recommendations are summarized in Table 1.

<table>
<thead>
<tr>
<th>Purpose of the assessment and the design</th>
<th>Structural issues</th>
<th>Possible solutions</th>
<th>Recommendations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purpose of the assessment and the design</td>
<td>Structural issues</td>
<td>Possible solutions</td>
<td>Recommendations</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>------------------</td>
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<td>-----------------</td>
</tr>
<tr>
<td>3. Functional and technological changes (changes in occupancy, new equipment)</td>
<td>1. Possibilities of the existing structure to meet the new function or the new equipment 2. New values of the loading 3. Seismic Performance 4. Strengthening</td>
<td>1. Choice of the most appropriate strengthening</td>
<td>1. If very extensive strengthening is necessary it is better to cancel the proposed changes 2. Different numerical models for the different variants of strengthening</td>
</tr>
<tr>
<td>4. Fixing of damages and defects (reduction of the bearing capacity, mistakes in the design or and the construction, unappropriated modifications)</td>
<td>1. Degree of the damages/defects 2. Rehabilitation or and Strengthening 3. Techniques for Rehabilitation and Strengthening</td>
<td>1. Choice of contemporary techniques</td>
<td>1. Different variants of the techniques for the design 2. Analysis of the advantages and disadvantages of the techniques for restoration and strengthening</td>
</tr>
<tr>
<td>5. Upgrading the performance</td>
<td>1. Possibilities of the existing structure 2. Reliability of the existing structure</td>
<td>1. New structure 2. Upgrade of the existing structure</td>
<td>1. Different numerical models should be studied and it is necessary to make a comparison of the obtained results</td>
</tr>
<tr>
<td>6. Changes of the facilities of the building (water supply, sewerage, air conditioning, heating systems, etc.)</td>
<td>1. Interventions on structural elements</td>
<td>1. Upgrading of the existing facilities</td>
<td>1. Application of contemporary systems 2. Minimizing the interventions on structural elements</td>
</tr>
<tr>
<td>7. Adaptation and re-use of existing buildings</td>
<td>1. Condition of the existing structure 2. Type of the existing structure</td>
<td>1. Upgrading the existing structure 2. Implementation of a new structure</td>
<td>1. Minimizing the interventions 2. It is not necessary to reach the safety level of the new built structures</td>
</tr>
</tbody>
</table>
The contemporary development of the retrofitting techniques gives the possibility to make the choice between many different variants. The development of the construction materials and the new technologies provide many opportunities.

For the RC and masonry structures (predominant types of existing structures in Bulgaria), the most popular retrofitting techniques in the national practice could be summarized in the following variants:

2.1 **Strengthening with reinforced concrete**

The method gives the possibility of relatively very easy connection between the new and the old concrete. Using reinforced concrete leads to an increasing of the bearing capacity and the stiffness of the elements and the structure. The concrete jacket, surrounding the whole section, gives a significant increasing of the deformation capacity of the element. The techniques with new shear walls for seismic upgrade improves the regularity of the whole structure. With the RC jacket improves also the behavior of the element of the case of deficiency in the lapping length.

This method of retrofitting requires the drilling of many holes for the execution of the jacket, necessity of scaffolding and special formwork. The construction process has a long duration and usually all finishes should be removed. Special attention for the interface mechanisms (adhesion, friction, dowels) should be carried out.

2.2 **Strengthening with steel**

The strengthening with steel is a relatively easy technique for execution, needs minimum wet processes and minimum interventions on finishes. The method is not so invasive for the architectural solution. The techniques for retrofitting with steel give the possibility for reducing some deformation using the prestressed steel. With steel jackets and braces the possibility for increasing the bearing capacity and the stiffness of the elements and the structure is possible. A very big advantage is the possibility for limitation of deformations in the transvers direction of the cross section and as a result to improve the ductile behavior –

<table>
<thead>
<tr>
<th>Purpose of the assessment and the design</th>
<th>Structural issues</th>
<th>Possible solutions</th>
<th>Recommendations</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. Interventions on heritage buildings</td>
<td>1. Low material characteristics</td>
<td>1. New structure to carry out the loadings</td>
<td>1. Minimizing the interventions</td>
</tr>
<tr>
<td></td>
<td>2. Non-engineered structures</td>
<td>2. New materials and techniques</td>
<td>2. Avoiding forgery of history: avoidance of speculative restoration or reconstruction</td>
</tr>
</tbody>
</table>

Table 1: Structural issues of the assessment and the design.
very significant for seismic performance. The steel braces provide also the possibility to improve the regularity of the structure.

The techniques based on steel need special measures against corrosion and fire. Sometimes some aesthetic conflict can occur. The application of steel jackets is a very popular technique but needs a special attention for the connection between the RC element and the steel jacket.

2.3 Strengthening with Fiber Reinforced Polymers (FRP)

The Fiber Reinforced Polymers (FRP) are one relatively contemporary method for rehabilitation and strengthening. The FRP materials have: high resistance in case of corrosion, high tensile strength and stiffness. They give an insignificant increasing of the self-weight and the size of the elements. The FRP have a very big diversity of geometry and size of the material. As a technique for retrofitting, it is a very easy and very fast way for strengthening without additional scaffolding and formwork. Very suitable for seismic strengthening because of the insignificant self-weight.

The FRP is an anisotropic material and has a significant reduction of the strength and the deformation capacity in alkaline conditions or in the case of high temperatures (fire). The behavior is typical linear behavior up to the ultimate deformations followed by brittle failure.

The strengthening with FRP increases the shear resistance and ductility of the elements, but without a serious contribution to the deflections. Usually there are specific requirements for application and a minimum grade of the concrete is required.

Many other technologies appear in the construction process (UHPFRC, seismic isolations, dampers, etc.) and sometimes it is very difficult to find the correct solution. How to make the choice of the appropriate solution? For the decision is necessary to take into account: the proper management of the natural resources such as timber, water, minerals etc.; the possibility to use renewable and recyclable materials – artificial, light weight aggregates, recycled aggregates, soil cement bricks, the development of technologies and strategies environmental friendly, the energy efficiency in concrete. The use of local materials, the use of nanomaterials as silica fume to improve the strength and the durability, the use of waste materials such as fly ash, slag is preferable.

3 EXAMPLES FROM THE NATIONAL PRACTICE

The following examples illustrate realistic project situations that make possible to identify and present the main difficulties and problems in the engineering practice.

3.1 Strengthening of the structure of industrial building because of technological changes [5]

The considered building is a warehouse for bulk materials and represents a part of a group of industrial buildings from the 70ies of the XX century.

For technological purposes it is necessary to make openings in the walls. For the realization of this project a part of the wall panels and two vertical steel braces should be removed. The removal of the panels and the braces is directly related to the global behavior of the building. New steel braces are provided for construction in other spans of the frames of the buildings.

The main structure consists of two spans frames in transverse direction and six spans frames in longitudinal direction with steel braces (axis I, VIII and Ж) (Fig.1). The span in longitudinal direction is 6m. The frame is composed by vierendeel columns till the level of the crane on axis I and VIII, RC columns with rectangular cross section on axis Ж till the lev-
el +19,20 and steel columns till the level +26,80. The roof structure is a steel truss structure with braces and purlins.

The new design solution requires to analyze the strengthening after the technological changes. For the purpose of the project, four numerical models are considered:
- M1 – the as-built situation (Fig.2, 3, 4);
- M2 – the structure after the removal of the panels;
- M3 – the structure after the removal of the panels and one of the vertical braces;
- M4 – the structure after the removal of the panels and one of the vertical braces and the addition of one new brace at different position (Fig.3,4).

The modal analysis shows that the planned removal of the panels do not decrease significantly the global stiffness of the building. In the opposite, the removal of the vertical steel braces increases the first period with 30 % and the execution of new braces will restorate the initial behavior.

The displacements in longitudinal direction increase by 20% after the removal of the panels, but after the removal of the braces, the displacements increase 3 times! The addition of the new braces restores the initial stiffness. For all these reasons, the removal of the panels has no influence on the global behavior of the building, but the removal of the braces should be executed after the placement of the new braces.

However, the real modernization should be in conformity of the actual regulations and standards but also in conformity of the actual condition of all structural elements. A detailed assessment is necessary for the final decision.

Figure 1: Typical cross section of the structure.

Figure 2: Model M1: a) View of the model; b) Section cut
3.2 Strengthening of existing sportive hall after expansion

The building of an existing sportive hall as a part of an existing school gymnasium is considered. The size in plan is 10,40 / 12,35 m. The structure consists of timber roof, supported by brick masonry walls. The building has a basement and the brick masonry is transformed in stone masonry in the basement. The vertical elements inside of the building are RC columns.

The modernization of the building represents an elongation in plan from 12,35 m to 18,85 m. The roof is completely redesigned. The analysis of the existing structure shows that the structure has enough capacity for gravity loads, but no sufficient capacity for seismic actions. The execution of vertical elements inside of the hall is not possible. All these peculiarities must be taken into account for the new design.

Three variants for strengthening are proposed (Fig.5): Variant 1 – Design of new RC frame in the existing masonry walls with a new steel structure for the roof, Variant 2 – Design of new steel frame with a new steel structure for the roof and Variant 3 – Design of new RC frame out of the plane of the existing masonry walls and new steel structure for the roof.
The advantages of the first variant are the preservation of the existing brick masonry and the strengthening of the masonry with the new RC elements. The disadvantage is the very difficult implementation of the new structure in the existing structure.

The second variant is very easy for execution and the time for the construction is shorter, but in this case the removal of the existing masonry walls is a serious problem.

The third variant is good solution because of the preservation of the existing masonry. The new RC frame in an outer plane makes the masonry vulnerable for seismic actions.

To make the choice of the most appropriate solution a special procedure for assessment is proposed. The assessment of the variants is based on the following criteria: regularity in plan, regularity in elevation, results from the modal analysis, maximal displacement, behavior of the non-structural elements, management of the construction stage and price of the variant.

3.3 Rehabilitation of the complex RC shell roof structure of an industrial building [6]

This case study presents the investigation and design of the rehabilitation of a storage structure for the cement industry and in particular of its roof (Fig. 6). The storage was designed and constructed within a two-year period between 1956 and 1958 as part of a cement plant in the northern part of Bulgaria. The storage is approximately 500 m long and is divided into seven separate structures by expansion joints.

![General view of the building](image)

Figure 6: General view of the building

The main structure is a single-span frame in transverse direction and multiple-span frames in longitudinal direction. The structural span width in transverse direction is 33 m, and the span width in longitudinal direction is generally 6 m. The roof structures of three of the storage blocks are steel trusses with a cover made of trapezoidal steel sheets. The roof of the other blocks is a reinforced concrete (RC) shell structure (Fig. 7). The RC shells span 33 m in transverse workshop direction and 6 m in the longitudinal direction. The shell structure is thin, elegant and unique for Bulgaria. It represents efficient implementation of the theoretical advantages of shell structures. The assessment includes visual observation, laboratory testing of the materials both on site and on samples, and computer modeling of the entire structure and its parts. Calculations and design checks were carried out according to the present-day codes that are valid in Bulgaria. Even so, the roof structure theoretically displayed perfect behavior. That behavior contrasts sharply to the real condition of the roof. The designers team considered two options—to replace the RC shell roof by means of steel trusses or to preserve the original roof structure. The owner chose to preserve the original structure.

Computer modeling of the entire storage building and a separate model for the roof alone were created by means of SAP2000 software. The results reveal the amazing behavior of the roof. The bending moments in the shell structure are practically zero for vertical loading due to self-weight, dust and snow. The above considerations led to the choice of a most “classic”
method for reinforcement, which appeared most suitable for the project. A layer of small-size aggregate concrete with a single rebar mesh on a cleaned and rough surface was applied.

![Figure 7: The roof from inside of building](image)

### 3.4 Redesign of constructed pile foundations of industrial facility related to the change of equipment [7]

The case study considers the main issues related to the redesign of constructed before few years reinforced concrete pile foundations for special facilities for purifying water in the region of city of Sozopol, Bulgaria. The original design has been changed and new equipment is provided. The loads are significantly increased. The new situation requires serious analysis, calculation and redesign of the structure. Different numerical models are created.

![Figure 8: 3D view of the model](image)

During the time of construction of the supporting system a decision for changing the equipment is taken. The purpose of the investigation is to answer the question if the constructed supporting structure is able to resist the loading from the new equipment for waste water treatment. The new mechanical devices with their corresponding equivalent loading are positioned according to the presented layout and transfer to the foundation the denoted characteristic loads. A finite element method numerical modeling is carried out by means of SAP2000 software (Fig.8). The slab is modeled by area (shell) finite elements and the piles – by linear (Frame) elements. The main problem is to model the structural support on the soil. The elastic characteristic of the soil is reflected in the model by springs. Several numerical models are developed for the clarification of the real soil conditions. The precise modeling is
necessary just for the final conclusions concerning the real bearing capacity of the piles and the necessity of strengthening.

4 CONCLUSIONS

- The process of rehabilitation and strengthening of existing buildings requires a deep knowledge on construction history, standards and strengthening techniques.

- The existing national technical regulations in Bulgaria are mainly turned on new structures. The presented review show that only particular articles in the codes give some general recommendations which are not enough for a detailed and correct design. The Eurocode 8-3 is the other possibility for the design of the rehabilitation and strengthening but the problems in the application of this standard create serious difficulties in the design practice.

- When the load path or the static scheme of the existing structure is not clear, full finite element model of the structure may be the best solution for the assessment. Several different numerical models should be developed.

- The final decision whether or not structural elements should be strengthened should be made not only based on the results from the numerical assessment, but also based on the actual state of the structure. Sometimes the numerical results show that the design checks are satisfied in general but the actual condition of the elements is crucial for the final decision for strengthening. The presented different realistic projects provide diverse approaches from the Bulgarian practice.

- In order to use the remaining capacity of the existing structure, rehabilitation should be undertaken before any strengthening interventions. Without this step, the adequacy of the final design solution is questionable.

- The choice of strengthening materials is dependent on the characteristics of the existing materials. Sometimes, however, technical limitations are the leading factor when choosing strengthening materials. In such cases verification of the new-old material connection is mandatory.

- The decision-making for the most suitable strengthening technique should be made on the basis of selected technical, socio-economical and architectural criteria that are predetermined. Weight factors should be assigned to each criteria, depending on the priorities of the decision-maker.

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Abstract

During the last decade, a set of new Structural Codes is in force, regarding the assessment, the redesign and the structural interventions (repair/strengthening) of reinforced concrete (RC) and plain/unreinforced (PM) buildings. As it is known, there is a strong influence and interrelation between the European Codes, and especially the more general EC8-3:2005 and the more specific (and lengthy) Greek ones (nGC’sSI). Nevertheless, there is a number of considerable (and rather strong) differentiations between the two sets of these modern Structural Codes, regarding relevant and interrelated aspects, as follows:

a) Investigation and documentation, collection and calibration of informational and experimental data, knowledge levels, basic assumptions, performance levels, reliability aspects, target behaviour etc,

b) Detailed structural analysis, checks, computational methods (linear & non-linear), estimation of action effects and of resistances (in terms of forces or of displacements),

c) Assessment and Redesign, and
d) Decision making, methods of interventions (techniques and materials) etc.

In this paper, a short but critical review of similarities and differentiations (with relevant comments) is attempted, while emphasis is given on future mutual decisions for an almost full harmonisation (at least form a practical point of view), regarding principles, provisions and application rules.

Keywords: Structural Codes, Assessment, Redesign, Models, Structural Interventions, Repair, Strengthening.
1 INTRODUCTION

Since existing old reinforced concrete (RC) and plain masonry (PM) buildings: i) Reflect the state of knowledge and of “traditional” building rules at the time of their (design and) construction, ii) Possibly contain hidden and combined gross errors (in their conception and/or construction, not to mention alterations, additions etc, and iii) Have been submitted to various actions during their life-time, including earthquakes (or other accidental actions), with unknown effects, structural evaluation and intervention (if needed) are typically subjected to a different and more complex level and procedure of design than that of new buildings, combined with a different degree of uncertainty.

To this end, different sets of performance levels and of material or structural safety factors are required, as well as different analysis, behaviour models and checking procedures (verifications), depending on the completeness and reliability of the structural information available, as foreseen by the framework of new Codes for assessment and retrofitting ([1] to [3] and especially [4] to [6]).

For many older buildings, seismic designs and evaluations in accordance with the present state of knowledge usually indicate the strong need for upgrading, while damages caused by earthquakes create the need for major and costly interventions (among other consequences).

Nevertheless, and although the principles and the provisions of the relevant Codes ([1] to [6]) are applicable to all categories of buildings, the (seismic) assessment and retrofitting of monuments and of traditional/historical buildings often requires different (and more strict) types of provisions and approaches, depending on the nature and the importance of the monuments.

2 PRINCIPLES AND MAIN PROVISIONS

The two sets of Codes, the European and the Greek one, are almost fully harmonised (as far as principles and provisions are concerned), while the new GC’sSI are detailed and lengthly, containing a lot of (supplementary and non-contradictory) application rules, for both the assessment and the retrofitting of existing buildings.

The first critical step (see next §3) is the collection and calibration of structural data, both informational and experimental (in situ and in lab), combined with a characterisation of their uncertainty and followed by a decision on the foreseen (or the targeted) performance level (PL), i.e. the tolerable degree and extent of damage for an expected earthquake action, possibly lower or higher than that of new buildings, designed in accordance with Codes [1] to [3], see next §4.

The next step is the detailed analysis of the building (followed by global and local safety verifications, see next §§5 and 6), for both situations:

- As it is, probably damaged, see next §7, and
- As it will be retrofitted and upgraded, see next §§8 and 9, possibly studying different schemes of structural interventions.

The final step is decision making, combining the efficiency of the approved scheme of interventions, its consequences, the cost and time needed etc. To this end, emphasis should be given on the urgency of structural retrofitting, depending on the level of reduced resistances and of damages (if any), see the Appendix at the end of this paper.

Note

The basic and innovative distinction between primary and secondary seismic elements for a new building (see [3]) is in force for existing buildings as well. According to the GC’s, the percentage (on terms of resistance) of secondary elements over total elements (primary and secondary) is 25%, instead of 15% according to the EC’s. In addition, there exists the distinc-
tion between “ductile” and “brittle” structural elements, to be checked/verified in terms of
displacement or of force, respectively.

3 COLLECTION AND CALIBRATION OF DATA

Instead of the rather overall knowledge level (KL), from limited to full (?), according to
the EC’s, and the corresponding confidence factor (CF), a more detailed uncertainties’ ap-
proach is followed according to the GC’s, that of data reliability level (DRL), from limited to
comprehensive, influencing not only the analysis to be followed but all partial safety factors
as well, regarding geometry (e.g. lengths, heights etc), actions (e.g. self weight), materials,
structural details and models (of both damaged and retrofitted elements), unavoidable con-
tribution of action effects (after damages and after interventions), final checks and verific
ations (depending on the target) etc.

Therefore, rather lengthy and early chapters of the GC’s are devoted on investigation and
documentation of the structural system (including the foundation and the subsoil), its compo-
nents and its elements, with their structural details and their materials. To this end, and be-
sides detailed inspection and measurements, collection of information etc, there are normative
provisions for testing of materials, for each type of primary seismic elements (mainly vertical),
mainly non-destructive combined with a minimum of destructive testing.

Finally, emphasis is given on the need for calibration and critical (engineering) cross-
checking of all data collected from different sources (including older Codes, similar buildings,
drawings and structural details etc), in order to minimise uncertainties and maximize data re-
liability level, influencing almost all steps and phases of assessment and retrofitting, not to
mention the final feasibility as well as cost/time issues. As an example, critical structural data
are the following:

- For RC buildings, the reinforcement layout and detailing (of primary seismic elements),
  including lap splices, arrangement and anchorage of stirrups and ties etc., not to mention
  the masonry infills (their presence, details, global and local influence etc.), and
- For PM buildings, the building details of walls (stones or bricks, volume of mortar, pre-
  sence of leaves and of transversal ties, if any), the presence and condition of embedded
  tie-beams (of any kind), the connections of walls between them as well as with the floors
  and roof etc.

4 TARGET BEHAVIOUR, PERFORMANCE LEVEL

Instead of the provision of Limit States (LS’s), according to the EC’s, there are normative
(and strict) application rules according to the GC’s, as follows: Depending on the (seismic)
significance and importance of the existing building (i.e. its importance factor, \( \gamma_i \), according to
[3]), there is a Target Behaviour (TB), a combination of the foreseen Earthquake (EQ)/ sei-
smic action (full, 100%, or reduced, 60%) and the expected Performance Level (PL) of the
building, i.e. the degree and extent of damages (for primary and not for secondary structural
elements, including infills, if any).

In Table 1 here below the Target Behaviour (TB) is presented (shortly), while in Tables 2a
and 2b the (seismic) importance classes are presented , according to [3] and [5] or [6], respec-
tively. As it is known, the TB’s are associated with different global (q) and local (m) beha-
vior factors as well as with different structural safety factors (regarding models, \( \gamma_{Rd} \)), see
Figure 1, not to mention that even the analysis is influenced by the TB. To this end, a linear
analysis is not suitable for TB C/Collapse Prevention and a non-linear analysis is not suitable
for TB A/Immediate Occupancy. In addition, the TB could be lower for assessment (e.g. C or
B) and higher for retrofitting (e.g. B or A). For Table 1, the following notes apply:
(1) Probability of exceedance (10% or 50%) in a life-time of 50 years 
(recurrence period of 475 or 75 years, respectively).
(2) Level A, Immediate Occupancy (IO), Limited Damage (LD) of a few primary elements.
(3) Level B, Life Protection (LP), Significant Damage (SD) of a lot of primary elements.
(4) Level C, Collapse Protection (CP), Near Collapse (NC), Heavy Damage of almost all elements.
(5) The Target Behaviour B1 is valid for new buildings ([3]).
(6) The minimum tolerable TB’s are as follows:

<table>
<thead>
<tr>
<th>EQ(1)</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A/IO(2)</td>
</tr>
<tr>
<td>10% / 50 years</td>
<td>A1</td>
</tr>
<tr>
<td>100%</td>
<td>A2</td>
</tr>
</tbody>
</table>

Table 1: Target Behaviour (TB), according to the GC’sSI ([5] and [6]), see the above note (6),
based on the Performance Level (PL) and the Seismic Action (EQ).

<table>
<thead>
<tr>
<th>Importance class</th>
<th>Buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Buildings of minor importance for public safety, e.g. agricultural buildings etc.</td>
</tr>
<tr>
<td>II</td>
<td>Ordinary buildings, not belonging in the other categories.</td>
</tr>
<tr>
<td>III</td>
<td>Buildings whose seismic resistance is of importance in view of the consequences associated with a collapse, e.g. schools, assembly halls, cultural institutions etc.</td>
</tr>
<tr>
<td>IV</td>
<td>Buildings whose integrity during earthquakes is of vital importance for civil protection, e.g. hospitals, fire stations, power plants etc.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Importance factor, ( \gamma_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,8</td>
</tr>
<tr>
<td>1,0</td>
</tr>
<tr>
<td>1,2</td>
</tr>
<tr>
<td>1,4</td>
</tr>
</tbody>
</table>

Table 2a: General Important Classes according to [4].

<table>
<thead>
<tr>
<th>Classes of importance</th>
<th>Buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Buildings with low importance for the public safety, <strong>such as:</strong> Agricultural buildings and agricultural warehouses, sheds, stables, cowsheds, pigsties, poultry farms etc.</td>
</tr>
<tr>
<td>II</td>
<td>Common buildings, <strong>such as:</strong> Houses and offices, industrial - manufacturing buildings, hotels (which do not include conference rooms), hostels, boarding houses, exhibition spaces, eating establishments and entertainment spaces (patisseries, cafeterias, bowings, billiards, arcade rooms, restaurants, bars etc), banks, clinics, markets, super markets, shopping centers, stores, pharmacies, barber shops, hairdressing salons, gyms, libraries, factories, garages, dyer's shops, carpentries, research laboratories, food preparation rooms, dry cleaners, I. T. centers, warehouses, parking lots, gas stations, windmills, public service's and local authorities buildings which do not belong to class IV etc.</td>
</tr>
</tbody>
</table>
Buildings including facilities with very high economic importance, along with public gathering buildings and buildings that host a lot of people during the whole day in general, such as: Airport halls, conference rooms, computer centers, special industries, educational buildings, teaching rooms, private tutoring rooms, kindergartens, concert halls, courts of law, temples, sports facilities, theaters, cinemas, entertainment centers, passenger waiting rooms, psychiatric hospitals, institutions of disabled, chronically ill institutions, nursing houses, nurseries, day care centers, playgrounds, juvenile reform schools, prisons, waste water treatment plants etc.

Buildings whose integrity during and after earthquakes is of vital importance, such as: Telecommunication buildings, energy production buildings, hospitals, clinics, sanitary stations, health centers, refineries, power stations, fire and police stations, public service buildings for handling emergency needs after an earthquake, a fire etc.

Buildings which host works of unique artistic value, such as: Museums, museums' warehouses etc.

Table 2b: Detailed Importance Classes according to [5] and [6].

<table>
<thead>
<tr>
<th>Performance level</th>
<th>Uniform factor $q = q_o \cdot q_d$</th>
<th>Design deformation, $d_u$ (or $\theta_u$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$q_o \cong 0.6 \cdot q_s$</td>
<td>$d_y / \gamma_{Rd}$ for primary structural elements. (1)</td>
</tr>
<tr>
<td>B</td>
<td>$q_o \cong 1.4 \cdot q_s$</td>
<td>$d_u / \gamma_{Rd}$ for secondary structural elements (2)</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>$d_u$ for infill walls</td>
</tr>
</tbody>
</table>

Figure 1: Skeleton Behaviour Diagram
(for individual structural elements, or for the structure as a whole).

The following comments apply to Figure 1:
1) For primary structural elements:
The ultimate design deformation ($d_d$), even for performance level C is less than that corresponding to quasi-failure ($d_u$), and with satisfactory reliability, expressed through $\gamma_{Rd}$.

2) For secondary structural elements:
For these elements, a greater degree of damage is acceptable (under earthquake) than for primary structural elements, depending on whether they are vertical or horizontal ones, for values of $d_d$ defined also through $\gamma_{Rd}$ (for performance level B but not C).
5 COMPUTATIONAL METHODS

According to [3], the model of the building shall adequately represent the distribution of stiffness and mass in it so that all significant deformation shapes and inertia forces are properly accounted for under the seismic action considered, while in the case of non-linear analysis it shall also adequately represent the distribution of strength.

The model should also account for the contribution of joints, connections etc to the deformability and the overall behaviour of the building while non-structural elements (e.g. infills), which may influence the response of the main seismic structure, should also be accounted for.

Therefore, and according to certain criteria and structural characteristics/conditions, the following two types of analysis may be used, for both the assessment and the redesign, following structural interventions:

- **Linear/elastic,**
  - i.e. the lateral force method or the modal response spectrum, and
- **Non-linear/inelastic,**
  - i.e. the static/pushover or the dynamic/time history analysis.

The reference method for determining the seismic effects is the modal response spectrum analysis, using appropriate and representative linear/elastic model of the structure and the relevant design spectrum given in [3]. Nevertheless, and especially for assessment purposes, non-linear/inelastic static/pushover analysis is encouraged, combined with a set of application rules regarding checks and safety verifications in terms of deformations or of forces (see also Figure 1/§4), depending on the Performance Level (PL).

Two relevant issues are covered in detail according to the Greek Codes (GC’s), that of plain/unreinforced (or reinforced) masonry infills of RC buildings ([5] and [7]) and that of global (overall) behaviour factors/q-values ([5] and [6]).

For masonry infills, a detailed bi-linear skeleton diagram is given, suitable for both linear and non-linear analysis and for both models, i.e. the equivalent strut and the equivalent isoparametric four noded panel.

For the behaviour factors (suitable for linear/elastic analysis), and especially as far as the assessment of the existing buildings is concerned, the provisions of GC’s are given below, in Tables 3 ([5]) and 4 ([6]), for the Performance Levels of LP-SD (PL B). For PL A, \( q_A = 0.6q_B \) but \( 1.0 \leq q_A \leq 1.5 \), while for PL C, \( q_C = 1.4q_B \), see also Figure1/§4.

<table>
<thead>
<tr>
<th>Standards applied for the design (and construction)</th>
<th>Substantial damage in primary elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995&lt;…</td>
<td>No</td>
</tr>
<tr>
<td>1985&lt;…&lt;1995</td>
<td>Yes</td>
</tr>
<tr>
<td>…&lt;1985</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 3: Behaviour factors, assessment of RC buildings, Performance Level B/LP-SD.

<table>
<thead>
<tr>
<th>Masonry walls</th>
<th>Substantial damage in primary elements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>Unreinforced</td>
<td>1.2</td>
</tr>
<tr>
<td>Reinforced</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 4: Behaviour factors, assessment of PM buildings, Performance Level B/LP-SD.
6 FINAL CHECKS/SAFETY VERIFICATIONS

According to Figure 1/§4, the final checks for assessment or retrofitting purposes, depend on the Performance Level (PL) as well: For PL’s A and B, linear/elastic analysis could be performed, followed by safety verifications in terms of forces (with quasi-characteristic values for the materials’ characteristics), while for PL’s B and C, non-linear/inelastic analysis shall be performed, followed by safety verifications in terms of forces for brittle elements and in terms of displacements for ductile elements (with quasi-mean values for the materials’ characteristics).

In addition to these provisions, and according to the GC’s ([5] and [6]), additional structural safety factors have to be used, namely $\gamma_{Ed}$ accounting for the uncertainty in estimating of action-effects (e.g. due to the unavoidable redistribution in case of heavy damages or of major interventions) and $\gamma_{Rd}$ accounting for the uncertainty in modelling of the resistance (mainly in terms of stiffness and of deformability) of both the damaged and the retrofitted structural elements. For relevant details and application rules, see [5] and [6].

To this end, the basic general safety inequality, for both verifications in terms of forces or of displacements, is formulated as follows:

\[
\begin{align*}
E_d & \leq R_d \\
E_d &= E (E_k \cdot \gamma_{f'}) \cdot \gamma_{Ed} \\
R_d & \leq R (R_k / \gamma_{m'}) / \gamma_{Rd},
\end{align*}
\]

where $\gamma_{f'}$ and $\gamma_{m'}$ are partial safety factors, for actions and materials, respectively, appropriately modified (for existing buildings), in order to cover increased and combined uncertainties.

7 INTERVENTIONS (STRUCTURAL)

According to the Greek Codes (GC’s, [5] and [6]), there are rather detailed provisions and application rules regarding repair (reinstatement) and strengthening (upgrading), covering not only modelling and design aspects but implementation as well, with provisions for the materials and techniques to be used, pilot applications, interfaces between existing and new layers or elements, detailing as well as rules for minimum/maximum thickness, diameters, sections, volumes etc, not to mention aspects of quality control, durability, fire resistance etc.

In the following §§8 and 9, some details are given for the strengthening (upgrading of resistances) of RC and PM buildings, based on [4] as well as on [5] and [6].

An interesting and innovative normative provision of the GC’s ([5] and [6]), although based on older approaches ([10];), is that of damaged structural elements and the estimation of their level/degree of damage as well as their residual structural characteristics (resistances).

In the Appendix, at the end of this paper, indicative values of resistance reduction factors are given for elements of conventional older RC buildings, including plain masonry infills, valid at least for Greece.

These values (residual resistances, $r_r$) for individual elements (primary ones) could be used for the estimation of an overall value $r_r$ for the critical storey of a building (e.g. the ground floor, especially in case of pilotis) and therefore rational and rapid decisions could be taken regarding (i) the need of strengthening and not only repair, and (ii) the urgency of interventions.

Notes

a) Even the more detailed new GC’s ([5] and [6]) are not covering aspects of interventions on buildings’ foundations (including basements) and/or on their subsoil, in cases of proven defi-
ciencies and/or characteristic damages. To this end, certain principles and provisions should be included in both sets of Codes, the EC’s and the GC’s.
b) In addition, in both sets of Codes there are no specific (and detailed) provisions and application rules regarding repairs and/or strengthening of precast (and, possibly, prestressed) concrete buildings, as well as of masonry structures such as bell-(or other) towers, chimneys etc.

8 STRENGTHENING OF RC BUILDINGS

The relevant new Greek Code ([5]) contains a lot of normative application rules, additional to those of EC8-3:2005 and 2009 ([4]), regarding various methods (materials, techniques etc) of upgrading and covering all structural characteristics, i.e. stiffness, strength and durability, of elements as well as of structures as a whole.

In this respect, various commonly used nowadays (or even special) methods are covered in the normative paragraphs of this GC (Chapter 8), such as:
- Strengthening against shear for linear elements,
- Application of external confinement (by means of steel or of FRP), continuous (jackets) or in the forms of collars, upgrading of local ductility and shear strength, improvement of lap splicing etc,
- RC jackets for vertical elements (columns and walls),
- Infilling of underdesigned RC frames, by means of various materials (mainly concrete) and techniques, transformation into low ductility shear walls, or arrangement of external (in contact) new high ductility shear walls, properly connected and founded (probably by means of heavy footings or of micro-piles),
- Strengthening of existing non-engineered masonry infills, by means of various materials and techniques (mainly by means of double light jackets), or arrangement of new engineered masonry infills, reinforced (with distributed reinforcement, steel or FRP) or even plain/unreinforced ones, possibly combined with local light jackets on columns, and
- Bracing of RC frames, arrangement of steel double diagonals, internal or external, concentric or eccentric, transformation into vertical hybrid trusses, properly connected and founded (see above, infilling of RC frames), probably equipped with energy dissipation devices.

9 STRENGTHENING OF PM BUILDINGS

The relevant new Greek Code ([6]) contains a lot of normative application rules, additional to those of EC8-3:2005 and 2009 ([4]), regarding various methods (materials, techniques etc) of upgrading and covering all structural characteristics, i.e. stiffness, strength and durability, of elements as well as of structures as a whole.

In this respect, various commonly used nowadays (or even special) methods are covered in the normative paragraphs of this GC (Chapter 8), such as:
- Light jacketing on both sides of the walls, with proper and strong anchorages, possibly combined with the arrangement of a grid of steel ties/connectors piercing the walls,
- Stitching/connecting of transversal walls (upgrading of corner bonds and of a box action), by means of bridging blocks or of steel ties/connectors (possibly lightly prestressed),
- Controlled and uniform mass impregnation (injections), use of special methods (mortars and techniques/equipment),
- Special strengthening be means of FRP’s, in the form of layers or of strips, combined with light mortar jackets,
- Strengthening my means of infilling of openings,
• Arrangement of steel ties and/or of masonry buttresses,
• Arrangement of concrete (or other) closed tie-beams, combined with strong connections to the floors and the roof, and
• Arrangement of new internal skeletons (of various materials), properly connected and founded.

REFERENCES

APPENDIX

Indicative values of reduction factors for the resistances of damaged structural elements of RC buildings (without any repair or strengthening)

1) The skeleton behaviour curve (F- d) of damaged (mainly due to earthquakes) structural elements, connections, joints etc, is generally degraded compared to its counterpart prior to damage (F- d), according to the figure below:

![Skeleton Behaviour Curve](image)

Specifically for damaged elements, and due to a lot of uncertainties, a residual strength branch is not foreseen after quasi-failure (i.e. $F_{\text{res}} \approx 0$).

2) Depending on the type and extent of damage, for structural elements, joints etc, reduction factors $r$ may be defined for the mechanical characteristics ("damage indices"), as follows:

$$r_K = \frac{K'}{K} \leq r_R = \frac{F_{\gamma'}}{F_\gamma} \leq r_{\text{du}} = \frac{d_{\gamma'}}{d_u}.$$  \hspace{1cm} (4)

Thus, values of the $r$ factor equal to 1 (or slightly lower) correspond to the initial state of the element prior to damage (or for damage with small impact), while values of $r$ closing on 0 correspond to full failure and in effect "loss" of the damaged element (exhaustion also of its ductility).

3) As substantial damages, i.e. for the purposes of [5] and [6], are defined those that have led to a reduction of the bearing capacity (in terms of forces) larger than 25%, i.e. $r_R \leq 0.75$.

Certainly, according to the provisions of Chapter 8 of [5] or [6], appropriate repair techniques (and materials) can be (or must be) applied in order to fully restore (under certain conditions) the mechanical characteristics of the damaged elements, i.e. $r \to 1$, regardless of possible strengthening.

4) For assessment purposes only, and to facilitate a possible parametric investigation of the consequences of the damage (and the extensive redistribution of the consequences of the actions that they entail), the values of the $r$ factors may be modified through appropriate (model) coefficients $\gamma_{Rd}$, i.e. through the relationship $r/\gamma_{Rd}$, with $\gamma_{Rd}$ values greater or less than 1 (to account for unfavourable or favourable effect) according to the justified decision of the Engineer.

5) Visual sketches and indicative values of reduction factors $r$ (damage "indices") are given in the following pages for damaged RC structural elements, without repair (or strengthening), as well as for infill walls, essentially after earthquake.

6) Because the skeleton behaviour curves (F- d and F'- d') involve mainly “force” F in terms of bending moment (M) or shear force (V), it is possible that reduction factors $r$ may be required also in terms of axial force only (i.e. $r_N$, generally greater than $r_R$ (R=M or V), if
depending on the type and extent of damage, according to the justified decision of the Engineer.

7) Also, because the earthquake “reveals”, as has been repeatedly observed, pre-existing wear (attack on materials) and impairment of the mechanical characteristics of the members, an additional reduction of the \( r \) factors may be required depending on the age, use and environment of the building, as well as the observed wear of the element, according to the justified decision of the Engineer.

8) Depending on the structural element, any damage due to (mainly) earthquake may be classified into characteristic typical degrees of damage, depending on which the reduction factors \( r \) may be estimated.

9) Thus, as already mentioned, depending on the structural element and the type/degree of its damage, the appropriate \( r \) values are estimated, with smaller values for more serious (and more “dangerous”) damage.

10) For columns, but also for beams, the damage may be classified as per Figure 3, while the corresponding \( r \) factors are given in Table 5. Especially for damage at column bases, in the area of starter bars / lap splices of longitudinal reinforcement bars, Table 6 gives the values of reduction factors \( r_M \) compatible with the damage, while \( r_V \) values may be taken as 85% of \( r_M \).

<table>
<thead>
<tr>
<th>Of limited importance</th>
<th>Light damage</th>
<th>B</th>
<th>B1</th>
<th>B2</th>
<th>&lt; 2mm</th>
<th>d=0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>&gt; 5mm</td>
<td></td>
<td></td>
<td>&lt; 3mm</td>
<td>d&lt;&lt;</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>C1</td>
<td></td>
<td>C2</td>
<td></td>
<td>d&lt;1%</td>
</tr>
<tr>
<td></td>
<td>D or D/E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>d&gt;2%</td>
</tr>
</tbody>
</table>

Figure 3: Typical degrees of damage of columns (and beams)  
(d: storey drift or drift of member ends).
<table>
<thead>
<tr>
<th>Typ. Degree</th>
<th>Damage description</th>
<th>$r_K$</th>
<th>$r_R$</th>
<th>$r_{du}$</th>
<th>$F(=R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Light flexural damage (no damage from shear). Single, isolated cracks, roughly perpendicular to member axis, &lt;2mm, absence of diagonal cracks.</td>
<td>0,95</td>
<td>1,00</td>
<td>1,00</td>
<td>M</td>
</tr>
<tr>
<td>A/B</td>
<td>Light damage, flexural or from shear. 1. Cracks (multiple rather than single) roughly perpendicular to member axis (&lt;2mm), diagonal cracks (&lt;1mm). Absence of visible permanent displacements or buckling. Absence of spalling. 2. Moderate cracks roughly perpendicular to member axis (3÷5mm), diagonal cracks (1÷2mm). Absence of visible permanent displacements or buckling. Light spalling.</td>
<td>0,90</td>
<td>1,00</td>
<td>1,00</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0,80</td>
<td>0,90</td>
<td>1,00</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0,70</td>
<td>0,90</td>
<td>0,95</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0,50</td>
<td>0,80</td>
<td>0,90</td>
<td>V</td>
</tr>
<tr>
<td>B</td>
<td>Serious flexural/moderate shear damage. Cracks roughly perpendicular to member axis (&gt;5mm), diagonal cracks (&lt;3mm). Absence of displacements or buckling. Spalling.</td>
<td>0,55</td>
<td>0,80</td>
<td>0,90</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0,40</td>
<td>0,60</td>
<td>0,80</td>
<td>V</td>
</tr>
<tr>
<td>C/D</td>
<td>Serious to heavy damage. 1. Flexural. Buckling of bars and spalling, core disintegration or intense side-to-side cracking, with slip, or permanent drift of member ends 1÷2% $l$. 2. Shear. Intense diagonal cracks (&gt;3mm), multiple rather than single, diagonal or crosswise, small but noticeable permanent drift of member ends.</td>
<td>0,30</td>
<td>0,50</td>
<td>0,70</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0,20</td>
<td>0,30</td>
<td>0,60</td>
<td>V</td>
</tr>
<tr>
<td>D (or D/E)</td>
<td>Total failure, loss of member. Buckling or fracture of bars, or opening (or fracture) of stirrups, or cracks &gt;10mm, or permanent drift of member ends &gt;2% $l$ (including potential slip).</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>M</td>
</tr>
</tbody>
</table>

Table 5: Reduction factors $r$ for damaged columns (and beams).
<table>
<thead>
<tr>
<th>Typ. Degree</th>
<th>Damage description</th>
<th>$r_K$</th>
<th>$r_R$</th>
<th>$r_{du}$</th>
<th>$F(=R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A/B</td>
<td>Moderated damage in lap splice areas. Cracking along bars. Short cracks roughly perpendicular to member axis. Light spalling.</td>
<td>0.70</td>
<td>0.70</td>
<td>0.90</td>
<td>M(*)</td>
</tr>
<tr>
<td>C/D</td>
<td>Heavy damage in regions of lap splices. Intense and deep spalling, bare segments of reinforcement bars (exposure).</td>
<td>0.50</td>
<td>0.50</td>
<td>0.70</td>
<td>M(*)</td>
</tr>
</tbody>
</table>

(*) It may be taken $r_V \approx 0.85 \ r_M$.

Table 6: Reduction factors $r$ for damaged lap splices at bases of columns (or other lap areas).

11) For shear walls, which are predominantly primary (under earthquake) structural elements, in the absence of other data, in principle the classification of damage according to Figure 3 as well as Table 5 may be used for the values of the reduction factor $r$.

- Simple slip, with cracks <3mm and displacement <10mm
  $$r_M \approx r_V, \ r_K \approx 0.40/ \ r_R \approx 0.60/ \ r_{du} \approx 0.70$$  (5)

- Intense slip, with cracks >5mm and displacement >15mm
  $$r_V \approx 0.90r_M, \ r_K \approx 0.20/ \ r_R \approx 0.30/ \ r_{du} \approx 0.50$$  (6)

12) Finally, for common unreinforced (existing) infill walls, with perforated bricks and poor (generally) grouts, the recommendations (in case of damage) of Figure 4 and Table 7 may be used, in the absence of more accurate and detailed data.

Reduction factors $r$ for infill walls relate to their shear resistance (or to the resistance of the equivalent diagonal strut in compression).

13) It is stressed that the definition of typical degrees of damage (in correspondence with those for reinforced concrete structural elements) is difficult and (largely) unreliable for existing infill walls. Thus, a simpler classification to degrees of damage is used (see Figure 4).
Miltiadis P. Chronopoulos and Petros M. Chronopoulos

detachment of infill wall

detachment of infill wall

Figure 4a: Characteristic light (to moderate) infill wall damage, with cracks < 2÷3mm (some of the damage may be due to permanent deformation of the structure, or the beam/slab system).

Figure 4b: Serious infill wall damage, with cracks > 5mm.
Miltiadis P. Chronopoulos and Petros M. Chronopoulos

Figure 4c: Heavy infill wall damage, with cracks > 10mm.

<table>
<thead>
<tr>
<th>Level of Damage</th>
<th>Damage description</th>
<th>$r_K$</th>
<th>$r_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>Light (to moderate) cracks, &lt; 2÷3 mm, around openings, or detachment of infills from the main structure.</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>Multiple light cracks, especially in walls with openings.</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>Serious</td>
<td>Intense cracking, diagonal or crosswise, with crack width &gt; 5mm, detachment from the main structure, cracking of the tie beams, absence of significant out-of-plane deformations (&lt;5mm).</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Heavy</td>
<td>Intense cracking, generally crosswise diagonal, with crack width &gt; 10mm, detachment from the main structure, damage of tie-beams and small out-of-plane deformations (&lt; 15mm).</td>
<td>0.20</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 7: Reduction factors $r_V (r_r)$ for damaged common unreinforced infill walls.

Note
Values of $r_{du}$, for the deformation at failure of damaged infill walls are not given. To this end, it is safer (and more reliable) to assume that “failure” coincides with “yielding” ($F_u \approx F_y$ and $d_u \approx d_y$, see skeleton behaviour curves).
RESILIENT SYSTEM MODELLING OF ANCHORAGE CONNECTION FOR SEISMIC STRENGTHENING APPLICATIONS

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Abstract

Strengthening applications form a significant field of research, which is reflected in significant relevant publications and dedicated scientific conferences, while practice-oriented guidance documents and standards have been published in the recent years and they substantially support the implementation of strengthening methods. At the same time, this very implementation strongly relies on the connection of new to existing components in order to achieve the desired strengthening outcome in stiffness, deformability, or load-bearing capacity. This transfer mechanism may often be realized through mechanical anchorage by use of post-installed anchors. Such anchors are generally expected to efficiently transfer seismic actions between steel and concrete, or within concrete interfaces. Moreover, many seismic retrofitting techniques would not be realizable without the use of post-installed anchors. This paper focuses on the structural performance of such connections in a challenging retrofitting method, namely a diagonal bracing with structural steel profiles, and by the use of high-performance undercut anchors. The investigation considers a concrete frame found in existing structures and it delivers insights on the structure’s performance before and after the strengthening application, based on a set of advanced, non-linear finite element model analyses.

Keywords: Seismic retrofitting, Strengthening, Steel braces, Finite Element Modelling, Fastening technology, Resilience.
1 INTRODUCTION

Strengthening applications form a significant field of research in recent years, which is reflected in numerous relevant publications and dedicated scientific conferences, while practice-oriented guidance documents and standards have lately been published and they substantially support the implementation of strengthening methods. The engineering and design of strengthening applications is strongly interwoven with challenging assessment exercises on both the pre-existing and the retrofitted structure, often involving highly complex numerical analyses supported by thorough experimental testing or on-site surveys. At the same time, this very implementation strongly relies on the connection of new to existing components in order to achieve the desired strengthening outcome in stiffness, deformability, or load-bearing capacity. This transfer mechanism is often realized by use of post-installed anchors. Such anchors are generally expected to reliably transfer seismic actions between steel and concrete, or within concrete interfaces. Moreover, many seismic retrofitting techniques would not be possible without the use of post-installed anchors.

However, the systematic complementation of anchors to the strengthening system is traditionally designed with basis on the individual anchoring points without a comprehensive consideration of the systematic response of the anchored component. This study makes use of experimental results from research and pre-qualification tests of individual anchors, and it draws information from MDOF shake table tests, in order to produce an appropriate constitutive model of anchorages used in seismic retrofitting. This model is then integrated to a Finite Element numerical model of a single anchor as well as a full-scale application, in particular for the strengthening of a concrete frame. The results of the model are juxtaposed to the actually expected response of the strengthened structure under realistic conditions. Based on this, design recommendations for the use of anchors in strengthening applications are delivered, considering the laboratory assessment of the reinforced concrete structural members and materials. Two different brace sizes have been used to that end. The first cross section is proposed for the retrofitting of the frame, under the consideration that the brace will reach its yielding limits prior to any damage at the anchors. The latter brace section is additionally analysed in order to present an upper bound strengthening intervention, where failure of the frame is simultaneous with the failure of the anchorage zone. The design recommendations (for the first brace section) satisfy the required reliability and robustness levels.

The following section of the paper (Section 2) discusses already existing studies from the perspective of structural strengthening with steel braces, and on the seismic performance of connections realized through anchors in concrete. An application is also selected from literature and it is used as a basis for the Finite Element (FE) model benchmarking. Section 3 discusses the details of the FE models built for the present investigation at three main levels, i.e. detail (small-scale modelling) modelling of the anchor, and structure (large-scale) modelling without and with strengthening intervention components. The analyses results are critically discussed in Section 4. Section 5 presents an analytical verification of the anchorage system based on current design standards for comparison purposes. The paper is summarized with overall conclusions from this study, and proposals toward resilient engineering design for seismic strengthening projects.

2 PREVIOUS INVESTIGATIONS IN INTERNATIONAL LITERATURE

2.1 Retrofitting of existing concrete frame structures – applications of diagonal braces

Several strengthening methods are available in modern earthquake engineering practice. A main classification can be carried out as regards the performance requirement under focus, as
for example strength increase, enhancement of ductility, redistribution of stiffness, or isolation from the seismic excitation. A broad scale strategy for redesign of existing structures is given in [1], where also the technical and conceptual challenges in strengthening design of existing, as compared to new structures are highlighted.

One of the applicable retrofitting applications is steel bracing of frames, usually in an X – arrangement, that can mainly increase stiffness, as well as bearing capacity and ductility of a structure. Such systems have been already been proven efficient historically in traditional building layouts, as e.g. discussed in [2]. Early investigations on external retrofitting of concrete frames with X-braces are reported in [3], where experimental investigations of scaled reinforced concrete (RC) frames indicate an increase of stiffness and maximum lateral load. The braces appear to carry approximately 70-60% of the entire shear, and the ultimate capacity of the system is governed by buckling and eventual rupture of welds and shear column failure. An evaluation through theoretical and parametric analyses the effect of X-braces in reducing the seismic vulnerability pilotis-frames is discussed in [4], while small scale experiments with 3 different configurations is given in [5]. Based on these studies, a clear shift of the plastic hinges from the ground floor to the upper floors is observed and X-braces are evaluated as an effective, feasible and low-cost method with a high ability to dissipate energy. Further experimental results of downscaled, moment-resisting RC frames retrofitted with direct X- and V- (knee shaped) braces under push-over and cyclic loading are presented in [6] and [7]. In [6] it is shown that both systems can be used to increase the yield capacity, yet with a reduction in global displacement and ductility; both systems can be used to achieve damage control criteria, while knee braces are more suitable for collapse prevention. A desired increase in load capacity with a drift reduction and substantial energy dissipation can be easily achieved by incorporating steel bracing, as discussed in [7]. In the same contribution, it is also proposed that an appropriate, although conservative, design can be carried out with conventional RC design methods. Akbari & Maheri [8] address the analytical evaluation of inverse V braced RC frames and the effect of some parameters influencing the behaviour index R. In this case, retrofitted frames have much larger ductility capacities, with a strong dependence on the structure’s height, and R factors for different ductility demands are proposed. Experimental investigations of different types of bracing are also reported in [9] for scaled samples under cyclic loading, showing decreased energy dissipation, but a more gradual and controllable failure mode.

In all cases discussed above, a safe load transfer between steel and concrete members became a crucial part of the application. In [10], particular focus is devoted to a customized solution for brace frame connection, introducing a dampening system, showing that the addition of a low friction material at the contact surface can regulate the proportion of shear to normal stresses in the anchorage, thus controlling the damage on the main frame. Besides, a similar response is indicated for dampening interfaces for masonry infills, as reported in [12]. The three different configurations of an X-brace connection are also treated in [11] based on numerical analyses, which exhibit similarly robust characteristics. Of particular interest is the proposal by [14] for the introduction of Buckling-Restraint-Braces (BRB), since the connection details are formed with post-installed anchorages, and tests are carried out in real scale. Both experimental and numerical investigations are provided, on the newly introduced retrofitting method and it is indicated that the overall performance is satisfactory, 60-70% of shear loads is borne by the brace (a similar result with [2]), while cracking can be contained by additional wire-mesh reinforcement. Tests in [15] prove that the concept is feasible and bonded expansion anchors are an effective proposed connection, and the effects of tension and closing corner are beneficial to the anchorage capacity.
2.2 Use of post-installed anchors in seismic applications and strengthening

Bracing may be applied to concrete indirectly through a steel frame fitted to the concrete frame envelope, or externally, by anchoring steel elements directly on the column/beam joints, and connecting those. However, a mostly efficient proposal is to connect steel braces directly on the concrete members through cast in place or post-installed anchoring means, and the present article discusses this case in more detail. Consequently, the load transfer mechanism and the type of anchor plays a very important role. Recently, significant steps have been taken to form a better understanding of the mechanical performance of anchors under cyclic and earthquake actions. Hoehler [16] provides significant insights on the seismic behavior of anchors, and lays a background for the assessment of anchors under seismic actions in typical, standardized tests. Rieder [17] provides a basis for the enhancement of safety of post installed anchors in seismic regions, and discusses advantages and disadvantages of damping elements with comparison of different damping systems. An advanced damping system at anchor heads, designed for seismic actions is presented in [18]. Rieder et al., [19] discuss the performance of post-installed anchors under earthquake excitations based on shake-table tests, and it shows that undercut anchors exhibit a superior post-damage performance. Based on this finding, and a respective case study, [19] concludes that such elements can be used for steel X-brace strengthening infills in concrete frames.

Significant steps have also been recently taken toward standardization. The newly published Eurocode 2 –Part 4 “Design of fastenings for use in concrete” [20] includes specific provisions for anchorage to concrete and seismic excitations on a design norm level. Simultaneously, [21] regulates the series of testing and the assessment procedures required for the pre-qualification of mechanical anchors for seismic use, as well as for the classification of the anchor’s load bearing performance under seismic events of various intensities. Although these standards primarily focus on the fixations of secondary or non-structural components to concrete (see also [22]), the use of mechanical anchors in the connection of (primary) structural elements to existing structures as part of a seismic strengthening intervention is rationalized, for example also in [23–25], and it is treated on a normative level in [26]. As seen below, the existing paradigms in [19] and [15] are expanded to more accurate numerical assessments of such an intervention.

3 FINITE ELEMENT MODELLING

3.1 Description of small-scale, anchor detailing modelling

In order to confirm that the modelling principles and parameters used in the FE assessment of the strengthened structure are reliable, individual mechanical anchors have been simulated in different load directions and geometric configurations, simulating deformation controlled tests. The focused failure modes where those of concrete failure, since steel rupture is generally well understood and efficiently modelled, and other failure modes where not applicable. The model results have been compared to own existing tests and predictive equations from literature, which also represent an accurate statistical regression on large testing datasets. For the concrete cone failure mode under tension Equation (1) and for concrete edge failure, Equation (2) have been considered as they are widely acknowledged and form the basis for design equations [20] [27]. Equation (3) represents a design criterion for combined tension and shear actions on anchors, when the governing verification is concrete related failure.

\[
N_{Ru,c}^0 = 15.5 \cdot \sqrt{f_c} \cdot h_{ef}^{1.5}
\]  \hspace{1cm} (1)
\[ V_{Ru,c}^0 = 3 \cdot d^a \cdot l_1^b \cdot \sqrt{f_c} \cdot c_1^{1.5} \]  
\[ \alpha = 0.1 \cdot \left( \frac{h_{ef}}{c_1} \right)^{0.5} \]  
\[ \beta = 0.1 \cdot \left( \frac{d}{c_1} \right)^{0.2} \]

where:  
- \( h_{ef} \): embedment depth  
- \( d \): the anchor diameter  
- \( f_c \): concrete compressive strength  
- \( c_1 \): edge distance of the anchor in the direction of shear

\[ \left( \frac{N}{N_{Ru,c}^0} \right)^{1.5} + \left( \frac{V}{V_{Ru,c}^0} \right)^{1.5} \leq 1 \]  

The values assigned to the parameters above were valid for all the simulations carried out. The steel quality assumed for the anchors was 8.8 which is widely used for existing anchors in the market with yield strength of 750MPa. The compressive strength of the concrete simulated was assumed to be 35MPa. The analyses were carried out with the commercial program package ANSYS. The modeling and analyses of the component and frame models of this study utilize the approach developed in [28-31] with respect to material parameters, interrelations among elements and materials as well as parameters for Explicit Dynamics Analyses. All the elements and material models used were implemented by the libraries included in the program. The element type that was chosen for all the analyses was a tetrahedron with 4 nodes and three degrees of freedom on each node. This type of element is preferred when very complicated geometries are simulated, especially geometries with gaps and holes such as in the presented case. The density of the mesh was higher for the elements in the region of the expected concrete cone (see Fig. 1). Also an even higher density was chosen for the narrow cross section of the anchor complying with the general rule of at least three elements to each interface or for the minimum dimension of different parts (or subparts). All the concrete specimens that were simulated were unreinforced as the case is for the tests compared.

Figure 1: Optimized mesh of the concrete block from FE model: External view (left), and section plane (right)
The anchors of the individual FE models were chosen to have embedment depth of 80 mm, and the distance from the edge was varied between 80 mm (to induce an edge influence on the concrete cone failure) and 150 mm (no edge influence). The design of the anchors approximated a self-cutting undercut anchors available in the market [32] (see Fig. 2). The anchor shaft diameter was 19 mm and the neck of the anchor had diameter of 10 mm, simulating the weak cross section of a typical M12 threaded rod. A nut was modelled, with diameter of 32 mm and thickness of 15 mm as a tension transfer connection of the fasteners and base plates (transverse sliding contact rule). The shear transfer is realized in the model by direct contact to the anchor shaft. The size of the modelled concrete block was 300x300x300mm. The boundary conditions were a fixed plane on the free surfaces and a hard restraint against the load direction at a perimeter of 160 mm (2·h_{ef} or 2·c_1) from the load application. Table 1 summarises the cases under investigation to validate the performance of the FE modeling and analysis.

![Figure 2: Undercut anchor as idealised in the FE model (left) and realistically installed in concrete (right).](image)

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Load angle (°)</th>
<th>Distance from the edge (mm)</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA1</td>
<td>0</td>
<td>150</td>
<td>Concrete cone</td>
</tr>
<tr>
<td>SA2</td>
<td>0</td>
<td>80</td>
<td>Concrete cone</td>
</tr>
<tr>
<td>SC1</td>
<td>45</td>
<td>150</td>
<td>Combined</td>
</tr>
<tr>
<td>SC2</td>
<td>45</td>
<td>80</td>
<td>Combined</td>
</tr>
<tr>
<td>SS1</td>
<td>90</td>
<td>150</td>
<td>Concrete edge failure</td>
</tr>
<tr>
<td>SS2</td>
<td>90</td>
<td>80</td>
<td>Concrete edge failure</td>
</tr>
</tbody>
</table>

Table 1: Investigated individual anchors. Load angle is 0° for load in the direction of the anchor axis, 90° for transverse shear load (toward the free edge where applicable), or a combination thereof accordingly.

3.2 Modelling of existing, and strengthened concrete frame

A similar methodology with the one followed in the validation of the anchor model was used for the validation of the RC frame model. After extensive research of the literature it was decided that the most suitable set of experiments to use as benchmark for the validation of the frame model is [15]. The tested RC frame in [15] had dimensions 3 x 3 m. This frame simulated a frame of an old existing structure and it does not comply with the modern designing rules. The frame’s columns were 500 mm wide and 300 mm deep reinforced with 10Φ25 in the longitudinal direction and stirrups of Φ10 /250 in the transverse direction. The beam was 550 mm wide by 500 mm deep, and had 10Φ19 longitudinal reinforcement and stirrups of Φ13/150 as shear reinforcement.

The mean values of the materials used in these analyses are as follows: the mean compressive strength of the concrete is 41.3 MPa, the mean yield stress of the longitudinal reinforce-
ment is 489 MPa and of the shear reinforcement 280 MPa. The yield strength of the anchors was assumed 750 MPa, while for the steel brace, the commonly used yield stress of 275 MPa with a fully plastic post-yield behaviour was assumed. The bottom foundation was excluded from the analyses to reduce the computational load and the base was assumed fully fixed. The element type that was used in the simulation of the RC frame is the same as in the simulations of the single anchor. The FE mesh optimization was identical around the anchoring region to obtain a more accurate insight of the interaction between the RC frame and the metal brace at local level. Additional analyses were conducted to investigate the influence of this FE mesh optimization on the overall behaviour of the frame. It was found that this mesh provides satisfactory results of frame behaviour also at global level.

Figs. 3 and 4 show the dimensions and detailing of the frames, of the bracing and of the gussets. The gussets’ dimensions are 480x250x20mm and the braces principally used in the investigations are the standardised cross sections SHS 60X60X4 and SHS 100x100x5. The anchors used for connecting the metal brace with the frame were assumed to have embedment depth of 125 mm, shaft diameter 21mm and a M12 bolt with weakest cross section of 10mm. All the anchors have close spacing and they are considered to work as anchor groups. This
forms a minor departure from the design standards, which requires that different anchor groups are connected with separate fixtures, but if the fixtures were separated, the stiff transverse gusset would still induce a common, constrained kinematic response. The spacing between the anchors is in the horizontal direction $s_1 = 70$ mm and in the vertical $s_2 = 240$ mm. The entire group is placed in adequate distance from the edge in all directions so that it can be considered that there is no edge influence in the concrete cone formation. During the designing of the specimen, great attention was given so that the anchors and the reinforcement do not intersect each other at any point.

4 RESULTS AND DISCUSSION

4.1 Verification at component level

For the verification of the anchor model, a comparison was made with the existing formulas (see also Section 3.1), which deliver the ultimate strength anchor. Table 2 indicates the ultimate loads per the analytical equations, juxtaposed with the single anchor FE model results. Furthermore, the failure patterns developed in the analyses are compared with those of respective experiments in Figs. 5 and 6. In all cases the model results are in agreement with the experimental results and the expected failure patterns seen in previous own laboratory experiments. A good agreement was also found for a combination of tension and shear loads, as presented in Fig. 7.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Load angle ($^\circ$)</th>
<th>Distance from the edge (mm)</th>
<th>CC-M Ultimate load</th>
<th>FE analysis Ultimate load</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA1</td>
<td>0</td>
<td>150</td>
<td>65.6</td>
<td>58.3</td>
</tr>
<tr>
<td>SA2</td>
<td>0</td>
<td>80</td>
<td>54.6</td>
<td>52.4</td>
</tr>
<tr>
<td>SC1</td>
<td>45</td>
<td>150</td>
<td>--</td>
<td>59.5</td>
</tr>
<tr>
<td>SC2</td>
<td>45</td>
<td>80</td>
<td>--</td>
<td>45.3</td>
</tr>
<tr>
<td>SS1</td>
<td>90</td>
<td>150</td>
<td>54</td>
<td>69.7</td>
</tr>
<tr>
<td>SS2</td>
<td>90</td>
<td>80</td>
<td>23.68</td>
<td>23.8</td>
</tr>
</tbody>
</table>

Table 2: Experimental and analytical results for investigated individual anchors.

Figure 5: FE model SA1 after failure (left), and concrete cone failure obtained previous experiments. Note for colour scale in FE results: blue indicates undamaged state, red indicates fully yielded material.
4.2 Global behaviour of the retrofitted RC frames

The first step for the verification of the overall model was to compare the results of the FE analyses with those of the experimental investigation of [15] for an the concrete frame discussed in Section 3.2 without strengthening measures. The RC-frame was compared to the analyses considering the ultimate load, the lateral displacement and the developed failure mode. The RC frame exhibited a maximum bearing horizontal load of 336 kN, which compares well with the load of 349 kN obtained from the 3D FE analysis with a uniform meshing (see Frame 1 in Table 3). Table 3 also summarises the ultimate displacement and drifts (hori-
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It is seen that the RC frame from experiments (90 mm) and the FE analysis (99.4 mm) are very close. This proves that the overall model of the frame is valid. The failure pattern derived from the RC frame (Frame 1) is shown in Fig. 8.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Ultimate load (kN)</th>
<th>Displacement (mm)</th>
<th>Drift (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE Frame 1 (uniform mesh)</td>
<td>349</td>
<td>99</td>
<td>0.3</td>
</tr>
<tr>
<td>FE Frame 2 (refined mesh)</td>
<td>299</td>
<td>56</td>
<td>1.9</td>
</tr>
<tr>
<td>FE Frame 2 + SHS60X4</td>
<td>440</td>
<td>42</td>
<td>1.4</td>
</tr>
<tr>
<td>FE Frame 2 + SHS100X5</td>
<td>769</td>
<td>18</td>
<td>0.6</td>
</tr>
<tr>
<td>Full scale test</td>
<td>336</td>
<td>90</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3: Comparison of experimental and numerical results

Figure 8: 3D FE analyses results at failure for RC frame without strengthening measures (Frame 1). Note for colour scale: blue indicates undamaged state, red indicates fully yielded material

For the numerical analyses including a diagonal brace with a detailed undercut anchors connection to the RC frame, required a locally refined mesh, in order to capture the anchorage response. Therefore, another analysis was made for the RC frame with the respective mesh in the anchorage zone (FE Frame 2). This allowed for an evaluation of this mesh sensitivity. Based on this model, it is observed that this local mesh had indeed a slight effect on the overall behaviour of the frame reducing the ultimate of the frame by 36 kN (11%), and the drift to 1.9%, as seen in Table 3 and Fig. 11. The experimentally obtained load-displacement curve lies between the respective curves obtain with and without mesh refinement and in small distance. This difference is considered of an acceptable magnitude, and it moreover lies on the conservative side as regards the strengthening design with FE Frame 2.

The retrofitting of the frame with the diagonal brace SHS 60x60x4 improved its lateral response by increasing the ultimate load to 374 kN, i.e. by 20%, while retaining the horizontal displacement at the slightly lower levels as the non-retrofitted RC frame. As seen in Fig. 9, the peak load is reached while the anchors are at a virtually undamaged state, at the initiation of concrete damage. At this stage, the anchors are also reaching their yield strength. However, the brace section is already fully plastified, and the post-peak behaviour of the frame is con-
trolled by the load retention of the steel brace in tension and the development of flexural damage in the weak concrete sections (column joints).

In the case of the frame retrofitted with the SHS 100x100x5 diagonal brace, the lateral load bearing capacity increases substantially at 769 kN, i.e. 2.5 times higher compared to the non-retrofitted RC frame. At peak load, the RC components are critically damaged, while the anchorage zone is failing with a distinctive concrete cone breakout, as seen in Fig. 10. This damage development occurs also at substantially smaller deformations, which indicates that this system has a much stiffer lateral response. In this case the brace remains slightly below its yielding limit.

Fig. 11 presents the results of the experiment and numerical analyses in form of lateral load–displacement diagrams.
ANALYTICAL VERIFICATION OF THE BRACE ANCHORAGE

As a comparison to the design of the anchorage verified by FE models, an analytically derived verification in the spirit of current design standards is performed below. The self-cutting undercut anchors are qualified for seismic applications according to [21] and [32], and the seismic design of the anchors was performed in the spirit of EN 1992-4 [20]. For fasteners designed under seismic loading, one of the following conditions must be fulfilled:

a) The anchorage is either designed for the minimum of the force corresponding to yield of the attachment taking into account over-strength, or the maximum force that can be transferred to the connection by the attached component

b) The fastener is designed for ductile steel failure, at a load level < 60% as of any other failure mode

c) For non-structural elements, brittle failure of the fastening occurs at a load level at least 2.5 times of the applied earthquake load. Brittle failure of the fastening is not allowed for structural elements, and the assessment below forms a departure from the code.

d) The verification for combined tension and shear must fulfil the criterion of Equation (4).

\[
\left( \frac{N}{N_{Ru,c}} \right) + \left( \frac{V}{V_{Ru,c}} \right) \leq 1
\]  

(4)

For the design of the anchors the maximum axial load was used according to the yield strength of the brace, \( F = 246.4 \) kN. For the verification of the anchorage the characteristic values of the materials. The characteristic compressive strength of the concrete was 33.3 MPa and for the anchors the characteristic failure strength was 800MPa. The anchorage was verified for steel failure in tension and in shear and for the combination of both. For the concrete
failure the verification was calculated for the concrete cone failure in tension, the concrete edge failure in shear, and their interaction. The rest of the failure modes were not examined since the necessary conditions to prevent them were fulfilled. All the partial factors applied were derived from [20]. Also additional factors for the seismic design resistance were applied.

<table>
<thead>
<tr>
<th>Verification</th>
<th>( \alpha_{eq} )</th>
<th>Partial factors</th>
<th>Resistances</th>
<th>Seismic load</th>
<th>Utilization factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel in tension</td>
<td>1</td>
<td>1.5</td>
<td>41.87</td>
<td>54.38</td>
<td>1.30</td>
</tr>
<tr>
<td>Steel in shear</td>
<td>1</td>
<td>1.25</td>
<td>110.83</td>
<td>54.38</td>
<td>0.49</td>
</tr>
<tr>
<td>Interaction</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.79</td>
</tr>
<tr>
<td>Concrete cone</td>
<td>0.75</td>
<td>1.5</td>
<td>32.37</td>
<td>217.5</td>
<td>6.72</td>
</tr>
<tr>
<td>Concrete edge</td>
<td>0.75</td>
<td>1.5</td>
<td>90.59</td>
<td>217.5</td>
<td>2.4</td>
</tr>
<tr>
<td>Interaction</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>9.11</td>
</tr>
</tbody>
</table>

Table 5: Overview of results with the analytical design verification of the brace anchorage. Note: \( \alpha_{eq} \) is an additional safety factor applied for seismic actions

6 CONCLUSIONS

This contribution focuses on the use of post installed anchors in concrete for the integration of seismic strengthening measures to existing structures. In the proposed retrofitting scheme, as the case is in numerous retrofitting applications, the connection detail between the existing and the additionally installed structural components plays a critical role in the design. Moreover, the presented investigation discusses the potential provided by highly detailed non-linear FE models in capturing the seismic response of an existing and a retrofitted structure, while the numerical simulations allow for an optimum calibration of the retrofitting scheme and connection detail.

A comparison with laboratory test results, indicates that the behaviour of anchors and the development of localised damage at an anchorage can be predicted very well by the FE simulations. Furthermore, the presented investigations use a full scale experiment on a reinforced concrete frame from international literature, as a basis to validate a non-linear model which also captures the actual response of a concrete frame under earthquake relevant lateral loading. Two different retrofitting configurations by use of steel braces connected to the RC frame are analysed. The first configuration implements a steel brace which yields prior to any damage developing at the anchorage zone. The second configuration uses a stronger brace cross section so as to ensure that failure of the system occurs simultaneously with a failure at the anchorage zone. These two cases form an envelope for the possible interactions between the three component system RC frame – steel brace – anchorage. It is shown, that a resilient anchorage can be formed, which can accommodate the introduction of a light steel brace, in order to considerably increase the existing structure’s stiffness and load capacity, by 20%. Introduction of a steel brace with a substantially larger cross section can lead to a much higher load bearing capacity (250%), but it can compromise the robustness of the retrofitting solution. The benefits of using advanced FE are also highlighted by the fact that current design standards can only offer conservative solutions, if the design relies on simple analytical calculations. Although the performance of the brace anchorage was confirmed by the FE results, it was proven impossible to verify this design based on the currently valid design provisions for anchorages in concrete. It is hereby proposed, that very challenging anchorage design can be largely facilitated by the use of well-built numerical models to minimize required experimental validation.
REFERENCES


ESTIMATING THE LEVEL OF SHEAR WALL CONTRIBUTION IN THE SEISMIC CAPACITY OF EXISTING RC BUILDINGS

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Keywords: Second-level earthquake assessment, Seismic vulnerability assessment, Existing RC buildings, RC shear walls, Earthquake engineering.

Abstract: A pilot study of the degree of relation between the “Shear Wall Contribution Level” (SWCL or $\alpha_T$) which is defined in the framework of the (under preparation) Greek Second Level Earthquake Assessment Methodology, and parameters which measure the participation of RC shear walls in RC buildings’ seismic load capacity, is presented. The parameters which were used for this study are the percentage of the base shear which is received by the RC walls ($\rho_{Vbw}$ ratio), and the ratio of the sum of the moment of inertia of RC shear walls’ cross-sections to the sum of the Moment of Inertia of all vertical members’ cross-sections at the base of building ($\rho_{MIw}$ ratio). In order to achieve the target of the present investigation three RC multistorey and symmetric in plan RC buildings were designed. A series of different versions of each one of these buildings were generated substituting columns with RC shear walls. Based on these samples and on regression analyses, expressions which relate the values of $\alpha_T$ with the $\rho_{Vbw}$ and $\rho_{MIw}$ ratios were extracted. The applicability of these relations was validated using three RC buildings different from the ones which were used for their development. This evaluation led to the conclusion that the relation of $\alpha_T$ with the $\rho_{MIw}$ ratio is relatively high, whereas the corresponding relation with the $\rho_{Vbw}$ ratio is not equally high but is in most cases acceptable. This conclusion is very significant, since the estimation of the $\rho_{MIw}$ ratio requires much simpler calculation procedure than the $\rho_{Vbw}$ ratio.
1 INTRODUCTION

The level of participation of shear walls in the seismic load capacity of RC buildings is a critical parameter of their seismic behavior (see e.g. [1,2]). In the present paper, the term “Shear Wall Contribution Level (SWCL)” is introduced and is defined as the ratio of the maximum shear force developed in the shear walls at the base of an existing building to the total base shear force at the failure stage. It is obvious that for the determination of SWCL for a particular building, recourse to detailed inelastic pushover or time-history analyses is necessary.

However, in the case of methodologies for the approximate assessment of the seismic capacity of existing buildings (such as those adopted in the – currently under final preparation – Greek Second Level Earthquake Assessment Methodology [3], aimed towards the prioritization of the existing building stock for further, more detailed investigations), or even in other, more detailed methodologies (such as the Greek Code of Interventions – KAN.EPE. [4]), the need arises for an approximate assessment of the SWCL without recourse to detailed and computationally expensive analyses.

The present paper is a primary effort to investigate whether such an approximate, yet reliable assessment of the SWCL of a building is feasible. Towards this end, several parametric analyses are performed on selected groups of buildings with different structural characteristics. An index of the degree of the seismic wall contribution of the shear walls for each case is obtained through several alternative approximate procedures, and the results are compared with the respective SWCL obtained from detailed numerical analyses in an attempt to establish correlation expressions between them, which may then be reliably used to estimate the SWCL in existing RC buildings.

2 DESCRIPTION OF THE INVESTIGATION PROCEDURE

2.1. Proposed method for the estimation of the Shear Wall Contribution Level

In the framework of the proposed draft of the methodology of the “Greek Second Level Earthquake Assessment” [3] the “Shear Wall Contribution Level (SWCL)” parameter \(\alpha_T\) is introduced. This parameter possesses significant role in the proposed methodology since it contributes to the grade of a RC building as regards the criteria which concern the influence of “short” columns and the mechanism of the distribution of seismic forces. The proposed expression for the estimation of the (SWCL) parameter is [3]:

\[
\alpha_T = \frac{\sum V_{R/C \_walls}^{R_i}}{V_{R_0}}
\]

Where:
\(\sum V_{R/C \_walls}^{R_i}\) is the sum the maximum shear forces (shear strengths) which the RC shear walls can receive at the base of the building,

\(V_{R_0}\) is the total shear force of the building’s vertical members (columns and shear walls) at the base. For the approximate estimation of \(V_{R_0}\) the proposed expression is [3]:

\[
V_{R_0} = \alpha_1 \cdot \sum V_{columns}^{R_i} + \alpha_2 \cdot \sum V_{R/C \_walls}^{R_i} + \alpha_3 \cdot \sum V_{short \_columns}^{R_i}
\]

Where:
\(\sum V_{columns}^{R_i}\) is the sum the maximum shear forces (shear strengths) which the columns can receive at the base of the building,
\[ \sum V_{Ri}^{\text{short\_columns}} \] is the sum the maximum shear forces (shear strengths) which the “short” columns can receive at the base of the building. 
\[ \alpha_1, \alpha_2, \alpha_3 \] are constants which can take the proposed values of Table 1 (it must be stressed that according to the draft of the Greek Second Level Earthquake Assessment Methodology [3], the values of the parameters are still a subject of research).

<table>
<thead>
<tr>
<th>Types of existing vertical members</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buildings with columns, “short” columns and RC shear walls</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Buildings with columns and RC shear walls</td>
<td>0.7</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>Buildings with columns and “short” columns</td>
<td>0.7</td>
<td>-</td>
<td>0.9</td>
</tr>
<tr>
<td>Buildings with columns</td>
<td>0.8</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Proposed values of the parameters \( \alpha_1, \alpha_2, \alpha_3 \) (expression for the estimation of \( V_{R0} \))

Regarding the estimation of the shear strengths of the vertical members (\( V_{Ri} \)) two cases are considered:

(a) The case of lack of data about the members’ reinforcement: In this case the proposed expression for (\( V_{Ri} \)) is:

\[
V_{Ri} = \left[ 35 \cdot k^{2/3} \cdot f_{ck}^{0.15} + 0.15 \cdot (N/A_c) \right] \cdot b_w \cdot d \tag{3}
\]

Where:
- \( f_{ck} \) is the characteristic compressive cylinder strength of concrete at 28 days (MPa)
- \( A_c \) is the cross-sectional area of concrete (m\(^2\))
- \( b_w \) is the width of the cross-section (m)
- \( d \) is the effective depth (in m) of the cross-section (which is considered equal to 90% of the actual cross-section depth in the current study)
- \( N \) is the axial force of the member due to the load combination \( G+\psi_2 Q \pm E \) (kN)
- \( k = 1 + \sqrt{0.2/d} \)

Alternatively, the expressions which are proposed in the Appendix 7\( \Gamma \) of KAN.EPE (considering that the plastic part of the ductility index \( \mu_{\theta_{pl}} \) is equal to 0) can be used.

(b) The case in which data about the members’ reinforcement are available: In this case the proposed expression for the estimation of (\( V_{Ri} \)) is:

\[
V_{Ri} = \min \left( V_{Rd,s}, V_M \right) \tag{4}
\]

The \( V_{Rd,s} \) is the shear resistance due the shear reinforcement (see e.g. Eq. (6.8) in [5]), whereas the \( V_M \) can be calculated using the following equation:

\[
V_M = 2M_R/L \quad (M_R = \text{the member's flexural resistance}) \tag{5}
\]

In the framework of the present study the Eq. (3) was used.

2.2. Investigation scheme for the relation between (SWCL) and parameters which estimate the participation of RC shear walls in seismic load capacity

In the present study the relation between the SWCL and the following two parameters was investigated:

(a) The percentage of the base shear which is received by the RC walls considering that the building responds elastically (\( \rho_{Vbw} \) ratio):

\[
\rho_{Vbw} = \frac{\sum V_{(R/C\_walls)}}{\sum (V_{base\_columns} + V_{base\_short\_columns} + V_{(R/C\_walls)})} \tag{6}
\]
(b) The ratio of the sum of the moments of inertia of RC shear walls’ cross-sections to the sum of the Moment of Inertia (MI) of all vertical members’ cross-sections at the base of building ($\rho_{MIw}$ ratio).

$$\rho_{MIw} = \frac{\sum \text{MI}_{\text{RC walls}}}{\sum (\text{MI}_{\text{RC walls}} + \text{MI}_{\text{columns}} + \text{MI}_{\text{short columns}})}$$  \hspace{1cm} (7)

All moments of inertia correspond to the cracked cross-sections of members (Table Σ4.1, [4]).

As regards the ratio $\rho_{Vbw}$ it must be stressed that its calculation requires the elastic analysis of the studied building, whereas for the calculation of the $\rho_{MIw}$ ratio only elementary calculations must be performed (on the basis either on in-situ measurements of the dimensions of the cross-sections of the vertical members or on data from the buildings’ drawings, if they are available).

The investigation for the relation between the $\alpha_T$ (SWCL) and the ratios $\rho_{Vbw}$ and $\rho_{MIw}$ was based on a procedure which consists of the following steps:

a) **STEP 1: Selection of RC buildings for the pilot development of relationships between $\alpha_T$ and $\rho_{Vbw}$ and $\rho_{MIw}$ ratios.**

Three (3) symmetric in plan multistorey RC buildings with frame bearing system (i.e. without RC walls) were selected and designed on the basis of the provisions of EC2-1-1 [5] and EC8-1 [2]. The configuration data of these RC buildings are presented in Figure 1.

### 5-storeys building

<table>
<thead>
<tr>
<th>Storey</th>
<th>Columns</th>
<th>Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st, 2nd</td>
<td>0.45/0.45</td>
<td>0.25/0.65</td>
</tr>
<tr>
<td>3rd</td>
<td>0.40/0.40</td>
<td>0.25/0.60</td>
</tr>
<tr>
<td>4th</td>
<td>0.35/0.35</td>
<td>0.25/0.55</td>
</tr>
<tr>
<td>5th</td>
<td>0.35/0.35</td>
<td>0.25/0.55</td>
</tr>
<tr>
<td>6th</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7th</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 7-storeys building

<table>
<thead>
<tr>
<th>Storey</th>
<th>Columns</th>
<th>Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st, 2nd</td>
<td>0.45/0.45</td>
<td>0.25/0.65</td>
</tr>
<tr>
<td>3rd</td>
<td>0.40/0.40</td>
<td>0.25/0.60</td>
</tr>
<tr>
<td>4th</td>
<td>0.35/0.35</td>
<td>0.25/0.55</td>
</tr>
<tr>
<td>5th</td>
<td>0.35/0.35</td>
<td>0.25/0.55</td>
</tr>
<tr>
<td>6th</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7th</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 3-storeys building

<table>
<thead>
<tr>
<th>Storey</th>
<th>Columns</th>
<th>Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st, 2nd</td>
<td>0.30/0.30</td>
<td>0.25/0.50</td>
</tr>
<tr>
<td>C0, C1: 0.35/0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>0.25/0.25</td>
<td>0.20/0.45</td>
</tr>
</tbody>
</table>

**Common data for all the selected buildings:**

- **Storeys’ heights:** $H_i = 3.2m$
- **Concrete:** C20/25 ($E_c = 3.0 \times 10^7 kN/m^2$, $v=0.2$, $w=25 kN/m^3$)
- **Steel:** S500B ($E_s = 2 \times 10^8 kN/m^2$, $v=0.3$, $w=78.5 kN/m^3$)
- **Slab loads:** Permanent G=1.0kN/m², Variable Q=2.0kN/m²
- **Weight per unit of the masonry area:** Perimetric beams 3.6kN/m², Internal beams 2.1kN/m²
- **Design spectrum for calculation of seismic forces:** (Paragraph 2.2.2.5 of EC8-1)
- **Reference PGA $a_{gr}=0.24g$**
- **Importance class:** II $\rightarrow \gamma_i = 1$  \hspace{1cm} $\gamma_d = 0.24g$  \hspace{1cm} **Ground type:** C

**Figure 1:** Data of the three selected symmetric in plan multistorey RC buildings
b) **STEP 2:** *Generation of a series of RC buildings with dual bearing system on the basis of the initially selected RC buildings.*

For each one of the 3 initially selected RC buildings a series of versions i.e. buildings with dual bearing system were configured. More specifically, substitutions of buildings’ columns by RC shear walls were made. The scope of these substitutions is the generation of RC buildings with a wide range of values regarding the $\rho_{Vbw}$ and $\rho_{MIw}$ ratios (Table 2).

<table>
<thead>
<tr>
<th>Version</th>
<th>3-storey building</th>
<th>5-storey building</th>
<th>7-storey building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Columns which are substituted</td>
<td>Cross-sections of the new RC walls</td>
<td>Columns which are substituted</td>
</tr>
<tr>
<td>1</td>
<td>C7</td>
<td>130/25</td>
<td>C3, C18</td>
</tr>
<tr>
<td>2</td>
<td>C7</td>
<td>140/25</td>
<td>C3, C18</td>
</tr>
<tr>
<td>3</td>
<td>C7</td>
<td>150/25</td>
<td>C3, C18</td>
</tr>
<tr>
<td>4</td>
<td>C7</td>
<td>160/25</td>
<td>C3, C18</td>
</tr>
<tr>
<td>5</td>
<td>C6, C7</td>
<td>130/25</td>
<td>C3, C18</td>
</tr>
<tr>
<td>6</td>
<td>C6, C7</td>
<td>135/25</td>
<td>C3, C18</td>
</tr>
<tr>
<td>7</td>
<td>C6, C7</td>
<td>140/25</td>
<td>C2, C4, C17, C19</td>
</tr>
<tr>
<td>8</td>
<td>C6, C7</td>
<td>150/25</td>
<td>C2, C4, C17, C19</td>
</tr>
<tr>
<td>9</td>
<td>C6, C7</td>
<td>160/25</td>
<td>C2, C4, C17, C19</td>
</tr>
<tr>
<td>10</td>
<td>C6, C7</td>
<td>180/25</td>
<td>C2, C4, C17, C19</td>
</tr>
<tr>
<td>11</td>
<td>C6, C7</td>
<td>200/25</td>
<td>C2, C4, C17, C19</td>
</tr>
<tr>
<td>12</td>
<td>C2, C3, C10, C11</td>
<td>130/25</td>
<td>C2, C4, C8, C13, C17, C19</td>
</tr>
<tr>
<td>13</td>
<td>C2, C3, C10, C11</td>
<td>135/25</td>
<td>C2, C4, C8, C13, C17, C19</td>
</tr>
<tr>
<td>14</td>
<td>C2, C3, C10, C11</td>
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<td>C2, C4, C8, C13, C17, C19</td>
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<tr>
<td>15</td>
<td>C2, C3, C10, C11</td>
<td>150/25</td>
<td>C2, C4, C8, C13, C17, C19</td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>-</td>
<td>C2, C4, C8, C13, C17, C19</td>
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<tr>
<td>20</td>
<td>-</td>
<td>-</td>
<td>C2, C4, C7, C9, C12, C14, C17, C19</td>
</tr>
</tbody>
</table>

Table 2: Columns which are replaced by RC shear walls in the three selected RC buildings of Figure 1

c) **STEP 3:** *Analyses of the initial selected RC buildings and of their versions.*

The 3 initially selected RC buildings as well as all their versions were analysed using the equivalent elastic static method (Lateral force method of analysis), following all the corresponding provisions/assumptions of EC8-1 [2] and KAN.EPE. [4]. For the sake of simplicity, the equivalent horizontal seismic forces were applied only along one direction (axis x, Figure 1).
d) **STEP 4: Post-processing of results of the previous step.**

For each one of the versions (Table 2) of the three selected RC buildings (Figure 1) the percentages of the seismic base shear that are received by the RC shear walls \((\rho_{Vbw})\) were calculated. Furthermore, for all these versions the values of parameter \(\alpha_T\) (SWCL) were calculated using Eqs. (1), (2) and (3). Thus, matrices which contain the values of the parameters \(\alpha_T\) and \(\rho_{Vbw}\) for all cases of Table 2 were formed. These tables were the basis for the investigation for the possible relation between \(\alpha_T\) and \(\rho_{Vbw}\). Respective matrices for the investigation of the possible relation between the parameters \(\alpha_T\) and \(\rho_{MIw}\) were also formed. Thus, specific expressions which relate \(\alpha_T\) with \(\rho_{Vbw}\) and \(\alpha_T\) with \(\rho_{MIw}\) were developed. The results of the aforementioned investigations as well as the corresponding expressions of the relations between \(\alpha_T\), \(\rho_{Vbw}\) and \(\alpha_T\), \(\rho_{MIw}\) are presented and evaluated in section 3.

e) **STEP 5: Evaluation of the relations between \(\alpha_T\), \(\rho_{Vbw}\) and \(\alpha_T\), \(\rho_{MIw}\) using testing buildings**

The expressions which relate \(\alpha_T\) with \(\rho_{Vbw}\) and \(\alpha_T\) with \(\rho_{MIw}\) were developed in the previous step using as basis the selected RC buildings of Figure 1 and their versions (Table 2). In the current step, the applicability of these expressions was tested using the 3 RC buildings which are presented in Figure 2.
3 RESULTS OF INVESTIGATION: EVALUATION AND ASSESSMENT OF THE DEVELOPED EXPRESSIONS FOR THE RELATIONS $\alpha_T$-$\rho_{Vbw}$ AND $\alpha_T$-$\rho_{MIw}$

3.1. Development of expressions for the relations $\alpha_T$-$\rho_{Vbw}$ and $\alpha_T$-$\rho_{MIw}$ using the three initially selected RC buildings

In the current subsection the application of the procedure for the development of the testing expressions for the relations between $\alpha_T$-$\rho_{Vbw}$ and $\alpha_T$-$\rho_{MIw}$ is presented. For this development the results from analyses of the three initially selected RC buildings (Figure 1) as well as their different versions (Table 2) were utilized.

The values of the parameters $\alpha_T$, $\rho_{Vbw}$ and $\rho_{MIw}$ (which were calculated performing the procedure that is presented in subsection 2.2) are presented in Table 3.

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Table 3: Values of parameters $\alpha_T$, $\rho_{Vbw}$ and $\rho_{MIw}$ for the versions of the three selected RC buildings of Figure 1

Based on Table 3, the diagrams of Figure 3 were created. From the study of these diagrams it is firstly concluded that the relation between the values of $\alpha_T$ and the values of the parameters $\rho_{Vbw}$ and $\rho_{MIw}$ depends on the number of RC shear walls, as well as on their location in the building’s plan. This conclusion can be extracted from the fact that the points in all diagrams of Figure 3 are grouped in groups on the basis of the buildings’ versions (see the different colors of points).

However, this conclusion does not prohibit an attempt to extract regression equations for the approach of relation between the values of $\alpha_T$ and the values of the parameters $\rho_{Vbw}$ and $\rho_{MIw}$. Thus, the regression analysis which was performed led to second order algebraic equations as it is presented in Figure 3. It is very significant to note that these regression equations relate parameter $\alpha_T$ with parameters $\rho_{Vbw}$ and $\rho_{MIw}$ adequately, bearing in mind that these relations are destined for implementation within the framework of the methodology of the “Greek Second Level Earthquake Assessment” [3] which is a de facto approximate method. As it can be seen from Figure 3, the correlation factor $R$ in every case is greater than 0.85.
Another conclusion which arises from the study of Figure 3 is that no great differences exist regarding the correlation degree between $\alpha_T$ and $\rho_{VBW}$ and between $\alpha_T$ and $\rho_{MIW}$.
The effectiveness of the regression equations of Figure 3 is more clearly illustrated with the aid of Table 4. In this Table the percentage differences between the values of $\alpha_T$ which are calculated using the Eq. 1 and the corresponding values which are calculated using the regression equations of Figure 3 are presented. It must be noted that the red colored values indicate the maximum and minimum values of the percentage differences.

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Table 4: Deviations between the $\alpha_T$ values extracted by Eq. 1 and the corresponding values extracted by the expressions of Figure 3

The main conclusion which arises from the study of Table 4 is that in the vast majority of the examined cases the absolute values of the percentage difference between the values of $\alpha_T$ which are calculated using the Eq. 1 and the corresponding values which are calculated using the regression equations of Figure 3 is less than 20%. This conclusion is significant, since it indicates that the values of the “Shear Wall Contribution Level” ($\alpha_T$) – which are calculated in terms of shear strength – can be reliably related with parameters which are based on stiffness.

3.2. Evaluation of expressions for the relations $\alpha_T$-$\rho_{Vbw}$ and $\alpha_T$-$\rho_{Mbw}$ using the three testing RC buildings

In order to investigate the general applicability of the regression equations of Figure 3 (which were developed in the step 4 of the investigation procedure (see subsection 2.2)), these equations were utilized for the estimation of the values of the “Shear Wall Contribution Level” ($\alpha_T$) for the testing RC buildings of Figure 2. In other words, in the current subsection the reliability of the regression equations of Figure 3 was tested using buildings different from the ones (Figure 1) which were used for their development.
Figure 4: Testing of the applicability of expressions of Figure 3 using the RC buildings of Figure 2

In Figure 4 the percentage differences between the values of \( \alpha_T \) which are calculated using Eq. 1 and the corresponding values which are calculated using the regression equations of Figure 3 in case of the testing buildings of Figure 2 are presented. It must be noted that for the calculation of \( \alpha_T \) by means of Eq. 1 two different methods for the estimation of the axial forces of vertical members (which are required for the estimation of the values of \( V_{Ri} \) through the Eq. 3) were implemented. More specifically, in Figure 4(a) the calculation of \( \alpha_T \) values by means of Eq. 1 was based on the vertical elements’ axial forces which were extracted using their areas of influence, whereas in Figure 4(b) the calculation of \( \alpha_T \) values was based on axial forces which were calculated by means of static analyses. The advantage of the estimation of the vertical members’ axial forces through use of their areas of influence is based on the fact that the procedure of this estimation can be made by means of simple geometrical calculations which are much faster and much simpler than the corresponding estimation by means of elastic analyses.

From the study of Figure 4 it can be concluded that the equations which relate the \( \alpha_T \) with the \( \rho_{\text{Vbw}} \) are generally more effective than the corresponding equations which are based on the relation between \( \alpha_T \) and \( \rho_{\text{MIw}} \). This conclusion is very significant, since the estimation of the \( \rho_{\text{MIw}} \) ratio requires a much simpler calculation procedure than the \( \rho_{\text{Vbw}} \) ratio.

4 CONCLUSIONS

In the current paper the results regarding the investigation of the degree of relation between the “Shear Wall Contribution Level” (SWCL or \( \alpha_T \)) which is defined in the framework of the (under preparation) Greek Second Level Earthquake Assessment Methodology, and parameters which represent the participation of RC shear walls in the RC buildings’ seismic load capacity, are presented. More specifically, the parameters which were used for this investigation are the percentage of the base shear which is received by the RC walls considering that the building responds elastically (\( \rho_{\text{Vbw}} \) ratio), and the ratio of the sum of the moment of inertia of RC shear walls’ cross-sections to the sum of the moment of inertia of all vertical members’ cross-sections at the base of building (\( \rho_{\text{MIw}} \) ratio). The \( \rho_{\text{Vbw}} \) ratio is widely used for the representation of RC shear walls’ participation to the seismic load capacity of RC buildings on the basis of provisions of seismic codes. On the other hand, the \( \rho_{\text{MIw}} \) ratio can be considered as an index which gives a measure of the contribution of RC walls to the total horizontal stiffness of RC buildings. In order to assess the contribution of RC walls to the RC buildings’ horizontal stiffness taking into consideration – ever approximately – the inelastic response of RC members, the cracked cross-sections of all structural members were used for the calculation of
ρ_{Vbw} ratio as well as for the calculation of ρ_{MIw} ratio. The benefits of the reliable relation of \( \alpha_T \) with parameters which are calculated using simple and not time-consuming procedures, are the validation of the expression of \( \alpha_T \) that is proposed in the draft of Greek Second Level Earthquake Assessment Methodology and, the estimation of its value by means of procedures which are compatible to the approximate nature of this methodology.

In order to achieve the target of the present investigation three RC multistorey and symmetric in plan RC buildings were selected and designed on the basis of the provisions of Eurocodes. A series of different versions of each one of these buildings were generated substituting columns with RC shear walls. Thus, RC buildings with different \( \rho_{Vbw} \) and \( \rho_{MIw} \) ratios were formed. Based on these series, regression analyses were performed. As a result of these analyses, expressions which relate the values of \( \alpha_T \) with the \( \rho_{Vbw} \) and \( \rho_{MIw} \) ratios were extracted. The application of these relations was tested using three multistorey and symmetric in plan RC buildings different from the ones which were used for their development. In this testing the axial forces of the vertical RC members (which is required for the calculation of the parameter \( \alpha_T \)) were calculated using two methods, namely the areas of influence of the vertical members and elastic static analyses. The tests led to the conclusion that the relation of \( \alpha_T \) with \( \rho_{MIw} \) is relatively high, whereas the corresponding relation with the \( \rho_{Vbw} \) is not equally high but is in most cases acceptable. This conclusion is very significant, since the estimation of the \( \rho_{MIw} \) ratio requires much simpler calculation procedure than the \( \rho_{Vbw} \) ratio.

Finally, it must be stressed that the results of the current investigation can be considered only as primary, since more generally valid conclusions can be extracted only after further extensive investigations using a greater number and types of RC buildings.

REFERENCES


NONLINEAR NUMERICAL PARAMETRIC STUDY OF DOWELS FOR THE SEISMIC STRENGTHENING OF RC FRAMES WITH RC INFILL WALLS

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Abstract

The parametric study of the contribution of dowels that connect a new reinforced concrete (RC) infill wall to the surrounding RC frame members was performed through nonlinear dynamic analyses of a numerical finite element (FE) model. The FE model was simulated in DIANA finite element analysis (FEA) software in order to study the effectiveness of the seismic retrofitting of existing structures with the conversion of selected bays into new infilled RC walls for the retrofitting of a multi-storey multi-bay RC frame building. A 2D frame was modelled and nonlinear transient analyses were performed in order to simulate the experimental results obtained from a full-scale experiment. The calibration of the FE model that simulated the experimental nonlinear cyclic behavior of the tested RC building is provided in [1]. Based on the calibration results it was concluded that the number of dowels used in the experiment resulted in a monolithic behavior of the RC infilled frame. In order to complement the experimental results and to study the interaction between RC infills and the bounding frame both in the global and local level, numerical simulation experiments were performed by reducing the number of dowels starting from a spacing of 100mm (monolithic) to a spacing of 380mm. Time-history was performed for each case and the results show that the maximum spacing of 380mm is sufficient to provide the required stiffness and ductility. In this paper, the FE model of the test specimen is described and presented along with a parametric study of the number of dowels connecting the wall to the bounding frame. These results contribute to the development of a general model for the application and the design of RC infills in existing RC frames.

Keywords: Finite Element Model, Dynamic Analysis, Dowels, Parametric Study, Seismic Strengthening.
1 INTRODUCTION

The addition of RC infilled walls in selected bays within existing RC frames, especially on the perimeter, is a popular method for seismic retrofitting and a simple and cost-effective method [2, 3]. According to [4] this is the most effective and economic method for retrofitting multi-storey RC buildings, especially those with pilotis (soft-storey). With the full infill of selected bays of an existing RC frame, the effectiveness of the retrofitting is increased, and the construction cost is reduced. The use of RC infill walls with the same thickness as the frame members that bound the wall for retrofitting RC buildings with openings is a relatively new retrofit method, which can be used to increase the strength, stiffness and ductility of the building. However, the RC infills as retrofitting method is commonly applied to guarantee monolithic behavior between the old and new members in order to design the new RC walls according to Eurocode 8 – Part 3 (EC8-3) [5]. This is achieved by the construction of new thicker web than the beams and the columns of the existing frame panel with the location of the new reinforced outside the existing members and the details of reinforcement as in a new wall [2]. In this way, the new infill walls are much stronger than what is needed for the strengthening of the structure, and this ‘over-strength’ causes additional issues like the weakening of the foundations of the existing buildings [2]. Hence, a significant rotation is expected at the foundation [6].

Even though the RC infills is a common retrofitting method and it is extensively applied, it is not addressed quantitatively by the codes, not even by EC8-3 [5]. On the other hand, [6] refers to the introduction of RC infills within a frame, only in terms of forces, providing tools for calculating their deformations (at yield and failure) and stiffness only if they are integral with the bounding frame. Although for other strengthening methods of existing structures there are guidelines regarding the retrofit design and certain aspects of the seismic response of the retrofitted structure, there are still open issues about the studies retrofit method. For example, their interaction with the bounding frame, their design and detailing between the new web and the surrounding frame members need to be regulated [2, 5]. The inadequacy of design codes in this respect is due to our poor knowledge of the behavior of walls created by infilling of a bay of an existing frame with RC. It is apparent that the codes or standards for seismic retrofitting do not provide proper guidance for the design and detailing of the attachment of new walls to existing frames. Furthermore, regulations do not exist for modeling or evaluation of frame bays converted into RC walls depending on the type and details of the connection.

Regarding the experimental research work that has been performed in the last decades, most of the experiments cover sufficiently the other frequently used typed of retrofitting, in particular the use of fibre-reinforced polymers (FRP) and the concrete jackets despite the common field practice of new walls which encapsulate the frame members [7]. There is no adequate experimental research work on the use of RC infill walls and most research has mainly targeted large specimens with high resistance [7]. The tests have been limited to small-scale specimens with new webs thinner than the surrounding beams or columns (possibly owing to the technical limitations of testing walls of very large shear-force resistance) [2, 8]. Another drawback of past investigations is that they did not propose or even follow a quantitative procedure for the design of the connection between the RC infilling and the surrounding frame members. Furthermore, they have not led to, or supported, any procedure or the quantification of the engineering properties of the RC infilled frame which is essential for its analysis and design in the context of modern performance based seismic design, that is the effective stiffness, the moment and shear resistances, the deformation at yielding and the cy-
clic deformation capacity [9]. Subsequently, data is lacking for taller full-scale specimens that reflect real applications [7].

In order to start filling the gap of knowledge regarding infilling of existing RC frames with RC walls, the effectiveness of seismic retrofitting of multi-storey multi-bay RC-frame buildings by converting selected bays into new walls through infilling with RC was studied experimentally through a full-scale pseudo dynamic (PsD) test within the project named SERFIN at the ELSA Laboratory of Structural Assessment (ELSA) facility at JRC, in Ispra. The research was under the project “Seismic Engineering Research Infrastructures for European Synergies” (SERIES). Further details can be found in [8, 10, 11]. This prototype model reflects correctly the real situation and its results and data are very useful. The results from the full-scale experiment that took place within the project SERFIN were studied and data from this test was used for the simulation of RC walls in FE software in order to study the behavior of the RC infills within RC frames.

In this paper the numerical simulation of the frame that was tested in Ispra is briefly described. A full description of the numerical model that simulates and validates the experimental results as well as the comparison of the numerical results to the experimental results can be found in [1]. This validated FE model was used to perform numerical experiments and various parametric studies were developed including the investigation of shear connectors (dowels) contribution. The aim of the FE simulation was a numerical parametric study by varying the number of dowels that connect the existing frame with the new infill wall and the web reinforcement of the wall. A parametric study that covers a range between the monolithic behavior and infilled frame, by varying the number of dowels connecting the wall to the bounding frame was performed and results of this study are presented and described in the following sections.

2 EXPERIMENTAL CASE STUDY

The subject of the project SERFIN was the retrofitting of a multi-storey multi-bay RC frame building by the conversion of selected bays into new infilled RC walls. Two parallel planar frames were infilled with RC infills and then they were unidirectionally pseudo-dynamically tested. The aim of the SERFIN experiment was to study the efficiency of the retrofitting method and to examine the amount of the web reinforcement in the walls and the connection details between the wall and the bounding frame.

The SERFIN specimen was a full-scale four storey prototype building structure that was designed to represent the two exterior three-bay frames of the prototype structure. A detailed description of the specimen geometry is given in [10, 11]. The RC infill walls in the two frames were in the central bays of the specimen and they had the same thickness of 0.25m equal to the width of the beams and the columns framing them. These frames were named North and South as it is defined in Figure 1. Hence, the direction towards the reaction wall of the experiment is East and the one in the opposite direction is West (Figure 1). For this paper, the results of the South frame of the experiment that was simulated and calibrated in DIANA FEA will be presented as the results of the validated model.

In order to facilitate the study of the effect of as many parameters as possible, the two frames of the specimen were reinforced with different amount and arrangement of reinforcement, with the North wall being the strongest of the two. More specifically, an elaborate and varying system of dowels and starter bars was used to join the walls with the frame. The differences are about the diameter and the length of reinforcement mainly in the ground and first floor. In the fourth floor the reinforcement was the same.
Moreover, it is important to mention that the tested model was designed using two different connection details between the new walls and the surrounding frame in order to evaluate the contribution of dowels that connect the new infill wall to the existing RC frame. These details are described in [10–12]. In all cases, the dowels were positioned along the centerline of the elements (i.e. at 0.125m from the face of the wall) and in the first connection scheme they acted as dowels since lap-splice bars were also provided to the wall reinforcement, while in the second scheme they acted both as lap-spices and dowels.

As it was mentioned before, the specimen was pseudo-dynamically (PsD) tested and within the testing campaign two PsD tests and one cyclic test were run. The second 0.25g acceleration test results were used for the calibration of the FE model. More details and results about the calibrated model and verified results of the FE model can be found in [1].

![Figure 1: Elevation of the specimen in the lab. The wall shown on the left is the South wall and the one on the right is the North wall.](image)

The validated model that was previously calibrated in DIANA FEA [1], had the same number of dowels like the SERFIN experiment (24 dowels connecting the wall to the columns and 20 dowels connecting the wall to the beams). It was decided to perform another six different cases of the number of dowels in the model. These parametric-study scenarios are shown in Table 1.

### 3 NUMERICAL SIMULATIONS – PARAMETRIC STUDIES

The experimental results of SERFIN project were complemented through numerical experiments in order to study the interaction between RC infills and bounding frame both in the local and global level. A parametric study that covers a range between the monolithic behavior and that of a non-integral infilled frame, by varying the number of dowels connecting the wall to the bounding frame was performed and is presented here. The second connection scheme was used as described above.

The validated model that was previously calibrated in DIANA FEA [1], had the same number of dowels like the SERFIN experiment (24 dowels connecting the wall to the columns and 20 dowels connecting the wall to the beams). It was decided to perform another six different cases of the number of dowels in the model. These parametric-study scenarios are shown in Table 1.
The same analysis procedure and a 0.25g earthquake record as explained in [1] were used for all the parametric-study scenarios.

4 NUMERICAL RESULTS

The global and local results from the parametric study of the effect of the dowels are presented in this chapter. Specifically, the top storey displacements of the frame and the base shear-forces of the frame for all the case scenarios that were performed are presented. In addition, the dowel’s axial forces that connect the foundation beam to the RC infill will be presented for the first three case scenarios.

4.1 Top Storey displacements

The top storey displacements of the frames are illustrated for all the case scenarios in Figures 2-4. The percentages of the increase or the decrease from the previous case scenario of the storey displacements are shown in Table 2 for all the case scenarios.

![4th Storey Displacements](image)

Figure 2: Top storey displacements for Cases 1,2 and 3.
Figure 3: Top storey displacements for Cases 3, 4 and 5.

Figure 4: Top storey displacements for Cases 5 and 6.

<table>
<thead>
<tr>
<th>Case Scenario</th>
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<td>-31.27</td>
</tr>
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<tr>
<td>Case 6</td>
<td>-24.23</td>
<td>+3.58</td>
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</table>

Table 2: Top storey displacement percentage increase (+) or decrease (-) from the previous case scenario.

It is shown that the top storey displacements in the positive direction are about the same for all case scenarios except the sixth case scenario (two dowels connecting only the beams of the surrounding frame to the RC infill wall) where the top storey displacement is decreased by 24.23% relative to the fifth case scenario. In the negative direction the top storey displace-
ment of the second case scenario is decreased by 31.27\% relative to the first case scenario. Then, the top storey displacement gradually increases for the third, fourth and fifth case scenarios. For the sixth case scenario, where it was previously mentioned that the positive maximum top storey displacement was decreased by 24.23\%, in the negative direction it is at the same level with the fifth case scenario. This is an indication that the building got some permanent deformations in the positive direction in this case scenario which is evident from Figures 2-4.

Another observation that is shown in Figures 5-7, is that the elastic characteristics of the frame have changed with the reduction of the number of dowels after the third case scenario. This is an indication that the stiffness and the fundamental frequency of the frame are reduced, with the reduction of the dowels.

4.2 Base shear-forces

The shear forces at the base of the frame were obtained from the numerical analysis and they are illustrated and discussed in this section. The base shear-force of the infilled frame for all the case scenarios are displayed in Figures 5-9. Furthermore, the base shear-forces of the frame versus the top storey displacements are illustrated in Figures 10-14. These results facilitate the comparison of the infilled-frame stiffness and energy absorption between the various cases studied.

The total base shear-forces shown in Figures 5-14 were obtained through the summation of the base shear of the four columns and the infill wall.

![Figure 5: Base shear-force of the frame for Cases 1 and 2.](image)
Figure 6: Base shear-force of the frame for Cases 2 and 3.

Figure 7: Base shear-force of the frame for Cases 3 and 4.

Figure 8: Base shear-force of the frame for Cases 4 and 5.
Figure 9: Base shear-force of the frame for Cases 5 and 6.

From Figures 5-9, it can be observed that the lower the number of dowels, the lower the base shear-force of the building. The second case scenario resulted to the highest total base shear-forces. The validated model (first case scenario), also reaches high base shear-forces relatively to the other case scenarios. Generally, it is shown that the first four case scenarios reach about the same values of the total base shear-force. The reduction of the maximum base shear-force is more obvious when the dowels are reduced to two.

Figure 10: Top storey displacements versus base shear of frames for Cases 1, 2 and 3.
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Figure 11: Top storey displacements versus base shear of frames for Cases 3, 4 and 5.

Figure 12: Top storey displacements versus base shear of frames for Cases 5 and 6.

From Figures 10-12, it is shown that the stiffness of the building is decreased with the reduction of dowels. In addition, it was observed that the energy absorption of the frame is less with the reduction of the dowels. These observations are more apparent after the fifth case scenario. For the first three case scenarios, the base shear-forces of the frames are higher in comparison with the next case scenarios whereas the top storey displacements are about the same.

It was also observed that for the first three cases the stiffness and the energy absorption of the buildings are about the same in the positive direction, whereas in the negative direction the validated model in the first case scenario clearly takes lower base shear-force and higher top storey displacement than the second, third and fourth case scenarios. For the fifth case...
scenario, even though the frame takes similar (little lower) base shear-force and top storey displacement with the previous case scenarios, the stiffness of the frame is reduced.

In general, it is shown that the stiffness and the energy absorption of the frame are not varying considerably for the first four case scenarios. However, when the dowels are reduced to two the decrease of the base shear-force of the frame is apparent and the stiffness and energy dissipation reduction is obvious.

4.3 Dowels behavior at the base

The axial load in all the dowels at the base interface when the total base shear-force of the frame is maximum (in both directions) is shown in Figures 13 and 14 for the first three case scenarios. As shown in the figures the maximum values occur at different instances during the analysis for each case.

![Figure 13: Dowels axial load at the interface when the base shear of the frame is maximum.](image)

![Figure 14: Dowels axial load at the interface when the base shear of the frame is minimum.](image)
It is shown that the dowels of the validated model in the first case scenario take the lowest forces in comparison with the other case scenarios. The dowels in the second and third case scenarios take higher forces (higher than their capacity). In addition, from the graphs in Figures 13 and 14, it is observed that more dowels reached their yield force with the reduction of the number of dowels.

In Figures 15-17, the axial load distribution in the dowels is shown for each case separately when the frame is moving towards both directions. For the first case scenario in (Figure 15 validated model) and the second one (Figure 16), it is shown that the location of the neutral axis is at about 0.55m for the negative direction and about 1.75m in the positive direction from the edge of the wall. With the reduction of the number of dowels in the third case scenario the position of neutral axis is moving to 0.40m in the negative direction and to 1.80m in the positive direction. This indicates that larger number of dowels have yielded resulting in increase of the lever-arm and is consistent with the results shown in Figures 13-14.

Figure 15: Dowels axial load at the interface when the base shear of the frame is maximum for Case 1.

Figure 16: Dowels axial load at the interface when the base shear of the frame is maximum for Case 2.
CONCLUSIONS

As it was mentioned at the beginning of this paper, even though the strengthening of existing structures with RC infills has been applied and accepted by engineers, further investigation is required regarding their design and detailing. There is no quantitative procedure for the design and construction of the new walls. The contribution of dowels that connect the new infill wall to the surrounding frame members have not been yet analyzed adequately.

Some general conclusions were drawn about this retrofitting method through the numerical experiments in this paper. The parametric study that was performed covers a range between monolithic behavior (first case scenario) and infilled frame, by varying the number of dowels connecting the wall to the bounding frame. From the parametric study of dowels that were performed to the full-scale FE model that was developed in DIANA FEA, it is shown that the dowels affect the behavior of RC infills and the overall shear resistance capacity of the building. More specifically, the results showed that the maximum spacing of 380mm is sufficient to provide the required stiffness and ductility to the building.

From the results of the parametric study, there were several indications that the building had a nonlinear behavior and that the fundamental characteristics of the frame changed with the reduction of the number of dowels after the third case scenario (dowels spacing of 380mm). More specifically, it was observed that the stiffness and the fundamental frequency of the frame are reduced with the reduction of dowels. Generally, it can be concluded that the lower the number of dowels, the lower the base shear-force, the stiffness and the energy absorption of the building. However, it is shown that these characteristics of the building are not varying considerably for the first four case scenarios (dowel spacing of 100mm to 630mm).

On the other hand, the top storey displacements of the frames were about the same. Regarding the local results of the dowels along the interface of the wall and the foundation, it is observed that the dowels of the first case scenario (spacing of 100mm) had the lowest forces in comparison with the other case scenarios, whereas the dowels of the second and third case scenarios reached their yield values (spacing of 210mm and 380mm, respectively). With the reduction of the number of dowels in the third case scenario the position of neutral axis is moving towards the edges of the wall in both directions, which indicates that larger number of dowels have yielded in this case resulting in increase of the lever-arm.

These results complement the experimental results and show that the number of dowels used in the experimental study can be reduced significantly, making the use of this method more cost effective. Further numerical parametric studies will be performed in order to obtain
a better understanding of this structural system that will allow the development of design guidelines.

REFERENCES


NUMERICAL MODELING OF MASONRY-INFILLED RC FRAME STRENGTHENED WITH TRM

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Abstract

The behaviour of masonry-infilled reinforced concrete (RC) frame structures during an earthquake has attracted the attention by structural engineers since the 1950’s. During the last decade, the use of textile reinforced mortar (TRM) as strengthening material for masonry-infilled RC frames under in-plane lateral loadings has been studied.

This paper presents a numerical model of the behaviour of existing masonry-infilled RC frame strengthened with TRM that was studied experimentally at the University of Patras. It was a 2:3 scale three-story structure with non-seismic design and detailing, subjected to in-plane cyclic loading through displacement control analysis. The objective of the present study is to identify suitable numerical constitutive models for each component of the structural system, in order to create a numerical tool to model the TRM strengthened-masonry infilled RC frames in-plane behaviour. A 2D TRM strengthened-masonry-infilled RC frame was developed in DIANA finite element analysis (FEA) software and an eigenvalue and nonlinear structural cyclic analysis were performed.

There is good agreement between the numerical model and experimental results. It has been found that the numerical model has the capability to predict the initial stiffness, the ultimate stiffness, the maximum shear-force capacity, cracking patterns and the possible failure mode of masonry-infilled RC frame with retrofitting. This model proves to be a reliable model of the behaviour of TRM strengthened-masonry-infilled RC frame under cyclic loading and can be used for further parametric studies.

Keywords: Masonry infills, Textile reinforced mortar, Cyclic loading, Finite element, Numerical modelling, Constitutive model
INTRODUCTION

Masonry-infilled RC frame structures are widely dispersed around the world. Past studies have shown that the in-plane strength and stiffness of the infill walls have influence on the global performance of a structure, subjected to seismic loads. The existence of infill walls in a RC frame can increase the strength, stiffness (relative to a bare frame) [1] and lateral capacity of the building [2]–[4] and at the same time it can introduce brittle shear failure mechanisms associated with the wall failure and wall-frame interaction. The failure mechanism and the load resistance of a masonry-infilled RC frame depend on the strength and stiffness of an infill with respect to those of the bounding frame. It is known that masonry structures are vulnerable to both in-plane and out-of-plane movements under the action of lateral loads. The in-plane and out-of-plane behavior of the masonry infill has been studied experimentally [5] and numerically [6]. The out-of-plane failures turn out to be more disastrous than the in-plane ones[7]. The in-plane failure mechanisms of masonry-infilled RC frames are identified according to ATC 43 [8], Asteris et al. [9] and Shing and Mehrabi [10]. The infill wall fails in a variety of modes, most often involving some combination of bed joint sliding, corner crushing, diagonal cracking (due to diagonal orientation of the tensile-compressive principal stress), diagonal compression [11] and frame failure modes. The exact mode of failure depends upon material properties, such as compressive strength, shear strength and friction and upon geometric constraints, such as frame-wall interface or window openings and other characteristics. In addition, infill walls in RC structures may cause several undesirable failure mechanisms under seismic loading due to the large concentration of ductility demand in a few members of the structure. For instance, the soft-story mechanism (concentration of inter-story drift demand and damage is in the first story) [12]), the short-column mechanisms (ductility demands on RC columns) [13]), and plan-torsion mechanisms (when infills are unsymmetrically located in plan). The negative effects are associated with plan or vertical irregularities introduced by the infill panels [14,15]. Fig. 1(a) and 1(b) illustrate the soft-story and short-column failure mechanism respectively.

Figure 1: a) Soft-story mechanism [14] and b) Short-column mechanisms [15].

Seismic rehabilitation of existing structural or non-structural elements is a challenging engineering problem nowadays. Several retrofitting techniques have been proposed in order to increase the strength, stiffness, deformation capacity and the ductility of masonry-infilled RC frame structures [16,17]. Most retrofitting techniques include an external coating or overlay to
one or both sides of the infill wall. The most recent retrofitting techniques include the use of fiber reinforced polymers (FRP) [18,19,20,21,22,23,24,25,26] ductile-fiber-reinforced cementitious composites (FRCM) [27,28,29] and textile reinforced mortar (TRM) [30,31]. TRM jacketing is an extremely promising solution for the strengthening of unreinforced infill walls subjected to either out-of-plane or in-plane loading [5]. In recent experimental and numerical studies provided by Koutas et al. [32,33] the use of TRM for strengthening masonry-infilled frames was studied. The study showed that in the retrofitted specimen an approximately 56% increase in the lateral strength, accompanied with a 52% higher deformation capacity at the top of the structure at ultimate strength state was achieved compared to the unretrofitted one. In addition, the TRM retrofitted specimen dissipated 22.5% more energy compared to the unretrofitted one, for the same loading history. Moreover, it was concluded that further studies are needed on masonry-infilled RC frame strengthened with TRM, due to the fact that this method is a newly established technique and the research is limited on this topic.

The purpose of this paper is to simulate the behavior of masonry-infilled RC frame strengthened with TRM under cyclic loading. To achieve this, 2D TRM-masonry-infilled RC frame model was developed in DIANA finite element analysis (FEA) software, using meso-level approach for modelling the infill wall, and an eigenvalue and nonlinear structural cyclic analysis were performed. The present study identifies suitable numerical constitutive models of each component of the structural system in order to create a numerical tool to model masonry infilled RC frames strengthened with TRM under in-plane cyclic loading. The calibration was based on the experimental test performed by Koutas [32] in his PhD study at the University of Patras.

2 DESCRIPTION OF EXPERIMENTAL CASE STUDY

In the experimental case-study carried out by Koutas et al. [32] the effectiveness of seismic retrofitting of existing masonry-infilled RC frames with TRM was studied. It was a 2:3 scale three-storey masonry-infilled RC frame with non-seismic design and detailing subjected to in-plane cyclic loading. Two masonry-infilled frames were designed and built with and without TRM (control specimen). In this part of the paper some details of the experimental case study regarding the masonry-infilled RC frame with the strengthening material TRM are presented for the benefit of the reader. Full details about the experimental case study can be found in Koutas et al. [32] [33].

2.1 Geometry of TRM strengthened masonry-infilled RC frame

The geometry of the masonry-infilled frame is shown in Fig.(2a). The columns were of rectangular cross section $170 \times 230$ mm and the beams were T-section. The column longitudinal reinforcement consisted of Y12 deformed bars lap-spliced only at the base of the first story. The transverse reinforcement for all concrete members consisted of Y6 plain bars with 90° hooks at the ends. The thickness of the concrete cover to stirrups was 10 mm. The infill had a length-to-height aspect ratio 1.36. The infill wall was constructed from perforated, fired clay bricks. The perforations of the bricks were running parallel to the unit’s length in the x-direction. The infill wall was composed of two independent wythes separated by a 60-mm gap. The wall was supported rigidly by the RC foundation beam plate with dimensions $0.4 \times 0.9 \times 4.0$ m at the bottom of the wall. The strengthening scheme for masonry-infilled RC frame is shown in Fig.(2b). The selection of the strengthening scheme was dictated by the behavior of the unretrofitted specimen. The strengthening scheme includes: carbon-TRM fully wrapped at the ends of columns at the first and second stories, glass-TRM externally bonded on the face of
the infill walls as shown in Fig. (2b) and in total, 11 and 8 anchors per side were placed at equal spaces along the interfaces (Fig.2b).

Figure 2 : (a) Geometry of the masonry-infilled RC frame and (b) Strengthening scheme : textile anchors of first and second story and TRM layer on the faces of masonry infill at the first, second and third story[32].

2.2 Material properties of TRM strengthened masonry-infilled RC frame

For the construction of the RC frame, C25/30 class of concrete was used and the compressive strength of concrete was equal to 27.8 MPa for control specimen and 27.2 MPa for the retrofitted specimen. The modulus of elasticity of the concrete was 24.1 GPa. The longitudinal reinforcement used was class B500C (yield stress equal to 550MPa) deformed steel bars, in the beams and columns, and smooth steel stirrups class S220 (yield stress equal to 270MPa).

Compression and diagonal test on masonry wallets were performed. The compressive strength of the masonry was 5.1 MPa and the elastic modulus of the masonry perpendicular to the bed joints was 3.37GPa. In addition, the mean value of diagonal cracking stress was 0.39 MPa and the shear modulus was 1.38 GPa.

The mortar used as the binding material of the textile was a commercial fiber-reinforced cement-based mortar The mean values of compressive and flexural strength were equal to 18.9 and 4.3 MPa, respectively. The closed carbon-fiber textile TRM jackets were used at the column ends of the first and second story. The tensile strength per running meter and the modulus of elasticity of carbon textile was 157kN/m and 225GPa, respectively. Commercial polymer-coated E-glass textile was used for the infill wall with tensile strength per running meter and modulus of elasticity equal to 115 kN/m and 736GPa, respectively.

2.2 Experimental campaign

The masonry-infilled RC frame with TRM was subjected to a sequence of quasi-static cycles of a predefined force pattern. A history of imposed cycles of displacements was defined to be applied at the top (Fig. 3b), while maintaining an inverted-triangular distribution of forces to all three floor levels until failure (in terms of global response) occurred. A total of seven cycles were finally applied to TRM strengthened specimen. A general view of the test setup is shown in figure below (Fig. 3a). Three servo-hydraulic actuators were mounted on the specimen, one per story. The strong foundation beam was fixed to the strong laboratory floor via 16 prestressing rods to provide specimen full clamping. Gravity loading of 80 kN per story
was considered to represent the fraction of permanent loads concurrent to the lateral loading action.

![Image](image1.png)

Figure 3: (a) Test set up [33] and (b) Imposed cyclic deformation history for all stories [33].

2.3 Experimental results

Free vibration test was performed in RC bare frame and in masonry-infilled RC frame with and without TRM to identify the experimental fundamental period of the structure in each phase of the construction. In order to perform free vibration test, the specimens subjected to a static displacement at the top of the specimen. For all free vibration tests the gravity loading of 80 KN per story was not considered. The fundamental period for masonry-infilled frame with and without TRM was equal to 0.06 and 0.047 seconds, respectively.

![Image](image2.png)

Figure 4: (a) Failure mode of infilled frame (first floor) at end of the test [32] and (b) Base shear force-displacement hysteresis curve for retrofitted specimen [32].

During early loading in masonry infilled RC frame strengthened with TRM, a dense cracking pattern was developed, with inclined cracks close to the corners of the infill panel and of sliding-type cracks at the central region of the panel at the first floor (Fig.4a) and few cracks parallel to the diagonal developed on the second story infill panel. The frame-infill separation occurred at the very early stages of cyclic loading in retrofitted and in unretrofitted specimen and therefore frame-infill separation was not avoided or eliminated after applying the textile layers. The maximum base shear force was attained during the fourth cycle of loading: for the two directions of loading a maximum base shear of 407KN and −395 KN was recorded at
corresponding top displacement of 40 mm and −38 mm (Fig. 4b). After the fourth cycle of loading, the lateral strength was decreasing due to complete debonding of the TRM from the beam surface on the back side of the first story and local crushing at the first story infill at the two upper ends neighboring the columns.

3  FINITE ELEMENT MODELING OF TRM MASONRY-INFILLED RC FRAME

This study used DIANA FEA software to model the masonry-infilled RC frame strengthened with TRM. The proposed meso-model for TRM strengthened masonry-infilled RC frame was implemented in DIANA FEA using available materials, sections and elements. It is important to be mentioned that the TRM anchors were not modeled. The DIANA FEA was selected for modeling, since it provided the elements, constitutive relationships and materials needed for TRM, concrete and masonry infill [34].

3.1  Constitutive model

In DIANA FEA software there are different available material models to simulate the TRM strengthened masonry-infilled RC frame. In this study, most of the material properties are taken from the experimental case study described above and other material parameters were taken from the literature [35,38]. The numerical results were compared to the experimental results and some parameters were adjusted to achieve reasonable results.

The concrete material model that was chosen is the Total Strain Crack model. The Total Strain Crack model [43] describes the tensile and compressive behavior of concrete as shown in Fig. (5a). Besides the definition of basic properties like Young’s modulus, the Total Strain Crack model requires only a small number of engineering parameters such as the tensile (2.15 MPa) and compressive strength based on the Maekawa Fukuura model [43] (27.2 MPa) and the fracture energy in tension (130 N/m). This model has no ability to reduce the stiffness due to early cracking of the concrete section and therefore the modulus of elasticity was reduced to 9.1 GPa. In addition, the tensile strength and fracture energy were obtained by the empirical equations according to the fib model code [35]. In this study, the approach which is used is the
Rotating crack model [36] which is one commonly used approach in which the stress–strain relations are evaluated in the principal directions of the strain vector.

Cyclic performance of RC elements highly depends on the nonlinear response of reinforcing bars under cyclic loading. The Menegotto-Pinto model is a special plasticity model for the cyclic behavior of steel and is available for embedded reinforcements. It consists of a finite stress-strain relationship for branches between two subsequent reversal points and the parameters involved are updated after each load reversal. The modulus of elasticity was 406GPa, and the yield stress was 549MPa and 295MPa for longitudinal reinforcement and stirrups, respectively.

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<td>Shear fracture energy (N/mm)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Friction angle (degree)</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total strain crack model with fiber reinforced concrete</th>
<th>Glass -TRM</th>
<th>Carbon-TRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GPa)</td>
<td>30.00</td>
<td>34.00</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Mass density (Kg/m³)</td>
<td>2400</td>
<td>2400</td>
</tr>
<tr>
<td>Total crack strain model</td>
<td>Crack orientation Rotating</td>
<td></td>
</tr>
<tr>
<td>Tensile behavior</td>
<td>Fib Fiber Reinforced Concrete</td>
<td></td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>2.72</td>
<td>5.57</td>
</tr>
<tr>
<td>Tensile stress point I (MPa)</td>
<td>2.72</td>
<td>5.57</td>
</tr>
<tr>
<td>Strain at point I (%)</td>
<td>0.00009</td>
<td>0.00017</td>
</tr>
<tr>
<td>Tensile stress point J (MPa)</td>
<td>2.72</td>
<td>5.57</td>
</tr>
<tr>
<td>Tensile strain point J (%)</td>
<td>0.0021</td>
<td>0.0011</td>
</tr>
<tr>
<td>Tensile stress point k(MPa)</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Tensile stress point K (%)</td>
<td>0.015</td>
<td>0.007</td>
</tr>
<tr>
<td>Ultimate strain (%)</td>
<td>0.015</td>
<td>0.007</td>
</tr>
<tr>
<td>Crack band width</td>
<td>Rotating</td>
<td></td>
</tr>
<tr>
<td>Compressive behavior</td>
<td>Fib model code for concrete structure 2010</td>
<td></td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Strain at maximum stress (%)</td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td>Strain at ultimate stress (%)</td>
<td>0.035</td>
<td>0.035</td>
</tr>
</tbody>
</table>

The masonry infill wall material model used is the Engineering Masonry model, which is a smeared failure model and it has a total-strain based continuum model that covers tensile, shear and compression failure modes. The Engineering Masonry model describes the unloading behaviour assuming a linear unloading for compressive stresses with initial elastic stiffness. In addition, a shear failure mechanism based on the standard Coulomb friction failure criterion is included in the model. The Engineering Masonry model requires large number of mechanical properties and most of these properties were not measured in the experimental case study. These mechanical properties were taken from the literature. The model is defined in DIANA FEA with the parameters as shown in Table 1. The tensile strength normal to the bed joint was taken as 0.5 MPa according to Lourenço [32] and Rots [33] and the residual tensile strength was specified at 40% of the tensile strength. The tensile strength of the joint is still a subject of
research and therefore the tensile behavior parameters have been assumed according to the information provided by the respective experimental testing reports or related references. The compressive fracture energy and the tensile energy were calculated according to Rots [40]. The cohesion was obtained 1.5 times larger than the tensile strength according to the relation that was proposed by Cur [41]. The shear fracture energy was equal to ten times smaller of the cohesion as proposed by Lourenço [39]. The friction coefficient was chosen so that the ratio between the specimen compressive and tensile strengths was about ten, which ratio is often found for masonry units.

The TRM material model that was chosen is the Total Strain crack model with Fiber reinforced concrete model (fib) for tensile behavior [35,43]. The fib Model Code for Concrete Structures 2010 model was chosen for the compressive behavior of the TRM. Besides the definition of basic properties, like Young’s modulus, the total crack strain model requires input parameters for the composite material behavior in tension and compression. The Fiber reinforced concrete model (fib) was specified as a function of the total strain. With the fiber reinforced concrete curve the cracking of concrete is initiated at the strain when the tensile strength is reached. In addition, the compressive and ultimate strain were obtained from CEB-FIP model code [35]. The model is defined in DIANA FEA with the parameters as shown in Table 2 and the tensile cyclic behavior of glass-TRM is shown in Fig. 5(b).

The gap between the frame and masonry infill could significantly influence the overall behavior of the masonry infill RC frame. An interface gap model, plasticity based and proposed by Lourenço and Rots [38] was used for the interface elements describing the connection between the masonry infill wall and the bounding RC frame. The model includes a tension cut-off for tensile failure (mode I), a Coulomb friction envelope for shear failure (mode II) and a gap mode for compressive failure. The fracture of the interface is controlled by its tension mode, shear behaviour by Coulomb friction behaviour and crushing by the gap in compression mode. One drawback regarding the use of this interface model is the lack of material properties, as no data were available regarding the behaviour of the interface between the masonry infill and the frame. Therefore, it was decided to define the material properties of the interface by fitting the numerical results to the experimental results obtained for the experimental case study. The interfaces normal modulus was 6KN/mm³ and 3 KN/mm³ for perpendicular (y-direction) and longitudinal (x-direction) direction, respectively. The interface shear modulus was 0.06 KN/mm³ and 0.03 KN/mm³ for y-direction and x-direction, respectively. Furthermore, the interface friction was 30º for both directions.

### 3.2 Type of element and mesh

DIANA offers a broad range of element types for modelling brittle and quasi-brittle materials. The concrete frame, masonry infill wall and TRM were modelled with eight-node quadrilateral isoperimetric plane stress elements (CQ16M). The steel reinforcement in the frame was modelled with two-node bar elements and they were connected to the eight-node concrete elements at the two external nodes. Fig. (6a) shows both elements.
The nonlinearity between masonry infill and RC frame zone was introduced with a 2D line interface element. The interface between the infill wall and the frame was modeled by the 3-point line interface element (CL12I) capable of modeling cohesion, separation, and cyclic behavior. The CL12I (Fig. 6b) element is an interface element between two lines in a two-dimensional configuration. The squared mesh is preferred in FE models [42] and therefore in this case study the shape of the 2D elements were kept rectangular with nearly equal sides.

The way that the TRM elements are connected with the infill wall and with the concrete elements influence the TRM-masonry-infilled frame model behavior. The glass-TRM plane stress elements were connected with the nodes of infill-wall plane stress elements. The carbon-TRM plane stress elements were connected with the nodes of concrete plane-stress element. At the interface of the first and second floor in the horizontal direction, the glass-TRM plane stress element was connected to the nodes of concrete plane stress element in order to take into account the anchors (full bond). In addition, in order to take into account the debonding of the TRM from the beam at the first floor, the glass-TRM elements were connected to the nodes of the infill wall element instead of the nodes of the concrete element.

3.3 Type of loading and constrains

The model was loaded with a constant axial load on the top of the columns to simulate the dead load and with imposed cyclic horizontal displacement as shown in Fig. 4(b). The loading process during the numerical analysis simulated as closely as possible the experimental loading by using point prescribed deformation load. All nodes at the base of the masonry-infill RC frame were restrained by preventing any translation in the x and y-directions to simulate the strong foundation beam that was used in the experimental case study.

3.4 Type of analysis and convergence

Two types of analysis were performed: eigenvalue analysis and nonlinear structural cyclic analysis (deformation control). To perform nonlinear cyclic analysis a two phased analysis was selected. In the first phase the self-weight and the additional dead load of the structure were imposed. In the second phase, a quasi-static implicit, material non-linear analysis was performed with the a secant iteration scheme. The automatic incrementation procedure is used in which both the number of steps and the corresponding step size are automatically computed. The energy-based convergence criterion is applied with standard DIANA FEA tolerance values (0.0001). The continuation option was activated. The numerical model was calibrated to the experimental results by varying the parameters of the engineering masonry and interface mode.

4 FINITE ELEMENT MODEL RESULTS

In this part of the paper, the results of the eigen value analysis and nonlinear structural cyclic analysis are presented. The fundamental period of the bare frame and of the masonry-infilled RC frame with and without TRM is presented in Table 3 and they are in good agreement with the experimental ones.

<table>
<thead>
<tr>
<th>Fundamental period (Seconds)</th>
<th>Bare frame</th>
<th>Masonry-infilled RC frame</th>
<th>TRM strengthened Masonry-infilled RC frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>0.24</td>
<td>0.06</td>
<td>0.047</td>
</tr>
<tr>
<td>Model</td>
<td>0.23</td>
<td>0.062</td>
<td>0.049</td>
</tr>
</tbody>
</table>
The global results obtained from TRM-strengthened masonry-infilled RC frame model, subjected to cyclic nonlinear analysis, are shown in Fig. 7(a), which illustrates the experimental (black line) and numerical model (red line) response curves for the TRM masonry-infilled frame. In addition, the base shear in relation to the load step and the top story displacement vs. load step are presented in Fig. 7(b) and Fig. 7(c), respectively.

![Figure 7: Comparison between experiment and model results in terms of (a) base shear-top floor displacement hysteric curves, (b) base shear in relation to the load step and (c) third story displacement in relation to the load step.](image)

A comparison between numerical and experimental results for masonry-infilled RC frame is given in Fig. 8(a) and (b) in terms of global lateral stiffness and cumulative hysteretic energy, respectively. As illustrated in Fig. 8(a) and (b) the agreement between modeling and test results is satisfactory.

![Figure 8: Comparison between analysis and experimental results for masonry-infilled frame in terms of the (a) global lateral stiffness per cycle and (b) cumulative global hysteretic energy.](image)
Numerical results and experimental data of the TRM strengthened masonry-infilled RC frame have been compared and are in excellent agreement with the experimental ones regarding initial stiffness, ultimate stiffness, maximum shear force capacity and energy absorption in a cycle.

Based on the results from Fig.(8c) and Fig.(9b), the shear-force capacity and the energy absorption for the last cycle of loading is overestimated 15% and 12% respectively. This might depend on the analysis convergence and on the nonlinearities that were introduced in the last cycle of unloading.

Figure (9a) shows the cracking that occurred at the first floor during the fifth cycle of loading and unloading in the experimental case study and Fig. (9b) shows the crack width in the numerical model during the fifth cycle of loading and unloading. The crack width in the numerical model shows that the cracks have the same location as observed in the experiment.

![Figure 9](image)

(a) Crack patterns of TRM strengthened masonry-infilled frame test specimen and (b) crack widths in the numerical model during the fifth cycle of loading (positive and negative).

The results from the numerical model show the damage of the first story after the end of the test (Fig.10), which is the same damage that was observed in the experiment upon test completion.
5 CONCLUSION

This paper presents a numerical model that simulates the nonlinear cyclic-behavior of a TRM strengthened masonry-infilled RC frame subjected to in-plane actions in DIANA FEA software. The present study identifies suitable numerical constitutive models of each component of the structural system in order to create a numerical tool to model masonry infilled RC frames strengthened with TRM under in-plane cyclic loading. The calibration was based on the experimental test performed by Koutas [32]. Some of the material properties, especially for the masonry and for the interface were obtained from the literature [35,38].

Simulation results of the TRM strengthened masonry-infilled RC frame have been compared to the experimental ones with excellent agreement regarding the fundamental periods the initial stiffness, the ultimate stiffness and the maximum shear-force capacity. The crack-patterns show in general good agreement with the experiment, with respect to location and orientation of the cracks. TRM jacketing proved by experimental and numerical studies that is effective in large shear deformations through the development of a multi-crack pattern and by introducing an efficient load transferring mechanism at the local level.

It can be concluded that this model is a reliable model of the behavior of TRM strengthened masonry-infilled RC frame, with acceptable mismatch between the test and simulation results is observed. In particular the energy absorption and maximum shear-force capacity in the last cycle of loading is overestimated compared to experimental results, which may be attributed to high nonlinearities at that stage. In the future, this proposed numerical model which simulates the nonlinear behavior of TRM strengthened masonry-infilled RC frame will be used to perform numerical experiments through a parametric study to quantify the effect of critical parameters which are capable of affecting the performance of masonry-infilled structures reinforced with TRM. This will expand the results’ database and will allow the development of design guidelines for a new strengthening technique on masonry-infilled RC frames using TRM.

6 REFERENCES


SEISMIC ASSESSMENT OF EXISTING URM BUILDINGS IN CODES: COMPARISON BETWEEN DIFFERENT LINEAR AND NONLINEAR STATIC PROCEDURES

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Abstract

Nonlinear static procedures are considered, at research level, the reference method for the seismic assessment of existing masonry buildings. However, at engineering practice level, the use of linear static procedures would be desirable, due to repeatability of results. The updated version of the Italian building code was released in 2018 and the review of Eurocode 8 Part 3 is going to be concluded. The aim of the paper is to analyze the outcome from the new linear static procedures and to compare it with the new nonlinear static procedures, when applied for the seismic assessment of existing unreinforced masonry buildings. This comparison is undertaken for both the new versions of the Italian and the European codes. Modelling issues such as the explicit consideration of the role of spandrels and of the flange effect will be also discussed.

Keywords: seismic assessment, linear static procedures, unreinforced masonry buildings, in-plane global response.
1 INTRODUCTION

A new generation of codes is being developed in Europe. The new version of the Italian code was released at the beginning of 2018 [1] and a draft of the new version of Eurocode 8 is currently available [2]. If compared with the older versions, several modifications are foreseen for the seismic assessment of existing unreinforced masonry (URM) buildings.

The aim of the paper is to apply the new linear static procedures adopted in the above cited codes for the in-plane global seismic assessment of existing URM buildings and then to compare the results obtained with the new nonlinear static procedures.

These procedures are applied to a simple 3-storey case study building that is representative of the residential URM buildings commonly present in Italy and in southern Europe. The case study building, described in Section 3, was modelled by using the equivalent frame approach technique and the consequences due to different modelling choices were also analyzed, such as: 1) the explicit modelling of URM spandrels or the consideration of cantilever walls connected by struts; 2) the contribution or neglection of the flange effect between orthogonal walls.

The main results, presented in Section 4, show that although the linear procedure was significantly improved, taking into account the research achievements of the last decade, it is still very conservative if compared with the nonlinear static one.

2 LINEAR AND NONLINEAR ANALYSES IN THE NEW ITALIAN AND EUROPEAN CODES

The last version of the Italian code [1] was published in 2018, however its commentary that contains several indications in addition to some strength criteria for the structural elements of existing URM buildings was published only few months ago [3]. The Eurocode 8 is currently under revision, however drafts of the revised versions are available: for Eurocode 8-1 [4] and Eurocode 8-3 [5].

The aim of the seismic assessment is to evaluate if the seismic capacity of a buildings is higher than the seismic demand in the specific site. The nonlinear static analyses (discussed in Section 2.2) are conducted in terms of displacements, thus considering the global behavior: it is checked that the maximum nonlinear displacement demand of a building is smaller than its displacement capacity. On the contrary, the verification through linear static analyses (discussed in Section 2.1) are usually conducted in terms of forces, properly reduced by the behavior factor: it is monitored that each structural element is not subjected to a force higher than its strength (therefore the verification is at element level). However, in [5], also the verification through linear static analysis is made through unreduced seismic actions in terms of deformation, but still at element level.

2.1 Linear static analysis

The linear static procedure consists in applying a system of static forces in each principal horizontal direction. For both the Italian and the European codes the total amount of these forces is based on the fundamental period of vibration of the building for lateral motion in the direction considered. Both codes proposed simplified equations in order to computer the fundamental period a building, however, since the aim of the present research is also to compare the results between the codes, the period of the first mode from a modal analysis was assumed as fundamental period for the procedures of both codes, this in order to reduce the number of variables that can influence the final result.
[1] and [5] adopt different philosophies for the linear analysis. The first adopts the so-called $q$-factor approach i.e., the design forces to be applied to the building under analysis derive from the elastic forces reduced by a factor $q$, and the safety check is conducted in terms of forces on any structural elements. On the contrary, the new Eurocode 8 allows also to check the occurrence of the limit state in a single element but in terms of deformation, with the condition to respect some force limitations. More precisely, also in [5] is allowed a safety check only in terms of forces ($q$-factor approach) but it may be applied only for the Significant damage SD limit state (and not for the near collapse - NC); the safety checks in terms of deformations is called Linear elastic analysis (LEA) and may be applicable for the verification of all the limit states.

Both the new Italian and European codes updated their strength criteria for both the piers and the spandrels of masonry walls and they distinguish two cases: 1) the masonry is formed by a regular pattern of brick, bed and head joints (regular masonry); 2) the masonry is formed by rubble stones (irregular masonry). The commentary of the new Italian code, [3], in case of full brick masonry with lime mortar, suggests the mechanical properties for both the regular and irregular masonry and they are reported in Table 1.

<table>
<thead>
<tr>
<th>Mean compressive strength [MPa]</th>
<th>Mean shear strength in absence of axial stress [MPa]</th>
<th>Young’s modulus [MPa]</th>
<th>Shear modulus [MPa]</th>
<th>Specific weight [kN/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>for irregular masonry ($\tau_0$)</td>
<td>for regular masonry ($f_{v0}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.45</td>
<td>0.09</td>
<td>0.20</td>
<td>1500</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 1: mean mechanical properties for full brick masonry with lime mortar as suggested in [3]

For both the regular and the irregular masonry it is necessary to conduct the flexural and the diagonal shear safety checks, then, for piers, in presence of regular masonry it is necessary to check also the shear sliding.

2.2 Nonlinear static analysis

The nonlinear static procedure requires the definition of proper constitutive laws for piers and spandrels. Differently than the past, both the new Italian code [1] and, in particular the draft of the new Eurocode 8-3 [5], adopt multilinear constitutive laws for both piers and spandrels, and the latter, with respect to the previous versions of the codes, are the structural elements that were updated the most. The load patterns to be applied to the building model and the procedure to convert the MDOF response into the equivalent SDOF are rather similar, the most notable difference is in the procedure adopted to evaluate the equivalent bilinear from the pushover curve. On the contrary, significantly different is the procedure adopted to evaluate the position of the limit states. Specifically, in [3] it is stated that, for existing masonry buildings, the ultimate displacement capacity $\Delta_u$ for the NC limit state should be taken as the lower control node displacement for which one the following conditions is attained: 1) the total base shear has dropped below 80% of the peak resistance of the structure or 2) when all the piers at any level of any masonry walls reaches a defined damage level. In [5], in addition to the two conditions above mentioned, it is stated that the NC condition is reached also if one masonry element has reached a drift ratio corresponding to 1.5 of that corresponding to the NC damage level. Then, and this is the main difference between the two codes, in [5], a partial factor is applied to the displacement capacity, i.e. the displacement capacity $\Delta_{NC}$ is evaluated as
\[ \Delta_{NC} = \Delta u / \gamma_{RD} \]  

where \( \gamma_{RD} \) is the partial factor accounting for uncertainty in the displacement capacity at NC and ranges from 1.7 to 1.9 based on the knowledge level assumed.

Then, with regard to the Italian code, the displacement at life safety is equal to 3/4 the displacement at NC, instead, in [5] no specific rules are defined in order to evaluate the displacement corresponding to the SD, but it is written: “the displacement capacity \( \Delta_{SD} \) for the SD limit state should be defined in terms of displacement, between the elastic limit and the ultimate state by using directly the displacement capacities at damage limitation and NC limit states, in order to include directly the effects related to uncertainties”. Therefore, in the present research, the displacement capacity \( \Delta_{SD} \) was computed as the average between the \( \Delta_{NC} \) and the displacement at the damage limitation.

Finally, the capacity is compared with the nonlinear displacement demand, and both codes adopt the N2 method as proposed by Fajfar [6].

### 3 MODELING STRATEGIES ADOPTED

Both the linear and nonlinear static procedure were applied to a rather simple case study building that is representative of the typical existing masonry buildings present in Italy and, more generally, in the Southern European countries. The base model, named Model 0 and depicted in Figure 1, is represented by a three-story full clay masonry building with lime mortar, rigid diaphragms and steel tie rods at each level. This model was already adopted and tested in previous researches, for a complete description of the geometry and the loads applied refer to [7]. It was modeled through the use of the equivalent frame approach and the use of the software Tremuri [8], however, for both piers and spandrels, the constitutive laws adopted in the codes above mentioned were used, therefore the results obtained and discussed in Section 4 would be likely similar if another software was used. It is worth noting that, in order to model an URM building, it is obviously also possible to use a finite element approach, however, the codes above mentioned adopt safety checks at the scale of the structural element, therefore it would be necessary to define the cross sections where to undertake the safety checks a posteriori.

[Figure 1: geometry of Model 0. Plan view (measurements in centimeters) and 3D view showing the equivalent frame idealization (orange = piers; green = spandrels; blue = rigid nodes).]

Both [1] and [5] do not provide specific rules on how to take into account the flange effect, moreover the possibility to neglect the masonry spandrels and model the piers as cantilevers
extending over the full height of the building is allowed. Therefore three further models were defined, for a total of four models, Table 2 summarizes their main characteristics.

<table>
<thead>
<tr>
<th>Spandrels modelled</th>
<th>Flange effect modelled</th>
<th>Period 1st mode [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 0</td>
<td>X</td>
<td>0.30</td>
</tr>
<tr>
<td>Model 1</td>
<td>X</td>
<td>0.33</td>
</tr>
<tr>
<td>Model 2</td>
<td>X</td>
<td>0.50</td>
</tr>
<tr>
<td>Model 3</td>
<td></td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table 2: main characteristic of the models analyzed

For all the models of Table 2 both the two type of masonry introduced in Section 2 were used, specifically in Section 4 the regular masonry is indicated by the letters reg whereas the irregular masonry by the letters irr.

Finally, it is worth highlighting that, as it is shown in Figure 1, most of the diaphragms loads only on the walls in X-direction, as a consequence the piers in Y-direction, at top story are subjected to a very low axial load. It was therefore decided that a small amount of their weight (10%) should be borne also by the walls in Y-direction. This choice, consistent from a structural point of view, was necessary because otherwise the shear sliding and the rocking strength computed on the cross section at the top of the piers at top story would be almost zero and therefore all linear analyses would have reached the collapse condition immediately for any horizontal load applied.

4 DISCUSSION OF THE MAIN RESULTS

In this section the results derived by the application of linear and nonlinear analyses as adopted in NTC18 [1] and proposed in EC8-1dr [4] and EC8-3dr [5] are discussed. In order to compare the results provided by the different methods, the reference parameter adopted is the intensity measure (IM) that causes the attainment of the considered limit state. Indeed, the IM synthetically summarizes the outcome of a seismic assessment procedure regardless of the complexity of the method of analysis. In the present research the PGA is the IM adopted and in the figures of this section the unit used is m/s². The limit state studied in the present research is the life safety limit state of [1] and the significant damage of [5]. This choice derives from the need to conduct a consistent comparison between the linear and nonlinear analyses, indeed the behavior factor (q) values used to undertake the linear analyses, adopted by the codes under study, were calibrated for the limit states above mentioned.

Figure 2 shows the results (in terms of IM) for the linear static procedure as adopted in: 1) NTC18, and 2) the q-factor approach and 3) the linear elastic analysis (LEA) adopted in the draft of the new EC8. It is possible to observe that often there are no differences between the regular and irregular masonry, it means that in those cases the flexural mechanism dominates the most vulnerable element response; then that several IMs are equal to zero. It means that the strength capacity of at least one element was reached already due to the application of the self-weight, it happened mainly when the EC8 was used, because: 1) the behavior factor adopted in the EC8 is significantly smaller (q = 1.5) than that of Italian code (q = 3.4); 2) the EC8 strength criteria are more conservative than the Italian code, i.e. the partial factors are applied to the whole shear strength of the structural element, rather than to only the strength of the material. However, by using both the European and the Italian codes, the IMs obtained are rather small, they never exceed 1 m/s². Finally, contrarily to what is expected, when the EC8 is applied, the spandrels usually are not the weakest elements.
Figure 2: IM (PGA in m/s$^2$) computed by using the linear static procedures adopted in NTC18 and in the draft of new EC8.

Figure 3 shows the results (in terms of IM) for the nonlinear static procedure as adopted in NTC18 and the draft of EC8. The coefficients $\Gamma$ and $m^*$ computed for the models analyzed are equal to: (1.36, 371954 kg), (1.38, 358384 kg), (1.39, 327876 kg) and (1.40, 315592 kg) for models 0, 1, 2 and 3 respectively. Differently than linear analysis, when the nonlinear analysis is adopted differences arise between the regular and irregular masonry, specifically, with the exception of Model 3, the irregular masonry provides higher IMs. This could seem not consistent with the reality, it is expected indeed a higher resistance in a regular masonry. It happens because for regular masonry also the shear sliding strength criterion needs to be undertaken and it results the most conservative. The Italian code always provides higher IMs than the European one (from 1.5 to 2.3 times higher), this is due to: 1) the significant damage limit state results more conservative than the life safety, since the first was computed as the average displacement between the near collapse and damage limitation displacements, on the contrary, the displacement at life safety was computed as $3/4$ the near collapse one; 2) the ratios between the elastic force over the equivalent bilinear yield strength are higher by applying the Italian code procedure than the EC8 one.
Figure 3: IM (PGA in m/s²) computed by using the nonlinear static procedures adopted in NTC18 and in the draft of new EC8.

It is not possible to define a trend, indeed there is not a model that is assessed as the strongest with all the nonlinear procedures analyzed: moving from Model 0 to Model 3, the strength reduces since the walls behave as independent cantilever beams rather than a frame, but the ductility significantly increases. Then, despite the constitutive laws for piers and spandrels adopted in the Italian code and the European code are different, the pushover curves obtained are almost identical, at least up to the first base shear reduction. In almost all cases the Eurocode-pushovers show a higher displacement capacity, this is due to the higher ultimate drifts adopted for masonry elements.

5 CONCLUSIONS

Comparing the results obtained by applying the linear static and the nonlinear static procedures proposed in the new generation of Italian and European codes, it is evident that it still remains a strong difference in the final outcome between the two types of procedures.

For the rather simple models discussed in the present paper the nonlinear procedure provides IM values corresponding to the attainment of the ultimate limit state that ranges from two up to five times higher when the draft of the new EC8 is adopted and from three up to ten times higher when the Italian code is used.

The fact the linear static procedures are overly in favor of safety comes from the use of a verification at local element level, which is not able to consider the global collapse mechanism that forms at ultimate limit states. Indeed, for an existing building it is not possible to assume a-priori a high value of the behavior factor and, for masonry buildings, of the overstrength ratio, which are actually not small. The use of a verification in terms of deformation overcomes only partially this critical issue. Results closer to the ones achieved with nonlinear static procedures may be obtained by using simplified models, which neglect spandrels and/or the flange effect; however, in this case, the model is conventional and far from the actual building behavior. It is worth noting that the seismic assessment of an existing masonry building in the present state is, usually, the first step for the design of strengthening intervention; therefore, the identification of the actual vulnerabilities should be advisable for the adoption of effective mitigation measures.
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REFERENCES


CALIBRATION OF THE HELLENIC SECOND-LEVEL SEISMIC CAPACITY PROCEDURE

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Abstract

In an effort to assess the integrity and the functionality of all the existing reinforced concrete (R.C.) buildings in Greece against moderate and strong earthquakes as a prerequisite for the seismic upgrade of the vulnerable cases the Hellenic Earthquake Planning and Protection Organization is preparing a comprehensive framework for second-level seismic assessment, allowing the rapid estimation of potential damages against the design earthquake for both individual structural members and the entire building and the approximation of the seismic capacity of the examined building. This paper investigates the accuracy and the ease in use of two recently developed second-level seismic assessment procedures, the Rapid Seismic Assessment Procedure for R.C. Buildings (RSA) and the Second-level Seismic Capacity Procedure for R.C. Buildings. The investigation is performed through application of the two assessment procedures in the case of a three-storey, full-scale, R.C. building, tested in the European Laboratory of Seismic Assessment (ELSA) of the Joint Research Center (JRC) in Ispra, Italy, in the framework of the Seismic Performance Assessment and Rehabilitation (SPEAR) European research program and by comparing the assessment results with the actual seismic response of the building.

Keywords: Second-level seismic assessment, Reinforced concrete buildings, Brittle failure modes, SPEAR building.
1 INTRODUCTION

A pressing social need in all countries that suffer from high seismicity is to check the integrity and functionality of existing buildings against moderate and strong earthquakes. Greece is facing a significant problem in managing the seismic risk of its existing building stock, among which more than 75% consists of old, reinforced concrete (R.C.) buildings, built until the early 1980s. These buildings were constructed using design codes of a former era, when the implications of capacity design philosophies were inadequate and were designed to seismic hazard levels below the current ones. Despite the fact that existing R.C. buildings possess a substantial amount of strength reserves, depending mainly on their redundancy and on the over-strength of individual members, existing R.C. buildings are vulnerable to strong earthquakes. To make things worse, many of these buildings suffer from deterioration due to their exposure to corrosive agents (either due to pollution or by exposure to airborne chlorides in the vast coastal regions of the country) setting a very negative background in terms of structural condition, for seismic upgrading and retrofit. Strong earthquakes which occurred in the past in different locations throughout Greece (Kalamata 1987, Kozani & Grevena 1995, Aigion 1995 [1], Athens 1999, Lefkada 2003 & 2015, Western Peloponnese 2008, Cephalonia 2014, Epirus 2016, Zakinthos 2018) demonstrate the deficiencies and the vulnerability of many of the existing R.C. buildings (Figure 1).

Figure 1: Structural damaged developed in R.C. buildings in Greece during past strong earthquakes (available from www.oasp.gr).
So far seismic assessment of existing R.C. buildings in Greece is performed by using first- and third-level assessment procedures. First-level assessment procedures involve rapid visual screening (RVS) procedures [2-3], during which experts first gather all available documentation and data on the buildings which will inspect and then go out into the field to quickly collect crucial structural characteristic information and other parameters, usually in the form of data collection sheets comprising yes/no answers. Rapid visual screening procedures are suitable for identifying a set of buildings that are potentially in the highest seismic risk group and are not suitable for determining if a specific building is, or is not, at seismic risk. They help to screen a building stock and identify the group of buildings that are more likely to be damaged during an earthquake and, therefore, need to be further investigated through checking the seismic capacity more precisely using a more detail seismic assessment. On the other hand, the third-level assessment procedures [4-8] involve simulation of the examined buildings as 3D finite element models and the performance of elastic or/and inelastic analyses procedures. These procedures yield results of the upmost possible detail in terms of estimated seismic response and capacity and they require high specialization on behalf of the Engineer conducting the assessment and high computational cost.

Acknowledging the problem of managing the seismic vulnerability of Greece’s built environment the Earthquake Planning and Protection Organization (EPPO) formed a Supervisory Committee with the task to prepare a comprehensive framework for second-level seismic assessment of all existing buildings, which will serve as a practical guide for the Hellenic State to allocate its available financial resources for the seismic upgrade of the vulnerable buildings. This framework will have the form of a second-level capacity procedure that similarly to the first-level assessment procedures will allow the in situ screening of potentially hazardous R.C. buildings, yet, rather than using a global vulnerability score the screening process will be based on the estimation of potential damages against the design earthquake for both individual structural members and the entire building, allowing the approximation of the seismic capacity of the examined building. To do so, simple calculations based on the knowledge of geometrical and material properties will be acquired, without losing the capability of rapid application to existing structures.

The present study investigates two second-level assessment procedures which are currently under investigation by the EPPO committee for the accuracy of their results and their ease in application, the Rapid Seismic Assessment Procedure for R.C. Buildings (RSA) [9-11] and the Second-level Seismic Capacity Procedure for R.C. Buildings [12]. The investigation is based on a pilot application of the two examined procedures in the case of a three-storey, full-scale, old-type R.C. building, tested in 2003 in the European Laboratory of Seismic Assessment (ELSA) of the Joint Research Center (JRC) in Ispra, Italy, in the framework of the Seismic Performance Assessment and Rehabilitation (SPEAR) European research program [13-15].

2 RSA PROCEDURE FOR R.C. BUILDINGS

The RSA procedure for R.C. buildings [9-11] has been developed to facilitate practitioners to rapidly identify existing R.C. buildings that are potentially hazardous in the event of a strong earthquake. Seismic assessment of buildings is conducted in terms of identifying the prevailing mechanism of failure of the structural elements of the critical storey of the examined buildings and then calculating the seismic capacity in terms of peak ground acceleration (PGA) that the examined buildings can sustain without developing any structural damages, or/and in terms of limiting value of inter-storey drift that can be developed at the buildings’ critical storey at the excess of their elastic response. The procedure, which is presented in the following sections, is consisting of three steps, namely:
- **Step 1**: Approximation of the deformed shape of the examined building at its peak seismic response and identification of its critical storey.
- **Step 2**: Identification of the prevailing failure mode of the vertical structural elements and calculation of the corresponding limiting resistance ratio.
- **Step 3**: Approximation of the PGA and/or the average drift ratio that the critical storey of the building may tolerate before developing structural damages.

### 2.1 Approximation of the deformed shape of the examined building at its peak seismic response and identification of its critical storey

The deformed shape that the examined R.C. building possesses at the instant of its peak seismic response may be approximated using different alternatives, depending on the required level of accuracy and the computation effort.

For buildings with typical plan configuration in all storeys the simplest alternative for approximating their deformed shape at the instant of peak seismic response is to assume one of the three deformed shapes illustrated in Figure 2, depending on the structural system of the examined R.C. building [16, 17]. Figure 2(a) corresponds to old-type R.C. buildings, usually design and constructed until the early 1980’s, comprising columns of small cross section dimensions and beams of significant height. This combination of weak columns – strong beams leads to concentration of lateral displacements in the lower storeys of the buildings at the instant of their peak seismic response. In the case of Pilotis type R.C. buildings, a conservative approach of the buildings’ deflected shape at peak seismic response is the assumption that all lateral deformations are concentrated in the buildings’ soft storey (Figure 2(b)), whereas, in the case of R.C. buildings designed and built according to modern seismic codes inter-storey drift may be assumed to be spread evenly among all storeys (Figure 2(c)).

![Figure 2: Simplified approximations of the deformed shapes at peak seismic response of (a) old-type, (b) pilotis-type and (c) contemporary R.C. buildings.](image)

A more accurate approximation of the deflected shape that up to four storeys high R.C. buildings with constant storey mass possess at the instant of their peak seismic response may derive from application of the expressions of Table 1 [9] with reference to the \( i \)-th storey translational stiffness, \( K_i \).
Table 1: Equations for approximating the deformed shape of up to 4 storey high buildings with constant storey mass at the instant of their peak seismic response.

The translational stiffness of the \( i \)-th storey of R.C. buildings with rigid diaphragms, \( K_i \), may derive as the sum of the translational stiffness of the \( N_c \) columns and the \( N_{inf} \) infill masonry piers of the corresponding storey \([18]\):

\[
K_i = \sum_{j=1}^{N_c} \alpha_{c,j} \cdot \frac{E_c}{H_{cl}} \cdot A_{col,j} \cdot \left( \frac{h_j}{H_{cl}} \right)^2 + \sum_{k=1}^{N_{inf}} A_{inf,k} \cdot \frac{f_{inf}^m}{\mu_{y,k} \cdot \theta_{y,k}^m \cdot \sqrt{1 + \frac{H_{inf,k}^2}{L_{inf,k}^2}}} \tag{1}
\]

where, \( E_c \) is the elastic modulus of concrete (\( \approx 4500 \cdot \sqrt{f_c} \), where, \( f_c \) is the concrete compressive strength), \( \alpha_c \) effectively reduces \( E_c \) in order to account for the extent of cracking (\( \alpha_c = 0.33 \) for \( 0 \leq \nu_{G+\psi/E Q} < 0.10; \alpha_c = 0.5 \) for \( \nu_{G+\psi/E Q} \geq 0.10; \alpha_c = 0 \) for columns in tension), \( \nu_{G+\psi/E Q} \) is the axial load ratio acting on the column as this derives from the gravity component of the earthquake loading combination, \( G+\psi/E Q \), \( H_{cl} \) is the clear (deformable) height of storey columns, \( h_j \) is the section height of the \( j \)-th column in the direction of the earthquake excitation, \( A_{col,j} \) is the \( j \)-th column cross sectional area, \( A_{inf,k} \) is the horizontal area of the \( k \)-th infill pier oriented parallel to the direction of seismic action, \( f_{inf}^m \) is the compressive strength of the masonry infills, \( \mu_{y,k} \) is the level of ductility attained by the masonry infill at the point of yielding of the surrounding R.C. frame, \( \theta_{y,k}^m \approx 0.2\% \) \([19]\), whereas, \( H_{inf,k} \) and \( L_{inf,k} \) are the height and length of the compression strut of the \( k \)-th masonry infill oriented parallel to the earthquake direction, defined according to Figure 3.
In the case of buildings with complex layout in plan (for example in buildings with setbacks in their higher storeys), or with significant variation of the magnitude of storey mass along their floors, the deflected shape at peak seismic response may be approximated by the buildings’ fundamental translational mode of vibration in each of the buildings’ principal plan directions, using simplified analysis methods (Rayleigh method, Stodola method, etc [16, 17]).

After approximating the deformed shape of the examined building at its peak seismic response, $\Phi$, its critical storey can easily be defined as the storey that attains the maximum value of interstorey drift, $\Delta \Phi_{cr} = \Phi_i - \Phi_{i-1}$.

2.2 Identification of the prevailing failure mode of the vertical structural elements and calculation of the corresponding limiting resistance ratio

In the second step of the RSA procedure the seismic capacity of the critical storey of the examined R.C. building is identified, via comparison of the storey’s limiting shear resistance, controlled by all different modes of failure that will develop at each of its column lines at peak seismic response, with the storey’s shear resistance corresponding to the onset of inelastic column response.

To do so, the shear strength of each column line of the critical storey corresponding to all possible mechanisms of failure that may be developed at an R.C. column under seismic excitation, namely, {F1: flexural yielding in columns, $V_{flex}$; F2: shear failure of the column web, $V_s$; F3: anchorage failure of longitudinal reinforcement of the column, $V_a$; F4: attainment of the development capacity of column lap splices, $V_{lap}$; F5: beam-column joint shear failure, $V_j$; F6: punching failure in slab-column connections, $V_{pn}$} are calculated, whereas, also considered is the ductile yielding of the longitudinal reinforcement of the beams adjacent to the examined column, $V_{by}$, which limit the shear force input to the column. In all failure mechanisms column shear is estimated considering the moment destitution along the height of the column under lateral sway. Equations for calculating $V_{flex}$, $V_s$, $V_a$, $V_{lap}$, $V_j$, $V_{pn}$ and $V_{by}$ are presented in the Appendix of the paper.

Next, the resistance ratios, $r$, of each column associated to the considered failure modes are determined by normalizing the corresponding shear strength with the column’s $V_{flex}$, as: $r_v = V_v/V_{flex}$, $r_a = V_a/V_{flex}$, $r_{lap} = V_{lap}/V_{flex}$, $r_j = V_j/V_{flex}$, $r_{pn} = V_{pn}/V_{flex}$ and $r_{by} = V_{by}/V_{flex}$. The prevailing mode of failure of each column line is identified by the resistance ratio $r_{fail}$:

$$r_{fail} = \min\{r_v, r_a, r_{lap}, r_j, r_{pn}\} \leq r_{by}$$  \hspace{1cm} (2)
If \( r_{\text{fail}} < r_{by} \) then in the case where \( r_{\text{fail}} \geq 1.0 \) the column is expected to develop ductile response as the result of yielding of its longitudinal reinforcement, whereas, if \( r_{\text{fail}} < 1.0 \) the column is expected to fail in a brittle manner after the exhaustion of its available strength which is lower than the flexural demand. If \( r_{\text{fail}} = r_{by} \), plastic hinges are expected to develop in the ends of the beams adjacent to the examined column, limiting the shear force input to the column and protecting it from developing other modes of failure.

Having identified the prevailing mode of failure of each column line of the critical storey, the seismic capacity of the same storey is expressed by the limiting resistance ration of the critical storey, \( R_{\text{fail,crs}} \), being equal to the average value of the resistance ratios \( r_{\text{fail}} \) of the \( N_c \) columns of the storey:

\[
R_{\text{fail,crs}} = \frac{\sum_{j=1}^{N_c} r_{\text{fail},j}}{N_c} \tag{3}
\]

### 2.3 Approximation of the PGA and/or the average drift ratio that the critical storey of the building may tolerate before developing structural damages

The last step of the RSA procedure refers to the approximation of the seismic capacity of the examined R.C. building. The seismic capacity may be estimated in terms of limiting ground acceleration that the building can sustain without developing structural damages in its critical storey, \( a_{g,\text{lim}} \). For buildings with constant mass, \( M \), and translational stiffness, \( K \), at all their storeys \( a_{g,\text{lim}} \) can be approximated from Eq. (4a), whereas, in the general case that storey mass and translational stiffness vary from one storey to the other \( a_{g,\text{lim}} \) may derive from Eq. (4b):

\[
a_{g,\text{lim}} = \left( R_{\text{fail,cr}} \times \frac{\theta_{c,y}}{\theta_{c,cr}} \right) \times \frac{H_{c,cr}}{\Delta \Phi_{c,cr}} \times \frac{1}{\omega^2} \times \frac{K}{M} ; \quad \Phi_s = \frac{\sum_{i=1}^{N} \Phi_i}{\sum_{i=1}^{N} \Phi_i^2} ; \quad \Omega = \sqrt{\frac{\sum_{i=1}^{N} \Phi_i^2}{\sum_{i=1}^{N} \Delta \Phi_i^2}} \tag{4a}
\]

\[
a_{g,\text{lim}} = \left( R_{\text{fail,cr}} \times \frac{\theta_{c,y}}{\theta_{c,cr}} \right) \times \frac{H_{c,cr}}{\Delta \Phi_{c,cr}} \times \frac{K^*}{L^*} ; \quad L^* = \sum_{i=1}^{N} M_i \cdot \Phi_i ; \quad K^* = \sum_{i=1}^{N} K_i \cdot \Delta \Phi_i^2 \tag{4b}
\]

In Eqs. (4), \( \theta_{c,y} \) is the column rotation at yielding of the longitudinal reinforcement (\( \theta_{c,y} = 0.5\% \) for columns with usual height to sectional dimension ratio of 6, \( \theta_{c,y} = 0.33\% \) for stocky columns with an aspect ratio of 4 or less and \( \theta_{c,y} = 0.75\% \) for slender columns with an aspect ratio of 8 or more), \( \theta_{c,cr} \) is the average value of \( \lambda_c \) of all columns of the critical storey, \( K \) and \( \Phi \) are estimated in the first step of the procedure and \( M_i \) is the \( i \)-th storey total mass deriving from the gravity component of the earthquake loading combination, \( G + y_\psi E \cdot Q \). Ratio \( \lambda_c = \lambda / (1 + \lambda) \) is the columns’ moment distribution factor and accounts for the relative stiffness of the columns in a given floor as compared to the beams and \( \lambda = (n_b \cdot E_c \cdot I_b \cdot H_c)/(n_c \cdot E_c \cdot I_c \cdot L_b) \) where \( E_c \cdot I_c \) and \( E_c \cdot I_c \) are the secant to yield sectional stiffnesses of these members respectively, \( n_b \) is the number of beams and \( n_c \) is the number of columns converging to a frame connection (excluding cantilevering members) and \( L_b \) is the typical length of beams. When one or more column lines of the critical storey exhibit yielding of the longitudinal reinforcement of the adjacent beams the product \( R_{\text{fail,crs}} \theta_{c,y}/\theta_{c,cr} \) in Eqs. (4) is replaced by \( \theta_b / \lambda_b \) where \( \theta_b = \mu \theta_{c,b}, \lambda_b \approx 1, \mu_b = \{1+\mu_p\} - 0.5 \cdot (T_1/0.50) \) for \( 0.15 \leq T_1 \leq 0.50 \) and \( 0.5 \cdot (\mu_p+1) \) for \( 0.50 < T_1 \leq 2.00 \) s and \( \mu_p \approx b_1 \cdot 0.0035/\rho_{c,y}, \) with \( b_1 \) ranging between 4 and 5 for usual types of beam cross sections. The fundamental translational mode of vibration, \( T_1 \), is calculated according to Eq.
or, for simplicity may be approximated as a function of the total height of the examined building measured from its foundation, or from the top of its rigid basement, $H_{\text{tot}}$, according to the empirical expression of EN1998-1 [20] presented in Eq. (5b):

$$T_i = 2 \cdot \pi \cdot \sqrt{\frac{M^*}{K^*}}; \quad M^* = \sum_{i=1}^{N} M_i \cdot \Phi_i^2; \quad K^* = \sum_{i=1}^{N} K_i \cdot \Delta \Phi_i^2$$  \hspace{1cm} (5a)

$$T_i = 0.075 \cdot H_{\text{tot}}^{0.75}$$  \hspace{1cm} (5b)

The seismic capacity of the examined building may also be estimated in terms of the average column drift ratio that the columns of the critical storey tolerate before failure. In this case the seismic demand from the columns of the critical storey of the building, $\Phi_{\text{crit}}^{\text{dem}}(i)$, may be estimated for buildings with constant storey mass, $M$, and translational stiffness, $K$, (Eq. (6a)), as well as for buildings where these parameters vary along the height (Eq. (6b)):

$$\Phi_{\text{crit}}^{\text{dem}} = A_{\text{cr}, \text{er}} \cdot \frac{\Delta \Phi_{\text{er}}}{H_{\text{cl}, \text{cr}}} \cdot S_a(T_i) \cdot \Omega^2 \cdot \frac{M}{K}$$  \hspace{1cm} (6a)

$$\Phi_{\text{crit}}^{\text{dem}} = A_{\text{cr}, \text{er}} \cdot \frac{\Delta \Phi_{\text{er}}}{H_{\text{cl}, \text{cr}}} \cdot S_a(T_i) \cdot \frac{L'}{K}$$  \hspace{1cm} (6b)

where, $S_a(T_i)$ is the spectral absolute acceleration at the building site corresponding to the fundamental translational period of vibration in the examined plan direction. The calculated value of $\Phi_{\text{crit}}^{\text{dem}}$, is then compared with the limiting value of the average column drift ratio that can be developed at the critical storey of the building, $\Phi_{\text{crit}}^{\text{lim}}$. If $\Phi_{\text{crit}}^{\text{dem}} < \Phi_{\text{crit}}^{\text{lim}}$, the building responds elastically, whereas, if $\Phi_{\text{crit}}^{\text{dem}} \geq \Phi_{\text{crit}}^{\text{lim}}$ structural damages at the columns of the critical storey are anticipated.

3 SECOND-LEVEL SEISMIC CAPACITY PROCEDURE FOR R.C. BUILDINGS

In May 2018 EPPO issued the pre-standard Second-level Seismic Capacity Procedure for R.C. Buildings [12]. The EPPO second-level assessment procedure is a semi-empirical procedure intended for narrowing down the number of buildings that are rendered seismically dangerous after conducting first-level seismic assessment by performing simplified calculations that lead to the rating of the examined buildings in a priority list for conducting third-level assessment procedures. The EPPO second-level seismic capacity procedure is consisting of three steps, namely:

- **Step 1:** Determination of the seismic demand from the examined building.
- **Step 2:** Calculation of the seismic resistance of the examined building, taking into account reduction factors depended on structural weak points.
- **Step 3:** Determination of the building’s structural coefficient, which leads to priority for conducting third-level seismic assessment.

3.1 Determination of the seismic demand from the examined building

The first step of the EPPO second-level assessment procedure refers to the determination of the seismic demand from the building that is investigated. According to the procedure, the seismic demand is expressed in terms of base shear developing at ground level of the examined building, $V_{\text{req}}$, in its $i$-th ($i = X$, or $Y$) plan directions,
\[ V_{\text{req},i} = M_{\text{tot}} \cdot S_{a,i}(T_1) \]  

where, \( M_{\text{tot}} \) is the total mass of the building corresponding to the gravity loads of the seismic combination, \( G + \gamma E \cdot Q \) and \( S_{a,i}(T_1) \) is defined in the previous section.

### 3.2 Calculation of the seismic resistance of the examined building

According to the EPPO second-level assessment procedure the seismic resistance of the examined building may be determined as the shear strength at ground level of the examined R.C. building, \( V_R \) in the building’s \( i \)-th plan direction:

\[ V_{R,i} = B_i \cdot V_{R0,i} \]  

In Eq. (8), \( V_{R0,i} \) is the maximum value of shear strength that may develop at ground level of the building, deriving as the sum of the shear strengths in the examined plan direction of the \( N_c \) columns \( V_{R0}^C \), the \( N_w \) walls \( V_{R0}^W \) and the \( N_{sc} \) short columns \( V_{R0}^{SC} \) of the first storey of the building,

\[ V_{R0,i} = a_1 \cdot \sum_{j=1}^{N_c} V_{R0,j}^C + a_2 \cdot \sum_{k=1}^{N_w} V_{R0,k}^W + a_3 \cdot \sum_{l=1}^{N_{sc}} V_{R0,l}^{SC} \]  

where, \( \{a_1 = 0.5 \ ; \ a_2 = 0.7 \ ; \ a_3 = 0.9\} \) if columns, walls and short columns exist simultaneously, \( \{a_1 = 0.7 \ ; \ a_2 = 0.9 \ ; \ a_3 = 0.0\} \) when only columns and walls exist, \( \{a_1 = 0.7 \ ; \ a_2 = 0.0 \ ; \ a_3 = 0.9\} \) in the case of frame structures having only columns, regular and short and \( \{a_1 = 0.8 \ ; \ a_2 = 0.0 \ ; \ a_3 = 0.0\} \) for frame structures having only regular columns. \( B_i \) is a coefficient accounting for the reduction of seismic capacity of the building in its \( i \)-th plan direction that derives as a function of the 13 inadequacy criteria presented in Table 2. Each of the \( n \) inadequacy criteria (\( n = 1 \) – 13) of Table 2 is ranked with a basic score, \( \sigma_n \), and is multiplied with the corresponding coefficient \( \beta_n \), which assumes values between 1 and 5 depending on the level of influence of each inadequacy criterion – the value of 1 corresponds to the extreme case of negative influence of the criterion to the reduction of the seismic capacity of the building, whereas, the value of 5 refers to zero influence of the criterion (optimal response).

<table>
<thead>
<tr>
<th>Criterion of reduction of seismic capacity</th>
<th>( \beta_n )</th>
<th>( \sigma_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR1. Existence of structural damages</td>
<td>1 ( \leq \beta_1 \leq 5 )</td>
<td>0.10</td>
</tr>
<tr>
<td>CR2. Corrosion of reinforcement</td>
<td>1 ( \leq \beta_2 \leq 5 )</td>
<td>0.10</td>
</tr>
<tr>
<td>CR3. Magnitude of axial load ratio, ( \nu )</td>
<td>1 ( \leq \beta_3 \leq 5 )</td>
<td>0.05</td>
</tr>
<tr>
<td>CR4. Regularity in plan</td>
<td>1 ( \leq \beta_4 \leq 5 )</td>
<td>0.05</td>
</tr>
<tr>
<td>CR5. Regularity of stiffness in plan – Torsion</td>
<td>1 ( \leq \beta_5 \leq 5 )</td>
<td>0.10</td>
</tr>
<tr>
<td>CR6. Regularity in height - section</td>
<td>1 ( \leq \beta_6 \leq 5 )</td>
<td>0.05</td>
</tr>
<tr>
<td>CR7. Existence of soft storey - Stiffness in height</td>
<td>1 ( \leq \beta_7 \leq 5 )</td>
<td>0.15</td>
</tr>
<tr>
<td>CR8. Mass distribution in height</td>
<td>1 ( \leq \beta_8 \leq 5 )</td>
<td>0.05</td>
</tr>
<tr>
<td>CR9. Short columns</td>
<td>1 ( \leq \beta_9 \leq 5 )</td>
<td>0.15</td>
</tr>
<tr>
<td>CR10. Structural discontinuities in height</td>
<td>1 ( \leq \beta_{10} \leq 5 )</td>
<td>0.05</td>
</tr>
<tr>
<td>CR11. Load transfer and structural weaknesses</td>
<td>1 ( \leq \beta_{11} \leq 5 )</td>
<td>0.05</td>
</tr>
<tr>
<td>CR12. Impact factors for adjacent buildings</td>
<td>1 ( \leq \beta_{12} \leq 5 )</td>
<td>0.05</td>
</tr>
<tr>
<td>CR13. Malfunctions - Slight damage</td>
<td>1 ( \leq \beta_{13} \leq 5 )</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 2: List of inadequacy criteria used in the approximation of the reduction of the seismic capacity of the examined building.
Coefficient $B_i$ is calculated in each of the $i$-th plan direction as:

$$B_i = \sum_{n=1}^{13} \sigma_n \cdot \beta_n$$  \hspace{1cm} (10)

### 3.3 Determination of the building’s structural coefficient

The EPPO second-level assessment procedure concludes with the ranking of the examined R.C. building in terms of the building’s Structural Coefficient, $\lambda$:

$$\lambda = 100 \cdot \max\{\lambda_X ; \lambda_Y\} ; \quad \lambda_X = \frac{V_{\text{req},X} + 0.30 \cdot V_{\text{req},Y}}{V_{R,X} + 0.30 \cdot V_{R,Y}} ; \quad \lambda_Y = \frac{V_{\text{req},Y} + 0.30 \cdot V_{\text{req},X}}{V_{R,Y} + 0.30 \cdot V_{R,X}}$$  \hspace{1cm} (11)

Coefficient $\lambda$ classifies the examined buildings in a list of priority for conducting third-level seismic assessment (note that coefficient $\lambda$ in the EPPO second-level assessment procedure has completely different physical meaning from coefficient $\lambda$ of the RSA procedure, which determines the percentage of the total storey rotation corresponding to the columns and the beams of the storey). Values of $\lambda$ smaller that 100% correspond to cases that are seismically adequate and do not require further examination, whereas buildings ranked with coefficient $\lambda$ equal, or greater than 100% require additional investigation of their seismic performance utilizing a third-level seismic assessment procedure; the priority for conducting third-level seismic assessment increases along with the increase of coefficient $\lambda$.

### 4 PILOT APPLICATION OF THE TWO SECOND-LEVEL ASSESSMENT PROCEDURES IN THE CASE OF THE SPEAR BUILDING

The accuracy and ease of application of the two presented second-level seismic assessment procedures are presented through their application to a three-storey, full-scale, R.C. building, representative sample of the R.C. buildings that were constructed in Southern Europe until the early 1980’s, which was tested in the European Laboratory of Seismic Assessment (ELSA) of the Joint Research Center (JRC) in Ispra, Italy, in the framework of the Seismic Performance Assessment and Rehabilitation (SPEAR) European research program [13-15].

#### 4.1 Description of the examined building

The SPEAR building was a full scale, 3-storey, 2 by 2 bay, torsionally sensitive R.C. building, representative example of the structures that were designed and constructed throughout Southern Europe from the 1950’s until the mid 1980’s (Figure 4(a)). The SPEAR building was designed for gravity loads alone, considering additionally to the self weight of the R.C. structural elements 0.5 kN/m$^2$ and 2.0 kN/m$^2$ gravity loads on slabs accounting for finishings and live loads, respectively. All storeys had a constant plan configuration with external dimensions of 12.825 m $\times$ 10.75 m, comprising 8 columns of 250 mm rectangular cross section and 1 strong column of 250 mm $\times$ 750 mm cross section dimensions. The columns were connected with beams of 250 mm $\times$ 500 mm web cross section dimensions. Slab thickness was 150 mm. Storey height in all cases was 3.00 m on centers (o.c.). Columns’ comprised as longitudinal reinforcement $\varnothing$12 mm smooth bars (Figure 4(b)), which were lap-spliced over 400 mm at the base of all three levels with a hook formation at the end of the bars. All columns had $\varnothing$8 mm perimeter stirrups, spaced at 250 mm o.c., which did not continued in the beam-column joints. Details of the longitudinal reinforcement at the ends of all beams are presented in Table 3. Mean concrete compressive strengths obtained from tests on cylindrical core specimen taken from the building after completion of the experiment, $f_{cm}$,
was found equal to 24.73 MPa, 26.70 MPa and 25.32 MPa for the 1st, the 2nd and the 3rd storey columns, respectively, whereas, from uniaxial tensile tests on steel bar coupons the yield stress of reinforcement, $f_y$, was found equal to 480.0 MPa and 474.0 MPa in the cases of the $\varnothing 8$ mm and the $\varnothing 12$ mm bars, respectively.

Figure 4: (a) Typical plan configuration of the SPEAR building; (b) Cross section details of the building’s columns.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Long. Reinforcement at the Start</th>
<th>Long. Reinforcement at the End</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Bottom</td>
</tr>
<tr>
<td>B1</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B2</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B3</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B4</td>
<td>4 $\varnothing 20 + 2 \varnothing 12$</td>
<td>3 $\varnothing 20$</td>
</tr>
<tr>
<td>B5</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B6</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B7</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B8</td>
<td>2 $\varnothing 20 + 4 \varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B9</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B10</td>
<td>4 $\varnothing 20 + 2 \varnothing 12$</td>
<td>2 $\varnothing 20$</td>
</tr>
<tr>
<td>B11</td>
<td>4 $\varnothing 12$</td>
<td>2 $\varnothing 12$</td>
</tr>
<tr>
<td>B12</td>
<td>3 $\varnothing 20 + 2 \varnothing 12$</td>
<td>2 $\varnothing 20$</td>
</tr>
</tbody>
</table>

Table 3: Longitudinal reinforcement at the start and the end of each beam of the SPEAR building; Notation: the start and the end of each beam are defined accordingly to the directions of the plan axes X and Y.
During the tests at ELSA a series of simulated ground motions of increasing intensity were applied through a pseudodynamic procedure. The basic accelerograms used were the two components of the Montenegro 1979 (Herceg Novi) ground motion, sequentially scaled upwards to 0.15 g and 0.20 g (namely s11 and s12 pseudodynamic test). Figure 5 illustrates the time-history of the basic accelerograms and the corresponding absolute acceleration and relative displacement elastic response spectra plotted for 5% damping.

Figure 5: (a) Time-histories of the two components of ground acceleration used in the pseudodynamic tests; (b1-b2) Absolute acceleration and relative displacement elastic response spectra of the s11 and s12 pseudodynamic tests, calculated for 5% damping.

4.2 Seismic assessment according to the RSA procedure

The first step in applying the RSA procedure is the identification of the critical storey of the examined building using the deflected shape of the building at its peak seismic response. To do so, first the translational stiffness of each storey of the SPEAR building is calculated, in both the X and the Y plan directions, using Eq. (1). The modulus of elasticity of each storey’s columns, $E_c \approx 4500 \cdot \sqrt{f_c}$, is estimated using the values of $f_{cm}$ listed above as 22.38 GPa for the first floor, 23.25 GPa for the second and 22.64 GPa for the third. $H_{cl} = 2.50$ m in all cases, $A_{col}$ is equal to 0.0625 m$^2$ for the square columns and equal to 0.1875 m$^2$ for column C2, whereas, $h = 0.25$ m in all cases, except for the column C2 when the building is subjected to earthquake excitation in its Y plan direction, where $h = 0.75$ m. Table 4 presents the calculation of the lateral stiffness, $k_{ci}$, for seismic action in both X and Y plan directions of the SPEAR building.

Considering the absence of masonry infills, the lateral stiffness of the $i$-th storey of the SPEAR building in its X and Y plan directions derive as the sum of $k_{ci}$ of columns C1 to C9. Table 5 presents the calculated values of $K_i$, the coordinates of the deflected shape, $\Phi$, of the SPEAR building in its X and Y plan directions approximated from application of the expressions of Table 1 and the corresponding relative storey deformations, $\Delta \Phi_i$. Also presented in Table 5 are the corresponding values of $\Phi$ and $\Delta \Phi_i$ deriving from the simplified approximation of Figure (2a), which corresponds to old-type R.C. buildings such as the
SPEAR building. As evidenced, both procedures for approximating the peak seismic response of the SPEAR building highlight the building’s first storey as the critical storey during seismic excitation in both X and Y plan directions.

<table>
<thead>
<tr>
<th>Storey</th>
<th>Column</th>
<th>Peak Seismic Response in X Direction</th>
<th>Peak Seismic Response in Y Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Simplified Approximation of Fig. 2(a)</td>
<td>Simplified Approximation of Fig. 2(a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K$ (kN/m)</td>
<td>$\Phi$ (norm.)</td>
</tr>
<tr>
<td>3</td>
<td>21700</td>
<td>1.000</td>
<td>0.237</td>
</tr>
<tr>
<td>2</td>
<td>24221</td>
<td>0.763</td>
<td>0.355</td>
</tr>
<tr>
<td>1</td>
<td>25175</td>
<td>0.409</td>
<td>0.409</td>
</tr>
</tbody>
</table>

Table 5: Deformed shape of the SPEAR building at peak seismic response in the X and the Y plan directions, approximated according to the procedures of the first step of the RSA method.

The second step of the RSA procedure refers to the determination of the prevailing mode of failure of each column line of the critical storey of the examined building (i.e. the 1st storey). First, the magnitude of shear resistance of all columns of the SPEAR building, corresponding to the mechanisms F1 to F5 and to the development of plastic hinges at the beams adjacent to the columns are calculated, using the expressions of the Appendix. Figure 6 illustrates the values of $V$ for all columns of each storey of the building, for seismic action parallel in each of the two plan directions. The resistance ratios, $r$, of all columns of the SPEAR buildings associated to the calculated shear resistances are illustrated in Figure 7. As illustrated in both Figs. 6 & 7 failure of anchorage of the columns’ longitudinal reinforcements prevails among all possible mechanisms of failure that may develop at the columns of the SPEAR building, except for the case of the strong column C2 when the building is swaying in the Y plan direction, where plastic hinges at the ends of beams B9 connected to C2 develop in all storeys. The results of RSA are in full accordance with the actual seismic of the SPEAR building, where after the s11 test bond splitting cracks developed at the top of the square columns of the second storey which deteriorate during the s12 test (Figure 8). The prevailing resistance ratio, $r_{fails}$ of the columns of the critical storey...
(the 1st storey) when the SPEAR building sways in the X plan direction varies between the values of 0.32 and 0.61 and $R_{\text{fail},X}$ equals to 0.46. When the SPEAR building is swaying in the Y plan direction the building is expected to sustain the seismic demand since at its critical storey yielding of longitudinal reinforcement of the beam B9 converging in the joint connection of the strong column C2 prevails, practically precluding the development of all brittle mechanisms of column failure.

Figure 6: Shear resistances of the SPEAR building columns corresponding to all possible modes of failure.
Figure 7: Resistance ratios of all storey columns of the SPEAR building calculated for all possible modes of failure.
The last step in the application of the RSA procedure is the determination of the seismic capacity of the SPEAR building. One option is to estimate seismic capacity in terms of average column drift ratio developing at the critical storey of the building at the onset of development of structural damages. In this case the seismic demand from the columns of the critical storey, $\theta_{c,\text{dem}}$, is estimated according to Eq. (6b). Coefficient $A_{c,cr}$ is calculated from the sectional stiffnesses of the beams and columns of the 1st storey acting in the X and the Y plan directions as $A_{c,1-X} = 0.91$ and as $A_{c,1-X} = 0.85$. Furthermore, $H_{cl,cr} = 2.50$ m. Table 6 presents the values of $\Delta \Phi_{cr}$, $L^*$, $K^*$ and $M^*$, calculated in both plan directions of the SPEAR building according to both the expression of Table 1 and the approximate shape of Figure 2(a).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Phi$ calculated according to Table 1</th>
<th>$\Phi$ calculated according to Fig. 2(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \Phi_{cr}$ (norm.)</td>
<td>0.409</td>
<td>0.500</td>
</tr>
<tr>
<td>$L^*$ (t)</td>
<td>160</td>
<td>175</td>
</tr>
<tr>
<td>$K^*$ (kN/m)</td>
<td>8464</td>
<td>9928</td>
</tr>
<tr>
<td>$M^*$ (t)</td>
<td>129</td>
<td>148</td>
</tr>
</tbody>
</table>

Table 6: Values of $\Delta \Phi_{cr}$, $L^*$, $K^*$ and $M^*$, calculated according to Table 1 and Figure 2(a).

The fundamental translational stiffness of the SPEAR building in its X and Y plan directions is estimated from Eq. (5a) as $T_{1,X} = 0.78$ s and $T_{1,Y} = 0.46$ s and the corresponding spectral absolute accelerations for the s11 test case are $S_{a,s11}(T_{1,X}) = 2.798$ m/s² and $S_{a,s11}(T_{1,Y}) = 3.793$ m/s² and for the s12 test case are $S_{a,s12}(T_{1,X}) = 3.732$ m/s² and $S_{a,s12}(T_{1,Y}) = 5.057$ m/s². Therefore, according to Eq. (6b), the seismic demand from the critical storey of the SPEAR building, $\theta_{c,\text{dem}}$, estimated according to $\Phi$ from Table 1, equals in the case of the s11 test to 0.79% and to 0.36% in the X and the Y plan directions, respectively, whereas the corresponding values for the s12 test case are $\theta_{c,\text{dem}-X} = 1.05%$ and $\theta_{c,\text{dem}-Y} = 0.48%$. Note that if an approximate approach was followed, according to Eq. (5b) $T_{1,X} = T_{1,Y} = 0.075 \cdot 0.75 = 0.39$ s and therefore in the case of the s11 test $S_{a,s11}(T_{1,X}) = 3.609$ m/s² and $S_{a,s11}(T_{1,Y}) = 3.750$ m/s², leading to values of $\theta_{c,\text{dem}}$ in the X and Y plan directions equal to 1.16% and 0.40%, whereas in the case of the s12 test $S_{a,s12}(T_{1,X}) = 4.812$ m/s² and $S_{a,s12}(T_{1,Y}) = 5.000$ m/s² and
the corresponding values of $\theta_{c,\text{dem}}$ are 1.54% and 0.53% in the X and Y plan directions. The limiting value of average column drift ratio of the critical storey of the SPEAR building, $\theta_{c,\text{lim}}^{\text{crit}} = R_{\text{fail}}\theta_{c,y}$, is calculated in the X plan direction of the first storey of the building for $\theta_{c,y} = 0.5\%$ as $\theta_{c,\text{lim}}^{\text{crit},X} = 0.23\%$, indicating that structural damages will develop in both s11 and s12 test cases, whereas, for seismic action in the Y plan direction development of plastic hinge at beam B9 converging in the column C2 will prevent the development of column damages.

If the seismic capacity of the SPEAR building is estimated in terms of peak ground acceleration that the building can tolerate before the development of structural damages Eq. (4b) will be used. Considering the values of Table 6 when $\Phi$ is estimated according to Table 1, the limiting ground acceleration that the SPEAR building can sustain in its X plan direction before the failure of the anchorage of the columns’ longitudinal reinforcement is $a_{g,\text{lim},X} = 0.829 \text{ m/s}^2 = 0.08 \text{ g}$. Note that if the deformed shape of the SPEAR building at peak seismic response is approximated using the shape of Figure 2(a), then $a_{g,\text{lim},X} = 0.730 \text{ m/s}^2 = 0.07 \text{ g}$. Also note that if the translational storey stiffness is assumed equal to $K = 21700 \text{ kN/m}$ in all storeys and considering that for all storeys $M = 73.84 \text{ t}$, $\Phi_1$ equals to 1.242 according to the expression of Table 1 and to 1.183 according to the shape of Figure 2(a) and that the values of $\Omega$ corresponding to the two versions of $\Phi_1$ are 2.240 and 2.231, respectively, then using Eq. (4a) the limiting ground acceleration that the SPEAR building can tolerate in its X plan direction is $a_{g,\text{lim},X} = 0.729 \text{ m/s}^2 = 0.07 \text{ g}$ if $\Phi_1$ is calculated according to Table 1 and $a_{g,\text{lim},X} = 0.631 \text{ m/s}^2 = 0.06 \text{ g}$ if $\Phi_1$ is approximated according Figure 2(a). With regards to the building’s Y plan direction, considering that in the case of the strong column, C2, the seismic input is limited by yielding of longitudinal reinforcement of the beam B9 converging in the column, the building is able to sustain the PGA of both s11 and s12 tests.

4.3 Seismic assessment according to the EPPO second-level assessment procedure

Assessment of the SPEAR building according to the EPPO second-level assessment procedure first requires the determination of the seismic demand from the examined building in terms of shear reaction developing at its foundation level, $V_{\text{req}}$. To calculate $V_{\text{req}}$ the fundamental translational modes of vibration of the SPEAR building in its principal plan directions, X and Y, were approximated using the empirical expression of EN1998-1 (Eq. (5b)), $T_{1,X} = T_{1,Y} = 0.075 \cdot 9.00.75 = 0.39 \text{ s}$. Considering the absolute acceleration response spectra of the s11 and s12 earthquake cases (Figure 5(b1)) the values of $S_a(T_1)$ used in calculating $V_{\text{req}}$ are for the s11 test case $S_a(T_1)_{s11,X} = 3.609 \text{ m/s}^2$ and $S_a(T_1)_{s11,Y} = 3.750 \text{ m/s}^2$ in the X and Y plan directions and for the s12 test case $S_a(T_1)_{s12,X} = 4.812 \text{ m/s}^2$ and $S_a(T_1)_{s12,Y} = 5.000 \text{ m/s}^2$. According to the EPPO second-level assessment procedure $V_{\text{req}}$ is the product of the SPEAR building total mass, $M_{\text{tot}} = 3 \cdot 73.84 = 221.52 \text{ t}$, multiplied by $S_a(T_1)$. To this end, for the s11 test case $V_{\text{req},s11,X} = 799 \text{ kN}$ and $V_{\text{req},s11,Y} = 831 \text{ kN}$ whereas, for the s12 test case $V_{\text{req},s12,X} = 1066 \text{ kN}$ and $V_{\text{req},s12,Y} = 1108 \text{ kN}$.

The second step of the EPPO second-level assessment procedure requires the calculation of the seismic resistance of the SPEAR building in its two plan directions. Table 7 presents the calculation of $V_{R0}$ for each column of the SPEAR building according to Eq. (9), considering the values of $f_1$ presented in section 4.1, $k = 1.951$ in all cases except for the C2 column subjected to earthquake action parallel to the Y plan direction, where $k = 1.527$ and the column loads associated to the building’s vertical loads. The total shear strength of the first storey columns of the SPEAR buildings, calculated as the sum of the shear strengths of columns C1 to C9, equals to $V_{R0,X} = 413 \text{ kN}$ and $V_{R0,Y} = 412 \text{ kN}$ in the X and the Y plan directions, respectively.
The values of $V_{R0,X}$ and $V_{R0,Y}$ are then effectively reduced by using coefficients $B_X$ and $B_Y$ to account for the influence of the 13 different criteria to the overall reduction of the building’s seismic capacity. Table 8 presents the estimated values of coefficients $\beta$ for each of the two plan directions of the SPEAR building. Coefficients $B_i (i = X \text{ or } Y)$, defined as $B_i = \sum(\sigma\beta_i)/5$, are calculated equal to $B_X = B_Y = 0.98$. Therefore, the seismic resistance at ground level of the SPEAR building, $V_{R,i} = a_1 B_i V_{R0,i}$, for $a_1 = 0.80$, equals to $V_{R,X} = 323 \text{ kN}$ and $V_{R,Y} = 322 \text{ kN}$.

<table>
<thead>
<tr>
<th>Criterion of reduction of seismic capacity</th>
<th>X direction $\beta_X \sigma \beta_X$</th>
<th>Y direction $\beta_Y \sigma \beta_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR1. Existence of structural damages</td>
<td>5 0.50 5</td>
<td>5 0.50</td>
</tr>
<tr>
<td>CR2. Corrosion of reinforcement</td>
<td>5 0.50 5</td>
<td>5 0.50</td>
</tr>
<tr>
<td>CR3. Magnitude of axial load ratio, $v$</td>
<td>5 0.25 5</td>
<td>5 0.25</td>
</tr>
<tr>
<td>CR4. Regularity in plan</td>
<td>5 0.25 5</td>
<td>5 0.25</td>
</tr>
<tr>
<td>CR5. Regularity of stiffness in plan – Torsion</td>
<td>5 0.50 5</td>
<td>5 0.50</td>
</tr>
<tr>
<td>CR6. Regularity in height - section</td>
<td>5 0.25 5</td>
<td>5 0.25</td>
</tr>
<tr>
<td>CR7. Existence of soft storey - Stiffness in height</td>
<td>5 0.75 5</td>
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</tr>
<tr>
<td>CR8. Mass distribution in height</td>
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<td>5 0.25</td>
</tr>
<tr>
<td>CR9. Short columns</td>
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<td>5 0.75</td>
</tr>
<tr>
<td>CR10. Structural discontinuities in height</td>
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<td>5 0.25</td>
</tr>
<tr>
<td>CR11. Load transfer and structural weaknesses</td>
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</tr>
<tr>
<td>CR12. Impact factors for adjacent buildings</td>
<td>5 0.25 5</td>
<td>5 0.25</td>
</tr>
<tr>
<td>CR13. Malfunctions - Slight damage</td>
<td>5 0.25 5</td>
<td>5 0.25</td>
</tr>
</tbody>
</table>

Table 8: Values of coefficients $\beta$ estimated for each of the 13 inadequacy criteria in each plan direction of the SPEAR building.

The seismic assessment of the SPEAR building using the EPPO procedure is concluded by evaluating the building’s structural coefficient, $\lambda$. In the case of the s11 test case the structural coefficient in the X plan direction equals to $\lambda_{X,s11} = (V_{req,s11,X}+0.3\cdot V_{req,s11,Y})(V_{R,X}+0.3\cdot V_{R,Y}) = 2.50$, whereas, in the Y plan direction $\lambda_{Y,s11} = (V_{req,s11,X}+0.3\cdot V_{req,s11,Y})(V_{R,Y}+0.3\cdot V_{R,X}) = 2.55$. The corresponding values of the structural coefficient in both plan directions of the SPEAR building for the s12 test case are $\lambda_{X,s12} = 3.33$ and $\lambda_{Y,s12} = 3.41$. Therefore, according to the
EPPO second-level assessment procedure, the SPEAR building is ranked with structural coefficient $\lambda = 100 \cdot \max\{\lambda_X; \lambda_Y\}$ equal to $\lambda_{s11} = 255\%$ and $\lambda_{s12} = 341\%$.

5 CONCLUSIONS

Two second-level seismic assessment procedures for R.C. buildings, the Rapid Seismic Assessment Procedure for R.C. Buildings (RSA) [9-11] and the Second-level Seismic Capacity Procedure for R.C. Buildings [12], are presented and evaluated for their accuracy and their ease in application through pilot applications in the case of the SPEAR test building [13-15].

With regards to the ease in application, both procedures require the knowledge of the layout of the examined building and of the cross-section dimensions and the locations of the structural elements. To this end, both procedures require the mapping of the examined building, which in both cases consumes the biggest percentage of the total effort required for conducting seismic assessment. Furthermore, both procedures require the knowledge of the compressive strength of the concrete of the structural elements, acquired from in-situ tests, or from access to the building’s design calculations. Additionally, the RSA procedure also requires knowledge of the details and the material properties of the longitudinal and the transversal reinforcement of the vertical structural elements, for allowing the determination of their prevailing mode of failure; this information may also derive from in-situ tests, from access to the building’s design calculations, or may be assumed in accordance to the construction practices used at the era of the building’s construction. In terms of computational effort, the EPPO second-level assessment procedure is easier than the RSA procedure, especially if performance of the last two steps of the RSA procedure does not use design charts developed for these stages [10, 11].

With regards to the accuracy of the assessment results, the RSA procedure yields results of very good correlation with the actual seismic response of the examined building, both in terms of estimating the anticipated response of the building’s structural elements near failure under lateral sway and in identifying the tendency for damage localization (i.e. the critical storey) of the building and in approximating the limiting lateral column drift and the limiting ground accelerating that the building could tolerate before developing structural damages. The EPPO second-level assessment procedure only provides ranking of the examined building in terms of the building’s Structural Coefficient, $\lambda$, which corresponds to the ratio of the developing base shear due to the considered earthquake to the shear strength at ground level of the building. Furthermore, by being associated with the design ground acceleration at the site of the building, $a_g$, coefficient $\lambda$ does not provide uniform evaluation of the seismic capacity of buildings of the same characteristics located at different seismic zones, making difficult the direct evaluation of buildings of similar seismic capacity – a building located in a seismic zone of high $a_g$ is ranked with higher score of $\lambda$ as compared to a similar building located at another seismic zone with lower $a_g$. Also noted from the results of the pilot application is that the reduction factors $\beta_i$ in the second step of the EPPO second-level assessment procedure require further calibration, as they fail to accurately represent the structural weaknesses of the examined building.
AKNOWLEDGEMENTS

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REFERENCES


APPENDIX

The individual strength terms of an R.C. column can be calculated using the following expressions, which represent the present state of the art at the field. Nevertheless, these expressions may be subject to change as the knowledge base in reinforced concrete leads to improved models for the individual mechanisms of resistance.

- Flexural shear demand:

\[ V_{\text{flex}} = \left[ \rho_{l,\text{tot}} \cdot \frac{f_y}{f_c} \cdot (1-0.4 \cdot \xi) + \nu \cdot \left( \frac{h}{d} - 0.8 \cdot \xi \right) \right] \cdot \frac{b \cdot d^2 \cdot f_c}{H_{cl}} \]  

(A1)

- Exhaustion of shear strength:

If \( \nu < 0.10 \):

\[ V_v = A_p \cdot f_u \cdot \frac{d \cdot (1-0.4 \cdot \xi)}{s} \cdot \cot \theta_v \]  

(A2a)

If \( \nu \geq 0.10 \):

\[ V_v = v \cdot b \cdot d \cdot f_c \cdot \tan \alpha + A_p \cdot f_u \cdot \frac{d \cdot (1-0.4 \cdot \xi)}{s} \cdot \cot \theta_v \]  

(A2b)

- Anchorage failure of longitudinal reinforcement:

Ribbed longitudinal reinforcement:

\[ V_a = \left[ \rho_{l,\text{tot}} \cdot \frac{\min \left\{ \frac{4 \cdot L_a \cdot f_b}{D_b} + \alpha_{\text{hook}} \cdot 50 \cdot f_b; f_y \right\}}{f_c} \cdot (1-0.4 \cdot \xi) + \nu \cdot \left( \frac{h}{d} - 0.8 \cdot \xi \right) \right] \cdot \frac{b \cdot d^2 \cdot f_c}{H_{cl}} \]  

(A3a)
Smooth longitudinal reinforcement:

\[
V_a = \min \left\{ 0.45 \cdot \frac{f_c}{\\sqrt{20}} \cdot N_b \cdot A_b \cdot L_a + \alpha_{\text{hook}} \cdot 50 \cdot N_b \cdot A_b \cdot f_y; N_b \cdot A_b \cdot f_y \right\} \cdot d \cdot (1 - 0.4 \cdot \xi) + \\
+ v \cdot b \cdot d^2 \cdot f_c \cdot (0.5 \cdot h/d - 0.4 \cdot \xi) \over H_{cl}/2 \right. (A3b)
\]

- Lap failure of longitudinal reinforcement:

\[
V_{\text{lap}} = \min \left\{ \mu_{j,\text{lap}} \cdot \frac{A_s}{s} \cdot f_{st} + \alpha_b \cdot (b - N_b \cdot D_b) \cdot f_y \right\} + \left\{ \alpha_{\text{hook}} \cdot 50 \cdot N_b \cdot A_b \cdot f_y \right\} \cdot d \cdot (1 - 0.4 \cdot \xi) + \\
+ v \cdot b \cdot d^2 \cdot f_c \cdot (0.5 \cdot h/d - 0.4 \cdot \xi) \over H_{cl}/2 \right. (A4)
\]

- Shear capacity of joints:

Unreinforced or lightly reinforced joints:

\[
V_j = \gamma_j \cdot 0.5 \cdot \sqrt{f_c} \cdot \sqrt{1 + \frac{v_j \cdot f_c}{0.5 \cdot f_c}} \cdot \frac{b_j \cdot d \cdot d_{\text{beam}}}{H_{cl}} (A5a)
\]

Well reinforced joints:

\[
V_j = \gamma_j \cdot 0.5 \cdot \sqrt{f_c} \cdot \sqrt{1 + \frac{v_j \cdot f_c}{0.5 \cdot f_c}} \cdot \frac{b_j \cdot d \cdot d_{\text{beam}}}{H_{cl}} \left[ 1 + \rho_{j,\text{horiz}} \cdot \frac{f_{st}}{f_t} \right] (A5b)
\]

- Punching shear of slab-column connections:

\[
V_{\text{pn}} = 0.12 \cdot \min \left\{ 1 + \frac{200}{d_{cl}}; 2 \right\} \cdot (100 \cdot \rho_{c,\text{sl}} \cdot f_c)^{1/3} \cdot d_{st} \cdot 0.25 \cdot u_{\text{crit}} \cdot (h + 4 \cdot d_{sl}) \over H_{cl} \right. (A6)
\]

- Limiting shear due to yield of beams' longitudinal reinforcement:

\[
V_{by} = 0.85 \cdot \rho_{\text{beam}} \cdot b_{\text{beam}} \cdot d_{\text{beam}}^2 \cdot f_y \over H_{cl} \right. (A7)
\]

where,

- \( \rho_{c,\text{sl}} = A_{c,\text{sl}} \cdot (b \cdot d) \) is the total longitudinal reinforcement ratio of a column with external dimensions \( h \times b \),
• \( A_{st,\text{tot}} \) is the total area of the longitudinal reinforcement at the column's critical section,
• \( d \) is the column effective depth,
• \( f_y \) is the longitudinal reinforcement yield stress,
• \( f_c \) is the concrete compressive strength,
• \( \xi = \frac{x}{d} \) is the normalized depth of compression zone,
• \( v \) is the axial load ratio acting on the cross section \( (N_g + 0.3q)(b \cdot d \cdot f_y) \),
• \( H_{cl} \) is the column's deformable length,
• \( \tan \alpha = \left( \frac{h}{d} - 0.8 \frac{\xi}{d} \right) \frac{d}{H_{cl}} \), where \( \alpha (\leq \theta_v) \) is the angle of inclination of the diagonal strut created between the centroids of the compression zones at the top and bottom column cross sections of the column (this represents the strut forming by the axial load acting on the column according to (Priestley et al. 1996))
• \( \theta_v = \{ 45^\circ \text{ when } v < 0.10, 30^\circ \text{ when } v \geq 0.25, \text{ whereas for } 0.10 \leq v < 0.25 \theta_v \text{ is calculated from linear interpolation} \} \) is the angle of sliding plane (i.e. \( \theta_v \) is the angle forming between the longitudinal member axis and a major inclined crack developing in the plastic hinge region of the column). It determines the number of stirrup legs that are intersected by the inclined sliding plane,
• \( h_{st} \) is the height of the stirrup legs,
• \( A_{tr} \) is the total area of stirrup legs in a single stirrup pattern, which are intersected by the inclined sliding plane,
• \( s \) is the stirrup spacing,
• \( f_{st} \) is the stirrup yield stress,
• \( L_a \) is the anchorage length of the longitudinal reinforcement,
• \( D_b \) is the diameter of longitudinal reinforcing bars,
• \( \alpha_{\text{hook}} \) is a binary index (1 or 0) to account for hooked anchorages \( (\alpha_{\text{hook}} = 0 \Rightarrow \text{no hooks}) \),
• \( f_b = 2 f_{b,o} \) is the concrete bond stress, where \( f_{b,o} = 1.80 \cdot (f_c / 20)^{0.5} \).
• \( \mu_{fr} \) is the friction coefficient \( (0.2 \leq \mu_{fr} \leq 0.3 \text{ for smooth bars; } 1.0 \leq \mu_{fr} \leq 1.5 \text{ for ribbed bars}) \),
• \( L_{\text{lap}} \) is the lap-splice length,
• \( \alpha_b \) is a binary index (1 or 0) depending on whether ribbed or smooth reinforcement has been used,
• \( N_b \) is the number of longitudinal bars in tension,
• \( A_b \) is the area of a single tension bar,
• \( f_t = 0.3 f_c^{2/3} \) is the concrete tensile strength,
• \( \gamma_j = \{ 1.40 \text{ for interior joints; } 1.00 \text{ for all other cases, whereas, for joints without stirrups these values are reduced to } 0.4 \text{ and } 0.3 \text{ respectively} \} \),
• \( v_j \) is the (service) axial load acting on the bottom of the column adjusted at the top of the joint (compression positive),
• \( b_j = (b + b_{beam})/2 \) is the joint width, where \( b_{beam} \) is the web width of the adjacent beam,
• \( d_{beam} \) is the beam depth,
• \( \rho_{b,\text{horiz}} = A_{tr}/(s \cdot b_j) \),
• \( d_{sl} \) is the slab depth,
• \( \rho_{\text{st,sl}} \) is the total slab reinforcement ratio at the critical punching perimeter around the column, \( u_{\text{crit}} \)
• \( \rho_{\text{beam}} \) is the tension longitudinal reinforcement ratio of the beam (i.e. the total longitudinal reinforcement ratio of the beam section adjacent to the column if an interior connection is considered, or in the case of exterior connections the value of the top or bottom beam reinforcement ratio (whichever is largest)),
• \( f_y \text{beam} \) is the yield stress of the beam longitudinal reinforcement.
FORMULATION OF A NOVEL OPENSEES ELEMENT FOR FPS BEARINGS WITH ENHANCED FRICTION MODEL

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Abstract

The new “CSSBearing_BVNC” element has been coded in the object-oriented finite element software program OpenSees to represent the behavior of the Friction Pendulum System® (FPS) comprising one concave sliding surface and a spherical articulation, accounting for an enhanced formulation of the friction behavior. In the novel element, the hysteretic force – displacement relationship of the FPS bearing in the horizontal direction is mathematically modelled using the theory of plasticity, and two yield conditions are used to account for either the static or the dynamic friction coefficient. Other features of the friction model are the change of the dynamic coefficient of friction with the instantaneous values of axial load and slide velocity, and its degradation due to heat generated during cyclic motion.

The primary assumptions in the development of the friction model are verified in a code-to-code comparison. A case study relevant to a base-isolated concrete, moment resisting frame is presented to demonstrate the improved prediction capability of the new bearing element over its standard counterpart, such as estimating a +40% increase in superstructure drift and column shear force and a +58% increase in displacement during high intensity earthquakes, and up to a +130% increase in internal forces and deformations of the structure under weak or medium intensity earthquakes due to the possible non-activation of the sliding isolators.

Keywords: Friction Pendulum, OpenSees, Bearing Element, Friction Model, Static Coefficient of Friction; Heating Effect.
1 INTRODUCTION

The Friction Pendulum System®, or FPS® [1-3], is one of the most popular isolation hardware worldwide thanks to its inherent simplicity, since it provides the four main functions required to the isolation system, i.e. carrying vertical load, lateral flexibility, energy dissipation and re-centering capability, in a single, compact design. In its basic configuration the FPS consists of a concave sliding plate and an articulated slider. The surface of the slider in contact with the concave plate is lined by a low-friction thermoplastic material, and a sliding interface is formed between the slider and the concave plate to accommodate the horizontal displacement of the superstructure (Figure 1). Though improved versions featuring multiple sliding surfaces have been proposed in recent years, like e.g., the Double Curved Surface Slider [4] and the Triple Friction Pendulum [5], their mechanical behavior follows the same fundamental principles. Number of sliding interfaces, coefficient of friction and radius of curvature determine the bearing performance: the concave surface provides a restoring force that is proportional to the horizontal displacement; the friction force developed during the accommodated sliding motion accounts for the energy dissipation, which reduces the horizontal force and the displacement demand, but increases the amount of residual displacement.

![Figure 1: Operation of the FPS: (a) undeformed configuration; (b) deformed configuration.](image_url)

Filled PTFE [6-7], Ultra High Molecular Weight (UHMWPE) [8], and Polyamide (PA) [9] have been used in applications as lining materials of the slider. The coefficient of friction of thermoplastic materials changes with a number of factors including the applied pressure and the velocity of sliding, the temperature, the roughness of the mating surface, and the wear and contamination of the sliding surfaces [10-15]. The typical dependency of the coefficient of friction on velocity and pressure is illustrated in Figure 2. Here $\mu_{st}$ denotes the static coefficient of friction observed in sticking, or pre-sliding, condition, and $\mu_{dyn}$ denotes the dynamic or kinetic coefficient of friction observed during sliding: $\mu_{dyn}$ in turns changes with velocity growing from a minimum value $\mu_{LV}$ at very low velocity to a maximum steady value $\mu_{HV}$ at high speeds. Static friction is developed during dwelling before the beginning of the motion and at any subsequent momentary sticking of the surfaces. The static coefficient of friction that manifests as the motion starts is denoted as the breakaway coefficient $\mu_B$; the drop of friction during the transition from sticking to sliding condition is generally large, and depending on the sliding material $\mu_B$ can be 1.5 to 4.5 times greater than $\mu_{LV}$. Based on experiments on steel – PTFE interfaces, Constantinou [16] suggested that for unworn PTFE a viable assumption for the ratio of static to low velocity coefficient of friction is about 4, and similar figures can be derived for current sliding materials, including filled PTFE, UHMWPE and PA [9, 14]. Research studies on the FPS also evidenced that friction decreases during sustained motion, as an effect of the heating of the sliding surface consequent to energy dissipation [17-19], which induces the melting of a thin layer of the thermoplastic liner material that acts a solid lubricant.
However, while the variation of the dynamic coefficient of friction with velocity and pressure has been recognized early on as significant and incorporated in software programs for structural analysis such as SAP2000 [20], OpenSees [21] and 3D-BASIS-ME [22], comprehensive models accounting for the effects of heating and breakaway are still not used in practice. An integrated friction model that incorporates the dependency on pressure, velocity, and heating was presented by Lomiento [19]. The three sources of variation are assumed independent of each other, and within this framework, the effect of heating is reproduced by means of a variable dependent on the histories of axial load and velocity that accounts for the accumulated heat supplied to the sliding surface from energy dissipation. Kumar [23-24] proposed an alternative three-function model, wherein the reference value of the coefficient of friction determined at a reference axial bearing pressure $p_0$, a reference temperature $T_0$ and high velocity $V_0$, is adjusted to the real application by three factors $k_p$, $k_T$ and $k_V$ that take account of the effects of instantaneous velocity, axial pressure and temperature at the sliding surface. This last friction model has been incorporated in the FPBearingPTV element formulated in the OpenSees software [25]. Gandelli [26] used mechanical fuses to reproduce the effect of static friction at the breakaway within the standard OpenSees framework.

The present study aims at proposing an enhanced friction model that incorporates the effects of axial load, velocity and heating, and includes new features such as the static coefficient of friction at the breakaway. This numerical formulation is developed and incorporated in a FPS bearing element coded in the object-oriented finite element OpenSees software. The advantages of the new OpenSees bearing element and the expected enhancement in the prediction capability of an ensuing response history analysis is discussed with reference to the case study of a reinforced concrete frame building.

2 FINITE ELEMENT MODEL OF FPS BEARING

2.1 Physical model

The SingleFPSimple3d element is coded in the finite element software OpenSees [25] to model a Friction Pendulum bearing comprising a single concave sliding surface and an articulated slider, like the model shown in Figure 1.

The physical model of the bearing is a two-node, twelve degrees of freedom, discrete element. The first node ($i$-Node) is located at the center of the concave sliding surface and the second node ($j$-Node) at the center of the spherical housing of the slider, with degrees of freedom in the global and the local coordinate systems defined as shown in Figure 3(a). The bear-
ing can displace in six directions, namely, translate in the vertical and in two horizontal directions, twist about the vertical axis, and rotate about two horizontal axes. In the vertical direction, the slider is considered rigid, but the vertical rigid-body motion of the slider accompanies the displacement in the horizontal direction.

Figure 3: Coordinate systems of the FPS element in OpenSees: (a) global and local systems; (b) basic system.
The response of the bearing can be conveniently formulated by introducing the basic coordinate system represented in Figure 3(b). The basic x-axis connects the centers of curvature of the lower and upper concave surfaces (Ci- and Cj-points, respectively), and the basic y- and z-axes follow the right-hand rule. In such representation, the bearing has six degrees of freedom corresponding to relative displacements and rotations between the auxiliary Ci- and Cj-Nodes, and the force – deformation response is formulated by assuming that the auxiliary nodes are connected by six springs that represent the mechanical behavior in the basic directions of the element: Axial, Shear 1, Shear 2, Torsion, Rotation1, and Rotation2. The discrete spring representation has the advantages of easy implementation and of being computationally efficient [27]; structural analysis programs that allow the user to add functionalities through user-created elements, like the OpenSees software, provide a framework for such implementation.

In the basic representation the general expression of the element stiffness matrix is [25]:

\[
[K_b] = \begin{bmatrix}
Axial & 0 & 0 & 0 & 0 & 0 \\
0 & Shear1 & Shear21 & 0 & 0 & 0 \\
0 & Shear12 & Shear2 & 0 & 0 & 0 \\
0 & 0 & 0 & Torsion & 0 & 0 \\
0 & 0 & 0 & 0 & Rotation1 & 0 \\
0 & 0 & 0 & 0 & 0 & Rotation2
\end{bmatrix}
\]

(1)

and the associate element force vector is

\[
\{q_b\} = \begin{bmatrix}
Axial \\
Shear1 \\
Shear2 \\
Torsion \\
Rotation1 \\
Rotation2
\end{bmatrix}
\]

(2)

The SingleFPSimple3d element has coupled friction properties with post-yield stiffening owing to the concave surface for the shear deformations, and linear force-deformation behaviors defined by UniaxialMaterial elastic models in the remaining four directions. To capture the uplift behavior of the bearing, the user-specified UniaxialMaterial in the axial direction is modified for no-tension behavior. Coupling between vertical and horizontal directions and between vertical direction and rotation is considered indirectly by using expressions for mechanical properties that are derived using explicit consideration for geometric nonlinearity due to large displacement effects [31]. By default, P-Delta moments are entirely transferred to the concave sliding surface, so that rotations of the concave surface affect the shear behavior of the bearing.

2.2 Numerical formulation

The element force, displacement, and stiffness matrices are formulated at the component level in the element’s basic coordinate system and transformation matrices are used to switch from basic to local and then from local to global coordinates. In the global system the contribution of each element is assembled to obtain the equations for the whole model, which are solved to obtain nodal forces and displacements. The nodal response quantities calculated in
the global system are then transformed back to the element’s local and basic coordinate systems to obtain forces and displacements in each element. The symbols \( \{ u_b \} \) and \( \{ \dot{u}_b \} \) are used hereinafter for the nodal displacement and nodal velocity vectors, while subscripts \( b \) and \( l \) will designate the quantities in basic and local coordinates, respectively.

The software performs a numerical procedure to calculate the internal forces of the element. The description is here limited to the procedure for calculation of the two shear forces, while for the other four components the standard \textit{UniaxialMaterial} model is adopted.

As a first step, the radius in the two basic shear directions is calculated accounting for the deformation of the bearing element:

\[
R_y = \sqrt{R^2 - (u_b(3))^2} \\
R_z = \sqrt{R^2 - (u_b(2))^2}
\]

where \( R \) is the radius of curvature of the bearing, and \( u_b(2) \) and \( u_b(3) \) are the displacements in the basic \( y \) and \( z \) directions defined in Figure 3(b).

Noting that for small incremental displacements the two vectors \( \{ u_b \} \) and \( \{ \dot{u}_b \} \) have the same direction, tangent to the concave surface, the absolute velocity is calculated as:

\[
|\dot{u}_b| = \sqrt{\left(\frac{u_b(2)}{R_y} \cdot \dot{u}_b(2) + \frac{u_b(3)}{R_z} \cdot \dot{u}_b(3)\right)^2 + \left(\dot{u}_b(2)\right)^2 + \left(\dot{u}_b(3)\right)^2}
\]

The hysteretic force – displacement relationship of the FPS bearing in the horizontal direction is mathematically modeled using the theory of plasticity [28-31]. When the shear force is smaller than the friction force at the sliding surface, sliding is prevented and the deformation is purely elastic. When the force exceeds the friction force, then sliding takes place.

An iterative procedure is performed to calculate shear forces and stiffnesses in basic \( y \) and \( z \) directions. At each iteration step, the normal force is first calculated

\[
N = -q_b(1) + q_{b,Old}(2) \cdot \frac{u_b(2)}{R_y} + q_{b,Old}(3) \cdot \frac{u_b(3)}{R_z} - q_{b,Old}(2) \cdot u_l(6) + q_{b,Old}(3) \cdot u_l(5)
\]

where \( q_b(1) \) is the axial force associated to displacement \( u_b(1) \) in axial direction through the \textit{UniaxialMaterial} model, and \( q_{b,Old}(2) \) and \( q_{b,Old}(3) \) are the values of shear force calculated at the previous iteration step.

Along each shear direction, the total resisting force in the plasticity model can be represented as the sum of an elastic and a hysteretic force component, as shown in Figure 4. The stiffness associated with the elastic component is given by the ratio between the normal force and the radius

\[
K_{2y} = \frac{N}{R_y} \quad K_{2z} = \frac{N}{R_z}
\]

and the initial stiffness of the hysteretic component is given as the difference between the (isotropic) initial stiffness of the isolator, \( K_1 \), and the elastic stiffness

\[
K_{0y} = K_1 - K_{2y} \quad K_{0z} = K_1 - K_{2z}
\]
The displacements of the hysteretic component are used as a state variable for the plasticity model, and the trial values of the hysteretic shear forces $q_{Trial}(2)$ and $q_{Trial}(3)$ are calculated as:

$$q_{Trial}(2) = K_{0,y} \cdot (u_b(2) - u_{b,PlasticOld}(2))$$
$$q_{Trial}(3) = K_{0,z} \cdot (u_b(3) - u_{b,PlasticOld}(3))$$

(8)

where $u_b,PlasticOld(2)$ and $u_b,PlasticOld(3)$ are the plastic displacements at the previous iteration.

Since FPS bearings with spherical surfaces exhibit isotropic behavior for bidirectional motion, a circular yield condition is adopted. A dummy parameter $Y$ is introduced to regulate the transition from elastic to plastic behavior, corresponding to the transition of the bearing response from sticking to sliding conditions:

$$Y = |q_{Trial}| - q_{yield}$$

(9)

where $q_{yield}$ is the yield force calculated in accordance with the assumed friction model and

$$|q_{Trial}| = \sqrt{(q_{Trial}(2))^2 + (q_{Trial}(3))^2}$$

(10)

is the resultant hysteretic shear force. The parameter $Y$ therefore represents the excess force above the yield strength of the bearing element.

When $Y \leq 0$ (elastic region), the shear forces are calculated as

$$q_b(2) = q_{Trial}(2) + K_{2,y} \cdot u_b(2) - N \cdot u_f$$

$$q_b(3) = q_{Trial}(3) + K_{2,z} \cdot u_b(3) + N \cdot u_f$$

(11)

and the tangent stiffness for coupled shear directions is

$$K_b(2,2) = K_b(3,3) = K_1$$

$$K_b(2,3) = K_b(3,2) = 0$$

(12)

When $Y > 0$ (plastic region), the software performs a return mapping algorithm [29] to calculate the resisting force. By assuming an associative plastic flow rule, the trial slip in either shear direction is obtained by dividing the dummy parameter $Y$ by the initial elastic stiffness of the hysteretic component

$$d\Gamma_y = \frac{Y}{K_{0,y}}$$

$$d\Gamma_z = \frac{Y}{K_{0,z}}$$

(13)
and the plastic displacement is then updated as

\[
\begin{align*}
&u_{b, \text{Plastic}}(2) = u_{b, \text{Plastic Old}}(2) + d\Gamma_y \cdot \frac{q_{\text{Trial}}(2)}{q_{\text{Trial}}} \\
&u_{b, \text{Plastic}}(3) = u_{b, \text{Plastic Old}}(3) + d\Gamma_z \cdot \frac{q_{\text{Trial}}(3)}{q_{\text{Trial}}}
\end{align*}
\]

Eventually the shear forces and associated components of the tangent stiffness are calculated:

\[
\begin{align*}
&q_{b}(2) = q_{\text{yield}} \cdot \frac{q_{\text{Trial}}(2)}{q_{\text{Trial}}} + K_{2,y} \cdot u_b(2) - N \cdot u_f(6) \\
&q_{b}(3) = q_{\text{yield}} \cdot \frac{q_{\text{Trial}}(3)}{q_{\text{Trial}}} + K_{2,z} \cdot u_b(3) + N \cdot u_f(5) \\
&K_{b}(2,2) = K_{0,y} \cdot q_{\text{yield}} \cdot \frac{q_{\text{Trial}}(3) \cdot q_{\text{Trial}}(3)}{q_{\text{Trial}}^3} - K_{2,y} \\
&K_{b}(3,3) = K_{0,z} \cdot q_{\text{yield}} \cdot \frac{q_{\text{Trial}}(2) \cdot q_{\text{Trial}}(2)}{q_{\text{Trial}}^3} - K_{2,z} \\
&K_{b}(2,3) = -K_{0,z} \cdot q_{\text{yield}} \cdot \frac{q_{\text{Trial}}(2) \cdot q_{\text{Trial}}(3)}{q_{\text{Trial}}^3} \\
&K_{b}(3,2) = -K_{0,y} \cdot q_{\text{yield}} \cdot \frac{q_{\text{Trial}}(2) \cdot q_{\text{Trial}}(3)}{q_{\text{Trial}}^3}
\end{align*}
\]

The procedure is run iteratively until convergence is achieved, i.e. the difference between the moduli of the resultant shear force in two consecutive runs is less than a set tolerance level. If the maximum number of iterations is exceeded, the algorithm fails and an error is returned. When convergence is achieved, the shear force components are used to formulate the element stiffness matrix in basic representation \([K_b]\) according to Equation (16). The element stiffness matrix is then transformed into the local coordinate system and “P-Delta” and “V-Delta” moment stiffness terms are added to the local force vector. The local stiffness matrix is eventually transformed into the global coordinate system and assembled to the contributions of the other elements to obtain the system of equations governing the response of the overall model.

### 2.3 Friction model

The standard SingleFPSimple3d element calculates the yield force based on the friction law coded in the associated FrictionModel command. Different friction model types are available in OpenSees libraries, where the coefficient of friction is either a constant (Coulomb friction) or it is a function of the sliding velocity, of the axial pressure, or of both of them.

The new element, hereinafter called the CSSBearing_BVNC element [32], has been implemented by modifying the source code of the standard element to introduce novel features such as the degradation of the coefficient of friction due to heating, and the static coefficient of friction at the breakaway. The constitutive modeling is similar to that of the SingleFPSimple3d element, otherwise.
It is noted that the absolute velocity of the bearing calculated according to Equation (4) includes the contribution of both elastic and plastic deformations. Therefore, the standard OpenSees element uses the total velocity, instead of the sliding velocity only, to calculate the current value of the coefficient of friction: this makes impossible to account, within the standard formulation, for the static friction coefficient, because the elastic velocity is never zero even during the sticking phase. The static friction has been incorporated in the novel formulation by introducing two distinct plastic material models to account for either the static or the dynamic friction and a switch condition triggered at the breakaway:

\[
q_{\text{yield}} = \mu_B \cdot N \quad \text{for} \quad h < 1
\]

\[
q_{\text{yield}} = \mu_{VNC} \cdot N \quad \text{for} \quad h \geq 1
\]

where \( h \) is a counter that is updated each time the yield condition \( Y > 0 \) is achieved. At the beginning of the analysis, the counter is initialized \( (h = 0) \), and the yield force \( q_{\text{yield}} \) is defined by a circular yield criterion according to a Coulomb material, i.e. \( q_{\text{yield}} = \mu_B \cdot N \), where \( \mu_B \) is the static coefficient of friction at breakaway (Figure 5(a)). Once yield occurs for the first time, the variable \( h \) is updated to \( h = 1 \), and from this time the plasticity algorithm switches to the user-defined \( VNC_{\text{Friction}} \) material model (Figure 5(b)). The second material model calculates the yield force as \( q_{\text{yield}} = \mu_{VNC} \cdot N \), where \( \mu_{VNC} \) is a function of the axial load, velocity and cumulated heat flux according to the expression

\[
\mu(N,V,c) = f_{NV}(N,V) \cdot f_c(c)
\]

Here \( f_{NV}(N,V) \) is a function that accounts for the effects of the instantaneous values of axial load \( N \) and velocity \( V \), while \( f_c(c) \) is a second function that accounts for the effect of the cumulated heat generated at the sliding surface through a degradation variable \( c \).

![Figure 5: (a) Bidirectional plasticity domain and (b) associated friction models.](image)

The function \( f_{NV}(N,V) \) is developed accordingly to the standard exponential formulation

\[
f_{NV}(N,V) = \mu_{HV}(N) - \left[ \mu_{HV}(N) - \mu_{LV}(N) \right] \cdot \exp\left( -\alpha |\dot{u}_b| \right)
\]

where \( \mu_{LV} \) and \( \mu_{HV} \) are the values of the dynamic coefficient of friction at low velocity and high velocity, respectively, and \( \alpha \) determines the rate of change of the dynamic coefficient of friction with the sliding velocity [16]. The coefficients \( \mu_{LV}, \mu_{HV} \) and \( \alpha \) depend on the instanta-
neous value of the axial load and are calculated at each iteration step according to power law expression [33]

\[
\begin{align*}
\mu_{HV}(N) &= A_{HV} \cdot N^{(n_{HV} - 1)} \\
\mu_{LV}(N) &= A_{LV} \cdot N^{(n_{LV} - 1)} \\
\alpha(N) &= \alpha_0 + \alpha_1 \cdot N + \alpha_2 \cdot N^2
\end{align*}
\] (20)

Here parameters \(A_{LV}, A_{HV} > 0\) and \(n_{LV} \leq 1, n_{HV} \leq 1\) determine the friction coefficient models, while the parameters \(\alpha_0, \alpha_1, \) and \(\alpha_2\) determine the effect of the axial load on \(\alpha\).

The effect of heating on the coefficient of friction is taken into account by means of a reduction factor expressed through the degradation function \(f_c\)

\[
f_c(c) = \exp\left(-\left(c / c_{ref}\right)^\gamma\right)
\] (21)

where \(c_{ref}\) is a parameter that regulates the rate of degradation (the smaller \(c_{ref}\), the higher the degradation), \(\gamma\) is a parameter that controls the shape of the function, and \(c\) is a variable that depends on the total power dissipated at the sliding surface and the distance travelled by the slider [32]:

\[
c = \int_0^t N \cdot |\dot{\textbf{u}}_b|^2 \, dt
\] (22)

At each time step, the increment \(\Delta c\) of the variable over the time interval \(\Delta t\) is calculated by numerical integration of Equation (20), and the variable is updated as \(c(t + \Delta t) = c(t) + \Delta c\).

Ten parameters are used in the implementation of the CSSBearing_BVNC element, namely \(\mu_B\) (static coefficient of friction), \(A_{LV}, A_{HV}, n_{LV}, n_{HV}\) (load-effect parameters), \(\alpha_0, \alpha_1, \) and \(\alpha_2\) (velocity-effect parameters), and \(c_{ref}\) and \(\gamma\) (degradation-effect parameters). Since in OpenSees software the parameters that characterize the element response are dimensionless [25], the units of the parameters must be specified consistently with the units adopted for the fundamental physical quantities, in accordance with Table 1. Different friction models, including those already coded in OpenSees, such as Coulomb, VelDependent, and VelNormalFrcDep materials [25], can be derived from the novel formulation by setting the relevant parameters, as shown in Table 2.

<table>
<thead>
<tr>
<th>parameter</th>
<th>unit</th>
<th>effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_B)</td>
<td>–</td>
<td>breakaway</td>
</tr>
<tr>
<td>(A_{LV})</td>
<td>((10^{-3} , \text{N})^{1-n_{LV}})</td>
<td>–</td>
</tr>
<tr>
<td>(n_{LV})</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(A_{HV})</td>
<td>((10^{-3} , \text{N})^{1-n_{HV}})</td>
<td>axial load</td>
</tr>
<tr>
<td>(n_{HV})</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(\alpha_0)</td>
<td>s mm(^{-1})</td>
<td>–</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>s mm(^{-1}) N(^{1})</td>
<td>velocity</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>s mm(^{-1}) N(^{2})</td>
<td>–</td>
</tr>
<tr>
<td>(c_{ref})</td>
<td>((10^{-3} , \text{N}) , \text{mm}^2/\text{s})</td>
<td>heating</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 1: Units of the friction model parameters of the CSSBearing_BVNC element.
Table 2: Friction models enveloped by the CSSBearing_BVNC element and equivalent FrictionModel objects coded in OpenSees software.

<table>
<thead>
<tr>
<th>source of variation</th>
<th>OpenSees FrictionModel</th>
<th>parameter set</th>
</tr>
</thead>
<tbody>
<tr>
<td>load, velocity, heating</td>
<td>= µB = A_{LV}</td>
<td>other parameters assigned</td>
</tr>
<tr>
<td>load, velocity, VelNormalFrcDep</td>
<td>µB = A_{LV}</td>
<td>c_{ref} = 10^{100}, γ = 1</td>
</tr>
<tr>
<td>velocity VelDependent</td>
<td>µB assigned</td>
<td></td>
</tr>
<tr>
<td>no variation Coulomb</td>
<td>µB assigned</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Friction model parameters assigned in code verification tests.

<table>
<thead>
<tr>
<th>parameter</th>
<th>unit</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>µB</td>
<td>–</td>
<td>0.10</td>
<td>0.05</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>A_{LV}</td>
<td>(10^{-3} N)^{(1-n_{LV})}</td>
<td>0.10</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>n_{LV}</td>
<td>–</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A_{HV}</td>
<td>(10^{-3} N)^{(1-n_{HV})}</td>
<td>0.10</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>n_{HV}</td>
<td>–</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>α₀</td>
<td>s mm^{-1}</td>
<td>10^{100}</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>α₁</td>
<td>s mm^{-1} N^{-1}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>α₂</td>
<td>s mm^{-1} N^{-2}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c_{ref}</td>
<td>(10^{-3} N) mm^2/s</td>
<td>10^{100}</td>
<td>10^{100}</td>
<td>10^{100}</td>
<td>5×10^{15}</td>
</tr>
<tr>
<td>γ</td>
<td>–</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The first step of code verification aims at verifying the capability of the novel element to reproduce the response of the standard element coded in OpenSees software. In Test 1, the
CSSBearing_BVNC element is compared to the SingleFPSimple3d element with constant (Coulomb) friction coefficient $\mu = 0.10$. In Test 2, comparison is made to the SingleFPSimple3d element with VelDependent friction model with assigned parameters $\mu_{LV} = 0.05$, $\mu_{HV} = 0.15$, and $\alpha = 0.05$ s/mm. Figure 6 demonstrates the complete agreement between the two codes, as shown by the complete overlapping of the horizontal force – displacement curves and displacement history plots.

![Figure 6: Code-to-code comparison between CSSBearing_BVNC element (new code) and SingleFPSimple3D element (standard code) with assigned (a) Coulomb friction model, and (b) VelDependent friction material model.](image1)

The second step of verification addresses the implementation of static friction and heating degradation in the new code. In Test 3, the contribution of the static coefficient of friction is introduced in the CSSBearing_BVNC element. The results of the analyses are illustrated in Figure 7. The force history presents one spike at the breakaway, due to the effect static friction, and no further spikes during sliding, demonstrating the correct switch of the yield threshold from $\mu_B$ to $\mu_{VNC}$ after the first yielding. The maximum force at breakaway is 297 kN, with a 1% deviation from the expected value of 294.3 kN (i.e. $0.30 \times 981$ kN).

![Figure 7: Code-to-code comparison between CSSBearing_BVNC element incorporating velocity and breakaway effects (BV element) and standard element with assigned VelDependent friction model (VelDep).](image2)

In Test 4, the heating effect is introduced in the code by assigning the parameters $c_{ref} = 5 \times 10^{15}$ $(10^{-3}$ N mm$^2$/s) and $\gamma = 1.0$, all other parameters being the same as in Test 3. Calculated force – displacement loops and response histories are illustrated in Figure 8. The degradation of the coefficient of friction is evident from the continuous decrease of the shear
force at each cycle, which is not predicted by the standard code with VelDependent material (model parameters: $\mu_{LV} = 0.05$, $\mu_{HV} = 0.15$, $\alpha = 0.05$ s/mm); comparison of shear forces between the two models registers a -4.4% difference at the first cycle, and -21% at the tenth cycle. Eventually Figure 9 compares for each cycle, at the instants of peak velocity, the coefficient of friction calculated from the software (red dots) to the expected value (solid line), where the degradation function $f_c$ is analytically calculated in accordance with Equations (19) and Eq. (20). The numerical prediction matches the analytical value with a deviation less than 0.5% at any point. It can be therefore concluded that also the incorporation of the heating effect in the new bearing element is verified.

![Figure 8](image-url)  
**Figure 8:** Code-to-code comparison between CSSBearing_BVNC element incorporating velocity, breakway and thermal effects (BVC element) and standard element with assigned VelDependent friction model (VelDep).

![Figure 9](image-url)  
**Figure 9:** Degradation of the coefficient of friction of the CSSBearing_BVNC element compared to the analytical solution according to Equations (19) and (20).

### 3 RESPONSE HISTORY ANALYSES

To numerically assess the performance of the CSSBearing_BVNC element and evaluate its potential to yield to a more accurate estimation, the new code has been used for the dynamic analyses of a multi-degrees-of-freedom base-isolated structure in accordance with the provisions of the Italian Building Code [34], and the results are compared to the response calculated through the standard SingleFPSimple3d element coded in OpenSees library [25].
3.1 Case study

A regular reinforced concrete, moment-resisting-frame building is considered. The structure is characterized by a double symmetric square plan of 18 by 18 m with three bays of 6 m length in both longitudinal and lateral directions, and four stores at 3 m each, for a total height of 12 m, and rests on a rigid base slab (Figure 10). Rectangular (600×300 mm) beams are used at every floor. The columns have square cross-section, with dimensions of 500×500 mm at the ground and the first floor, and 400×400 mm at the second and at the third floor. Seismic masses were evaluated by taking into account the full permanent loads plus 30% of the live loads for residential buildings [34]. The total seismic weight of each floor and of the base slab is 2400 kN, resulting in a cumulative weight of the whole building of 12000 kN.

![Figure 10: Section of the case-study building, base-isolated with FPS bearings, where $m_{\text{floor}}$ is the mass of each floor and $M_{\text{BS}}$ is the mass of the base slab.](image)

The structural model is implemented in the OpenSees v.2.5.4 software [25]. A moment-resisting frame structure with rigid joints is assumed in either direction of the building. *ElasticBeamColumn* elements [25] are used for the structural members, and the superstructure is considered to behave as a linear elastic system according to the provisions of the Italian Building Code [34] for base-isolated structures. The assigned modulus of elasticity of concrete is $E_c = 31476$ MPa (C25/30 concrete class), and the moments of inertia of the homogenized cross-sections are $I_{c1} = 530512\times10^4$ mm$^4$, $I_{c2} = 165627\times10^4$ mm$^4$ and $I_b = 129872\times10^4$ mm$^4$ for the 500×500 mm columns, the 400×400 mm columns and the 600×300 mm beams, respectively. Eventually, a *RigidFloorDiaphragm* multi-points constraint [25] is introduced at each story to account for the in-plane stiffness of the floor slabs. The fundamental period of the superstructure is $T_{SS} = 0.31$ s. The internal structural damping is modeled as a stiffness proportional damping [35], with parameters assigned to achieve 5% damping ratio at a 3.5 s period.

The building is isolated at the foundation level by means of sixteen FPS bearings, one bearing underneath each column. A floor diaphragm composed of stiff beams is created above the isolation units in order to prevent differential displacements. The nodes at foundation level are constrained by means of rigid joints and subjected to the application of an *UniformExcitation* seismic input [25].
3.2 Isolation system

The isolation system consists of sixteen FPS isolators, with effective radius \( R = 2500 \text{ mm} \), corresponding to a design period \( T_{iso} = 3.17 \text{ s} \). The initial stiffness of the bearings is 479.52 kN/mm. The \texttt{CSSBearing\_BNVC} element is used to model the isolators and the parameters of the friction model for its implementation are given in Table 4, where a ratio \( \mu_B / \mu_{LV} = 4 \) is assumed. For simplicity, in the case study the load effect is disregarded \( (n_{LV} = n_{HV} = 1) \). To enlighten the influences of static friction and heating, two friction models enveloped by the \texttt{CSSBearing\_BNVC} element are assumed: the BV model, accounting for breakaway and velocity effects, and the BVC model accounting for breakaway, velocity and heating effects.

<table>
<thead>
<tr>
<th>Friction model</th>
<th>( \mu_B )</th>
<th>( A_{LV} )</th>
<th>( n_{LV} )</th>
<th>( A_{HV} )</th>
<th>( n_{HV} )</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( c_{ref} )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BV</td>
<td>0.12</td>
<td>0.03</td>
<td>1</td>
<td>0.075</td>
<td>1</td>
<td>0.055</td>
<td>0</td>
<td>0</td>
<td>10^{100}</td>
<td>1.0</td>
</tr>
<tr>
<td>BVC</td>
<td>0.12</td>
<td>0.03</td>
<td>1</td>
<td>0.075</td>
<td>1</td>
<td>0.055</td>
<td>0</td>
<td>0</td>
<td>3.49 \times 10^{14}</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 4: Friction model parameters assigned in the case study; units in accordance with Table 2.

Baseline references for comparison are derived from nonlinear analyses of the same structure performed by using the \texttt{SingleFPSimple3d} element with \textit{VelDependent} friction model [25] and assigned parameters: \( \mu_{LV} = 0.03, \mu_{HV} = 0.075 \), and \( \alpha = 0.055 \text{ s/mm} \).

3.3 Seismic inputs

The nonlinear history response analyses of the base-isolated building are performed assuming an ordinary structure (functional class II [35]) with nominal life of 50 years, corresponding to a reference period of 50 years, located in Naples, South Italy (14.28° longitude, 40.86° latitude), topographic category T1, soil type A (rock or other rock-like geological formation). Target elastic spectra were determined in accordance with the Italian Building Code [35] provisions for Damage Limitation (SLD) and Human Life Saveguard (SLV) hazard levels.

For either hazard level, a set of 21 independent bidirectional ground motions consistent with the Italian Building Code [35] was selected with REXEL v3.4 beta [36] software from the European Strong-motion Database [37]. The magnitude (Mw) of the ground motions was chosen within the interval (5–8), and the epicentral distance (Rep) in the range (0–50 km), and more precisely 7 accelerograms with Rep in the range (0–10 km), 7 accelerograms with Rep in the range (10–20 km), and 7 accelerograms with Rep in the range (20–50 km), in order to cover different fault distances. The selected waveforms were scaled to the design Peak Ground Acceleration of either 0.059 g (SLD level) or 0.168 g (SLV level) calculated according to the code [34]. At either limit state, the average spectrum of the accelogram set matches the Italian Building Code spectrum with a tolerance of -10/+30% in the period range 0.15 – 4.0 sec, with due consideration of the fundamental period of the base-isolation system [34].

The vertical component of the ground motion has been neglected. Though the assumption is acknowledged unrealistic in practice, it has been adopted in order to directly compare the results of the analyses performed using the BVNC friction model to the baseline results obtained through the \textit{VelDependent} model.

3.4 Results

A first important result of the analyses is the confirmation that the static coefficient of friction at breakaway can prevent weak or moderate intensity ground motions from triggering the FPS isolators. Sliding begins when the shear force through the bearings, which is the resultant of the inertial forces acting on the masses of the superstructure and the base slab, exceeds the frictional force at the sliding surface. If the contribution of the breakaway friction is consid-
erred though the CSSBearing_BNVC element with either the BV or the BVC model, 18 out of the 21 selected ground motions at SLD intensity level prove to be unable to trigger sliding (Table 5); on the contrary at SLV level sliding of the bearings is always triggered.

<table>
<thead>
<tr>
<th>Friction model</th>
<th>hazard level</th>
<th>Epicentral distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0-10</td>
</tr>
<tr>
<td>BV, BVC</td>
<td>SLD</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>SLV</td>
<td>7</td>
</tr>
<tr>
<td>VelDependent</td>
<td>SLD</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>SLV</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 5: Trigger of FPS bearings for either hazard level predicted by the assumed friction models.

The response of the structure is appraised by means of the following quantities: (i) the horizontal displacement of the base slab $d_{iso}$; (ii) the ratio between the overall shear force $V_{iso}$ carried by the FPS bearings and the total vertical load ($N_{SS} + N_{BS}$), where $N_{SS}$ and $N_{BS}$ are the seismic weights of the superstructure and the base slab, respectively; (iii) the inter-story drift ($\Delta$); (iv) the superstructure seismic coefficient ($SC$) defined as the ratio between the column shear and the supported seismic weight; (v), the maximum floor acceleration in the superstructure $a_{SS}$. During the post-processing of the analysis results, the maxima of the response parameters have been identified for each history analysis and the mean and standard deviation values calculated for each set of 21 ground motions. The results are summarized in Figure 11.

The improvements in the accuracy of the analyses introduced by the new CSSBearing_BNVC element are discussed hereinafter by comparison with the results provided by the standard element with VelDependent friction model.

The static coefficient of friction at breakaway has an insignificant effect on the maximum displacement at SLV level, as it can be observed by comparing the results provided by the BV and by the VelDependent models. The effect of heating, which is accounted for in the BVC friction model introduced by the CSSBearing_BNVC element, is to reduce damping and hence to increase the displacement: at SLV level, a +58% increase with respect to the baseline value is indeed predicted, whereas at SLD level a comparison of the displacement demands provided by the various models is not feasible because of the large number of ground motions which failed to activate the isolators.

The outcome of the breakaway friction on the shear force though the bearings is shown in the second panel of Figure 11, with a +155% increase at SLD level and a +95% increase at SLV level. The degradation of the coefficient of friction due to heating is insignificant not only at SLD level, but also at SLV level in spite of the increased displacement: in these circumstances due to the huge increase of the frictional resistance at the onset of the motion, the shear force at breakaway is larger than the force developed at the maximum displacement of the bearings. Eventually, it is worth noting that the average shear force predicted at SLD by the CSSBearing_BNVC element with breakaway is 71% higher than the value calculated by the standard SingleFPSimple3d element at SLV, and only 13% lower that the shear force at SLV according to the new element. Dynamic analyses performed using conventional bearing elements and friction models are likely to largely underestimate the maximum force in the isolation system, and this result should be taken into account in the design of the bearings and the supporting structural elements. In the situations where the sliding isolators are not triggered, the superstructure behaves as a fixed base structure, subjected to higher accelerations than expected according to the design of the isolation system. This is well evident from the analyses at SLD level, which demonstrate a +128% increase in peak floor acceleration,
+130% in shear force in the most stressed columns, and a +134% in inter-story drift over the baseline values. As already noted for the force through the FPS bearings, internal forces and deformations of the superstructure at SLD are larger than the values predicted by the standard element at SLV, namely +57% for drift, +64% for column shear and +59% for floor acceleration. Although for the investigated case study the levels of inter-story drift and shear force are within the serviceability limits [34], attention is drawn to the possible consequences of large drifts and floor accelerations on the performance of nonstructural elements [38].

At SLV level, accounting for the static friction at breakaway entails again a large increase in superstructure drift (+41%), column shear (+43%), and maximum floor acceleration (+75%) when compared to the response predicted through the VelDependent friction model. The influence of heating is insignificant, demonstrating that the superstructure experiences the maximum acceleration when the isolators are in the sticking condition before breakaway.

Figure 11: Response of the base-isolated structure at SLD and SLV hazard levels.
4 CONCLUSIONS

• A novel CSSBearing_BVNC element has been formulated in OpenSees software by modifying the standard SingleFPSimple3d element. The new features introduced by the formulation are the static friction at the breakaway and the degradation of the coefficient of friction induced by the heat generated during the sliding motion, while the dependence of the friction coefficient on the instantaneous values of axial load and velocity follows accepted practice within the literature. The consistency of the new element has been verified in a code-to-code comparison by means of numerical tests considering unidirectional histories.

• A comparative evaluation performed through a case study highlights the potential of the newly developed isolator element to yield a more accurate estimation. Nonlinear response history analyses of a base-isolated building help to quantify the improved prediction capability over the standard element when applied to real situations, with a +40% increase in estimate of superstructure drift and column shear force and a +58% increase in estimate of displacement demand at Human Life Safeguard hazard level, and possible non-activation of the sliding isolators at Damage Prevention level.

• Since current elements for FPS bearings coded in software programs for structural analysis do not account for static friction, this can lead to underestimate the forces and accelerations induced in the superstructure, especially under weak or medium intensity earthquakes which are unable to trigger the isolators. The proposed CSSBearing_BVNC element is expected to overcome this issue.

• The degradation of the coefficient of friction promoted by heating has an important effect in determining the maximum displacement of the isolation system; however its influence on the response of the superstructure may be negligible even under strong earthquakes, as it has been found in the assumed case study, when sliding of the isolators is delayed due to the large breakaway frictional resistance, and the peak response occurs while the isolators are in sticking condition.

• Further studies should present an experimental validation of the element by simulating actual experimental observations.

REFERENCES


OPTIMIZATION OF NONLINEAR FLUID VISCOUS DAMPERS FOR BUILDING STRUCTURES: ENERGY-BASED DESIGN APPROACH UNDER STOCHASTIC SEISMIC EXCITATION

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Abstract

It is well known that there are certain advantages in using nonlinear (rather than linear) fluid viscous dampers, primarily, the achievement of a desired seismic performance with reduced damper forces in comparison with linear counterparts. However, the excitation-dependent constitutive behavior of such nonlinear devices may complicate the related optimal design procedure so that, generally, multiple-step approaches are pursued by exploiting the concept of “energy equivalent” dampers after a preliminary optimization of linear viscous dampers is carried out. As an alternative, this contribution presents a stochastic-based numerical procedure in which the nonlinear behavior of the dampers is fully integrated in a single-step optimization process. In particular, a non-Gaussian stochastic linearization technique is used in the optimal design procedure to linearize the equations of motion of structures equipped with NFVDs and subject to a Kanai-Tajimi filtered random excitation. The optimal dampers are identified by addressing an energy-based objective function representing the energy dissipated by the devices out of the total input energy from the earthquake excitation. Therefore, the procedures aim to maximize the energy dissipation performance of the devices subject to a given constraint condition, namely a (desired) target added damping ratio. The effectiveness of the proposed energy-based stochastic design approach is assessed by comparison with alternative procedures through stochastic dynamic analysis as well as by performing nonlinear response history analysis under a suite of ground motion records.

Keywords: Fluid Viscous Dampers, Optimal Damper Design, Energy Dissipation Devices, Stochastic Earthquake Excitation, Numerical Optimization, Structural Control.
1 INTRODUCTION

Besides seismic base isolation [1], [2], supplemental energy dissipation devices in the form of viscous dampers have been increasingly used for earthquake protection of new and existing structures. In this context, damper optimization has been a subject of investigation for decades with a large number of analytical, heuristic, and numerical methodologies developed thus far. A review of some of these procedures can be found in the monograph by Takewaki [3], in the book by Lagaros et al. [4] and in the recent review paper by De Domenico et al. [5]. However, most of the developed strategies imply a linear behavior of the dampers, whereas a nonlinear constitutive model would be more suitable for fitting the experimental results [6], with a power law relation governed by a velocity exponent \( \alpha \) typically in the range 0.1-0.5 [7]. Additionally, there are certain advantages in employing nonlinear dampers, primarily the reduction of damper forces for accomplishing a comparable seismic performance to linear dampers [8]. Since high peak forces of the devices have a great impact in terms of economic implications, manufacturing companies currently strive for achieving such a nonlinear power law behavior in their production process. Consequently, numerical procedures for the optimal design of nonlinear fluid viscous dampers (NFVDs) represent an important research topic that is still of great interest.

For NFVDs the constitutive behavior is excitation-dependent, meaning that the damper response is related to the intensity of the earthquake ground motion. Thus, the determination of the optimal parameters of NFVDs is not straightforward but is usually pursued within a multiple-step approach [9], [10], by exploiting the concept of “energy-equivalent” dampers [11]. As a first step, the optimal characteristics of linear fluid viscous dampers (LFVDs) are identified, as if they had linear behavior; then, the maximum velocity at the ends of the LFVDs is evaluated via a set of preliminary time-history analyses consistent with a given earthquake scenario; finally, the optimal LFVDs are converted into “energy-equivalent” NFVDs associated with a comparable amount of energy dissipation. A drawback of this approach is the need for a preliminary set of time-history analyses to determine the maximum relative velocity of each damper. Moreover, the hypothesis underlying the energy-equivalence between LFVDs and NFVDs is that the earthquake excitation gives rise to a harmonic motion, whereas the earthquake-induced motion is a random process, which can be better captured via stochastic-based approaches.

Building on these motivations, this paper presents a stochastic-based numerical procedure for the optimal design of NFVDs. The proposed optimal design procedure explicitly emphasizes the primary role of NFVDs as energy dissipation devices. Indeed, the numerical procedure aims to maximize the energy dissipation capacity of these devices under a given earthquake excitation described through a power spectral density (PSD) function within a stochastic framework. The objective function is derived from the energy balance equations of the system, and it represents the energy dissipated by the NFVDs in stochastic terms. In this way, the optimal set of dampers are identified as the ones that dissipate the largest possible amount of energy out of the total input energy from the earthquake excitation, under a given constraint condition related to a (desired) target added damping ratio. Moreover, unlike the multiple-step approaches outlined before, the proposed procedure is a single-step approach in which the nonlinear behavior of the dampers is fully integrated in the optimization process. Indeed, to deal with the nonlinear constitutive behavior of the dampers through simplified and amenable tools of the linear random vibration theory, a novel energy-based stochastic linearization technique recently developed by the authors [12] is employed and incorporated in the optimal design process. With this expedient, one has to deal with linearized damping coefficients of the dampers that, though, account for their actual nonlinear behavior.
The performance of the proposed optimal design procedure is assessed via both stochastic dynamic analysis under a Kanai-Tajimi filtered stationary stochastic excitation and nonlinear response history analysis under real earthquake ground motions. Comparison of the proposed optimal design approach against alternative distribution procedures that are not based on energy principles is finally discussed in a critical manner.

2 EQUATIONS OF MOTION AND STOCHASTIC LINEARIZATION

The resisting force of the $j^{\text{th}}$ NFVD in a structure can be expressed as

$$f_{dj} = c_{dj} |\dot{w}_j|^{\alpha_j} \text{sgn}(\dot{w}_j)$$

(1)

where $c_{dj}$ and $\alpha_j$ represent the damping coefficient and the velocity exponent, respectively, and $\dot{w}_j$ is relative velocity at the ends of the $j^{\text{th}}$ damper. The equations of motion of a planar lumped-mass structure equipped with NFVDs can be written as follows

$$M\ddot{u} + C\dot{u} + Ku + F_{d}^{\text{NL}} = -M\tau\ddot{u}_g$$

(2)

where $M$, $C$, $K$ represent the mass, (inherent) damping and stiffness matrices of the structure, respectively, $\dot{u}$ denotes the system vector displacement and over-dots indicate derivatives with respect to time, $\tau$ is the influence vector (collecting ones in the case of planar structural systems), $\ddot{u}_g$ is the horizontal ground motion acceleration, while $F_{d}^{\text{NL}}$ is the vector of forces of NFVDs in the global coordinate system. This vector is obtained from the vector of the damper forces $f_d$ expressed in the damper local coordinates system (collecting the $f_{dj}$ contributions reported in (1) of all the dampers) through the following transformation relation

$$F_{d}^{\text{NL}} = R^T f_d$$

(3)

where the transformation matrix $R$ establishes a relation between the vector of relative displacements at the ends of each damper $w$ and the displacement vector $u$

$$w = Ru$$

(4)

The ground motion acceleration $\ddot{u}_g$ is modeled as a random process in the stochastic framework of the proposed optimal design procedure. In order to incorporate the earthquake frequency content in a simplified manner, the Kanai-Tajimi PSD function is used, with a second filter in series as proposed by Clough and Penzien [17] to remove the unrealistic effects in the low-frequency range

$$S_{\ddot{u}_g}(\omega) = \frac{\omega^4 + 4\zeta_g^2\omega^2_0\omega^2}{(\omega_0^2 - \omega^2)^2 + 4\zeta_g^2\omega_0^2}\frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + 4\zeta_f^2\omega_0^2\omega^2}S_w$$

(5)

where $\omega$ is the circular frequency, $\omega_0$, $\zeta_g$, $\omega_0$ and $\zeta_f$ represent the filter parameters that depend upon the surface soil characteristics (in this paper the values reported in [18] are
assumed), and \( S_w \) denotes the white-noise intensity level that can be related to the peak ground acceleration (PGA) \( u_{g0} \), for instance, through the expression adopted in [19].

Linear random vibration theory cannot be applied to Eq. (2) because of the presence of the nonlinear force vector \( F_d^{NL} \). An effective manner to handle the nonlinear terms is to use the stochastic linearization technique (SLT), after which the linearized equations read

\[
\mathbf{M}\ddot{\mathbf{u}} + \left( \mathbf{C} + \mathbf{C}_{d,\text{eq}} \right) \dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\dot{\mathbf{u}}_g
\]

where \( \mathbf{C}_{d,\text{eq}} \) is the vector of the linearized forces of the dampers. The linearization coefficients \( c_{d,\text{eq}} \) entering the equivalent viscous damping matrix \( \mathbf{C}_{d,\text{eq}} \) are usually determined through a Gaussian SLT, by minimizing the mean-square difference between the nonlinear and linearized force [14], [20]. An alternative non-Gaussian SLT was developed by the authors in [12] based on an equal-energy equivalence criterion proposed by Elishakoff and Zhang [21]. This SLT was found to be more effective and accurate for the kind of nonlinearity of NFVDs, and is therefore employed in the proposed optimal design procedure. According to this equal-energy non-Gaussian SLT, the linearization coefficients of the \( j \)th damper \( c_{dj,\text{eq}} \) are related to the damping coefficients of the \( j \)th NFVD \( c_{dj} \) through the following relationship [22]

\[
c_{dj,\text{eq}} = c_{dj} \left[ \frac{2^{1-\alpha_j/2} \sqrt{\Gamma(3+2\alpha_j)}}{\sqrt{3(1+\alpha_j)}} \right] \sigma_{u_j}^{\alpha_j-1}
\]

where \( \Gamma(\cdot) \) is the Gamma function and \( \sigma_{u_j} \) is the standard deviation of the relative velocity at the ends of the \( j \)th damper. Note that due to the presence of \( \sigma_{u_j} \) in (7), which is implicitly dependent upon the unknown linearization coefficients \( c_{d,\text{eq}} \), the search procedure is iterative and involves a repeated use of input-output relations in the frequency domain.

3 FROM THE ENERGY BALANCE EQUATIONS TO THE OBJECTIVE FUNCTION

3.1 Energy balance equations

The objective function (OF) that has to be minimized in the proposed optimal design procedure is called “filtered energy index” (FEI). The FEI represents the part of the earthquake input energy that is not dissipated by the NFVDs – as such, it may be viewed as the portion of energy that “filters” (penetrates) into the structure. This OF has been extensively used in recent research works for the optimal design of different vibration control systems, including hysteretic dampers [23], tuned mass dampers (TMD) [24]-[26], and TMDs combined with inerters as mass-amplification devices [27]-[29]. Before introducing this energy-based OF, it is convenient to recall the energy balance equations of the system [30].

Multiplication of the (linearized) equations of motion (6) by \( \dot{\mathbf{u}}^T \) and integration over the time produces the energy balance equations of the structure with NFVDs

\[
E_k(t) + E_{d,\text{kin}}(t) + E_{d,\text{visc}}(t) + E_e(t) = E_i(t)
\]

where the various energy contributions are the kinetic energy (\( E_k \)), the viscous energy dissipated by the structure (\( E_{d,\text{kin}} \)) and by the dampers (\( E_{d,\text{visc}} \)), the elastic strain energy (\( E_e \)), and
the input energy from the earthquake ($E_i$). From Eq. (8) it is possible to write the equation of power balance, which represents the energy balance equation for unit time

$$e_i(t) + e_{ds}(t) + e_{dd}(t) + e_s(t) = e_i(t)$$  \hspace{1cm} (9)$$

where the generic power term $e_i$ represents the rate of energy $E_i$ at the specific time instant $t$. At any time $t$ the rate of the input energy must be equal to the sum of the terms on the left-hand-side of Eq. (9). In line with the stochastic framework of the present approach, the rate of the input energy is evaluated by applying the expectation operator $E[ ]$ to the terms in Eq. (9).

Considering that in stationary conditions the expected values of $(d, eq)$ and $(e, eq)$ are zero, the power balance equation of a structure with NFVDs in stochastic terms is

$$E[e_i] = E[e_{ds}] + E[e_{dd}]$$  \hspace{1cm} (10)$$

where the operator .*$^*$ represents the MATLAB element-wise multiplication, $A(i,j)$ denotes the $i^{th}$ row-$j^{th}$ column element of the $A$ matrix, $\dot{u}_i$ is the $i^{th}$ component of the velocity vector $\dot{u}$ representing the velocity of the $i^{th}$ floor, $n$ is the total number of degrees of freedom of the system (e.g. the number of floors in a lumped-mass planar system) and finally $\Sigma_{uu}$ is the covariance matrix of the velocity response. The latter is calculated as

$$\Sigma_{uu} = E[\dot{u}\dot{u}^T] = \int_{-\infty}^{\infty} \omega^2 H_U(\omega)S_u(\omega)H_U^*(\omega) d\omega$$

$$= \left[ -\omega^2 M + i\omega(\mathbf{C} + \mathbf{C}_{deq}) + \mathbf{K} \right]^{-1} \mathbf{M} \tau$$  \hspace{1cm} (11)$$

where $H_U(\omega)$ represents the displacement transfer function vector, $i = \sqrt{-1}$ is the imaginary unit and $(\cdot)^T$ the complex conjugate transpose operator. According to Eq. (10), in stationary conditions the expected value of the seismic input energy $E[e_i]$ is balanced by two terms, namely the energy dissipated by the structure $E[e_{ds}]$, which is related to the inherent damping of the building (e.g. Rayleigh damping), and the energy dissipated by the NFVDs $E[e_{dd}]$. If all the input energy were dissipated by the NFVDs $E[e_i] = E[e_{dd}]$, then the remaining part that the structure should dissipate would be ideally null. This, in fact, occurs only for disproportionately large values of the damping coefficients of the NFVDs. To clarify this, let us consider a single degree of freedom (SDOF) system (having mass $m = 1000\text{kg}$, inherent damping $c = 400\text{Ns/m}$ and lateral stiffness $k = 100000\text{N/m}$) equipped with a NFVD. The damping coefficient $c_d$ is the free design variable to optimize, while the velocity exponent $\alpha$ is fixed to 0.3 (this value is typically related to the hydraulic circuit and to the orifice design). The energy terms defined in Eq. (10), namely $E[e_i]$ (thick line), $E[e_{ds}]$ (dashed line) and $E[e_{dd}]$ (thin line) are shown in Figure 1 in a semi-logarithmic plot for a broad range of damping coefficients $c_d$ spanning four orders of magnitude and considering two different
PSD functions for the earthquake excitation (associated with firm and soft soil characteristics). It is seen that with very low values of the damping coefficient $c_d$ all the input energy is absorbed by the structure as $E[e_i] \approx E[e_{d,i}]$ and $E[e_{d,i}] \approx 0$. As $c_d$ increases the energy balance changes, with $E[e_{d,i}]$ decreasing and the complementary contribution $E[e_{d,e}]$ increasing accordingly, up to a maximum dissipation point (yellow circle) corresponding to $c_{d,opt} = 1628.1 \text{Ns/m}$ for firm soil and $c_{d,opt} = 2478.1 \text{Ns/m}$ for soft soil conditions.

Therefore, this point depends upon the frequency content of the earthquake excitation because it is around 1.5 higher for soft soils than for firm soils for the given data of the SDOF. Although the $E[e_{d,i}]$ is monotonically decreasing with increasing $c_d$, for damping coefficients larger than the maximum dissipation point this reduction is negligible. Therefore, the maximum dissipation points is a convenient design choice that maximizes the energy dissipation of the damper, while values of $c_d > c_{d,opt}$ would be non-economical and non-convenient (producing limited reduction of $E[e_{d,i}]$ at the expense of a larger, ineffective damper). Nevertheless, the maximum dissipation point is often a non-feasible design choice because of constraint conditions of the optimization problem. These constraint conditions represents upper-bound values of the damping coefficients that should not be exceeded (or equality conditions enforced in the problem). As an example, a reasonable choice is to design the dampers to achieve a (desired) target added damping ratio $\zeta_d$ (in line with performance-based earthquake engineering concepts), which can be computed through the simplified FEMA 356 formula as follows

$$\zeta_d = \frac{T_1}{4\pi} \sum_{j=1}^{n_d} c_{d,j,eq}(\phi_j - \phi_{j-1})^2 = \frac{T_1}{4\pi} \frac{\phi^TR'D_{eq}R\phi}{\phi^T M\phi} \tag{12}$$

where $n_d$ is the number of NFVDs in the structure, $\phi_j$ is the first eigenvector of the undamped structure ($K\phi_j = M\phi_j \omega_j^2$), $T_1$ is the first natural period and $D_{eq} = \text{diag}(c_{d,eq})$ is a diagonal matrix collecting all the equivalent linearization coefficients $c_{d,eq}$ ($j = 1, \ldots, n_d$). As an example, in Figure 1 we report the values of $c_d$ corresponding to damping ratios equal to 5%, 10%, 20% and 30%, from which one can conclude that the point $c_{d,opt}$ corresponds to
\( \zeta_d \approx 23\% \) for firm soil conditions and to \( \zeta_d > 30\% \) for soft soil conditions. Therefore, the maximum dissipation point would not be an admissible solution in either soil condition if the design problem is such that the target added damping ratio of the NFVD is, for instance, \( \zeta_{\text{d, target}} = 20\% \). In such a constrained optimization problem the optimal damping coefficient would be the one that provides the largest energy dissipation subject to the constrain condition.

### 3.2 Energy-based objective function and constrained optimization problem

Building on the previous remarks, the OF of the proposed energy-based optimal design procedure is the FEI that in stochastic terms can be defined as

\[
\text{FEI} = \frac{E[e_i] - E[e_{dd}]}{E[e_{dd}]} = 1 - \frac{E[e_{dd}]}{E[e_i]} \quad (\text{filtered energy index})
\]  

(13)

and the optimal design procedure can be expressed through a constrained optimization problem formulated in mathematical terms as follows

\[
\min_{\zeta_d} \text{FEI} = 1 - \frac{\tau^T (C_{\text{d, eq}} \ast \Sigma_{\text{uu}}) \tau}{\tau^T ((C + C_{\text{d, eq}}) \ast \Sigma_{\text{uu}}) \tau}
\]

subject to \( \zeta_d = \frac{T_1}{4\pi} \frac{\phi_i^T D_{\text{eq}} R \phi_i}{\phi_i^T M \phi_i} = \bar{\zeta}_d \)

(14)

![Figure 2 Expected value of the energy dissipated by NFVDs for a two-story structure](image)
The problem of constrained optimization of the dampers is clarified by considering a multi-degree of freedom (MDOF) system. Let us consider a two-story structure (with story mass \( m = 1000 \text{kg} \) and lateral story stiffness \( k = 200000 \text{N/m} \) ) with two interstory NFVDs both with velocity exponent \( \alpha = 0.3 \) such that there are two design variables, namely \( c_{d_1} \) and \( c_{d_2} \). A stiffness proportional inherent damping matrix \( C \) is assumed that is associated with a damping ratio \( \zeta = 0.02 \) for the two modes of vibration. The relevant surface and contour plots of \( E[e_{dd}] \) (for soft soil conditions) are depicted in Figure 2. Similarly to the SDOF case discussed before, we superimpose four (red) dotted lines in the contour plot corresponding to \( \zeta_d \) (computed through Eq. (12)) equal to 5,10,20,30\%. The unconstrained optimization point (or maximum dissipation point) has a damping ratio \( \zeta_d \approx 46\% \). If a constraint condition of the type \( \zeta_d = 20\% \) is enforced in the problem (14), then the solution of the constrained optimization problem is the point represented by the (cyan) square marker in Figure 2, which corresponds to the highest value of \( E[e_{dd}] \) along the curve \( \zeta_d = 20\% \) . The same results can be obtained by applying the concept of gradient derivatives to a multi-variable \( E[e_{dd}] \) surface in case of more than just two design variables (more than two floors).

It is worth noting that in the above constrained optimization problem (14) the damping coefficients \( c_{d_j} \) of the NFVDs are implicitly recalled in the design process, as they are present in the \( D_{eq} \) and \( C_{d,eq} \) matrices through the SLT relationships (7). Therefore, unlike other multiple-step approaches outlined before, there is no need of performing preliminary time-history analyses to determine the maximum relative velocity of the devices. In other words, the nonlinearity of the NFVDs is incorporated a priori in the optimal design procedure within a single-step approach.

4 ASSESSMENT OF THE EFFECTIVENESS OF THE PROPOSED PROCEDURE

4.1 Stochastic dynamic analysis under Kanai-Tajimi filtered random excitation

The effectiveness of the proposed optimal design procedure is first assessed considering a Kanai-Tajimi filtered random excitation defined by the PSD \( S_u \) in (5). A set of performance indices (called “stochastic performance indices” SPIs) are introduced here to quantitatively evaluate the seismic performance:

- standard deviation of top-story displacement \( \text{SPI}_1 = \sigma_{u_n} \);
- standard deviation of top-story absolute acceleration \( \text{SPI}_2 = \sigma_{\dot{u}_n} \);
- standard deviation of the base shear \( \text{SPI}_3 = \sigma_{V_b} = k_i \sigma_{\Delta u_j} \);
- average standard deviation of interstory drift ratios (IDRs) \( \text{SPI}_4 = \frac{1}{n} \sum_{j=1}^{n} \sigma_{\Delta u_j} / h_j \) where \( h_j \) denotes the interstory height and \( \Delta u_j = u_j - u_{j-1} \) the interstory displacement;
- sum of the amplitudes of the transfer functions of the IDRs computed at the fundamental frequency \( \omega_1 \), \( \text{SPI}_5 = \sum_{j=1}^{n} \left| H_{\Delta u_j}(\omega_1) \right| \);
- amplitude of base shear transfer function computed at the fundamental frequency \( \omega_1 \), \( \text{SPI}_6 = \left| H_{V_b}(\omega_1) \right| \).
The proposed energy-based distribution (EBD) is compared with alternative distribution techniques based on a fixed (target) added damping ratio $\tilde{\zeta}_d = \zeta_d$. In particular, the following four distribution methods proposed by Hwang et al. [31] are considered in this comparison: i) uniform distribution (UD); ii) story shear proportional distribution (SSPD); iii) story shear strain energy distribution (SSSE); iv) story shear strain energy to efficient stories distribution (SSSEES). Analytical details of the mentioned distribution methods can be found in [31]. Finally, the stiffness proportional damping distribution (SPD) is also considered as a fifth distribution method in this comparative study. These distribution methods, proposed in the field of LFVDs by Hwang et al. [31], are here properly modified to incorporate NFVDs according to the conversion formula of the SLT in (7). Indeed, once the linearization coefficients $c_{i,j,eq}$ are determined for each distribution method, they are converted into damping coefficients of the NFVDs through the knowledge of the standard deviation of the relative velocity response $\sigma_{v,j}$ via expression (7).

![Figure 3 Values of SPIs for different methods of distribution of NVFDs for firm soil conditions](image)

Two reference six-story shear type structures are considered for this comparative study, called Type A (regular in elevation) and Type B (irregular in elevation) structure (whose pertinent numerical data are provided in the Takewaki 1997 paper [32]), with an inherent damping ratio $\zeta = 0.05$ for all the 6 modes of vibration. A target added damping ratio $\zeta_d = \tilde{\zeta}_d = 20\%$ is considered in the distribution methods and a velocity exponent $\alpha = 0.3$ is assumed for the NFVDs. The seismic performance is evaluated through the above set of SPIs in dimensionless (normalized) form:

$$SPI_i = \frac{SPI_i}{SPI_i^{(0)}} \quad (i = 1, \ldots, 6)$$

(15)
where $\text{SPI}_{i}^{(0)}$ represents the value of the SPI$_i$ indicator in the structure without dampers (uncontrolled configuration). Consequently, all the values SPI$_i$ are expected to be less than 1, and the lowest values are associated with the most effective distribution methods.

Some comparisons among the resulting SPIs for the six distribution methods (the five described above and the proposed EBD) are illustrated in Figure 3 (for Kanai-Tajimi PSD and filter parameters related to firm soil conditions). It can be seen that the various distribution methods produce similar results in terms of SPIs, which is consistent with the identical added damping ratio constraint $\bar{\xi}_d = 20\%$. Therefore, the histograms in Figure 3 do not reveal any clear difference between the different design procedures, although the energy-based design procedure in most cases leads to the lowest values of the SPIs. To demonstrate this from a quantitative point of view, we have calculated the product of all the SPIs for the two structures. Relevant results are listed in Table 1.

Table 1. Values in bold denote the lowest product for that specific soil condition and for that specific structure. It is observed that in four out of six cases the proposed energy-based design procedure is associated with the lowest product of the response indicators SPIs, which means that, for the given constrain condition, produces the most convenient distribution choice of the NVFDs.

<table>
<thead>
<tr>
<th>Soil condition</th>
<th>Structure</th>
<th>UD</th>
<th>SSPD</th>
<th>SSSE</th>
<th>SSSEES</th>
<th>SPD</th>
<th>EBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>WN</td>
<td>Type A</td>
<td>2.69e-4</td>
<td>2.75e-4</td>
<td>2.92e-4</td>
<td>3.93e-4</td>
<td>2.69e-4</td>
<td><strong>2.54e-4</strong></td>
</tr>
<tr>
<td></td>
<td>Type B</td>
<td>4.05e-4</td>
<td>2.52e-4</td>
<td>2.68e-4</td>
<td>5.64e-4</td>
<td>2.46e-4</td>
<td><strong>2.19e-4</strong></td>
</tr>
<tr>
<td>Firm</td>
<td>Type A</td>
<td><strong>2.46e-4</strong></td>
<td>2.59e-4</td>
<td>2.81e-4</td>
<td>3.93e-4</td>
<td>2.46e-4</td>
<td>2.53e-4</td>
</tr>
<tr>
<td></td>
<td>Type B</td>
<td>3.52e-4</td>
<td>2.21e-4</td>
<td>2.34e-4</td>
<td>5.97e-4</td>
<td><strong>2.17e-4</strong></td>
<td>2.20e-4</td>
</tr>
<tr>
<td>Soft</td>
<td>Type A</td>
<td>1.75e-4</td>
<td>1.73e-4</td>
<td>1.74e-4</td>
<td>1.85e-4</td>
<td>1.75e-4</td>
<td><strong>1.71e-4</strong></td>
</tr>
<tr>
<td></td>
<td>Type B</td>
<td>2.82e-4</td>
<td>1.79e-4</td>
<td>1.89e-4</td>
<td>2.37e-4</td>
<td>1.74e-4</td>
<td><strong>1.64e-4</strong></td>
</tr>
</tbody>
</table>

We investigate whether the six analysed distributions imply a significant difference in terms of cost of the damper system. A rough estimator of the initial cost of the dampers is the sum of the damping coefficients $c_{d,\text{tot}} = \sum c_{d_j}$ [31], [33], although more appropriate (though more sophisticated) cost functions have been developed in the recent literature [34]. Instead, the performance associated with each distribution method can be evaluated, as said before, though the product of the SPIs, i.e. $\prod_{i=1}^{7} \text{SPI}_i$. Each distribution method is therefore characterized by two coordinates in such cost-versus-performance plane, as shown in Figure 4. A reasonably good trade-off between performance and cost is achieved by those distribution methods having low product of SPIs (good performance) and low cost. It is noted that the proposed EBD leads to the best performance but a relatively high cost for the Type A structure (although less than SSSEES and SPD), whereas for the Type B structure all the distributions methods (except for UD and SSSEES) have a comparable cost but the EBD is associated with the best performance. To sum up, it seems that the better seismic performance of the EBD does not imply a higher cost of the damper system because it optimizes the dissipation capacity of the dampers (to achieve a more effective dissipation behaviour).
4.2 Nonlinear response history analysis under natural ground motions

The performance of the EBD is finally evaluated through nonlinear response history analysis [35]. The FEMA P695 far-field record set [36] comprising 44 ground motions having PGA between 0.21g and 0.82g is considered. The records are first scaled so as to have a common PGA = 0.3 g (which is consistent with the assumed PSD function in the stochastic-based approach): the original and scaled pseudo-acceleration response spectra of the 44 ground motions are depicted in Figure 5.

These earthquake components are related to soil class C or D (stiff soils), thus they can be associated with firm soil conditions in the corresponding Kanai-Tajimi PSD function for the design of the NFVFDs in the stochastic-based optimization problem (14). We assume again velocity exponent $a = 0.3$ and a target added damping ratio $\bar{\zeta} = 20\%$. A Runge-Kutta algorithm is employed to integrate the nonlinear equations of motion (2), then the
representative root-mean-square (RMS) and maximum (MAX) values of some response indicators are calculated through average of the results for the 44 ground motions.

We limit ourselves to analyse the seismic performance for the Type B structure of the proposed EBD against other design philosophies for the same target damping ratio. In the present comparison, in addition to the proposed EBD, we analyse the following strategies: a displacement-based approach minimizing the SPI_{1}; an acceleration-based approach minimizing the SPI_{2}; a base-shear-based approach minimizing the SPI_{3}; the UD and the SSSEES distribution proposed by Hwang et al. [31]. Relevant results, in terms of profiles of average MAX and RMS displacements, accelerations and IDR, are comparatively displayed in Figure 6 for the six design strategies. It is clearly seen that the energy-based design procedure proposed in this paper leads to very low values of the response indicators, representing in most cases the lowest values among the six considered design procedures (see for instance the RMS of the absolute acceleration response). This analysis definitely confirms the effectiveness of the proposed EBD procedure under real (non-stationary) earthquake ground motions. Moreover, the comparison shows the superiority of the EBD procedure over alternative design strategies that minimize specific response indicators (e.g. displacements, accelerations, etc.) that are only indirectly related to the energy dissipation capabilities of the NFVDs (maximized in the proposed procedure).

**Figure 6** Profiles of average RMS and MAX displacements, accelerations and IDR for Type B structure and different distributions of NFVDs subject to the 44 ground motions of the FEMA P695 far-field record set.
5 CONCLUSIONS

A stochastic-based numerical procedure aimed at finding the optimal distribution of NFVDs for optimal earthquake protection of structures has been presented. The nonlinear behavior of the dampers has been fully integrated in a single-step optimization process through the aid of the stochastic linearization technique. A novel non-Gaussian stochastic linearization, previously developed by the authors, has been adopted. It was demonstrated that, for the kind of (power law) nonlinearity of the viscous dampers, this variant of stochastic linearization offers improved accuracy over the (commonly used) Gaussian linearization formula without implying any additional computational effort. The design procedure has been formulated as a constrained optimization problem, considering an energy-based objective function called filtered energy index, which physically represents the portion of the total input energy that is not dissipated by the dampers and, thus, filters into the structure. A constraint condition in terms of achievement of a (desired) target added damping ratio of the NFVDs has been incorporated in the design procedure, which is in line with performance-based earthquake engineering methodologies.

The effectiveness of the proposed energy-based procedure has been assessed through stochastic dynamic analysis (under a Kanai-Tajimi filtered random excitation) as well as under a suite of 44 natural ground motions extracted from the FEMA P659 far-field record set. The procedure has also been compared to other distribution methods or design procedures in order to highlight advantages over alternative design philosophies in terms of seismic performance while accounting for the overall cost of the damper system, here roughly related to the sum of the damping coefficients.

The analysis has been limited to linear elastic structures (where the nonlinearity is concentrated in the NFVDs) and to stationary seismic input. In future research work, we plan to extend the proposed procedure to deal with non-stationary excitations and nonlinear building structures.

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REFERENCES


RESPONSE VARIABILITY OF STRUCTURES WITH HYBRID BASE ISOLATION SYSTEMS

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Abstract. This paper investigates the effect of random parameter variation on the dynamic response of structures equipped with hybrid base isolation systems (HBIS). The considered HBIS combines high damping rubber bearings with low friction sliders modeled by a five-parameter trilinear hysteretic and a single-parameter Coulomb friction model, respectively. A shear beam type, four-degree-of-freedom model is used to simulate the superstructure. The calibration of the six-parameter mechanical model describing the HBIS is based on five initial-displacement, free vibration tests performed on two R/C residential buildings in Solarino, Sicily. The calibrated parameters values from all the experiments are used as mean values, while the standard deviation for each parameter is deduced employing best-fit optimization for each experiment separately. Monte Carlo simulation is used to compute the response variability of both the HBIS and the superstructure. It is shown that the response is highly non-stationary as its first two moments are significantly evolving with time. Moreover, a magnification of uncertainty is observed since the coefficient of variation of the response is in most cases larger than the coefficient of variation in the parameter values. The aforementioned effects can be attributed to the high level of nonlinearity in the base isolation system, as observed in the amplitude of vibration brought about by large initial displacements.
1 INTRODUCTION

Base isolation has been extensively used over the last decades for the protection of structures against earthquakes. The concept behind base isolation is the introduction of a flexible layer between the superstructure and its foundation \[1\]. The goal of base isolation is simply to reduce the transmission of energy from the ground to the superstructure \[2\]. To this end, the mechanics behind an isolation system are: (i) a flexible support in order to elongate the natural period of the structure, (ii) energy dissipation in order to control the relative displacements and (iii) sufficient rigidity under service loads (wind, minor earthquakes, ambient vibrations) to avoid unnecessary motion \[3\]. The first mode of an isolated structure involves only deformations in the base isolation level, while the higher modes do not contribute to the response due to orthogonality conditions \[4\].

The first systematic efforts for Italian buildings to be retrofitted with base isolation started in 2004 \[5\]. Among those buildings were two four-story R/C residential buildings in Via Baden Powell 23-25, Solarino, Eastern Sicily \[6\]. The retrofit included a hybrid base isolation system, which combined 12 high damping rubber bearings with 13 low friction sliding bearings \[6\]. In July 2004, static and dynamic tests were performed in one of the two Solarino buildings, see Fig. 1. The static tests were used for the identification of the static friction force, while the dynamic ones were in the form of free vibration tests following application and instantaneous release of a displacement close to the design value \[7\]. More specifically, the identified initial displacements vary from 86mm to 113mm. The basic set-up for the free vibration tests is shown in Fig. 2. A hydraulic jack is supported at one end against a rigid wall, while at the other end supports a device which pushes against the base of the isolated building. The device is designed in such a way so as to transform into a mechanism at a specific force and release the building. The idea behind the design of this device is based on the articulated quadrilateral mechanism.

In the years following these experiments, research efforts were made towards dynamic identification of the Solarino hybrid base isolation system by using several mechanical models and various identification techniques. In \[8\], a Kelvin model was used to model the high damping rubber bearing component and a constant Coulomb friction model was used to model the low friction sliding bearing, while the least squares method in the frequency domain was chosen as the identification procedure. In \[9\], the high damping rubber bearing component was modeled by a bilinear hysteretic model, the low friction sliding bearing component was modeled by Coulomb friction, while the least squares method was once more used as the dynamic identification technique. In \[7\], a new trilinear hysteretic model was introduced to simulate the behavior of the high damping rubber bearing component. A comparative study appearing in \[10\] involved a fractional derivative Zener model to describe the behavior of the high damping rubber bearing component and showed that the trilinear hysteretic model outperforms both bilinear hysteretic model and Zener models.

Recent studies have clearly highlighted that although two different trilinear hysteretic models exist stemming from a common mechanical model, only the variant used herein is able to describe satisfactorily the high damping rubber bearing response, see \[11\]. As was pointed in Ref. \[11\], the advantages of the new trilinear hysteretic model compared to the old one can be summarized as follows: (a) it accounts for large energy dissipation at larger strain amplitudes; (b) it better describes the different behaviour observed during the loading and unloading paths in high damping rubber bearings and (c) the trilinear
hysteretic mechanical model, through a proper choice of parameters, can be reduced to simpler models comprising a trilinear elastic spring and an elastoplastic element, which are the basic two mechanical components of existing trilinear hysteretic models. Further studies based on laboratory-generated experimental results on a spare high damping rubber bearing from the Solarino project, showed that alternative mechanical models also exist for the description of the behavior of the device [12, 12]. In the present study, the five-parameter trilinear hysteretic model will be used for the high damping rubber bearing response, while the single-parameter constant Coulomb friction model will be used for the low friction sliding bearing response, for a total of six model parameters.

Uncertainties inherently exist in the loading as well as in the material and geometric parameters of engineering systems. Within the framework of safe engineering design, papers in the literature primarily deal with the effect of stochastic earthquake excitation on the structural response. For instance, Ref. [13] studies the stochastic response of secondary systems attached to a base isolated structure undergoing random ground motions described by a filtered white noise model. In Ref. [14], randomness in earthquake loads is considered, but a parametric investigation with regard to deterministic structural and isolator parameters is also conducted. The stochastic response of base-isolated buildings, considering uncertainty in the characteristics of the earthquake loading is investigated in [13]. The effect of different isolator models on the seismic response of base-isolated liquid storage tanks using recorded earthquake ground accelerations is investigated in [16]. In [17], only the properties of the superstructure are treated as random variables in an optimization procedure. The effect of stochastic ground motion excitation and/or stochastic system parameters on the reliability-based design and control of base-isolated systems is explored in [18, 19, 20, 21]. In fact, very few publications consider uncertainty in the base isolator parameters. For instance, the stochastic response and seismic fragility of base isolated liquid storage tanks is computed in [22, 23, 24] using a polynomial chaos expansion and a response surface model, respectively, to represent the uncertainty in the characteristic parameters of a laminated rubber bearing isolator. Finally, the authors of Ref. [25], [26] performed a robust optimum design of base isolation systems taking into account the uncertainty in the isolator parameters.

In the above papers, various assumptions were made regarding the statistical characteristics of the base isolator parameters. In the present paper, the parameters of the adopted hybrid base isolation system are calibrated by using experimental evidence [7]. Specifically, the aforementioned five free vibration tests performed in Solarino will be used to define the mean value and standard deviation of the six-parameter mechanical model. The effect of parameter variation on the response of the supported superstructure comprising the Solarino building will be investigated within the framework of Monte Carlo simulations, leading to useful conclusions about the probabilistic characteristics of the response. It is noted that an alternative formulation based on stochastic energy measures was recently introduced by the authors in [27].

2 MECHANICAL MODELLING

Two possible trilinear hysteretic model formulations based on different mechanical representations exist, but as it was shown in [11], only one is able to describe the high damping rubber bearing response satisfactorily. This trilinear hysteretic model comprises three elements, a linear elastic spring of stiffness $K_0$ (element 1) in series with a parallel system, namely a plastic slider of characteristic force $f_s$ (element 2) connected in parallel
Figure 1: One of the Solarino buildings in Via Baden Powell 25, Solarino, Sicily.

Figure 2: Section view of the basic set up of the free vibration tests.
Figure 3: Trilinear hysteretic model: (a) mechanical model (b) $f_e - u_e$ graph of element 1 (c) $f_e - u_h$ graph of element 2 (d) $f_e - u_h$ graph of element 3 and (e) overall $f_T - u$ response graph.

with a trilinear elastic spring with stiffnesses $K_1, K_2$ and characteristic displacement $u_c$ (element 3), see Figs. 3(a),(b),(c),(d). The compatibility, equilibrium and constitutive equations of the trilinear hysteretic model are presented in Table 1. As shown in Fig. 3(e) the trilinear hysteretic model has three plastic phases (1-3) and one elastic phase. Plastic phase 1 has stiffness $k_2$ (shown in yellow), plastic phase 2 has stiffness $k_1$ (shown in green), plastic phase 3 has stiffness $k_2$ (shown with blue color) and the elastic phase has stiffness $k_0$ (shown with red color), Fig. 3(e). In the same figure the two characteristic displacements are also shown, namely the first yield displacement $u_y$ and the second yield displacement $u_3$. The force at zero displacement after yielding ($F_2$) and the force at second yield displacement $u_3$ in the loading phase with positive displacement, ($F_3$) are defined as follows, see Fig. 3(e):

$$F_2 = (k_0 - k_1)u_y, \quad F_3 = k_0u_y + k_1(u_3 - u_y)$$

The resulting trilinear hysteretic model is a five-parameter system and the relationships between the mechanical parameters ($K_0, K_1, K_2, f_s, u_c$) shown in Fig. 3(a) and the mathematical ones ($k_0, k_1, k_2, u_y, u_3$) shown in Fig. 3(e) are listed in Table 2.

The constant Coulomb friction model is used for the description of the behavior of the low friction sliding bearing component, Fig. 4. This model is defined by the characteristic force $f_f$ and its constitutive equation after initiation of motion ($\dot{u} \neq 0$) as follows:

$$f_F = f_f \text{sgn}(\dot{u})$$

When motion stops ($\dot{u} = 0$) the friction force $f_F$ can take any value between $-f_f < f_F < f_f$. 
Table 1: Compatibility, equilibrium and constitutive equations for the trilinear hysteretic model.

<table>
<thead>
<tr>
<th>Compatibility</th>
<th>$u = u_e + u_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium</td>
<td>$f_T = f_{e1} = f_{e2} + f_{e3}$</td>
</tr>
<tr>
<td></td>
<td>$f_{e1} = K_0u_e$</td>
</tr>
<tr>
<td></td>
<td>$f_{e2}(\dot{u}_h \neq 0) = f_s \text{sgn}(\dot{u}_h)$</td>
</tr>
<tr>
<td></td>
<td>$f_{e2}(\dot{u}<em>h = 0) = f</em>{e1} - f_{e3}$</td>
</tr>
<tr>
<td></td>
<td>$f_{e3}(</td>
</tr>
<tr>
<td></td>
<td>$f_{e3}(</td>
</tr>
</tbody>
</table>

Table 2: Relationships between mechanical and mathematical parameters appearing in the trilinear hysteretic model formulation, see Figs. 3(a),(e).

- $K_0 = k_0$
- $K_1 = k_1 \frac{k_0}{k_0 - k_1}$
- $K_2 = k_2 \frac{k_0}{k_0 - k_2}$
- $f_s = k_0u_y$
- $u_c = (u_3 - u_y)\frac{k_0 - k_1}{k_0}$

The base isolator studied herein is produced by combining the rubber bearing and sliding friction components, yielding a six parameter mechanical model.

3 EQUATION OF MOTION UNDER FREE VIBRATION

In terms of numerical modeling, a single-degree-of-freedom system is used, see Fig. 5. The equation of motion of the single-degree-of-freedom-system system under free vibration excitation is given by:

$$m\ddot{u} + f_T + f_F = 0$$

where $f_T$ denotes the force in the trilinear model of the high damping rubber bearing component and $f_F$ denotes the force in the friction model of the low friction sliding bearing component.

3.1 Constitutive equations for the trilinear hysteretic model

The restoring force in the trilinear hysteretic model, $f_T(u, \dot{u})$, takes different forms according to whether the system experiences an elastic phase or a plastic phase of motion. The force-displacement relationship for the elastic phases is given by the following expression:

$$f_T(u, \dot{u}) = F_T^e(\dot{u}) + k_0(u - u_T^e)$$

Figure 4: Constant Coulomb friction model: (a) mechanical model (b) overall $f_F - u$ response graph.
where \((F_J^e, u_J^e)\) is the starting point of the elastic phase. The three plastic phases are governed by the following equations:

\[
f_T(u, \dot{u}) = F_J sgn(\dot{u}) + h_J(u - u_J sgn(\dot{u})), \quad (J = 1, 2, 3)
\]  

(5)

where \((F_J, u_J)\) are characteristic points of the upper plastic phases. As it may be seen from Fig. 3(e), \(u_2 = 0\), \(h_1 = h_3 = k_2\) and \(h_2 = k_1\).

3.2 Constitutive equation for the constant Coulomb friction model

Independently of the phase of motion, the resisting force in the slider, \(f_F(u, \dot{u})\), is always given by Eq. 2. At times when the system stops, the friction force must satisfy the following inequality:

\[
|\tilde{f}_F(u_R, 0)| \leq f_f \tag{6}
\]

where \(u_R\) is the residual displacement.

3.3 Rest conditions

The system will come to rest if the following conditions are satisfied:

\[
sgn(\ddot{u}_\text{\_}) = -sgn(\dot{u}_\text{\_}) \tag{7}
\]

\[
|\ddot{u}_\text{\_}| \leq 2 \frac{|f_F|}{m} \tag{8}
\]

where \(\ddot{u}_\text{\_}\) and \(\dot{u}_\text{\_}\) denote the acceleration and the velocity just before the stoppage. At the point when the system stops \((\dot{u} = 0)\), it can reach a position of static equilibrium different from the original unstrained one, as long as the following equation is satisfied:

\[
f_T(u_R, 0) + \tilde{f}_F(u_R, 0) = 0 \tag{9}
\]
Figure 6: Recorded acceleration time histories for (a) test 3, (b) test 5, (c) test 6, (d) test 7 and (e) test 8.
Table 3: Floor masses for the Solarino building.

<table>
<thead>
<tr>
<th>floor</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (kN s²/m)</td>
<td>255.11</td>
<td>243.30</td>
<td>240.58</td>
<td>223.10</td>
</tr>
</tbody>
</table>

Table 4: Stiffness matrix of the superstructure $K_{ij}$ (kN/m).

<table>
<thead>
<tr>
<th>i/j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17309668</td>
<td>-8954021</td>
<td>567817</td>
<td>164039</td>
</tr>
<tr>
<td>2</td>
<td>-8954021</td>
<td>16582807</td>
<td>-8964305</td>
<td>759394</td>
</tr>
<tr>
<td>3</td>
<td>567817</td>
<td>-8964305</td>
<td>16161822</td>
<td>-7791256</td>
</tr>
</tbody>
</table>

4 BASE ISOLATED STRUCTURE RESPONSE VARIABILITY

In this section, the response variability in the superstructure of the Solarino building is investigated. The acceleration time histories presented in Fig. 6 are imposed at the base of the superstructure, just above the base isolation system. Due to the fact that the superstructure was a low-rise R/C building, which behaves like a rigid block, a simple shear beam model was deemed adequate to use in the present study. A four-degree-of-freedom system covers the response of the three floors and of the base of this structure, see [28]. The masses of each floor are presented in Table 3, while the stiffness matrix was obtained from a SAP2000 [29] detailed finite element model and it is given in Table 4. A modal damping of 2% was used for all four modes of vibration. The results for the acceleration versus time graphs are plotted for a separate (independent) variation of each of the six model parameters in Fig. 7. Consider that the parameters $u_y$ and $u_3$ are defined in Fig. 3, while the rest of the parameters are defined as follows:

$$\omega_i = \sqrt{\frac{k_i}{m}}; \quad f_i = \frac{\omega_i}{2\pi}; \quad u_f = \frac{f_f}{k_0}; \quad (i = 0, 1, 2)$$

Furthermore, Figs. 8, 9 respectively plot the complete temporal evolution of the mean and the std for the top floor absolute acceleration as each material parameter of the hybrid base isolation system is varied. It should be noted that the acceleration signals have stop-jump patterns due to change of sign of the friction force in the sliding friction component of the base isolation system, which means that the force jumps from a positive value to a negative one and vice versa. Figs. 7, 8 show that $u_y$, $u_3$ and $f_1$ are the most critical parameters in terms of response variability of the superstructure, although the influence of $f_2$ seems to be also important. In terms of hybrid base isolation system effectiveness in filtering the ground motions, which here is simply a large initial displacement, we observe that the maximum absolute acceleration values at the top floor of the Solarino building are lower when compared to the amplitude recorded at the base, even in the presence of uncertainty.

5 CONCLUSIONS

In this paper, Monte Carlo simulations have been employed in order to investigate the effect of the uncertainty in the values of a six-parameter mechanical model used to simulate the response of hybrid base isolation systems supporting low-rise R/C buildings. The hybrid base isolation system examined herein was used in practice in the Solarino retrofit project, and numerical values for its mechanical parameters were calibrated from
Figure 7: Acceleration time history at the top of the superstructure for test 5 with randomly varying material parameters (the remaining kept constant at their mean value): (a) $u_f$, (b) $u_y$, (c) $u_3$, (d) $f_0$, (e) $f_1$, (f) $f_2$. 
Figure 8: Mean acceleration value variation with time for the acceleration time history at the top of the superstructure of test 5 with randomly varying material parameters (the remaining kept constant at their mean value): (a) $u_f$, (b) $u_y$, (c) $u_3$, (d) $f_0$, (e) $f_1$, (f) $f_2$. 
Figure 9: Standard deviation of the acceleration with time for the acceleration time history at the top of the superstructure of test 5 with randomly varying material parameters (the remaining kept constant at their mean value): (a) $u_f$, (b) $u_y$, (c) $u_3$, (d) $f_0$, (e) $f_1$, (f) $f_2$. 
experimental data. The results of the Monte Carlo simulations have shown that variation in the material parameters of the isolation system and of the superstructure produce a non-stationary effect in the response, which can be traced by the time evolution of the standard deviation computed from the response at different time intervals. In addition, there was a magnification effect, due to the fact that the coefficient of variation of the response was larger than the coefficient of variation of the parameter itself, and this holds true for most of the parameters. The high level of nonlinearity in the base isolation system amplitude of vibration brought about by large initial displacement helps explain the previously described effects. The above observations can serve as guidelines and indicators in the design of base isolation systems. Concerning the effectiveness of the isolation system in filtering the ground motion, it was observed that the maximum acceleration values at the top floor of the superstructure were lower when compared to the amplitude recorded at the base, even in the presence of uncertainty in the hybrid base isolation system.

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REFERENCES


SEISMIC RETROFIT OF THE STUDENT HALL OF RESIDENCE OF MESSINA THROUGH BUCKLING RESTRAINED BRACES

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Abstract

This contribution describes a seismic retrofitting intervention designed and currently ongoing in the earthquake-prone area of Messina, Italy, where local strengthening techniques are combined with supplemental energy dissipation devices. This intervention concerns a 5-story building that represents the main student hall of residence of Messina. The structure is made of confined brick masonry in the first four stories, built in the 1930s, with an added fifth story together with a light appendage in reinforced concrete frame, built around 40 years later than the original masonry building. The building has a C shape in plan and exhibits moderate torsional effects, which are undesirable. Buckling restrained braces (BRBs) placed in selected frames of the structure have been adopted as dissipative elements to improve the seismic performance of the building. BRBs reduce torsional effects and increase the dissipation capacity of the building. Furthermore, to reduce the vulnerability of the last-floor reinforced concrete frame, pre-tensioned stainless steel ribbons are used in the beam-column joints of the last elevation, thus enhancing the confinement effect and inducing a beneficial pre-compression state that increases the ductility. The effectiveness of the retrofitting interventions is assessed through pushover analysis on the original and retrofitted structure comparatively. Acceptance tests on the employed hysteretic dampers carried out at the laboratory CERISI of Messina are also described.

Keywords: Seismic retrofit, Buckling Restrained Braces, Hysteretic Dampers, Energy Dissipation, Pre-tensioned stainless steel ribbons, Confined Masonry-RC Building.
1 INTRODUCTION

Several buildings placed in earthquake-prone areas were designed according to past seismic regulations and, consequently, may not comply with current seismic codes in force today. The constantly evolving map of seismic hazard, based on past seismic events, has in most cases increased the peak ground acceleration (PGA) of many installation sites, which makes seismic retrofitting interventions on existing buildings urgent. Furthermore, principles of modern building codes, such as capacity design and ultimate limit states, have been introduced in seismic regulations only in relatively recent times; therefore, it is very likely that existing buildings were originally designed ignoring such principles and concepts. Retrofitting operations are particularly important for those building with high importance class, i.e. strategic because of the underlying social and/or economic implications of an eventual collapse (hospitals, university buildings, schools, etc.). The case-study building discussed in this contribution belongs to this class of strategic buildings and represents the main student hall of residence of the city of Messina, Italy.

Seismic retrofitting of existing buildings can be carried out following two main families of strategies: 1) local strengthening of the structure by increasing the load-carrying capacity of specifically selected structural members, for instance with fiber reinforced polymers [1] or similar ones; 2) modification of the seismic performance of the structure through seismic protection devices, including seismic base isolators [2], [3] or other supplemental energy dissipation devices, like viscous dampers [4], [5], hysteretic dampers, dissipative braces in general [6], [7], or exploiting the advantageous properties of tuned mass dampers [8], [9].

While local strengthening operations are preferable for limited parts of the structure, they may become too expensive for large buildings requiring interventions on many structural members (beams, columns). As a result, in most practical cases these local strengthening operations, applied to critical zones or to specific structural members (deemed to be particularly weak and vulnerable), are conveniently combined with the above-mentioned seismic protection devices for a more effective seismic retrofit of the structure. Such a combined seismic retrofit strategy has also been adopted in the case-study building described in this contribution.

The earthquake events of L’Aquila (Italy) in 2009 led to the dramatic collapse of many public buildings, among which the university hall of residence. This catastrophic and emblematic collapse has induced a number of regional authorities in other parts of Italy to carry out surveys and structural analyses of similar student accommodation buildings, unveiling considerable structural deficiencies in many instances. Indeed, the student hall of residence of the University of Messina is one of such buildings that requires urgent retrofitting interventions. Of particular interest to the present paper, it is worth recalling that the area of Messina strait experienced one of the most disastrous seismic events of modern history, occurred on December 28, 1908 and associated with moment magnitude 7.1 Mw: more than 100000 people died, and around 91% of structures were significantly damaged by the ground motion shaking and concurrent tsunami. After this tragic event, the city of Messina was reconstructed to a large extent. One of the most popular structural configurations adopted in the reconstruction stage of Messina consists of a confined brick masonry scheme, with confining reinforced concrete (RC) beams and columns casted after the erection of the masonry walls. Originally built in 1930, the student hall of residence of Messina adopted this structural scheme for the first four elevations. The remaining two elevations (fifth floor and a light appendage) were built subsequently (in the early 1970s) with a more modern RC framed structure, which causes a structural heterogeneity in elevation. Moreover, the building has a C shape in plan, with a long front side and two shorter wings, which triggers undesirable torsional effects when subject to horizontal ground motion accelerations. The recent Italian
seismic standards NTC08 [10] have increased the seismic demand parameters and this has required urgent seismic retrofitting interventions.

To this aim, buckling restrained braces (BRBs) are adopted as replacement of some selected masonry walls of the existing building. Besides enhancing the overall dissipation capacity through the incorporated hysteretic dampers, the BRBs are placed in some specific frames that are chosen in an attempt to reduce torsional effects of the building caused by the nonsymmetrical configuration in plan. The BRBs (adopted at all stories of the selected frames) are combined with local strengthening interventions applied to the beam-to-column joints of the last elevation, which were particularly vulnerable to shear actions. In this case, pre-tensioned stainless steel ribbons are employed to create a beneficial triaxial compression stress state and to improve the overall ductility of the RC frame. The effectiveness of the proposed retrofitting intervention is assessed through nonlinear static analysis on the original and retrofitted structure comparatively. Acceptance tests on the employed hysteretic dampers carried out at the laboratory CERISI of Messina are also described.

2 DESCRIPTION OF THE CASE-STUDY BUILDING

The case-study building is located in the city center of Messina. With reference to the photographs and sketches reported in Figure 1, it is noted that the structure has a C shape with a long front side and two shorter wings. The building consists of 5 stories (with a floor area of approximately 1300 m² per story) and a light appendage built in a limited zone of the overall floor area. The first four stories (basement, ground floor, first and second level) were constructed in the early 1930s. The structural configuration for this part of the structure is represented by a confined masonry-RC scheme widely adopted in the reconstruction of the city of Messina after the 1908 earthquake, which is illustratively sketched in Figure 2. In this configuration, the masonry brick walls were originally built with courses left staggered along the wall height and reinforcing bars simultaneously prepared for the column casting. However, both RC beams and columns were casted only after the masonry walls. The mutual collaboration between the two systems (toothing between masonry and RC) increased the out-of-plane resistance of the panel and was effectively exploited to prevent the overturning of the masonry walls, which was frequently observed during the 1908 Messina earthquake in most masonry buildings. In this way, the masonry walls were utilized as formwork of the confining RC beams and columns. Beams with variable sections (increasing near the beam ends), commonly employed at the time of construction, were realized by leaving staggered bricks of the walls as depicted in Figure 2. The resulting hybrid masonry-RC scheme presents a box-shaped configuration and an effective structural collaboration of masonry and RC even for small displacements. This structural arrangement was among the schemes recommended by the seismic regulations (R.D. n. 2089) in force at the time of the reconstruction of Messina [11] and extensively adopted until the Second World War in the area of the Messina strait. On the other hand, the fifth story (third floor) and the overlying appendage (marked in red in Figure 1) were realized nearly 40 years later than the original masonry building (early 70s) with a conventional RC framed structure (and internal partitions in hollow brick masonry walls). The steel reinforcement bars of the added story were welded to the upper portions of the bars of the bottom story of the masonry building. Finally, in the internal courtyard there is another structure (“Mensa building”), which was built in more recent times and is separated from the case-study building by a seismic gap (i.e., it is structurally independent from the analyzed building and will be not considered in the sequel of the paper).

The foundations of the building were realized with a bi-directional grid of inverted T-beams (RC strip foundation). The geotechnical properties of the installation site deposit (identified through two continuous surveys, twenty-four standard penetration tests and
multichannel analysis of surface waves) can be classified as being of a soil type B according to Italian seismic regulations NTC08 ($360 \text{m/s} < V_{30} < 800 \text{m/s}$).

The mechanical properties of the materials were determined through 48 extractions of concrete core cylinder samples, 20 extractions of steel reinforcing bars and 46 pull-out tests (all equally distributed on beam and column members). The average compressive strength of concrete cores $R_c$ in the first four stories was 13.9 MPa for columns and 14.2 MPa for beams, while the yield stress of bars was $f_y = 222.8$ MPa. On the other hand, the analogous quantities

Figure 1 Case-study building: (A) aerial view; (B) plan view; (C) and (D) sections A-A and B-B (front views) with added RC frame highlighted in red
for the added stories of the RC frame were 18.9 MPa, 20.7 MPa and 301.9 MPa, respectively. These values clearly demonstrate the different structural configuration of the two parts of the structure: in the lower part, the RC beams and columns were placed primarily for confining purposes of the masonry walls rather than to represent the main structural system, which instead occurs in the upper parts of the structure (added RC framed structure).

The cross-section details of RC beams and columns as well as the masonry wall thickness are listed in Table 1. The floor slabs of the first four stories were realized with a RC plate 8cm thick with stiffening beams at every 1.50 m. The floor slabs of the last two elevations (added RC framed structure) were realized with conventional one-way RC slabs with hollow brick as internal lightening elements.

Since original drawings and documentations of the existing building were found only for the added RC framed structure (last two elevations), a series of simulated calculations have been preliminarily performed to identify reasonable reinforcement arrangement of beams and columns of the first four floors. To this aim, the prescriptions of the seismic code in force at the time of construction, R.D. 2089 [11], were followed for the simulated calculations. The materials considered in the calculations are Aq42 steel (having admissible tensile stress of
1600 kg/cm$^2$) and concrete Rck150 (having admissible compressive stress of 50 kg/cm$^2$). All the other principal guidelines reported in the seismic code R.D. 2089 [11], here omitted for the sake of brevity, were followed. Two representative frames are considered in the simulated calculations, one on the main front side and one on the wing side of the structure. Assuming the design loads as per R.D. 2089 [11], reinforcement bars are computed for RC columns and beams and subsequently compared to actual reinforcement bar amounts determined through pachometer experimental measurements (rebar locator) and visual inspection after removal of the concrete cover. Selected comparisons are shown in Figure 3. Overall, the simulated calculations lead to results that are in good agreement with actual reinforcement arrangement determined in-situ. All the previous surveys and tests have led to a knowledge level LC2 according to the Italian seismic code §8.5.4 [10], which is associated with a confidence factor (FC) equal to 1.20.

![Figure 3 Identification of steel re-bars in RC beams and columns of the first four stories through visual inspection (A) and simulated calculations (B)](image)

**Figure 3 Identification of steel re-bars in RC beams and columns of the first four stories through visual inspection (A) and simulated calculations (B)**

### 3 SEISMIC PERFORMANCE OF THE EXISTING BUILDING

The seismic performance of the building in its original configuration (prior to any retrofitting operation) was evaluated through static nonlinear analysis (pushover analysis – N2 method) [12]. The design loads were established according to the Italian seismic code NTC08 [10], assuming a reference life of the building $V_R = V_N \times C_u = 50 \times 1.5 = 75$ years (relevant to an importance class III, $C_u=1.5$). Both damage limit states (SLD in Italian seismic code) and ultimate limit states (SLV in Italian seismic code) are considered, which are related to
probability of exceedance equal to 63% and 10% in the reference life of the building, respectively. The associated PGA values are 0.102g and 0.296g, respectively.

Figure 4 3D finite element model of the original building with masonry walls modelled through the equivalent pin-jointed strut (macro-modelling approach)

A 3D finite element model (FEM) has been developed to simulate the structural behaviour of the existing building, which is shown in Figure 4. After a preliminary assessment of the in-plane stiffness of the existing floor slabs, it has been verified that the conditions of the Eurocode 8 §4.3.1 about the rigidity of the diaphragm are satisfied. Therefore, diaphragm constraints have been applied at each level to simulate the behaviour of a rigid floor slab, which significantly reduces the computational effort of the overall model. The contribution of the brick masonry is incorporated in the model through a macro-modelling approach, by introducing equivalent pin-jointed struts related to the actual geometry and mechanical characteristics of the masonry walls. There exists a broad variety of empirical expressions in the literature to determine the dimensions of the equivalent strut in a consistent manner [14]-[16]. In the present paper, the simplified approach incorporated in the Italian guidelines [17] is used, whereby the nonlinear response of the masonry wall is modelled through an elastoplastic idealized behaviour. The shear strength of the masonry without any applied vertical load $f_{vk}$, the Young’s modulus of the masonry $E_w$ and the wall thickness $t_w$ represent the input parameters of the adopted model to determine the ultimate strength of the strut, the lateral stiffness and the yield and ultimate displacement of the elastoplastic model [17].

A plastic hinge approach has been adopted to incorporate the nonlinear behaviour of RC beams and columns, accounting for the confinement effect due to the presence of stirrups as per Italian regulations NTC08 [10]. The moment-curvature relationship is determined through
conventional analysis of the flexural plastic behaviour of the section. The curvature capacity of the plastic hinges is evaluated in accordance with the Italian code, assuming the definitions of the deformation capacity in terms of the chord rotation as reported in § C8A.6.1 [10].

According to the pushover method, the structure is subject to vertical loads and a set of monotonically increasing lateral loads. Two load distributions are considered, the mode distribution and the mass proportional distribution. From the base shear versus roof displacement curve of the multi-degree-of-freedom (MDOF) system, energy equivalence is applied to determine an equivalent single-degree-of-freedom (SDOF) model with a bilinear idealization [12]. Then, seismic performance of the building is assessed by comparing seismic demand (from the response spectrum) with seismic capacity (from the resulting SDOF pushover curve). A series of 16 different combinations are considered (four directions of the ground motion, two profiles of lateral loads and two additional eccentricities), from which the verification is carried out in terms of \( \frac{\text{PGA}_{\text{capacity}}}{\text{PGA}_{\text{demand}}} \) (ratios lower than one indicate that the structure is not safe). For damage limitation requirement (SLD, return period 75 years) the lowest ratio was 0.536; for no-collapse requirement (SLV, return period 712 years) the lowest ratio was 0.20. As a result, in either case the structure is not safe, meaning that the displacement demand exceeds its displacement capacity. Consequently, the structure needs retrofitting interventions.

4 SEISMIC RETROFIT OF THE BUILDING

The most critical aspects of the original building detected by the previous analysis concerns the premature shear failure of the T-beams in the foundation, the moderate torsional behavior due to the non-symmetrical configuration of the building in plan (along the \( x \) axis) and the unsatisfactory dissipation capacity of the structural members due to a generalized lack of steel reinforcement amounts, especially in the critical zones (beam-column joints). In essence, the structure does not comply with capacity design principles.

To retrofit the foundation, the cross-sections of the inverted T-beams were enlarged to increase the stiffness and strength. This has involved the following steps: removal of the concrete cover, introduction of new reinforcement bars connected to the existing T-beams through rheoplastic resin, and final casting of concrete. Details of these retrofitting phases are here omitted for the sake of brevity and also because they follow conventional retrofitting procedures of existing RC members and are not of particular interest to this contribution.

What is more interesting to discuss here is the introduction of BRBs (incorporating hysteretic dampers) to improve the seismic performance of the building by reducing torsional effects and increasing the overall dissipation capacity. The underlying theoretical principles of BRBs date back to the early 1970s, although devices with a stable force-displacement curve were developed and tested in the following years [18]. BRBs dissipate energy by hysteresis (plastic deformation and yielding) of an internal steel core element, placed within a concrete-filled steel tube. The risk of buckling of such steel element for high compressive loads is prevented by the surrounding concrete or grout, which ensures a stable (symmetric) cyclic force-displacement curve in both tension and compression. However, low-friction coating materials are introduced in the device to prevent the transmission of axial force from the steel core to the surrounding concrete/grout. In this way, the steel core is axially disconnected from the filling material. After attainment of the yielding deformation of the steel core, BRBs undergoes large deformations without decreasing strength. BRBs Nowadays, BRBs have been extensively applied worldwide, especially for steel structures in Japan [19] and in the United States [20], [21].
The BRBs were inserted as replacement of the original confined masonry walls at all stories of the building (from ground floor to the top floor). The adoption of BRBs was motivated by the limited flexibility of the masonry-RC structure (first natural period equal to 0.4s). In this regard, the use of alternative supplemental energy dissipation devices like viscous dampers [22]-[26] might be not effective because they may not be fully engaged due to the small interstory drifts experienced by such stiff structures. On the other hand, the plastic deformation of BRBs is activated even for relatively small displacements (the yield displacement of the internal steel core may be of few millimeters), which renders these devices particularly effective for stiff structures, as in the masonry-RC building here analyzed.

The BRBs were introduced in the 3D FEM as shown in Figure 5 through an idealized elastoplastic constitutive behavior, based on the force-displacement characteristics identified in the experimental tests, which will be described below. A series spring model incorporating the brace stiffness $k_b$ along with the damper stiffness $k_d = F_y / u_y$ is assumed. The position of the BRBs in plan has been carefully chosen so as to align the center of mass (CM) and the center of stiffness (CS). In Figure 6 the old CS (of the existing building) and the new CS (of the retrofitted building with BRBs) are depicted against the CM. Since the building is symmetric with respect to the $y$ axis but not with respect to the $x$ axis, BRBs were mostly concentrated near the two wings of the building to compensate for the asymmetrical stiffness distribution on that side.

Figure 5 3D finite element model of the retrofitted building with BRBs modelled with elastoplastic behavior
The parameters of the hysteretic dampers were preliminarily estimated through a direct displacement-based design procedure [27], [28], combining pushover analysis of the real MDOF structure with response spectrum analysis of an equivalent SDOF system. For practicality reasons, only one family of hysteretic dampers has been considered, having yield displacement $d_y = 1.63$ mm, corresponding force $F_y = 737$ kN, ultimate displacement $d_u = 15$ mm and corresponding force $F_u = 1065$ kN. More details of the adopted hysteretic dampers will be provided below. As already said, the yield displacement of the hysteretic dampers is lower than 2 mm, which ensures that such devices are fully engaged even for small interstory displacements (as in the case study building), provided the terminals of the device are effectively attached to the existing structure with the necessary torque moment in order to avoid any loosening effect.

Figure 6 Selected frames for the introduction of BRBs (to reduce torsional effects of the existing building)
The installation of the BRBs as replacement of the masonry walls is illustrated in Figure 7 through design drawings as well as photographs taken in-situ. The installation consists of the following steps: 1) removal of existing masonry wall; 2) installation of steel anchoring plates at the two terminals of the braces, which are connected to the masonry structure via epoxy resin and bolts; 3) installation of steel connecting frames hosting the BRBs (consisting of L-shaped steel members that are welded to one another); 4) installation of BRBs connected to the steel anchoring plates of point 2) via a series of bolts; 5) closure with lightweight concrete from either side of the BRBs (to facilitate maintenance of the devices in the future).

Figure 7 Installation of BRBs as replacement of the masonry walls, design drawings and photographs in-situ
However, a preliminary pushover analysis of the retrofitted structure with BRBs shed light on the vulnerability of the beam-column joints of the last elevation. This is motivated by the different flexibility characteristics of the two portions of the structure, namely the bottom confined masonry-RC structure and the upper RC framed structure. Evidently, the upper portion of the building has lower stiffness than the lower part, thus behaving as a more flexible SDOF system appended at the top of the underlying confined masonry-RC building. Consequently, large plastic deformations are concentrated in the beam-column joints of the last elevation, which represents the principal cause of collapse due to the low amount of steel stirrups. Due to such premature (brittle type) shear failure of the beam-column joints, the structure with BRBs still did not meet the requirements of the Italian code, as the seismic demand exceeded the seismic capacity.

Therefore, in addition to the installation of BRBs, the seismic retrofit of the building also involved pre-tensioned stainless steel ribbons applied to the beam-column joints of the last elevation to increase the shear capacity. The pre-tensioned stainless steel ribbons, applied for an overall length of 100 cm above the beam-column joint, offer a beneficial triaxial compression stress state and act as external stirrups – see design drawing and photographs in Figure 8. Design and verification of the strengthening system was carried out using a macro-element software package. Without going into details of the calculations, the shear capacity of the beam-column joints increased of 230% and 260%, passing from 86kN and 75kN to 191kN and 165kN in the two main directions, respectively.

The seismic performance of the retrofitted structure has been re-assessed through pushover analysis. In Figure 9 we report the results in acceleration-displacement response spectrum (ADRS) format for the two main directions and two load profiles. Comparing the seismic capacity with the seismic demand, it emerges that the retrofitted structure satisfies the requirements of the Italian seismic code [10] for both the SLD (damage-limitation requirement) and SLV (no-collapse requirement). The progressive formation of the plastic hinges was quite in line with a (desired) global collapse mechanism, with development of plastic zones in the beams prior to those in the columns. The ratio of the capacity PGA to the demand PGA exceeds one for all the 16 design scenarios analysed (the lowest ratios were 1.089 for SLD and 1.021 for SLV). Instead the safety index with respect to brittle failure modes (shear failure of beams) was found to be greater than 1.25 for all the combinations.

The hysteretic dampers of the present structure were realized by the Italian manufacturing company C.M.M. F.Lli Rizzi s.r.l.. In line with the prescriptions of the European regulations for antiseismic devices EN 15129 [29], factory production control (FPC) tests were performed before installation in situ. These dampers are classified as displacement-dependent devices and, according to [29], they were tested under 5 fully reversed cycles with amplitude

![Figure 8 Strengthening of beam-column joints of the last elevation through pre-tensioned stainless steel ribbons](image)

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\[ d_u/4 = 3.75 \text{ mm}, \text{ 5 fully reversed cycles with amplitude} \ d_u/2 = 7.5 \text{ mm} \text{ and 15 fully reversed cycles with amplitude} \ d_u = 15 \text{ mm}. \text{ The FPC tests were performed at the laboratory Eurolab of the CERISI – Centre of Excellence Research and Innovation of Large Dimensions Structures and Infrastructures, whose main hydraulic and mechanical characteristics can be found in [30]. We limit ourselves to recall the load capacity up to 3100 kN, the stroke of the actuators up to ±550 mm and the maximum allowed velocities up to 1100 mm/s in the main direction (x axis) of the testing equipment.}

![Figure 9 Pushover curves of the retrofitted structure for different loading scenarios](image)

A sketch of the hysteretic damper, some photographs of the testing equipment of the CERISI, and experimental force-displacement curves for the three above-mentioned tests (at the three increasing displacement amplitudes) are illustrated in Figure 10. The damper is more than 1 m long, with a rectangular cross-section of 290 mm x 390 mm. Considering the relatively limited displacement (yield displacement of 1.63 mm), it was necessary to install linear variable displacement transducers (LVDTs) at the two ends of the device to measure the actual axial deformation of the internal steel core. Indeed, the displacement measures from the hydraulic actuators were found to be significantly influenced by loosening effects of the bolt connections and, thus, affected by a series of additional spurious displacements. These spurious displacements had to be totally eliminated from the force-displacement curves for a correct assessment of the hysteretic characteristics. This also implies that the effectiveness of this kind of devices is strongly related to careful installation operations that allow the transfer of the interstory displacement from the two terminals of the braces to the internal steel core. By inspection of the force-displacement curves shown in Figure 10, the experimental values
of the hysteretic parameters \((d_y, F_y, d_u, F_u)\) were found to be in reasonable agreement with the theoretical design parameters adopted in the calculations.

Figure 10 FPC tests according to EN 15129 [29] of hysteretic dampers performed at the laboratory EUROLAB of the CERISI, Messina, Italy

5 CONCLUSIONS

This contribution has summarized a seismic retrofitting intervention designed and currently ongoing in the earthquake-prone area of Messina, Italy. After the tragic collapse of the university hall of residence of L’Aquila during the recent seismic events of 2009, similar student accommodation buildings in other Italian cities were studied to investigate their structural safety against seismic loads. The analyzed case study building represents, indeed,
the main student hall of residence of Messina. The area of Messina strait experienced one of the most catastrophic seismic events of modern history in 1908, with more than 100000 deaths. Despite this, many public buildings are still highly vulnerable to seismic actions and need to be retrofitted urgently. The case study building was structurally deficient because of a number of reasons among which we mention: 1) inadequate foundations; 2) moderate torsional behavior due to the C-shape in plan; 3) heterogeneity of the structural configuration in elevation, with a RC framed structure added on the underlying confined masonry-RC structure around 40 years later than the original construction; 4) unsatisfactory dissipation capacity, due to low transverse reinforcement (especially in the critical zones such as beam-column joints) and absence of capacity design principles.

The retrofitting interventions have considered these critical aspects. More specifically, foundation structures have been strengthened through a set of additional RC plates connected to the existing inverted T-beams (although this was not discussed extensively in this contribution for the sake of brevity). A series of BRBs were added in specifically selected frames of the structures at all the floors, as replacement of the brick masonry walls (at the first three elevations) and the masonry infills (at the last two elevations) in order to minimize torsional effects as well as increase the dissipation capacity of the structure. The frames where the BRBs are installed were selected in an attempt to align the center of mass with the center of stiffness as close as possible. The choice of hysteretic dampers (in place of other supplemental energy dissipation devices) was motivated by the nature of the existing building, which has low flexibility characteristics (quite stiff). Since the yield displacement of such devices is of few millimeters, they are engaged even for small interstory drifts experienced by the structure under design seismic loads. Finally, pre-tensioned stainless steel ribbons were applied to the beam-column joints of the last elevation in order to increase the shear capacity and to compensate for the lack of adequate amount of steel stirrups in such critical zones. These zones were found to be particularly critical with high concentration of plastic deformations due to the high flexibility characteristics of the added RC frame that behaves as a flexible mass oscillating on the underlying masonry building. FPCs tests on the employed hysteretic dampers were also described.

The above set of retrofitting interventions has improved the seismic performance of the structure significantly. The seismic capacity of the retrofitted building, evaluated through the pushover analysis, now exceeds the seismic demand imposed by the Italian seismic code, both for SLD (damage limitation requirement) and for SLV (no-collapse requirement) for all the design combinations analyzed. Although limited to the presentation of a particular case study building, the authors think that the retrofitting interventions proposed here are applicable to other buildings having similar structural configurations and heterogeneity of materials in elevation, which may be the case of several structures constructed around 50 years ago in other cities.

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LIFE-CYCLE COST OPTIMIZATION OF TUNED MASS DAMPERS FOR TALL BUILDINGS SUBJECTED TO WINDS AND EARTHQUAKES

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Abstract

In the last decades, much research has been dedicated to developing methodologies to optimize dampers for wind or seismic control. However, little investigation has been performed to deal with multi-hazard demands. Nevertheless, previous works have shown that from a life-cycle loss perspective winds and earthquakes may be equally relevant to many tall buildings. Thus, methodologies for multi-hazard loss optimization of tall buildings with dampers are of much need. To join these hazards in a sole optimization procedure, we adopt the life-cycle cost (LCC) resultant from wind and earthquakes as a unified design criterion. This work presents a multi-hazard optimization methodology of Tuned Mass Dampers (TMDs) in tall buildings. The methodology is applied to a 76-story building, employing the LCC as objective function. A Multiple TMDs system composed of four TMDs is considered on the top floor, assigned to dampen the first four modes. The TMDs mass, damping ratio and frequency are taken as design variables, and different constraints are imposed on the total added mass and individual TMDs frequencies. The linear dynamic analysis results are used to calculate the LCC through a platform based on the PEER equation. An efficient genetic algorithm combined with a pattern search algorithm permits to achieve the optimal solutions. A purely intuitive MTMDs design based on modal analysis would suggest largest masses should be assigned to the dominant modes. However, the results reveal this rationale could be misleading, demonstrating the need for optimization techniques to obtain adequate dampers designs. This innovative design procedure can improve long-time performance and deliver an optimal design from economic standpoint.

Keywords: TMD optimization, multi-hazard analysis, life-cycle cost analysis, tall buildings.
1 INTRODUCTION

It is widely recognized that the structural design of tall and supertall buildings is dictated by their lateral loads behavior. Natural hazards such as wind and earthquakes are the main generators of horizontal loads. To present day, conventional structural design usually considers only the hazard suspected to be the most restrictive. Nevertheless, there is no consensus about an objective criterion to determine the dominant hazard. Moreover, from a life-cycle cost perspective, often both wind and earthquakes are prone to provoke damage and produce losses independently of which one appears to dominate the design. To deal with these hazards [1]–[3] recommended to adopt a multi-hazard approach in the analysis.

The modern trend to mitigate wind and seismic-related losses consists in employing energy dissipation devices, usually known as dampers. Among the different available technologies, the use of passive Tuned Mass Dampers (TMDs) seems to be a favorable alternative. TMD concept consists in attaching a secondary mass to the main structural mass through a spring and a dashpot [4]. The choice of the spring stiffness and dashpot damping define the frequency of the TMD. Den Hartog first investigated a single TMD optimal parameters when linked to a single-degree-of-freedom (SDOF) system [5]. He suggested that tuning the TMD frequency to a value close to the structural natural frequency is efficient to attenuate vibrations. TMD optimization in SDOF systems was investigated using distinct methods such as numeric searching techniques [6], [7] and evolutionary algorithms [8]. Other researchers studied optimization of TMDs connected to multiple-degree-of-freedom (MDOF) systems employing evolutionary algorithms such as Genetic Algorithm (GA) [9], Differential Evolution (DE) [10], and gradient-based techniques [11].

The single TMD solution has been proved effective for structures whose response is dominated by a single vibration mode, thus its use to mitigate wind vibrations is widespread [12], [13]. Nevertheless, this is not the case of civil structures with significant higher modes contribution, such as systems subjected to seismic loads. For these applications, [14] proposed to employ multiple TMDs (MTMDs), splitting a TMD into various masses that may be tuned to different frequencies and placed in distinct locations. Optimal MTMDs parameters were investigated adopting distinct zero-order methods such as numerical searching techniques [15], [16]; pattern search algorithm [17]; fully-stressed design ideas [18] and GA [19]. Gradient-based techniques were adopted for MTMDs optimization as well [20], [21], showing good computational performance. All previously mentioned works on optimal design, nonetheless, are concerned with only one hazard – either wind or seismic effects.

Because of the differences in terms of frequency range, intensity and structural responses affected [22], an important obstacle to unify winds and earthquakes in a sole optimization procedure is finding a common parameter for these hazards. Previous investigations have pointed towards using economic criteria such as damage-related losses [23] or life-cycle cost (LCC) [3], [24]. The LCC seems to be an attractive option to work as optimization criteria due to its capability of describing losses associated to different performance levels consistently with performance-based engineering (PBE) concepts [25], encompassing ideas of risk probability and structural reliability within the losses assessment [26]. The LCC estimation has been performed in last years employing the methodology developed by the Pacific Earthquake Engineering Research (PEER) Center by a number of researchers [3], [27], and has been successfully adapted for wind loads [28], [29].

2 GOVERNING EQUATIONS

In this section we will initially introduce the assumptions about TMD modelling and its governing equations. We then present the equations of motions for a system equipped with
MTMDs. Subsequently we expose the main concepts and formulation of LCC as developed in the PEER framework.

2.1 Tuned Mass Dampers (TMDs)

A typical TMD consists in a mass connected to the main building structure by means of a spring and a dashpot very often at the top floor level. The devices parameters are its mass \( m_{TMD} \), stiffness \( k_{TMD} \), and damping \( c_{TMD} \). A MTMDs system installed at the top floor of a building is schematically represented in Figure 1.

![Figure 1 - A MTMDs system with 4 TMDs at the top](image)

The individual TMD frequency \( \omega_{TMD} \) and damping ratio \( \xi_{TMD} \) are related to previous parameters through the equations:

\[
(k_{TMD})_i = (m_{TMD})_i \cdot (\omega_{TMD})_i^2 \tag{1}
\]

\[
(c_{TMD})_i = 2 \cdot (m_{TMD})_i \cdot (\xi_{TMD})_i \cdot (\omega_{TMD})_i \tag{2}
\]

When employing multiple TMDs, each one of them may be tuned independently using distinct stiffnesses and damping coefficients. The resulting system is governed by the following equation of motion:

\[
M \ddot{u}(t) + C \dot{u}(t) + K u(t) = Pw(t) \tag{3}
\]

For a structure whose planar behavior only is considered, the mass matrix is:
\[
\mathbf{M} = \begin{bmatrix}
\mathbf{M}_{\text{original}} & 0 \\
0 & \mathbf{m}_{\text{TMD}}
\end{bmatrix}
\]  \hspace{1cm} (4)

where \([\mathbf{M}_{\text{original}}]\) is the mass matrix of the original structure without supplemental damping and \([\mathbf{m}_{\text{TMD}}]\) is a diagonal matrix containing all terms \((\mathbf{m}_{\text{TMD}})_i\) in its diagonal. The damping matrix is given by:

\[
\mathbf{C} = \begin{bmatrix}
\mathbf{C}_{\text{original}} + \mathbf{B}_d^T \mathbf{c}_{\text{TMD}} \mathbf{B}_d & -\mathbf{B}_d^T \mathbf{c}_{\text{TMD}} \\
-\mathbf{c}_{\text{TMD}} \mathbf{B}_d & \mathbf{c}_{\text{TMD}}
\end{bmatrix}
\]  \hspace{1cm} (5)

where \([\mathbf{C}_{\text{original}}]\) is the damping matrix of the original structure without supplemental damping, \(\mathbf{B}_d\) is a transformation matrix assigning TMDs positions within the system and \([\mathbf{c}_{\text{TMD}}]\) is a diagonal matrix containing all terms \((\mathbf{c}_{\text{TMD}})_i\) in its diagonal.

Finally, the stiffness matrix is constructed as:

\[
\mathbf{K} = \begin{bmatrix}
\mathbf{K}_{\text{original}} + \mathbf{B}_d^T \mathbf{k}_{\text{TMD}} \mathbf{B}_d & -\mathbf{B}_d^T \mathbf{k}_{\text{TMD}} \\
-\mathbf{k}_{\text{TMD}} \mathbf{B}_d & \mathbf{k}_{\text{TMD}}
\end{bmatrix}
\]  \hspace{1cm} (6)

where \([\mathbf{K}_{\text{original}}]\) is the stiffness matrix of the original structure without supplemental damping, \(\mathbf{B}_d\) is the same transformation matrix aforementioned and \([\mathbf{k}_{\text{TMD}}]\) is a diagonal matrix containing all terms \((\mathbf{k}_{\text{TMD}})_i\) in its diagonal.

The vector \(u(t)\) describes respectively the displacements of the structure degrees-of-freedom (DOFs). It may be written as:

\[
\mathbf{u}(t) = \begin{bmatrix}
\mathbf{u}_{\text{original}}(t) \\
\mathbf{u}_{\text{TMD}}(t)
\end{bmatrix}
\]  \hspace{1cm} (7)

where \([\mathbf{u}_{\text{original}}(t)]\) describes the displacements at the original DOFs and \([\mathbf{u}_{\text{TMD}}(t)]\) describes the displacements of the TMD-related DOFs.

The external loads amplitude vector is calculated as:

\[
\mathbf{p} = \begin{bmatrix}
\mathbf{p}_{\text{original}} \\
\mathbf{p}_{\text{TMD}}
\end{bmatrix}
\]  \hspace{1cm} (8)

where \([\mathbf{p}_{\text{original}}]\) is a vector of loads amplitude at the original system and \([\mathbf{p}_{\text{TMD}}]\) is a vector of loads amplitude at the TMDs. For seismic hazards, \(\mathbf{p}_{\text{original}} = -\mathbf{M}_{\text{original}} \mathbf{e} \) and \(\mathbf{p}_{\text{TMD}} = -\mathbf{m}_{\text{TMD}} \) with \(\mathbf{e}\) representing the excitation influence vector. In such a case, \(w(t)\) in equation (3) is a scalar denoting the ground acceleration and its time variation.

2.2 Life-cycle cost (LCC) assessment

The model employed for calculating the LCC in this study is based on the same assumptions of [3]: the structure is restored to its previous state after each damage event; multiple hazards never occur simultaneously and damage does not occur while the building is still in its damaged state. Considering that the probability of exceeding damage states is time-independent, then the expected LCC of a non-structural component may be defined as ([3], [24]):
\[ E(C) = C_0 + \frac{1 - e^{-\lambda t}}{\lambda} \sum_{j=1}^{k} (C_j v P_j) \]  

(9)

where \( E(\cdot) \) is the expected value, \( C_0 \) is the initial cost, \( \lambda \) is the discount rate per year, \( t \) is the design life time of the structure, \( k \) is the number of considered damage states \( C_j \) is the cost of the \( j \)-th damage state, \( v \) is the mean rate of occurrence of the hazard, and \( P_j \) is the probability of exceeding the \( j \)-th damage state associated with the occurrence of hazard.

The equation (9) was adapted by [3] to calculate the LCC of a component exposed to two different hazards – winds and earthquakes. Considering the restoration cost normalized with respect to its initial cost and calculated for any period of \( t \) years:

\[ E\left(\frac{C - C_0}{C_0}\right) = \frac{1 - e^{-\lambda t}}{\lambda t} \left\{ \sum_{j=1}^{k} -c_j^w \left[ \ln\left(1 - P_{tj}^w\right) - \ln\left(1 - P_{tj+1}^w\right) \right] \right. 
\left. + \sum_{j=1}^{k} -c_j^e \left[ \ln\left(1 - P_{tj}^e\right) - \ln\left(1 - P_{tj+1}^e\right) \right] \right\} \]  

(10)

The equation (10) considers the fact that although the same component may be subjected to attain distinct damage levels during a hazard event, the restoration cost should evidently be calculated only from the most severe one. The terms \( c_j^w \) and \( c_j^e \) represent the restoration cost related to the \( j \)-th damage state provoked respectively by wind and earthquakes. The terms \( P_{tj}^w \), \( P_{tj}^e \), and similar ones denote the probability of exceeding the \( j \)-th damage state caused by wind and seismic loads during a period of \( t \) years. These probabilities may be calculated from their equivalent annual probabilities, using the equation:

\[ P_{tj}^h = 1 - (1 - P_{aj}^h)^t \]  

(11)

where \( h \) denotes the hazard and may be replaced by \( w \) or \( e \) to account for wind or seismic-related damage states. \( P_{aj}^h \) is the annual probability of exceeding the \( j \)-th damage state induced by the hazard \( h \). The probabilities \( P_{aj}^h \) can be calculated using the well-known PEER equation in its formulation adapted by [3] from the former versions presented by [25], [29], [30]:

\[ P_{aj}^h = \int \int P(DS_j \| EDP) \frac{dP(EDP \| IM^h)}{dEDP} \frac{dH^h(IM^h)}{dIM^h} d(EDP) d(IM^h) \]  

(12)

In the previous equation, \( DS_j \) denotes the \( j \)-th damage state, \( EDP \) is a vector of engineering demand parameters (usually structural responses related to the damage), \( IM^h \) represents the intensity measure of the hazard, and \( H^h(IM^h) \) is the annual mean frequency of exceeding the intensity measure \( IM^h \) at least once. This PEER formulation neglects the variability of structural parameters, taking them as deterministic.

As pointed out by [30], the damage probability assessment through the PEER equation is a Markov process, where distinct conditional probabilities can be evaluated independently:
$P(\text{DS}_j|\text{EDP})$ is the complementary cumulative distribution function (known as fragility curve) of damage state $\text{DS}_j$ as function of structural response $\text{EDP}$; $dP(\text{EDP}|\text{IM}^h)/d\text{EDP}$ is the probability density function (PDF) of attaining a certain $\text{EDP}$ for given intensity measure $\text{IM}^h$, and is obtained by structural analysis procedures. The function $dH^h(\text{IM}^h)/d\text{IM}^h$ is the PDF of the hazard intensity measure.

### 2.3 Hazard modelling

For seismic loads, the PDF of the intensity measure $\text{IM}^e$ is materialized by a Seismic Hazard Curve (SHC), which must be obtained by means of a Probabilistic Seismic Hazard Analysis (PSHA) [31], [32]. The typical outcome of a PSHA is a hazard curve relating a certain intensity measure $\text{IM}$ with its mean annual frequency of exceedance $\nu^e(\text{IM})$ at a specific site. Assuming that seismic events consist in a Poisson process whose annual mean occurrence rate is $\nu^e$, the probability of exceeding the intensity $\text{IM}$ at least once is given by [33]:

$$H^e(\text{IM}) = 1 - e^{-\nu e(\text{IM})}$$

The intensity measure $\text{IM}$ can be for instance the peak ground acceleration or the spectral acceleration at a given period [3]. In any case, to obtain probability distributions of structural responses it is necessary to provide a set of realistic ground motion records and scale them to match the chosen $\text{IM}$.

The wind loads modelling process is very similar to seismic modelling: it is necessary to establish a Wind Hazard Curve (WHC) for the site and dispose of set of wind load time histories [3]. Consistently with [3], the reference wind velocity $V_{\text{ref}}$ is taken as the only random parameter representative of the local load intensity. [34] define the reference wind velocity as the largest annual mean wind speed at a 10 m in open terrain condition that can be appropriately modelled by the Gumbel distribution. Then, the WHC for a particular site may be computed considering the extreme wind speeds. In the present paper, the intensity measure $\text{IM}$ is the extreme annual value of $V_{\text{ref}}$. Assuming the Gumbel distribution for $V_{\text{ref}}$, the WHC can be directly obtained by [3]:

$$H^W(V_{\text{ref}}) = 1 - \exp \left[ -\exp \left( \frac{V_{\text{ref}} - \mu}{\beta} \right) \right]$$

where $\mu$ is the mean value and $\beta$ is the standard deviation of the distribution.

### 3 CASE STUDY

#### 3.1 The structure

The structure taken to exemplify the methodology is a 76-floor benchmark building introduced by [35], shown in Figure 2.
The building is modelled as a classic Euler-Bernoulli cantilever beam with one DOF per floor, resulting in 76 DOFs. The resulting matrix of mass $M$, damping $C$ and stiffness $K$ are extracted from [36]. The damping matrix $C$ actually was constructed considering the Rayleigh damping [37] adopting a damping ratio of 1% of the critical of each vibration mode. In the following analysis and optimization procedures, the structure location assumed is Otira, New Zealand.

3.2 Seismic loads

The spectral acceleration corresponding to the first natural period $S_a(T_1)$ is selected as seismic IM. To obtain the SHC, this paper adopts the hyperbolic model in the double logarithm plane proposed by [38] in the adapted format presented by [3]:

$$\ln(v^e) - \ln(v_{asy}^e) = \frac{\alpha}{\ln(S_a) - \ln(S_{a,asy})}$$ (15)

where $v^e$ is the mean annual rate of exceedance of a spectral acceleration $S_a$; $v_{asy}^e$, $\alpha$ and $S_{a,asy}$ are local constants. In their article, [3] established the typical constants to be adopted when using the $S_a$ as IM, considering the difference between the arbitrary period $T_1^* = 1.5$ s assumed by [38] and the benchmark building effective period $T_1 = 6.25$ s. The SHC parameters are shown in Table 1, and the resultant SHC is presented in Figure 3.

<table>
<thead>
<tr>
<th>Location</th>
<th>$v_{asy}^e$</th>
<th>$S_{a,asy}$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Otira, New Zealand</td>
<td>16.54</td>
<td>1.49</td>
<td>29.75</td>
</tr>
</tbody>
</table>

Table 1 - SHC parameters
The effects of seismic loads is computed based on 20 accelerograms relative to Los Angeles area, with 10% probability of exceedance in 50 years, as suggested in [38]. These records are conveniently scaled to match the same spectral acceleration $\bar{S}_a(T_1)$, thus following the same procedure as in [3].

### 3.3 Wind loads

To compute the WHC, the wind reference velocity $V_{\text{ref}}$ previously mentioned is calculated from the local 3-s gust reference wind speed given by the code AZ/NZS 1170:2:2011 (2011), and the coefficient of variation is taken arbitrarily as 0.2 [3]. The WHC parameters are shown in Table 2. The Figure 4 represents the resultant WHC.

**Table 2 - WHC parameters**

<table>
<thead>
<tr>
<th>Location</th>
<th>$V_{\text{ref}}^{3-s \text{gust}}$ (m/s)</th>
<th>$V_{\text{ref}}^{10-min \text{gust}}$ (m/s)</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Otira, New Zealand</td>
<td>30</td>
<td>20.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The wind loads are generated from the wind tunnel test results performed by [39] on a 1:400 model of the benchmark building. To represent the stochastic variability of wind forces, the one-hour equivalent time of load application adopted at the wind tunnel is transformed into 20 independent wind load time history series, as described by [3]. As in the case of accelerograms, the wind records also are scaled to match the same mean wind velocity $\bar{V}_{\text{ref}}$.

### 3.4 Structural analysis

As common practice for wind load analysis of tall buildings and very frequent for seismically excited tall buildings as well [40], a linear behavior is considered. Further checks described in details by [3] confirm the validity of such assumption for the present benchmark building example. A time-history analysis based on Newmark's Method for linear systems [37] is employed to obtain structural responses.

It is assumed that all damage induced by wind and earthquakes is related to two types of EDPs: inter-story drift ratio (IDR) and peak acceleration ($a$); consequently, these responses are adopted as EDPs for LCC analysis. To compute their Probability Density Functions, first the Newmark's Method is executed for each one of the 20 ground motion records (for earthquakes) or 20 wind forces time history series (for wind). At every floor a response mean value $\bar{EDP}$ and
standard deviations are calculated. Because the accelerograms and wind forces time histories were previously scaled as stated before, the obtained \( \overline{EDP} \)s correspond to the particular IM values \( S_a(T_1) \) and \( V_{ref} \). Due to linear behavior assumption, the peak EDP for any desired IM level can be calculated simply by scaling the initially calculated \( EDP \)s, as follows respectively for seismic and wind hazards [3]:

\[
EDP^e | S_a(T_1) = \overline{EDP}^e \frac{S_a(T_1)}{S_a(T_1)}
\]

\[
EDP^w | V = \overline{EDP}^w \left( \frac{V_{ref}}{V_{ref}} \right)^2
\]

Once the EDP mean and standard deviation values are known, its PDF for any given IM may be deduced assuming a lognormal distribution for EDP curve.

### 3.5 Damage and cost analysis

Consistently with [3], it is considered that fragility curves characterizing the probability \( P(DS_j | EDP) \) of exceeding a certain damage state as a function of attaining a certain EDP are lognormal functions. The typical values governing these functions depend upon the EDP provoking the damage. The Table 3 shows mean and standard deviation values describing fragility functions related to four different damage states, for drift-sensitive or acceleration-sensitive components. They are extracted from [41], and are assumed as valid for both seismic and wind effects [3].

**Table 3 - Statistical parameters for fragility functions of non-structural components [41]**

<table>
<thead>
<tr>
<th>Damage level</th>
<th>IDR (_m) (%)</th>
<th>( \sigma_{IDR} )</th>
<th>( a_m (g) )</th>
<th>( \sigma_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS1 - (IDR or a): slight</td>
<td>0.4</td>
<td>0.5</td>
<td>0.25</td>
<td>0.6</td>
</tr>
<tr>
<td>DS2 - (IDR or a): moderate</td>
<td>0.8</td>
<td>0.5</td>
<td>0.50</td>
<td>0.6</td>
</tr>
<tr>
<td>DS3 - (IDR or a): extensive</td>
<td>2.5</td>
<td>0.5</td>
<td>1.00</td>
<td>0.6</td>
</tr>
<tr>
<td>DS4 - (IDR or a): complete</td>
<td>5.0</td>
<td>0.5</td>
<td>2.00</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The restoration cost ratios associated with the four damage levels referred are taken as the mean values of cost distributions presented by [41], and are shown in Table 4. These mean values are taken as deterministic [3].

**Table 4 - Cost ratios for restoration of non-structural components [41]**

<table>
<thead>
<tr>
<th>Damage level</th>
<th>( C_{IDR} )</th>
<th>( C_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS1 - (IDR or a): slight</td>
<td>0.025</td>
<td>0.02</td>
</tr>
<tr>
<td>DS2 - (IDR or a): moderate</td>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td>DS3 - (IDR or a): extensive</td>
<td>0.6</td>
<td>0.36</td>
</tr>
<tr>
<td>DS4 - (IDR or a): complete</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

### 4 PRELIMINARY ANALYSIS

A goal of this work is to provide optimal solutions of MTMDs systems for this particular structure. As it will be formally stated in next section, the optimization problems will assume a system of four distinct TMDs placed on the top floor of the benchmark building. These TMDs are intended to dampen the first four natural vibration modes of the original structure. Therefore,
before proceeding to the optimization, preliminary analysis is carried out to get insight into the relative importance of vibration modes in the structure under investigation, before adding any supplemental damping.

4.1 Wind scenario

To investigate the modes participation and estimate their contributions for top floor responses, a modal analysis procedure is adopted. The classic dynamic equilibrium equation shown in equation (3) is re-written in modal coordinates rather than classic cartesian ones, using matrix transformation. Further details on this procedure may be found in [11]. The resulting problem is then separated into independent problems for each one of the 76 vibration modes, and solved for all 20 wind load time histories by the linear Newmark's Method [37]. For every SDOF analysis, the 76th floor peak displacement and peak acceleration are identified. For each wind force record, the modal participation is estimated by calculating the percentage of the squared peak response divided by the sum of all squared modal responses.

The following Tables 5-6 show respectively the top response obtained from each independent mode analysis averaged among the 20 records, and the average relative participation of each mode at the estimated total response. It should be noted that the percentages shown are averages among percentages obtained for the distinct records, so are not directly calculated from the presented average peak responses.

<table>
<thead>
<tr>
<th>Mode</th>
<th>(\omega) (rad/s)</th>
<th>Peak modal displacement (m) (average of 20 records)</th>
<th>Modal particip. - sum of squares (average of 20 records)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode</td>
<td>1.01</td>
<td>0.73</td>
<td>99.97%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>(\omega) (rad/s)</th>
<th>Peak modal acceleration (m/s^2) (average of 20 records)</th>
<th>Modal particip. - sum of squares (average of 20 records)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode</td>
<td>1.01</td>
<td>0.73</td>
<td>75.7%</td>
</tr>
<tr>
<td>2nd mode</td>
<td>4.81</td>
<td>0.28</td>
<td>11.6%</td>
</tr>
<tr>
<td>3rd mode</td>
<td>12.52</td>
<td>0.21</td>
<td>6.9%</td>
</tr>
<tr>
<td>4th mode</td>
<td>23.81</td>
<td>0.12</td>
<td>2.2%</td>
</tr>
<tr>
<td>5th mode</td>
<td>40.18</td>
<td>0.09</td>
<td>1.2%</td>
</tr>
<tr>
<td>6th mode</td>
<td>59.42</td>
<td>0.07</td>
<td>0.6%</td>
</tr>
<tr>
<td>7th mode</td>
<td>83.24</td>
<td>0.06</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

The Table 5 shows clearly that practically the whole top floor displacement is related to 1st mode behavior. The other modes participation is neglectable. As may be seen in Table 6, the 1st mode also dominates the 76th floor top acceleration response. However, in this case certain higher modes offer relatively significant contributions. From the 8th vibration mode and higher modes the participation is systematically inferior to 0.5%.

4.2 Earthquake scenario

The exact same procedure described for wind loads is employed to understand the modes influence on 76th floor top displacement and acceleration under earthquakes. The 20 accelerograms previously described are used to generate seismic loads for dynamic analysis carried independently for SDOFs representing the modes, and the peak floor responses are identified.
The following Tables 7-8 present the seismic equivalent responses of the results shown for wind analysis: top responses obtained from each independent mode analysis averaged among 20 accelerograms, and the average relative participation of vibration modes at the estimated total response.

Table 7 - Peak top floor displacement under seismic loads, independent vibration modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\omega$ (rad/s)</th>
<th>Peak modal displacement (m)</th>
<th>Modal particip. - sum of squares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(average of 20 records)</td>
<td>(average of 20 records)</td>
</tr>
<tr>
<td>1st mode</td>
<td>1.01</td>
<td>1.19</td>
<td>77.0%</td>
</tr>
<tr>
<td>2nd mode</td>
<td>4.81</td>
<td>0.58</td>
<td>21.7%</td>
</tr>
<tr>
<td>3rd mode</td>
<td>12.52</td>
<td>0.13</td>
<td>1.2%</td>
</tr>
<tr>
<td>4th mode</td>
<td>23.81</td>
<td>0.03</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Table 8 - Peak top floor acceleration under seismic loads, independent vibration modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\omega$ (rad/s)</th>
<th>Peak modal acceleration (m/s$^2$)</th>
<th>Modal particip. - sum of squares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(average of 20 records)</td>
<td>(average of 20 records)</td>
</tr>
<tr>
<td>1st mode</td>
<td>1.01</td>
<td>1.20</td>
<td>0.8%</td>
</tr>
<tr>
<td>2nd mode</td>
<td>4.81</td>
<td>13.34</td>
<td>21.3%</td>
</tr>
<tr>
<td>3rd mode</td>
<td>12.52</td>
<td>20.59</td>
<td>35.9%</td>
</tr>
<tr>
<td>4th mode</td>
<td>23.81</td>
<td>15.47</td>
<td>19.5%</td>
</tr>
<tr>
<td>5th mode</td>
<td>40.18</td>
<td>12.95</td>
<td>11.0%</td>
</tr>
<tr>
<td>6th mode</td>
<td>59.42</td>
<td>8.34</td>
<td>5.6%</td>
</tr>
<tr>
<td>7th mode</td>
<td>83.24</td>
<td>5.34</td>
<td>2.4%</td>
</tr>
<tr>
<td>8th mode</td>
<td>110.04</td>
<td>3.67</td>
<td>1.3%</td>
</tr>
<tr>
<td>9th mode</td>
<td>143.36</td>
<td>2.94</td>
<td>0.7%</td>
</tr>
</tbody>
</table>

The Table 7 demonstrates that the 1st mode dominates largely the seismic peak displacement, but the 2nd mode also contributes expressively on this response. The higher modes participation decreases rapidly and becomes neglectable for the 4th and higher modes. Nevertheless, a quite different distribution is observed in Table 8 concerning the top floor peak acceleration. No single mode is clearly predominant in this case: while the largest participation comes from the 3rd mode, the 2nd and 4th mode offer equally expressive contribution for the final response. The 1st mode contribution for the peak acceleration is very low. Moreover, smaller shares come from modes 6th – 9th, resulting in a combined 10% participation.

5 OPTIMIZATION PROBLEMS

A historical trend at research on MTMDs is the concept that because distinct TMDs are designed to dampen the structure natural modes [14] the MTMDs optimal distribution is highly related to the relative modal importance for the responses of interest. The first optimization problems to be solved are proposed to check this assumption. In these problems, the structural peak responses investigated by modal analysis are minimized. Subsequently, a multi-hazard optimization problem is solved, considering the LCC as objective function.

For all cases, it is assumed a system of four TMDs placed at the 76th floor of the benchmark building. The MTMDs parameters taken as variables are their masses $m_{TMD,i}$, frequencies $\omega_{TMD,i}$, and damping ratio $\xi_{TMD,i}$. The design variables are subjected to the following constraints: the MTMD total added mass is limited to 5000 ton, corresponding to 3.27% of the initial structure mass, and any individual mass $m_{TMD,i}$ is limited to this same value; the individual frequencies $\omega_{TMD,i}$ are constrained to a range between 0.8 and 1.2 times the natural mode
frequency intended to be dampened; and the damping ratios $\xi_{TMD,i}$ are constrained to values between 0 and 1.

The optimization problems are solved by a two-step procedure corresponding to distinct zero-order methods. First, an optimal solution is obtained employing a Genetic Algorithm (GA) [42]; then, the optimal solution resulting from GA is adopted as starting point for application of the Pattern Search (PS) algorithm [43]. In both cases, ready-to-run MATLAB codes [44] are used to execute the algorithms.

### 5.1 Top floor structural responses for wind loads

Initially, optimization problems related to wind loads are treated. The minimization of the building 76th floor structural responses is investigated. The problem of finding an optimal MTMD configuration resulting in the minimum top floor displacement can be described as:

$$
\begin{align*}
\min_{m_{TMD,i}, \omega_{TMD,i}, \xi_{TMD,i}} & \quad \bar{\ddot{d}}_{76}^w \\
\text{s.t.} & \quad 0 \leq m_{TMD,i} \leq 5000, \quad i = 1,2,3,4 \\
& \quad \sum_{i=1}^{4} m_{TMD,i} \leq 5000 \\
& \quad 0.8 \cdot \omega_{n,i} \leq \omega_{TMD,i} \leq 1.2 \cdot \omega_{n,i}, \quad i = 1,2,3,4 \\
& \quad 0 \leq \xi_{TMD,i} \leq 1, \quad i = 1,2,3,4
\end{align*}
$$

where $\bar{\ddot{d}}_{76}^w$ is the average 76th floor peak displacement among 20 load records, and $\omega_{n,i}$ are the uncontrolled structure natural frequencies. The optimal MTMDs configuration obtained is shown in Table 9.

**Table 9 - Optimum MTMDs design for wind-related top floor displacement**

<table>
<thead>
<tr>
<th>Mode</th>
<th>$m_{TMD}$ (ton)</th>
<th>% of total mass</th>
<th>$\omega_{TMD}$ (rad/s)</th>
<th>$\xi_{TMD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode</td>
<td>3024.3</td>
<td>60.5%</td>
<td>0.92</td>
<td>0.10</td>
</tr>
<tr>
<td>2nd mode</td>
<td>997.0</td>
<td>19.9%</td>
<td>3.85</td>
<td>0.23</td>
</tr>
<tr>
<td>3rd mode</td>
<td>505.5</td>
<td>10.1%</td>
<td>10.31</td>
<td>1.00</td>
</tr>
<tr>
<td>4th mode</td>
<td>473.2</td>
<td>9.5%</td>
<td>26.33</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The TMD assigned to dampen the 1st mode clearly absorbs the majority (60.5%) of the total MTMDs mass. Nevertheless, comparing this optimal design with the modal contributions shown in Table 5 the 1st mode dominance of the optimal solution is less expressive than one could expect. While the 1st mode is practically the only one contributing to the top floor displacement of the uncontrolled structure, the optimal configuration also exhibits significant shares of higher modes.

The problem of minimizing the 76th floor peak acceleration due to wind loads is defined as:

$$
\begin{align*}
\min_{m_{TMD,i}, \omega_{TMD,i}, \xi_{TMD,i}} & \quad \bar{\ddot{a}}_{76}^w \\
\text{s.t.} & \quad 0 \leq m_{TMD,i} \leq 5000, \quad i = 1,2,3,4 \\
& \quad \sum_{i=1}^{4} m_{TMD,i} \leq 5000 \\
& \quad 0.8 \cdot \omega_{n,i} \leq \omega_{TMD,i} \leq 1.2 \cdot \omega_{n,i}, \quad i = 1,2,3,4 \\
& \quad 0 \leq \xi_{TMD,i} \leq 1, \quad i = 1,2,3,4
\end{align*}
$$

where $\bar{\ddot{a}}_{76}^w$ is the average 76th floor peak acceleration among 20 load records, and $\omega_{n,i}$ are the uncontrolled structure natural frequencies.
\[ 0 \leq m_{TMD,i} \leq 5000, \quad i = 1, 2, 3, 4 \]
\[
\sum_{i=1}^{4} m_{TMD,i} \leq 5000
\]
\[ 0.8 \cdot \omega_{n,i} \leq \omega_{TMD,i} \leq 1.2 \cdot \omega_{n,i}, \quad i = 1, 2, 3, 4 \]
\[ 0 \leq \xi_{TMD,i} \leq 1, \quad i = 1, 2, 3, 4 \]

where \( \bar{a}_{76} \) is the average top floor peak acceleration among 20 load records. The optimal design achieved is presented in Table 10.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( m_{TMD} ) (ton)</th>
<th>% of total mass</th>
<th>( \omega_{TMD} ) (rad/s)</th>
<th>( \xi_{TMD} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} mode</td>
<td>2148.5</td>
<td>43.0%</td>
<td>0.88</td>
<td>0.16</td>
</tr>
<tr>
<td>2\textsuperscript{nd} mode</td>
<td>1426.0</td>
<td>28.5%</td>
<td>4.48</td>
<td>0.08</td>
</tr>
<tr>
<td>3\textsuperscript{rd} mode</td>
<td>623.4</td>
<td>12.5%</td>
<td>14.21</td>
<td>0.29</td>
</tr>
<tr>
<td>4\textsuperscript{th} mode</td>
<td>802.1</td>
<td>16.0%</td>
<td>22.80</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Once more the 1\textsuperscript{st} mode absorbs a larger mass than higher modes, but its share (43.0%) is less than a half of the overall MTMDs mass. A very expressive mass (28.5%) is assigned to the 2\textsuperscript{nd} mode and significant portions go to higher modes as well. Interestingly, the 4\textsuperscript{th} mode receives a mass larger than the 3\textsuperscript{rd} mode. As for the top displacement, the mass distribution does not follow the relative contribution of vibration modes to the top acceleration seen in Table 6. The mass assigned to the 1\textsuperscript{st} mode is proportionally inferior to its modal contribution, and the exact opposite happens for higher modes.

### 5.2 Top floor structural responses for seismic loads

The same problems of minimizing top floor peak responses are formulated for seismic loads. The optimization of the 76\textsuperscript{th} floor peak displacement provoked by earthquakes is formulated as:

\[
\min_{m_{TMD,i}, \omega_{TMD,i}, \xi_{TMD,i}} \bar{d}_{76}
\]
\[
\begin{array}{l}
0 \leq m_{TMD,i} \leq 5000, \quad i = 1, 2, 3, 4 \\
\sum_{i=1}^{4} m_{TMD,i} \leq 5000 \\
0.8 \cdot \omega_{n,i} \leq \omega_{TMD,i} \leq 1.2 \cdot \omega_{n,i}, \quad i = 1, 2, 3, 4 \\
0 \leq \xi_{TMD,i} \leq 1, \quad i = 1, 2, 3, 4
\end{array}
\]

(20)

where \( \bar{d}_{76} \) is the average 76\textsuperscript{th} floor peak displacement among 20 accelerograms. The resultant optimal configuration is shown in Table 11.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( m_{TMD} ) (ton)</th>
<th>% of total mass</th>
<th>( \omega_{TMD} ) (rad/s)</th>
<th>( \xi_{TMD} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} mode</td>
<td>599.1</td>
<td>12.0%</td>
<td>1.06</td>
<td>0.00</td>
</tr>
<tr>
<td>2\textsuperscript{nd} mode</td>
<td>3280.3</td>
<td>65.6%</td>
<td>4.80</td>
<td>0.12</td>
</tr>
<tr>
<td>3\textsuperscript{rd} mode</td>
<td>827.5</td>
<td>16.5%</td>
<td>11.14</td>
<td>0.11</td>
</tr>
<tr>
<td>4\textsuperscript{th} mode</td>
<td>293.2</td>
<td>5.9%</td>
<td>21.74</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Here, the discrepancy between the modes influence on the uncontrolled 76th floor displacement presented in Table 7 and the optimal TMDs mass distribution is even more noticeable. The 1st mode largely controls the peak displacement but receives only 12.0% of the MTMD optimal configuration mass. However, the 2nd mode absorbs nearly two thirds of the entire added mass. A disproportionally relevant mass is also assigned to the 3rd mode, and even the 4th mode receives an unexpected share of TMDs mass.

The minimization of the top floor peak acceleration under seismic loads is formally written as:

\[
\min_{m_{TMD,i}, \omega_{TMD,i}, \xi_{TMD,i}} \bar{\alpha}^{s}_{76}
\]

\[
\begin{align*}
0 \leq m_{TMD,i} \leq 5000, & \quad i = 1,2,3,4 \\
\sum_{i=1}^{4} m_{TMD,i} \leq 5000 \\
0.8 \cdot \omega_{n,i} \leq \omega_{TMD,i} \leq 1.2 \cdot \omega_{n,i}, & \quad i = 1,2,3,4 \\
0 \leq \xi_{TMD,i} \leq 1, & \quad i = 1,2,3,4
\end{align*}
\]

where \(\bar{\alpha}^{s}_{76}\) is the average top floor peak displacement among 20 accelerograms. The Table 12 presents the MTMDs optimal design relatively to this response.

<table>
<thead>
<tr>
<th>Mode</th>
<th>(m_{TMD}) (ton)</th>
<th>% of total mass</th>
<th>(\omega_{TMD}) (rad/s)</th>
<th>(\xi_{TMD})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode</td>
<td>0.0</td>
<td>0.0%</td>
<td>1.21</td>
<td>1.00</td>
</tr>
<tr>
<td>2nd mode</td>
<td>671.2</td>
<td>13.4%</td>
<td>5.77</td>
<td>1.00</td>
</tr>
<tr>
<td>3rd mode</td>
<td>588.7</td>
<td>11.8%</td>
<td>11.87</td>
<td>0.02</td>
</tr>
<tr>
<td>4th mode</td>
<td>3740.1</td>
<td>74.8%</td>
<td>28.57</td>
<td>0.42</td>
</tr>
</tbody>
</table>

The results show that a vast majority (74.8%) of the MTMDs mass should be assigned to the 4th mode to reach the minimum peak acceleration. It should be noted that the modal contributions presented in Table 8 differs significantly from other structural responses, for it is the only case for which the 1st mode is not the largest contributor. Notwithstanding, once more the optimal TMDs mass modal distribution is inconsistent with the modal shares for the structural response, and higher modes absorb mass shares much larger than expected.

There is a general trend of disagreement between the modal participations at structural responses presented in Tables 5-8 and the optimal MTMDs configuration resultant from optimization procedures. Shorter vibration modes controlling the structural response systematically absorb mass shares inferior to their contribution, and higher modes with little impact on structural responses receive TMDs mass disproportionally large. This led to strong indications that the optimal MTMDs design relative to LCC would not necessarily follow relative importance of vibration modes for a given structural response.

5.3 Multi-hazard life-cycle cost optimization

Both wind and seismic loads are considered in an integrated framework to estimate the life-cycle cost (LCC) of a building relatively to these two hazards. The methodology and assumptions described in Sections 2 and 3 are adopted into the LCC formulation. Briefly, the LCC in
the present paper is the sum of the expected restoration cost of non-structural components of the building caused by winds and earthquakes.

The LCC optimization problem is formulated as:

\[
\min_{m_{TMD,i}, \omega_{TMD,i}, \xi_{TMD,i}} \ LCC^{total} = LCC^w + LCC^e \\
\text{s.t.} \\
0 \leq m_{TMD,i} \leq 5000, \quad i = 1, 2, 3, 4 \\
\sum_{i=1}^{4} m_{TMD,i} \leq 5000 \\
0.8 \cdot \omega_{n,i} \leq \omega_{TMD,i} \leq 1.2 \cdot \omega_{n,i}, \quad i = 1, 2, 3, 4 \\
0 \leq \xi_{TMD,i} \leq 1, \quad i = 1, 2, 3, 4
\]  

Solving the problem by the two-step procedure comprehending a Genetic Algorithm and a Pattern Search, the obtained optimal MTMD solution is presented in Table 13.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( m_{TMD} ) (ton)</th>
<th>% of total mass</th>
<th>( \omega_{TMD} ) (rad/s)</th>
<th>( \xi_{TMD} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^{st}) mode</td>
<td>1686.1</td>
<td>33.7%</td>
<td>0.91</td>
<td>0.09</td>
</tr>
<tr>
<td>2(^{nd}) mode</td>
<td>747.9</td>
<td>15.0%</td>
<td>4.59</td>
<td>0.04</td>
</tr>
<tr>
<td>3(^{rd}) mode</td>
<td>1142.5</td>
<td>22.8%</td>
<td>11.17</td>
<td>0.06</td>
</tr>
<tr>
<td>4(^{th}) mode</td>
<td>1423.6</td>
<td>28.5%</td>
<td>22.71</td>
<td>0.23</td>
</tr>
</tbody>
</table>

No single vibration mode may be considered as dominant in this optimal design. The 1\(^{st}\) and 4\(^{th}\) modes absorb the largest masses, but significant amounts are assigned to other modes as well. This optimal design represents a set of masses well distributed among the four modes. This result confirms the indications from the first optimization problems that the relative contribution of vibration modes to relevant structural responses does not provide enough information to predict an optimal MTMD design.

6 CONCLUSIONS

In this paper, the adoption of LCC as common design criterion for a multi-hazard optimization is successfully implemented and results in an efficient MTMDs design from economic standpoint. The LCC calculation employing the PEER framework allows to consider both winds and earthquakes probabilistically and provides a continuous measure of economic loss. The implementation presented supports the case for employing a multi-hazard approach rather than assuming arbitrarily one hazard as the dominant one, and points toward adopting the LCC as a convenient common parameter to unify winds and earthquakes in a unique design procedure.

The modal analysis results and modal relative participations on structural responses cannot work as references to predict the optimal design configurations, differently from what intuition would suggest. Comparisons between modal analysis and optimal design obtained for same structural responses show that higher modes with no apparent relevance in modal participation are systematically assigned with significant TMD masses. This fact stresses the importance of employing optimization procedures to design MTMDs systems.

Because optimization methodologies are of so much need, further research is necessary to develop efficient and computationally viable procedures. Though zero-order methods can be easily implemented and do not require sensitivities, their computational cost may become an
obstacle when treating larger or more complex structural problems. Moreover, future research should also clarify the reasons why intuitive design solutions based on modal analysis results do not coincide with optimal MTMDs configurations.

REFERENCES


EFFECT OF THE DYB ON THE SEISMIC RESPONSE OF STEEL CONCENTRIC BRACINGS

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Abstract

Even though steel Concentric Braced Frames (CBF) provide buildings with large strength and stiffness and the use of pinned beam-to-column connections makes their fabrication economical, the poor dissipation capacity of steel braces and the low redundancy of the structure make CBFs prone to storey collapse mechanisms.

In this paper, the Double-stage Yield BRB (DYB) is proposed to replace conventional steel concentric braces in order to improve the ductility of CBFs. This damper yields in tension and compression exhibiting fat hysteresis loops and large ductility capacity. In addition, it is characterized by a two-stage yielding. The double-stage yielding mitigates the lack of redundancy of the structural type and promotes a global collapse mechanism. In fact, even if the DYBs of one storey yield before the others, the internal forces will increase in the whole structure between the first and the second yield stages of DYB, thus forcing yielding of the DYBs of the other stories.

The goal of this paper is to investigate the effect of the insertion of the DYB on the seismic performance of CBFs. To this end, a steel frame with DYBs is designed according to a method derived from that stipulated in Eurocode 8 for conventional CBFs. The behaviour factor \( q \) is set equal to 4.5. To examine the influence of the properties of the DYBs on the structural response of the frame, different values of the ratio \( \rho_{YS} \) between first and second yield forces of the DYBs are considered. Finally, the seismic response of the obtained frames is evaluated by nonlinear dynamic analysis and verified according to Eurocode 8 acceptance criteria.

Keywords: Steel frames, buckling restrained braces, hysteretic dampers, seismic design, collapse mechanism.
1 INTRODUCTION

In Europe, the design of steel frames is generally based on two design concepts: (1) the Moment Resisting Frame (MRF), and (2) the Concentric Braced Frame (CBF). MRFs resist the seismic force by members acting in an essentially flexural manner, while CBFs resist the seismic forces mainly by means of members subjected to axial forces. In the occurrence of strong seismic events, MRFs experience widespread yielding and exhibit a ductile seismic response if members are properly designed and detailed [1, 2]. Based on this, the European seismic code (Eurocode 8, EC8) [3] rewards MRFs with a high behaviour (reduction) factor $q$ (up to 6.5) for the evaluation of the design seismic forces. However, EC8 stipulates also strict requirements on the storey drift demand in the occurrence of moderate earthquakes to control the damage to non-structural elements. These requirements often cause the increase in size of the structural members of MRFs. Therefore, the lateral strength actually provided to the structure is significantly larger than the design seismic shear force and the large $q$-factor stipulated by the code reveals to be deceptive. On the contrary, CBFs are inherently very stiff structural systems, which allow designers to easily satisfy the limits imposed on storey drifts. On the other hand, CBFs are characterized by a poor inelastic response at (i) member [4] and (ii) frame level [5-6]. Indeed, braces subjected to cyclic load exhibit buckling, stiffness and strength degradation, pinching, and severe concentration of plastic deformation in a small portion of the member. These phenomena reduce both the displacement ductility capacity and the energy dissipation capacity of the brace. Further, because of the low redundancy of the CBF, the lateral stiffness of the frame is drastically reduced when compression and tension braces of one storey have buckled or yielded, respectively. Hence, internal forces cannot increase further in the structure and the effect of the earthquake is the increase of the drift demand at the storey where braces have buckled and yielded, while the rest of structure remains elastic. In the occurrence of strong ground motions, this imposes the use of large design forces to compensate for the low levels of available ductility and dissipation capacity. In fact, EC8 – part 1 provides a $q$-factor equal to 2.5 for high ductility CBFs with chevron braces.

To overcome the limitations of MRFs and CBFs, the insertion of a device, named Double-stage Yield Buckling restrained brace (DYB), can be a solution. The DYB has been conceived by Pan et al. [7, 8] as a development of the conventional Buckling Restrained Braces (BRB) [9, 10]. The DYB is composed of two BRB units connected in series with different yield forces. In both tension and compression, the DYB behaves elastically until first yielding is achieved. Then, the device deforms plastically until the maximum deformation dictated by the buckling-restraint mechanism is obtained (first yield stage). Later, the device recovers stiffness and deform elastically again, until the second yielding is achieved (second yield stage). Experimental tests prove that DYBs possess good ductility capacity and exhibit cyclic behaviour characterised by stable and fat hysteresis loops. The geometrical configuration of the DYB and the data of experimental testing provided by Pan and co-workers are analysed to (i) individuate the parameters that govern its cyclic response and (ii) evaluate the mechanical properties (axial stiffness, yield force, etc.) needed for its design. A numerical model able to reproduce the cyclic response of the DYB is built in OpenSees environment [11] to fit the results of the experimental test. Different values of the ratio $\rho_{YS}$ between first and second yield forces of the DYBs are considered.

The objective of this paper is to show the advantages provided by the insertion of DYBs in a concentric braced frame. To this end, a steel bracing with DYBs is designed according to a design method derived from that stipulated in Eurocode 8 for conventional concentric bracings. Finally, the seismic response of the obtained frames is determined by nonlinear dynamic analysis and is verified according to EC8 acceptance criteria.
2 THE DOUBLE-STAGE YIELD BUCKLING RESTRAINED BRACE

2.1 Concept of the DYB

Figure 1 shows the DYB tested by Pan and co-workers [8]. The dissipative steel core is en-cased in the buckling-restraint mechanism. The core (Fig. 1a) is a steel plate composed of five segments: two segments with the reduced cross-section (yielding segments) are designed to yield under loading, while the other three segments with full cross-section (connection segments) remain elastic and serve for the connection with the buckling-restraint mechanism. The yielding and connection segments are connected by means of elastic segments with variable cross-sections (transition segments). The two yielding segments, together with the respective transition and connection segments, are called small BRB and large BRB and behave as two BRBs connected in series. The buckling-restraint mechanism is connected to the external connection segment of the small BRB by means of bolts and elongated holes (Fig. 1b). When the DYB is loaded, the small BRB yields first and the deformation occurs only in the small BRB. The plastic shortening/lengthening of the small BRB ends when the bolts of the external connection segment of the small BRB touch the inside of the elongated holes. The clearance of these holes defines the threshold on the deformation of the small BRB and it is sized to maintain the ductility demand of the small BRB below the capacity. When this threshold is achieved, the force overpasses the small BRB by means of the buckling-restraint mechanism and transfers to the large BRB. Hence, the DYB gradually recovers stiffness and, for a larger force level, the large BRB starts yielding. The connection between the large BRB and the relevant external connection segment is made by means of elongated holes with large clearance (Fig. 1b), so that the buckling-restraint mechanism does not prevent the deformation of the large BRB. Buckling-restraint mechanism and dissipative core should be made by standard and low yield point steel grades, respectively. The Q345 and LY225 steel grades fab-
ricated in China, with nominal yield strength of 345 and 225 MPa, were used for the buckling-restraint mechanism and dissipative core of the DYB tested in [8].

The geometrical characteristics of the DYB are indicated in Figure 1a. The threshold on the deformation of the small BRB is equal to 5 mm. Details about the specimen can be found in [8]. The hysteresis loops of the specimen shown in Figure 1d (black line) were determined by applying the loading protocol plotted in Figure 1c [8]. The experimental results show the two stages of yielding and evidence both the isotropic and kinematic hardening of the DYB, even though the latter is quite small. Despite several cycles of loading [8], the hysteretic behaviour was very stable and the specimen did not show any degradation of the mechanical properties even at an axial displacement ratio of 1/100. Considering that yielding of the small BRB occurred for a displacement ratio of 1/2000 (Fig. 1d), the specimen exhibited a displacement ductility capacity, defined as the ratio of the maximum displacement to the displacement at yielding of the small BRB, equal to 20, in accord with other studies regarding conventional BRBs [12].

A numerical model of the DYB has been developed in OpenSees using the nonlinear-BeamColumn element and the uniaxial material model proposed by Zona and Dall’Asta [13]. The parameters of the model components were calibrated based on the geometrical and mechanical features of the specimen (Fig. 1a) in order to fit the experimental hysteretic response of the DYB. The numerical model was subjected to the same loading protocol of the experimental test and the obtained force-displacement relationship is compared to the actual response of the specimen in Figure 1d. The hysteresis loops obtained by the numerical model (red line) almost overlap those of the test (black line). Hence, the numerical model accurately reproduces the main features of the hysteretic response of the tested DYB.

2.2 Evaluation of tension and compression strength adjustment factors

The forces transmitted by the BRBs to the surrounding frame after yielding are larger than their nominal yield strength. In DYB, assuming the yield strength $N_y$ as the axial force corresponding to first yielding of the small BRB, the increase of the axial force above the yield strength value is caused by the double-stage yielding. The maximum forces transmitted in tension ($N_{\text{max}}^+$) and compression ($N_{\text{max}}^-$) are different and are usually derived from the yield strength $N_y (N_y = A_s f_y)$ by means of the tensile strength adjustment factor $\alpha$ and the compression strength adjustment factor $\beta$. These factors depend on the characteristics of the considered BRB and, for the DYB, their value is determined by a parametric analysis.

A set of DYBs is designed considering five values of the ratio $\rho_{YS}$ of the yield strength of the large BRB to that of the small BRB (yield strength ratio), i.e. from 1.0 to 1.8 in steps of 0.2. In case of $\rho_{YS} = 1.0$, the small and large BRBs yield simultaneously and the relevant DYB behaves like a traditional BRB. For all the DYBs, the dissipative core is shaped like the specimen tested by Pan and co-workers and is made of S235 steel grade (with nominal yield strength $f_y$ equal to 235 MPa).

For each DYB, a cyclic pushover analysis is run in displacement control. The total ductility demand $\mu_B$ at the single loading step is defined as the ratio of the total elongation/shortening at the relevant step to the elongation of the DYB corresponding to yielding of the small BRB. The loading protocol is defined so that the total ductility demand of the DYB $\mu_B$ is equal to 1 in the first step of loading, increases to 3 in the second step, and then gradually increases up to 30 in steps of 3 up. For each value of $\mu_B$, two cycles of loading are applied, the values of $N_{\text{max}}^+$ and $N_{\text{max}}^-$ are recorded and the strength adjustment factors are evaluated as
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The values of $\omega$ are evaluated for all the imposed levels of ductility demand and are represented in Figure 2 as a function of $\mu_B$ (grey triangles). The same procedure is repeated for all the considered values of $\rho_{YS}$. Independently of the value of $\rho_{YS}$, the factor $\omega$ rapidly increases with $\mu_B$. For values of $\mu_B$ not larger than 6, the increase of $\omega$ is due to the isotropic hardening of the small BRB, while the large BRB basically remains elastic. For values of $\mu_B$ larger than 6, the inelastic deformation extends to the large BRB and the increase of $\omega$ is primarily due to the isotropic hardening of the large BRB. However, the isotropic hardening saturates for a value of $\mu_B$ close to 12, thus in the range from 12 to 30 the strength adjustment factor $\omega$ still increases, but very slowly. Based on this, the values of $\omega$ may be determined by the following relations

$$\omega = 1 + \left(1.15 \rho_{YS} - 1\right) \frac{\mu_B - 1}{9} \quad \text{for } 1 \leq \mu_B \leq 10$$

$$\omega = 1.15 \rho_{YS} + 0.005 (\mu_B - 10) \quad \text{for } \mu_B > 10$$

that are represented for the five considered values of $\rho_{YS}$ in Figure 2 by black lines. The compression strength adjustment factor $\beta$ is set equal to 1.1, according to the experimental results. The value of $\beta$ is assumed to be independent of the value of $\rho_{YS}$.

3 SEISMIC PERFORMANCE OF FRAMES WITH DYBS

The effectiveness of the DYBs in enhancing the seismic response of steel frames and its relation with the yield strength ratio $\rho_{YS}$ is herein investigated. To this end, a set of steel frames equipped with DYBs characterised by different values of $\rho_{YS}$ is designed following a force-based procedure derived from that proposed in [12] for steel frames with BRBs. Hence, the seismic performances of the frames with DYBs are evaluated by nonlinear dynamic analysis and compared in terms of ductility demand and residual drift.

3.1 Design of steel frames with DYBs

The DYBs are the dissipative members of the structural system, while beams and columns are considered non-dissipative members and should remain elastic even in the case of strong ground motions. The proposed design procedure is controlled by three parameters: the behaviour factor $q$ and the yield strength ratio $\rho_{YS}$, that determine the yield strength of the small and
large BRBs, respectively, and the design storey drift angle $\Delta_d$, which determines the ductility capacity that the DYBs should possess. Capacity design criteria are incorporated into the procedure to design the non-dissipative members.

The effect of the design seismic force is determined by a linear elastic analysis. The elastic response spectrum with probability of exceedance of 10% in 50 years is reduced by the behaviour factor $q$ to obtain the design response spectrum. The building structure is modelled by an elastic centreline numerical model with rigid floor diaphragms. Columns are modelled as continuous along the height and pinned at their base, while beams and DYBs are pinned at their ends. Hence, the seismic force of the building is mainly resisted by the braced frames. DYBs are modelled as trusses with equivalent area $A_{eq}$ calculated as follows

$$A_{eq} = \frac{A_{eq,S} A_{eq,L}}{A_{eq,S} L_w + A_{eq,L} L_w,S}$$

where $A_{eq,S}$ and $A_{eq,L}$ are the equivalent area of the small and large BRB unit, respectively, while $L_w$, $L_w,S$ and $L_w,L$ are the total length of the DYB, the length of the small BRB and the length of the large BRB, respectively. $P-\Delta$ effects are treated according to the provisions given in Section 4.4.2.2 of EC8 – part 1 [3]. The DYBs are designed to fulfil three requirements. First, the yield strength of the small DYB $N_{pl,Rd,S}$ should not be smaller than the design axial force $N_{Ed}$, which is evaluated as the summation of the axial forces caused by the gravity loads in the seismic design situation and by the design seismic forces. The cross-sectional area of the yielding core of the small BRB $A_{c,S}$ is equal to

$$A_{c,S} = \frac{\gamma_{M0} N_{Ed}}{f_y}$$

where $\gamma_{M0}$ is the partial safety coefficient for resistance of cross-sections stipulated in Euro-Code 3 (EC3) [14]. Second, the cross-sectional area $A_{c,L}$ of the yielding core of the large BRB is calculated as $\rho_{YS}$ times $A_{c,S}$. At the same time, the overstrength factor $\Omega$ of DYB, i.e. the ratio $N_{pl,Rd,S}$ to $N_{Ed}$, should not exceed at any storey the minimum value $\Omega_{min}$ by more than 25%. Third, the ductility capacity $\mu_{B,d}$ of the DYB is determined so that the DYB subjected to cyclic loading should accommodate a ductility demand equal to twice that ($\mu_{B,d}$) corresponding to the attainment of the design storey drift $\Delta_d$, evaluated as follows

$$\mu_{B,d} = 2 \mu_{B,d}^d = 2 \frac{\Delta_d - (\Delta_y - \Delta_y^d)}{\Delta_y^d}$$

The storey drift $\Delta_y$ corresponding to yielding of the DYB is determined as that caused by the application of the design seismic force. Indeed, DYBs are expected to yield for the design seismic force. Thus, they are sized so that the yield strength be as close as possible to that required by the design analysis. Furthermore, the contribution of the axial deformations of the DYBs $\Delta_y^d$ to $\Delta_y$ is calculated according to this equation

$$\Delta_y^d = \frac{A_{c,S} f_y L_w}{\gamma_{M0} E_y A_{eq} \cos \alpha_B}$$

where $\alpha_B$ is the angle of inclination of the brace with respect to the longitudinal axis of the beam. Usually, the required ductility capacity is almost constant along the height of the frame and the largest value can be assumed for all the DYBs.
According to the capacity design criteria, non-dissipative members and connections of dissipative members (DYBs) are sized to sustain the maximum forces transmitted by the dissipative members. Here, the simplified equations for the design of beams and columns of frames with DYBs are derived from those stipulated in EC8 – part 1 for conventional chevron braces. In particular, the design axial force of these members is determined by the relation

\[ N_{Ed} = N_{Ed,G} + 1.1 \gamma_{ov} \frac{1+\beta}{2} \omega \Omega_{\min} N_{Ed,E} \]  

(7)

where \( N_{Ed,G} \) is the axial force due to the gravity loads in the seismic design situation, \( N_{Ed,E} \) is the axial force due to the design seismic force, \( \gamma_{ov} \) is the steel overstrength factor and \( \omega \) is the tension strength adjustment factor calculated by Eq. (2) and assuming \( \mu_B = \mu_{B,r} \). If the DYBs are arranged in the chevron configuration, the beams are subjected to an unbalanced vertical force equal to

\[ F_{V,unb} = 1.1 \gamma_{ov} (\beta - 1) \omega N_{pl,Ed,S} \sin \alpha_B \]  

(8)

that acts upwards at the mid-span. This force causes a bending moment that is calculated assuming the beam as pinned at its ends.

### 3.2 The analysed frames

A set of five 8-storey buildings, located in a high seismicity zone, with reference peak ground acceleration \( a_{g,E} \) equal to 0.35 g and founded on soil type C (soft soil) (EC8 – part 1), is designed according to the described procedure. In each building, the geometric and mass properties are equal at all storeys and the DYBs, arranged in the chevron configuration, are located in the middle span of the perimeter frames (Figure 3a). The five buildings differ for the considered values of \( \rho_{YS} \), which are assumed equal to 1.0, 1.2, 1.4, 1.6 and 1.8. The DYBs are shaped as those presented in Section 2.1 and made of Steel grade S235. The partial safety coefficient \( \gamma_{M0} \) is assumed equal to 1. Since the selected cross-sectional area of the core of DYBs is that strictly required in design, the brace overstrength ratio \( \Omega \) is always equal to 1.

Columns are continuous along the height and oriented as shown in Figure 3b. Beam-to-column connections are pinned. The values of gravity loads are resumed in Figure 3a. Since the building is symmetric, the numerical model adopted for the design represents half of the structure. Figure 3c shows the geometrical scheme of the numerical model. Mass is lumped at floor levels and is equal to 146.8 t at each floor. To consider the effect of the floor slab, the nodes of the same floor are constrained by rigid diaphragms. Modal response spectrum analysis is used to evaluate the internal forces in members and the storey drifts, and the response is then amplified to account for the \( P-\Delta \) effects. The design storey drift angle \( \Delta_d \) is assumed equal to 1.5% because it leads to a required ductility capacity \( \mu_{B,S} \) of DYBs similar to that obtained in the experiment by Pan and co-workers. The maximum value of \( \mu_{B,r} \) is obtained for the braces of the first storey and it is assumed as the ductility capacity that DYBs have to be provided with. The adopted behaviour factor \( q \) is equal to 4.5. This value is chosen because is intermediate between the value stipulated in EC8 for conventional chevron braces \( (q = 2.5) \) and the largest value \( (q = 6.5) \) allowed by EC8 for steel structures. Furthermore, the value \( q = 4.5 \) is larger than that \( (3.6) \) required for steel frames with conventional BRBs [12] to fulfil the performance requirements of EC8 when \( \Delta_d \) is 1.5%.

Beams and columns are sized based on strength and stability verifications (EC3), considering partial safety coefficients \( \gamma_{M0} \) and \( \gamma_{M1} \) equal to 1. The design axial force of the columns of the braced frames is determined by Eq. (7) while the bending moment is obtained by the design analysis. In Eq. (7), \( \gamma_{ov} \) is assumed equal to 1 and \( \Omega_{\min} \) is equal to 1. The beams of the
braced frames are designed to sustain the bending moment caused by the unbalanced force evaluated by Eq. (8). The columns that do not belong to the braced frame are designed considering two loading conditions: (i) the axial force determined by the gravity loads in non-seismic design situation evaluated according to the tributary area, (ii) the axial force caused by the gravity loads in seismic design situation plus the bending moment evaluated by the seismic design analysis. The beams that do not belong to the braced frame sustain only the bending moment caused by the gravity loads in the non-seismic design situation and their size is determined based on the (i) ultimate limit state, and (ii) serviceability limit state verifications stipulated in EC3. European wide flange sections are used for beams and columns. Steel grade S235 is used for beams, steel grades S235, S275 ($f_y = 275$ MPa) and S355 ($f_y = 355$ MPa) are used for columns. The same column cross-section is adopted for two consecutive storeys.

### 3.3 Nonlinear dynamic analyses

The nonlinear dynamic analyses are carried out considering the seismic excitation levels corresponding to probabilities of exceedance of 10% and 2% in 50 years. The seismic excitation is simulated by means of the 20 accelerograms derived from the FEMA/SAC project with probability of exceedance of 10% in 50 years in the Los Angeles area [15]. These accelerograms are scaled by a factor equal to 0.88 so that the average spectrum of the 20 accelerograms matches the elastic spectrum with probability of exceedance of 10% in 50 years (Figure 4) of EC8 – part 1 [3]. Finally, these accelerograms are scaled by a factor of 1.71 to obtain the set of

### Table 1: Load combinations (kN/m²)

<table>
<thead>
<tr>
<th>Description</th>
<th>Combination</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Seismic design combination</td>
<td>$G_k + 0.3Q_k$</td>
<td>5.00</td>
</tr>
<tr>
<td>Non seismic design combination</td>
<td>$1.4G_k + 1.5Q_k$</td>
<td>9.16</td>
</tr>
<tr>
<td>Serviceability limit state design</td>
<td>$G_k + Q_k$</td>
<td>6.40</td>
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</tbody>
</table>

**Figure 3.** (a) Gravity loads and loads combinations, (b) Plan layout of the pilot building and (c) geometrical scheme of the numerical model

**Figure 4.** Elastic response spectra (a) for each accelerogram and (b) average spectrum vs EC8 spectrum

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**Table 1**

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<tr>
<td>Dead Load $G_k$</td>
<td>$4.40$ kN/m²</td>
<td></td>
</tr>
<tr>
<td>Live Load $Q_k$</td>
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<td></td>
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**Figure 4.** Elastic response spectra (a) for each accelerogram and (b) average spectrum vs EC8 spectrum

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**Figure 4.** Elastic response spectra (a) for each accelerogram and (b) average spectrum vs EC8 spectrum

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**Figure 4.** Elastic response spectra (a) for each accelerogram and (b) average spectrum vs EC8 spectrum
accelerograms with probability of exceedance of 2% in 50 years according to EC8 – part 1 [3].

The nonlinear dynamic analyses are carried out by the OpenSees computer code. The geometrical scheme, mass properties, and floor constrains of the numerical model are equal to those used in the design analysis. The Rayleigh viscous damping is adopted and the equivalent viscous damping ratio is set at 3% for the first two modes of vibration of the system. P-Δ effects are included in the analysis. The numerical model introduced in Section 2.1 is used for the DYBs. Beams and columns of the braced frame, which are non-dissipative members, are simulated by elastic (elastic BeamColumn) elements, whilst members that do not belong to the braced frame are simulated by inelastic (beamWithHinges) elements. The length of the plastic hinge is equal to the depth of the cross-section of the member. The cross-section of the plastic hinges is discretised into fibres. The uniaxial steel material proposed by Menegotto and Pinto (Steel 02 Material in OpenSees) with kinematic hardening ratio equal to 0.003 is used in the design analysis. The Rayleigh viscous damping is adopted and the viscous damping ratio is set at 3% for the first two modes of vibration of the system.

3.4 Results

The structural performance provided by the five frames are investigated by the nonlinear dynamic analyses. To this end, two parameters have been considered: (i) the ductility demand μB of the DYBs and (ii) the residual drift angle ΔRes of the frame. The response of the analysed frames in terms of ductility demand of DYBs is assessed considering the Near Collapse (NC) and Significant Damage (SD) limit states defined in EC8 – part 3 [16], which should not be exceeded for seismic excitations with probability of exceedance of 2% and 10% in 50 years, respectively. For each seismic excitation level, the peak values of μB determined by the 20 time-histories are recorded at each storey of the frame. Hence, assuming that the parameter under investigation is log-normally distributed, the median of the 20 values is assumed as the demand of the earthquake in the relevant verification. The ductility demands are divided by the values of μB,LS corresponding to the attainment of the relevant limit state. For the verification of the NC performance objective, μB,LS is assumed equal to the ductility capacity μB,r of the DYBs. Instead, the value of μB,LS corresponding to the attainment of the SD limit state is taken equal to 3/4 of μB,r, in keeping with the capacity models stipulated in EC8 – part 3 for RC and steel beams in the absence of specific criteria. The ratio of μB/μB,LS is called hereinafter normalised ductility demand μ̃B of DYBs. The ratio μ̃B is calculated at each storey of the frame and plotted in Figures 5a and 5b for the five analysed frames and for both the performance objectives. Here, different data markers pinpoint the results obtained for frames equipped with DYBs with different ρYS. The comparison between the curves of each figure illustrates the effect of the double-stage yielding on the normalised ductility demand. For both the SD and NC performance objectives, increasing ρYS reduces the normalised ductility demand of DYBs and makes its distribution along the height of the frame more uniform. The

Figure 5. Effect of the double-stage yielding on (a, b) the normalised ductility demand of DYBs and (c) on the normalised residual drifts
verification of the NC performance objective is the most demanding. Indeed, it leads to larger values $\bar{\mu}_s$ and is fulfilled for $\rho_{YS}$ not smaller than 1.6. The effectiveness of the DYB reduces for smaller values of $\rho_{YS}$ and when $\rho_{YS} = 1.0$ the values of $\bar{\mu}_s$ for the verification of the NC performance objective attains very large values, that exceed the full-scale of the diagram. In fact, the response of this frame diverged for many of the accelerograms with probability of exceedance of 2% in 50 years, because of the small post-yield stiffness and the drift concentration at the lower storeys.

Despite the lack of indication in EC8, studies available in literature [17] point out that the recovery of the functionality of the building after a seismic excitation is strongly related to the residual drifts. In this study, the maximum admitted storey residual drift angles $\Delta_{Res}$ caused by a seismic excitation with probability of exceedance of 10% in 50 years is assumed equal to $0.5\%$, as reported in FEMA 356 [18]. Considering the probability of exceedance of the seismic excitation, this verification is included in the SD performance objective. The peak values of $\Delta_{Res}$ were determined by the 20 time-histories and their median value is calculated. The median values are then divided by the limit value $0.5\%$ to obtain the normalised residual drift $\bar{\Delta}_{Res}$, that is shown in Figure 5c for the five analysed frames. The comparison shows that increasing $\rho_{YS}$ has a beneficial effect also on the normalised residual drift. In fact, the larger $\rho_{YS}$ the smaller the normalised residual drift, even though the SD performance requirement is satisfied for $\rho_{YS}$ not smaller than 1.6.

4 CONCLUSIONS

This paper proposes the use of DYBs as a possible tool to enhance the structural performance of the steel concentric braced frame, able to mitigate (i) the effect of the low structural redundancy and (ii) the low energy dissipation capacity of the conventional braces. The fat-shaped and stable hysteresis loops of the DYB ensure large energy dissipation capacity. Furthermore, the double-stage yielding of the DYBs prevents the localization of structural damage and allows a better exploitation of the ductility capacity of the dissipative members.

The numerical investigation presented in this paper evidences the benefits obtained by means of the use of DYBs. A design procedure is formulated for this structural type in keeping with the architecture of EC8. A set of five steel frames was designed by varying the yield strength ratio of the DYBs. The structural response of the designed frames was determined by nonlinear dynamic analysis both at the NC and SD limit states. The results showed that the double-stage yield of the DYBs leads the frame to a more effective seismic response by (i) making the heightwise distribution of ductility demand of the DYBs more uniform and (ii) reducing the peak values of both the ductility demand and the residual drift. Particularly, increasing the yield strength ratio $\rho_{YS}$ affects positively the response of frames with DYBs both in terms of brace ductility and residual drift. A behaviour factor $q = 4.5$, which is larger than that found in [12] for steel frames with BRBs with similar ductility capacity, leads to frames with DYBs that fulfil the considered SD and NC performance objectives, provided that the value of $\rho_{YS}$ is not smaller than 1.6.

REFERENCES


STATIC CONDENSATION PROCEDURE OF FINITE ELEMENT MODELS FOR FAST NON-LINEAR TIME HISTORY ANALYSES OF BASE-ISOLATED STRUCTURES

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Abstract

Non-linear time history analysis represents the most realistic simulation of the effects of selected seismic events on a structural system. Especially when natural recorded signals are applied, it is possible to evaluate maximum values of internal forces for all elements, as well as if the studied structure had been subjected to the considered earthquakes. On the other hand, Finite Element Models of building structures in real application may have a large number of elements, and consequently higher and higher computational times can be reached. In addition, when seismically base-isolated structures are implemented, the isolation system is generally characterized by non-linear constitutive behaviors, so that convergence failure can occur.

In this work a static condensation procedure is proposed for building structures, in order to define a Multi Degree of Freedom system, with same dynamic properties of the Finite Element Model of the considered base-isolated structural system. Through the fast direct integration of the lumped-mass model, it is possible to define a number of time instants, which corresponds to the worst cases for the overall structure; thus, static deformed shapes can be applied to the Finite Element Model, in order to evaluate internal forces for all elements. Results have shown a fairly good agreement between the outcomes of the proposed procedure and the envelope of internal forces returned by the non-linear time history analysis of the full Finite Element Model.

Keywords: Static condensation, Base isolation, Non-Linear time history analysis, non-linear isolation device.
1 INTRODUCTION

Structural design of base-isolated buildings has gained during last years due to the high level of knowledge achieved through both numerical and experimental research works. The post-processing of the outcomes of several testing campaigns, carried out on full scale physical prototypes according to standard codes for anti-seismic devices [2, 3], have led to the definition of a number of analytical models, able to capture the effective behavior of isolation devices. In addition, commercial software provide non-linear link elements which implement the hysteretic force response of the most common typologies of devices. Consequently, very realistic numerical simulation of the seismic behavior of base-isolated structural systems can be performed, and the uncertainty on the computed response quantities is significantly reduced.

On the other hand, in some cases such simulations may imply long run times and frequent problems in achieving convergence at all time steps of a Non-Linear Time History Analyses: especially when highly non-linear hysteretic devices are implemented, such as Concave Surface Slider isolators [7, 11, 12], adaptive integration methods could excessively reduce the time step aiming at achieving convergence, increasing the total duration of the analysis, if no error occurs. Furthermore, run times are also increased whenever the considered structural system have significantly large sizes, and a set of spectrum-compatible seismic events has to be studied, according to a national standard code.

In this work a static condensation procedure is proposed, in order to carry out fast Non-Linear Time History Analyses on a full 3D FEM model, by considering an equivalent Multi Degree of Freedom oscillator. Such a model is able to return the displacement response of each floor of the system, by implementing the actual non-linear hysteretic behavior of the isolation system. In order to estimate peak internal forces for all elements, a set of time instants have been defined, which correspond to the worst-cases, in terms of maximum drift of the MDOF model, for all levels of the structure: at those time instants, static deformed shapes have been defined and applied to the full 3D FEM model, and internal forces have been compared to maximum values returned by the full time history analyses. Results have shown a very good agreement between the reference case (NLTHA) and the presented procedure, which provide a significant run time reduction.

2 STATIC CONDENSATION PROCEDURE

In order to consider the actual behavior of the considered building, a special operating procedure has been defined, which returns a condensed stiffness matrix of the system [1]. Precisely, the proper stiffness values of all the structural members have to be accounted for, with no additional modeling assumptions (such as shear-type scheme). To do so, the location of the centre of mass has been defined for all the floors of the 3D F.E.M. model: such points are connected to the related storey by means of a rigid diaphragm constraint, so that the out-of-plane stiffness of beams is considered. Hence, the centre of mass of all storeys is restrained, with respect to the horizontal translational degree of freedom, related to the analyzed direction of motion. Hence, in order to compute the i-th column of the condensed stiffness matrix, the i-th level of the system is unrestrained, and a floor horizontal force is lumped at the centre of mass, as shown in Figure 1.
Then, in order to obtain the i-th column of the matrix, the vector containing the reactions and the lumped force is divided by the horizontal displacement at the i-th unrestrained floor, originated by the applied lumped load:

\[ K(i, i-th) = \frac{1}{d_{i-th}} \begin{pmatrix} R_0 \\ R_1 \\ \vdots \\ F_{i-th} \\ \vdots \\ R_{N-1} \\ R_N \end{pmatrix} \]  \hspace{1cm} (1)  

It has to be noted that no force response is considered for the implemented devices at the isolation level, since the hysteretic behavior is separately modeled by means of actual non-linear constitutive laws: thus, the resulting stiffness matrix just accounts for the linear elastic behavior of the superstructure, and refers also to the horizontal rigid body translational motion. This procedure is expected to lead to the same dynamic response of the full 3D model of the structure, since out-of-plane beams deformations, transverse translational directions and torsional rotations at each floor are unrestrained. Finally, the whole model is reduced to a Multi Degree of Freedom oscillator, with a full stiffness matrix; concerning the mass matrix, the assembled mass values at each point for a single storey are summed up: the formulation of the dynamic system accounts for degrees of freedom, with respect to the ground displacement: such an assumption implies a diagonal mass matrix.

3 CASE STUDY STRUCTURE

In the present work a three storey building has been studied (Figure 2). Precisely, a reinforced concrete frame structure has been considered, with a square plan development; four spans along both x and y directions are designed, 6m long. At the ground level of the building, a reinforced concrete slab represents the interface between the building itself and the isolation system: the thickness is 500mm and plan dimensions have been obtained by considering the plan development of the superstructure, increased by 1.5m along all sides (27m x 27m).
Figure 2: Case study structure.

The interstorey height is 3m and all floors can be considered and modeled as rigid diaphragms, that is with infinite stiffness from the in-plane point of view, with a finite stiffness for the out-of-plane behavior. In Table 1 results of the aforementioned static condensation procedure are reported, in terms of mass and stiffness matrices.

<table>
<thead>
<tr>
<th>Mass (tons)</th>
<th>Stiffness (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>941.3</td>
<td>236656</td>
</tr>
<tr>
<td>0.0</td>
<td>-329731</td>
</tr>
<tr>
<td>0.0</td>
<td>111851.3</td>
</tr>
<tr>
<td>0.0</td>
<td>-18776.6</td>
</tr>
</tbody>
</table>

Table 1: Mass (left [tons]) and stiffness (right [kN/m]) condensed matrices for the case study structure.

Through the presented procedure, an approximately symmetric stiffness matrix is computed: for all the numerical analyses, differences between the correspondent symmetric components have been eliminated, by considering an average value. The isolation system consists of Double Concave Surface Slider devices, with a design friction coefficient equal to 6% and an equivalent radius of curvature equal to 3m. Actually, the overall reinforced concrete structure is laid on a grid of 5x5 devices, and isolators are installed at the same position of the building columns.

4 SEISMIC INPUT

In order to perform Non-Linear Time History Analyses, a spectrum-compatible set of ground acceleration time series has been adopted, in agreement with what is prescribed by the Italian Building Code 2018 [5] and selected by means of the Software REXEL [10]. The construction site of the system is located at L’Aquila (Italy), and the following characteristics have been assumed:

- Soil class: C;
- Topographic category: T1;
- Limit state: Collapse Limit State (return period: 975 years).

Furthermore, the special matching software SeismoMatch [15] has been used for every single record, aiming at obtaining response spectra closer to the target spectrum provided by the
code. In Figure 3 the selected and matched records set are shown and in Table 2 the main characteristics are listed.

As can be noted response spectra related to selected records are very close to the target spectrum, and consequently spectrum-compatibility is ensured within the period range provided by the Italian Building Code for base-isolated structures (i.e. between 0.15s and 1.2Ti, being Ti the design isolation period, assumed equal to 2.5s); precisely, according to the considered standards, spectrum-compatibility is checked if the mean spectrum among the selected records falls within 90% and 130% of the target spectrum.

The fairly good approximation of all response spectra with respect to the target one will provide more robust results. It has to be noted that seismic events have been considered as the application of unidirectional motions, since previous studies have shown a good agreement between the radial and bidirectional responses of base-isolated structures equipped with CSS devices [8].

<table>
<thead>
<tr>
<th>Waveform</th>
<th>Earthquake Name</th>
<th>Date</th>
<th>Mw</th>
<th>Epicentral Distance (km)</th>
<th>PGA (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14_x</td>
<td>Friuli earthquake 1st shock</td>
<td>06/05/1976</td>
<td>6.4</td>
<td>21.72</td>
<td>3.39</td>
</tr>
<tr>
<td>91_x</td>
<td>Friuli earthquake 3rd shock</td>
<td>15/09/1976</td>
<td>5.9</td>
<td>5.23</td>
<td>3.18</td>
</tr>
<tr>
<td>104_y</td>
<td>Friuli earthquake 4th shock</td>
<td>15/09/1976</td>
<td>5.9</td>
<td>4.70</td>
<td>2.50</td>
</tr>
<tr>
<td>166_x</td>
<td>Irpinia earthquake</td>
<td>23/11/1980</td>
<td>6.9</td>
<td>21.80</td>
<td>1.27</td>
</tr>
<tr>
<td>790_y</td>
<td>L’Aquila mainshock</td>
<td>06/04/2009</td>
<td>6.3</td>
<td>4.39</td>
<td>4.37</td>
</tr>
<tr>
<td>870_x</td>
<td>L’Aquila earthquake</td>
<td>06/04/2009</td>
<td>5.6</td>
<td>15.14</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 2: Characteristics of the adopted selection of seismic events.
5 NON-LINEAR TIME-HISTORY ANALYSES

According to the considered base-isolated system, Non-Linear Time History Analyses have been performed, by means of the full 3D F.E.M. model. The superstructure has been modeled with linear-elastic elements, whereas isolation devices have been implemented through non-linear link, able to reproduce the actual hysteretic response of the considered isolators. No additional viscous damping has been defined, in order not to underestimate any response parameters, since isolation devices already provide the hysteretic damping to the overall system.

In the present endeavor a static condensation procedure is presented, in order to carry out fast NLTH analyses, by considering the Multi Degree of Freedom oscillator equivalent to the case study structure. Thus, according to the previously defined mass and stiffness matrices for the case study structure, the following dynamic system of the MDOF model can be obtained:

\[
\begin{bmatrix}
\dddot{\bar{u}}_0 \\
\dddot{\bar{u}}_1 \\
\dddot{\bar{u}}_2 \\
\dddot{\bar{u}}_3
\end{bmatrix} + M \begin{bmatrix}
\dddot{u}_0 \\
\dddot{u}_1 \\
\dddot{u}_2 \\
\dddot{u}_3
\end{bmatrix} + K \begin{bmatrix}
\dddot{u}_0 \\
\dddot{u}_1 \\
\dddot{u}_2 \\
\dddot{u}_3
\end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \times \dddot{x}_g
\]

(2)

Being:

- \( \bar{M} \) the condensed mass matrix of the system;
- \( \bar{K} \) the condensed stiffness matrix of the system;
- \( u_i \) the translational degrees of freedom at the centre of mass location of the i-th floor;
- \( \dddot{x}_g \) the considered ground acceleration time series;
- \( \{F_{is}\} \) the isolation force response.

The force response of the isolation system \( \{F_{is}\} \) is added in the first equation only. In order to consider the actual hysteretic behavior of the Double Concave Surface Slider devices, provided by the stepwise sliding motion, the summation of two individual contributions has been implemented, as reported in the following equation:

\[
\{F_{is}\} = W_{tot} \cdot \frac{\dddot{u}_0}{R_{eq}} + \mu \cdot f_{NF}
\]

(3)

Precisely, the force response of such isolators is provided by the combination of a recentering contribution, returned by the projection of the vertical load applied to the devices and the spherical shapes of the implemented sliding surfaces; in addition, the sliding motion generates frictional forces, which are represented by a rigid-plastic hysteretic constitutive law [7, 14]. In order to numerically model such a behavior, an equivalent elastic-plastic parameter \( f_{NF} \) has been implemented (Figure 4): the rigid behavior is modeled by means of a linear stiffness, by considering a yielding displacement small enough to obtain almost vertical loading/unloading branches. Such a parameter is bounded between ±1, since represents the normalized frictional force of the devices. It has to be noted that the isolation force response is a function of the total weight of the structural system \( W_{tot} \), since a single device equivalent to the whole set of isolators is implemented.
In this work, the friction coefficient has been considered as a constant value, with no dependency with respect to sliding velocity, vertical load and cyclic effect [4, 6, 9, 11, 12]. The presented MDOF model provides a direct comparison of the displacement responses of all the levels of the structural system; however, no information about internal forces is provided. Thus, the displacement responses have been analyzed, in order to determine time instants which lead to the maximum drift for all floors: then, the instantaneous deformed shapes returned by the MDOF model at such time values have been statically applied to the full 3D F.E.M. model, in order to obtain internal forces. Maximum values for axial load, shear force and bending moment of the overall time history analyses are expected to occur at the time instant which maximizes the drift value for the considered floor. In this way, thanks to the static condensation procedure, the time integration of the dynamic system can be carried out by considering the condensed MDOF model, whereas maximum internal forces can be obtained by applying static deformed shapes to the full FEM model, and consequently the overall run time is significantly reduced.

6 RESULTS

In this section results are analyzed in terms of:

- comparison between the statically condensed MDOF model with respect to the full 3D FEM model (modal analysis, displacement and drift responses);
- comparison between internal forces returned by the full 3D FEM model, considering time history analyses and the application of static deformations, related to maximum drift time instants of the MDOF model.

6.1 Modal analysis

In Figure 5 modal shapes are reported, related to both fixed base (FB) and the base isolated (ISO) configurations, returned by the full 3D FEM and the MDOF models; for the analysis of the isolated structure, the isolation system has been linearly modeled, by considering the secant stiffness at the design displacement of the device.
Graphical results show overlapped modal shapes for all modes: thus, the computed condensed stiffness and mass matrices seem to fairly capture the same dynamic properties of the full 3D FEM model. In order to have a comprehensive comparison, in Table 3 vibration period and modal participating mass ratio values are listed.

### Table 3: Modal parameters comparison.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fixed-base structure</th>
<th>Base-isolated structure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FEM</td>
<td>MDOF</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; mode</td>
<td>T [s] 1.000, Mₚ [%] 83.88</td>
<td>T [s] 1.000, Mₚ [%] 84.12</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; mode</td>
<td>T [s] 0.292, Mₚ [%] 12.41</td>
<td>T [s] 0.293, Mₚ [%] 12.38</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt; mode</td>
<td>T [s] 0.151, Mₚ [%] 2.10</td>
<td>T [s] 0.152, Mₚ [%] 3.50</td>
</tr>
<tr>
<td>4&lt;sup&gt;th&lt;/sup&gt; mode</td>
<td>T [s] -</td>
<td>T [s] -</td>
</tr>
</tbody>
</table>

Vibration periods are exactly the same between the analyzed models, and also modal participating mass ratios are very well approximated for all modes.

### 6.2 Displacement response

In Figure 6 the comparison of the displacement response is reported.

Figure 5: Results – Modal shapes comparison.

Figure 6: Results – Displacement response comparison (Event r3)
Actually, time series returned by the condensed MDOF and the full 3D FEM models are compared for all levels of the structure, by considering the seismic event which leads to the highest displacement demand at the isolation level ($r_3$). Displacement time histories are referred to the centre of mass location for all the floors. As can be noted, for the whole duration of the considered seismic event the displacement time series are approximately overlapped, at all levels of the structure. In Figure 7 variation percentages of the maximum displacement demand at all levels between the MDOF and the full 3D FEM model are reported.

At all levels variation percentages are lower than 10% for all the considered seismic events: precisely, both positive and negative variations have been noticed, and consequently the proposed MDOF model is expected to averagely capture the displacement response of the structure, within acceptable tolerances.

### 6.3 Inter-storey drift response

Thus, since the floor displacement response is fairly described by the MDOF model, the interstorey drift time histories have been analyzed, in order to consider the behavior of the superstructure only, which is more significantly influenced by the definition of the presented static condensation procedure. In Figure 8 the interstorey drift time series are shown at all levels of the superstructure, by considering again the third record, which leads to the highest displacement demands, whereas in Figure 9 variation percentages related to maximum drift at all levels are provided, for all records. The comparison of the time histories reveals a very good agreement between the response of the statically condensed model with respect to the full 3D FEM model, hence, the computed stiffness matrix is effectively able to reproduce the real behavior of the structure at all levels, even though a single translational degree of freedom is accounted for each storey.
Concerning variation percentages, at some location a maximum variation can be detected (+13% or -15%); on the other hand, if an average variation is considered among the applied seismic events, very low values can be obtained.

Being the interstorey shear a direct function of the interstorey drift response at all levels, and given the shown previous results, the evaluation of the static deformed shapes through the MDOF model and the application to the FEM model is expected to provide good estimations of maximum internal forces of the full NLTH analyses.
6.4 Internal forces

In this section the internal forces evaluation is analyzed. For both beams and columns, a comparison of internal forces diagrams for the most stressed element at all levels is provided, by considering the time instant which returns the maximum drift value. Actually, the MDOF model is used for the evaluation of the instant at the numerical maximum drift and the consequent computation of the static deformed shape to be applied to the full 3D FEM model; on the other hand, the reference case diagram is obtained from the time history of the full 3D FEM model, at the time instant which maximizes the drift value. In Figure 10 the comparison is reported for the most stressed beams at all levels, in terms of shear force and bending moment diagrams (seismic event r3).

As can be noted, the application of the static deformed shape defined according to the MDOF model response leads to exactly the same internal forces diagrams of the full time history of the FEM model, at the time instant related to the peak drift value for all levels of the superstructure. Such results have been expected, since a very good agreement between MDOF and full 3D FEM models have been previously noticed and discussed from the displacement response point of view; thus, the main expectation is that the application of the static deformation defined according to the MDOF model can also provide the best estimation of the...
maximum internal forces for the whole duration of each considered seismic events. In order to validate this aspect, in Figure 11 variation percentages between the maximum internal forces of the full time histories and the applications of the static deformed shapes are provided.

![Figure 11: Results – Percentage variation of maximum shear and moment responses for beams](image)

Variation percentages look very low at all level (generally lower than ±10%) and averagely null among the considered seismic events. Only in event r4 an exceptionally higher variation has been found (+17%); however such a case represents an overestimation of the bending moment, if the static deformation defined with the MDOF model is applied, which provide a precautionary estimation.

Same comparison is provided for internal forces in the most stressed column of the building: in Figure 12 the internal forces diagrams are compared for record r3 whereas in Figure 13 variation percentages are provided for the estimation of the peak values.

![Figure 12: Results – Column maximum internal forces comparison at levels P1 (bottom), P2 (middle) and P3 (top) (Event r3)](image)
Diagrams are almost overlapped for all internal forces, respectively axial load, bending moment and shear force. The maximum discrepancies can be detected for shear diagram at level 2, where the shear force is overestimated (approximately +15%) by the application of the static deformation procedure. Nonetheless, a very good agreement has been noticed.

Axial load variations are approximately null at all levels, even though small values can be found in some cases (max +3%). Concerning shear force and bending moment much higher variation percentages can be found (within -27% and +20% for shear at level 3, within ±20% for bending moment); however the average variation among the selected seismic events looks small. Such discrepancies could be due to the comparison between internal forces returned by the application of the static deformation of the MDOF model, in comparison to the maximum absolute value of the full time history analysis: probably a separated study of both maximum positive and minimum negative drift for all levels could lead to better results, and minor variation percentages could be found.

**7 CONCLUSIONS AND FUTURE DEVELOPMENTS**

In this endeavor a static condensation procedure is presented, in order to carry out Non-Linear Time History Analyses on a base isolated structural system. Precisely, the definition of a statically condensed stiffness matrix is provided, which is able to reproduce the actual dynamic behavior of a considered structure, just by considering a Multi Degree of Freedom (MDOF) oscillator. One horizontal translational degree of freedom has to be defined at the centre of
mass of each floor of the system, with a single value of storey mass. In order to compute the stiffness matrix all the floors have to be initially restrained at the correspondent centre of mass. The i-th column of the stiffness matrix is then obtained by unrestraining the i-th storey and by applying a horizontal force (which leads to a horizontal displacement at the same location) and to reaction forces at all the other levels of the structure: the numerical values of the components of the i-th column are then calculated by dividing the vector which contains reactions and the applied force by the displacement originated at the i-th floor. A case study structure has been considered, in order to validate the proposed procedure. A comparison of modal analyses returned by the statically condensed MDOF and the full 3D FEM models is provided: overlapped modal shapes have been computed between the considered models, in addition to approximately same period and modal participating mass ratio values for all modes of both fixed-base and base-isolated configurations. Then a spectrum-compatible set of ground acceleration time series has been adopted in order to carry out Non-Linear Time History Analyses on both the MDOF and full 3D FEM models, and a comparison of the displacement and interstorey drift responses has been provided for all levels of the system. The analysis of displacement and drift time series has been shown for the record which leads to the highest seismic demands, and a very good agreement has been found, with almost overlapped signals. Then, variation percentages have been analyzed, for all the selected records and variations within ±10% and ±15% for displacement and drift respectively have been found. For the computation of internal forces, the MDOF model has been used to determine static deformed shapes to be applied to the FEM model, which are able to return maximum internal forces computed by the full non-linear time history analysis, so that run time can be significantly reduced. Different deformed shapes have been chosen in order to individually maximize internal forces for all levels of the building, by considering the time instants correspondent to the maximum absolute value of the interstorey drift. Diagrams are directly compared for record r3, whereas variation percentages are provided for all records. Such a procedure leads to fairly good estimation of internal forces for the most stressed elements of the structure, with slightly higher variations for columns, concerning shear force and bending moment (averagely ±25%). As discussed in the results sections, variations could be reduced if individual time instants for the definition of the static deformed shapes are considered, for minimum and maximum drift at all levels. Further research has to be carried out, in order to validate the proposed procedure by considering structural systems with irregular plan development [13], and by considering bi-directional motions, with two individual ground acceleration components.

REFERENCES


CONSEQUENCES OF MECHANICAL PROPERTIES VARIABILITY OF SEISMIC ISOLATION SYSTEMS ON THE STRUCTURAL RESPONSE OF BUILDINGS

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Abstract

Structural vulnerability of buildings can be significantly reduced if seismic isolation systems are implemented at the base of the overall system. Thanks to the low stiffness characteristics, reduced forces are experienced during a seismic event, whereas displacements are limited by the provided energy dissipation of the installed devices. In the design phase, for the assessment of the response of a considered structure, deterministic mechanical parameters are generally assumed, which correspond to the mean values of the real experimental response. On the other hand, outcomes of several testing campaigns have revealed non negligible variability of the main response characteristics of isolators, which may lead to unexpected behaviors of the isolated structure.

In the present endeavor a parametric study has been performed, in order to evaluate the consequences of the variability of the main mechanical characteristics of the implemented devices on the response of a base-isolated structure. Concave Surface Slider devices have been considered: results have led to preliminary guidelines for the correct assumptions in the design phase of isolation devices in real applications.

Keywords: Response variability, parametric study, Isolation devices, base isolation, non-linear time history analysis.
1 INTRODUCTION

Base isolation represents a very effective solution for the seismic protection of both building and bridge structural systems. Research works in the recent past have allowed the definition of a number of extremely realistic modeling strategies, for the most common isolation devices, and consequently the uncertainty of the structural behavior prediction is reduced. Thanks to a higher period of the system, forces in the superstructure can be significantly limited; even though high displacement demands are lumped at the isolation level, lower values can be achieved, by gaining the hysteretic dissipative properties of the implemented devices. One of the most efficient isolator is represented by the Concave Surface Slider device [6, 7, 12]. Such an isolator allows to accommodate large displacements, with potentially high dissipative capacities, provided by the frictional response of the sliding motion; in addition recentering contributions are experienced, thanks to the stepwise projection of the applied vertical load, with respect to the curved sliding surfaces. A number of both research and commercial testing campaign [2, 4] have highlighted several dependencies of the frictional properties on some important parameters, such as sliding velocity, vertical load and temperature or repetition of cycles [8, 10, 11, 13]. Such issues lead to variations of the friction coefficient value during motions: the velocity effect generally increases the friction coefficient, as the velocity increases; the vertical component of a seismic event, together with overturning effects may lead to non-negligible variations of the vertical load/contact pressure at the sliding interfaces, and lower (pressure increase) or higher (pressure decrease) values of the stepwise friction coefficient can be found; in addition, repetition of cycles and the increase of temperature at the sliding material cause the continuous decrease of the friction coefficient, and consequently higher displacement demands can be obtained. However, recent studies have shown that frictional properties can not be considered as deterministic quantities, as generally happens in the common practice, but a certain distribution has to be assumed [3]. Precisely, the ratio between friction coefficient and the correspondent design value can be considered as a random variable, described by a Gaussian’s (normal) distribution, with unitary mean and 30% of coefficient of variation, for the most common sliding materials. Such a variability can lead to unexpected structural responses, if not properly modeled with bound analyses, even though guidelines for upper and lower bounds are not available.

In this work the influence of the variability of the frictional properties on the seismic response of a case study structure has been assessed, by means of Non-Linear Time History Analyses, as ruled by the Italian Building code. A spectrum-compatible set of seven records has been adopted, in order to analyze the mean value for all the quantities, and up to 70'000 numerical simulations have been performed (10’000 for each seismic event). Results have provided useful information about the distributions of the main response parameters of the base-isolated system (displacement, drift, isolation force and building base shear responses), as a direct consequence of the assumed variability of the frictional properties of the implemented devices.

2 CASE STUDY STRUCTURE

For the analysis of the influence of the variability of mechanical properties of Concave Surface Slider devices on the main response parameters, a three storey building has been analyzed: precisely, a reinforced concrete frame structure has been implemented, by means of linear-elastic frame elements and a concrete slab at the interface level between the superstructure and the isolation system, through shell elements (Figure 1).
Four spans along both x and y directions are designed, 6m long, and an interstorey height of 3m has been considered. At the ground level of the building, a reinforced concrete slab represents the interface between the building itself and the isolation system: the thickness is 500mm and plan dimensions have been obtained by considering the plan development of the superstructure, increased by 1.5m along all sides (27m x 27m). Flexural stiffness coefficients have been reduced by means of scale factors (lower than 1), in order to fit the linear-elastic branch of the bi-linear approximation of the capacity curve. The non-linear capacity curve has been obtained through a pushover analysis carried out by using the software SeismoStruct [14], which allows to model force-based frame elements, with distributed plasticity and non-linear constitutive laws for materials (Mander’s model for Concrete and Menegotto-Pinto’s model for reinforcement steel); furthermore, it is possible to account for confinement effects in the concrete core of columns, in order to consider the effective ductility and strength of the sections. Isolation devices are represented by Concave Surface Slider devices. Since a large number of analyses have to be performed, the overall system has been reduced to a Multi Degree of Freedom oscillator, by applying a static condensation procedure to the full 3D FEM model of the structure. In Table 1 mass and stiffness matrices are reported.

<table>
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<td>90295.5</td>
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<td>141894.8</td>
</tr>
</tbody>
</table>

Table 1: Mass (left [tons]) and stiffness (right [kN/m]) condensed matrices for the case study structure.

As can be noted, the stiffness matrix is a full matrix, with non-null components: this result is a direct consequence of the adopted static condensation procedure, which allows to account for the actual flexural behavior in the out-of-plane direction for beams, together with torsional behavior of each floor. It has to be noted that the stiffness matrix does not account for the force response of the isolation system, since it has been separately modeled by using an actual hysteretic constitutive law, which has been discussed in the description of the adopted dynamic system.

3 MECHANICAL PROPERTIES OF ISOLATORS

Concave Surface Slider devices have been considered for the base-isolation system of the present study. In the common practice mean values of the mechanical properties of isolation de-
vices are considered in the design phase, and no variability is generally assumed. In some case, bound analyses are performed, in order to evaluate variability of the main response parameters, by assuming the mean, upper and lower bounds of the isolators parameters, even though no guidelines are provided, for the definition of such limits. For CSS devices the friction coefficient variability has been studied in recent research works and the distribution shown in Figure 2 can be generally adopted for the most common sliding materials [3].

Precisely, a Gaussian distribution can be assumed, with mean value equal to 1 and a coefficient of variation equal to ±30%. Thus, for all the analyses of the present study, the numerical value of the ratio between the actual and the design friction coefficients has been randomly extracted by a numerical simulator, according to the assumed probability density function, and then the variability of the main response parameters of the overall system has been studied.

4 DEFINITION OF THE DYNAMIC SYSTEM

The considered base-isolated structure has been considered as an equivalent multi degree of freedom (MDOF) oscillator [1], with statically condensed stiffness matrix. Such a stiffness matrix is able to reproduce the actual dynamic properties of the system, and approximately same results of a full time history analysis on a 3D FEM model can be obtained. Thus, the dynamic system can be expressed as follows:

$$\bar{M} \cdot \ddot{\bar{u}} + \bar{K} \cdot \bar{u} = -\bar{M} \cdot \ddot{x}_g$$

(1)

Being:

- $\bar{M}$ the condensed mass matrix of the system;
- $\bar{K}$ the condensed stiffness matrix of the system;
- $u_i$ the translational degrees of freedom at the centre of mass location of the i-th floor;
• $\bar{x}_g$ the considered ground acceleration time series;
• $\langle F_{is} \rangle$ the isolation force response.

No additional viscous damping matrix has been accounted for, since the non-linear hysteretic behavior of the isolation system already provides hysteretic damping, in order not to underestimate the structural response. The isolation system has been modeled as a single device, equivalent to the whole set of isolators, by means of the following equations:

$$\langle F_{is} \rangle = W_{tot} \cdot \left( \frac{u_0}{R_{eq}} + k_{\mu} \cdot \mu \cdot f_{NF} \right)$$

Being:
• $W_{tot}$ the structural weight of the system;
• $R_{eq}$ the equivalent radius of curvature of the device (3m);
• $u_0$ the translational degree of freedom at the centre of mass location of the concrete slab (isolation level);
• $\mu$ the design friction coefficient (5%);
• $f_{NF}$ a normalized frictional hysteretic parameter;
• $k_{\mu}$ a variability scale factor of the friction coefficient.

The hysteretic parameter $f_{NF}$ allows to model the frictional hysteretic response of the isolation system, by assuming an elasto-plastic rule (Figure 3).

![Figure 3: Normalized friction force.](image)

In order to consider the actual variability of the friction coefficient, in agreement to what highlight by recent research studies, a variability scale factor has been adopted: such a scale factor for each analysis is returned by a numerical simulator which follows the probability density function reported in Figure 2, and represents the ratio between the real friction coefficient and the design value. By considering a large number of simulations for each analyzed seismic input, it is possible to obtain a significantly robust set of data, and statistical analyses can be performed. In the results sections, the variability of the main response parameters has been evaluated, for the mean value of each quantity, within a set of spectrum-compatible seismic events. For each record, 10'000 simulations have been performed, and for each set analysis friction coefficients are scaled by different scale factors, in order to consider the most general cases.
5 SEISMIC INPUT

According to the Italian Building Code 2018 [5], a spectrum-compatible set of ground acceleration time series have been analyzed, in order to study the consequent variability of the mean response, by assuming the friction coefficient as a random variable, instead of a deterministic quantity. Natural records have been adopted, and scaled, in order to obtain better agreement of the single event response spectrum with respect to the target one, provided by the code. Precisely, the following seismic hazard parameters have been assumed:

- Construction site: L’Aquila
- Soil class: C;
- Topographic category: T1;
- Limit state: Collapse Limit State (return period: 975 years).

Records have been selected through the software REXEL [9], which allows to obtain spectrum-compatible set of seismic events, according to several standard codes. Thus, scale factors bounded between 0.5 and 2 have been considered, in order not to obtain unrealistic ground motion time series, in terms of frequency and amplitude. The mean spectrum has been bounded between 90% and 130% of the target one, as ruled by the standard code, in a period range between 0.15sec and 120% of the isolation period (upper bound: 3.0 sec). In Figure 4 results of the spectrum-compatibility study are reported.

![Figure 4: Spectrum-compatibility graphical results.](image)

In Table 2 the selection of the adopted records is listed.
With a selection of at least 7 records, the Italian Building Code allows to consider the mean response as reference quantities in order to check the structural behavior of the designed system.

6 RESULTS

In this section results are provided, for the main response parameters of the base-isolated system. Firstly, reference cases have been computed, which corresponds to the time integration of the dynamic system, by considering each selected record, and assuming the design value of friction coefficient (5%), as a deterministic quantity. Then results have been considered as distributions of mean response parameters, by considering 10’000 numerical simulations of the friction coefficient value for each record of the set, according to the assumed probability density function: consequently, 70’000 non-linear time history analyses have been performed, and 10’000 mean quantities for each parameter have been obtained. Results have been normalized with respect to the correspondent value of the reference case, so that the actual variability about the mean value can be directly evaluated. Special attention has been focused on displacement, interstorey drift, isolation and base shear forces responses.

6.1 Reference case

The reference case has been initially studied, by considering the adopted spectrum-compatible set, and the design value of the friction coefficient. In Figure 5 and Figure 6 displacement and interstorey drift responses are reported for each level of the structural system, whereas in Figure 7 the force response of both the isolation system and the building base are provided.
Thanks to the good level of spectrum-compatibility of the selected records, variability of the single-event quantity with respect to the mean value is very limited, and consequently comparable loading conditions are ensured among the applied seismic events, especially concerning the drift response at all levels.
Both the isolation system and the building base force response variation about the mean reference case looks even lower, in comparison to displacement variability. The building average base shear is significantly lower (63%) of the strength of the building (Figure 1), and consequently the linear elastic response of the superstructure is ensured.

### 6.2 Mean displacement response

In Figure 8 the mean displacement response is analyzed.
For each set analysis, records have been applied to the dynamic system, by assuming different scale factors for the friction coefficient, up to 10'000 simulations per event (total number of NLTHA: 70'000). Then, mean displacements for all cases have been normalized by the reference case value. The first aspect which can be noted from graphical results is that the bar plot of the mean displacement response suggests a log-normal distribution as the best probability density function, for all levels of the system. Thus, parameters have been obtained, by computing the mean and the standard deviation of the natural logarithmic of the mean displacement response, divided by the reference case value and the corresponding curve has been overlapped. The mode (most frequent value), median and mean of the obtained distributions are close to one, as expected. Variability looks to slightly decrease, if higher levels of the structure are considered, and standard deviations value are significantly lower than the assumed value for the friction coefficient variability (±30%); it has to be noted that the standard deviation can be considered as the coefficient of variation (c.o.v.) since, the mean value is close to 1 for all cases. For the present case study, variability of the friction coefficient seems to have a smaller influence on the displacement response, in comparison to the assumed c.o.v. of the frictional properties of the devices.

6.3 Mean inter-storey drift response

In Figure 9 the mean interstorey drift distributions are provided.

Interstorey drift mean responses show symmetric distributions, which can be assumed as Gaussian’s probability density functions, as well as the adopted variability of the friction coefficient. As can be noted, also for drift responses the variability looks lower than the coefficient of variation of the frictional properties of the implemented devices: this may imply also lower variability for internal forces of all structural elements of the frame structure, since drift responses are directly related to internal forces. In addition, variability looks to slightly increase if higher floors of the building are analyzed.

6.4 Mean Isolation system & Building Force responses

Finally, in Figure 10 results for mean isolation system and building force responses are shown.
Alberto Pavese and Marco Furinghetti

Figure 10: Mean isolation system and building force responses.

The mean isolation force distribution presents a very low variability, and graphical results suggest that the best probability density function is represented by a log-normal distribution. On the other hand, the mean building base shear variability can be fully described by a Gaussian’s distribution, and a slightly higher coefficient of variation can be detected.

6.5 Summary of variability results

In order to summarize all the obtained results, Table 3 provide the main characteristics of the defined distributions.

<table>
<thead>
<tr>
<th>Distribution</th>
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<th>Std</th>
</tr>
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<td>1.022</td>
<td>1.022</td>
</tr>
</tbody>
</table>

Table 3: Summary of variability parameters.

Results of the presented case study show that the best distribution for all the response quantities, which are directly affected by the influence of the isolation system (displacements and isolation force), can be considered as log-normal probability density function, whereas the a normal (Gaussian’s) distribution can be adopted for building parameters (interstorey drift and building base shear). Since the isolation system is equipped with Concave Surface Slider devices, a fixed lower bound of the force response can be considered, which corresponds to the
single contribution of the recentering force, with no frictional behavior: this aspect leads to a non-symmetric distribution of mean quantities. On the other hand, concerning the building parameters, a symmetric distribution can be assumed, since no lower or upper bound of the building response can be defined. For normal distributions mode, median and mean values coincide, whereas individual values are defined for log-normal distributions: nonetheless, all values are very close to 1.0, as expected, since the reference case provide the most frequent results. Standard deviation for all the considered quantities are lower than the assumed value of the frictional properties of isolators: thus, results seems to suggest that even though properties of the implemented CSS devices are considered as random variables with non negligible variability, the mean response is not significantly affected. However, much more research has to be carried out on the topic.

7 CONCLUSIONS AND FUTURE DEVELOPMENTS

In the present endeavor the variability of the seismic response of a case study structure has been numerically assessed, by assuming a probability density function for the friction coefficient of the implemented Concave Surface Slider devices. Precisely, a three storey reinforced concrete frame structure has been considered, and CSS isolation devices with equivalent radius of curvature equal to 3m and 5% of friction coefficient have been modeled, by means of an actual non-linear constitutive law. The Superstructure has been implemented as a Multi Degree of Freedom oscillator, with mass and stiffness matrices obtained by a static condensation procedure, which returns exactly the same dynamic properties of the system. No additional viscous damping has been modeled, since the hysteretic rule of the isolation system already provide the hysteretic damping to the overall structure. Non-Linear Time History Analyses (NLTHA) have been performed, by applying a spectrum-compatible set of natural seismic events, according to the Italian Building Code 2018, and the variability of mean values of the main response parameters has been studied. The friction coefficient has been multiplied by a variability scale factor, according to the initially assumed distribution for frictional properties: such a parameter, in all simulations, has been randomly extracted by a numerical simulator, and 10’000 analyses have been performed for all records (total number of NLTHA: 70’000). Results have been normalized with respect to the value correspondent to the reference case, which has been obtained by considering design values of the mechanical properties for the isolation devices, as deterministic quantities. Thanks to the fairly good definition of the spectrum-compatibility of the considered set of records, variability of the single-event parameters with respect to the mean is very low, so that same loading conditions can be assumed for all the seismic events. Results have shown much lower variability in the considered mean response parameters, with respect to the assumed coefficient of variation of the friction coefficient (±30%). In addition, for all parameters directly affected by the isolation system (displacement and isolation force) the best distribution is represented by the log-normal probability density function; on the other hand, response quantities related to the building behavior have a normal (Gaussian’s) distribution, as well as the assumed probability density function for the frictional properties of isolation devices. Such an aspect could be due to the definition of the force response of the isolation system, which has actually a fixed lower bound, corresponding to the recentering contribution only, with no frictional force: this leads to non-symmetric distributions.

The present study has to be extended, and much more research has to be carried out, in order to generalize the drawn conclusions, by considering several cast study structures. Furthermore,
also other isolation technologies have to be investigated, such as Rubber Bearings and Lead Rubber Bearings, so that possible discrepancies in the structural behavior among the isolations prototypes may be highlighted, and specific guidelines can be drawn for practitioners, for the execution of bound analyses.

REFERENCES


SEISMIC RELIABILITY-BASED DESIGN OF HARDENING STRUCTURES EQUIPPED WITH DOUBLE SLIDING DEVICES

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Abstract

This study deals with seismic reliability-based design (SRBD) relationships in terms of behavior factors and displacement demands for hardening structures equipped with double friction pendulum system (DFPS) bearings. An equivalent 3dof system having a hardening post-yield slope is adopted to describe the superstructure behavior, whereas velocity-dependent laws are assumed to model the responses of the two surfaces of the DFPS. The yielding characteristics of the superstructures are defined for increasing behavior factors in compliance with the seismic hazard of L’Aquila site (Italy) and with NTC18 assuming a lifetime of 50 years. Considering several natural seismic records and building properties under the hypothesis of modelling the friction coefficients of the two surfaces of the DFPS as random va-
riables, incremental dynamic analyses are performed to evaluate the seismic fragility and the seismic reliability of these systems. Finally, seismic reliability is evaluated and seismic reliability-based design (SRBD) curves for the two surfaces of the double sliding devices are described.

**Keywords:** behavior factor, ductility demand, friction pendulum bearing, post-yield hardening stiffness, seismic isolation, seismic reliability.
1 INTRODUCTION

A very effective technique for the seismic isolation[1] of building frames and infrastructure is represented by the sliding pendulum bearings [2]-[3] examined by several literature studies (e.g., [4]-[7]). Probabilistic analyses and reliability-based analyses have also been presented by [8]-[9] as well as reliability analysis and reliability-based optimization of base-isolated systems including the main uncertainties have been performed by [10]-[14]. A non-dimensionalization of the motion equations governing the dynamic response of equivalent two-degree-of-freedom (2dof) models equipped with friction pendulum system (FPS) isolators has been proposed by [15]. In the hypothesis that the friction coefficient and the earthquake main characteristics are the relevant random variables, seismic reliability analyses of a 3D base-isolated r.c. system have been developed in Castaldo et al. [16] and Palazzo et al. [17] to propose a method useful to design the isolator dimensions in plan. The life-cycle cost analysis (LCCA) of a r.c. 3D structure isolated by FPS bearings has been examined by [18] to evaluate the dependence on increasing isolation degrees. The approach for a seismic reliability-based design (SRBD) of elastic systems isolated by FPS has been generalized in Castaldo et al. [19] for a wide range of structural properties. A robustness analysis in reliability terms of a r.c. 3D building frame isolated by FPS devices in presented in [20] proposing the failure scenarios if a malfunction affects a seismic device together with the design solution. The literature studies of [21] and [22] proposed, respectively, the optimal values of the friction coefficient, on the one hand, as a function of the system properties and of the soil condition in order to minimize the superstructure response and, on the other, as a function of the ground motion characteristics by means of the ratio PGA/PGV (peak ground acceleration/velocity). In [23], a robust design optimization (RDO) of base isolation system considering random system parameters characterizing the structure, isolator and ground motion model, is performed by minimizing the weighted sum of the expected value of the maximum root mean square acceleration of the structure as well its standard deviation. In [24], an optimal design of frictional devices is proposed by applying a Pareto-type optimization approach.

The seismic performance of bridges or structures isolated with FPS or DFPS has been investigated in [25]-[31]. Specifically, [28]-[30] provide useful relationships, according also to experimental results, for the evaluation of the seismic response of structures isolated by DFPS together with the equations governing the dynamic behaviour of these devices. The principal benefit of the DFPS bearing is its capacity to accommodate substantially larger displacements compared to a traditional FP bearing of identical plan dimensions as discussed in [28]. In [26] and [31], the seismic performance of isolated bridge and liquid storage tanks are respectively investigated, considering different combinations of radii of curvature and friction coefficients.

As for the design of base-isolated systems under strong earthquake events, seismic code provisions [32]-[36] are based on low values of the strength reduction factor [32]-[36] or behavior factor [33]-[34] to ensure a safety level against the non linear dynamic amplification phenomenon (partial resonance) [37]. Precisely, NTC18 [34], Eurocode 8 [33] and the Japanese building code [35] provide a maximum behavior factor value of 1.5, without explicitly distinguishing between the ductility and overstrength factor terms, ASCE 7 [32] prescribes a value equal to 0.375 times the one for corresponding fixed-base systems and no larger than 2. In this context, Vassiliou et al. [38] obtained that the displacement ductility demand of the inelastic base-isolated structure is 3 times the strength reduction factor confirming that, for base-isolated structures, it is not possible to adopt the formulas relating the strength reduction factor R and the displacement ductility demand μ of Newmark and Hall [39] and of Miranda and Bertero [40]. Then, seismic reliability-based relationships between the ductility-dependent strength reduction factors and the displacement ductility demand, respectively, for
equivalent perfectly elastoplastic and hardening structural systems equipped with FPS depending on the structural properties have been proposed in [41],[42].

Inspired by [41],[42], this study proposes reliability-based design regressions relating the behavior factors and the displacement ductility demands for hardening structural systems equipped with double friction pendulum system (DFPS) devices and considering a high seismic hazard site like L’Aquila (Italy). By means of an equivalent 3dof system, different elastic and inelastic structural system properties are investigated. Specifically, the yielding characteristics of the hardening superstructures are designed in compliance with the life safety limit state and with the seismic hazard of L’Aquila site (Italy) assuming a lifetime of 50 years and increasing behavior factors [32]-[35]. The model developed by [4] is used to describe the non-linear velocity-dependent behavior of the two surfaces of the DFPS. The study is also based on the hypothesis of assuming the both friction coefficients of the two surfaces of the DFPS and the characteristics of the records as the relevant random variables. In detail, appropriate Gaussian probability density functions (PDFs) are adopted to characterize the aleatory uncertainties of the both sliding friction coefficients and, by means of the Latin Hypercube Sampling (LHS) method [43]-[45], the input data have been generated.

Then, several incremental dynamic analyses (IDAs) are performed for increasing seismic intensity levels in compliance with the site seismic hazard to derive the seismic fragility curves related to the different degrees of freedom of the equivalent (3dof) system. Finally, by means of the convolution integral between the fragility curves and the seismic hazard curves of L’Aquila site (Italy), in the hypothesis of a design life of 50 years for the equivalent base-isolated systems, the corresponding reliability curves are derived.

2 EQUATIONS OF MOTION FOR NON-LINEAR HARDENING STRUCTURAL SYSTEMS WITH DOUBLE CONCAVE SLIDING BEARINGS

The equivalent model, herein employed and depicted in Fig. 1, is a 3dof system with a dof representative of the superstructure behaviour and two dofs representative of the responses of the two surfaces of the DFPS. The model takes into account the inelastic hardening response of the superstructure and non-linear behaviours of the two surfaces of the DFPS [28].

Regarding the free body diagram of the DFPS, the bearing restoring force, considering only the horizontal component of the displacement on each surface, is:

$$f_{b,1} = \frac{W_1}{R_1} u_{b,1} + \mu_{d,1} W_1 \text{sgn}(\dot{u}_{b,1})$$

$$f_{b,2} = \frac{W_2}{R_2} u_{b,2} + \mu_{d,2} W_2 \text{sgn}(\dot{u}_{b,2})$$

(1)
where \( W_1 = (m_b + m_s)g \) is the weight on the upper surface (surface 1) of the bearing, \( W_2 = (m_b + m_s + m_d)g \) is the weight on the lower surface (surface 2) of the bearing, \( g \) is the gravity constant, \( R_1 \) and \( R_2 \) are the radii of curvature of the two surfaces of the device, \( u_{b,1} \) denotes the displacement of the surface 1 with respect to the slider, \( u_{b,2} \) represents the slider displacement with respect to the ground as well as \( \mu_{d,1} \) and \( \mu_{d,2} \) are the sliding friction coefficients of the two surfaces and \( \text{sgn} \) is the signum function of the sliding velocity for each surface. In this study, the upper surface (surface 1) is characterized by higher values of the friction coefficient and of the radius of curvature. Specifically, \( \mu_{d,1} \) is selected as \( 4\mu_{d,2} \) and \( R_1 = 2R_2 \) [28]-[31]. The force of the bearing coincides with the force of each surface response \( f_b = f_{b,1} = f_{b,2} \). For each surface, the friction coefficient is given as a function of the sliding velocity [4]-[6]:

\[
\mu_{d,i} = f_{\max,i} - (f_{\max,i} - f_{\min,i}) \exp(-\alpha\dot{u}_{b,i}) \quad \text{for} \quad i = 1,2
\]

where \( f_{\max,i} \) and \( f_{\min,i} \) are, respectively, the friction coefficients at high and very low sliding velocities of the i-th surface, \( \alpha \) is a constant set equal to 30 as well as the ratio \( f_{\max,i} / f_{\min,i} \) equal to 3 for each surface [15]-[21],[41].

A bilinear constitutive law describes the inelastic hardening behaviour of the superstructure, which responses in elastic phase if Eqn.(3) is satisfied and the restoring force \( f_{s,i} \) is expressed by Eqn.(4):

\[
|u_{s,i} - u_{0,i-1}| < y(u_{s,i})
\]

\[
f_{s,i}(u_{s,i}) = k_s(u_{s,i} - u_{0,i-1})
\]

where \( f_{s,i} \) is the restoring force at time instant \( i \), \( u_{s,i} \) is the superstructure deformation at the same instant, \( u_{0,i-1} \) is the maximum plastic excursion at time instant \( (i-1) \) and \( k_s \) is the elastic stiffness of the superstructure. The function \( y(u_{s,i}) \) is the yielding condition in function of the displacement and is non-univocally defined due to the translation of the elastic domain [46]. Defining \( u_y \) as the yield displacement, whose yield force is \( f_y \), \( H \) denotes the ratio between the hardening post-yield and the elastic stiffness [47]-[48], evaluated as:

\[
H = S = \frac{k_y}{k_s}
\]

The superstructure response is plastic if Eqn.(6) is satisfied and the restoring force applies according to Eqn.(7):

\[
|u_{s,i} - u_{0,i-1}| \geq y(u_{s,i})
\]

\[
f_{s,i}(u_y) = k_s(u_{s,i} - y(u_{s,i})) \text{sgn}(u_{s,i} - u_{0,i-1})
\]

Therefore, the equations which describe the response of an inelastic 3dof system, isolated by DFPS devices, to the seismic input \( \dot{u}_s(t) \), without any viscous property for the DFPS, are:
\[ m \ddot{u}_s + (m_s + m_b) \ddot{u}_{b,1} + (m_s + m_b + m_d) \ddot{u}_{b,2} + \frac{W_s}{R_2} u_{b,2} + W_s \mu_s \text{sgn} \left( \dot{u}_{b,2} \right) = -(m_s + m_b + m_d) \ddot{u}_s \]

\[ m \ddot{u}_s + (m_s + m_b) \ddot{u}_{b,1} + (m_s + m_b) \ddot{u}_{b,2} + \frac{g}{R_1} u_{b,1} + (m_s + m_b) g \mu_s \text{sgn} \left( \dot{u}_{b,1} \right) = -(m_s + m_b) \ddot{u}_s \]

\[ m \ddot{u}_s + m \ddot{u}_{b,1} + m \ddot{u}_{b,2} + c \ddot{u}_s + f_s(u_s) = -m \ddot{u}_s \]

where \( m_s \), \( m_b \) and \( m_d \) are respectively the mass of the superstructure, of the isolation level and of the slider, \( c_s \) is the viscous damping factor of the superstructure. Dividing Eqn.(8a) by \( m_s + m_b + m_d \) as well as Eqn.(8b) by \( m_b + m_s \) and Eqn.(8c) by \( m_s \), defining the mass ratios as 
\[ \gamma_s = m_s / (m_s + m_b + m_d) \quad \gamma_b = m_b / (m_s + m_b + m_d) \quad \gamma_d = m_d / (m_s + m_b + m_d) \] [49], the isolation \( \omega_{b,i} = \sqrt{g/R_i} \) and structural \( \omega_s = \sqrt{k_s/m_s} \) circular frequency, the structural damping ratio \( \xi_s = c_s / 2 m_s \omega_s \), the non-dimensional equations apply:

\[ \gamma_s \ddot{u}_s + (\gamma_s + \gamma_b) \ddot{u}_{b,1} + \ddot{u}_{b,2} + \alpha_s g \mu_s \text{sgn} \left( \dot{u}_{b,2} \right) = -\ddot{u}_s \]

\[ \gamma_s \ddot{u}_s + (\gamma_s + \gamma_b) \ddot{u}_{b,1} + (\gamma_s + \gamma_b) \ddot{u}_{b,2} + (\gamma_s + \gamma_b) \omega_b^2 u_{b,1} + (\gamma_s + \gamma_b) g \mu_s \text{sgn} \left( \dot{u}_{b,1} \right) = -(\gamma_s + \gamma_b) \ddot{u}_s \]

\[ \ddot{u}_s + \ddot{u}_{b,1} + \ddot{u}_{b,2} + 2 \omega_s \xi_s \ddot{u}_s + a_s(u_s) = -\ddot{u}_s \]

where \( a_s(u_s) = f_s(u_s) / m_s \) is the dimensionless force of the superstructure that depends, respectively, on the stiffness \( k_s \) in the elastic phase and on the yielding condition in the plastic phase. Note that the elastic isolation period of vibration varies if the sliding movement occurs along surface 1 or surface 2 or along the both surfaces simultaneously [30]. Specifically, if the sliding movement is developed along only a surface, the isolation period depends only on the radius of curvature of the spherical surface \( R_i \) (i.e., typically the radius of the surface with the lower friction coefficient) and the bearing behaves like a simple FPS [19], whereas when the both surfaces are involved, the isolation effective period applies [30]:

\[ T_b = 2 \pi \sqrt{\frac{R_1 + R_2}{g}} \]

The change of the vibration period shows the adaptive behavior to the seismic intensity that characterizes these devices [28]-[30]. It follows that the ratio between the variable isolation period and structural period of vibration, which defines the seismic isolation degree [52] cannot be a constant during an earthquake event. Moreover, when the both surfaces slide simultaneously the restoring force of the DFPS device can be evaluated as \( \mu_s W_t \) neglecting the mass of the slider [28], where \( \mu_s \) is the effective sliding coefficient given by:

\[ \mu_s = \frac{\mu_{s,1} R_1 + \mu_{s,2} R_2}{R_1 + R_2} \]

2.1 Inelastic properties of the superstructure

The inelastic behavior of the superstructure is assumed as an equivalent sdof system [50]-[51] having a hardening post-yield stiffness. The behavior factor, \( q \), and displacement ductility, \( \mu \), are defined, respectively, as:

\[ q = \frac{f_{s,el}}{f_s} = \frac{u_{s,el}}{u_s} \]
where $f_{s,el}$ and $u_{s,el}$ are, respectively, the peak elastic response values required to the superstructure, whereas $u_{s,max} = \upsilon_{s}(t)|_{t = \text{max}}$ is the peak inelastic displacement during a ground motion. This behaviour factor $q$ is consistent with the codes [32]-[36] and with the study [42].

3 UNCERTAINTIES RELEVANT TO THE PERFORMANCE ASSESSMENT

For the seismic reliability assessment of a building frame, within the structural performance (SP) evaluation method [53]-[55], specific correlations between the SP levels [56] and appropriate exceeding probabilities during its design life [57]-[58] as well as the relevant (aleatory and/or epistemic) uncertainties with the corresponding PDFs have to be defined. According to the PEER-like modular approach [59] and performance-based earthquake engineering (PBEE) approach [60]-[61], distinguishing the aleatory uncertainties related to the seismic input intensity from those corresponding to the characteristics of the record by means of an intensity measure (IM), this work evaluates and quantifies the seismic reliability of hardening systems equipped with DFPS, located in L’Aquila site (Italy), assuming also the friction coefficients as other relevant random variables. Other aleatory uncertainties are not modelled since their effects can be neglected as discussed in [41],[62]. The epistemic uncertainties are not considered in this study. Specifically, a Gaussian PDF truncated on both sides to 2% and 6% with a mean equal to 4% for the upper surface ($\mu_{d,1}$) and a Gaussian PDF truncated on both sides to 0.5% and 1.5% with a mean of 1% for the lower surface ($\mu_{d,2}$) are used to model, respectively, the sliding friction coefficients at large velocities of the two surfaces of the DFPS bearings [41]-[42]. These values are in compliance with [28]-[31] and chosen in order to obtain a mean value of the effective friction coefficient equal to 3% and, so to allow a comparison with the FPS analysed in [41]-[42]. Then, using the LHS technique [43]-[45], 15 sampled couples of the friction coefficients at large velocities are defined.

As for the uncertainty on the characteristics of the seismic records (record to record variability), according to PBEE approach [60]-[61] and similarly to [41]-[42], the spectral displacement $S_D(\xi_b, T_b)$, related to the equivalent effective period $T_b = 2\pi / \omega_b$ (Eq.(10)) and to damping ratio $\xi_b$ [19],[41] is chosen as $IM$ [64]-[66]. Considering $\xi_b$ equal to zero [15],[41],[67], the corresponding IM is hereinafter denoted as $S_D(T_b)$ in the range from 0 m to 0.45 m according to the seismic hazard of L’Aquila site (Italy) [34]. The record-to-record variability is taken into account by means of 30 ground motion records, corresponding to 19 different earthquake events, selected form different national and international databases. A detailed description may be found in [41].

4 INCREMENTAL DYNAMIC ANALYSES: RESULTS AND COMPARISON

The performance of hardening systems isolated with DFPS is evaluated through incremental dynamic analyses (IDAs), considering several structural parameters combination and L’Aquila (Italy) as the reference site.

4.1 Design of the elastic and inelastic properties of the structural systems

An extended parametric analysis is carried out considering the following deterministic parameters: isolation degree $I_d$, varying between 2, 4, 6 and 8 with respect to the equivalent
effective isolated period; the equivalent effective isolation period \( T_b \), varying between 3s, 4s, 5s and 6s; the mass ratio \( \gamma_s \), assumed equal to 0.6 and 0.8 with \( \gamma_d \) equal to 0.001 and so \( \gamma_b \) equal to 0.399 and 0.199; the behaviour factor \( q \), ranging from 1.1 to 2, with a step of 0.1, according to the codes [32]-[35], and the post-yield hardening stiffness ratio \( H \), set equal to 0.03 [47]-[51]. It follows that 384 equivalent 3dof systems, with isolation damping ratio \( \xi_b \) and superstructure damping ratio \( \xi_s \), respectively equal to 0% and 2%, are properly defined. These abovementioned 384 equivalent 3dof systems derive from 32 different 3dof systems (with the different values of \( I_d \), of \( T_b \) and of the mass ratio) by modifying the behavior factor \( q \).

In the hypothesis of \( \mu_{d,1} \) and \( \mu_{d,2} \) equal to 4% and 1%, respectively, and a ratio equal to 2 between \( R_1 \) and \( R_2 \) [28]-[31], the yielding characteristics of 32 3dof elastic systems, necessary to perform IDAs, are evaluated considering the average elastic responses to the 30 seismic inputs scaled to the \( IM=0.311 \) m for \( T_b=3 \), 4, 5 s and 0.26 m for \( T_b=6 \) s (NTC18 [34]). In this way, the average values in terms of both yield strength \( f_{y,average} \) and displacement \( u_{y,average} \) of the superstructure have been computed in Matlab-Simulink [72] and, the yielding properties are finally defined for each value of \( q \), according to Eqn.(14):

\[
 u_{y,average} = \frac{f_{y,average}}{k_q} = \frac{f_{s,el,average}}{k_q} = \frac{u_{s,el,average}}{q} \quad (14)
\]

4.2 Incremental dynamic analysis (IDA) curves

This section describes the responses of the 384 equivalent 3dof hardening systems having different properties (i.e., \( I_d \), \( T_b \), \( \gamma_s \), \( q \), \( H \)) combined with the 15 sampled couples of the friction coefficients, to the 30 seismic inputs scaled to the different \( IM=S_D(T_b) \), ranging from 0 m to 0.45 m. A total number of 450 numerical analyses has been performed for each \( IM \) level and parameter combination. The isolated non-linear hardening systems are modelled in Matlab-Simulink [72], by employing the Runge-Kutta-Fehlberg integration algorithm to solve the coupled equations (Eqn.(9)) and determine the responses of each degree of freedom. The results of the non-linear IDAs have made it possible to estimate the collapsed system cases as well as the superstructure and isolation response parameters, expressed, respectively, in terms of displacement ductility demand \( \mu \) and of displacements for the DFPS (i.e., peak value for each surface or peak value of their sum computed at each time instant). These response parameters are assumed as the engineering demand parameters (EDPs) and their peak values have been fitted with lognormal distribution [15],[16]-[21],[41],[60],[67], by estimating the sample lognormal mean, \( \mu_{ln}(EDP) \), and the sample lognormal standard deviation \( \sigma_{ln}(EDP) \), or dispersion \( \beta(EDP) \), through the maximum likelihood estimation technique, to determine the 50th, 84th and 16th percentile of each lognormal PDF [15]. Note that other uncertainties as well as aftershock events [73]-[83] and the contribution provided by the infills [84]-[89] are not considered in this study.

Figures 2-7 show the IDA results corresponding to some parameters (\( I_d=2 \) and 8, \( T_b=3s \) and 6s and mainly related to \( \gamma_s \) equal to 0.6).

Fig.s 2-3 show the IDA results regarding the isolation level EDP \( u_{b,max} \), which is the peak value of the sum of \( u_{b,1} \) and \( u_{b,2} \) in each time instant, in the case of hardening structures. This
response parameter is important to design the elements and components at the isolation level and to estimate the maximum displacement of the isolation system. Therefore, the displacement $u_{b,max}$ shown in Figs. 2-3 is the maximum displacement recorded during the non-linear dynamic analysis, and generally is not concomitant with the maximum displacement recorded at each single surface. The lognormal mean of the EDP $u_{b,max}$ increases by increasing $T_b$ and $I_d$. Both the statistical values are also influenced by $\gamma$, because the isolation displacement increases for lower values of $\gamma$. Moreover, the decrease of $q$ has a slight effect on the isolation response leading to a slight increase of $u_{b,max}$, especially for higher values of $I_d$. The results are roughly in accord with the ones obtained in [42] considering a single surface FPS.

Figs. 4-5 show, respectively, the response in terms of the peak displacement of each surface of the DFPS: the displacement of the upper surface are higher than the displacement of the surface 2, characterized by a lower value of the radius of curvature.

Figs 6-7 show the IDA curves regarding the hardening superstructure EDP $\mu$. The statistical parameters of the EDP $\mu$ highly depend on $q$ because an its increase leads to a very high displacement ductility demand $\mu$. In addition, the statistical value $\mu_{ln}(\mu)$ highly increases by increasing $T_b$ but decreases for higher values of $I_d$, especially for high values of $T_b$. Moreover, with reference to the effects of $\gamma$, on the both superstructure statistics, the increase of mass ratio leads to an increase of the superstructure response.

Figure 2: IDA curves of the isolation level with $\gamma_s=0.6$ for $I_p=2$, $T_b=3$ s, $H=0.03$ (a), $I_p=2$, $T_b=6$ s, $H=0.03$ (b), $I_p=8$, $T_b=3$ s, $H=0.03$ (c), $I_p=8$, $T_b=6$ s, $H=0.03$ (d).
Figure 3: IDA curves of the isolation level with $\gamma^s = 0.8$ for $L_d=2$, $T_b=3$ s, $H=0.03$ (a), $L_d=2$, $T_b=6$ s, $H=0.03$ (b), $L_d=8$, $T_b=3$ s, $H=0.03$ (c), $L_d=8$, $T_b=6$ s, $H=0.03$ (d).

Figure 4: IDA curves of the sliding surface 1 with $\gamma^s = 0.6$ for $L_d=2$, $T_b=3$ s, $H=0.03$ (a), $L_d=2$, $T_b=6$ s, $H=0.03$ (b), $L_d=8$, $T_b=3$ s, $H=0.03$ (c), $L_d=8$, $T_b=6$ s, $H=0.03$ (d).
Figure 5: IDA curves of the sliding surface 2 with $\gamma_s = 0.6$ for \( I_d=2, T_b=3 \text{ s}, H=0.03 \) (a), \( I_d=2, T_b=6 \text{ s}, H=0.03 \) (b), \( I_d=8, T_b=3 \text{ s}, H=0.03 \) (c), \( I_d=8, T_b=6 \text{ s}, H=0.03 \) (d).

Figure 6: IDA curves of the superstructure with $\gamma_s = 0.6$ for \( I_d=2, T_b=3 \text{ s}, H=0.03 \) (a), \( I_d=2, T_b=6 \text{ s}, H=0.03 \) (b), \( I_d=8, T_b=3 \text{ s}, H=0.03 \) (c), \( I_d=8, T_b=6 \text{ s}, H=0.03 \) (d).
5 SEISMIC FRAGILITY CURVES

Defined the limit states, respectively, in terms of the radii in plan for the two surfaces of the DFPS device, \( r_i [m] \) and \( r_j [m] \), and of the displacement ductility for the superstructure, \( \mu [\cdot] \), the seismic fragility, representative of the probabilities \( P_f \) exceeding the different limit states at each level of the \( \text{IM} \), is evaluated. Tables 1-2 report, respectively, the failure probabilities in 50 years [54],[55] with the corresponding \( \text{LS} \) thresholds, related to the \( \text{LS} \)s provided by the codes [33]-[34]: the failure probability in 50 years [18],[54],[55] corresponding to the collapse \( \text{LS} \) [34] for the DFPS; whereas, the failure probability in 50 years [18],[54],[55] corresponding to the life safety \( \text{LS} \) [34] for the superstructure in compliance with the design. The limit state thresholds of Table 1 are also used to assess the fragility in terms of the overall displacement demand to the DFPS. For the both \( \text{LS} \)s, several thresholds are considered with the aim to provide reliable \( \text{LS} \) thresholds for these systems. For each parameter combination (384 equivalent 3dof systems), the probabilities \( P_f \) exceeding the different \( \text{LS} \)s at each \( \text{IM} \) level, are numerically computed and then fitted through lognormal distributions [19] with a R-square value higher than 0.8.

\[
\begin{array}{cccccccccccc}
\text{r}_i [m] & \text{for } i=1,2 & 0.05 & 0.1 & 0.15 & 0.2 & 0.25 & 0.3 & 0.35 & 0.4 & 0.45 & 0.5 \\
p_f (50 \text{ years})=1.5 \times 10^{-3} \\
\end{array}
\]

Table 1: Limit state thresholds for the two surfaces of the DFPS with the associated exceeding probability.
Table 2: Limit state thresholds for the superstructure with the associated exceeding probability.

<table>
<thead>
<tr>
<th>$\mu ; [\cdot]$</th>
<th>$LS_{\mu,1}$</th>
<th>$LS_{\mu,2}$</th>
<th>$LS_{\mu,3}$</th>
<th>$LS_{\mu,4}$</th>
<th>$LS_{\mu,5}$</th>
<th>$LS_{\mu,6}$</th>
<th>$LS_{\mu,7}$</th>
<th>$LS_{\mu,8}$</th>
<th>$LS_{\mu,9}$</th>
<th>$LS_{\mu,10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pf ; (50 ; \text{years})=2.2 \cdot 10^{-2}$</td>
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Figure 8: Seismic fragility curves of the isolation level related to $LS_{r,4} = 0.2 \; \text{m}$, for $I_d = 2$, $T_b = 3 \; \text{s}$, $H = 0.03$ (a), $I_d = 2$ and $T_b = 6 \; \text{s}$, $H = 0.03$ (b), $I_d = 8$ and $T_b = 3 \; \text{s}$, $H = 0.03$ (c), $I_d = 8$ and $T_b = 6 \; \text{s}$, $H = 0.03$ (d).

Fig.s 8-11 depict the fragility curves (i.e., the exceeding probabilities $P_f$ (complementary distribution functions (CCDFs))) versus the $IM$ for hardening. Precisely, the curves corresponding to the different structural properties of interest and related only to some $LS$ thresholds ($LS_{r,4}$ and $LS_{\mu,3}$) and to $I_d = 8$ and $T_b = 3 \; \text{s}$, are represented. Generally, the seismic fragility of each degree of freedom decreases for increasing the corresponding $LS$ threshold.

For the all limit states, the exceeding probabilities slightly increase for decreasing $\gamma_s$. Then, especially for high limit state thresholds, the fragility decreases by decreasing $T_b$, $I_d$ and increasing $q$. Note that the probability exceeding a limit state is quite low for the single surface, with a lower probability for the surface 2 characterized by a lower friction coefficient with a lower radius of curvature, in compliance with the IDA results.

The fragility curves of the nonlinear hardening superstructures are shown in Fig. 11. The exceeding probabilities are slightly higher as $\gamma_s$ increases but highly increase for increasing values of $q$. 

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Figure 9: Seismic fragility curves of the sliding surface 1 related to $LS_r = 0.2$ m, for $I_p = 2$, $T_b = 3$ s, $H = 0.03$ (a), $I_p = 2$ and $T_b = 6$ s, $H = 0.03$ (b), $I_p = 8$ and $T_b = 3$ s, $H = 0.03$ (c), $I_p = 8$ and $T_b = 6$ s, $H = 0.03$ (d).

Figure 10: Seismic fragility curves of the sliding surface 2 related to $LS_r = 0.2$ m, for $I_p = 2$, $T_b = 3$ s, $H = 0.03$ (a), $I_p = 2$ and $T_b = 6$ s, $H = 0.03$ (b), $I_p = 8$ and $T_b = 3$ s, $H = 0.03$ (c), $I_p = 8$ and $T_b = 6$ s, $H = 0.03$ (d).
Conversely, higher values of $T_b$ for fixed $I_d$ lead to a decrease of the seismic fragility because an increase of the period $T_s$ means an increase of the correlated yielding displacement as well as lower values of $T_b$ for fixed $I_d$ lead to higher values of the seismic fragility. In fact, the coupling between $I_d$ and $T_b$ is a very important parameter because it defines $T_s$ and the corresponding yielding displacement. Therefore, as also discussed in [41]-[42], in the case of systems with low $T_s$, the ensuing dynamic amplification can cause disproportioned superstructure responses and, so a high seismic fragility derives. These resonance effects, are reduced with respect to the outcomes of [41] in the case of the post-yield hardening stiffness ratio. Comparing the result with the outcomes of [42], the hardening systems have slightly higher probability exceeding a limit state for higher value of $T_b$, and the influence of $\gamma_s$ is more marked.

6 SEISMIC PERFORMANCE OF INELASTIC STRUCTURES WITH DFPS

The convolution integral between the previously achieved seismic fragility curves and the seismic hazard curves expressed in terms of the same IM, $S_D(T_b)$, related to the reference site (L’Aquila (Italy)), allows the evaluation of the mean annual rates exceeding the limit states for each parameter combination. Then, by using a Poisson distribution, the seismic reliability of the all hardening structures isolated by DFPS in the time frame of interest (e.g., 50 years) have been computed. In this work, the seismic hazard of L’Aquila site (Italy), soil class B,
with geographic coordinates 42°38’49’’N and 13°42’25’’E, has been considered, as widely described in [41].

As for the DFPS devices, the seismic reliability evaluation makes it possible to define SRBD curves to design the dimensions in plan of each surface of these devices and the overall dimension of the isolation level as a function of the expected reliability level and of the structural properties.

Fig. 12 depicts the linear regressions representative of the seismic reliability of the overall dimension of the isolation level, in the semi-logarithmic space. The value of R-square is higher than 0.9. These curves show that an exceeding probability of $P_f = 1.5 \cdot 10^{-3}$ (corresponding to the collapse LS, in 50 years) [54]-[57] requires a global dimension ranging from about 0.3 m to about 1 m depending on the structural properties. The overall dimension of the isolation level estimated with the above described curves, can also be useful to define the radius in plan of each surface of the DFPS. In fact, SRBD curves of each surface, evaluated and not represented due to space constrains, highlighted that around 1/3 of the global dimension can be attributed to the surface 2 (having a lower friction coefficient with a lower radius of curvature) and 2/3 to the surface 1 for low $T_b$, whereas for high $T_b$, these ratios become 1/4 and 3/4, respectively. This aspect is a very important design feature because if high displacements are required to the isolation level are divided between the two sliding surfaces reducing the geometric encumbrance of the itself device and of the structural elements directly connected as also highlighted in [28]-[30].

Fig. 13 shows the results, representative of the SP curves of the hardening superstructure in 50 years, in the logarithmic scale for the different LS thresholds in terms of $\mu$ and for the dif-
ferent structural properties. The seismic reliability of the superstructure increases for low values of $\gamma_s$, $I_d$, $q$ and for high values of $T_b$. Comparing the results with the outcomes achieved by [42], the seismic reliability of systems equipped with DFPS, with different friction coefficients for the two surfaces, is slightly lower respect the systems equipped with simple FPS due to the reasons previously explained for the fragility assessment.

![Seismic reliability curves](image)

Figure 13: Seismic reliability curves of the superstructure related to $L_d=2$, $T_d=3$ s, $H=0.03$ (a), $L_d=2$ and $T_d=6$ s, $H=0.03$ (b), $L_d=8$ and $T_d=3$ s, $H=0.03$ (c), $L_d=8$ and $T_d=6$ s, $H=0.03$ (d). The arrow denotes the increasing direction of $q$.

7 CONCLUSIONS

This study describes the seismic reliability-based performance of hardening structural systems equipped with double concave sliding devices isolators on varying the elastic and inelastic building properties, seismic intensity levels with the hypothesis of the friction coefficients and of the characteristics of the seismic records assumed as the relevant random variables. By means of an equivalent 3dof system with a non-linear velocity-dependent model for the two surfaces of the DFPS, incremental dynamic analyses are carried out considering several natural seismic records, the seismic hazard of L’Aquila site (Italy), increasing behavior factors and different post-yield stiffness ratios. Then, the seismic fragility curves are derived for the hardening superstructure and for the isolation level taking also into account the dynamic failure cases. After that, assuming a design life of 50 years, seismic reliability-based design (SRBD) curves are proposed useful to design the radii in plan of the two surfaces as well as the maximum demand to the DFPS. The results have highlighted the negative effects of the post-yield stiffness as well as the possibility to reduce the encumbrance of the devices and of the struc-
tural elements directly connected because the seismic demand is divided on the two surfaces. This aspect is a very important design feature of the DFPS representing an its advantage.

Regarding the non-linear hardening superstructures, the seismic reliability-based linear regressions demonstrate the positive influence of the overstrength hardening properties able to reduce the displacement ductility demand. From the analysis of the seismic reliability-based results, an upper values of 1.5 for the behaviour factor is compatible with all the structural properties for hardening systems with DFPS. The proposed SRBD curves can be useful for a preliminary and reliable design or verification of regular frames, not sensitive to the $P-\Delta$ effects, equipped with DFPS and located in a high seismic hazard area.

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ANALYSIS OF THE INFLUENCE OF VISCOUS DAMPERS PROPERTIES VARIABILITY VIA RELIABILITY-BASED OPTIMIZATION METHOD

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Abstract

The response of viscous devices is notably influenced by the values assumed by the two constitutive parameters governing the damper behaviour, namely: the viscous coefficient $c$ and velocity exponent $\alpha$. At this regard, devices manufacturing process may introduce some level of variability on the expected nominal values of $c$ and $\alpha$. In order to control the damper response deviation from the nominal conditions, the main international codes provide some acceptance tolerance criteria to be checked during prototype tests. However, no admissible ranges of variability are directly specified for the damper constitutive parameters. In light of this, the aim of this study is that of analyzing the sensitivity of the seismic risk of structural systems with respect to uncertain damper properties, in order to assess the effectiveness of the tolerance criteria provided by the international seismic regulations in controlling the structural performance parameters variation. An innovative approach [1] based on the solution of a reliability-based optimization (RBO) problem is used, in which the optimization is carried out to search for the worst combination of the uncertain damper parameters, within the allowed range of variation, that maximize a probabilistic objective function expressing the variation of the mean annual frequency of exceedance of some relevant performance demand thresholds. A moment resisting steel building with linear and nonlinear viscous dampers is analysed. The study results show that the considered source of variability can have non-negligible effects on the structural seismic response, with some EDPs showing higher sensitivity than others.

Keywords: Reliability-based optimization, Viscous Dampers, Uncertain Viscous Parameters, Seismic risk, Structural engineering
1 INTRODUCTION

Seismic reliability analysis of structural systems requires the characterization of the uncertainty in the seismic input as well as in the structure geometrical and mechanical properties. The propagation of such uncertainties allows the estimation of the structural failure probability, which is typically expressed as the probability of exceeding specified levels of the monitored response parameters [2,3].

It is well known that the seismic input uncertainty significantly influences the seismic risk [4–7]. On the other hand, the effect of uncertain structural properties must be evaluated on a case-by-case basis [7–10]. Isolated structures or structures equipped with dampers are an example of systems whose performance may be significantly affected by uncertainties other than the seismic one, because their seismic response depends mainly on the characteristics of a few number of devices. Furthermore, seismic isolation and energy dissipation devices have properties which can vary significantly as a function of manufacturing process, time, temperature, load history, strain-rate and velocity, among others [11], thus differing from the nominal ones considered for the design.

This paper concerns fluid viscous dampers (FVDs), a class of dissipation devices effective in increasing the damping capacity of structural systems [12,13] and thus widely used as seismic vibration control strategy.

The response of viscous devices is notably influenced by the values assumed by the two constitutive parameters governing the damper behaviour, namely: the viscous coefficient $c$ and velocity exponent $\alpha$. In this regard, it is worth noting that devices manufacturing process may introduce some level of variability on the expected nominal values of $c$ and $\alpha$. In order to control the damper response deviation from the nominal conditions, the main international codes [14,15] provide some acceptance tolerance criteria to be checked during prototype tests. However, no admissible ranges of variability are directly specified for the damper constitutive parameters, and moreover as explicitly declared in ASCE/SEI 41-13 [14] (Section C14.3.2.4, Upper- and Lower-Bound Design and Analysis Properties) and implicitly assumed by EN 15129 [15] (Section E.2, Design Requirements), the effects of dampers’ uncertainty are only considered on the viscous coefficient $c$, while the exponent $\alpha$ is assumed as fixed. Such assumption (variability on $c$ only) is not justified in either of the two aforesaid seismic codes, and furthermore, in ASCE/SEI 41-13 [14] (Section C14.3.2.4) it is explicitly written that: “The authors recognize that much of the data needed to rationally develop the required individual factors for energy dissipation devices do not exist at the time of writing”.

In light of the aforementioned gap in the regulatory framework, and given the scarce literature available on this topic [1,10,16], the aim of this study is that of analyzing the sensitivity of the seismic risk of structural systems with respect to uncertain damper properties (including the variability of $\alpha$), in order to assess the effectiveness of the tolerance criteria provided by the international seismic regulations in controlling the structural performance parameters variation.

To this aim, an innovative approach recently proposed in [1] is used to assess the seismic risk sensitive with respect to uncertain parameters for which a statistical characterization is not available [17,18]. This approach exploits a reliability-based optimization (RBO) tool, in which the optimization is carried out to search for the worst combination of the uncertain damper parameters, within the allowed range of variation, that maximize a probabilistic objective function expressing the variation of the mean annual frequency of exceedance of some relevant performance demand thresholds.

A benchmark case study is assumed, consisting of a 3-storey steel building [19] equipped, alternatively, with linear and nonlinear viscous dampers. The seismic risk sensitivity to the dampers properties uncertainty is thus analysed by solving the RBO problem for different
global and local engineering demand parameters (EDPs) such as: interstory drifts, absolute accelerations, forces and strokes on dampers.

The study results show that the considered source of variability can have non-negligible effects on the structural seismic response, with some EDPs showing higher sensitivity than others.

Moreover, this work, along with some previous studies made by the authors [1,3], provide useful insights for a future revision of the reliability factors proposed by the current seismic codes, whose values could be better calibrated in order to properly account for the tolerances allowed on the devices’ constitutive parameters.

2 SEISMIC RISK ESTIMATION METHOD

Seismic design and assessment of structures aim at ensuring acceptable levels of failure probability. With reference to ordinary civil structures, conventional limit values of mean annual frequencies (MAFs) of exceedance are provided, vary from 1·10⁻² yrs⁻¹ for serviceability limit states to 1·2·10⁻³ yrs⁻¹ for ultimate limit states, while safety checks against collapse should be oriented to ensure a mean annual failure rate lower than 10⁻⁵·10⁻⁶.

In this work, a rigorous approach is used for the seismic performance assessment, which explicitly provide the seismic demand hazard function \( v_d(d) \), expressing the MAF of exceeding different values of a global or local EDP relevant to the performance of the analyzed system. In evaluating \( v_d(d) \), all the sources of uncertainty involved in the problem shall be accounted for.

For what concerns the systems equipped with viscous dampers, it may be convenient to consider two separate vectors for describing these uncertainties: \( X \in \Omega \) is the vector collecting the random variables representing the ground motion and the structural system uncertainties, which can be described by assigning a probability density function, and \( \theta = [\theta_1, ..., \theta_i] \in \Gamma \) is the vector collecting the viscous damper’s parameters (i.e., \( c \) and \( \alpha \) of all the devices installed within the structure), for which a probabilistic model is not available. These parameters, assumed independent from \( X \), are hereinafter referred to as design variables (DVs), and the corresponding nominal values are denoted as \( \theta_0 \).

In particular, if \( x \) denotes the realization of \( X \), and \( d(x|\theta) \) denotes the generic demand, conditional to a given combination of model parameters \( \theta \), the corresponding demand hazard function \( v_d(d|\theta) \) can be estimated via direct probabilistic approach [20], i.e., by solving the following reliability integral:

\[
\nu_d(d|\theta) = \nu_{min} \int_{\Omega} I_d(x|\theta) p_X(x) dx
\]  

(1)

where \( p_X(x) \) is the joint probability density function (PDF) of \( X \), and \( I_d(x|\theta) \) is an indicator function, such that \( I_d = 1 \) if \( d(x|\theta) > d^* \), otherwise \( I_d = 0 \). The multiplicative term \( \nu_{\text{min}} \) is the MAF of occurrence of a seismic event of any significant magnitude [21].

The main drawback of direct simulation methods is represented by the high number of numerical analyses required for achieving confident estimates, in particular at the lower exceedance probabilities. To overcome with the aforesaid issue, conditional (IM-based) methods have been developed in the last 20 years, starting from the seminal work of [22]. The main purpose of these methods is to make seismic risk estimation a more practice-oriented and affordable task.

According to this approach, the reliability problem of Eq. (1) can be solved through the following convolution integral [23]:

\[
\int_{\Omega} I_d(x|\theta) p_X(x) dx = \int_{\Omega} I_d(x|\theta) p_X(x) dx
\]
where, \( v_{IM} \) denotes the hazard curve of an Intensity Measure (IM) used for describing the ground motion intensity at the site of the structure, while \( G_{DIM}(d|\theta, im) \) expresses the probability of exceeding the demand value \( d \), conditional to \( \theta \) and to the seismic intensity level \( im \), and can be estimated by assuming a lognormal distribution for describing the response \( d \) for each value of \( \theta \) and each IM level.

In this study, a hybrid probabilistic approach is used, based on a conditional method for evaluating the seismic demand at different seismic intensity levels, and on the use of Subset Simulation [20] for defining the seismic hazard. More precisely, a stochastic ground motion model is considered, and Subset Simulation (SS) is employed to derive the IM hazard curve, \( v_{IM}(im) \), up to very small rates of exceedances. SS also provides a set of stochastic ground motion samples conditional to different non-overlapping IM intervals, which are considered to build the conditional seismic demand model via multiple-stripe analysis (MSA) [24].

Such hybrid probabilistic method has the advantage of having the high computational efficiency of a conditional approach and at the same time the accuracy of a robust direct simulation approach (i.e., Subset Simulation or a plain Monte Carlo analysis).

3 RBO METHOD APPLIED TO VISCOUS DAMPERS

3.1 Probabilistic objective function

The influence of the uncertain parameters \( \theta \) on the system seismic demand hazard is investigated by means of the constrained reliability-based optimization (RBO) method recently proposed in [1]. According to this approach, the objective function \( f_{Obj} \) that must be maximized is expressed as the variation of the MAF of exceedance of an EDP value \( d^* \), normalized with respect to the MAF of exceedance corresponding to the reference design values \( \theta_0 \)

\[
f_{Obj}(\theta, d^*) = \frac{v_D(d^*|\theta) - v_D(d^*|\theta_0)}{v_D(d^*|\theta_0)}
\]

In order to specialize the objective function of Eq. (3) to the case of viscous dampers with uncertain constitutive parameters, the force-velocity constitutive law [25] for this type of dissipation device is reported below

\[
F_d(\dot{u}) = c|\dot{u}|^\alpha sgn(\dot{u})
\]

where \( \dot{u} \) is the velocity between the device’s ends, \( F_d \) is the damper resisting force, \(|\dot{u}|\) is the absolute value of \( \dot{u} \), \( sgn \) is the sign operator, \( c \) and \( \alpha \) are two constitutive parameters: the former is a multiplicative factor, while the latter describes the damper nonlinear behaviour.

Eq. (3) consequently becomes

\[
f_{Obj}(c, \alpha, d^*) = \frac{v_D(d^*|c, \alpha) - v_D(d^*|c_0, \alpha_0)}{v_D(d^*|c_0, \alpha_0)}
\]

where \( c = [c_1, \ldots, c_n] \) and \( \alpha = [\alpha_1, \ldots, \alpha_n] \) denote the perturbed viscous constitutive properties of \( n \) fluid viscous dampers, whose values are allowed to vary in compliance with the code-based constraints defined in the next subsection, while \( c_0 \) and \( \alpha_0 \) identifies the nominal values of the same parameters. It is worth clarifying that \( c \) and \( \alpha \) are design variables within this optimization framework.
3.2 Code-based constrains

The main international seismic codes \[14,15\] acknowledge that the manufacturing process is characterized by some uncertainty affecting the viscous constitutive parameters, whose actual values might differ from the nominal ones used in the design. To cope with such uncertainty, some acceptance criteria are provided. In particular, the ASCE/SEI 41-13 \[14\] and the European code EN 15129 \[15\] require that the maximum experimental force \(F_d(\dot{u})\) exhibited by the damper tested under harmonic displacement time-histories, deviates from the expected (design) value, \(F_d^*(\dot{u})\), by no more than a tolerance \(p\) within a range of velocities \(v\) spanning from zero to the maximum design one \(\dot{u}^*\). This requirement can be formulated in terms of the following inequality:

\[
(1 - p)F_d^*(\dot{u}) \leq F_d(\dot{u}) \leq (1 + p)F_d^*(\dot{u}), \quad 0 \leq \dot{u} \leq \dot{u}^*
\]

where \(p = 15\%\) according to the abovementioned seismic standards.

The condition expressed by Eq. (6) is consequently assumed as nonlinear constrain within the RBO problem, which can be finally formulated as follows,

\[
\begin{align*}
\text{max}_{c, \alpha} \quad & f_{\text{obj}}(c, \alpha, d^*) = \frac{v_D(d^*|c, \alpha) - v_D(d^*|c_0, \alpha_0)}{v_D(d^*|c_0, \alpha_0)} \\
\text{subject to} \quad & |(c_{0,i} + \hat{c}_i) \hat{u}(\alpha_{0,i} + \hat{\alpha}_i) - c_{0,i}v^{\alpha_{0,i}}| \leq 0.15c_{0,i}\hat{u}^{\alpha_{0,i}} \quad \forall \quad 0 \leq \dot{u} \leq \dot{u}^* \quad (i = 1, ..., n)
\end{align*}
\]

in which the constraint is expressed in terms of damper forces within the range of velocity from 0 to \(\dot{u}^*\) (maximum design velocity).

The RBO framework defined above allows to find the worst combination of design variables (\(c\) and \(\alpha\) allowed to vary while satisfying the acceptance criterion) leading to the highest increments of the MAF of exceedance for the reference monitored performance measure.

To better explain the physical meaning of the constraint defined in Eq. (7) (which leads to a not straightforward link between the dampers force variations and the perturbed viscous parameters), an example is provided in Figure 1, showing the normalized force-velocity relations of a nonlinear damper with design nominal parameters \((\alpha_0, c_0)\) (black solid line). On the same figure, the upper and lower bounds of the allowed response variability are illustrated as well (red solid lines), corresponding to the case with viscous coefficient variations \(\hat{\alpha} = +/−15\%\) and \(\hat{\alpha} = 0\). Moreover, the varied response curves obtained for two specific pairs of admissible perturbed parameters combinations \((\hat{\alpha}, \hat{\alpha})\) are superimposed: the dashed blue curve represents the maximum admissible positive variation of the exponent \(\alpha\) corresponding to the condition in which the normalized response variation attains the upper bound value (i.e., \(F_d/F_d^* = 1.15\) at the normalized design velocity (i.e., \(\nu/v^* = 1\)); the dotted blue curve represents the maximum admissible negative variation of the exponent \(\alpha\), corresponding to the condition in which the response variation attains the lower bound value (i.e., \(F_d/F_d^* = 0.85\)) at the design velocity (i.e., \(\nu/v^* = 1\)). Such combined variations of the viscous properties are both complying with the tolerances for velocity values lower than the design one (\(\dot{u}/\dot{u}^* = 1\) in the normalized axis), as required by the code \[14\]. However, it is worth noting that, for velocity values beyond the design one (i.e., \(\dot{u}/\dot{u}^* > 1\)), the perturbed force assumes values outside the upper/lower bounds (red solid lines), and the specific trend depends on the sign of \(\hat{\alpha}\), which governs the rate of change of the nonlinear response, with non-homogeneous effects along the range of velocity.
3.3 Details on the optimization tool

The RBO problem of Eq. (7) is solved by employing a nested double-loop approach [1,21], consisting of an outer loop, where optimization is carried out, and an inner loop which is used at each iteration to evaluate $v_D(d^*|\theta)$. This solution approach is computationally very expensive, particularly because the inner loop must be invoked many times before converging to the optimal solution. This aspect motivated the use of the hybrid approach discussed above, which is efficient to achieve accurate estimates of the $v_D(d^*|\theta)$ while limiting the number of simulations.

For what concerns the optimization solver, the COBYLA (Constrained Optimization By Linear Approximation) gradient-free optimization algorithm is used, developed by Powell (2007) [26] and implemented in OpenCossan [27]. Further details on COBYLA can be found in Powell (2007) [26] and Altieri et al. (2017) [21].

4 CASE STUDY

4.1 Seismic scenario

The Atkinson-Silva (2000) [28] source-based ground motion model for California region is used to characterize the seismic hazard at the building’s site. The seismic scenario is described by two seismological parameters, the moment magnitude $M$, and the source-to-site (epicentral) distance $r$, which are modelled as random variables. The magnitude is characterized by the Gutenberg-Richter probability density function $f_M(m)$ bounded within the interval $[m_0, m_{\text{max}}] = [5, 8]$ and with parameter $\beta=2.303$; the epicentral distance is modelled under the hypothesis that the source produces random earthquakes with equal likelihood anywhere within a distance from the site $r_{\text{max}} = 50$ km, beyond which the seismic effects are assumed to become negligible.

The soil condition is described by a deterministic value of the shear-wave velocity parameter $V_{S30} = 310$ m/s, representative of average soil condition in the considered area [52].

Further details about the parameters of the stochastic model used in the present study can be found in [1,3].

The pseudo-spectral acceleration at the fundamental period $T_1 = 1.0$ s, $Sa(T_1)$, is assumed as IM, and the corresponding hazard curve is provided by a single-run of Subset Simulation, carried out by considering 20 simulation levels, each having a target intermediate exceedance probability $p_0 = 0.5$, and $N=500$ samples per level. Figure 2 shows the IM hazard curve with the discretization levels at which MSA analysis is performed highlighted by grey vertical lines.
Among the \( N = 500 \) ground motion time-series generated at each simulation level, a subset of 20 samples is selected for performing MSA. It is noteworthy that number of simulation levels has been defined based on an extensive preliminary parametric study [29]. Based on this study, it was concluded that considering the hybrid reliability analysis method with 21 stripes and 20 samples per IM level (for a total amount of 420 nonlinear structural analyses to be performed) allows to obtain results comparable in terms of accuracy to those provided by the application of direct Subset Simulations, requiring over 3000 simulation for each monitored EDP.

4.2 Structural system and dampers design

The case study consists of a 3-storey steel moment-resisting (MR) frame building, designed within the SAC Phase II Steel Project, and widely used as benchmark structure in several other works concerning structural response control [19]. The structural system was designed for gravity, wind, and seismic loads in order to conform to local code requirements in Los Angeles, California region.

The real structural system consists of perimeter moment-resisting frames and internal gravity frames with shear connections, while the structural model for analysis purposes is a two-dimensional frame representing one half of the structure in the north–south direction. The main geometrical details and the size of the steel members (wide-flange sections are used for both columns and beams) are shown in Figure 3. Further details concerning the structural geometry and loads can be found in [19].

The finite element model of the system is developed in OpenSees [30] following the approach described in [31] and briefly recalled below. A distributed plasticity approach is adopted, with nonlinear force-based elements and fiber sections with Steel02 uniaxial material. An elastic fictitious P-delta column (not shown in Figure 3) is introduced to account for the nonlinear geometrical effects induced by the relevant vertical loads, those carried by the inner (not explicitly modelled) gravity frames included. A corotational approach is used to perform large displacement analysis. The first fundamental period \( T_1 \) of the system is equal to 0.995 s.
The dampers are designed to achieve an enhanced building performance level according to ASCE/SEI 41-13 [14], consisting of meeting the immediate occupancy requirements at the BSE-2E seismic hazard level (i.e., with probability of exceedance equal to 5% in 50 years, corresponding to the annual rate of exceeding ν₀ = 0.001).

The dampers are placed into the structural frame (Figure 4) connected in series with steel supporting braces, and two different cases are studied: linear viscous dampers (α₀ = 1.0) and nonlinear viscous dampers (α₀ = 0.3). For the purpose of the dampers’ design, the structural performance is described in terms of a global EDP, represented by the interstory drift ratio, whose limit value at the Immediate Occupancy Limit State (IO-LS) is assumed equal to 0.01 as suggested in FEMA-350 [32] (Table 4-10 of the aforementioned code) for low-rise ordinary moment-resisting steel buildings.

Dampers’ viscous coefficients c₀ᵢ (i = 1, 2, 3 floor levels) are calibrated to reduce the mean value of the maximum interstory drift demand evaluated for a set of 7 accelerograms whose intensity (expressed in terms of IM= Sₐ(T₁)) is consistent with the reference hazard level (i.e., with ν₀ = 0.001 yrs⁻¹). The IM design value is equal to IM(ν₀) = Sₐ(T₁) = 0.77g. The subset of 7 simulated ground motion time-histories is selected from the set of samples stored during execution of Subset Simulation, and the selection criterion is such that it satisfies (without scaling) the spectrum compatibility at the building’s first period T₁. The target drift, achieved with both the linear and nonlinear dampers, is equal to 0.0097 < 0.01, corresponding to a 40% reduction with respect to the bare frame performance.

The viscous coefficients c₀ᵢ distribution along the storeys is assumed proportional to the shear distribution of the first mode of vibration. The nominal properties of the viscous coefficients are reported in Table 1 (with labels related to Figure 4) together with the axial stiffness values of the steel braces.

Assuming a S275 steel grade cold-formed profiles with the following hollow squared cross-sections are employed for the supporting braces: 350x16mm for the braces connected to linear dampers, and 300x10 mm for those connected to the nonlinear ones.

In the finite element analyses, a Maxwell element consisting of an elastic spring (with stiffness kₘ) and a viscous dashpot in series is used to model the damper-brace systems.

![Figure 4: Viscous dampers location and coefficients numbering.](image)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>α₀ = 1.00</th>
<th>α₀ = 0.30</th>
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<tr>
<td>c₀₀₁ [kN·s²/m²]</td>
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<td>2350</td>
</tr>
<tr>
<td>c₀₀₂ [kN·s²/m²]</td>
<td>7310</td>
<td>2021</td>
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<tr>
<td>c₀₀₃ [kN·s²/m²]</td>
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<td>1269</td>
</tr>
<tr>
<td>kₘ [kN/mm]</td>
<td>450</td>
<td>244</td>
</tr>
</tbody>
</table>

Table 1: Properties of dampers and steel braces.

The performance of the structure is evaluated by monitoring the following global EDPs: the maximum interstory drift among the various storeys, IDR, the maximum absolute base-shear

3316
carried by the steel frame only, $V_b$, and the maximum absolute acceleration among the various floors, $A$. The dampers performance is monitored by considering the following two local EDPs, accounting for the cost, the size and the failure of the devices: the maximum absolute force of the dampers placed at the first storey, $F_{d1}$, which carries the largest forces; and the maximum stroke, $\Delta d_1$, of the damper at the first storey.

Table 2 collects the design values $d^*$ of the five monitored EDPs, all obtained under the design seismic scenario except for the stroke and force of the dampers, whose design values are further amplified according to the rules provided in ASCE/SEI 41-13 [14]. The design values of the maximum dampers’ velocity, necessary to define the tolerance range described by Eq. (6), are $\dot{u}^* = 0.27\,\text{m/s}$ and $\dot{u}^* = 0.30\,\text{m/s}$ for, respectively, linear and nonlinear dampers.

## 5 SEISMIC RISK WITH DAMPERS WITH NOMINAL VISCOS PARAMETERS

The performance of the systems corresponding to linear and nonlinear dampers with nominal properties $(\alpha_0, i, c_0)$ is evaluated by performing MSA. The demand hazard curves of the EDPs of interest are illustrated in Figure 5a-d (with grey solid lines for linear dampers and black dashed lines for nonlinear dampers). For each curve, the reference performance point corresponding to the design condition, represented by the coordinates \( \{d^*, v_0(d^*)\} \), is illustrated with a marker (a grey circle for linear dampers and a black circle for nonlinear ones). The values of the coordinates of these points are also reported in Table 2. The values $v_0(d^*)$ are required to define the objective function of the RBO problems, since they represent the reference values (corresponding to dampers with nominal properties) with respect to which the maximum increment of the risk is evaluated. It is worth to observe that the reference MAF values estimated via probabilistic approach (i.e., MSA) are always higher than the design ones obtained through the deterministic design approach, as also observed in [31].

For an more extensive discussion of the differences observed between the response of linear and nonlinear dampers, the readers are advised to refer to [1] and [3]. In this works, indeed, the reliability of the simplified approach for damper design provided by the seismic codes is critically assessed.

<table>
<thead>
<tr>
<th>IDR</th>
<th>$d^*$</th>
<th>$v_0 \times 10^3$ [yrs$^{-1}$]</th>
<th>$d^*$</th>
<th>$v_0 \times 10^3$ [yrs$^{-1}$]</th>
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<td>$A_{dd}$</td>
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<tr>
<td>$\Delta d_1$</td>
<td>0.0509</td>
<td>0.242</td>
<td>0.0426</td>
<td>0.404</td>
</tr>
<tr>
<td>$F_{d1}$</td>
<td>3614.20 kN</td>
<td>0.137</td>
<td>1811.82 kN</td>
<td>0.316</td>
</tr>
<tr>
<td>$V_b$</td>
<td>7988.56 kN</td>
<td>1.619</td>
<td>7615.73 kN</td>
<td>1.612</td>
</tr>
<tr>
<td>$A$</td>
<td>4.65 m/s$^2$</td>
<td>1.929</td>
<td>5.49 m/s$^2$</td>
<td>1.723</td>
</tr>
</tbody>
</table>

Table 2: Design values $d^*$ and corresponding MAF of exceedance $v_0$. 

3317
6 SEISMIC RISK WITH DAMPERS WITH VARIABLE VISCOS PARAMETERs

This section investigates the influence of variable damper constitutive parameters. The design variables corresponding to the viscous parameters perturbations are collected in the vectors $\mathbf{\tilde{c}} = [\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3]^T$ and $\mathbf{\tilde{\alpha}} = [\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3]^T$, and they are assumed to vary independently among the dampers at the different storeys, while respecting the tolerance bounds of Eq. (6). This RBO problem, mathematically formulated in Eq. (7), is solved for each of the five monitored EDPs of Table 2.

The results obtained for the RBO problems performed on all of the considered EDPs are summarized in Table 3, reporting, for each EDP and for both the linear and nonlinear dampers, the combinations of the most critical percentage variations of the viscous constitutive parameters, and the corresponding percentage increment of the MAF of exceedance. The negative variations of the design variables are highlighted in red, while the positive variations in black solid font.
Based on the results reported in Table 3, the following observations can be made on the influence of variable viscous parameters:

- In four cases out of 10, the worst combinations of viscous parameters involve the velocity exponents variability ($\alpha \neq 0$). These cases are: the EDPs $F_{dl}$ and $A$ for the buildings with linear dampers, and the EDP $\Delta_{dl}$ for both the linear and nonlinear damped systems. In the remaining six cases, the RBO problem yields variations on $c$ only, thus revealing a reduced sensitivity of the monitored objective functions to the velocity exponent variability.

- The maximum admissible positive variation observed on the viscous coefficients is equal to +33.9%, corresponding to a velocity exponent variation of +11.6%. The maximum admissible negative variation observed on the viscous coefficients is equal to +27.0%, corresponding to a velocity exponent variation of -11.2%.

- The higher sensitivity is observed on the damper force demand parameter ($F_{dl}$).

- The seismic demand hazard sensitivity to viscous parameters variation is generally low for the most of the EDPs, with percentage increments of the MAF of exceedance below the 24%.

- Notably higher is the sensitivity shown by the damper force, with increments of 63.0% and 271.7% of the MAF of exceedance attained for, respectively, the linearly and nonlinearly damped building.

In light of the results obtained thus far, it is possible to state that the variability of the velocity exponent ($\alpha$), usually disregarded by seismic codes [14,15], may affect the seismic hazard of some EDPs. In particular, the study presented in this paper focuses on the effect of such variability on the seismic response variations at the design condition. However, the structural performance should be checked also at different conditions, corresponding to higher and lower values of the design MAF of exceedance.

In this regard, an extension of the study presented in the current paper can be found in [1], which goes in deep on the following two aspects: 1) the effect of the variability on the viscous coefficients only (as currently considered by the main international seismic codes [14,15]) is compared to the effect of a combined variability involving both $c$ and $\alpha$; 2) the sensitivity of the seismic risk to variations of the viscous parameters is investigated not only at the design level but over a wide range of MAF of exceedance values, from the serviceability limit state condition up to the attainment of the collapse conditions.
7 CONCLUSIONS

In this paper, the approach based on the solution of Reliability-Based Optimization (RBO) problems (proposed in Scozzese et al. [1]) is exploited for investigating the seismic risk sensitivity of structural systems with respect to uncertain viscous damper’s parameters having a bounded range of variation. In particular, a specific application is considered to investigate the influence of the variability of the constitutive parameters, $c$ and $\alpha$, of linear and nonlinear fluid viscous dampers on the seismic risk of a low-rise steel building.

According to the results of this study, the following conclusions can be drawn:

- The variability on the damper properties may lead to the increment of the seismic risk of the structural systems.
- This effect varies depending on the type of dampers (linear or nonlinear) and the specific EDP considered.
- The most sensitive EDP is represented by the damper force, whose design value may experience increments of the corresponding MAF of exceedance spanning from 63% to 272% for, respectively, linear and nonlinear dampers, because of admissible combined variations of the viscous parameters $c$ and $\alpha$.
- The effect of the variation of the damper constitutive parameters is found to be limited on the other monitored EDPs.

Moreover, the proposed method (based on the use of the RBO tool) can be employed not only to further investigate the problem of viscous dampers, but also to analyze the effect of the uncertainties of other types of seismic devices, such as hysteretic dampers or isolation bearings.

REFERENCES


Testing Requirements of Hysteretic Energy Dissipating Devices According to Italian Seismic Code

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Abstract

Anti-seismic displacement dependent devices are used to improve the dynamic characteristics of the structural system of buildings and bridges. Performance and functional requirements of energy dissipating devices are specified in most new seismic design codes. In this paper, nonlinear dissipative capacity of hysteretic dampers is evaluated by specific experimental tests and compared with the Italian code testing requirements. Quasi-static and dynamic experimental testing have been performed on U-shaped flexural steel plates (UFPs) devices. Controlled-displacement tests considering a specific loading protocol have been performed to define the cyclic behavior of UFPs. Shaking table testing considering a random sequence of seismic inputs have been carried out on a 3D, 2:3 scaled, three-storey post-tensioned timber framed building with dissipative bracing systems based on UFPs. The reliability of the testing procedures required by codes for prototype and production control tests is discussed in terms of number of cycles and of cycles to failure.

Keywords: Displacement dependent devices, Hysteretic dampers, Testing requirements, Experimental testing, Cycles to failure.
1 INTRODUCTION

Passive control systems of earthquake and wind excitations based on energy dissipation techniques have achieved significant progress for existing buildings over the last three decades. One of the most effective and inexpensive solutions to minimize damage of structures is dissipating energy through the yielding of metallic materials [1], [2]. The elements are so shaped as to have a uniform strain distribution in their parts devoted to the energy dissipation function and, then, to maximize their low-cycle fatigue resistance [3], [4]. Several metallic dampers have been developed and installed in structural systems, and their effectiveness and reliability has been widely demonstrated also for stronger earthquakes [5], [6], [7].

U-shaped Flexural Plate (UFP) device is a flexural metallic damper initially proposed and experimentally tested by Kelly et al. [8] to provide energy dissipation between structural walls and adjacent floors. UFPs have been successfully used in several structural dissipation applications [9], including for timber building such as installation in coupled walls [10], at the column base connection [11], [12] or into dissipative bracing systems [13].

The capability of UFPs to dissipate energy is due to the nonlinear behavior when stressed beyond elastic limit, moving the yielding point along the plate. In particular, when one side of UFP is subjected to a displacement relative to the opposite side, the semi-circular section rolls along the plate and work is done at two points where the radius of curvature is changed from straight to curve and vice-versa [14]. The typical nonlinear hysteretic force-displacement behavior of UFP is shown in Figure 1a [15] [16]. A large range of force and displacement levels could be reached, by varying the plate thickness ($t_u$), width ($b_u$) and diameter ($D_u$), as shown in Figure 1b.

The yielding force ($F_1$), the yielding displacement ($d_1$) and maximum force ($F_2$) provided by a single UFP (Equation 1) depends on the geometric and mechanic characteristics, and are defined as [8], [14]:

$$F_1 = \frac{2}{3} F_2 \quad d_1 = \frac{27\pi F_1 D_u^3}{16 E b_u t_u^3} \quad F_2 = \frac{f_y b_u t_u^2}{2 D}$$

(1)

where $f_y$ is the steel yield stress and $E$ is the steel elastic section modulus. The first branch stiffness ($K_1$) of single UFP can thus be defined as follows:

$$K_1 = \frac{16 E b_u}{27\pi} \left( \frac{t_u}{D_u} \right)^3$$

(2)
The seismic design requirements and hysteretic performance of displacement dependent devices are included in the Italian seismic code [15], mainly inspired by European code [16], and in most international regulations, such as the American provisions [17].

In this paper, testing procedures defined by the new Italian seismic code for prototype and factory production control tests for UFP devices designed for dissipative bracing systems are investigated. Quasi-static cyclic tests on UFP devices have been performed to characterize the force displacement behavior. Then, several shaking table tests on UFP dampers considered as dissipative bracing devices for a post-tensioned timber framed building have been performed. A sequence of almost 40 seismic inputs, composed by 7 natural earthquakes at various intensities, has been dynamically applied. The cycles to failure, i.e. the number of fatigue cycles at which the rupture of the test piece happened, has been experimentally found. The number of hysteretic cycles of UFP dampers performed at different ductility levels is estimated and compared with codes requirements.

2 CODE PROVISIONS

The current seismic codes use different empirical approaches for determining the performance characteristics of a nonlinear devices (NLD). The conformity of mechanical characteristics of NLD with the performance requirements shall be verified by the outcome of specific tests, representing the working conditions and fixings of the device.

Following the code [15] the maximum displacement $d_2$, corresponding to the design displacement ($d_{bd}$) taking into account the reliability factor $\gamma_x \cdot d_{bd}$ by [16], shall be considered for the design and analysis at the collapse prevention limit state of structures with energy dissipative systems. The mechanical tests of the devices shall be carried out by imposing cyclic deformations according to the schedule and the procedures indicated as Type Tests (TT) and Factory Production Control Tests (FPCT) of devices.

Type tests shall be carried out on at least one specimen in order to reproduce the actual working conditions of the devices up to a maximum displacement, at least equal to $\pm d_{bd}$ [16]. For type tests the Italian seismic code [15] refers to [16], increasing amplitude cycles shall be imposed at 25%, 50% and 100% of $d_{bd}$ applying five cycles for each intermediate amplitude and at least ten cycles for the maximum amplitude. Moreover, a ramp test shall be performed for the static evaluation of the failure displacement up to a displacement not less than the maximum displacement taking into account the partial factor $\gamma_b \cdot d_2$. The number of cycles imposed to the devices is related to both the duration of the earthquake and to the fundamental mode of vibration of the structure given in the design specifications [16]. Similarly, ASCE 7 [17] requires type testing on at least two prototype devices at 33%, 67% and 100% of the displacement at the maximum considered earthquake ($d_{MCE}$) applying ten, five and three cycles, respectively.

Factory production control tests of devices shall be always carried out on the devices prior to their installation on at least 20% of the supply, and not less than 4 devices [15]. Quasi-static test shall be performed imposing five cycles at the maximum displacement $d_2$ [15]. Differently, Eurocode [16] reduces the devices number to tests to at least 2% with a minimum number of one device with the same increasing amplitude cycles applied for type testing. According to ASCE7 [17], the production test shall validate the nominal properties by testing 100% of the devices for three cycles at 67% of the $d_{MCE}$ at a frequency equal to $1/(1.5 T_1)$. 
3 EXPERIMENTAL TESTING

In this section, the results of controlled displacement tests and shaking table tests performed at the structural laboratory of the University of Basilicata, are described.

The case study is an experimental model consisted of a three dimensional, 2:3 scaled, three storey timber framed building with dissipative bracing systems [1].

The specimen has been designed according to the current seismic codes [15], [18] to represent an office structure (live load of 3kPa for level I and II) with a rooftop garden (2kPa for level III). The inter-storey height is 2 m and the rectangular plan is single bay in both directions with dimensions of 4 m by 3 m. Dissipative bracing systems consisting of six dissipative braces, two for each storey, were characterized and then inserted within the bays of two parallel frames along the test direction (Figure 2a). The dissipative bracing system is composed by six V-inverted timber rods in series with two UFP (Figure 2b) dampers, made of C60 stainless steel (Figure 2c), designed for each storey to yield in a controlled manner. The design displacement of the UFPs is $d_{bd} = 26\text{mm}$, corresponding to ductility $\mu = 4$, and the maximum displacement is $d_2 = 30\text{mm}$.

3.1 Characterization tests

Quasi-static tests have been performed to characterize the hysteretic behavior of U-shaped Flexural Plates steel dampers. Three different UFPs, as representative of the device installed at each storey of the braced model (named as UFP1, UFP2 and UFP3), have been tested. A test sequence composed by increasing amplitude cycles was performed applying two cycles at different amplitudes of 10%, 25%, 50%, 75% and 100% of $d_{bd}$ and one cycle up to the amplitude of 200% of $d_{bd}$. Figure 3 shows the testing apparatus and the testing protocol of UFPs compared with the minimum number of cycles required for Type Tests (TT) by the codes (TT [NTC2018 - EN 15129] and TT [ASCE 7]).
Figure 3: Quasi-static testing setup with four UFPs in parallel and testing procedure for nonlinear devices

Figure 4 shows the test results obtained for a couple of UFPs in parallel. All tests showed a stable hysteretic cyclic behavior of UFPs without any sign of failure, over the maximum displacement.

Figure 4: Quasi-static testing results in terms of Force-displacement behavior of UFP1, UFP2 and UFP3

### 3.2 Shaking table tests

For shake table testing, a set of seven natural earthquakes was selected from the European strong-motion database. The normalized elastic acceleration and displacement spectra are reported in Figure 5. To ensure consistency with the scale of the experimental model, the recordings were scaled down in time by a factor of $1/\sqrt{(3/2)}$. In order to match the real acceleration inputs to the code spectrum it was necessary to scale four of the earthquakes by means of an appropriate scale factor, see Table 1.

<table>
<thead>
<tr>
<th>ID Code</th>
<th>Location</th>
<th>Date</th>
<th>Mw</th>
<th>PGA (g)</th>
<th>Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1228</td>
<td>Izmit, Turkey</td>
<td>17/08/1999</td>
<td>7.6</td>
<td>0.357</td>
<td>1.5</td>
</tr>
<tr>
<td>196</td>
<td>Montenegro Serbia</td>
<td>15/04/1979</td>
<td>6.9</td>
<td>0.454</td>
<td>1</td>
</tr>
<tr>
<td>535</td>
<td>Erzican Turkey</td>
<td>13/03/1992</td>
<td>6.6</td>
<td>0.769</td>
<td>1.5</td>
</tr>
<tr>
<td>187</td>
<td>Tabas, Iran</td>
<td>16/09/1978</td>
<td>7.3</td>
<td>0.926</td>
<td>1</td>
</tr>
<tr>
<td>291</td>
<td>Campano Lucano, IT</td>
<td>23/11/1980</td>
<td>6.9</td>
<td>0.264</td>
<td>1.5</td>
</tr>
<tr>
<td>4673</td>
<td>South Iceland</td>
<td>17/06/2000</td>
<td>6.5</td>
<td>0.716</td>
<td>1.5</td>
</tr>
<tr>
<td>4677</td>
<td>South Iceland</td>
<td>17/06/2000</td>
<td>6.5</td>
<td>0.227</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 - Seismic inputs selected for experimental testing
Figure 5: Selected earthquake inputs for shaking table testing and design spectra

The seismic sequence is composed by 20 seismic inputs applied by increasing the intensities from 10% to 100% of PGA. The schedule of the complete sequence is reported in Table 2. From test 1 to test 16 (blue line) shaking table tests were performed twice. Displacement potentiometers and load cells were used for recording the force-displacement behavior of the UFP dampers at each storey for both sides of the braced model.

<table>
<thead>
<tr>
<th>Test name</th>
<th>Input ID</th>
<th>PGA [%]</th>
<th>PGA [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1’</td>
<td>1228</td>
<td>10</td>
<td>0.036</td>
</tr>
<tr>
<td>Test 2’</td>
<td>196</td>
<td>10</td>
<td>0.045</td>
</tr>
<tr>
<td>Test 3’</td>
<td>535</td>
<td>10</td>
<td>0.077</td>
</tr>
<tr>
<td>Test 4’</td>
<td>1228</td>
<td>25</td>
<td>0.089</td>
</tr>
<tr>
<td>Test 5’</td>
<td>196</td>
<td>25</td>
<td>0.113</td>
</tr>
<tr>
<td>Test 6’</td>
<td>535</td>
<td>25</td>
<td>0.192</td>
</tr>
<tr>
<td>Test 7’</td>
<td>1228</td>
<td>50</td>
<td>0.178</td>
</tr>
<tr>
<td>Test 8’</td>
<td>196</td>
<td>50</td>
<td>0.227</td>
</tr>
<tr>
<td>Test 9’</td>
<td>535</td>
<td>50</td>
<td>0.384</td>
</tr>
<tr>
<td>Test 10’</td>
<td>187</td>
<td>50</td>
<td>0.463</td>
</tr>
</tbody>
</table>

* test repeated twice

Table 2 - Seismic sequence of shake table testing

Figure 6 shows the time-history of relative displacement of two UFP1 for the complete seismic sequence recorded at the first storey brace (west side). The results pointed out that the system exhibits a complete re-centering capability, without residual displacement accumulated during the tests. As can be observed, when the levels of the PGA intensity increases, the ductility demand reaches the design value (μ=4).

Figure 7 shows the force-displacement behavior of a couple of UFP1 for the case of seismic inputs 535 at different intensities (from 10% to 100% of PGA). The UFPs remain elastic up to intensities of 10% and 25% of PGA, while showed excellent hysteretic response corresponding to higher intensities (50%, 75% and 100% of PGA) without significant degradation in strength and stiffness.

In Table 3 the total number of cycles at different ductility levels, for all dynamic tests and for tests performed at 100% of PGA level, are reported with the corresponding mean and standard deviation values.

One of the UFP of the first storey (UFP1) reached the failure condition for fatigue during Test 20 (187-100% of Figure 7) occurred through fracture of the yielding section at displacement of 50 mm (μ>4). The rupture of the UFP device happened after 36 shaking table tests performed on the braced model and more than 150 cycles with ductility μ > 2 sustained by the device.
Figure 6: Time history of the displacement of UFP1 under the seismic sequence

Figure 7: Force displacement cycles of UFP1 at different intensity levels

<table>
<thead>
<tr>
<th>Ductility</th>
<th>All dynamic tests</th>
<th>Dynamic tests at PGA 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Mean</td>
</tr>
<tr>
<td>$\mu \leq 1$</td>
<td>1675</td>
<td>43</td>
</tr>
<tr>
<td>$1 &lt; \mu \leq 2$</td>
<td>121</td>
<td>9</td>
</tr>
<tr>
<td>$2 &lt; \mu \leq 4$</td>
<td>39</td>
<td>4</td>
</tr>
<tr>
<td>$\mu &gt; 4$</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3 – Number of cycles of UFP1 from dynamic tests
4 RESULTS AND DISCUSSION

Figure 8 shows the estimation of the number of cycles $N_c$ of the UFP1 as function of different ductility levels (from $\mu=1$ to $\mu>4$) of all shaking table tests (Dynamic tests) and of controlled-displacement tests (Static tests). As can be observed, the average of number of cycles (Mean Dynamic tests) decrease at increasing levels of required ductility and confirm the trend imposed by the considered procedure for characterization tests.

![Graph showing the number of cycles versus ductility for Dynamic and Static tests.](image)

Figure 8: Experimental values of number of cycles of the complete seismic sequence compared with quasi-static tests results.

In the same way, Figure 9a shows the number of cycles for the seismic inputs at 100% of PGA only. It should be noted that the number of cycles averaged on dynamic tests performed at PGA 100% (Mean Dynamic tests) reconfirm the trend imposed by procedure for static tests. In Figure 9b the number of cycles from dynamic and quasi-static tests is compared with minimum code requirements for Type Tests (TT) and Factory Production Control Tests (FPCT) of devices [16], [15], [17].

![Graph showing the comparison of number of cycles with code provisions.](image)

Figure 9: a) Experimental values of number of cycles of seismic sequence at 100% of PGA compared with quasi-static tests and b) with minimum code provisions.

For the case study, it can be observed that the procedure imposed by American recommendations (TT [ASCE 7] and FPCT [ASCE 7]) confirm the trend of the mean dynamic tests. On the contrary, the minimum number of cycles required by Italian [15] and European [16] codes (TT [NTC2018 - EN 15129]) seems to overestimate the number of cycles at the design displacement ($\mu=4$). Moreover, the minimum number of cycles required for FPCT by Italian code (FPCT [NTC2018]) at the maximum displacement ductility ($\mu \geq 4$) is two times more than the corresponding average values observed for dynamic tests at PGA 100%. 

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5 CONCLUSIONS

In this paper, the number of cycles at different ductility levels of U-shaped flexural plate steel dampers is investigated by comparing static tests with dynamic tests and current code provisions. Controlled-displacement cyclic tests were carried out on UFP devices and shaking table tests were performed on a post-tensioned timber frame building with dissipative bracing systems based on identical UFP devices.

In order to develop more reliable prototype and production control test procedures for displacement depending energy dissipation devices, based on the experimental outcomes the following comments and recommendations are reported.

The considered procedure for static tests confirms the decreasing trend of the number of cycles at increasing ductility obtained by dynamic tests, coherently with American provisions [17].

The Italian [15] and European [16] codes overestimate the minimum number of cycles required for type tests and factory production control tests at higher ductility ($\mu > 2$).

The number of devices required for factory production control testing by the Italian code should be redundant and uneconomical, especially in case of steel based dampers.

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EXPERIMENTAL AND NUMERICAL INVESTIGATION OF BASE ISOLATED SDOF SYSTEM IMPACT AGAINST BUMPERS UNDER HARMONIC BASE EXCITATION

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Abstract

During strong earthquakes, base isolated systems (buildings, bridges, strategic facilities, equipment etc.) can impact against the surrounding moat wall because of the deformation of the isolator if the available separation distance is limited. A possible mitigation measure is the interposition of shock-absorbers (bumpers). In this work some of the results of an experimental laboratory campaign are presented. The experimental tests were carried out to investigate the dynamic response of a base-isolated single-degree-of-freedom (SDOF) oscillator, excited by a harmonic base acceleration and symmetrically constrained by two unilateral deformable and dissipative bumpers. Three different peak values of table acceleration (A), four amplitudes of the total gap between mass and bumpers (G) and four types of bumpers (B) were considered. In this work the attention is focused on some characteristics of the dynamics with impact, such as force and time of contact between mass and bumpers, coefficient of restitution and energy dissipated by the bumpers during the impact. The results of the experimental tests were used to identify the parameters of stiffness and damping of a numerical model able to simulate the behavior of the system by using a general-purpose computer code.

Keywords: Shaking table test, Numerical model, Base-isolated SDOF, Bumpers, Base excitation, Impact parameters.
1 INTRODUCTION

Base-isolated structures can undergo large displacements when subjected to severe earthquakes, because of the lengthening of their fundamental periods induced by seismic isolation [1,2]. These large displacements, confined at the isolation system, can damage the isolators themselves or can lead to poundings with the surrounding moat wall or adjacent structures, if the width of the available seismic gap is not sufficient. The problem of pounding doesn’t concern only structures, but also bridges and equipment [3,4]. The consequences of such pounding, that can range from damage of acceleration-sensitive equipment housed in the structure to severe structural damage, can be mitigated by reducing the impact stiffness. This can be done, for example, by introducing shock absorbers (bumpers), made of deformable and dissipative material, between the structure and the obstacle.

The dynamic response of a nonlinear base-isolated single-degree-of-freedom (SDOF) oscillator subjected to a harmonic base acceleration and constrained by two symmetrically arranged unilateral bumpers has been numerically investigated by Andreaus and De Angelis in [5], where possible scenarios of the system’s response have been highlighted. Subsequently, the experimental response of a physical model of the SDOF has been studied in [6,7].

In this work, some of the results of an experimental laboratory campaign of tests, concerning the study of the SDOF oscillator, are presented. In particular, the evaluation of the impact parameters (time and force of contact, coefficient of restitution and energy dissipated by the bumpers during the impact) was made on the basis of the experimental data (times and velocities immediately before and after contact). Three different peak values of table acceleration $A_i$ ($i = 1, 2, 3$), four amplitudes of the total gap between mass and bumpers $G_j$ ($j = 1, 2, 3, 4$) and four types of bumpers $B_k$ ($k = 1, 2, 3, 4$) were considered.

The results of the experimental tests were also used to identify the parameters of a numerical model able to simulate the behavior of the system by using a general-purpose computer code.

The paper is organized as follows: the experimental setup is described in Sect. 2; the impact parameters, calculated from the experimental data, are reported in Sect. 3; a numerical model able to simulate the behavior of the system is presented in Sect. 4; the main conclusions are drawn in Sect. 5.

2 EXPERIMENTAL SETUP

The physical model of the SDOF system consists of a rigid body, composed of six plates of mild steel jointed through bolts, that can be treated as a lumped mass $M$, an elastomeric isolator (damper) and a couple of elastomeric shock absorbers (bumpers) symmetrically installed on steel stands which are bolted onto the base plate. The damper is bolted to the lower plate of the mass in a central position. The mass is supported by four spherical bearings, rotating within unidirectional guides (Figure 1).

The experimental campaign was performed using a shaking table, and the measured parameters were absolute accelerations and displacements of the table and of the mass.

Four different values of the distance between bumpers and mass (total gap, defined as the sum of left and right gaps), obtained by adjusting the screws at the fronts of the stands, were considered ($G_1 = 15 \text{ mm}$, $G_2 = 20 \text{ mm}$, $G_3 = 25 \text{ mm}$, $G_4 = 30 \text{ mm}$).
Four different types of bumpers were employed (B1, B2, B3, B4), with different contact lengths and obtained from three types of cross section (Figure 2).

The system was excited by a step-wise forward and backward Sine Sweep signal in displacement control, in the frequency range $f = 0.5 – 5.0$ Hz with step $\Delta f = 0.1$ Hz, in order to impose three different values of peak table acceleration ($A_1 = 0.03$ g, $A_2 = 0.04$ g, $A_3 = 0.05$ g, where g is the gravity’s acceleration), with a sufficient number of cycles to reach steady state.

3 EXPERIMENTAL EVALUATION OF IMPACT PARAMETERS

Among the various aspects investigated, in this work the attention is focused on the results obtained in the presence of bumpers which were used to identify some characteristic parameters of the dynamics with impact, such as force and time of contact between mass and bumpers, coefficient of restitution and energy dissipated by the bumpers during the impact. These parameters were evaluated from the experimental data, considering the steady-state in correspondence to the resonance frequency. In case of rigid body impact, the contact time is very small, and the contact force prevails over other involved forces [8].

By analyzing the time histories of absolute acceleration in steady-state resonance condition (Figure 3), it was possible to identify the times of beginning and end of contact (respectively $t_i$ and $t_o$), and thus calculate the duration of the contact interval:

$$\Delta t_c = t_o - t_i \quad (1)$$
Figure 3: Time histories of absolute acceleration and relative velocity in steady-state resonance condition (B4-G4-A3).

The coefficient of restitution $s$ describes the loss of kinetic energy due to wave propagation and deformation of the contact area caused by the impact. It is defined as the ratio between the relative velocities immediately after, $v_o$, and immediately before, $v_i$, the contact (Figure 3):

$$s = \frac{-v_o}{v_i}$$

It depends on material, geometry of impact bodies and relative velocities of the colliding solids [9,10]. The coefficient, so defined, varies within the range $s \in [0, 1]$. The limit values correspond to perfectly inelastic and perfectly elastic impact, respectively.

Using the impulse momentum law, with reference to the time interval $\Delta t_c$, the mean contact force, acting on the mass, can be evaluated as follows:

$$F_c = I/\Delta t_c$$

where $I$ is the change in momentum during impact:

$$I = M \left(1 + s\right) v_i$$

Finally, the energy dissipated during the impact, equal to the difference between the kinetic energies of the mass $M$ immediately before ($E_{ki}$) and immediately after ($E_{ko}$) the contact with the bumper, can be calculated from the coefficient of restitution through the following expression [11]:

$$E_d = (1 - s^2) \ E_{ki}$$

In the following figures, the trends of the impact parameters are depicted as a function of the peak table acceleration A (Figure 4), the amplitude of the total gap G (Figure 5) and the stiffness of the bumper (Figure 6). Missing values correspond to combination of the parameters A-G-B for which the impact has not occurred.
To each bumper B corresponds a color (red for B1, blue for B2, black for B3 and green for B4); to each value of total gap G corresponds a marker (square for G1, diamond for G2, triangle for G3 and circle for G4); to each value of peak table acceleration A corresponds a line style (dotted for A1, dashed for A2 and solid for A3). From Figure 4 it can be observed that all the impact parameters increase with the peak table acceleration A. While bumpers B2 and B3, and often also B4, behave in a similar way, bumper B1 exhibits quite different trends. As concerns the coefficient of restitution s, from Figure 4a it can be observed that it always assumes values lower than 1. The lowest values are those associated with the bumper B1, whereas the curves corresponding to the other three bumpers are very close to each other, especially for large values of A. Consequently, it is expected that, for those three bumpers, the influence of the total gap G on s decreases with increasing of A.

![Figure 4](image_url)

Figure 4: Impact parameters vs. A with the variation of B and G: a) coefficient of restitution; b) contact time; c) contact force; d) dissipated energy.

As concerns the contact time $\Delta t_c$ (Figure 4b), it is very short, of the order of milliseconds. The increase of $\Delta t_c$ with A is due to the fact that the higher A, the greater the penetration of the mass in the bumpers. Furthermore, in the case of bumper B4, it is worth noting that not only the contact time does not vary significantly with A, but also G does not influence too much the duration of contact because the curves, corresponding to the four values of G, are very close to each other. As the stiffness of the bumper decreases, the influence of A increases, and it is more evident the difference in $\Delta t_c$ when varying G, especially for bumper B1. Bumpers B2 and B3 exhibit values of $\Delta t_c$ very close to each other.
As concerns the mean contact force $F_c$, from Figure 4c it can be observed that it increases almost linearly with $A$. The lower values are those associated with the more deformable bumper B1, whereas the larger values are those corresponding to the stiffer bumper B4. B2 and B3 are placed in an intermediate position. As concerns the energy dissipated during impact $E_d$, from Figure 4d it can be observed that, except for some cases, it increases with $A$. The values of $E_d$ associated with the bumper B1 are always larger and differ significantly from the other bumpers, especially for large values of $A$. The curves corresponding to the other bumpers are very close to each other.

![Figure 5: Impact parameters vs. G with the variation of B and A: a) coefficient of restitution; b) contact time; c) contact force; d) dissipated energy.](image)

From Figure 5 it can be observed that also the amplitude of the total gap $G$ influences the impact parameters. The coefficient of restitution (Figure 5a) does not vary significantly with $G$, especially for large values of $A$. As concerns $\Delta t_c$ (Figure 5b), it is not significantly influenced by $G$ for bumpers B2, B3 and B4, whereas it decreases with $G$ for bumper B1. The curves associated with bumper B4 are very close to each other, indicating a small influence of $A$. This influence increases as the stiffness of the bumper decreases. The mean contact force $F_c$ (Figure 5c) does not vary significantly or increases slightly with $G$. The higher values are associated with the pair B4-A3. The energy dissipated (Figure 5d), attains value in the range 0.5-3.5 kN mm for all values of $G$. The curve corresponding to the pair B1-A3 distances itself significantly from the others and it is associated with the larger values of $E_d$. The same pair of parameters corresponds to the greater value of $\Delta t_c$. 

As concerns the influence of the stiffness of the bumper B, from Figure 6a it can be observed that, as it increases, the coefficient of restitution first increases and then it stands at a constant value or decreases. The reduction increases as A decreases. Only the curve corresponding to the pair G2-A1 gradually diverges more and more from the others as B increases. Passing from B1 to B4, that is increasing the stiffness of the bumper, the penetration of the mass into the bumper decreases. Consequently, $\Delta t_c$ decreases (Figure 6b) because the mass remains in contact with the bumper for a shorter time. It is worth noting that all the curves are gradually closer to each other as B increases. The mean contact force $F_c$ increases with the stiffness of the bumper (Figure 6c) and the larger values are associated with the higher values of peak table acceleration. As concerns the energy dissipated during the impact (Figure 6d), it is not significantly influenced by B for small values of A. With increasing of A, $E_d$ first decreases with B, with a reduction that is all the more significant the greater is A, then stands at a constant value or does not vary significantly. Finally, as previously observed, also from it can be noticed that bumpers B2 and B3 correspond more or less the same values of the parameters.
4 NUMERICAL MODEL

The results of the experimental tests were used to identify the parameters of stiffness and damping of a numerical model able to simulate the behavior of the SDOF system, by using a general-purpose computer code (Sap2000 v.20). The model, depicted in Figure 7a, consists of a mass $M$, a nonlinear isolation damper, in the following text denoted by the subscript “d”, and two bumpers, in the following text denoted by the subscript “b”.

![Figure 7: a) Numerical model; b) Trilinear constitutive law of damper.](image)

$D_d$ and $D_{bj}$ ($j = R, L$) denote the relative displacement of mass, right and left bumper respectively with respect to the table; in the following equations the superimposed dot denotes derivation with respect to time. The gap, that is the distance between the mass M and the single bumper, is denoted by $\Delta_j(t)$ ($j = R, L$). The system is subjected to a harmonic base acceleration $A(t) = A_G \sin(\Omega t)$.

The experimental and numerical results where compared considering the combination of investigated parameters B4-G4-A3. The piece-wise linear elastic behavior of the damper [12] (Figure 7b) was modeled with stiffness $K_{ed1} = 38$ kN/m between $D_d = 0$ and $D_d = D_{d1} = 8$ mm, $K_{ed2} = 18$ kN/m between $D_d = D_{d1} = 8$ mm and $D_d = D_{d2} = 40$ mm, $K_{ed3} = 2$ kN/m for $D_d > D_{d2} = 40$ mm; the linear viscous behavior of damper was identified by the damping coefficient $C_d = 1.1$ kNs/m. The linear elastic behavior of the bumper was modeled with a stiffness $K_{eb} = 2200$ kN/m; the linear viscous behavior of the bumper was identified by the damping coefficient $C_b = 5$ kNs/m.

The dynamic response of the system is governed by the following equations of motion. Three situations exist, when the mass is oscillating, and they can be described as follows:

- the mass is not in contact with any of the bumpers

\[
\begin{align*}
&M \ddot{D}_d + C_d \dot{D}_d + R_d = -M A_G \sin(\Omega t) \\
&C_{bj} \dot{D}_{bj} + K_{eb} D_{bj} = 0
\end{align*}
\]

(6a)

- the mass is in contact with the right bumper (RB)

\[
\begin{align*}
&M \ddot{D}_d + C_d \dot{D}_d + R_d + C_{bR} \dot{D}_{bR} + K_{eb} D_{bR} = -M A_G \sin(\Omega t) \\
&C_{bL} \dot{D}_{bL} + K_{eb} D_{bL} = 0
\end{align*}
\]

(6b)
• the mass is in contact with the left bumper (LB)

\[
\begin{align*}
M \ddot{D}_d + C_d \dot{D}_d + R_d + C_{bL} \dot{D}_{bL} + K_{eb} D_{bL} &= -M A_G \sin(\Omega t) \\
C_{bR} \dot{D}_{bR} + K_{eb} D_{bR} &= 0
\end{align*}
\]  

(6c)

The definition of \( R_d \) appearing in Eqs. (6) above will be given in the Eqs. (7) below.

The piecewise-linear restoring force of the damper obeys the following constitutive law in the \((D_d \geq 0, R_d \geq 0)\)-region (Figure 7b):

\[
\begin{align*}
R_d &= K_{ed1} D_d & 0 \leq R_d \leq R_{d1} & \quad \text{1st elastic branch} \\
R_d &= R_{d1} + K_{ed2}(D_d - D_{d1}) & R_{d1} \leq R_d \leq R_{d2} & \quad \text{2nd elastic branch} \\
R_d &= R_{d2} + K_{ed3}(D_d - D_{d2}) & R_{d2} \leq R_d & \quad \text{3rd elastic branch}
\end{align*}
\]  

(7a, 7b, 7c)

where \( D_d, R_d \) are the current values of displacement and force; \( D_{d1} \) and \( D_{d2} \) are the displacements corresponding to \( R_{d1} \) and \( R_{d2} \), Figure 7b. Analogous laws hold in the \((D_d \leq 0, R_d \leq 0)\)-region.

It is worth noting that each bumper relaxes to its original state after the impact and it may not be at rest when a new contact occurs. This depends on the ratio between the relaxation time of the bumper and the time interval between two subsequent impacts.

The comparison between experimental and numerical results, corresponding to the combination B4-G4-A3, was made with respect to the hysteresis loop of inertia force (Figure 8a) and the phase portrait (Figure 8b) in steady-state resonance condition. The solid lines represent the experimental results, and the dashed lines the numerical ones. The two vertical dashed lines identify the position of the bumpers. It can be observed that there is a good agreement between numerical and experimental results.

![Figure 8: Numerical vs. experimental results at resonance (B4-G4-A3): a) hysteresis loop; b) phase portrait.](image-url)
5 CONCLUSIONS

In this paper the experimental dynamic response of a base-isolated SDOF system, constrained by two symmetrically arranged deformable and dissipative bumpers and subjected to harmonic base excitation, is investigated. Three different peak values of table acceleration $A$, four amplitudes of the total gap $G$ between mass and bumpers and four types of bumpers $B$ were considered. The impact parameters (force and time of contact, coefficient of restitution and energy dissipated by the bumpers during the impact) were calculated from the experimental data. It was observed that they are influenced by the investigated parameters ($A$, $G$, $B$).

In particular, all the impact parameters, for almost all $B$-$G$ pairs, increase with the peak table acceleration $A$. While bumpers $B2$ and $B3$, and in some cases also $B4$, behave in a similar way, the values of the parameters associated with $B1$ differ significantly.

Slightly different trends with the amplitude of the total gap $G$ were observed; $G$ appears to influence the impact parameters to a lesser extent, except for some combination $B$-$A$, that show quite different trends.

Finally, the more the bumper is rigid (from $B1$ to $B2$), the lower the penetration of the mass into the bumper. Consequently, both the contact time and the energy dissipated decrease, while the contact force increases. As concerns the coefficient of restitution, it shows an increase passing from $B1$ to $B2$ then it stands at a constant value or decreases.

The experimental results were used to identify the parameters of a numerical model that has been shown to reproduce in a sufficiently accurate manner the results obtained experimentally.

REFERENCES


SEISMIC BEHAVIOUR OF A RC FRAME ISOLATED BY HDNR BEARINGS UNDER INCREASING INTENSITY LEVELS

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Abstract

High Damping Natural Rubber (HDNR) bearings are characterized by stiffness and damping capacity that significantly depend on the shear deformation amplitude. More in details, at low deformations stiffness and damping increase, whereas at large deformations the stiffness remarkably increase but the damping capacity decreases. Additionally, this kind of bearings show a loading hysteresis dependence, due to the internal damage of the rubber occurring as the deformation history progresses. This effect, also known as stress-softening, becomes significant for large deformation amplitudes and represents a source of uncertainty, which has recently caused a limitation of the use of this kind of isolators. However, consequences of this nonlinear behaviour of HDNR bearings on the response of isolated structures are not comprehensively investigated, primarily because advanced models have been only recently developed. In this paper some investigations are carried out by using a nonlinear constitutive law recently developed by some of the authors describing the behaviour of a HDNR with significant stress-softening complying with the limits of European code on anti-seismic devices. Analyses are carried out on a multi-degree of freedom system by considering different seismic intensity levels and different response parameters, including floor response spectra. A linear visco-elastic model calibrated at each seismic intensity level is also adopted in the analyses. The obtained results show that some response amplifications happen due to the higher modes of the superstructure, which are underestimated by linear models and may cause damages to non-structural components and equipment.

Keywords: Seismic isolated structures, High damping rubber bearings, Mullins effect, Stress-softening, Equivalent linear models
1 INTRODUCTION

High Damping Natural Rubber (HDNR) bearings are widely used for seismic isolation because their low lateral stiffness and dissipating capacity. However, both stiffness and damping capacity strongly depend on the shear deformation amplitude. Moreover, in some rubber compounds the introduction of filler, aimed at increasing the dissipation capacity, also induces a degradation of the cyclic response of the devices due to an internal damage process occurring in the rubber. This effect, known as stress-softening or Mullins effect [1], characterizes the virgin rubber (i.e. never subjected to deformation) or a rubber that has recovered the original properties after a sufficient time of rest [2]. The performance of structural and non-structural components of isolated system based on HDNR bearings has been investigated in the past by using simplified models, such as linear viscoelastic models [3] or elasto-plastic models [4]. Thus, there is a need of more in-depth investigations on the potential effects of the behaviour of HDNR bearings and this requires the use of advanced models and their implementation within a finite-element framework. Some models [5]-[7] are currently available in the OpenSees software [8] even if they show some approximations. Recently, an accurate nonlinear and history dependent constitutive law [9],[10] for describing the HDNR material behaviour with significant stress-softening but complying with the European code on anti-seismic devices [11] has been developed. This model, which uses multiple damage parameters to simulate the stress-softening including its direction dependence, has been calibrated against the results of experimental tests carried out on several virgin rubber pieces and can be used to simulate the HDNR bearings response under service and design conditions.

In this paper, the HDNR model has been used to simulate the behaviour of a base-isolated multi-degree of freedom (M-DOF) system representing a realistic building equipped with a hybrid isolation system consisting of HDNR bearings and low friction flat sliders. In particular, the system is designed according to the European design codes [11][12], i.e. by considering the seismic action at the Ultimate Limit State (ULS), with return period $TR = 475$ years. Nonlinear dynamic analyses are carried out under a set of real ground motion records scaled to different hazard levels, typical of the design limit state and two serviceability limit states, i.e. the Operational Limit State (OLS with $TR = 30$ years according the Italian design code [13]) and the Damage Limit State (DLS with $TR = 50$ years according to the Eurocode [12]). Different response parameters are evaluated: the isolation system displacement, the superstructure interstory drifts and absolute accelerations. Floor response spectra are also derived to investigate the behaviour of flexible contents of the superstructure. In order to highlight the effects of the non-linear behaviour of HDNR bearings, results are also compared with those obtained by using a linear visco-elastic model for the isolation system calibrated at each seismic intensity levels.

2 HDNR BEHAVIOUR

The HDNR compound adopted in this study exhibits a significant dissipation capacity, associated to a significant stress-softening. Nevertheless, the compound behaviour complies with the prescription of the European code on anti-seismic devices [11] about the stability of the shear properties under repeated cycling, requiring a ratio between the minimum and the maximum shear modulus measured between the first and the tenth cycle of imposed harmonic strains not less than 0.6. The experimental characterization of the rubber are illustrated in detail in [9]and [10] together with the calibration of the constitutive shear model, linking the shear stress $\tau_b$ to the shear strain $\gamma_b$. Numerical hysteresis loops of virgin HDNR samples subjected each one to a maximum shear deformation ranging from 25% to 250% are reported in Figure 1. It can be observed that for low shear strain amplitudes (Figure 1a) the ecant stiffness to maximum
deformation decreases for increasing strain amplitudes and the stress-softening is limited. Differently, at large strain amplitudes, an hardening behaviour is observed and the stress-softening is more significant (Figure 1b). Cycles are carried out with a period equal to 2.5 s, anyway, this rubber compound shows a negligible dependence on the shear strain rate [9]. Figure 2 shows the equivalent linear proprieties of the HDNR compound for different strain amplitudes and loading cycles. In particular, Figure 2a reports values of the secant shear modulus ($G_{\alpha}$), whereas Figure 2b reports the values of the equivalent damping ratio ($\xi_{\alpha}$), defined according to [11] by the following expressions:

$$G_{\alpha} = \frac{\tau_{b,\text{max}} - \tau_{b,\text{min}}}{\gamma_{b,\text{max}} - \gamma_{b,\text{min}}}$$  \hspace{1cm} (1a)$$

$$\xi_{\alpha} = \frac{2W_D}{\pi G_{\alpha} \left(\gamma_{b,\text{max}} - \gamma_{b,\text{min}}\right)^2}$$ \hspace{1cm} (1b)$$

where $W_D$ is the energy dissipated in each cycle.

![Figure 1: Hysteresis loops of virgin HDNR at different strain amplitudes: (a) 25%, 50% and 100% and (b) 150%, 200% and 250%.](image1)

![Figure 2: Equivalent linear parameters at different strain amplitudes and cycles: (a) equivalent shear modulus and (b) equivalent damping ratio](image2)
3 STUDY CASE BUILDING

3.1 Isolation system design

The case study considered in this paper consists in a 6-storey reinforced concrete building frame (Figure 3). The beams and columns have a rectangular transverse cross section with height of 500 mm and width of 300 mm. The floors are assumed rigid in plane and have a mass of 200kNms\(^{-1}\). A hybrid isolation system, consisting of 6 HDNR bearings and 9 low friction flat sliders providing mainly a vertical support (Figure 3), is considered.

![Figure 3: (a) 6-storeys isolated building, and (b) isolation system configuration](image)

The isolation system is dimensioned to attain under the design seismic action a value of the shear deformation equal to \(\gamma_{b,d} = 1.5\), corresponding to a value of the equivalent shear modulus close to the minimum (Figure 2a), and to a value of the equivalent damping ratio close to the maximum (Figure 2b). Thus, the value of \(\gamma_{b,d}\) is close to optimal and it is lower than the limit of \(2.5/\gamma_x\) imposed by the European code on anti-seismic devices [11], where \(\gamma_x\) is the reliability factor equal to 1.2. According to European code [12], a design action corresponding to the Ultimate Limit State (ULS) of the Eurocode 8 has been considered, characterized by an exceedance probability of 10% in 50 years or a return period TR=475 year. In particular, a Type 1 spectrum with a peak ground acceleration on stiff soil of \(0.35g\) and soil C conditions are considered for the ULS seismic action. The corresponding peak ground acceleration at the building site is equal to \(a_g = 0.35g \cdot 1.15 = 0.403g\). Moreover, following the indications given by the EN15129 [11], the design of the isolation system has been carried out by considering nominal values of the equivalent linear properties calculated at the third cycle of imposed cyclic deformations carried out at the selected design shear strain and design period. For the considered rubber compound, the design equivalent shear modulus and the design equivalent damping ratio are respectively \(G_{is,d}=1.015\) MPa and \(\xi_{is,d}=16.1\%\). By considering the superstructure as infinitely rigid and lumping the total mass M over the isolation system, the structure reduces to a S-DOF system and the isolation system displacement can be estimated from the displacement spectrum corresponding to the target isolation period (\(T_{is,d}=2.5s\)) and the damping ratio of the isolation system (friction of sliders has been neglected). The obtained displacement is \(u_{b,d} = \ldots\)
0.207 m and the total rubber thickness is \( t_{\text{r}} = u_{\text{b,d}}/\gamma_{\text{b,d}} = 0.138 \) m. Consequently, the total rubber area, \( A_{\text{is}} \), can be obtained through the following expression:

\[
A_{\text{is}} = \left( \frac{2\pi}{T_{\text{is,d}}} \right)^2 \frac{t_{\text{r}}}{G_{\text{is,d}}} \times M
\]  

(2)

The obtained value is \( A_{\text{is}} = 0.120 \) m\(^2\), which leads to 6 HNDR bearings \( (n_{\text{is}} = 6) \) with diameter \( D_{\text{is}} = 504 \) mm and secondary shape factor \( S_2 = D_{\text{is}}/t_{\text{r}} \) equal to 3.6. The bearings are supposed to be composed by 20 rubber layers of thickness \( t_{s} = 6.9 \) mm yielding to a primary shape factor \( S_1 = D_{\text{is}}/4t_{s} \) equal to 18.3. The selected HNDR bearings agree with indications about primary and secondary shape factors given by the standard for buildings isolation [14] and their buckling load capacity at the design displacement \( P'_{\text{cr}} \) (calculated according to the theory of the stability of multi-layered rubber compression springs under large lateral displacements [15]) is equal to 5400 kN. The axial forces due to vertical loads are about 830 kN and 440 kN for central and lateral bearings respectively, thus an ample margin is adopted with respect to the buckling load capacity, such that the horizontal behaviour is not influenced by axial loads even under the design seismic events. This makes possible to directly derive the horizontal force-displacement relationship of HNDR bearing \( (F_{\text{b}-u_{\text{b}}}) \) from the shear constitutive behaviour of the HDNR material and bearings dimensions \( (F_{\text{b}} = A_{\text{is}} \times \gamma_{\text{b}}/n_{\text{is}}, u_{\text{b}} = t_{\text{r}} \gamma_{\text{d}}) \). The analyses have been performed by considering the advanced HDNR material model with stress-softening for the bearings. In addition, an equivalent Linear Visco-Elastic (LVE) model of the isolation bearings is defined and adopted, as illustrated in the following section.

### 3.2 Limit states and LVE models

In order to investigate the response of the isolation system at serviceability limit states seismic hazard levels lower than the design seismic action have been considered. Values of the peak ground acceleration \( (a_{g}) \) at return periods other than the design one have been obtained by considering the following curve:

\[
v(a_{g}) = k_0 \left( \frac{a_{g}}{a_{g0}} \right)^{-k_1}
\]  

(3)

where \( k_1 \) is set equal to \( 1/0.35 = 2.857 \) according to [16], and \( k_0 \) is set equal to 0.013. The selected parameters provide a peak ground acceleration \( a_{g} = 0.403 \) g for \( \nu = 0.0021 \) yrs\(^{-1} \) (corresponding to \( T_R = 475 \) yrs), coherently with the design of the isolation system carried out in the previous section. Table 1 reports the values of \( a_{g} \) corresponding to the considered limit states, (OLS, DLS and ULS) and the corresponding return periods and mean annual frequencies \( (\nu = 1/T_R) \).

As known, linear equivalent properties of HDNR bearings are displacement dependent, thus are different for each limit state. Therefore, the LVE model of bearings have to be calculated for each limit state. More in details, the stiffness \( k_{\text{is}} \) and damping constant \( c_{\text{is}} \) of the LVE model can be evaluated as follows:

\[
k_{\text{is}} = \frac{G_{\text{is}} A_{\text{is}}}{t_{\text{r}} n_{\text{is}}}
\]  

(4)

\[
c_{\text{is}} = \frac{2\xi_{\text{is}} \omega_{\text{is}}}{\omega_{\text{is}}} k_{\text{is}} = \frac{\xi_{\text{is}} T_{\text{is}}}{\pi} k_{\text{is}}
\]  

(5)
where $G_{is}$, $\xi_{is}$ and $T_{is}$ are evaluated in correspondence of the shear deformation of bearings at the considered limit state, by means of values reported in Figure 2 relevant to the 3rd cycle of imposed deformation. At the design limit states (ULS) values are known ($G_{is}=G_{is,d}=1.015$ MPa, $\xi_{is}=\xi_{is,d}=16.1\%$ and $T_{is}=T_{is,d}=2.5s$) since the design shear deformation of bearing is set equal to $\gamma_{b,d}=1.5$. Differently, at serviceability limit states $G_{is}$, $\xi_{is}$ and $T_{is}$ are evaluated as a function of the isolation system displacement, by using an iterative procedure. By assuming the same spectral shape for all the return periods, the obtained LVE parameters are respectively $G_{is}=1.327$ MPa and $\xi_{is}=17.8\%$ for the DLS and $G_{is}=1.625$ MPa and $\xi_{is}=17.1\%$ for the OLS. The corresponding isolation periods of the reduced S-DOF system are $T_{is}=2.19$ s at the DLS and $T_{is}=1.98s$ at the OLS. In Table 1 the pseudo-acceleration ($S_a$) and the spectral displacement ($S_d$) evaluated using the isolation period ($T_{is}$) and damping ratio ($\xi_{is}$) calculated for each hazard level are reported. The corresponding shear deformation of bearings ($\gamma_{b}$) and the properties of the LVE model ($k_{is}$ and $c_{is}$) are also included in the Table.

<table>
<thead>
<tr>
<th></th>
<th>$T_r$</th>
<th>$\nu$</th>
<th>$a_g$</th>
<th>$S_a(T_{is}, \xi_{is})$</th>
<th>$S_d(T_{is}, \xi_{is})$</th>
<th>$\gamma_{b}$</th>
<th>$k_{is}$</th>
<th>$c_{is}$</th>
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<td>0.207</td>
<td>1.50</td>
<td>1474</td>
<td>189.0</td>
</tr>
</tbody>
</table>

Table 1: Considered hazard levels

3.3 Modal analysis at the design limit state

The model of the superstructure is built by considering a Young modulus of concrete equal to 32000 MPa and a cracking reduction coefficient equal to 0.5 for the beams and 0.7 for the columns of the superstructure. Moreover, a stiffness proportional damping is assumed. The mass-proportional component is set equal to zero because it would lead to underestimate the isolated system response [17]. The damping constant for the stiffness-proportional damping matrix is set to provide a damping ratio equal to 2% at the first vibration period of the fixed-base superstructure. By adopting a LVE model for the bearings, the complex modal analysis can be carried out on the LVE model of the M-DOF system. In order to verify the outcomes of the isolation system design, the LVE model calibrated at the ULS is considered for the modal analysis. Figure 4 illustrates the absolute values of the undamped eigen-modes of the first three vibration modes of the isolated MDOF system at the design condition.

![Figure 4: First three modal shapes of the isolated MDOF system for the LVE bearing model at the design condition ($\gamma_{b,d}=1.5$, 3rd cycle).](image-url)
The relevant mass participation ratios are 0.995, 0.004 and 0.0002 respectively. The vibration period of the two significant modes (the first two modes) are $T_1=2.65$ s and $T_2=0.55$ s. The associated damping ratios are $\xi_1=13.4\%$ and $\xi_2=10.1\%$. As expected the difference between the target isolation period and the actual first period $T_1$, due to the influence of the superstructure flexibility, is not very significant.

4 SEISMIC ANALYSES

4.1 Seismic input

A set of 20 ground motion records is employed to describe the record-to-record variability effects. These records have been selected from the PEER strong motion database [18] based on the following criteria: they are associated to the site class C as defined in Eurocode 8 [12], have a source-to-site distance $R$ varying in the range between 20 km and 50 km (thus records do not contain any pulse) and a moment magnitude $M_w$ varying in the range between 6.5 and 7.5. All the records matching the selected scenario have been scaled in amplitude to match the ULS spectrum at the design isolation period and damping ratio. Among all the records available for the selected scenario, the 20 selected ones are characterized by scale factors closest to 1. Record details and scale factors are reported in Table 2, whereas the response spectra of the scaled records are plotted in Figure 5 together with the average and the design spectrum.

In order to describe the seismic action at lower hazard levels, the same ground motions have been further scaled by a factor given by the ratio between the spectral ordinate at the considered limit state and at the design limit state, as reported in Table 1.

<table>
<thead>
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<th>Earthquake Name</th>
<th>Station Name</th>
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<th>Comp.</th>
<th>M.</th>
<th>$R_{rup}$ [km]</th>
<th>Mechanism</th>
<th>RSN</th>
<th>SF</th>
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<tr>
<td>8</td>
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<td>6.9</td>
<td>27.9</td>
<td>Rev. Ob.</td>
<td>776</td>
<td>1.54</td>
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<td>Rev. Ob.</td>
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<td>1.12</td>
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<td>Sunnyvale - Colton Ave.</td>
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<td>Eureka - Myrtle &amp; West</td>
<td>0.17</td>
<td>337</td>
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<td>7.0</td>
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<td>884</td>
<td>1.29</td>
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</table>

Table 2: Records details and values of the scale factor (SF) for the design earthquake level
4.2 Analyses results

This section summarizes results of the analyses carried out on the base-isolated MDOF system at the design and serviceability limit states. The analyses have been performed by considering for the isolation system both the advanced HDNR model with stress-softening and the LVE model calibrated for each limit state. In particular, Table 3 reports mean values of the maximum response in terms of base displacement \( u_b \) and superstructure floor displacements \( u_{s,i} \), Table 4 in terms of shear deformation of bearings \( \gamma_b \) and superstructure inter-storey drifts \( d_{s,i} \), and Table 5 in term of absolute base acceleration \( \ddot{u}_b \) and floor accelerations \( \ddot{u}_{s,i} \). The same results are shown in Figure 6, Figure 7 and Figure 8, in terms of mean values (continuous lines) ± standard deviation (horizontal bars) for each floor.

<table>
<thead>
<tr>
<th>LVE</th>
<th>HDNR</th>
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<tr>
<td>( u_0 )</td>
<td>0.083</td>
</tr>
<tr>
<td>( u_{s,1} )</td>
<td>0.090</td>
</tr>
<tr>
<td>( u_{s,2} )</td>
<td>0.098</td>
</tr>
<tr>
<td>( u_{s,3} )</td>
<td>0.104</td>
</tr>
<tr>
<td>( u_{s,4} )</td>
<td>0.110</td>
</tr>
<tr>
<td>( u_{s,5} )</td>
<td>0.113</td>
</tr>
<tr>
<td>( u_{s,6} )</td>
<td>0.115</td>
</tr>
</tbody>
</table>

| OLS | 0.89 | 0.91 |
| SLS | 0.89 | 0.91 |
| ULS | 0.89 | 0.91 |

Table 3: Mean values of the floor displacement \( u_b \) [m] and superstructure floor displacement \( u_{s,i} \) [m]

<table>
<thead>
<tr>
<th>LVE</th>
<th>HDNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_b )</td>
<td>0.60</td>
</tr>
<tr>
<td>( d_{s,1} )</td>
<td>0.26%</td>
</tr>
<tr>
<td>( d_{s,2} )</td>
<td>0.27%</td>
</tr>
<tr>
<td>( d_{s,3} )</td>
<td>0.23%</td>
</tr>
<tr>
<td>( d_{s,4} )</td>
<td>0.19%</td>
</tr>
<tr>
<td>( d_{s,5} )</td>
<td>0.14%</td>
</tr>
<tr>
<td>( d_{s,6} )</td>
<td>0.08%</td>
</tr>
</tbody>
</table>

Table 4: Mean values of isolator shear deformation \( \gamma_b \) [-] and superstructure inter-storey drifts \( d_{s,i} \) [-]

<table>
<thead>
<tr>
<th>LVE</th>
<th>HDNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ddot{u}_b )</td>
<td>0.95</td>
</tr>
<tr>
<td>( \ddot{u}_{s,1} )</td>
<td>0.94</td>
</tr>
<tr>
<td>( \ddot{u}_{s,2} )</td>
<td>0.94</td>
</tr>
<tr>
<td>( \ddot{u}_{s,3} )</td>
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</tr>
<tr>
<td>( \ddot{u}_{s,4} )</td>
<td>1.01</td>
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<tr>
<td>( \ddot{u}_{s,5} )</td>
<td>1.12</td>
</tr>
<tr>
<td>( \ddot{u}_{s,6} )</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Table 5: Mean values of the base floor acceleration \( \ddot{u}_b \) [m/s²] and superstructure floor accelerations \( \ddot{u}_{s,i} \) [m/s²]
With reference to the performance of the isolation system, at the design limit state the average value of the bearings shear deformation obtained with the LVE model is 1.51, which is almost equal to the design value (1.5). Differently, the value obtained with the HDNR model is lower (1.39), confirming the importance of the stress-softening affecting the seismic response at the ULS which makes, on average, the isolation system stiffer with respect to the condition assumed in the design (i.e. equivalent properties at the third cycle at the design shear). On the contrary, for frequent events, mean values of the shear deformation obtained with the LVE model are larger than those estimated in Table 1, due to the smaller isolation ratio and thus the larger the influence of the superstructure deformability at these limit states. However, the obtained displacements are close to results of the HDNR model, confirming the lower importance of the stress-softening for the DLS and OLS.

![Figure 6: Mean value ±standard deviation of maximum displacements for (a) LVE models and (b) HDNR model (dotted lines refer to LVE model for comparison purpose)](image1)

![Figure 7: Mean value ±standard deviation of maximum inter-storey drifts for (a) LVE model and (b) HDNR model (dotted lines refer to LVE model for comparison purpose).](image2)
The importance of the stress-softening at the ULS and its lower importance at serviceability limit states, is also evident by observing results reported in Figure 8, where similar values of the record to record variability are obtained between the LVE and HDNR models for serviceability limit states, whereas significantly different values result at the ULS. In particular, at this limit state, the variability obtained with the HDNR model is higher, due its the nonlinear and history dependent behaviour, leading to record-dependent dynamic properties of the isolation system different from those assumed in the design.

Regarding the superstructure performance, floor inter-storey drifts are lower at higher floor, as expected. Differently, floor absolute accelerations are larger at the base and top floors, due to the significant influence of the second mode of vibration of the isolated building, whose shape is illustrated in Figure 4. More in detail, for the HDNR model, the maximum values of the accelerations observed for the OLS and DLS intensity levels, attained at the top floor, are respectively 1.29 m/s$^2$ and 1.66 m/s$^2$, and the maximum values of the inter-storey drifts, attained at the second storey, are respectively 0.26% and 0.32%. At the design limit state intensity level, the maximum absolute acceleration is 2.41 m/s$^2$ and the maximum inter-storey drift is 0.43%, which is still lower than the limit imposed by the Eurocode 8 [12] for the limitation of damages of non-structural components, i.e. 0.5%. It is worth to note that results of the LVE and HDNR models are in good agreement for drifts at the OLS and SLS, whereas drifts at the ULS are underestimated by the LVE model. The absolute accelerations are also underestimated by the LVE model at all the hazard levels considered. In particular, the absolute accelerations relevant to the HDNR model are significantly higher at the lowest and highest floors, confirming that the nonlinear behaviour of bearings amplifies the response contribution of the second mode of the base-isolated system. Also for the superstructure, the response variability is large with the HDNR model at the ULS, as a consequence of the variability of the isolation properties with records.

In order to evaluate the exposure of acceleration-sensitive flexible equipment inside the building, the mean Floor Response Spectrum (FRS) under the different seismic records is also evaluated for all the building storeys and for all the limit states. Figure 9a, 9b and 9c illustrates the mean FRS at the OLS, DLS and ULS intensity level for each floor, whereas Figure 9d illustrates the mean FRS of the top floor for different seismic intensities. Two major peaks are observed, in correspondence of the first and second vibration period of the isolated system. It is interesting to observe that the first mode peak slightly increases by passing from the base to
the top floor. Differently, in correspondence of the second mode, the peaks of the base and top floors and also of the 1\textsuperscript{st} and 5\textsuperscript{th} floors are larger than the peaks of the 2\textsuperscript{nd} and 4\textsuperscript{th} floors. This is consistent with the shape of the second mode (Figure 4), resulting in different demands at various floors. For the same reason, the 3\textsuperscript{rd} floor does not exhibit a peak in correspondence of the second vibration period, since it is located in correspondence of the node of the second modal shape. It is also worth to note that, as shown in Figure 9d referring to the top floor, the peak in correspondence of the first vibration mode changes in shape, value, and location with the seismic intensity level, due to nonlinear and record-dependent behaviour of the isolation system. Moreover, for serviceability limit states (OLS and DLS) the peak in correspondence of the second vibration period is similar in amplitude to the peak in correspondence of the first vibration mode and becomes significantly larger for the design seismic action. In this last Figure 9d response floor spectra obtained with the LVE model calibrated for each limit state are also reported (dotted lines). The comparison shows that peaks in correspondence of the first mode obtained with the HDNR model are similar or lower in amplitude with respect those obtained with the LVE model, but different in shape, especially at the ULS, due to the record-dependent dynamic properties of isolation HDNR bearings. On the contrary, peaks in correspondence of the second vibration mode obtained with the HDNR model are almost twice those obtained with LVE models, which further confirms that the higher modes response is significantly increased by the nonlinear behaviour of the HDNR model.

![Figure 9: FRS of superstructure floors at (a) ULS, (b) DLS, (c) OLS and (d) FRS of the top floor for different limit states (dotted lines refer to LVE model for comparison purpose).](image-url)
5 CONCLUSIONS

This paper investigates the seismic response of structures isolated by HDNR bearings under increasing seismic intensity levels up to the design one. In particular, the effects of the change in stiffness and damping with the strain amplitude and of the stress-softening are investigated. To this purpose, results obtained by using an advanced HDNR model are compared with results obtained by adopting a LVE model calibrated for each limit state. Obtained results show that the performance of the superstructure at the serviceability and design limit states is satisfactory, i.e. absolute acceleration and relative displacements of the superstructure do not show excessive amplifications. However, while the LVE model provides a good estimate of the base-isolated system response at serviceability limit states, at the ULS the bearings displacement is overestimated and the superstructure response is underestimated. In particular, floor accelerations are significantly underestimated at the lowest and highest floors, due to the nonlinear behaviour of the rubber, which amplify the contribution of second mode to the base-isolated system response. The variability of the superstructure and isolation system response at the ULS is also notably underestimated by the LVE model, due to the stress-softening of HDNR bearings making the dynamic properties of the isolation system strongly record-dependent. Finally, floor response spectra obtained with the HDNR model show peaks in correspondence of the second vibration mode two times larger than corresponding peaks obtained with the LVE model, confirming the higher modes response amplification due to the nonlinear HDNR behaviour. Peaks in correspondence of the first mode are more similar in amplitude but different in shape, due to the influence of the stress-softening on the dynamic properties of bearings, if modelled by using the advanced rubber model.

REFERENCES


CASE-STUDY OF A COST-BASED SEISMIC DESIGN FOR A R.C. FRAME WITH ADDITIONAL DISSIPATIVE BRACE SYSTEMS

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Abstract

The employment of cost-based design framework in seismic engineering is an important issue researchers are dealing with in last years. It mainly assumes loss indicators as design parameters, aiming to make a designer having greater consciousness about losses to expect due to seismic loads. In the state-of-art there are many proposals of loss-based seismic design methods. As well, it is possible to read about loss assessment analysis of several case-study systems. Although, very poor indications are provided about loss assessment of structural systems equipped with dissipative braces, that today represent a valid seismic design alternative. This work aims to present a case-study structure of a reinforced concrete building equipped with shear link dampers supported by a bracing system. The loss assessment analysis of the structure is realized through the use of PACT software (FEMA P58). The shear link’s fragility curve is constructed starting from experimental test results, while the corresponding repair cost curve is generated through engineering judgment. Main loss results are commented and analyzed in the view of implementing a cost-based design framework.

Keywords: Loss assessment, Performance-based, Dissipative seismic system, Shear Link, Earthquake Engineering.
1 INTRODUCTION

Seismic structural design is a branch of civil engineering of fundamental importance in all the earthquake-prone countries. It was first adopted in the 1920s and 1930s, when rough and rudimentary definitions of lateral loads acting on structures were introduced. In the 1960s, thanks to the availability of the first accelerograms, more information about ground shaking allowed to deepen the study of seismic engineering. Thanks to the advancements in new technologies and introduction of first calculators, the experience of earthquakes and the consequent reconnaissance process allowed to develop new consciousness about seismic structural behavior, letting the scientific community set the fundamentals of the capacity design philosophy [1]. The main principles at the base of modern earthquake engineering have been condensed in the discipline of the Performance-Based Earthquake Engineering (PBEE), outlined in the SEAOC Vision 2000 in 1995 [2], which introduces the concept of limit states. In particular, according to the design earthquake intensity and the building’s use of destination, the minimum performance objectives are defined. Seismic code provisions, constantly updated thanks to continuous new research achievements, drive professionals in the application of a correct earthquake-resistant structural design.

In last decades, the occurrence of new earthquakes and the observations of their effects on modern seismically designed structures have allowed to put in evidence the high economic and social impact of significant structural and non-structural damages, generally admitted in the capacity design philosophy [3]-[4]. This new perspective has favored the development, on one hand, of new seismic design technologies limiting the amount of allowed damaging [5]-[6], while, on the other hand, of the loss-based design. The latter was first introduced by Krawinkler et al. [7], proposing a design framework based on the limitation of losses to an acceptable level. Through the definition of a new loss-based performance matrix, Nuzzo et al. [8] suggested a seismic design approach in which a loss parameter, namely a characteristic value of the Probable Maximum Loss (PML), is set as design objective. The aim of the present work is to apply this design approach to the case of a r.c. structure equipped with hysteretic braces. As a matter of fact, many loss assessment studies can be found in the state-of-art [9]-[11], but few of them refer to braced structures equipped with energy dissipation devices. An interesting contribution is provided by Kim and Shin [12], who assessed seismic loss performance of a hybrid control device, composed by a slit damper in parallel with friction pads. In this study a particular passive hysteretic device, namely the shear link (SL) damper, is employed, assessing its performances in terms of economic losses.

The implementation of the loss-based design approach suggested in [8] requires the development of a loss assessment analysis, performed by the authors through the application of a specific tool, called PACT – Performance Assessment Calculation Tool [13]-[14]. The main issue concerning loss assessment analysis concerns the identification of all the structural, non-structural components and furnishings susceptible of damage within the building and, for each of them, the definition of the corresponding fragility curve. In many studies adopted fragility curves are taken from the state-of-art, sometimes implying some important simplifications. In order to support users, PACT tool provides a wide library including more than 700 fragility components. Although, in some case-study applications, fragility curve of specific elements cannot be found in existing studies. In general, this is the case of most of the seismic control systems, which can be adopted as additional source of energy dissipation. Porter et al. [15] suggested several methodologies to evaluate fragility curves for a specific element according to the existing level of knowledge about it. In this work, the fragility curve of the SL damper is constructed. This device, consisting in a steel plate milled in correspondence of its web, where energy dissipation is concentrated, has been recently object of a wide experimental
I. Nuzzo

Gathered data are used in order to get the corresponding fragility curve. This specific damper is then assumed as energy dissipation device in the case-study braced structure analyzed in this work.

2 USE OF LOSS PERFORMANCE MATRIX IN SEISMIC DESIGN FRAMEWORK

In the perspective of introducing a seismic design approach providing higher awareness of economic losses induced by a severe seismic event, a loss performance matrix was proposed in [8]. The use of this new tool in seismic design framework is herein illustrated.

The loss performance matrix (Figure 1) associates different levels of Repair Cost Ratio (RCR) to the probability of loss, expressed as the probability that a certain RCR value is overcome given the occurrence of a certain seismic intensity. This definition of the probability of loss corresponds to the parameter known in literature as the Probable Maximum Loss (PML). The RCR parameter, defined as the ratio between the repair cost RC and the total replacement value RPLV, is presented within a range between 0% (no loss) and 50%, considered as the maximum loss threshold for which it is economically feasible to repair the facility [13].

According to the values of RCR and corresponding probability of loss, the loss performance matrix identifies different loss performance design levels. In particular, the lower are both RCR and corresponding probability of loss, the better is the facility's performance in terms of losses under the design earthquake level, yielding towards an ideal design. Conversely, when a system provides low probability of loss in correspondence of high values of RCR, it means that it is susceptible of significant damage, as well as a system characterized by high probability of loss associated to small repair cost values. When PML values are not satisfactory because RCR and/or its probability of exceedance are too high, the building's design is defined unacceptable, since economically unrepairable.

The loss performance matrix allows to implement a seismic design framework in which a loss parameter, namely the PML, becomes the new design objective. As a matter of fact, after assuming a loss design level, through the implementation of a loss assessment analysis it is possible to overlap the loss curve on the performance matrix, thus verifying if it passes through the performance point or beneath it (Figure 1-b). In this way, through a calibrated loss performance matrix, the stakeholder can immediately appreciate the (expected) probability of loss associated to the design RCR value and decide if it is deemed acceptable. In some cases it could happen that there is no correspondence between the allowable RCR value and the acceptable probability of loss, given IM. Then it can be necessary to implement low-damage technologies to reduce the probability of loss.

The use of this new loss performance matrix could imply that seismic design might not necessarily be carried out in order to limit a specific EDP, as it is in the traditional PBEE, rather it will pursue pre-defined level of economic losses associated to a certain level of confidence. In more practical terms, a combination of an EDP-based design approach (i.e. Displacement-Based Design) and loss-based charts/spectra would allow to integrate the two approaches and enhance the current mechanical-based approach with some more explicit economical considerations.
3 COST-BASED DESIGN FRAMEWORK APPLIED TO A R.C. STRUCTURE EQUIPPED WITH SHEAR LINK DAMPERS: A CASE-STUDY

In this section the seismic design of a regular 5-storey 3-bay r.c. structure equipped with dissipative braces (Figure 2), synthetically indicated in the following as Braced Frame (BF), is realized within a loss-based framework, providing added energy dissipation through the adoption of steel shear links (SL) supported by tubular braces [16]. In particular, the loss design parameter assumed is the characteristic value of PML (i.e. the 5th percentile, PMLk) equal to 15%, that means a maximum value of RCR=15% associated to a probability of being exceeded equal to 5%, given the occurrence of the design seismic intensity. In other words, the structure has to be designed in such a way that, if the design earthquake occurs, the maximum losses admitted are the 15% of the total RPLV, with a probability of being overcome of 5%. In order to meet this requirement, the structural design at Life Safety (LS) performance level, in the respect of the Italian [17] seismic code, is realized through a displacement-based design approach [18], admitting a maximum interstorey drift (IDR) of 0.5%, thus limiting structural and non-structural damage.

The building’s occupancy is supposed to be commercial office, located in Norcia, central Italy, soil type B, topography class T2. The total replacement cost is assumed to be 2551 $/m², that corresponds to a total of 7.75 MS, considering the total area of 506 m² and the number of levels (which is 6 considering also the ground floor).

Beams and columns, respectively of dimensions 300x400 mm and 500x500 mm, are designed only for gravitational loads, given that the seismic action is withstand by dissipative braces,
having the accuracy of verifying columns against eventual traction axial stresses induced by braces. Minimum and maximum dimensions of tubular braces are 168.3x4 and 219.1x5.9. More details about braces and dampers dimensions can be found in [18].

3.1 Structural analysis

Given the regularity and symmetry of the case-study structure, the 2D frame has been considered. It has been modelled in OpenSees [19], adopting “elasticBeamColumn” elements for beams and columns, verifying successively that yielding is never achieved in any of them. Dissipative braces are modeled as "twoNodeLink" elements characterized by "uniaxialMaterial Steel01". The first vibrational period of the 2D system is $T_1=0.55$ s. Dynamic nonlinear analysis have been performed in correspondence of the set of selected records [20] listed in Table 1. The displacement profile envelope in correspondence of each record is plotted in Figure 3, showing also the mean trend. Engineering Demand Parameters (EDP) required for the loss estimation analysis are IDR, PFA (Peak Floor Acceleration) and RIDR (Residual Interstorey Drift), whose envolopes in correspondence of each record are shown from Figure 4 to Figure 6.

<table>
<thead>
<tr>
<th>EQ</th>
<th>Waveform ID</th>
<th>EQ ID</th>
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<th>PGA_y [g]</th>
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<td>0.22</td>
<td>1.18</td>
<td>1.59</td>
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<td>0.10</td>
<td>2.73</td>
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</tr>
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<td>4</td>
<td>432</td>
<td>83</td>
<td>Parkfield</td>
<td>6</td>
<td>0.36</td>
<td>0.23</td>
<td>0.98</td>
<td>1.54</td>
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<td>5</td>
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<td>83</td>
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<td>46</td>
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<td>0.18</td>
<td>0.16</td>
<td>2.02</td>
<td>2.24</td>
</tr>
</tbody>
</table>

Table 1 Set of 7 couples of spectro-compatible records

![Figure 3 Displacement profile envelope of BF system](image3.png)

![Figure 4 IDR envelope of BF system](image4.png)
Incremental Dynamic Analysis (IDA) are performed selecting 42 records from SIMBAD database [21], in the attempt of reducing uncertainties of the collapse fragility curve, assuming soil type B, covering a range of moment magnitude Mw between 5 and 7 and epicentral distance R between 15 km and 35 km (so including mid-to-far field events). Resulting IDA curves are plotted in Figure 7. The fragility curve, given in Figure 8, is constructed selecting the IM values that cause collapse and plotting the relative Cumulative Distribution Function (CDF). In particular the attainment of failure is considered when an IDR value of 0.8% is reached. This value represents the attainment of an interstorey displacement of 25 mm, which is estimated to be distributed between the SL damper and steel braces in the ratio of 20:5. Indeed the maximum allowable deformation that SL dampers can admit is experimentally demonstrated to be 20 mm.

3.2 Fragility components definition
In order to perform the loss estimation analysis it is necessary to identify all the structural and non-structural (including furnishings) elements which may contribute to determine losses and define for each of them possible damage states and corresponding fragility curves. In the specific case-study, the structural (S), non-structural (NS) and contents (C) fragility components have been selected from PACT database and are listed in Table 2.
The deformation capacity of the SL devices is significant but not collapsed. Anyway, it would be substituted never reached if not when the damper is already severely damaged, the only damage state identified is when the SL undergoes a deformation corresponding to its maximum capacity. Actually, if under a moderate earthquake a SL, which is designed to yield for very small values of displacement, is significantly deformed but not collapsed, anyway it would be substituted. For this reason the damage state is selected to be at the attainment of the 75% of the SL's deformation capacity. The Engineering Demand Parameter (EDP) selected to represent SL damage state is the IDR. Actually, the measure observed in experimental tests was the relative deformation of the devices, so it is necessary to convert it into the selected EDP. It is

\[
\mu = \min_{i} \left( \frac{1}{\ln(1-\beta)} \right) 
\]

\[
\beta = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left( \frac{d_i}{\mu} \right)^2} 
\]

Table 2 Fragility components from PACT library

As far as SL dampers are concerned, no fragility curves exist in literature. In [15], several methodologies for the development of fragility curves are proposed. Among them, the one better fitting the specific case is the Actual Demand Data derivation method, usable when data are available from m individual specimen tests, and each tested specimen experienced the damage state of interest at a known value of demand, \(d_i\). The fragility parameters to be defined are the median value of the demand at which the damage state is likely to initiate and the value of the random dispersion, respectively derived as follows:

\[
\mu = \min_{i} \left( \frac{1}{\ln(1-\beta)} \right) 
\]

\[
\beta = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left( \frac{d_i}{\mu} \right)^2} 
\]
satisfactorily admissible that when SL dampers are yielded, given that their post-yielding stiffness is significantly lower than support braces' elastic stiffness, IDR corresponds to the relative deformation within dissipative devices. With this assumption, IDR corresponding to the attainment of the maximum deformation capacity of specimens can be easily derived dividing the experimental observations (SL’s relative displacement) by the interstorey height of the case-study structure. The definition of fragility parameters is given in Table 3, while the final fragility curve is plotted in Figure 9. The cost for the realization of a singular device is estimated to be around 300 $, considering the cost per kg of steel according to Italian price list for Central regions [22], including labor cost. The repair intervention after a seismic event, consisting in the substitution of dampers, is assumed equal to the cost of the device itself increased of the 30%. Actually the structural architecture has to be defined so that SL dampers result located in easily accessible locations, so not requiring any further significant cost for the substitution intervention. Considering also an increment of the cost per device in the case in which a very small number (2) of elements have to be substituted, the final economic consequence function of Figure 10 is determined.

<table>
<thead>
<tr>
<th>specimen #</th>
<th>$d_{	ext{collapse}}$ [mm]</th>
<th>$d_{	ext{collapse}}/h$ [rad]</th>
<th>$3/4 d_{	ext{collapse}}/h$ [rad]</th>
<th>$\mu$ [rad]</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0063</td>
<td>0.0048</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>22.8</td>
<td>0.0071</td>
<td>0.0053</td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td>20</td>
<td>0.0063</td>
<td>0.0047</td>
<td></td>
<td></td>
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<td>4</td>
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<td>0.0060</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Definition of fragility parameters for SL dampers

Figure 9 Fragility curve of SL damper

Figure 10 Economic consequence function of SL damper

3.3 Loss estimation analysis and results

The intensity-based loss estimation analysis is performed by PACT software, implementing a Monte Carlo simulation with 500 realizations. The pie chart of Figure 11-a shows the contribution of structural and non-structural components and contents to the overall repair cost (269M$) of all the realizations. Data observed confirm that the main part of repair costs is attributed to non-structural components, which cover a percentage of 67% of the total, followed by contents (27%) and structural components (6%). It is interesting to observe that the cost attributed to structural elements is significantly limited, confirming that beam-column joints are preserved from damage. The very low cost of SL devices also has a positive impact on this result. Note that in no simulation the structure is collapsed and never irreparability is reached, so the total replacement cost is never observed.
The probability of loss, expressed as the probability of exceeding a certain RCR value, given the intensity measure, is plotted in Figure 11-b. It is evident that the minimum loss design objective of $P_{ML}^{k}=15\%$ has been pursued, given that it results a value of $P_{ML}^{k}=12.5\%$, that means lower than the maximum admissible.

![Figure 11: a) Contribution of different components to the overall repair cost (269 M$); b) Probability of loss](image)

**4 CONCLUSIONS**

The present paper represents the case-study application of the loss-based seismic design of a r.c. structure equipped with dissipative braces. The importance of accounting for repair costs in the design process is emerging, with the urgent need of providing a new design framework involving economic losses. To this aim, the use of a loss performance matrix is investigated. In particular, it considers the Probable Maximum Loss (PML) as new performance measure, instead of, or in addition to, typical discrete engineering demand parameters suggested by traditional PBEE framework. The probability of loss, corresponding to the probability of exceeding a certain loss once the seismic intensity is fixed, is expressed as a function of repair costs (RC) adimensionalized with respect to the total Re PLacement Value (RPLV). The introduction of the loss performance matrix allows the implementation of a cost-based design approach, according to which seismic design is satisfactory when the maximum allowable Repair Cost Ratio (RCR), corresponding for example to the risk insurance coverage, is associated to a probability of loss sufficiently low, given the seismic design intensity.

The application of the loss-based design approach to a dissipative braced structure has the aim of providing a case-study not often found in the state-of-art, thus giving the opportunity of showing a possible methodology to derive the fragility curve of a specific passive protection system, namely the Shear Link damper. In particular, mean and standard deviation defining the fragility curve have been derived on the basis of recent experimental investigations.

The analyzed case-study has shown that if the structure is opportune ly designed limiting its deformation, it is possible to preserve its damaging, significantly reducing economic losses associated to structural components. This perfectly reflects the philosophy of adding energy protection systems has sacrificial fuses, thus modifying the capacity design hierarchy. Moreover, if the control devices are opportune ly located and connected to the structure, their substi-
I. Nuzzo

Istitution after the occurrence of a severe earthquake should be easy and not invasive, allowing a prompt restoration of the building’s functionality.

In on-going developments, the authors aim to perform significant high number of parametric analysis in order to give range of values in the new loss performance matrix for different structural typologies. In particular, it would be very useful to know which are the possible PML values associated to a specific seismic-resistant system, depending on initial design considerations, technology and earthquake design intensity, so that a more direct cost-based design approach could be performed.
REFERENCES


Prezzario Unico del Cratere del Centro Italia, Ordinanza n°7 del Commissario del Governo per la Ricostruzione del 14/12/2016 (in Italian)
AN INNOVATIVE FRICTION BASE RESTRAINT TO REDUCE STRUCTURAL DEMAND TO WIND TURBINES

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Abstract

This work proposes the use of an innovative base restraint to reduce the structural demand to wind turbine towers due to exceptional wind loads. The base of the tower has been designed as connected to the foundation thanks to (i) a hinge that allows rotation in one direction, (ii) a Rotational Friction Device (RFD) that dissipates energy when the external load overpasses the friction value, and (iii) a rotational spring able to re-center the tower during and after the history load case. The RFD and the rotational spring are placed in parallel. The effectiveness of such passive control system has been assessed comparing the structural demand – base bending moment and top displacement – between the benchmark NREL 5 MW wind turbine in absence (i.e. fixed base) and in presence of control. The control strategy has turned out to be very promising. It leads, in fact, to a significant reduction of the bending moment at the base without involving an increasing in the displacement at the top. Moreover, it also favors the re-centering of the tower after the end of the wind action.

Keywords: Rotational Friction Device, NREL 5 MW Wind Turbine, Structural Control, Passive Control, Wind Exceptional Load.
1 INTRODUCTION

An elastic dissipative rotational base restraint for wind turbine towers is proposed herein. The aim is that to reduce the bending moment at the base of the tower, hence limit the size of the latter as well as time and costs for construction. Such control strategy may be useful to make even more convenient the use of higher wind turbines able to produce more energy by capturing stronger winds. By reducing the cross section of the tower, it is possible to avoid incurring in the cost increase due to the building of the turbine in situ rather than in the workshop.

Rotational Friction Dampers have been used to control the dynamic response of structures in Civil Engineering. These devices consist, in their simpler form, of a central vertical plate, two side horizontal plates and two circular friction pad discs made of high tech composite materials placed in between the steel plates, see Figure 1. All the plates are clamped together by a prestressed steel bolt to form a T-shaped device. The energy dissipation can be increased by adding more layers of steel plates and friction pads. Further developments and commercial supply with improved configurations have been also elaborated and demonstrate effectiveness in the vibration structural control [1, 2].

This kind of passive control device is relatively cheap, easy to be installed, used and maintained. Depending even on their location into the structure, friction devices can be even affected by temperature effects and the friction coefficient can be affected by variation (reduction) over time. Their response is non-linear and displacement dependent. The mechanism used by RFDs to mitigate harmonic vibrations, hence top displacements and base stress, is simply that of the energy dissipation due to friction.

Many other passive control systems have been developed during the last decades in horizontal axis wind turbine (HAWT) with the purpose of the vibrational structural control [3 -12], in this context, this paper exposes the research work about the use of a RFD for the same purpose for the first time. The turbine used as reference has been the benchmark NREL 5 MW wind turbine [13]. The tower has been thought as structurally connected to the foundation by a hinge that allows rotation in one direction while denying any other movement. The RFD, even applied at the base of the tower, has been thought as applied in parallel with a rotational spring, this able to recenter the tower after an exceptional wind load defined in accordance with the IEC 61400-1 [14] and defined as acting in one direction only, perpendicular to the blades plane and to the axis of the tower allowed to rotate around, see Figure 2. The control system design mainly consists in the definition of the elastic stiffness of the rotational spring \(k_r\) and of the strength of the RFD \(M_{fr,y}\).
It has been optimized with reference to the case-study wind turbine, finding a suitable couple of values for $k_s$ and $M_{r,y}$. This one allows to achieve a good compromise between the two conflicting performance objectives since it leads to a strong reduction in the base bending moment demand keeping at the same time top displacement lower than that in the fixed base condition.

The proposed technique turns out to be very attractive, since it leads to a significant reduction of the bending moment, also being able to re-center the tower once the wind action ends.

Figure 2: Wind turbine equipped with the RFD. a) Lateral view with the application of the forces along the height of the tower, b) section AA.
2 CASE-STUDY APPLICATION: THE NREL 5 MW WIND TURBINE

The structural model of the RFD designed in parallel with the rotational spring is indicated in Figure 3, where \( m, c \) and \( k \) represent the mass, damping and stiffness of the wind turbine, \( c_m \) is the damping of a non-linear Maxwell element with velocity exponent \( \alpha = 10^{-3} \) adopted – to avoid numerical problems – to model an equivalent friction of strength \( M_{fr,y} \left( c_m = M_{fr,y} \right) \); \( k_m \) is its stiffness assumed high enough so as to behave like a rigid link, and whose value is defined considering \( k_m/c_m = 10^6 \) Hz, \( k_s \) represents the stiffness of the rotational spring. This special restraint allows reducing the stress at the base of the tower dissipating energy thanks to the friction, while re-centering the tower thanks to the spring after and during the history load case.

![Figure 3: Model of the base restraint at the base of the tower.](image)

The equations describing the behavior of the spring and the RFD are respectively Eq. 1 and Eq. 2:

\[
M_s(t) = -k_s \times \Theta(t) \quad (1)
\]
\[
M_{fr}(t) = -c|\Theta(t)|^{\alpha} \text{sign} (\dot{\Theta}(t)) \quad (2)
\]

where \( \Theta(t) \) is the rotation of the base in the instant of time \( t \) and \( \dot{\Theta}(t) \) is the velocity.

2.1 Optimal calibration of the base restraint

The design of the proposed control system mainly consists in finding a suitable couple of values for stiffness of the rotational spring at the base \( k_s \) and strength of the rotational friction device \( M_{fr,y} \). The bigger the stiffness of the spring, the closer they get to the case of the fixed base. The same happens with high strength friction. On the other hand, installing a poorly rigid spring could mean inducing large displacements at the top and dangerous P-Delta effects. Even the use of low resistance frictions can be harmful. In fact, in this case, even if the base of the tower would rotate more often and due to low forces, the energy dissipation that would be gained thanks to the opposition of the friction would be very limited. It is therefore necessary to identify a pair of \((k_s, M_{fr,y})\) values that achieve a good compromise. The two - generally conflicting - performance objectives are the significant reduction of the base bending moment demand and the limitation of top displacement within the peak value you would achieve in the fixed base case.

The procedure herein adopted to find the above trade-off solution, given an extreme load assumed as reference for wind action, starts from the evaluation of the Fixed Base (FB) response and the identification of the peak response value for base bending moment \( M_{FB,max} \) and top displacement \( u_{FB,max} \). Then the optimal couple of parameters \((k_s, M_{fr,y})\) for the Rotating Base
case (RB) has to be found within those falling in a reasonable range (0.5 to 2 times the lateral stiffness of the FB tower, for $k_s$; 0.05 to 0.5 times $M_{FB,\text{max}}$, for $M_{fr,y}$) as the one that fulfill the following requirements:

1. the best solution has to be the one leading to the highest reduction of bending moment demand in comparison to the FB case while meeting the criteria 2 and 3 below;

2. the maximum top displacement is lower than or equal to that in the FB case;

3. the residual top displacement, associated to the residual base rotation at the end of the wind load, is lower than or equal to $H/300$, being $H$ the hub height expressed in meters.

The latter is a limit value defined according to engineering judgements to limit P-Delta effects and to ensure the re-centering of the tower.

It should be noted that, given two values of $M_{fr,y}$ and $k_s$, the ratio $M_{fr,y}/k_s$ corresponds to a critical value of the base rotation, say $\theta_{cr}$. Once the wind action is over, if the base rotation is greater than $\theta_{cr}$ (neglecting for a moment the influence of P-Delta effects), the spring can no longer overcome the resistance of the friction and then it is no longer able to re-center the base of the tower. This observation allows to discard pairs of values $(k_s, M_{fr,y})$ which lead to values of $\theta_{cr}$ for which the top displacement (roughly, $\theta_{cr} \cdot H$) exceeds the limit of $H/300$. It can therefore be concluded that the pairs of $(k_s, M_{fr,y})$ values for which $M_{fr,y}/k_s > 1/300 = 0.0033$ radians should be discarded a priori.

3 CASE-STUDY APPLICATION: THE NREL 5 MW WIND TURBINE

The turbine used as reference is the benchmark one, NREL 5 MW from [13]. Some technical details are reported in the following Table 1. The wind turbine has been investigated and modeled with SAP2000 [15]. Hub, nacelle and blades have been considered as mass applied at the top of the tower, the effects due to the wind on these structural components have been taken into account as force and moment applied at the top.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
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</tr>
<tr>
<td>Tower height</td>
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</tr>
<tr>
<td>Blades length</td>
<td>61.5 m</td>
</tr>
<tr>
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<td>126 m</td>
</tr>
<tr>
<td>External tower top diameter</td>
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</tr>
<tr>
<td>External tower base diameter</td>
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</tr>
<tr>
<td>Thickness bottom</td>
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</tr>
<tr>
<td>Thickness top</td>
<td>0.0019 m</td>
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<td>240000 kg</td>
</tr>
<tr>
<td>Blades mass</td>
<td>53220 kg</td>
</tr>
<tr>
<td>Tower mass</td>
<td>347460 kg</td>
</tr>
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</table>

Table 1: NREL 5 MW [13] wind turbine essential data.
3.1 Tower modeling

The tower, an hollow circular one, made of steel, has been modeled according to the methodology that authors introduced in the past [16 – 18] with reference to smart dissipation associated to the base rocking.

As previously defined hub, blades, and nacelle have been considered as mass applied at the top. Each element of the tower has uniform section and diameter, assumed equal to those at half height of that piece. The last element is more stiff and having stiffness equal to the double of the second last element before the base, in order to identify the stiffness due to the link with the nacelle at the top. The structural model has 75 degrees of freedom (DOFs), since each of the 37 joints distributed from the base, that has only 1 DOF being able only to rotate, up to the top has 2 DOFs (translation along x axis, rotation around y axis). The mass has been modeled as continuous along the height, associated to the local cross section of the tower and to the weight for unit volume of the steel 7850 KN/m$^3$. The lumped mass at the top is equal to 350 t and accounts for the hub, blades and nacelle.

Two models of the tower have been generated: one “fixed base” (FB), where the base of the tower is fully restrained against rotation and translation, the other one where the rotation of the joint at the base is allowed (RB) in only one direction, the same of the wind, and faced off by the presence of the rotational spring and friction. In the last case, the base restraint has been modeled as a non-linear Maxwell element in parallel with the rotational spring, as said above (Figure 3).

3.2 Wind load and main results

The load applied to the tower has been a parking one defined according to IEC 61400-1 [14]. This is an exceptional wind load with peak of about 70 m/s. In this condition the blades are stopped. Naturally, the wind changes not only during time, but also along the height of the tower hence the representation of the wind at predefined heights as shown in Figure 4.

Figure 4: Wind load applied along the height of the tower.
According to the criteria given in Section 2, the optimal values for $k_s$ and $M_{fr,y}$ in this case resulted to be:

- $k_s = 14'524$ MNm/rad, that corresponds to about 1.6 times the lateral stiffness of the FB tower;
- $M_{fr,y} = 48$ MNm, that is around 0.15 times the $M_{FB,max}$ value ($M_{FB,max}=343$ MNm).

Note that, for the selected values, it results $M_{fr,y}/k_s = 0.0033$ radians, therefore still acceptable.

Figures 5, 6 show the results obtained by the application of the wind along the height of the tower. A time window of 50 seconds has been added after the end of the load to observe the free oscillations of the tower and the residual top displacement. The latter results to be equal to 10 mm, so well below the limit of $H/300=300$ mm.

The results obtained are summarized in Table 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Top displacement</th>
<th>Base moment</th>
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</thead>
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<tr>
<td>Fixed Base</td>
<td>2.61 m</td>
<td>343 MNm</td>
</tr>
<tr>
<td>Rotating Base</td>
<td>2.54 m</td>
<td>206 MNm</td>
</tr>
<tr>
<td>Percentage variation</td>
<td>-3 %</td>
<td>-40 %</td>
</tr>
</tbody>
</table>

Table 2: Comparison of peak response value for FB and RB cases.

Figure 5: a) Elastic moment of the spring, b) friction moment of the RFD, c) base rotation, d) cyclic response of the RFD.
Figure 6: Comparison between a) top displacement ($u_{\text{top}}$) and b) base moment ($M_{\text{base}}$) for the NREL 5 MW wind turbine subjected to parking load in the cases of fixed and rotating base.

As expected, Figures 5a and 5c are similar since $\Theta$ and $M_s$ are proportional according to the scale factor $k_S$. Looking at Figures 5b and 5c, it is worth noting that each time $|M_{fr}|$ reaches $M_{fr,y}$, the base rotation changes. When $|M_{fr}|$ is less than $M_{fr,y}$, conversely, the base rotation does not change. This confirms the effectiveness of the numerical model in reproducing the response of the RFD. Finally, the number and width of the hysteretic loops in Figure 5d demonstrate the high capability of the RFD in dissipating energy. The last aspect is of fundamental importance and explains how it is possible, thanks to the high dissipation of the input energy, to relax the constraint at the base of the tower - and strongly reduce the moment demand - without incurring in undesired increasing of displacements.

4 CONCLUSIONS

The numerical study conducted for an exceptional wind load such that of parking on the reference wind turbine NREL 5MW showed that the application of a new passive base restraint composed by a RFD and a rotational spring in parallel may lead to strongly mitigate structural
demand to a wind turbine tower. After having calibrated the parameters of the base restraint, the case-study application led to the reduction of the moment at the base of the tower of about the 40% even containing the top displacement compared to the FB response, also allowing the final re-centering of the tower.

Due to these results, the use of friction dampers to control the structural response of wind turbine towers demonstrates to be a good and cheap solution compared to active and semi-active ones, but also to other passive techniques proposed in literature. The main limit in the use of this kind of base restraint at this moment is that best combo of values for friction strength and spring stiffness, load depending. Future developments will see the authors investigate the effectiveness of the control technique for different wind loads, as well as when the load is applied in two horizontal directions simultaneously. Even the study of the long-term behavior of the RFD should be investigated, in order to understand if and how the use and the environmental exposure may affect the response over time.

ACKNOWLEDGMENTS

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THE SEISMIC RETROFIT BY EXTERNAL DISSIPATIVE SYSTEMS: A CASE STUDY

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Abstract

External dissipative systems are seismic retrofit solutions for existing building frames that present some feasibility advantages with respect to more classic solutions involving dissipative devices installed within existing frames. This study analyses and compares the effectiveness of three alternative external retrofit configurations based on the use of fluid viscous dampers (FVDs) with reference to a case study involving the seismic upgrading of a typical school r.c. frame. In particular, the three considered systems are: a rocking base system consisting of a pinned-rocking braced frames equipped with fluid viscous dampers at their base, a fixed base system consisting of external fixed-base braced frames connected to the frame through dampers located at floor levels and a diagonal bracings system similar to the traditional arrangement, but installed adjacent to the frame. All the proposed retrofit solutions are analysed taking into account the non-linear behaviour of the r.c. frame building (designed with inadequate seismic detailing) and both the linear and non-linear behaviour of the FVDs. The obtained results show that all the systems are effective solutions, even if performances in terms of interstorey-drifts, absolute floor accelerations, forces and strokes of the dampers are different.

Keywords: External Retrofitting Systems, Pinned-Rocking Bracings, Fixed Base System, Diagonal Bracings System, Non-Linear behaviour, Fluid Viscous Dampers.
1 INTRODUCTION

During the last three decades, passive protection techniques have been proved to be efficient solutions for the seismic upgrading of existing structures as well as for new realizations [1]-[5]. Among them, fluid viscous dampers (FVDs) are particularly diffused, due to their large capability of dissipating the seismic input energy without any damage to the devices even for small displacements [4].

Traditional arrangements for the protection of r.c. frame building involve the installation of FVDs on diagonal or V-shaped braces, connecting adjacent floors within the building frames. This type of configuration is widely studied and tested [6]-[10], but it has some disadvantages especially in the case of existing buildings. Generally, tensile and compressive forces increase in the columns leading to fragile ruptures [11], [12] and localized strengthening of beam-column joints or foundations may be necessary. Moreover, in the case of existing buildings the demolition and reconstruction of internal partition walls, the installation of bracings and possible structural reinforcement works, cause downtime of buildings and related indirect costs for the temporary relocation of the activities. For these reasons, recently, the interest in external passive control systems has increased thanks to their lower interference with both the existing frame and the activities carried out in it [13]. More in detail, with outer arrangements the interventions on the existing frame mainly concern the areas of connection with the external bracings and protection systems have an independent foundation. Furthermore, the external systems are more versatile and can be applied in different configurations, allowing a large flexibility in the control of the structural behaviour of the building to be protected.

In some recent studies, [14] and [15], the dynamic behaviour of frame structures coupled with external systems has been studied, accounting for the non-classically damped nature of the coupled system. In [15] a performance comparison between two external system configurations is done, first in terms of dynamic properties and then in terms of seismic response of different engineering parameters. However, these studies are limited to plane problems and linear behaviour of both the coupled system components.

In this paper three different external bracing configurations are considered and applied to a real three-dimensional case study, consisting of a r.c. school building designed without seismic details and with irregularities both in plan and in height. The non-linear behaviour of both the r.c. frame and the viscous dampers is taken into consideration. In particular, in the first configuration, named fixed-base (FB) system, FVDs are located at the floor level between the existing frame and an external stiff contrasting structure. In the second configuration, named diagonal bracing (DB) system, FVDs are located on diagonal bracings outside the building. The last arrangement is the rocking base (RB) system and consists of an external stiff bracing structure pinned at the base and rigidly linked to the existing frame at the floor levels, which exploits the rocking motion of the base, where FVDs are located in vertical position. For all the three configurations, a preliminary design is carried out accounting for the nonlinear behaviour of the existing r.c. frame and considering both the cases of linear and nonlinear behaviour of FVDs. Then, the seismic response of the coupled systems is evaluated in all the proposed solutions, by performing nonlinear dynamic analyses. Different demand parameters have been considered in order to compare the seismic performance of both structural and non-structural components of the existing frame, as well as the external dissipative systems.

2 EXTERNAL BRACING CONFIGURATIONS

The external arrangements analysed in this paper are depicted in Figure 1. They are characterized by different kinematic behaviours and can be realized either with three-dimensional bracings, effective in both the two main directions of the frame, either with planar trusses placed
in the two main directions of the building. In the latter case, the dissipative system can be specialized for each direction and can be realized adjacent to the frame to reduce the external space and the overall dimensions.

The first configuration (Figure 1a) is the FB system, where FVDs deformation are proportional to the floor displacements and their efficiency is strictly related to the stiffness of the external bracings. A similar solution can be achieved in the case of dissipative connection between two adjacent structures with different dynamic properties, as described in [16], [17]. Generally, the FB system does not significantly modify the modal properties of the existing frame. In the case of linear FVDs, whose constants are proportional to the floor masses, the vibration modes of the bare frame do not change and the system is classically damped. Figure 1 b) depicts the DB configuration, where the viscous devices deformation is proportional to the interstorey-drifts. In this case, too, the external system does not significantly modify the modal properties of the existing frame and in the case of linear FVDs, whose constants are proportional to the floor stiffness, the vibration modes of the bare frame do not change and the system is classically damped. Moreover, in this configuration, the external system may also have its own stiffness, which can be modulated at each floor level if necessary, in order to achieve a regularization of the building deformation.

The last arrangement, shown in Figure 1 c), deals with a quite recent solution, known in its three-dimensional configuration as “Dissipative Tower” [18]. This system consists of an external stiff bracing system pinned at its base and rigidly linked to the existing frame at the floor levels in order to exploit the rocking motion of the base (RB). The FVDs are located in vertical position at the base of the truss and their deformations are proportional to the vertical displacements of the basement, induced by the rocking motion. As highlighted in [19], during the rocking motion, the self-weight of the external bracing can determine an overturning destabilizing contribution; therefore, it is of extreme importance to minimize the mass of the external structure, by realizing it with steel members. Moreover, the stiffness of the external bracing is a useful tool to modify the deformed shape of the frame, leading to a linear distribution of floor displacements. Despite the RB system is highly non-classically damped, this does not affect the dynamic response of the coupled system.

More details about the dynamic behaviour of external dissipative systems in the RB and FB configurations, can be found in [14] and [15] in the case of linear and plane structural systems.

Figure 1: a) fixed base system, b) diagonal bracing system, c) rocking base system.

3 CASE STUDY BUILDING

3.1 Description of the existing building

The design criteria and the seismic performance of the three systems under investigation are discussed with reference to the r.c. frame building highlighted in green in Figure 2, which is part of the “Parrozzani” school complex of Isola del Gran Sasso, in the central Italy area.
The case study is an r.c. frame building, realized between the ‘60s and ‘70s without seismic detailing, and it consists of two elevations above the ground level and one partially underground with reduced dimensions, for an overall height of 11.25 m (each elevation is $h_i=3.75$ m). The building has already been studied within the ReLUIS project and further information related to the unretrofitted structure assessment can be found in [20].

The building has girder beams on the perimeter and in the internal transverse X direction, while the internal central ones in the longitudinal Y direction have the same thickness of the floor slab. An eccentric body scale is located in the part of the building characterized by three elevations. In the same area columns are oriented with maximum inertia along Y direction, while in the remaining portion of the building, they have opposite orientation (maximum inertia in X direction). A normal level of knowledge (KL2) [21] of the building has been achieved thanks to the tests performed on the materials, providing useful information regarding the reinforcement of the members and the average compressive strength of concrete ($f_{cm}=16.6$ MPa) and tensile yielding strength of steel rebars ($f_{ym}=390.8$ MPa). The vulnerability assessment of the building has been carried out by considering the seismic action referring to the building site and relevant local conditions (soil category B and topographical category $T_1$).

3.2 Structural model of the existing building

Figure 3 depicts the finite element model (FEM) of the r.c. frame building, where beams and columns are modelled as linear elastic elements with plastic hinges located at their ends. The same figure reports the first three vibration modes obtained by considering linear properties of the structural members. Modal properties of the bare frame in terms of vibration period ($T$) and participant masses ($M_x$, $M_y$ and $M_0$) are reported in Table 1. The first mode is purely translational in the longitudinal Y direction, while the second and third are coupled in the X-0 directions.
As illustrated in the next sections, the non-linear behaviour of the building, described through flexural ductile plastic hinges located at the ends of beams and columns, has been considered through nonlinear static (pushover) analyses for both the frame vulnerability assessment and dampers design procedures. Successively, nonlinear dynamic analyses of the coupled systems have been carried out, also accounting for the degrading mechanism typical of r.c. members with smooth rebars, through the pinching effect.

### 3.3 Vulnerability assessment of the structure

The seismic assessment of the existing building has been conducted through non-linear static analysis (pushover) which results in capacity curves expressing the base shear (V) versus the top floor displacement (d). A force profile consistent with the first vibration mode in the considered direction has been adopted. Modal coefficients \( \phi_{1,i} \) of the i-th floor centre of mass, normalized with respect to the top floor, are reported in Table 2 and refer to the first and third modes reported in Table 1. Also the corresponding inter-storey drifts \( \delta_{1,i} \) are reported in Table 2. The generalized mass \( m_i^* \), the first mode equivalent mass \( m_1 \) and the coefficient of participation \( \Gamma_1 \), can be calculated according to the following relationships:

\[
m_i^* = \sum_i m_i \phi_{1,i} \tag{1}
\]

\[
m_i = \sum_i m_i (\phi_{1,i})^2 \tag{2}
\]

\[
\Gamma_1 = m_i^*/m_i \tag{3}
\]
where $m_i$ symbolizes the $i$-th floor mass. For the X direction $m_1^* = 744.1 \text{ kN}s^2/m$ and $\Gamma_1 = 1.28$, while for the Y direction $m_1^* = 876.3 \text{ kN}s^2/m$ and $\Gamma_1 = 1.25$.

<table>
<thead>
<tr>
<th>level</th>
<th>height [m]</th>
<th>$m_i$ [kN$s^2$/m]</th>
<th>$\phi_{1,i}$ [-]</th>
<th>$\delta_{1,i}$ [-]</th>
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</tbody>
</table>

Table 2: Features of the first vibration mode in the transverse X and longitudinal Y directions.

The assessment of the seismic vulnerability of the existing frame requires the identification of the target displacement ($d_u$), related to the level of damage considered acceptable for the design seismic action. Successively, it is necessary to determine the bilinear curve of the single degree of freedom (S-DOF) system equivalent to the capacity curve of the M-DOF one, in terms of $V^*_1 = V/\Gamma_1$ and $d^*_1 = d/\Gamma_1$. Figure 4 illustrates the curve referring to the most critical situation in the Y direction. In this analysis case, the target displacement is $d_u = 0.067$ m and corresponds to the activation of a local ductile mechanism involving the staircase. The corresponding elastoplastic curve, determined by means of the equivalence of dissipated energy proposed in [22], is also reported in Figure 4. From the bilinear curve, the elastic limit of the base shear $V_1^*$ and the ductility $\mu_b = d_u^*/d_y^*$ of the bare frame associated to the selected target displacements can be obtained.

![Figure 4: Capacity curve and equivalent elasto-plastic system in the longitudinal Y direction.](image)

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In order to combine the response of the bare frame with the response of the external viscous bracing system it is useful to identify the equivalent S-DOF visco-elastic system. The equivalent stiffness $K_b^*$ and the equivalent damping $\xi_b$ of the bare frame can be derived as follows:

$$K_b^* = V_1^* / d_u^*$$

$$\xi_b = 0.05 + k \frac{2(\mu_b - 1)}{\pi \mu_b}$$

where the reduction factor $k$, which varies in the range $0.33$-$1$, can be assigned according to [23]. For existing buildings realized without seismic details the recommended value is $k = 0.33$, which accounts for a degrading cyclic behavior of the r.c. frame. Table 3 shows the characteristics of the equivalent viscoelastic systems, obtained according to the procedure previously described, for both the transversal X and longitudinal Y direction.

Figure 5 depicts elastic spectrum at the ULS (black curve) representing the seismic demand in the acceleration-displacement response spectrum (ADRS) plane, referring to the case study building (Isola del Gran Sasso, soil category B and topographical category $T_1$). The reduced demand spectrum (blue curve) consistently with the damping of the bare frame obtained for the Y direction ($\xi_b = 0.11$) is also reported. The capacity of the viscoelastic system equivalent to the bare frame (orange curve) is lower than the reduced demand spectrum, confirming the need of increasing the seismic performance of the structure.

<table>
<thead>
<tr>
<th>direction</th>
<th>$d_u$ [m]</th>
<th>$K_b^*$ [kN/m]</th>
<th>$T_b^*$ [s]</th>
<th>$\omega_b^*$ [rad/s]</th>
<th>$\xi_b$ [-]</th>
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<td>X</td>
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<td>39852</td>
<td>0.859</td>
<td>7.32</td>
<td>0.12</td>
</tr>
<tr>
<td>Y</td>
<td>0.067</td>
<td>26792</td>
<td>1.136</td>
<td>5.53</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 3: Equivalent viscous systems for the two main directions (X and Y).

Figure 5: Visco-elastic equivalent system in the ADRS plane for the longitudinal Y direction.

4 DESIGN OF THE EXTERNAL RETROFIT SYSTEMS

Preliminary sizing criteria with velocity-dependent devices (viscous or visco-elastic) and displacement-dependent devices (elasto-plastic) are widely treated in literature and some references can be found in [24]-[26]. In this application, external dissipative bracings contribute
only in terms of added damping $\xi_d$, without modify the overall stiffness of the existing frame. Consequently, global dimensions of dampers can be easily determined by finding the additional damping necessary to meet the seismic demand. In the Y direction the required additional damping is $\xi_d = 0.19$, which added to the damping associated with the frame provides a total damping equal to $\xi_{tot} = \xi_b + \xi_d = 0.30$. In the X direction the contribution required for the FVDs is relatively low, equal to $\xi_d = 0.05$, for a corresponding total damping of $\xi_{tot} = 0.17$. However, due to building asymmetry in the X direction, a total damping equal to $\xi_{tot} = 0.13 + 0.12 = 0.25$ is assigned in order to reduce rotational effects in this direction.

Form the additional damping $\xi_d$, dampers dimension for each considered direction can be determined by using the general expression proposed by [27]:

$$\xi_d = \frac{\sum_{j=1}^{N} E_j}{4\pi E_f} 
$$

where

$$E_j = m_i \left( d_{o} \omega_b \right)^2 / 2 
$$

expresses the energy dissipated by the $j$-th device (characterized by the viscous constant $c_d$) in one complete vibration cycle at frequency $\omega_b$ and amplitude $s_j$, which is the relative displacement between the ends of the device. It is worth to note that damper displacements can be expressed as a function of the floor displacements of the coupled system, through a linear operator $L$ accounting for their position. By collecting dampers displacement ($s_j$) in the vector $s$ and floor displacements ($u_i$) in the vector $u$, the following relationship can be defined:

$$s = Lu 
$$

where, in the FB and DB systems which do not modify the modal shapes of the existing building, the first mode floor displacement ($d_{o} \phi_1$) of the bare frame in the considered direction can be assumed as the $i$-th component of the vector $s$. Differently, the RB arrangement leads to a linear displacement distribution of the coupled system, due to the stiffening effect of the external truss. Thus, in this latter case, the $i$-th component of the vector $s$ can be expressed as $d_{o} z_i / H$, where $z_i$ is the $i$-th floor height normalized with respect to the total height of the building ($H$).

In the case of dampers having all the same properties and linear behavior, expression (6) can be rearranged to provide the viscous constant $c_d$ of the dampers as follows:

$$c_d = \xi_d \frac{2m_i d_{o}^2 \omega_b}{\sum_j s_j^2} 
$$

Figure 6 shows the position and the nomenclature ($X_A$, $X_B$ and $Y_A$) of the external structures designed for each direction, namely 3 in transversal X direction (blue lines) and 4 in the Y direction (red lines). In the FB system, 2 FVDs per floor are installed for each external bracing, for a total amount of 12 devices in the X direction and 16 in the Y direction. In the RB system,
there are 2 FVDs for each lattice girder, located vertically at its base, for a total amount of 6 and 8 devices in X and Y directions, respectively. For the DB configuration 2FVDs are installed on diagonal braces for each external structure, for a total amount of 6 and 8 devices in X and Y directions, respectively. For all the proposed configurations, the external truss called $X_A$ adjacent to the existing stairwell extends for three elevations, but only the last two levels are connected to the bare frame. The viscous constants obtained for the three considered cases are reported in Table 4.

The plan dimensions of the bracings are consistent with the dimensions of the spans of the existing frame and they are detailed in Figure 7. The same figure also describes steel trusses adopted for each investigated configuration, whose steel members have been designed so that the forces are lower than buckling loads. By adopting a S275 steel, the HE260A profile is used for the FB system and HE200A profiles for the RB and DB configurations.

![Figure 6: Position of the external dissipative bracings for all the configurations proposed.](image)

![Figure 7: Geometry and number of the FVDs for each type of external structure.](image)
Finally, in the case of nonlinear behaviour the energy dissipated in a cycle by the nonlinear device with constant $\alpha$ can be expressed as:

$$E_{dj}^{NL} = 2\sqrt{\pi} c_d^{NL} (\omega_b)^{\alpha} \left( s_j \right)^{1+\alpha} \frac{\Gamma \left( 1 + \alpha / 2 \right)}{\Gamma \left( 3 / 2 + \alpha / 2 \right)}$$

(11)

where $\Gamma(\cdot)$ is the gamma function. The nonlinear dissipative system has been designed in order to be equivalent to the linear one at the design condition. In particular, the equivalence criterion used requires that the energy dissipated by the two systems is the same for a cycle with angular frequency $\omega_b$ and amplitudes $s_j$. In the case of dampers having all the same properties, the equivalence criterion leads to a viscous constant $c_d^{NL}$ of the nonlinear dampers given by the following expression:

$$c_d^{NL} = 2\sqrt{\pi} c_d \left( \omega_b \right)^{1-\alpha} \left( \sum_j s_j^2 \right)^{1+\alpha} \frac{\Gamma \left( 3 / 2 + \alpha / 2 \right)}{\Gamma \left( 1 + \alpha / 2 \right)}$$

(12)

Nonlinear devices with $\alpha = 0.15$ are adopted and the obtained constants are reported in Table 5.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$c_d^{NL}$ [kNs/m]</th>
<th>$c_d^{NL}$ [kNs/m]</th>
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<tr>
<td>FB</td>
<td>53</td>
<td>39</td>
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<tr>
<td>DB</td>
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<td>216</td>
</tr>
<tr>
<td>RB</td>
<td>321</td>
<td>334</td>
</tr>
</tbody>
</table>

Table 5: Values of the damping constants for the nonlinear devices.

5 NONLINEAR DYNAMIC ANALYSES RESULTS

The seismic performance of the bare frame and of the proposed upgrading configurations (Figure 8) is evaluated by means of nonlinear dynamic analyses (time-histories) performed with the SAP2000 program [28][23]. The pivot hysteretic model with a remarkable pinching effect is adopted for the plastic hinges, to describe the degrading mechanism typical of r.c. members with smooth rebars. The FVDs are modelled by two joints links, having a linear behaviour in the case of linear devices and a nonlinear behaviour (exponential Maxwell dampers with $\alpha=0.15$) for nonlinear devices.
The seismic scenario is described by seven groups of artificial earthquakes, generated by the SIMQKE_GR software, according to the design spectrum at the ULS. The following parameters are monitored to evaluate the seismic performance: floor displacements \( (u_i) \), interstorey-drifts \( (\delta_i) \), floor absolute accelerations \( (a_i) \) and stroke and force of the FVDs \( (s_j, F_j) \). All the results are in terms of mean values of the maximum response parameter obtained for the seven time-history analyses performed. Figure 9 highlights the nomenclature and positioning of the investigated columns. The control of three different verticals, that is the centre of mass (CM) and two points located at opposite ends of the frame (A and B), permits to provide information on the rotation component of the motion of bare frame.

5.1 Linear FVDs

Table 6 contains the results in terms of floor displacements of the bare frame and the three retrofit configurations analysed (RB, FB and DB systems). For all the three systems, a significant displacements reduction is observed, especially in the longitudinal Y direction, which is the most deformable one. In particular, all the systems lead to the same reduction of the top floor displacement of CM, which is nearly 45% in the Y direction and about 15% in the X direction.

<table>
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<th>FB</th>
<th>DB</th>
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<td></td>
<td></td>
<td>( u_{x,i} )</td>
<td>( u_{y,i} )</td>
<td>( u_{x,i} )</td>
<td>( u_{y,i} )</td>
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<tr>
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<td>0.082</td>
<td>0.027</td>
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</table>

Table 6: Floor displacements for all the analysed configurations.
Table 7 shows the results in terms of the interstorey-drift (IDR). It is important to note that, the bare frame shows IDRs significantly larger than the 1% of the floor height ($h_i$), which is the limit for structural damages proposed by the standards [29]. On the contrary, all the investigated configurations are able to guarantee IDRs significantly lower, with maximum values of about 0.7% for the RB system (in both the directions), of 0.86% for the FB system (in the Y direction) and of 0.84% for the DB system (in the Y direction). Thus, limited damages are expected for all the external systems. The larger effectiveness of the RB system is due to its ability to make the floor displacements distribution approximately linear and consequently, to obtain uniform values of IDRs. Differently, as already explained, the DB and FB configurations does not change the displacements distribution of the original frame.

<table>
<thead>
<tr>
<th>Vertical alignment</th>
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<td>0.0059</td>
<td>0.0102</td>
<td>0.0035</td>
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</tbody>
</table>

Table 7: IDRs for all the analysed configurations.

In order to highlight the damaging reduction, the comparison between hysteretic loops before and after the retrofit of a plastic hinge (located at the base of a column, close to the vertical alignment B) is shown in Figure 10. The bare frame performance is reported with grey lines, while the three external arrangements are depicted with, respectively blue (RB), violet (FB) and red (DB) lines. It can be observed that all the external systems are able to reduce the amplitude of the loops with values of the moment larger than the cracking limit, but sensibly smaller than the yielding one. The FB and DB configurations provide similar performances, while the RB system seems to provides the best result.

Table 8 contains the results in terms of floor absolute accelerations, which are a particularly sensitive parameter with reference to the protection of non-structural components, such as machinery and contents. In this case, differently from previous response parameters, the external FB and DB systems are more effective with respect to the RB one in reducing the absolute accelerations. In particular, in the case of RB configuration floor accelerations slightly reduce
in the X direction, while they increase in the Y direction. Differently, for the FB configuration and even more for the DB one, accelerations reduce in both the X and Y directions.

<table>
<thead>
<tr>
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<th>FB</th>
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<td>5.27</td>
<td>2.96</td>
<td>4.33</td>
<td>3.13</td>
</tr>
</tbody>
</table>

Table 8: Floor absolute accelerations for all the analysed configurations.

Finally, Table 9 shows values of strokes and forces recorded on the FVDs placed at the base of the RB system and at the second and third level (first and second row of the table) for the FB and DB configurations. It is useful to recall that the FB system has a double number of devices with respect to the other systems; however, forces are notably lower than half of those obtained for the other configurations. On the other hand, strokes are notably larger because devices work for floor displacements and not for interstorey-drifts or rocking displacements. In particular, for the FB configuration, stokes span from 18.7 mm to 51 mm, whereas for the DB configuration values are in the range of 11.2 mm-21 mm. The smallest strokes are obtained for the RB configuration, where the vertical dampers stroke is 11.7 mm in the Y direction and 13.6 in the X direction. At this regard, it is recalled that very low strokes are difficult to be guaranteed in a real application.

<table>
<thead>
<tr>
<th></th>
<th>RB</th>
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<tr>
<td>$F_y$</td>
<td>-</td>
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</tr>
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</table>

Table 9: Dampers stroke and force for all the analysed configurations.

5.2 Non-linear FVDs

In this section, results achieved by using nonlinear FVDs are reported. As expected, the nonlinear system provides similar performances with respect to the linear one, due to the energy equality criterion applied. However, from the results comparison it can be observed that in the case of nonlinear dampers slightly smaller displacements are obtained, both on the frame (Table 10) and on the dampers (Table 11) for all the three configurations. It is worth to note that smaller displacements of the frame is an positive result because they yield to smaller damages on the frame, while smaller strokes on the devices may make further difficult to guarantee their performance. On the other hand, the absolute accelerations on the frame tend to be larger, especially for the FB and DB configurations at the last elevation in the Y direction (Table 12).
Table 10: Floor displacements in the case of nonlinear FVDs.

<table>
<thead>
<tr>
<th>Vertical alignment</th>
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<th>RB $u_{y,i}$</th>
<th>FB $u_{x,i}$</th>
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Table 11: Nonlinear FVDs strokes and forces at SLV.

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Table 12: Absolute accelerations in the case of nonlinear FVDs.

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6 CONCLUSIONS

In this work, three different configurations of external retrofitting systems, equipped with fluid viscous dampers, have been adopted for the seismic upgrading of a school r.c. frame building, by assuming the same amount of additional damping. Moreover, for each configuration both the cases of linear and nonlinear dampers has been analysed and compared, assuming an equivalence energy criterion at the design condition. With reference to the linear case, it is found that the three configurations proposed (RB, FB and DB) exhibit similar performances. However, the effectiveness of the FB and DB arrangements is higher in limiting the absolute accelerations, whose impact is significant especially for machinery and contents. The RB system, instead, provides smaller interstorey-drifts, thanks to the stiffness of the external bracing, which tends to linearize the distribution of floor displacements. For what concerns the response of FVDs, the FB configuration seems to be the better solution providing the smallest forces and
the largest strokes. Differently, in the DB and RB arrangements dampers work for higher forces and for very small displacements, which may be difficult to guarantee in a real application. Finally, the comparison between linear and non-linear fluid viscous dampers has shown that for all the three configurations analysed, similar performances are obtained. However floor displacements are slightly lower, whereas absolute accelerations slightly larger. Obviously, a comprehensive comparison between the linear and nonlinear cases requires to investigate the performances of the coupled systems at more limit states different from the design one, where the equivalence criterion is applied.

REFERENCES


OPTIMAL DISSIPATIVE COUPLING DESIGN OF TWO OSCILLATORS BASED ON NONLINEAR STOCHASTIC RESPONSE

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Abstract

The work proposes an optimization procedure for the design of a nonlinear dissipative coupling device of two oscillators subjected to stochastic dynamic excitation. Two simple oscillators connected by Bouc-Wen type hysteretic device describe the system. Zero-mean filtered white noise is the base excitation. The knowledge of the stationary response variances, evaluated by equivalent linearization permits to define the optimization problem to design the device. Finally, Monte Carlo simulations using filtered white noise and natural earthquake records at different levels of seismic magnitude conclude the evaluation of the procedure efficacy.

Keywords: Structural optimization, nonlinear response, dissipative coupling, stochastic load
1 INTRODUCTION

One of the mainly target pursued by structural design methods is to obtain a solution that assure an optimal performance. Structural optimization procedures are often proven to be an efficient tool to balance different competing design objectives [1]. Moreover, several loads that have to be taking into account in the structural design process are stochastic and dynamic in nature (like wind and seismic actions) but usually analyses related to optimization techniques are based on dynamic excitation represented by equivalent static load [2]. Many of research works on structural optimization have been driven using Monte Carlo simulations that, of course, can be time consuming and computationally demanding because several simulations are required to get converged response statistics [3]. However, scientific papers have applied random vibration theory to solve structural optimization problems. Some examples are reported in [4], [5] where the aim of the authors is to optimize elements’ size for a linear five-story shear structure subject to stochastic seismic ground motion. This work proposes the optimization of the nonlinear stochastic responses of a simple model composed by two linear oscillators linked by a dissipative element [6]. A depth analysis of synthetic and analytical systems could be very useful for both to explain experimental phenomena and to develop design procedures [7], [8].

2 NONLINEAR ANALYTICAL MODEL

Let to consider a simple nonlinear analytical model composed by two simple oscillators coupled by a nonlinear dissipative devise as reported in Figure 1. They have mass $M_i$ and stiffness $K_i (i = 1, 2)$ and, moreover, are linked together by a dissipative element. Indicating with $U_1$ and $U_2$ the relative displacements and $F$ the force delivered by the damper, the equations of motion of the two-degree-of-freedom model, shown in Figure 1, are governed by the following system:

$$
M_1 \ddot{U}_1 + K_1 U_1 - F = -M_1 U_g \\
M_2 \ddot{U}_2 + K_2 U_2 - F = -M_2 U_g
$$

(1)

where dot indicates the derivative with respect to time $t$. Then, defining the subsequent dimensionless variables and parameters

$$
u_i = \frac{U_i}{L}, \quad \nu_g = \frac{U_g}{L}, \quad \omega_i^2 = \frac{K_i}{M_i}, \quad \beta = \frac{\omega_g}{\omega_i}, \quad 
\rho = \frac{M_2}{M_1}, \quad f_d = \frac{F}{\omega_i^2 M_1 L}, \quad \tau = \omega_i t
$$

(2)

where $L$ is a convenient reference length, $\rho$ and $\beta$ are the mass and frequency ratio between the uncoupled oscillators, the expression of the dimensionless equations of motion written in the synthetic matrix form can be derived:

$$
M \ddot{u} + K u + r f_d = -\Gamma \ddot{u}_g
$$

(3)

In such system $u$ is the displacements vector while $M$ and $K$ are the mass and stiffness matrices. Moreover, the vectors $r$ and $\Gamma$ have been introduced to allocate the control and external forces. The scalar variable $\ddot{u}_g$ represents the seismic ground acceleration. The form of these parameters will be the following:
After formulating the linear system, the nonlinear equation of motion can be generated considering a hysteretic behavior for the dissipative force of the damper. The new nonlinear system looks as following:

\[
\ddot{u} + \mathbf{M} \dot{u} + \mathbf{K} \mathbf{u} + \mathbf{C} \alpha \mathbf{K} \mathbf{d} \mathbf{u} + (1 - \alpha) \mathbf{K} \mathbf{d} \mathbf{u} + \mathbf{g} = \mathbf{M} \dot{\mathbf{g}}
\]

(5)

where looking to the restoring force can be recognized two component related to the linear part modelled as a Kelvin-Voigt model (linear spring and viscous element arranged in parallel) while the nonlinear one is regulated by an adjunct variable \( z \) and moreover the coefficient \( \alpha \) is the post- to pre- yield stiffness ratio for the damper. The linear matrices describing the dissipative constitutive law assume the following form:

\[
\mathbf{K} = \begin{bmatrix} \eta & -\eta \\ -\eta & \eta \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \gamma & -\gamma \\ -\gamma & \gamma \end{bmatrix}
\]

(6)

where \( \eta \) and \( \gamma \) are the dimensionless parameters for spring and dashpot whose form is:

\[
\eta = \frac{K}{\omega_i^2 M_i}, \quad \gamma = \frac{C}{2 \omega_i M_i}
\]

(7)

in which \( K \) and \( C \) are the elastic and viscous coefficient, respectively.

The dynamic evolution of the variable \( z \) is described by a Bouc-Wen model where the coefficients \( \gamma_b \) and \( \beta_b \) control the shape of the hysteresis loop, \( A \) the restoring amplitude and \( n \) the smooth transition from elastic to plastic response (for large values of \( n \) the model tends to an elasto-plastic behavior).

3 NONLINEAR STOCHASTIC RESPONSE BY EQUIVALENT LINEARIZATION

It is well-known that for a state-space model the linear stationary stochastic response can be evaluated through the covariance matrix \( \Gamma \). Indeed, assuming a system written in state-space formulation as in the following:
where $w$ is a zero-mean stationary Gaussian process, $A$ and $B$ are the state-space matrices, $x$ the state-space vector, usually composed by the displacement and velocity of the system, the stationary stochastic responses can be obtained solving the following equation [9]:

$$ 0 = A \Gamma + \Gamma A + 2\pi B S B $$

which is the well-known Lyapunov equation in the unknown covariance matrix. The variable $S$ is the power of the white noise spectral density. It right to remember that the covariance is defined as:

$$ \Gamma_x(t) = \mathbb{E}[x(t)x^T(t)] = \begin{bmatrix} \sigma_{x_1}^2 & \mathbb{E}[x_1 x_2] & \cdots & \mathbb{E}[x_1 x_n] \\ \vdots & \ddots & \ddots & \vdots \\ \mathbb{E}[x_n x_1] & \cdots & \sigma_{x_n}^2 \end{bmatrix} $$

It contains in the main diagonal the expected values of the displacements and velocities (i.e. variances and standard deviations) while out of diagonal are located the mixed expected values.

The nonlinear stochastic response can be well approximate by an equivalent linear system that will allows to easily and quickly evaluate the nonlinear solution solving the previous Lyapunov equation. The evolution of the variable $z$ can be reproduced by the following first-order differential equation:

$$ \dot{z} + C_{eq} \Delta \dot{u} + K_{eq} z = 0 $$

where $\Delta \dot{u} = \dot{u}_2 - \dot{u}_1$.

In [9] the authors evaluated the equivalent of $C_{eq}$ and $K_{eq}$ for Bouc-Wen model in the special case $n = 1$

$$ C_{eq} = \gamma_b \mathbb{E} \left[ z \frac{\partial \Delta \dot{u}}{\partial \Delta \dot{u}} \right] + \beta_b \mathbb{E} [z] - A $$

$$ K_{eq} = \gamma_b \mathbb{E} [\Delta \dot{u}] + \beta_b \mathbb{E} \left[ \Delta \dot{u} \frac{\partial z}{\partial z} \right] $$

Assuming a Gaussian excitation, the equivalent terms can be assessed through second moments:

$$ C_{eq} = \sqrt{\frac{2}{\pi}} \left[ \gamma_b \frac{\mathbb{E}(\Delta \dot{u} z)}{\sigma_{\Delta \dot{u}}} + \beta_b \sigma_z \right] - A $$

$$ K_{eq} = \sqrt{\frac{2}{\pi}} \left[ \gamma_b \sigma_{\Delta \dot{u}} + \beta_b \frac{\mathbb{E}(\Delta \dot{u} z)}{\sigma_z} \right] $$

where $\mathbb{E} (\cdot)$ is the expectation operator while $\sigma$ represents the standard deviation. Moreover, expression for a general case of $n \neq 1$ can be found in Giaralis and Spanos [10].
### 3.1 Structural responses

Defining the state-space vector as

\[ x_s = [u^T \; \dot{u}^T \; z]^T \]  

(16)

the equivalent linear system given putting together the Eq. (5a) and Eq. (11) and subjected to a stochastic excitation assumes the following form:

\[ \dot{x}_s = A_s x_s + B_s w \]

\[ y_s = C_s x_s + D_s w \]  

(17)

The state-space matrices \( A_s \) and \( B_s \) look as:

\[
A_s = \begin{bmatrix}
0 & I & 0 \\
-M^{-1}(K+\alpha K_d) & \alpha M^{-1} C_d & -(1-\alpha)K_d \\
0 & -C_{eq} r^T & -K_{eq} r^T \\
\end{bmatrix}, \quad B_s = \begin{bmatrix}
0 \\
\Gamma \\
0 \\
\end{bmatrix}
\]  

(18)

The matrices related to the output \( y_s \), \( C_s \) and \( D_s \), can be defined to contain arbitrary information about the system (generally \( D_s = 0 \)).

Based on this linearized process, a reasonable evaluation of the nonlinear stochastic response can be carried out solving the Lyapunov equation through an iterative procedure since the equivalent coefficients, \( C_{eq} \) and \( K_{eq} \), depend by the standard deviations.

The robustness of this procedure has been preliminary carried out comparing the results obtained with the linearized procedure and direct integration for a nonlinear oscillator. In this case the hysteretic component of the restoring force is described through a Bouc-Wen model. The equation of motion looks as following:

\[
\ddot{x} + 2\zeta \omega \dot{x} + \alpha \omega^2 x + (1-\alpha)\omega^2 z = w(t) \\
\dot{z} = -\gamma_b |x|^\alpha |z|^{\alpha-1} - \beta_b \dot{x} |z| + A \dot{x}
\]  

(19)

In the below Figure 2 are illustrated two direct integration analyses related to two cases with low and high noise power. Indeed, in the Tables 1 and 2 are reported the results of the standard deviations evaluated with both methods, direct integration and linearized procedure, for

![Figure 2: Direct Integration of the nonlinear oscillator. Parameters: \( \omega = 2\pi1.35, \zeta = 0.05, \alpha = 0.05 \), Bouc-Wen: \( \gamma_b = 0.5, \beta_b = 0.5, n = 1, A = 1 \). Noise Power: (a), (b), (c) \( S = 0.01 \); (d), (e), (f) \( S = 1 \).](image)
The stochastic excitation can be represented as a filtered white noise described, for example, through a Kanai-Tajimi filter:

\[ \ddot{x}_f + 2\zeta\omega_g \dot{x}_f + \omega_g^2 x_f = w(t) \]

\[ \ddot{u}_g = \omega_g^2 x_f + 2\zeta\omega_g \dot{x}_f \]  \hspace{1cm} (20)

The Kanai-Tajimi model is a white noise random process \( w(t) \) passed through a second-order filter, \( \omega_g \) and \( \zeta \) are the site natural frequency and damping of the soil. The latter are properties that depend by local site conditions, seismic hazard maps, etc. [11].

The Eq. (20) can be written in the state-space formulation:

\[ \begin{align*} 
\dot{x}_f &= A_f x_f + B_f w(t) \\
\dot{u}_g &= C_f x_f 
\end{align*} \hspace{1cm} (21) \]

where \( x_f \) is the filtered state vector while the matrices \( A_f, B_f, \) and \( C_f \) are chosen to represent the characteristic of the excitation.

The combined representation between system and excitation can be obtained through an augmented state vector given by

\[ x_a = \begin{bmatrix} x_f^T \\ x_g^T \end{bmatrix} \]  \hspace{1cm} (22)

and so, combining together the equation of the structural system (Eq. 17) and the ones for the loading model (Eq. 20), both written in the state-space formulation, it is possible to get a new form of the augmented system:

\[ \begin{align*} 
\dot{x}_a &= A_s x_a + B_s w(t) \\
y_a &= C_a x_a 
\end{align*} \hspace{1cm} (23) \]

where the new expression of the augmented state-space matrices is the following

\[ A_s = \begin{bmatrix} A_f & B_f C_f \\ 0 & A_f \end{bmatrix}, \quad B_s = \begin{bmatrix} 0 \\ B_f \end{bmatrix}, \quad C_a = [C_f, D, C_f] \]  \hspace{1cm} (24)
The enlarged dynamical system permits to evaluate the stochastic structural response, taking into account the soil interaction.

### 3.3 Structural optimization formulation

The optimization problem is formulated as follows:

Find the design variables \( \theta = \{ \theta_1, \theta_2, \ldots, \theta_N \}^T \in \mathbb{R}^n \) such that it is minimized the objective function \( J(\theta) \). The optimization procedure will have to respect also the requirements of a constraint function given by

\[
\begin{align*}
\theta_j > \theta_{j,\text{min}}, & \quad j = 1, 2, \ldots, N \\
E[G_k(\mathbf{y}_s, \theta)] \leq 0, & \quad k = 1, 2, \ldots, m
\end{align*}
\]

(25)

where \( \theta_{j,\text{min}} \) represents the lower bound of the jth design variable while \( G_k(\mathbf{y}_s, \theta) \) is the kth constraint function in the optimization procedure.

In general, in dealing with an optimization analyses different performance competing objectives should be minimized. For example, in the control design the optimal simultaneous reduction of both displacements and absolute accelerations is a target practically impossible to reach. For this reason, in the research will be introduced a Pareto optimal front that will constitute a tool for multi-objective optimization and decision making. This optimal frontier is defined by the following expression

\[
J_p = (1-\xi) \cdot J_{\text{disp}}(\theta) + \xi \cdot J_{\text{acc}}(\theta)
\]

(25)

in which \( \xi \) is a parameter that provides a tradeoff between two competing objective. It will be included between 0 and 1. When \( \xi = 0 \) only the optimization of the displacements will be take into account while for \( \xi = 1 \) will be optimize only the accelerations. When \( \xi \in (0,1) \) the resulting target will be a linear combination of both performance objectives.

### 4 CONCLUSIONS

The paper proposes an iterative procedure to evaluate the nonlinear response of a simplified model subject to stochastic excitation. The procedure is also suitable for a filtered white noise excitation that takes into account the soil interaction. The main findings permit to develop a design method for hysteretic device based on multi-objective optimization based on a Pareto optimal frontier.

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### REFERENCES


RINTC-E PROJECT: THE SEISMIC RISK OF EXISTING ITALIAN RC BUILDINGS RETROFITTED WITH SEISMIC ISOLATION

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Abstract. This paper reports on the results of an ongoing Research Project, funded by the Italian Civil Protection Department, aimed at the evaluation of the risk of collapse and usability-preventing performance levels for existing buildings. This paper, in particular, describes the results of nonlinear time-history analyses (NTHA) carried out on two different RC buildings retrofitted with isolation systems. The two buildings are located in two different sites, Naples and L’Aquila characterized by medium and high seismicity for Italy, and are designed for gravity loads only and according to outdated seismic codes respectively. The buildings have been retrofitted using three different isolation systems: (i) high damping rubber bearings; (ii) rubber bearings and flat sliding bearings; (iii) curved surface sliders. The results point out that all isolation systems work effectively in limiting the onset of damage of non-structural members for seismic intensities much higher than the action required by the current design code. On the other hand, they show a little margin towards collapse, beyond the design intensity level. In particular, the collapse of the superstructure turns out to be the dominant collapse mode, especially for the building designed for gravity loads only.
1 INTRODUCTION

Experience from earthquakes worldwide as well as numerical studies highlight the vulnerability to damage or collapse of a large proportion of the existing reinforced concrete buildings during a strong earthquake. In particular, existing structures designed by old codes, have no sufficient strength or ductility to sustain earthquake, even due to lack of maintenance, improper construction method and material. Replacement of existing damaged buildings by reconstruction is generally avoided due to high economic costs.

It has long been known that seismic isolation technology is one of the most effective solution to retrofit existing buildings while they are in service [1], [2]. Base isolation is a passive anti-seismic system currently adopted to reduce the fundamental frequency of vibration of buildings, decoupling the superstructure from the horizontal components of the ground motion.

The behaviour under service and design conditions of isolated structures and the effectiveness of different isolation systems have been widely investigated by several authors [3], [4]. It has been demonstrated [5], [6] that correctly designed base isolated buildings subjected to earthquakes with intensity comparable to the design one can continue to be used immediately in quake aftermath with no loss of building functionality. Differently, there is a lack of information about the behaviour under collapse conditions of isolated structures and experimental investigations about the actual collapse condition of isolation devices are also limited [7], [8].

This paper describes research activities carried out on two different RC infilled frame buildings retrofitted by using the seismic isolation technique. In particular the buildings consist of two existing six-storey RC infilled frame buildings, one designed for gravity loads only during 1970s (gravity load designed, GLD) and the other for moderate seismic loads during 1990s (seismic load designed, SLD). They are located in two different sites, Naples and L’Aquila, respectively, characterized by medium and high seismicity for Italy. Different isolation systems have been considered in this study in order to compare their performances: i) High Damping Rubber Bearings (HDRBs), ii) Hybrid isolation system composed by both HDRBs and Flat Sliding Bearings (FSBs) and iii) Curved Surface Sliders (CSS). In all the cases the isolation systems have been designed according to current Italian seismic code [9].

The aim of the research is to identify critical aspects of the retrofit design and to provide the basis for the computations of the implicit risk of seismic collapse and of damage of the non-structural components of the retrofitted buildings [10]. To this purpose NTHA have been performed under bidirectional ground motions considering twenty couple of natural earthquakes for ten different earthquake intensity levels characterized by a return period ranging from 30 to 100000 years. The numerical models account for the nonlinear behaviour of both the isolation system and the superstructure. Collapse and damage conditions are pointed out for each record. For both the collapse and the usability-preventing performance level results are compared with those of the corresponding new isolated buildings analysed in the previous RINTC project [6] in order to highlight the different vulnerability.

2 DESIGN OF THE CASE STUDIES

2.1 Superstructure

The superstructure of the building located in L’Aquila represents a 6-storeys existing building designed according to pre-96 seismic code (SLD building). The superstructure of the building located in Napoli represents a 6-storeys existing building designed for gravity loads only (GLD building). Details about the buildings in the fixed base configuration can be found in [11].
Figure 1 shows the typical floor plan of the (a) SLD building and (b) GLD building. For the latter, the internal beam are only in the y-direction while in the x-direction there are no internal frames. The arrows indicate that the slabs are all one-way. The outer beam are all deeper than the slab, while all internal beams are flat. All the stories have the same slab with 25cm of total thickness (including hollow bricks). The buildings are intended for residential use and are characterized by a regular plan of approximately 240 square meters and 6 stories above ground. The height of the ground level is 3.4m while that of all the other stories is equal to 3.05m. The building structure includes the staircase, designed with knee beams. All floor plans are identical except for column and beam dimensions and reinforcements. The minimum column size is 30 cm. The 3D frame includes infill panels modelled for design and analysis purposes.

In the base-isolated configuration, a supplementary floor has been added at the bottom of the first storey columns and a grid of RC beams has been implemented at the same level.

2.2 Isolation system

The seismic design of the isolation systems has been performed through modal response spectrum analysis, according to the Italian Seismic Code [9]. Figure 2 shows the elastic response spectra for the sites and the soil conditions considered.

The isolation system has been designed with the aim of protecting the buildings from any structural damage at the design seismic intensity defined by the NTC18 [9], i.e. the Life-safety Limit State (LLS). To this purpose the elastic limit of the base shear (\( V_e \)) of the superstructure (in the fixed-base configuration) has been identified for the two buildings in the pushover curves at the occurrence of the first plastic hinge in the weaker direction, as reported in Figure 3. Once the elastic limit of the superstructure has been identified, the isolation system has been designed.
by entering the design spectrum of the site for the LLS using the spectral acceleration associated with the occurrence of the first plastic hinge ($S_e = V_e/M_{tot}$), where $M_{tot}$ is the total mass of the superstructure. In order to take into account the structural uncertainties, when possible, the aforesaid spectral acceleration has been divided by a safety factor equal to 1.1. As a result, the minimum value of the fundamental period of the base isolated system is obtained ($T_{is,min}$). In correspondence of that period the maximum displacement ($S_{dmax}$) has been evaluated using the displacement spectrum at Collapse Limit State. The latter value has opportune increased using a specific coefficient to account for torsional effects. Finally, commercial devices have been selected from the manufacturers’ catalogues.

![Figure 3: Elastic limit for (a) SLD building at L’Aquila and (b) GLD building at Napoli in the fixed base configuration.](image)

For the case study buildings with HDRBs, by adopting a damping ratio of the isolation system equal to 15% and without adopting any safety factor, minimum periods equal to $T_{is,min}=2.3$ sec and $T_{is,min}=2.9$ sec have been obtained respectively for the SLD building at L’Aquila and the GLD building at Napoli. The associated spectral acceleration are equal to 0.13g and 0.057g for L’Aquila and Napoli (see Figure 4a and Figure 5a). In Figure 4b and Figure 5b displacement spectra at the Collapse limit state are reported. The obtained maximum displacements ($S_{dmax}$) opportune increased using a coefficient equal to 1.2 to account for torsional effects are $S_{dmax}=0.255$ m for the SLD building at Aquila and $S_{dmax}=0.150$ m for the GLD building at Napoli.

By adopting a configuration with all HDRBs it is not possible to reach the minimum isolation period obtained for the GLD building at Napoli, because there is a strong interdependency of vertical load capacity, period of vibration and displacement capacity for this type of devices. More in detail, the low displacement demand and the low total stiffness of the isolation system lead to very small isolators, unable to support the vertical loads. As a consequence, only the SLD building located in L’Aquila (case SLD-Aq) is considered for HDRBs case, for which an isolation period of 2.49 sec is adopted, which is slightly larger than the minimum one.

![Figure 4: Design procedure for the SLD building at Aquila](image)
The isolation system has been designed following the indications suggested in §7.10.4.2 and C11.9 of the NTC2018. More in details, isolation bearings have been designed according to the following limitation for displacements and forces:

\[ \gamma_s \leq \gamma^* / 1.5 \leq 2 \]

where \( \gamma_s \) is the shear deformation of rubber layers due to the total seismic displacement (included torsional effects) and \( \gamma^* \) is maximum shear deformation obtained from qualification tests aimed to assess the effective rubber-steel adhesion. The other limitation considered in the design is:

\[ \gamma_t = \gamma_c + \gamma_s + \gamma_a \leq 5 \]

where \( \gamma_t \) is the total deformation, \( \gamma_c \) is the shear deformation of rubber layers due to axial load and \( \gamma_a \) is the shear deformation of rubber layers due to angular rotation. Additionally, for the critical devices it has been checked that the maximum tensile stress is lower than the minimum between \( 2G_{\text{din}} \) and 1 MPa, as stated in §7.10.4.2 of NTC2018 [9], in order to avoid cavitation phenomena. Also the maximum compression acting on the bearings has been checked to be lower than \( V_{\text{max,c}}/2 \), where \( V_{\text{max,c}} \) is buckling load evaluated as reported in C11.9.7:

\[ V_{\text{max,c}} = \frac{G_{\text{din}} A_s D}{t_c} \]

Figure 6a shows the device configuration for the SLD building, with bigger devices placed under the columns with larger axial loads. The HDRBs are identified by a two number code, the first number defines the diameter (\( \phi \)) and the second one the total rubber layer thickness (\( t_e \)). Table 1 reports the main design results. In particular, \( T_i \) is the isolation period (including the superstructure deformability), \( T_i/T_{bf} \) is the isolation ratio between the periods of the isolated and superstructure in the base fixed configuration (including infill panels), \( d_{\text{max,HDRB}} \) is the maximum displacement of the bearings leading to the maximum shear strain of the rubber \( \gamma_{\text{max}} \), D/C is the demand/capacity ratio in terms of shear strain (D/C shear), vertical compression load (D/C compr), vertical tensile stress (D/C tens) and inter-story drift at Damage Limit State (D/C drift DLS).
For the hybrid (elastomeric + steel/PTFE sliders) isolation system, the device configuration examined is illustrated in Figure 6b. A number of sliders equal to 1/3 of the total devices has been assumed to ensure a suitable horizontal deformability. In this case, spectral acceleration equal to 0.12g and 0.051g have been used to determine minimum isolation period for L’Aquila and Napoli respectively. They are obtained by adopting a damping ratio of the isolation system equal to 15% and a safety factor equal to 1.1. The corresponding minimum periods are $T_{is,min} = 2.5$ sec and $T_{is,min} = 3.1$ sec. The case study building located in L’Aquila (case SLD-Aq) is characterized by an isolation period of 2.7 sec whereas the case study building located in Napoli (case GLD – Na) features an isolation period of 3.3 sec, which are both larger than the obtained minimum periods. The design of the isolation bearings has been conducted based on the same limits and equations used for HDRBs. The main design outcomes for the case study buildings with hybrid isolation systems are reported in Table 2.

Finally, the design of the isolation system based on CSS [11], [13] has been performed through an equivalent linear response spectrum analysis. Since the isolation system equivalent properties are dependent from the design displacement value ($d_d$), an iterative procedure is adopted, as described in § 7.10.5.2 of NTC2018. In particular, to determine the effective stiffness $K_e$, the effective period $T_e$ and effective damping $\xi_e$ of the equivalent linear model, the following expressions have been used:

$$K_e = N \left( \frac{1}{R_e} + \frac{\mu}{d_d} \right), \quad T_e = \pi \sqrt{\frac{1}{g \left( \frac{1}{R_e} + \frac{\mu}{d_d} \right) + \frac{1}{\mu R_e}}}, \quad \xi_e = \frac{2}{\pi} \frac{1}{\frac{d_d}{\mu R_e} + 1}$$

(4 a,b,c)

where $N$ is the actual vertical load, $R_e$ is the effective radius, $\mu$ is the friction coefficient, $g$ is the gravity acceleration. Two different cases are designed: the first one (case SLD-Aq) with effective radius $R_e = 3700$mm and nominal value of the friction coefficient equal to 2.5%, the second one (case GDL-Na) with larger effective radius $R_e = 4500$mm and a lower friction coefficient equal to 2.0% (see Table 3). At the LLS in both the cases the effective damping is equal to the maximum value admitted for the spectral analysis ($\xi_e = 28\%$) and effective periods are respectively 2.75 sec and 2.83 sec. In both the cases the obtained effective periods are larger than minimum values (equal to $T_{is,min} = 2.1$ for L’Aquila and $T_{is,min} = 2.7$ for Napoli) obtained by adopting the same procedure of the hybrid isolation system with a damping ratio equal to 28% instead of 15%. Finally, the characteristics of the CSS systems at CLS are shown in Table 3, together the design displacement at the CLS ($d_{dc}$) and the displacement capacity ($d_m$, on the base of suppliers’ data). In terms of vertical load capacity, 2 different model have been selected for external and internal bearings, in order to ensure the ratio $N_{SD}/N_{Ed} > 0.5$, where $N_{SD}$ is maximum vertical load capacity in static conditions and $N_{Ed}$ is the maximum vertical load capacity in dynamic conditions (Figure 6c).
3 MODELLING ASSUMPTIONS

A nonlinear model has been developed in OpenSees [14], including the superstructure and the isolation system. In particular, for the isolation devices accurate nonlinear models have been selected in order to correctly predict the response of the isolated building. On the contrary, in the superstructure modelling pragmatic choices have been done due to the size of the building and the number of analyses to be carried out. In this section, a description of the adopted models is reported for both the isolation system and the superstructure.

3.1 Isolation system

To describe the cyclic behaviour of the HDRBs, the model recently developed by Kumar et al. 2014 [15] and implemented in OpenSees as HDR Bearing Element has been selected. The physical model is as a two-node, twelve degrees-of-freedom discrete element. The two nodes are connected by six springs that represent the mechanical behaviour in the six basic directions of a bearing. The coupling of the two shear springs is considered directly by using a coupled bidirectional model. All other springs are uncoupled. The coupling of vertical and horizontal directions are partially considered (in an indirect way) by using expressions for mechanical properties in the vertical direction that are dependent on the response parameters in the horizontal direction (but properties in the horizontal direction does not depend on the response in the vertical direction in the current version of the model). Linear uncoupled springs are considered in the torsion and the two rotational springs, as they are not expected to significantly affect the response of an elastomeric bearing. The material model in the axial direction is based on a mathematical model developed by Kumar [15] that captures the cavitation and post-cavitation behaviour in tension and the variation of the critical buckling load and the vertical axial stiffness with horizontal displacement in compression. The bidirectional model proposed by Grant et al. 2004 [16] is adopted to describe the behaviour under the two shear directions. This model is able to capture the degradation of bearing stiffness and damping due to scragging effects in shear, which is of particular importance for high dissipative rubbers [16][17]. For the torsional behaviour in the two rotational directions, a linear elastic model is assumed.
An iterative procedure has been followed to calibrate the model parameters, based on the fitting of experimental tests carried out on real scale HDRBs (provided by the university of Basilicata) made by a soft rubber with nominal shear modulus $G = 0.4$ MPa. Detail about the calibration procedure may be found in [6]. The model parameter used to simulate the rubber with equivalent linear parameters $G = 0.4$ and $\xi = 0.15$ are reported in the first row of Table 4. In the second row the parameters used to simulate the less dissipative rubber used in the design ($G = 0.4$ and $\xi = 0.10$) are reported. The differences between the two rubbers are illustrated by the hysteresis loops reported in Figure 7.

<table>
<thead>
<tr>
<th>Rubber</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\chi_1$</th>
<th>$\chi_2$</th>
<th>$\chi_3$</th>
<th>$\chi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 0.15$</td>
<td>276.03</td>
<td>17.19</td>
<td>3.606</td>
<td>170.37</td>
<td>5.094</td>
<td>0.01287</td>
<td>0.086</td>
<td>0.8306</td>
<td>0.00005</td>
<td></td>
</tr>
<tr>
<td>$\xi = 0.10$</td>
<td>314.94</td>
<td>17.19</td>
<td>3.606</td>
<td>100.22</td>
<td>5.094</td>
<td>0.01287</td>
<td>0.086</td>
<td>0.8306</td>
<td>0.00005</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: model parameters for the shear behaviour of HNDP bearings

![Figure 7: comparison between the two design rubbers](image)

Actually Grant model [16] has 10 parameters which define the behaviour of the entire bearing ($a_1, a_2, a_3$ for the elastic component, $b_1, b_2, b_3$ for the inelastic component, $c_1, c_2, c_3, c_4$ for the damage). Thus, a procedure to convert rubber parameters to bearing parameters has been developed, by implementing the following relations, where $A$ is the rubber area and $T_r$ is the total rubber thickness:

$$a_1 = a_1 \cdot A / T_r$$

$$a_2 = a_2 \cdot A / T_r^3$$

$$a_3 = a_3 \cdot A / T_r^5$$

$$b_1 = b_1 \cdot A$$

$$b_2 = b_2 \cdot A / T_r^2$$

$$b_3 = b_3 \cdot A / T_r^3$$

$$c_1 = c_1 / T_r^3$$

$$c_2 = c_2 / T_r^3$$

$$c_3 = c_3$$

$$c_4 = c_4 / T_r^3$$

(5 a,b,c) (6 a,b,c) (7 a,b,c)

The finite element friction-pendulum isolator was used for the numerical nonlinear analysis of the CSSs. The friction and pendulum forces are directly proportional to the compressive axial force in the element which cannot carry axial tension [18]. The cyclic nonlinear behaviour is expressed through the initial stiffness $K_i$ (before sliding, with a quasi-rigid behaviour), and the restoring stiffness $K_r = N / R_c$, as shown in Figure 8. The velocity dependence of the friction coefficient is described by the Constantinou et al. [19] model:

$$\mu = \mu_{\text{fast}} - \left( \mu_{\text{fast}} - \mu_{\text{slow}} \right) e^{-\alpha \vert v \vert}$$

(8)

Where $v$ is the sliding velocity; $\mu_{\text{fast}}$ and $\mu_{\text{slow}}$ are the sliding coefficients of friction at maximum and minimum velocity respectively; $\alpha$ is a rate parameter that controls the transition from $\mu_{\text{slow}}$ to $\mu_{\text{fast}}$. The axial load dependence of the coefficient of friction is not considered in this study.
Table 5 reports the main parameters of the CSS numerical model. For each case study, the values of $\mu_{\text{slow}}$ is the friction coefficient as declared by the manufacturer. The other parameters ($\mu_{\text{fast}}$, $\alpha$ and $K_i$) have been calibrated on the base of experimental characterization tests on similar bearings. Large values have been used for vertical stiffness, $K_V$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Radius (mm)</th>
<th>$\mu_{\text{slow}}$ (%)</th>
<th>$\mu_{\text{fast}}$ (%)</th>
<th>$\alpha$ (s/m)</th>
<th>$K_i$ (kN/m)</th>
<th>$K_V$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>case SLD-Aq</td>
<td>3700</td>
<td>2.5</td>
<td>5.0</td>
<td>5.0</td>
<td>5000</td>
<td>1E^10</td>
</tr>
<tr>
<td>case GLD-Na</td>
<td>4500</td>
<td>2.0</td>
<td>4.0</td>
<td>5.0</td>
<td>5000</td>
<td>1E^10</td>
</tr>
</tbody>
</table>

Table 5: CSS model parameters

3.2 Superstructure

A lumped plasticity model has been chosen for beam and column members of the superstructure, whereas elastic beams have been used for the base floor grid above the isolation system. The choice of representing the superstructure with a nonlinear model is justified by the results of recent studies on this topic [20], which pointed out the effects of the inelastic behaviour of the superstructure on the seismic response of base isolated structures. The model also includes the staircase structure (knee beams and cantilever steps) as well as masonry infill panels. The contribution of the masonry infill panel to the response of the reinforced concrete frame is modelled by replacing the panel with an equivalent strut acting only in compression. The equivalent diagonal strut is a coherent engineering model for infilled frames and a modified version of Decanini et al. model [21] was selected for modelling the struts and the effects of the openings were accounted for through reduction factors. Considering that the modelling choices concerning the structural (and non-structural) elements of the superstructure have been derived from those adopted by WP working on existing RC frames, reference to [11] can be made for all the modelling details.

4 NONLINEAR DYNAMIC TIME HISTORY ANALYSES

The seismic vulnerability of the designed structures was assessed by means of multi-stripe non-linear dynamic analysis carried out by considering 10 intensity measure levels (IMLs), as reported in Table6. A number of 20 ground motions per stripe has been considered. Reference to [10] and [22] can be made for details about the seismic input.

For base isolated buildings, the collapse condition could be related to the collapse of the superstructure or the collapse of the isolation system, which are both described in this section.
Table 6: Spectral acceleration $S_a(T=3s)$ for the IMLs (expressed in $g$)

<table>
<thead>
<tr>
<th>IML</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tr (years)</td>
<td>10</td>
<td>50</td>
<td>100</td>
<td>250</td>
<td>500</td>
<td>1000</td>
<td>2500</td>
<td>5000</td>
<td>10000</td>
<td>100000</td>
</tr>
<tr>
<td>L’Aquila</td>
<td>0.0002</td>
<td>0.011</td>
<td>0.031</td>
<td>0.062</td>
<td>0.11</td>
<td>0.177</td>
<td>0.271</td>
<td>0.384</td>
<td>0.576</td>
<td>1.053</td>
</tr>
<tr>
<td>Naples</td>
<td>0.001</td>
<td>0.009</td>
<td>0.026</td>
<td>0.044</td>
<td>0.067</td>
<td>0.093</td>
<td>0.126</td>
<td>0.162</td>
<td>0.216</td>
<td>0.348</td>
</tr>
</tbody>
</table>

4.1 Isolation system collapse conditions

The global collapse condition of the isolation system depends on the typology and the associated failure modes of the specific devices composing the system. For what concerns HDRBs, in the present study the collapse of a singular elastomeric device has been associated to the occurrence of one of the following failure modes: i) cavitation ii) buckling iii) shear. About the first failure mode, recent experimental tests have shown that elastomeric bearings can sustain large tensile strains of up to 100% following cavitation, without rupture of the bearing [22]. In this work an axial tensile strain equal to 50% has been prudently assumed as reference threshold and the global collapse condition has been conventionally fixed when the 50% of elastomeric devices reaches an axial tensile strain greater or equal to the assumed threshold. For the buckling failure, a step-by-step value of the critical buckling load of each device has been recorded during the analyses in the OpenSees environment. Therefore, the $P/P_{cr}$ ratio between the current axial load and the critical buckling load has been evaluated in order to identify the collapsed devices i.e. when $P/P_{cr}=1$. The global collapse condition has been conventionally fixed when the 50% of elastomeric devices simultaneously reaches a value of the axial compressive force equal to the critical buckling load. Finally, with regard to the shear failure, recent studies [24] pointed out a lower bound limit for rubber failure in terms of shear deformation ($\gamma$) of about 260%, regardless the shape factor value and the applied pressure. Such value seems to be excessively precautionary if compared to the experimental results obtained by Muramatsu et al. [25] and Kawamata and Nagai [26], which propose values of the order of 400-500%. All that considered, a limit value equal to 350% has been assumed in the present study, which also corresponds to the maximum shear strain of the experimental tests used to calibrate the horizontal shear behaviour of the numerical HDR model. The global collapse condition has been conventionally fixed when the 50% of elastomeric devices reaches a shear strain greater or equal to the assumed limit.

<table>
<thead>
<tr>
<th>FAILURE MODE</th>
<th>COLLAPSE CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buckling</td>
<td>50% of elastomeric devices (simultaneously) reaches a value of the axial compressive force equal to the critical buckling load; $P/P_{cr}=1$</td>
</tr>
<tr>
<td>Cavitation</td>
<td>50% of elastomeric devices reaches an axial tensile strain ($\varepsilon_t$) greater or equal to 50%; $\varepsilon_t \geq 50%$</td>
</tr>
<tr>
<td>Shear</td>
<td>50% of elastomeric devices reaches a shear strain ($\gamma$) greater or equal to 3.5; $\gamma \geq 3.5$</td>
</tr>
<tr>
<td>Sliders</td>
<td>The center of gravity of the base floor reaches an horizontal displacement equal to the device capacity increased by an extra-displacement. $d_a = d_{max,slide} + \phi/2$</td>
</tr>
<tr>
<td>CSS</td>
<td>The external angle of the base floor reaches an horizontal displacement equal to the device capacity increased by an extra-displacement. $d_a = 1.1 d_m$</td>
</tr>
</tbody>
</table>

Table 7: collapse Conditions for the isolation system
For what concerns the steel/PTFE sliders, the failure has been associated to a horizontal displacement value equal to the device capacity increased by an extra-displacement equal to the bearing radius. The current displacement of centre of gravity of the base floor has been assumed as displacement demand. Similarly, the collapse of a singular CSS device is associated to a horizontal displacement value equal to the maximum displacement capacity $d_m$ increased by an extra-displacement capacity (+10% of $d_m$). The current displacement at the external angle joint of the base floor has been assumed as displacement demand. All the collapse conditions are summarized in Table 7. Obviously in the case of HDRBs or hybrid systems, the collapse of the isolation system is deemed attained when one of the associated collapse conditions is reached.

### 4.2 Superstructure collapse condition

For the superstructure it was pragmatically chosen to employ a global but simplified collapse criterion, based on a global capacity obtained from pushover analysis. Criticism of this choice is known [27] and [28], since a capacity value related to the collapse mechanism occurring in the pushover analysis is adopted rather than motion-specific values. In particular, the collapse of the superstructure is associated to the occurring of the first of the following conditions on the pushover curve: (i) a 50% decrease of the lateral strength; (ii) the attainment of the zero strength point in a shear-critical column’s backbone [11].

Therefore, collapse is deemed to occur when, in one of the two main directions, the maximum roof displacement derived from NTHA, is greater than (or equal to) the roof displacement corresponding to the first of the aforesaid limit conditions. Obviously, the fixed-base configuration was considered to perform the pushover analysis of the examined building. Two values of collapse are determined, one in the X and the other in the Y direction (single value, no significant difference was detected between ‘+’ and ‘-’). Push over curves are reported in Figure 9 and Figure 10 and refer to mass proportional loads.

![Figure 9](image1.png)  
Figure 9: Pushover curves (case SLD-Aq) and displacement thresholds in the X (a) and Y (b) directions.

![Figure 10](image2.png)  
Figure 10: Pushover curves (case GLD-Na) and displacement thresholds in the X (a) and Y (b) directions.
4.3 Results of NTHA for the collapse performance level

The analyses results for the global collapse performance levels are summarized in this section in terms of number of failures. In other words, the total number of records determining a collapse condition for the base isolated building is reported as a function of the seismic intensity level. Moreover, the failure modes that caused the collapse are pointed out for each earthquake intensity level by using different colours.

The results presented in Figure 11a refer to the case study building with HDRBs only located at L’Aquila (SLD-Aq). Figure 11b shows, for comparison purpose, results of the new building with HDRBs analyzed in the previous research project [6]. It is reminded that the new building is equal to the retrofitted one in terms of global geometry (e.g. spans, floors) and it is isolated with a similar bearings configuration, but structural elements and reinforcements are significantly different.

The obtained results show that for the retrofitted building, one failure occurs for an intensity measure level corresponding to the Life-safety Limit State (IML 5 with return period equal to 500 years). Some failure occur at IML6 and a significant number of failure cases is recorded from IML7. For all the IMLs, the superstructure collapse is the first failure mode recorded. This suggest that the retrofitting design procedure (carried out without any safety factor about the elastic limit of the superstructure base shear, provides a safety margin for the global collapse of superstructure significantly lower with respect to the isolation system and lower than prescriptions provided by the design code [9]. In fact, by comparing the retrofitted building with the new isolated building it appears that the number of collapse are lower for the new building and the first collapse mode changes because in most of the cases the collapse of the isolators for buckling anticipates that associated with the superstructure. The reason is that the superstructure design of a new isolated building according to the NTC18 [9] leads to a superstructure with a better performance, both in terms of maximum shear strength and ductility capacity, with respect to a superstructure that in the fixed based configuration has been designed according to pre ‘96 seismic code, also due to the minimum requirements for concrete reinforcement. Thus, for new buildings the low margin with respect to the buckling load capacity of HDRBs is the most critical aspect of the design procedure, whereas for existing buildings the superstructure becomes the most vulnerable component if an adequate safety margin towards the elastic limit of the superstructure behaviour is not assumed in the design procedure.

The analyses results for the SLD-Aq case study building retrofitted by hybrid system are summarized in Figure 12a. Like for the case study building with HDRBs, Figure 12b shows the results for the new building, analysed in the previous project [6], featuring similar geometrical characteristics and the same isolation system. Figure 12a points out that the first collapse cases
occur for an earthquake intensity level with return period equal to 1000 years (IML 6), which corresponds to the earthquake intensity level assumed by NTC18 for the verification of the Collapse Limit State of the isolation system (for residential buildings). A significant number of failure cases is recorded from IML7. As expected, considering that the structure in the fixed based configuration was designed according to an older seismic code, the superstructure collapse is the prevalent failure mode. Also in this case, comparing the two case studies (retrofitted building vs. new base-isolated building), it is apparent that the number of collapse remains almost the same but the dominant collapse mode changes because the collapse of the superstructure anticipates that of the isolation system. In some cases, however, the ultimate displacement capacity of the sliders is attained before the collapse of the superstructure.

Figure 13a shows the results relevant to attainment of collapse conditions for the GLD case study building retrofitted by hybrid system and located at Naples. Figure 13b shows the same results for the new building with similar geometrical characteristics and similar isolation system. As can be seen, no collapse is recorded up to an earthquake intensity level with return period equal to 1000 years (IML 6). Some collapse cases are recorded from IML7 and they tend to saturate only for higher intensity level. Comparing the two case studies, it is apparent that the number of collapse increase for the retrofitted building and, in line with the results obtained for the case study SLD-Aq, the collapse of the superstructure is the prevalent failure mode.

Finally, results presented in Figure 14 and Figure 15 are related to the case study buildings with CSS devices. Differently from HDRB and HDRB+FSB, results are here given in terms of number of records leading to the superstructure collapse, to the CCS collapse or to the collapse of both the isolated building components. In particular in Figure 14a results of the case SLD-
Aq are reported, whereas in Figure 14b results refer to the new building analysed in the previous research project [6], isolated with the same devices and located in the same site. Also in this case, the comparison between the two case studies (retrofitted building vs. new base-isolated building), shows that the number of collapse remains almost the same. However, for the new base-isolated building the dominant collapse mode is associated to the attainment of the CCS displacement capacity (due to the low margin with respect the bearing displacement capacity assumed in the design), whereas for the retrofitted building the number of superstructure failures is also significant.

Figure 15a shows the results of the GLD case study located at Naples, while Figure 15b shows the same results for the new building located at the same site and analysed in the previous research project [6], having similar geometrical characteristics but CCS devices with different radius. Also in this case it is confirmed that in the retrofitted case the superstructure is the prevalent failure mode, whereas in the new one the isolation system is the most vulnerable component.

### 4.4 Results of NTHA for the usability-preventing performance level

Generally speaking, the usability-preventing performance level can be defined as the performance level where the structure, including structural and nonstructural elements, and machines relevant to its functions, exhibits damage that does not expose its occupants to any risk, and that does not compromise the strength and stiffness of the structure with respect of the vertical and horizontal loads. The structure is immediately usable even if some equipments are not fully operational.
In this study, a multi-criteria approach has been adopted to define the usability-preventing performance level from a practical point of view. The proposed multi-criteria approach combines concepts related to easy reparability of non-structural components and no service interruption with the requirement of protecting structural members from any damage under frequent earthquakes. The selected multi-criteria approach is described in detail in the Appendix A of ReLUIS-RINTC2018 report [29]. The attainment of the performance level under consideration has been assessed by comparing the Demand top displacement (i.e. the maximum top displacement derived from NTHA) with the Capacity top displacement obtained from Pushover analysis (see Table 8). D/C ratios larger than 1 are associated to failure condition.

Table 8 Limit values of top displacement associated with the attainment of the Usability-preventing Performance Level, as derived from Pushover analysis.

<table>
<thead>
<tr>
<th>Limit top displacement (mm)</th>
<th>X dir</th>
<th>Y dir</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLD - Naples</td>
<td>37</td>
<td>49</td>
</tr>
<tr>
<td>SLD - L’Aquila</td>
<td>51</td>
<td>64</td>
</tr>
</tbody>
</table>

The results of the usability-preventing performance level are summarized in the following diagrams in terms of D/C ratios associated with each single record pair, in the two directions. As can be seen, for an earthquake intensity level corresponding to a return period equal to 50 years (IML2), corresponding to the design level for the verification of the damage limit state, according to Italian seismic code, no failure cases are observed. Indeed, the D/C ratios are much lower than 1. The first failure cases (D/C ≥ 1) are recorded for seismic events characterized by return periods greater than 250 years (IML 4). The obtained results confirm that also existing buildings retrofitted with seismic isolation works effectively in limiting damage to non-structural components for seismic intensities much higher than the design earthquake intensity level. It is worth to note that there are many records, especially at large IMLs, where the limit top displacement associated with the attainment of the UPD is not reached (in the considered direction) but the isolated building is collapsed for the attainment of the superstructure failure (in the other direction) or the isolation system failure.

Figure 16 D/C ratios for Case SLD-Aq retrofitted with HDRBs in the (a) X direction and (b) Y direction.
Figure 17 D/C ratios for Case SLD-Aq retrofitted with the hybrid isolation system in the (a) X direction and (b) Y-direction.

Figure 18 D/C ratios for Case GLD-Na retrofitted by the hybrid system in the (a) X direction and (b) Y direction.

Figure 21 D/C ratios for Case SLD-Aq retrofitted with CSS in the (a) X-direction and (b) Y direction.

Figure 22 D/C ratios for Case GLD-Na retrofitted with CSS in the (a) X direction and (b) Y direction.
5 CONCLUSIONS

This paper presents the results of an ongoing project on the implicit risk of seismic collapse and damage of two existing reinforced concrete (RC) buildings located in two different sites (L’Aquila and Naples) and retrofitted using three different isolation systems, based on: (i) rubber bearings only, (ii) rubber bearings and flat sliding bearings, (iii) curved surface sliders. Collapse conditions and usability-preventing performance levels have been evaluated through nonlinear dynamic analyses performed under bidirectional ground motions considering twenty couple of natural earthquakes for ten different earthquake intensity levels with return period ranging from 30 to 100000 years. The results point out that all isolation systems work effectively in limiting the onset of damage in non-structural components for seismic intensities much higher than the design earthquake. On the other hand, similarly to new isolated buildings, they show a little margin to the occurrence of collapse beyond the design earthquake intensity level. However, differently from new base-isolated buildings where the isolation system collapse is always the prevalent failure mode, for retrofitted buildings the collapse of the superstructure turns out to be the dominant collapse mode, especially for the building designed for gravity loads only.

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MITIGATION OF GROUND VIBRATIONS BY CIRCULAR ARRAYS OF RIGID BLOCKS

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Abstract. Pile driving and other activities in the built environment cause ground vibration at low frequencies. This may result in annoyance to people as well as damage to civil structures. It is known that masses added on the ground surface can have an impact on the vibration levels in the surrounding environment. Hence, employing a semi-analytical model for rigid blocks on the surface of a layered ground, this paper investigates whether circular arrays of such blocks can be used as a means of vibration mitigation. The frequency range 0–80 Hz is considered, since this is relevant to whole-body vibrations of humans as well as the fundamental modes of resonance in building elements, e.g., floors and walls. Two different soil profiles are analysed: a soft dry sand layer over a till half-space and a soft wet clay layer over a lime half-space. Further, three configurations of the block arrays are taken into consideration, and for the first soil profile also the height of the blocks is varied to test its influence on the insertion loss in a zone 20–40 m away from the source. The aim is to quantify the overall insertion loss that can be expected using the proposed methodology. Further, the variation in insertion loss caused by changes in the block array configuration is examined.

Keywords: Soil, Wave Propagation, Layered Half-Space, Insertion Loss, Wave Impedance.
1 INTRODUCTION

Ground vibrations from traffic or construction work may cause damage to structures. They may as well cause annoyance to inhabitants in urban environments and have an impact on their health. To reduce the problems associated with ground vibrations, several ideas have been proposed for mitigation. This includes trenches and barriers as well as embedded wave-impeding blocks (WIBs) or masses placed on the ground surface. These can be placed in front of a vibration source to prevent vibrations from spreading to the surroundings, or they can be placed in front of a receiver to protect it, or a combination can be used. In any case, the idea is to insert one or more obstacles within the wave propagation path that will redirect, reflect or absorb the waves.

The literature reports several studies of single blocks or barriers [1, 2, 3] as well as, for example, double sheet pile walls [4, 5]. Especially, the idea of using blocks placed on the ground surface for mitigation of ground vibrations was proposed already in the 1970s by Warburton et al. [7, 8]. Later, Peplow et al. [8] proposed the inclusion of WIBs in the ground, and this concept was validated experimentally by Masoumi et al. [9].

To improve the solution that can be achieved with a single barrier or block, arrays of such barriers and blocks can be introduced. The idea of using structural periodicity to mitigate wave propagation was originally proposed by Mead [10] who analysed a periodically supported beam. Recently, Van et al. [11] developed a methodology to design a one-dimensional wave barrier by topology optimization of a one-dimensional medium. More relevant for the present problem of ground vibration, Persson et al. [12] studied the effect of having repeated hills and/or valleys, and Andersen [13] analysed the effect of linear arrays of rigid blocks embedded in the ground using a two-dimensional model of a periodic structure. This work was extended to three-dimensional analysis of nearly-periodic structures [14, 15, 16]. Furthermore, Peplow et al. [17, 18] established simplified modelling approach to analysis of linear arrays of blocks placed on the ground surface. As a common finding of the reported research, periodic inclusions of blocks or other inhomogeneities have an effect on the ground vibrations and may, if designed properly, produce band gaps with (almost) no wave propagation.

As an alternative to the linear barriers or arrays of masses, typically considered, the present paper proposes the use of circular arrays of rigid blocks. Such arrays can be placed, for example, around a pile during driving or around a machine foundation—or it can be placed around a building to partially prevent vibration from reaching it. Only the first scenario is analysed in the paper, using a semi-analytical model of a layered viscoelastic half-space to represent the ground. Rigid blocks are placed on the ground surface in concentric circles surrounding the source point with the aim of reducing the vibration level in a shielded zone 20 m to 40 m from the centre of the source. A parameter study is performed regarding the number of circles, the number of blocks in each circle, and the height of the blocks. In any case, the blocks have a footprint of 1 m by 2 m. It is deemed realistic that such blocks could be produced on a given site, for example, by filling in steel containers or construction-waste bags with soil that can be excavated locally. Thus, it is not necessary to place blocks made, for example, of reinforced concrete that may be difficult and expensive to produce at or move to the site.

In Section 2, the computational model is described. This includes a description of the semi-analytical model of the ground and the rigid blocks as well as definition of the various array configurations. Sections 3 and 4 present two case studies. In Case 1, a soft dry sand layer over a till half-space is considered, whereas Case 2 concerns a soft wet clay layer over a lime half-space. These strata are considered to be representative for soil conditions that could typically cause problems with ground vibrations in the environment surrounding a construction site. Finally, Section 5 lists the main conclusions of the paper.
2 COMPUTATIONAL MODEL FOR THE SOIL AND BLOCKS

2.1 Model of a layered ground

The soil is modelled as a linear viscoelastic solid half-space with surface at $x_3 = 0$. In the time domain and using Cartesian coordinates, the displacement response $u_i(x_1, x_2, t)$ at time $t$ and position $(x_1, x_2)$ on the ground surface to a surface traction time series $p_j(y_1, y_2, \tau)$ acting at the position $(y_1, y_2)$ can be found as

$$ u_i(x_1, x_2, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{t-\tau}^{t} g_{ij}(x_1 - y_1, x_2 - y_2, t - \tau) p_j(y_1, y_2, \tau) \, d\tau \, dy_1 \, dy_2, \quad (1) $$

where $g_{ij}(x_1 - y_1, x_2 - y_2, t - \tau)$ is the Green’s function for the half-space. This formulation assumes invariance of the Green’s function to temporal and spatial translation. The latter requires that the half-space is homogenous in the $x_1$ and $x_2$ directions, which in turn requires that any layers are horizontal. However, even with these constraints, a general analytical expression for the Green’s function cannot be established in the time–space domain. Instead, a semi-analytical solution can be obtained by first solving the problem in frequency–horizontal wavenumber domain. Thus, by a triple Fourier transformation, Eq. (1) is reformulated into

$$ U_i(k_1, k_2, \omega) = \tilde{G}_{ij}(k_1, k_2, \omega) \tilde{P}_j(k_1, k_2, \omega), \quad (2) $$

where $U_i(k_1, k_2, \omega)$ is the triple Fourier transform of $u_i(x_1, x_2, t)$, $\tilde{P}_j(k_1, k_2, \omega)$ is the triple Fourier transform of $p_j(y_1, y_2, \tau)$, and $\tilde{G}_{ij}(k_1, k_2, \omega)$ is the triple Fourier transform of $g_{ij}(x_1, x_2, t)$. Unlike $g_{ij}(x_1, x_2, t)$, a closed-form solution exists for $\tilde{G}_{ij}(k_1, k_2, \omega)$ in the case of a horizontally layered linear viscoelastic half-space, as first proposed by Thomson [19] and Haskell [20], and as further explained in the work by Andersen and Clausen [21]. For a given combination of wavenumbers and angular frequency, a transfer matrix for a single layer can be determined in closed form, and the transfer matrices for all layers are then multiplied in order to obtain a transfer matrix for the layered half-space. This can finally be rearranged into a Green’s function tensor relating the surface displacement to the surface traction.

The computational strategy is to discretize the problem into a number of discrete angular frequencies $\omega_n, n = 1, 2, \ldots, N_\omega$. For each frequency, $\tilde{G}_{ij}(k_1, k_2, \omega_n)$ and $\tilde{P}_j(k_1, k_2, \omega_n)$ are evaluated at a number of discrete wavenumbers. Next, double inverse discrete Fourier transformation is performed with respect to the horizontal wavenumbers. That brings the solution back into frequency–space domain. Hence, for a given frequency, the displacement $U_i(x_1, x_2, \omega)$ at any point on the ground surface due to the traction $P_j(y_1, y_2, \omega)$ at any point on the surface can be found. As pointed out by Andersen and Clausen [21], rotational symmetry of the load can be utilized to speed up the calculations. Further, inverse discrete Fourier transformation can be used to obtain the time-domain solution. However, the present analyses concern the frequency domain with focus on the range 0–80 Hz that is relevant to whole-body vibration as well as the first modes of resonance in typical building structures and soil profiles.

It is noted that rate-independent material dissipation is assumed in the semi-analytical model of the ground. Hence, hysteretic damping, represented by the loss factor $\eta$, is employed. For each layer of soil, the material properties are further specified in terms of the shear modulus $G$, Poisson’s ratio $\nu$, and the mass density $\rho$.

As an example, Figure 1 shows the steady state ground displacement in a model of a layered soil with 3 m deep soft dry sand overlying a till half-space. Details on the material properties can be found in Section 3. The load is applied as a uniformly distributed vertical traction over a circular area with a radius of 1 m, and the displacements are plotted at the frequencies 25 Hz and 30 Hz, respectively, for the zone up to 40 m away from the centre of the load.
2.2 Dynamic soil–structure interaction for an array of rigid blocks

To model the dynamic soil–structure interaction (SSI) for $N_B$ rigid blocks placed on the ground surface, the interface between each block and the soil is discretized into $N_{ssi}$ points. In the present analyses, rectangular blocks are considered with 6 points in one direction and 12 points in the other direction, i.e. $N_{ssi} = 72$. Hence, the total number of soil–structure interaction points will be $N_{SSI} = N_{ssi} N_B = 72 N_B$. At each of the SSI points, traction is assumed to be distributed on a small area with rotational symmetry around the point. With reference [21], a so-called “bell-shaped” traction is applied, i.e. a double Gaussian distribution in the $(x_1, x_2)$ plane.

Further, load is applied at a source point, adding another three degrees of freedom (d.o.f.) to the problem, i.e. the discretized ground surface as $3 + 3 N_B N_{ssi}$ displacement d.o.f.. A global flexibility matrix $C_{SSI}(\omega_n)$ can now be established, employing the Green’s function $g_{ij}(x_1 - y_1, x_2 - y_2, \omega_n)$ which relates $U_i(x_1, x_2, \omega_n)$ to $P_j(y_1, y_2, \omega_n)$. Each of the rigid blocks has six d.o.f., i.e. a total of $6 N_B$ rigid-body modes can be identified for the blocks. In addition to this, the three d.o.f. for the source point must be considered. Thus, one by one, a unit displacement or rotation is prescribed for each of the $6 N_B + 3$ d.o.f. while fixing the remaining d.o.f. for the blocks and the load. The displacement components are determined for each of the $N_{ssi}$ points of the local interface between the soil and the block with prescribed unit rigid-body motion. This provides the matrix $U_0$, where each column defines one rigid-body mode of a block or the displacement of the source point in one direction. The magnitude of the traction at each SSI point due to each unit displacement or rotation can be found as the solution to $C_{SSI}(\omega_n) P_0(\omega_n) = U_0$. Finally, for each discrete frequency, a stiffness matrix for the SSI problem can then be expressed as $D_{SSI}(\omega_n) = U_0^T P_0(\omega_n)$. This matrix will have the dimensions $(6N_B + 3) \times (6N_B + 3)$.

Once, the $6N_B + 3$ rigid-body translations and rotations of the blocks and source point have been determined for a given load, the displacement at a number of discrete points on the ground surface can be determined by using the Green’s function. In the present analyses, a circular zone with radius 40 m is considered, as already discussed in the previous subsection. The discrete points are organized in a regular pattern with a 1.0 m distance in the radial direction and approximately 1.0 m distance in the azimuthal direction. More precisely, 6 points are placed in the circle with radius 1 m, 12 points are placed in the circle with radius 2 m, and so on. This provides a total of 4921 observation points on the ground surface.
2.3 Block array configurations

In order to assess the potential of using circular block arrays to mitigate ground vibration from a single source on a layered ground, three different overall configurations of the blocks will be considered. In any case, the blocks are placed on the ground surface in one, two or three concentric rings with the radii 4 m, 8 m and 12 m, respectively. The radii refer to the distance between the centre of the loaded circular area and the midpoints of the SSI interfaces between the individual blocks and the soil. The three configurations are:

a) “Stonehenge” configurations with 6, 12 and 24 blocks in Rings 1, 2 and 3, respectively, cf. Figure 2a. The blocks in the inner Ring 1 have the footprint 2 m × 1 m with the longer side placed tangentially to the perimeter of the circle. The side lengths of the blocks in Rings 2 and 3 are scaled by a factor of $0.5^{1/3}$ and $0.5^{2/3}$, respectively, and the height of the blocks are scaled by the same factors. Hence, the volume (and hence the mass) of the blocks in Ring 2 will be one-half compared to that of the blocks in Ring 1, whereas the volume (and mass) of the blocks in Ring 3 will be scaled by a factor 0.25. Given that each ring contains twice the number of blocks, the total mass of the blocks in each ring is therefore the same.

Figure 2: Three configurations of rigid blocks on the ground surface: a) “Stonehenge” with 6, 12 and 24 blocks in Rings 1, 2 and 3, respectively; b) “turned rings” with six identical blocks in each ring and Ring 2 turned 30°; c) six “linear arrays” with three blocks in each array. The total mass of all blocks in each ring is identical in all configurations. The radius of the rings are 4 m, 8 m and 12 m, respectively, and the radius of the model is 40 m.
b) “Turned rings” with six blocks in each ring and with Ring 2 turned by 30° around a vertical axis through the load centre relative to Rings 1 and 3, cf. Figure 2b. Thus, when all rings are present, the middle ring “closes the gaps” between the blocks in the inner and outer rings. In this configuration, the blocks are identical, all having the footprint 2 m × 1 m, again with the longer side placed tangentially to the perimeter of the circle. Given that six blocks are present in each ring, the total mass of the blocks in each ring is therefore again the same.

c) “Linear arrays” with one, two or three blocks lined up along six radii, cf. Figure 2c. Except for the relative orientation of Ring 2, this configuration is identical to the “turned rings”.

The blocks are rigid and have a mass density of 2000 kg/m³. The height of all blocks (in the inner Ring 1 for the “Stonehenge” configuration” and in all rings for the other configurations) is either 1 m, 2 m or 3 m. As indicated above, the blocks in Rings 2 and 3 in the “Stonehenge” configuration will be downscaled to provide the same total mass in each ring.

Figure 3 shows example results for the “Stonehenge” configuration with three rings of blocks placed on a soft dry sand layer over a till half-space (the soil properties are given in Section 3). These results should be compared with the green-field response shown in Figure 1. Clearly, for the present case, the “Stonehenge” provides excellent mitigation of ground vibrations in the surrounding environment at the frequency 25 Hz, whereas the effect at 30 Hz is limited.

![Figure 3: Snapshots of the steady state ground displacement response to a harmonic vertical load applied uniformly over a circular area with radius 1 m: a) “Stonehenge” at 25 Hz; b) “Stonehenge” at 30 Hz.](image)

The radius of the model is 40 m and the depth of the model is 12 m.

2.4 Calculation of insertion loss and transmission loss in protected zone

To quantify the effect of block arrays with different configurations on the ground vibration level in the surrounding environment, the insertion loss (IL) is calculated. For a given position \((x_1, x_2)\) and angular frequency \(\omega\), the IL is determined as the difference in dB in the vibration levels before and after insertion of the blocks, defining a reduction in vibration level as a positive insertion loss. For displacement component \(i\) and block array configuration \(j\),

\[
IL_j^i(x_1, x_2, \omega) = 20 \log_{10} \left( \frac{U_j^i(x_1, x_2, \omega)}{U_0^i(x_1, x_2, \omega)} \right) - 20 \log_{10} \left( \frac{U_j^i(x_1, x_2, \omega)}{U_0^i(x_1, x_2, \omega)} \right),
\]

where \(U_j^i(x_1, x_2, \omega)\) is the displacement amplitude after insertion of the block array, and \(U_0^i(x_1, x_2, \omega)\) is the reference displacement amplitude before insertion of the block array, i.e. the vibration level in the green field. The present analyses consider vertical vibrations caused by a unit-magnitude vertical load applied uniformly over a circular area with radius 1 m. Hence, \(IL_j^i(x_1, x_2, \omega)\) is calculated at the 4921 observation points on the ground surface.
A high value for IL is advantageous. However, array configurations that provide high IL in frequency ranges with low vibration levels, but low IL in frequency ranges with high vibration levels, are not effective solutions. Hence, to assess the quality of an array, the reference level (RL) for vibrations at a given observation point is important. The present analyses employ the definition:

$$RL_i(x_1, x_2, \omega) = 20 \log_{10} \left( \frac{|U^0_i(x_1, x_2, \omega)|}{U_{\text{ref}}} \right) - 20 \log_{10} \left( U_{\text{ref}} \right),$$  \(3\)

where \(U^0_i(x_1, x_2, \omega)\) is the displacement amplitude in the greenfield when a unit-magnitude (1 N) vertical load is applied, and \(U_{\text{ref}} = 1 \text{ pm}\). Again, the vertical vibration level is in focus; hence, \(RL_3(x_1, x_2, \omega)\) is evaluated for the observation points. It should be noted that the present definition of the RL corresponds to a transfer function for the layered ground. Thus, it contains no information about the frequency content of a source—it only provides information about the capacity of the ground to transfer vibration from a position at the surface to another.

In the following sections, the IL and RL will be determined for two different stratifications of the soil and for the various configurations of the block arrays described in the previous sub-section. In order to provide single-value measures of the IL and RL at a given frequency, the mean values of \(IL_i(x_1, x_2, \omega)\) and \(RL_i(x_1, x_2, \omega)\) are determined for the donut-shaped zone, coined here as the “shielded zone” bounded by the two circles with radii 20 m and 40 m, respectively. Further, the 10% and 90% quantiles are calculated in order to quantify the variation of the quantities within the shielded zone.

3 CASE 1: A SOFT DRY SAND LAYER OVER A HALF-SPACE OF TILL

The first case concerns a 3 m deep soft dry sand layer overlying a half-space of till. Material properties for the two soil layers are listed in Table 1. Three subcases are considered: Case 1.1 with 1 m high blocks, Case 1.2 with 2 m high blocks and Case 1.3 with 3 m high blocks.

<table>
<thead>
<tr>
<th>Soil layer</th>
<th>Shear modulus (G) [MPa]</th>
<th>Poisson’s ratio (v) [-]</th>
<th>Mass density (\rho) [kg/m(^3)]</th>
<th>Loss factor (\eta) [%]</th>
<th>Thickness (h) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand, soft, dry</td>
<td>35.72</td>
<td>0.3330</td>
<td>1553</td>
<td>4.50</td>
<td>3.0</td>
</tr>
<tr>
<td>Till (half-space)</td>
<td>500.0</td>
<td>0.3500</td>
<td>2100</td>
<td>2.00</td>
<td>(\infty)</td>
</tr>
</tbody>
</table>

Table 1: Soil properties for Case 1.

3.1 Case 1.1: 1 m high blocks in three configurations

Table 2 shows the resonance frequencies of the blocks in Rings 1, 2 and 3 for the various configurations of the 1 m high blocks in Case 1.1. The resonance frequencies are calculated from the static stiffness of the ground and the inertia of the blocks. With reference to Subsection 2.3, subcases a, b and c refer to the “Stonehenge”, “turned ring” and “linear array” configurations, respectively. Only the vertical rigid-body translation (or heave mode) and the rotation around the longer main axis of the SSI interface (or pitch mode) of the blocks are considered. These modes are deemed to be important regarding the interaction with the surface wave propagating away from the source.

As a first observation, the resonance frequencies are all in the considered frequency range 0–80 Hz, and the pitch mode has a resonance frequency slightly below that of the heave mode. Further, the smaller blocks in Rings 2 and 3 of the “Stonehenge” configurations (Case 1.1a) have higher resonance frequencies than the larger blocks in Ring 1. This is expected, since the SSI areas are reduced less than the volumes, given that the same scaling is applied in all directions (length, width and height).
Table 2: Resonance frequencies (in Hz) of blocks in Case 1.1.

<table>
<thead>
<tr>
<th></th>
<th>Blocks in Ring 1</th>
<th>Blocks in Ring 2</th>
<th>Blocks in Ring 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heave</td>
<td>Pitch</td>
<td>Heave</td>
<td>Pitch</td>
</tr>
<tr>
<td>Case 1.1a</td>
<td>29.64</td>
<td>19.72</td>
<td>34.27</td>
</tr>
<tr>
<td>Case 1.1b</td>
<td>29.64</td>
<td>19.72</td>
<td>29.60</td>
</tr>
<tr>
<td>Case 1.1c</td>
<td>29.64</td>
<td>19.72</td>
<td>29.60</td>
</tr>
</tbody>
</table>

Figure 4 shows the mean values of the IL and RL in the shielded zone (20 m to 40 m from the centre of the load) in Case 1.1a for the frequency range 0–80 Hz and for “Stonehenge” configurations with either 1, 2 or 3 rings. Figure 5 shows the RL (top left quadrants of the subplots) and IL (remaining quadrants of the subplots) on the ground surface at nine selected frequencies. Figure 6 and Figure 7 show similar results for Case 1.1b, the “turned rings”, while Figure 8 and Figure 9 show the results for Case 1.1c, the “linear arrays”.

Generally, for Case 1, the RL of vibration in the shielded zone is low for frequencies below 10 Hz, and it increases significantly with an increase of the frequency in the range 10–20 Hz. This is due to the presence of a cut-on frequency for wave propagation in the sand layer around 12.5 Hz. Below this frequency, the far-field response is dominated by wave propagation in the much stiffer till half-space, while the soft topsoil only influences the local response near the source. As the frequency is further increased in the range 20–80 Hz, the mean RL in the shielded zone decreases. It should be noted that results are presented for the displacement. Obviously, the frequency dependencies for velocity and acceleration amplitudes are different.

In terms of the mean IL in the shielded zone, the “Stonehenge” and “turned ring” configurations provide similar results. A small peak in the IL is observed around 22 Hz, and the IL is generally high, around 10 dB in the frequency range 25–40 Hz. A 5 dB increase of the IL is achieved by insertion of three rings instead of just one ring. However, by a closer inspection, it can be observed that Cases 1.1a and 1.1b are different in terms of the variation of the IL within the shielded zone. For example, at 25 Hz and 30 Hz the “Stonehenge” with 2 rings or 3 rings provides IL distributions with a high degree of rotational symmetry cf. Figure 5, whereas the “turned rings” provide more variation in the IL, cf. Figure 7.

The third configuration, the “linear array” provides a result that is different from the two other configurations. Thus, Figure 8 shows a significant IL around 22 Hz, which was not the case for the other configurations. However, around 40 Hz, the IL is low compared to the other configurations of the block array, and there is almost no gain in IL by inclusion of a second or third ring. As an interesting observation in Figure 9, it is noted that the linear arrays do not provide maximum IL in the parts of the shielded zone that lie directly behind the blocks. Due to wave scattering and interference, the IL at some frequencies is 10–15 dB higher in the gaps between the linear arrays than just behind the linear arrays.
Figure 4: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of
the loaded area. Case 1.1a: “Stonehenge” with 1 m high blocks (inner circle) on soft dry sand over till.
The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 5: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.1a, “Stonehenge”
with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 6: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.1b: “Turned rings” with 1 m high blocks (inner circle) on soft dry sand over till. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 7: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.1b, “turned rings” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 8: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.1c: “Linear arrays” with 1 m high blocks (inner circle) on soft dry sand over till. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 9: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.1c, “linear arrays” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
3.2 Case 1.2: 2 m high blocks in three configurations

Table 3 shows the resonance frequencies of the blocks in Rings 1, 2 and 3 for the various configurations of the 2 m high blocks in Case 1.2. Compared with the resonance frequencies listed in Table 2 (for Case 1.1 with 1 m high block), the values in Table 3 are considerably lower, especially for the pitch mode. This can be explained by the relatively larger increase of the rotational inertia compared to the increase of the mass. It can be observed that the resonance frequency for the pitch mode is now below the cut-on frequency for wave propagation in the top layer, but the resonance frequencies for the heave mode a closer to the peak of the RL.

<table>
<thead>
<tr>
<th>Blocks in Ring 1</th>
<th>Blocks in Ring 2</th>
<th>Blocks in Ring 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heave Pitch</td>
<td>Heave Pitch</td>
<td>Heave Pitch</td>
</tr>
<tr>
<td>Case 1.2a</td>
<td>20.96 7.562</td>
<td>24.23 9.977</td>
</tr>
<tr>
<td>Case 1.2b</td>
<td>20.96 7.562</td>
<td>20.93 9.128</td>
</tr>
<tr>
<td>Case 1.2c</td>
<td>20.96 7.562</td>
<td>20.93 7.555</td>
</tr>
</tbody>
</table>

Table 3: Resonance frequencies (in Hz) of blocks in Case 1.2.

Figure 10 shows the mean values of the IL and RL in the shielded zone (20 m to 40 m from the centre of the load) in Case 1.2a for the frequency range 0–80 Hz and for “Stonehenge” configurations with either 1, 2 or 3 rings. Figure 11 shows the RL (top left quadrants of the subplots) and IL (remaining quadrants of the subplots) on the ground surface at nine selected frequencies. Figure 12 and Figure 13 show similar results for Case 1.2b, the “turned rings”, while Figure 14 and Figure 15 show the results for Case 1.2c, the “linear arrays”.

Compared to Case 1.1, the general observation is that the 2 m high blocks provide significantly higher IL than the 1 m high blocks in the frequency range 17–30 Hz. Especially, a very pronounced peak is present in the mean IL around 18 Hz. The IL at higher frequencies is at the same time slightly reduced. This results from the lower resonance frequencies of the blocks due to the increase of inertia when the blocks become higher.

Little improvement is obtained by including Ring 3 compared to having only two rings. However, for the “Stonehenge” configuration, the IL in the frequency range 28–35 Hz is increased by about 5 dB by insertion of Ring 3. A smaller gain in the IL, about 2–3 dB, is achieved in Case 1.2b within the frequency range 23–32 Hz after insertion of the third ring.

Compared to Cases 1.2b and 1.2c (cf. Figure 12 and Figure 14), the IL is significantly higher in Case 1.2a (cf. Figure 10) for the configurations with two or three rings in the frequency range 23–33 Hz. Here, the “Stonehenge” configuration provides an IL of about 20 dB, which can be considered a very large reduction in the vibration level. Furthermore, inspection of Figure 11 shows that the IL is fairly homogeneous, though with variation between 15 dB and 30 dB. There are no spots with low IL in the shielded zone. For the IL peak near 20 Hz, an even more uniform distribution of the IL can be observed.

Finally, comparing Cases 1.2b and 1.2c it can be observed that the “linear arrays” with 2 m high blocks tend to provide higher IL just behind the arrays at 30 Hz, whereas the IL is on average higher for the “turned rings”, but with more local variations. This demonstrates that the orientation of the individual rings can be important in the design of the array, when the goal is to protect certain smaller zones, rather than the entire larger zone.
Figure 10: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.2a: “Stonehenge” with 2 m high blocks (inner circle) on soft dry sand over till. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 11: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.2a, “Stonehenge” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 12: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.2b: “Turned rings” with 2 m high blocks (inner circle) on soft dry sand over till. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 13: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.2b, “turned rings” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 14: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.2c: “Linear arrays” with 1 m high blocks (inner circle) on soft dry sand over till. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 15: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.2c, “linear arrays” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
3.3 Case 1.3: 3 m high blocks in three configurations

Table 4 lists the resonance frequencies of the 3 m high blocks in Case 1.3. These are lower than those in Case 1.2 and even lower than those of Case 1.1. Of particular interest to the present problem, the resonance frequencies of the 3 m high blocks (for the heave and pitch modes) fall below 21 Hz where the RL of vibration peaks. Hence, it may be expected that the 3 m high block will not interact dynamically with the soil to the same extent as the smaller blocks.

<table>
<thead>
<tr>
<th></th>
<th>Blocks in Ring 1</th>
<th>Blocks in Ring 2</th>
<th>Blocks in Ring 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heave</td>
<td>Pitch</td>
<td>Heave</td>
</tr>
<tr>
<td>Case 1.3a</td>
<td>17.11</td>
<td>4.185</td>
<td>19.79</td>
</tr>
<tr>
<td>Case 1.3b</td>
<td>17.11</td>
<td>4.185</td>
<td>17.09</td>
</tr>
<tr>
<td>Case 1.3c</td>
<td>17.11</td>
<td>4.185</td>
<td>17.09</td>
</tr>
</tbody>
</table>

Table 4: Resonance frequencies (in Hz) of blocks in Case 1.3.

Figure 16 shows the mean values of the IL and RL in the shielded zone (20 m to 40 m from the centre of the load) in Case 1.3a for the frequency range 0–80 Hz and for “Stonehenge” configurations with either 1, 2 or 3 rings. Figure 17 shows the RL (top left quadrants of the subplots) and IL (remaining quadrants of the subplots) on the ground surface at nine selected frequencies. Figure 18 and Figure 19 show similar results for Case 1.3b, the “turned rings”, while Figure 20 and Figure 21 show the results for Case 1.3c, the “linear arrays”.

Comparing Case 1.3 to Case 1.2, the main observation is that the 3 m high blocks perform poorly compared to the 2 m high blocks. It must be emphasized that this is an observation for the present configurations of the blocks and for the present stratification of the soil. However, it cannot be generalized, and in other situations, larger masses may be preferred. Also, increasing the mass by using higher density or increasing the footprint instead of the height may have a different influence on the result.

Apart from the lower IL compared to Case 1.2, the “Stonehenge” configuration with two or three rings (Figure 16 and Figure 17) performs significantly better than the “turned rings” and “linear arrays” (Figures 18 to 21) in the frequency range 25–30 Hz, whereas the opposite observation can be made in a narrow range near 16 Hz. Further, at 25 Hz the “Stonehenge” configuration with three rings provide an almost axisymmetric IL distribution in the shielded zone, whereas the “turned rings” provide a radial (or ray-formed) pattern with very large IL in smaller zones. Finally, at 25 Hz the “linear array” with three rings provides very low IL. Generally, the addition of Ring 3 provides very little improvement in Case 1.3c with “linear arrays” of 3 m high blocks on the soft dry sand layer over the till half-space.
Figure 16: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.3a: “Stonehenge” with 3 m high blocks (inner circle) on soft dry sand over till.

The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 17: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.3a, “Stonehenge” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 18: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.3b: “Turned rings” with 3 m high blocks (inner circle) on soft dry sand over till. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 19: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.3b, “turned rings” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 20: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 1.3c: “Linear arrays” with 3 m high blocks (inner circle) on soft dry sand over till. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 21: Contours of the reference vibration levels and insertion losses (in dB) for Case 1.3c, “linear arrays” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
4 CASE 2: A SOFT WET CLAY LAYER OVER A HALF-SPACE OF LIME

The second case concerns a 5 m deep soft wet clay layer overlying a half-space of lime. Material properties for the two soil layers are listed in Table 5. The main difference compared to the previous case, i.e. Case 1, is the larger depth of the top layer combined with the slightly lower shear modulus. This provides a cut-on frequency of about 7 Hz in Case 2, whereas the cut-on frequency was around 12.5 Hz in Case 1. Hence, it can be expected that higher mitigation effect can be obtained by using higher blocks with lower resonance frequencies in this case, compared to Case 1. Hence, in this section, only results for 3 m high blocks will be discussed. The resonance frequencies for these blocks are given in Table 6.

<table>
<thead>
<tr>
<th>Soil layer (from top)</th>
<th>Shear modulus $G$ [MPa]</th>
<th>Poisson’s ratio $\nu$ [-]</th>
<th>Mass density $\rho$ [kg/m$^3$]</th>
<th>Loss factor $\eta$ [%]</th>
<th>Thickness $h$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand, soft, dry</td>
<td>30.03</td>
<td>0.4942</td>
<td>1694</td>
<td>4.50</td>
<td>5.0</td>
</tr>
<tr>
<td>Till (half-space)</td>
<td>4300</td>
<td>0.3500</td>
<td>2100</td>
<td>2.00</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Table 5: Soil properties for Case 2.

<table>
<thead>
<tr>
<th>Soil layer (from top)</th>
<th>Shear modulus $G$ [MPa]</th>
<th>Poisson’s ratio $\nu$ [-]</th>
<th>Mass density $\rho$ [kg/m$^3$]</th>
<th>Loss factor $\eta$ [%]</th>
<th>Thickness $h$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand, soft, dry</td>
<td>30.03</td>
<td>0.4942</td>
<td>1694</td>
<td>4.50</td>
<td>5.0</td>
</tr>
<tr>
<td>Till (half-space)</td>
<td>4300</td>
<td>0.3500</td>
<td>2100</td>
<td>2.00</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Table 6: Resonance frequencies of blocks in Case 2.

Figure 22 shows the mean values of the IL and RL in the shielded zone (20 m to 40 m from the centre of the load) in Case 2a for the frequency range 0–80 Hz and for “Stonehenge” configurations with either 1, 2 or 3 rings. Figure 23 shows the RL (top left quadrants of the subplots) and IL (remaining quadrants of the subplots) on the ground surface at nine selected frequencies. Figure 24 and Figure 25 show similar results for Case 2b, the “turned rings”, while Figure 26 and Figure 27 show the results for Case 2c, the “linear arrays”.

Compared to Case 1, the RL of vibration is lower at frequencies below the cut-on frequency. This is caused by the higher stiffness of the lime half-space in Case 2 compared to that of the till half-space in Case 1. However, the peak RL is slightly higher in Case 2 compared to the peak RL in Case 1, since the clay is slightly softer than the sand.

For the soil profile considered in Case 2, the various block array configurations mainly provide mitigation of ground vibrations in the frequency range 12–27 Hz. The “turned rings” have a good performance around 12–13 Hz and again near 20 Hz. However, outside these narrow peaks, the IL is low. The “linear arrays” have a similar performance, but the IL in the intermediate range 13–18 Hz is better than for the “turned rings”. As can be seen from Figure 25, there is a clear radial pattern in the IL at 15 Hz for both these configurations.

The “Stonehenge” configuration is about 5 dB less effective than the other configurations at frequencies near 12–13 Hz. On the contrary, it provides 10 dB higher IL at frequencies around 25 Hz—however only when all three rings are present. A possible explanation to this is the somewhat higher resonance frequency of the heave mode for the smaller masses in Ring 3 of the “Stonehenge” configuration. Whereas the resonance frequency for the larger blocks in the inner ring, and in all rings of Cases 2b and 2c, are below 20 Hz, the resonance frequency of the heave mode related to the smaller blocks in Ring 3 of Case 2a is above 24 Hz. Similarly, it can be seen that the “Stonehenge” with two rings provides slightly higher TL for frequencies just above 20 Hz, but lower IL at 20 Hz than the array with only one ring.
Figure 22: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 2a: “Stonehenge” with 3 m high blocks (inner circle) on soft wet clay over lime. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 23: Contours of the reference vibration levels and insertion losses (in dB) for Case 2a, “Stonehenge” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 24: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 2b: “Turned rings” with 3 m high blocks (inner circle) on soft wet clay over lime. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 25: Contours of the reference vibration levels and insertion losses (in dB) for Case 2b, “turned rings” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
Figure 26: Mean insertion loss in dB for the donut-shaped zone between 20 m and 40 m away from the centre of the loaded area. Case 2c: “Linear arrays” with 3 m high blocks (inner circle) on soft wet clay over lime. The black line shows the reference level of vibration. Thin lines show 10% and 90% quantiles.

Figure 27: Contours of the reference vibration levels and insertion losses (in dB) for Case 2c, “linear arrays” with 1, 2 and 3 rings. Pale yellow/dark blue shades represent unfavourable/favourable values, respectively.
5 CONCLUSIONS

Circular arrays of rigid blocks interacting dynamically with a horizontally layered linear viscoelastic half-space have been analysed. Especially, the mitigation efficiency of such arrays placed around a load acting on the ground surface has been quantified. Two different cases have been considered: Case 1) a soft dry sand layer over a till half-space; Case 2) a soft wet clay layer over a lime half-space. For each case, three different configurations of the circular block arrays have been examined: a) a “Stonehenge” configuration with up to three concentric rings of blocks; b) a “turned rings” configuration with the middle ring turned relatively to the inner and outer ring; c) a “linear array” configuration similar to the “turned rings”, but with all rings oriented in the same way to obtain six rays of linear block arrays.

For Case 1, blocks of different heights have been considered. Only 3 m high blocks have been analysed for Case 2. In all the analyses, the frequency range 0–80 Hz has been considered due to its relevance to annoyance caused by whole-body vibration as well as structural damage caused by resonance in the fundamental modes of a building. Results have been presented in terms of the insertion loss that occurs after insertion of one, two or three rings of blocks in each considered combination of soil and array configuration. The mean value of the insertion loss has been calculated for a shielded zone, 20 m to 40 m away from the centre of the circular loaded area. The main conclusions of the paper are as follows:

- Circular arrays of blocks can have a significant mitigation effect. Insertion losses up to about 20 dB have been observed at frequencies that are relevant to whole-body vibration as well as the first modes of resonance in buildings.
- In the cases considered in the paper, the circular block arrays have been found to be particularly effective in the frequency range between the cut-on frequency for wave propagation in the soft topsoil layer and up to about 30–40 Hz. This is an important finding, since this is typically the frequency range in which problems with ground vibrations can arise due to the frequency contents of sources—e.g., traffic or pile driving—combined with the ability of the ground to transfer vibrations at different frequencies.
- The height of the blocks can be adjusted to change the resonance frequencies of the heave and pitch mode. This may be useful in order to calibrate the blocks for optimal performance of the circular arrays in terms of ground vibration mitigation.
- In many cases, the “Stonehenge” configurations have been found to provide higher insertion losses that the two other configurations of the block arrays. This suggests that combination of blocks with different sizes can be a good alternative to using blocks of equal size.

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WAVE PROPAGATION IN POLAR PERIODIC STRUCTURES USING FLOQUET THEORY AND FINITE ELEMENT ANALYSIS

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Abstract

In this paper, a generalised approximated approach to study wave propagation in structures that exhibit radial and/or circumferential periodicity is presented. Only a circular sector of the structure is studied, which could be a circumferential period or an arbitrary slice according to the kind of periodicity of the structure (radial, circumferential, both radial and circumferential). The slice is then approximated using piecewise Cartesian waveguides, whose wave characteristics are obtained by the theory of wave propagation in periodic Cartesian structures and Finite Element analysis. Wave amplitudes change due to the changes in the geometry of the slice are accommodated in the model assuming that the energy flow through the interfaces of each Cartesian waveguide is the same. Results are validated considering the response of an infinite isotropic thin plate excited by a point harmonic force, showing the accuracy of the method and the computational advantage compared to a standard FE harmonic analysis for infinite structures. A numerical example of a polar periodic structure, mimicking a spider web, is also presented to investigate the potential applications of the method.

Keywords: periodic structures; polar coordinates; wave propagation; finite element analysis
1 INTRODUCTION

Periodic structures have attracted attention since a long time ago, perhaps due to both the facts that they naturally represent a finite difference approximation of continuous structures and that many structures in nature show spatial repetitive organization.

One of the milestone studies on periodic structure is the book written by Brillouin [1], where Floquet theory, and its generalization for three dimensions - Block’s theorem, are applied to study wave propagation in lattices. Since then, periodicity effects in electromagnetic wave propagation have been largely investigated, and their properties have been exploited especially for optical and electromagnetic applications.

In structural engineering and vibro-acoustics, the studies on the periodicity effects are “younger”, e.g. [2]. In particular, numerical methods based on Finite Element (FE) analysis to predict the free and forced wave motion in continuous and periodic engineering structures have been developed relatively recently, e.g. [3-6]. These methods have shown advantages compared to other approaches due to their ability to model complex periodic structures, accuracy up to high frequency, and low computation cost compared to standard FE method. The aim of this paper is to extend the application of one of these methods, named the Wave and Finite Element (WFE) Method [6], to periodic structures in polar coordinates. Although applications of the method to axisymmetric, helical, and slowly varying waveguides have been presented [7-9], the method assumes wave propagation in 1 dimensional and 2-dimensional periodic structures posed in Cartesian coordinates [10].

This paper deals with structures that exhibit periodicity in the circumferential and radial directions, whose cylindrical wave propagation is estimated using the Floquet theory formulation for an infinite periodic structure in one dimension. Approximation of the Floquet theory for radially periodic structures has been recently presented in [11]. Compared to this work, the approximation is here achieved in terms of the FE discretisation of a period of the structures to which the theory of wave propagation in periodic structures is applied according to the formulation of the WFE method. Only a circular sector (slice) of the structure is studied, which could be a circumferential period or an arbitrary slice according to the kind of periodicity of the structure (radial, circumferential, both radial and circumferential). The slice is then approximated using piecewise Cartesian waveguides, whose wave characteristics are obtained by the WFE method. Wave amplitudes change due to the changes in the geometry of the slice are then accommodated in the model assuming that the energy flow through the interfaces of each Cartesian waveguide is the same.

The paper is organized as follows. In section 2 the approach is outlined. Section 3 is devoted to the validation of the method through comparison of the results with those obtained analytically for the response of an infinite isotropic thin plate to a harmonic point harmonic force. In section 4 a numerical example of a spider-web inspired structure is presented to show the potential applications of the method to more complicated polar periodic structures.

2 WAVE FINITE ELEMENT APPROXIMATION OF RADIALLY PERIODIC STRUCTURES

The analysis of wave propagation in the near-field of a lossless radially periodic structures is here approximated using a stepwise WFE approach. Figure 1 depicts an infinite radially periodic structure and an arbitrary circular sector of the periodic structure. If the structure is periodic in the circumferential direction the circular sector must be taken according to this circumferential periodicity. This periodic slice is approximated using a series of rectangular segments in Cartesian coordinates as shown in Fig. 2, whose wave characteristics are obtained
using the WFE method. Periodically varying characteristics are assumed along the slice approximated in Fig. 2.

![Figure 1: a) radially periodic structure; b) slice of the radially periodic structure](image1)

**Figure 1:** a) radially periodic structure; b) slice of the radially periodic structure

![Figure 2: approximation of the radially periodic waveguides in piecewise Cartesian periodic waveguides.](image2)

**Figure 2:** approximation of the radially periodic waveguides in piecewise Cartesian periodic waveguides.

### 2.1 Wave propagation characteristics in each Cartesian waveguide segment

Wave characteristics of each waveguide, denoted as 1, 2, 3, etc. in Fig. 2, are here obtained using the WFE method [10]. In this method, a period of the waveguide is discretised using standard FE elements. The FE mass and stiffness matrices are reduced using the theory of wave propagation in periodic structures [1] and wave characteristics are numerically evaluated from an eigenvalue problem. In particular, wavenumbers and frequencies, viz. dispersion curves \((k, \omega)\), nodal displacements \(\Phi_q\) and nodal forces \(\Phi_f\) under the passage of a wave are predicted.

Wave properties evaluated at frequency \(\omega\) are grouped into positive and negative going waves. For example, considering waveguide 1, they are \((k_1^+, \Phi_1^+=[\Phi_{q,1}, \Phi_{f,1}]^T, a_1^+)\) and \((k_1^-, \Phi_1^-=[\Phi_{q,1}, \Phi_{f,1}]^T, a_1^-)\), where \(k^+, k^-\) and \(a^+, a^-\) are wavenumbers and waves’ amplitudes travelling in the positive and negative direction, while \(\Phi_q^\pm\) and \(\Phi_f^\pm\) are the FE discretisation of wavemodes.
Wave amplitudes are related at two points \( x_1 \) and \( x_2 \), located at the distance \( L \) between them, by
\[
\mathbf{a}_1^+ (x_2) = \mathbf{T}_i^+ (L) \mathbf{a}_i (x_1) \quad \text{and} \quad \mathbf{a}_i^+ (x_2) = \mathbf{T}_i^- (L) \mathbf{a}_i^+ (x_1) ,
\]
where
\[
\mathbf{T}_i^+ (L) = \text{diag} \left[ \exp (-i \mathbf{k}_i^+ L) \right] \quad \text{and} \quad \mathbf{T}_i^- (L) = \text{diag} \left[ \exp (-i \mathbf{k}_i^- L) \right] .
\]
A similar notation is used for waveguides 2, 3, etc.

The relation between the nodal displacements \( \mathbf{q} \) and nodal internal forces \( \mathbf{f} \) and the wave amplitudes in the piecewise waveguides is given by
\[
\begin{bmatrix}
\mathbf{q}_1 \\
\mathbf{f}_1
\end{bmatrix} = \begin{bmatrix}
\Phi_{q,1}^+ & \Phi_{q,1}^- \\
\Phi_{f,1}^+ & \Phi_{f,1}^-
\end{bmatrix} \begin{bmatrix}
\mathbf{a}_1^+ \\
\mathbf{a}_1^-
\end{bmatrix},
\begin{bmatrix}
\mathbf{q}_2 \\
\mathbf{f}_2
\end{bmatrix} = \begin{bmatrix}
\Phi_{q,2}^+ & \Phi_{q,2}^- \\
\Phi_{f,2}^+ & \Phi_{f,2}^-
\end{bmatrix} \begin{bmatrix}
\mathbf{a}_2^+ \\
\mathbf{a}_2^-
\end{bmatrix},
\]
where the subscripts 1, 2, etc. indicate the waveguide.

The time average energy flow in each segment can be obtained from
\[
\Pi_i = \frac{1}{2} \text{Re} \left\{ \mathbf{f}_i^T i \omega \mathbf{q}_i \right\}; \quad \Pi_2 = \frac{1}{2} \text{Re} \left\{ \mathbf{f}_2^T i \omega \mathbf{q}_2 \right\}; \quad \ldots
\]

### 2.2 Coupling of the segments and wave amplitude decay

Compared to the corresponding Cartesian periodic structure, in the radially periodic structures, wave amplitude attenuation is dependent upon the distance from the centre. Wave amplitude change due to changes in the geometry can be easily accommodated in the model from energy conservation consideration. Therefore, in the following, it is assumed that the energy flowing through the cross sections at the segments is constant along the slice.

First, nodal displacements and nodal forces are evaluated for waveguide 1 at the interface \( x_2 \), that is
\[
\begin{bmatrix}
\mathbf{q}_1 (x_2) \\
\mathbf{f}_1 (x_2)
\end{bmatrix} = \begin{bmatrix}
\Phi_{q,1}^+ & \Phi_{q,1}^- \\
\Phi_{f,1}^+ & \Phi_{f,1}^-
\end{bmatrix} \begin{bmatrix}
\mathbf{T}_1^+ (L) & 0 \\
0 & \mathbf{T}_1^- (L)
\end{bmatrix} \begin{bmatrix}
\mathbf{a}_1^+ \\
\mathbf{a}_1^-
\end{bmatrix}
\]
and continuity of displacement and equilibrium of nodal forces between waveguide 1 and 2 at \( x_2 \) are then introduced, \( [\mathbf{q}_2 (x_2) \mathbf{f}_2 (x_2)]^T = [\mathbf{q}_1 (x_2) \mathbf{f}_1 (x_2)]^T \). Using Eq. (1), the first attempt to obtain the wave amplitude in 2 gives
\[
\begin{bmatrix}
\tilde{\mathbf{a}}_2^+ \\
\tilde{\mathbf{a}}_2^-
\end{bmatrix} = \begin{bmatrix}
\Phi_{q,2}^+ & \Phi_{q,2}^- \\
\Phi_{f,2}^+ & \Phi_{f,2}^-
\end{bmatrix} \begin{bmatrix}
\mathbf{q}_2 (x_2) \\
\mathbf{f}_2 (x_2)
\end{bmatrix}
\]
where \( \dagger \) denotes pseudoinverse (note that left eigenvectors as described in [10] are typically used in order to avoid numerical errors). The time-averaged energy flow in waveguide 1 and 2 at \( x_2 \) is then evaluated using Eq.(2). Equation (4), however, does not take into account the waves decay due to the change in the geometry of the waveguide.

Assuming that the energy flowing through the cross sections at the interface between the two waveguides is the same, the wave amplitude decay \( \alpha_2 \) in waveguide 2 is evaluated as [9]
\[
\alpha_2 = \frac{\text{Re} \left\{ \mathbf{f}_1 (x_2)^T i \omega \mathbf{q}_1 (x_2) \right\}}{\text{Re} \left\{ \mathbf{f}_2 (x_2)^T i \omega \mathbf{q}_2 (x_2) \right\}}
\]
The wave amplitudes in waveguide 2 are therefore approximated to
\[
\begin{bmatrix}
a_2^+ \\
a_2^-
\end{bmatrix} = \alpha_2 \begin{bmatrix}
\tilde{a}_2^+ \\
\tilde{a}_2^-
\end{bmatrix}
\] (6)

Eqs. (3)-(6) are repeated for the other segments in order to obtain \( \alpha_1, \alpha_4, \ldots \) according to the number of segment used to approximate the slice as in Fig. 2.

3 NUMERICAL EXAMPLES

3.1 Response of an infinite plate excited by a point transverse force

In this numerical example, a thin aluminium plate is considered, which has thickness \( h=1 \text{mm} \), density of \( \rho = 2700 \text{ kg/m}^3 \), Young’s modulus \( E= 710 \text{ GPa} \) and a Poisson’s ratio of \( \nu =0.3 \). The flexural waves in the plate are excited by a central force \( F(t) = F e^{i\omega t} \) and the complex out of plane displacement is determined as a function of the radial coordinate evaluated at a distance \( \bar{r} \) from the force.

A continuous uniform structure can be considered as an arbitrary periodic structure. Therefore, the plate is modelled as a periodic structure with arbitrary radial and circumferential periodicity. A small slice of the plate, spanned by a small angle \( \theta \), is taken under the assumption that the energy flow associated with each wave component (bending wave in this case) is preserved. The small slice is approximated as a piecewise rectangular waveguide as in Fig. 2. Wave characteristics of each of these segments are obtained using the WFE approach. As an example, the periodicity used for the WFE discretisation of each segment can be equal to the height of the corresponding segment, so that the period is a square plate of dimensions \( a=b=2x_1 \tan(\theta/2) \). The choice of the period, however, is here arbitrary under the assumptions of no element distortion and limits for avoiding dispersion errors and numerical artefacts [12-13]. The FE model of the period is a 4-noded plane element in bending having three degrees of freedom per node: translation in the \( z \)-direction and rotations around the \( x \)- and \( y \)-directions.

A small circle around the point force is removed and the left nodes of the first cell are shifted at a distance \( \bar{R}_0 = x_1 = 6 \text{mm} \) from the point force. According to this approximation, external nodal forces \( f_i(x_1)_{\text{ext}} \) and external displacements \( q_i(x_1)_{\text{ext}} \) can be found from analytical solution, FE model, or approximated by an equivalent system of forces applied at the nodes along the circumference \( 2\pi R_0 \). Waves are induced only in the positive \( x \) direction (radial direction) and \( a_1^- = 0 \).

The excited wave amplitudes generated by the external nodal forces in waveguide 1 are given by
\[
\Phi_{i,1}^\dagger a_i^+ = f_i(x_1)_{\text{ext}} \rightarrow a_i^+ = \left(\Phi_{i,1}^\dagger\right)^\dagger f_i(x_1)_{\text{ext}}
\] (7)

The nodal displacements and nodal forces at the interface between waveguides 1 and 2 are then recovered from Eq. (3), that is \( q_i(x_2) = \Phi_{q,1}^\dagger T_{i,1}^\dagger (L_1)a_i^+; f_i(x_2) = \Phi_{f,1}^\dagger T_{i,1}^\dagger (L_1)a_i^+ \). Wave amplitude decay in the next waveguides is evaluated following the passages described in Section 2.2 and applying Eqs. (3)-(6) to each segment up to the segment \( j \) which includes the point distant \( \bar{r} \) from the force.

The response is then evaluated from
\[ q_j(x_r) = \Phi^+ T^+(x_r) a^+_j, \quad x_r = \bar{r} - (R_0 + L_1 + L_2 + \ldots + L_{j-1}) \] (8)

Figure 3 shows, as an example, the real and imaginary part of the transverse response at a distance \( \bar{r} = 0.3m \) from a transverse point force of magnitude \( F = 100N \). Comparison between the numerical and analytical results is shown. The latter are evaluated from
\[
w = -\frac{iF}{8Dk} \left( H^2_0(kr) - i\frac{2}{\pi} K_0(kr) \right),
\]
where \( D \) is the plate bending stiffness per unit length, \( k = \sqrt{\alpha^2 \rho h / D} \) is the flexural wavenumber, \( H^2_0(kr) \) is the zero-order Hankel function of the second kind and \( K_0(kr) \) is the zero-order modified Bessel function of the second kind [14].

![Figure 3: response of a thin plate excited by a harmonic transverse point force. Comparison between the approximated WFE and analytical results.](image)

The WFE approximation is obtained for a slice with \( \theta = 6^\circ \) and \( R_0 = 10mm \). The external force is approximated by two transverse nodal forces equal to \( F_{\text{ext}} = F / p \), where \( p \) is the number of nodes at the circumference \( 2\pi R_0 \). Discretisation by only three segments has been found sufficient for convergence to the approximated results in Fig. 3 and no significant differences are noticed decreasing the distance \( R_0 \) further. It can be seen that results are in very good agreement. Small errors at lower frequency are due to the singularity of the Bessel functions in the analytical solution when \( k = 0 \) or, equivalently, when \( f = 0Hz \).

3.2 Polar periodic structure

In this section, a polar periodic structure, which imitates a spider web, is investigated. Figure 4 shows the structure and the slice studied using the approach described in Section 3. The polar periodicity is \( \alpha = 2\pi / 60 \) and \( \Delta_r = 0.01m \), while the radius of the internal hole is \( R_0 = 0.01m \). Each wire has a square cross-section of 1mm. The same material properties as in the previous example are used, and the WFE model is obtained approximating the slice in 3 piecewise waveguides as shown in Figure 4(c).
The FE model of each period is realised using four beam elements with 6 degrees of freedom per node: translations in the \( x \), \( y \) and \( z \)-direction and rotations around the \( x \), \( y \), and \( z \)-directions. Figure 5 shows the dispersion curves of each segment in figure 4(c) for waves propagating in the positive \( x \) direction obtained by the WFE method. The dotted line in Figure 5 shows the flexural waves propagating in a corresponding plate for comparison.

Compared to the low-frequency dispersion curves for propagating waves in a plate, which show three branches corresponding to flexural, shear and axial waves, Fig. 5 shows that a further wavemode, involving displacements mainly in \( z \)-direction, can propagate in the waveguides and that the shear and the flexural mode differ from those propagating in the corresponding continuous plate/beam while the axial wavemode is similar.

The real and imaginary part of the transverse response (out-of-plane), evaluated at a distance \( \bar{r} = 0.3m \) from the centre of the structure, are presented in Fig. 6. This figure shows a comparison between the numerical results obtained considering only the flexural mode propagating in the structure and those obtained including the contribution of the all wavemodes propagating in the frequency range considered. Results for the corresponding plate, similarly excited by a central force, are also shown. It can be noticed that the results differ in terms of magnitude and phase. In particular, the effect of the other modes propagating in the structure and involving...
displacement in the out-of-plane direction can affect significantly the results, even at low frequency.

Figure 6: response of the polar periodic structure excited by a harmonic transverse point force. Comparison between the approximated WFE results considering only the flexural wavemodes and all the wavemodes propagating in the structure. Comparison with the response of a corresponding thin plate is also given in the figure for reference.

4 CONCLUSIONS

In this paper, free and forced wave propagation in a polar periodic structure have been studied using an approximated finite element discretisation and the theory of wave propagation in Cartesian periodic structures. The method proposed is an extension of the Wave Finite Element method to structures showing radial, circumferential, or both radial and circumferential periodicities. The method has been validated through comparison of the results obtained for an infinite isotropic uniform thin plate excited by a point harmonic force. Results are in a very good agreement with the analytical solutions. A structure that imitates a spider-web, has then been studied as a more complicated example of a polar periodic structure for which no analytical solution to the equations of motion exists. Results have been obtained assuming that the energy flow associated with each wavemode is maintained. Further studies are concerned with the analysis of the wave propagation and forced response when wavemodes cut-off and the wave propagation features stop-band behaviour.

REFERENCES


FLOQUET THEORY ANALYSIS OF A WEAKLY NON-LINEAR PERIODIC STRUCTURE

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Abstract

In the paper, the question of applicability of Floquet theory in a weakly non-linear structure is considered. Usually, references contain numerical approaches for the simple non-linear spring masses case. Nevertheless, there is no readily available analytical solution for that problem. Moreover, the theory expansion to a continuous to a non-linear case is still questionable. In the paper, the structure consisting of a waveguide connected with a non-linear spring is considered. One can use the eigenfrequency of a symmetrical periodicity cell approach, which has no restrictions on a periodic structure form. Two approaches for the stop-band predictions in the infinite periodic weakly nonlinear structure on a simple example of rods connected with the non-linear spring are considered. The approaches considered in the paper such as force balance and symmetrical cell eigenfrequencies could be used for more complicated structures.

Keywords: Periodic structure, vibro-isolation, Floquet theory, non-linear spring
1 INTRODUCTION

The periodic structures are of interest from the scope of vibro-isolation applications starting from the work of D. Mead [1]. However, the methods described in the references (the transfer matrix method, Floquet theory) work only for linear cases. The main property for the Floquet analysis in the translational symmetry of the structure. Therefore, for non-linear structures theory extension is required in order to implement classical methods for periodic structures.

In the paper the question of applicability of Floquet theory in a weakly non-linear structure. Weakly nonlinear periodic structures are of interest of some classical [2] and recent papers [3-4]. Usually, references contain numerical approaches for the simple non-linear spring masses case. Nevertheless, there is no readily available analytical solution for that problem. Moreover, the theory expansion to a continuous to a non-linear case is still questionable. Theoretically, Floquet theory is not working in structures, where the property of translational symmetry is not fulfilled.

In the paper, the structure consisting of waveguides connected with a non-linear spring is considered. Such a structure is obviously having no property of translational symmetry. However, with some adjustments, Floquet theory can be expanded to a weakly non-linear case. In the paper range of applicability of the classical Floquet theory is assessed. To implement the Floquet theory harmonic balance method [5] is used.

From the other hand, one can use the eigenfrequency of a symmetrical periodicity cell approach as described in [6-7], which has no restrictions on a structure type. In [7] it is used for radially periodic structure, which also does not have the translational symmetry property. Therefore, it can be used for Floquet theory predictions quality assessment.

On a balance in the paper, the vibro-isolation properties of the infinite periodic weakly non-linear structure are considered. It is done from two points of view (energy flow analysis and symmetrical cell eigenfrequency analysis) on a simple example of rods connected with the non-linear spring. The approaches considered in the paper such as force balance and symmetrical cell eigenfrequencies could be used for more complicated structures.

The paper is organized as follows: Section 2 contains the more detailed problem formulation, Section 3 is dedicated to the energy flow analysis, Section 4 is about the symmetrical periodicity cell eigenfrequency analysis and Section 5 concludes the paper.

2 THE PROBLEM FORMULATION

Within the paper, we consider structure, periodic in the axial direction. It is assumed that each segment is connected to the left and the right neighbor with the non-linear spring. The structure is shown in Fig.1 we will refer to as an infinite periodic structure within the paper.

![Periodic structure](image)

Figure 1: Periodic structure.

Every segment can have its own governing equation. No matter how different governing equations for every segment are, they should have periodically repeating pattern, starting from certain segment pattern of governing equation choice is repeating.

For simplicity matters in the paper, we assume that every part of the structure has following governing equation that represents axial rod vibration.
\[ c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \]  

(1)

Between segments following interfacial conditions, that are representing nonlinear spring between the adjacent parts, are stated in the form

\[ u_i(1,t) = \kappa(u_i(1,t) - u_j(1,t))^N \frac{N}{\kappa}(u_i(1,t) - u_j(1,t))^2 \]

(2)

\[ f_i(1,t) = f_j(1,t) \]

, where \( f(x,t) = \frac{\partial u(x,t)}{\partial x} \) - is the axial force.

Usually periodic structures are considered in the vibroisolation scope, however, usual methods that are applicable in linear theories like Floquet theorem are not suitable for this case, since Floquet theorem require translational symmetry. The waveguide properties, in this case, could be analyzed for example with energy flow analysis.

3 THE ENERGY FLOW ANALYSIS

The existence and position of Floquet zones can be proved with numerical energy flow analysis of forcing problem for non-linear interface case. Schematically forcing problem can be illustrated as shown in Fig.2

![Energy flow analysis scheme.](image)

In forcing problem boundary conditions are replaced with harmonic force at the left boundary and Sommerfeld condition at the infinity, both conditions, in this case, have the form Eq. 3.

\[ f_i(0,t) = (b_{i,1} - 1)^* \cos(\omega t) \]

(3)

\[ b_{last,2} = 0 \]

We use following time dependence \( u_i(x,t) = u_i(x)^* \left( \frac{1}{2} \exp(-i\omega t) + \frac{1}{2} \exp(i\omega t) \right) = u_i(x) \cos(\omega t) \), where \( i \) is the number of the segment shown in Fig.2. The general solution for the axial rod vibration case has the form Eq.4.

\[ u_i(x,t) = (b_{i,1} \exp(-ix) + b_{i,2} \exp(ix)) \cos(\omega t) \]

(4)

It should be noted that time-dependence \( \cos(\omega t) \) is chosen for the brevity of evaluation, one could consider an arbitrary combination of exponents as time dependence and obtain the same result.

After Eq.4 is substituted into conditions Eq.2, Eq.3, one obtain non-linear system, that can be written in form Eq.5

\[ (L + F) * \cos(\omega t) + C * \cos^3(\omega t) = 0 \]  

(5)
In Eq.5 with $L, F$ shown parts of the system Eq.2, Eq.3 with respect to the unknown displace-ments $b_{i,j}$ that contain linear term $\cos(\omega t)$ as a multiplier $C$ is the part of the system with a cubic multiplier $\cos^3(\omega t)$.

Using harmonic balance method [5] in the expression $\cos^3(\omega t) = \frac{1}{4}(3\cos(\omega t) + \cos(3\omega t))$ term $\cos(3\omega t)$ can be omitted and then $\cos(\omega t)$ is canceled giving a final form of the equation

$$L + \frac{3}{4}C = -F$$

Eq.6 is the system of polynomial equations. First $n - 1$ equations of the system with $n$ periodic inserts are cubic with respect to the unknown displacements $b_{i,j}$, whereas other equations of the system are linear.

After force balance method is implemented and time-dependency is canceled out, one can use numerical methods in order to find unknown displacement amplitudes. The solution of the system with $n - 1$ a cubic equation obviously gives $3(n - 1)$ sets of solutions of Eq.6. Only one set that gives real-valued energy flow should be chosen.

When $2n$ unknown displacements for every segment are known, energy flow through the structure with $n$ inclusions can be found as Eq.7.

$$E_n(x, \Omega) = -\frac{1}{2}i\Omega * u(x, \Omega) * f(x, \Omega)$$

It should be noted that the energy flow is obtained in assumption $f \rightarrow 0$, i.e. the energy flow value is not depended on forcing chosen when a number of periodic insertions tend to infinity.

Usually, vibro-isolation effects are shown with insertion loss function, that usually defined as Eq.8.

$$IL_n(x, \Omega) = \frac{1}{10} \log_{10}(\frac{E_0(\Omega)}{E_n(x, \Omega)})$$

In Eq.8 with $E_0(\Omega)$ energy flow through the homogenous rod without nonlinear inclusions is designated. It should be noted that energy is not conserved in a non-linear case and therefore below the values are considered at the point $x = n + 1/2$, where the periodic insertion ends.

In the linear case, the insertion losses values are depended on the amount of the periodic insertions. The larger is the number of insertions the larger is the stop-band attenuation rate. However, it should be noted that in the linear case the eigenfrequencies of the periodic insertion introduce “anti-peaks”, that are amplifying the waves in pass-bands, which could be considered as a negative effect. The number of “anti-peaks” is depended on a number of periodic insertions as shown in Fig.3.

![Figure 3: Insertion loss for a linear case (N=0) for a number of insertions n=3,5,10 (blue, green, red).](image-url)
It should be emphasized that in non-linear case the “anti-peaks” are smoothed out by the non-linearity of the periodic structure and thus it may be considered as more beneficial to use the non-linear spring in pass-bands. As in the linear case, attenuation level increases with the increasing of the number of cells as shown in Fig.4.

From Fig.3-Fig.4 is seen that the width of attenuation zone (with position insertion loss value), which can be considered as stop band shrinks with increasing the number of insertions, however, the maximum of the insertion loss value grows. That means that the stop-band position predicted by the classical Floquet theory will always be “underpredicted” with respect to the real energy flow picture.

In Fig.5 shown influence of non-linearity parameter \( N \) on the energy flow for \( n = 10 \) periodic insertions.

The attenuation zones are shifted by the non-linearity parameter \( N \). Attenuation zones could be as extended as well as reduced. Since the energy flow evaluation is computationally expensive it is required to build the approximation that allows one to assess attenuation zones without computing insertion losses. One of the approximations are the eigenfrequencies of single periodicity cells \([6-7]\). The process of eigenfrequency obtaining is described below.

On a balance, the energy flow analysis shows that attenuation zones in periodic structures exist in the non-linear case. In contrast to the linear case, the width of the attenuation zone depends on the non-linearity parameter \( N \). Additionally, a sufficient amount of periodicity cells is required in order to estimate stop-band properly. It should be emphasized that the force balance method gives an only approximation of the actual energy flow and attenuation zones boundaries cannot be determined clearly from the energy flow analysis.

4 THE EIGENFREQUENCY ANALYSIS

The second step in the analysis in consider finite symmetrical counterpart, most interesting is to consider structures, that have a one-period length as shown in Fig.6
Finite structure, shown in Fig.6 is called symmetrical periodicity cell. When symmetrical periodicity cell is supported with the symmetrical boundary conditions, it has eigenfrequencies that are located on the stop-band boundaries [6]. Despite the structure is non-linear, one can still find eigenfrequencies of the linearized with the harmonic balance method system. In case of governing equation Eq.1, symmetrical boundary conditions have the form

\[
\begin{align*}
  u_1(1/2) &= 0 \\
  u_2(3/2) &= 0
\end{align*}
\] (9)

\[
\begin{align*}
  f_1(1/2) &= 0 \\
  f_2(3/2) &= 0
\end{align*}
\] (10)

Following Mead [1], conditions Eq.9 are called A-type conditions and Eq.10 B-type conditions. Together with interfacial conditions, they are forming A-type symmetrical boundary problem and B-type symmetrical boundary problem. It is shown in [6-7] that the symmetrical A and B type problems are covering all Floquet stop-band boundaries in the linear case.

In the linear case, a sufficient number of cells is required in order to energy flow attenuation zones were places on stop-band boundaries as it is shown in Fig.7.

![Figure 7: Eigenfrequencies of A and B types (linear case)](image)

Unlike the linear case, the determinant for the system with cubic polynomials is not defined. One can obtain resultant [8] of the systems Eq.2, Eq.9, and Eq.2, Eq.10. However, we stop the procedure on a single permutation resultant and take real part. For the systems Eq.2, Eq.9 and Eq.2, Eq.10 this approach is invariant to the permutation choice and thus one can conclude that it is the final form of the resultant.

The backbone eigenfrequency curves for a symmetrical periodicity cell, shown in Fig.2, the final form of the non-linear A- and B-type eigenfrequency equation is Eq.11.

\[
\begin{align*}
  D_A(\beta, \Omega) &= -4\Omega \cos(\frac{\Omega}{2}) \left( \Omega \cos(\frac{\Omega}{2}) + 2\kappa \sin(\frac{\Omega}{2}) \right) + 48 \beta \kappa \Omega \sin^2(\frac{\Omega}{2}) \sin(\Omega) \\
  D_B(\beta, \Omega) &= 4\Omega^3 \sin(\frac{\Omega}{2}) \left( -2\kappa \cos(\frac{\Omega}{2}) + \Omega \sin(\frac{\Omega}{2}) \right) - 12 \beta \kappa \Omega^3 \csc^2(\frac{\Omega}{2}) \sin^3(\Omega)
\end{align*}
\] (11)

In the linear case \(D_A\) and \(D_B\) are covering all stop- and pass-band boundaries. It can be proved directly when one is considering the linear case (\(N=0\) or \(\beta = 0\)). In general case it is
not proved, however, this fact is also valid for the radial periodic membrane case [7]. Dependencies \( D_A = 0 \) and \( D_B = 0 \) could be plotted as shown in Fig.8.

In Fig.8 vertical lines are corresponding to eigenmodes without spring deformation. Therefore, they are not changing as the non-linearity parameters \( \beta \) grow. In the linear case, such eigenfrequencies also exist.

When comparing both approximations is can be seen that they are giving different predictions. However, in a low-frequency region, they are both agree. When moving to higher frequencies the A and B-type boundary problems start “under predicting” stop-band boundaries as it is shown in Fig.9.

Since standard Floquet theory is working only for operators with translational symmetry, Fig.8 and A- and B-type problems backbone curves can be used for analytical stop-band estimation in non-linear case. Fact, that in linear case eigenfrequencies on a symmetrical periodicity cell with A- and B-type conditions are placed on a stop-band boundary gives one a tool that predicts stop-bands on a larger amplitudes magnitude as shown in Fig. 9.

5 CONCLUSIONS

Within the paper, two approaches to a vibro-isolation properties assessment of a weakly non-linear structure are considered. Following results are obtained:

- The energy flow analysis with the harmonic balance method can be used to model the vibro-isolation properties
- The energy flow analysis “underpredicts” the classical stop-bands. To obtain a correct picture of Floquet zones sufficient number of periodic insertions are required in the forcing problem
- The eigenfrequency analysis is another method to analyze vibroisolation properties of a weakly nonlinear structure
- The eigenfrequency analysis requires both harmonic balance and resultants in order to obtain the picture of stop-bands
• Both methods are agreeing on a low-frequency range

As the next step, the third method – general Floquet theory for a weakly non-linear structure is proposed. Such formulation will allow one to consider the waveguide properties in a more correct way.

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MODELLING OF PERIODICITY-INDUCED PRESSURE PULSATION SUPPRESSION IN PIPES EXPOSED TO INTERNAL HEAVY FLUID LOADING

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Abstract

The internal pressure pulsations always accompany functioning of a liquid pumping system. These pulsations generate forces acting at its structural components and eventually produce undesirable noise emission. The level of noise emitted from such a system as a by-product of its operation needs to be minimized due to comfort regulations without hampering the mean pressure it produces. In the case of narrow band pressure pulsations, a periodic dampener (a soft cylindrical insert) made of a specially engineered acoustic metamaterial may be used. To support this design, a mathematical model of performance of such a system, which fully accounts for the fluid-structure interaction, is formulated and numerical analysis of periodicity-induced wave attenuation is performed.

Keywords: Periodic cylindrical shell, Heavy fluid loading, Wave propagation, Reduced-order modelling, Pass- and stop-bands.
1 INTRODUCTION

In acoustics of exhaust and ventilation systems, pressure pulsation damping and, therefore, sound emission reduction is the classical subject, thoroughly discussed in many textbooks with [1] being just an example. Use of Helmholtz resonators and quarter-wave tubes is a standard tool to suppress pressure pulsations at distinct frequencies [2]. For broadening frequency range of pulsation suppression, ensembles of Helmholtz resonators with different parameters are often used. It is also common practice to employ a compliant, solid lining in combination with resonators. Another type of dampeners contains perforated plates or shells of revolution (cylinders and cones), which are in contact with an acoustic medium experiencing pressure fluctuations [3-4]. All these well-established and well-understood measures become challenged as soon as the fluid-borne sound is conveyed by water, rather than by air. The main reason for that is that the fluid-structure interaction ceases to be one-way, and the fluid-borne sound must be considered alongside with the structure-borne sound [5]. Considerably less attention has, therefore, been directed towards the predictive mathematical modelling of pressure pulsation dampeners for hydraulic systems [6]. As stated in reference [7], analysis of performance of these devices is absent in the scientific literature.

The paper is concerned with periodicity-induced suppression of pressure pulsation in a fluid-filled pipe. The possibility to use periodicity effects in this situation is challenged by the fact that the acoustic medium inside the pipe is uniform. Thus, a periodic alternation of parameters may be done only in structural components. Specifically, we consider one component as a stiff (metallic) shell, and the other component as a membrane (rubber) shell, which has a negligible bending stiffness. To assess feasibility of periodicity-induced attenuation of wave propagation in a pipe with constant inner diameter, alternating material properties, and filled by the homogeneous acoustic medium, we employ the canonical model of thin cylindrical shell [8] and linear acoustics. We consider axisymmetric wave motion in a cylindrical shell under heavy internal fluid loading and pre-tension. Such a formulation may be found in many references with [9-12] being just a few examples. We consider a periodic fluid-filled shell as has been done in [12]. To facilitate solving the transcendental dispersion equations we employ the finite product method [13].

The reduced-order model of wave propagation in homogeneous fluid-filled cylindrical shells is presented in Section 2. The canonical Floquet analysis of wave propagation in a periodic shell composed of high-contrast cells is accomplished and discussed in Sections 3. The results are summarized in Conclusions.

2 THE MODEL OF A TWO-COMPONENT PERIODIC PRE-STRESSED FLUID-FILLED CYLINDRICAL SHELL

As explained in the Introduction, we explore possibilities of suppression of unwanted pressure pulsations by means of periodic insertion of compliant cylindrical segments in a stiff fluid-conveying pipe. Strictly speaking, behavior of ‘compliant’ and ‘stiff’ segments is governed by the same equations of a shell theory. However, due to the high contrast between properties of constituents of such a periodic pipe, each one may be described by simpler means as formulated in this Section.

In the framework of the canonical thin shell theory, the governing equations, which describe propagation of free axisymmetric waves in a pre-stressed elastic cylindrical shell with fluid loading, are (time dependence is taken as $\exp(-i\omega t)$ and this multiplier is omitted):
Equations (1a,b) describe wave motion in the shell of the radius $R$ and thickness $h$ exposed to an axial pre-tension $xN$. In these equations, $u(x)$ is the axial component of displacement and $w(x)$ is the radial one. The material of the shell has a Young’s modulus $E$ and a Poisson ratio $\nu$. Equation (1c) is concerned with the velocity potential $\varphi(x, r)$ in a compressible fluid with the sound speed $c_{\beta}$, and equation (1d) formulates the continuity of wave motion between structural and acoustical components of a waveguide.

A solution to the set of equations (1) has the standard form, which defines free waves:

$$\begin{align*}
\varphi(x, r) &= \hat{\varphi}(r) \exp(k_{\dim} x), \\
w(x) &= C \exp(k_{\dim} x), \\
u(x) &= \hat{\nu}(r) \exp(k_{\dim} x). 
\end{align*}$$

(2)

Here $k_{\dim}$ is a dimensional wavenumber.

The function $\hat{\varphi}(r)$ is governed by the Bessel equation:

$$\frac{\partial^2 \hat{\varphi}}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{\varphi}}{\partial r} + \left( k_{\dim}^2 + \frac{\omega^2}{c_{\beta}^2} \right) \hat{\varphi} = 0$$

(3)

Since a fluid occupies the interior of the cylindrical shell, the velocity potential is:

$$\hat{\varphi}(r) = -i\omega J_0 \left( r \sqrt{k_{\dim}^2 + \frac{\omega^2}{c_{\beta}^2}} \right)$$

(4)

Here $J_0$ is the standard notation for Bessel function of the zeroth order.

The sound speed in the shell material is: $c^2 = \frac{E}{\rho(1-\nu^2)}$. The scaled co-ordinates are: $\tilde{r} = \frac{r}{R}$, $\tilde{x} = \frac{x}{R}$. Besides, the following non-dimensional parameters are used hereafter:

$$\Omega^2 = \frac{\rho(1-\nu^2)\omega^2 R^2}{E}, \quad \hat{\rho} = \frac{\rho_{\beta}}{\rho}, \quad \hat{\gamma} = \frac{c}{c_{\beta}}, \quad k = k_{\dim} R, \quad \kappa^2 = k^2 + \hat{\gamma}^2 \Omega^2, \quad \eta = \frac{N_x(1-\nu^2)}{Eh}.$$
This equation is exact (in the framework of linear acoustics and classical thin shell theory), and may be used for formulation of a mathematical model of the periodic two-component pipe. However, for several reasons this equation is unnecessarily complicated to describe each component. First, equally valid for both, is that we consider the low-frequency range, where compressible fluid contributes with only one travelling wave to the composite waveguide. Therefore, it is expedient to formulate reduced order models, which accurately capture this contribution of fluid to the propagating waves and, by use of the finite product method (see [13] for details), convert the dispersion equation to the polynomial form. It is sufficient to employ the single term finite product approximation:

\[
\frac{1}{2} \left( k^2 + \tilde{j}^2 \Omega^2 \right) \left[ -\nu^2 k^2 + \left( k^2 + \Omega^2 \right) \left( 1 + \frac{h^2}{12R^2} k^4 - \eta k^2 - \Omega^2 \right) \right]
\]

\[
+ \tilde{\rho} \Omega^2 \frac{R}{h} \left( k^2 + \Omega^2 \right) \left( 1 - \frac{k^2 + \tilde{j}^2 \Omega^2}{j_{0,1}} \right) = 0
\]

(6)

Here \( j_{0,1} \) is the first zero of the Bessel function \( J_0(z) \), \( j_{0,1} \approx 2.404 \).

Then two important simplifications are used for modelling a membrane pre-stressed cylindrical shell. The first one is concerned with setting to zero the parameter \( \frac{h^2}{12R^2} \) in equation (6). This assumption implies a negligible bending stiffness of the shell, and it is standard for the membrane shell theory, see [8].

The second simplification is to neglect the effect of Poisson contraction. As can be seen from (6), three waves are captured when \( \nu = 0 \), but the longitudinal wave defined by equation \( k^2 + \Omega^2 = 0 \) becomes completely decoupled with the other two. Thus, the propagation of waves with fluid-structure coupling is characterised by the bi-modal dispersion relation:

\[
\frac{1}{2} \left( k^2 + \tilde{j}^2 \Omega^2 \right) \left( 1 - \eta k^2 - \Omega^2 \right) + \tilde{\rho} \Omega^2 \frac{R}{h} \left( 1 - \frac{k^2 + \tilde{j}^2 \Omega^2}{j_{0,1}} \right) = 0
\]

(7)

One of these modes is dominantly acoustic one, while the second one, which has the cut-on frequency \( \Omega_{\text{cut-on}} = \sqrt{\frac{2\tilde{\rho}}{h^2 \tilde{j}^2}} \) (obtained from (7) by setting \( k = 0 \)), is related to the transverse deformation of a pre-stressed membrane shell’s wall coupled to fluid’s motion. We do not elaborate on technicalities of manufacturing of a cylindrical insert made of materials having \( \nu = 0 \). However, given the recent advances in use of 3D printing technologies for production of acoustic meta-materials, such a task may be easily accomplished. To this end, it should
be observed that we are aiming at a light and compliant rubber-type metamaterial. A high contrast of its properties with the material of the stiff segment is requested to reach the attenuation effect.

The wave motion in a stiff segment may be modelled just as in a rigid acoustic duct:

$$k^2 + \gamma^2 \Omega^2 = 0 \quad (8)$$

This is an elementary single-mode model, in which the flexibility of walls of a fluid-filled cylindrical shell is neglected. For a stand-alone metallic shell its relevance is questionable. However, for a bi-component periodic shell with soft membrane components, this model may be used as a simplified one for the stiff component.

3 FLOQUET ANALYSIS OF WAVES IN AN INFINITE PERIODIC SHELL

We consider an infinite cylindrical shell composed of the periodicity cells shown in Figure 3 and use the canonical Floquet theory. The non-dimensional frequency is introduced as $$\Omega = \frac{\omega R}{c_m}$$, the scaled wavenumber is $$k = k_{\text{dim}} R$$, and $$L_{b/m} = \frac{L_{b/m}}{R}$$.

For the membrane segment, the dispersion equation (7) is used, and solution at the boundaries, $$x = L_b, x = L_b + L_m$$ (see Figure 1) is:

$$w_m(x) = \sum_{j=1}^{2} \left[ C_{mj}^+ \exp(k_{mj}^+(x - L_b)) + C_{mj}^- \exp(k_{mj}^-(x - L_b - L_m)) \right] \quad (9a)$$

$$\varphi_m(x,r) = \sum_{j=1}^{2} \left[ \beta_{mj}^+ \left(r\right) C_{mj}^+ \exp(k_{mj}^+(x - L_b)) + \beta_{mj}^- \left(r\right) C_{mj}^- \exp(k_{mj}^-(x - L_b - L_m)) \right] \quad (9b)$$

As seen from (9), the membrane shell supports two pairs of waves.

The wave motion in a fluid-filled stiff segment is described by the single mode model with the dispersion equation (8) - i.e., an acoustic duct with rigid walls. Then the scaled velocity and pressure are formulated via two wave amplitudes

$$v_b(x) = C_{b}^+ \gamma \Omega \exp(i \gamma \Omega x) - C_{b}^- \gamma \Omega \exp(-i \gamma \Omega (x - L_b)) \quad (10a)$$
\[ p_b(x) = C^+_b \sqrt{\Omega} \exp(i \sqrt{\Omega} x) + C^-_b \sqrt{\Omega} \exp(-i \sqrt{\Omega} (x - L_b)) \]  

(10b)

There are four unknown amplitudes \( C^+_m, C^-_m, \quad j = 1, 2 \) of free waves in the membrane segment and two unknown amplitudes \( C^+_b, C^-_b \) of free waves in the rigid segment. To generate the system of homogeneous linear algebraic equations with respect to these amplitudes, interfacial and periodicity conditions should be formulated.

The continuity conditions are concerned with acoustic pressure and velocity

\[
\int_0^1 p_m(L_b, r) r^2 dr = \int_0^1 p_b(L_b, r) r^2 dr; \quad \int_0^1 v_m(L_b, r) r dr = \int_0^1 v_b(L_b, r) r dr
\]

(11a,b)

Likewise, the periodicity conditions are formulated as

\[
\int_0^1 p_b(L_b + L_m, r) r^2 dr = \int_0^1 p_m(L_b + L_m, r) r^2 dr = \lambda \int_0^1 p_b(0, r) r^2 dr
\]

(12a)

\[
\int_0^1 v_b(L_b + L_m, r) r dr = \int_0^1 v_m(L_b + L_m, r) r dr = \lambda \int_0^1 v_b(0, r) r dr
\]

(12b)

The additional conditions for the ‘excessive’ state variables are

\[ w_m(L_b) = w_m(0) = 0; \]

(13)

Equations (11-13) are assembled into the system of six linear algebraic equations with respect to the two modal amplitudes \( C^+_r, C^-_r \) and four modal amplitudes \( C^+_m, C^-_m, \quad j = 1, 2 \). As its determinant is equated to zero, the quadratic equation in \( \lambda \) emerges:

\[
\lambda^2 + z_1 \lambda + 1 = 0
\]

(14)

The coefficient \( z_1 \) in this quadratic equation has an explicit analytical form, which features all parameters involved in the problem formulation. However, it is very cumbersome and, therefore, not presented here. As known from the Vieta’s theorem, \( \lambda_1 \lambda_2 = 1 \). It means that a wave characterized by the propagation constant \( \lambda_1 \) and travelling/decaying in the positive direction of the axial coordinate has its counterpart with \( \lambda_2 = \frac{1}{\lambda_1} \) and exactly the same properties with respect to the negative direction of the axial coordinate. In the pass-band, \( |\lambda_1| = |\lambda_2| = 1 \). In the stop-band, \( |\lambda_1| \neq |\lambda_2| \neq 1 \). Figure 2 illustrates brief parametric studies of location of pass and stopbands for a periodic pipe filled with water. The material of a membrane shell has density \( \rho_m = 921 \text{ kg/m}^3 \) and Young’s module \( E_m = 1.2 \text{ MPa} \) (Poisson ratio is set
to zero). The fluid parameters are $\rho_f = 1.2 \frac{kg}{m^2}$, $c_p = 343 \frac{m}{s}$. The non-dimensional parameters are $\frac{h}{R} = 0.113$, $\tilde{\rho} = 1.09$, $\gamma = 0.025$.

Figure 2: Location of pass- and stop-bands

In Figure 2a, the pre-tension parameter is set to $\eta = 0.23$. Red curve is plotted for $L_m = L_b = 1$, green curve for $2L_m = L_b = 1$, and blue one for $L_m = 2L_b = 1$. As seen, the relation between lengths of components of a periodic shell strongly influences location of stop- and pass bands. In Figure 2b, the lengths of segments are set to $L_m = L_b = 1$, while pretension is $\eta = 0.115$ (green), $\eta = 0.23$ (red), and $\eta = 0.345$ (blue). The decrease in pre-tension results in the shift of the stop-band to lower frequencies, that is useful for various applications. Experimental validation of these preliminary results constitutes the subject of our on-going work.

4 CONCLUSIONS

In this paper, the reduced order model is formulated to explore the possibilities to suppress wave propagation in a periodic cylindrical shell of the constant inner diameter under heavy fluid loading conditions. For the frequency range of interest, polynomial dispersion equations derived with the first order finite product approximation of the fluid loading term may be used to find wavenumbers of propagating waves. The location of stop-bands may be shifted towards lower frequencies by reducing the pre-tension of membrane inserts. The positions of boundaries between pass- and stop bands may be further adjusted by a proper choice of the lengths of stiff and membrane segments.

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DEVELOPMENT OF A BRAIN EMOTIONAL LEARNING BASED CONTROLLER FOR APPLICATION TO VIBRATION CONTROL OF A BUILDING STRUCTURE UNDER SEISMIC EXCITATION

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\textbf{Keywords:} BEL Controller, Vibration Control, Structural Dynamics

\textbf{Abstract.} In this paper, a numerical simulation of a semi-active neuroemotional based control system for vibration reduction of a 3-story framed building structure under seismic excitation is presented. The Brain Emotional Learning Based Intelligent Controller (BELBIC) is used to design a closed-loop control system that determines the required control action (emotional response) based on the desired and actual system response (sensory input). In this case, the control signal is used to adjust in real time the damping force of a MagnetoRheological (MR) damper to reduce the system response. The results obtained from the numerical simulation validate the effectiveness of the brain emotional learning semi-active controller in improving the overall response of the structural system.
1 Introduction

The Brain Emotional Learning Based Intelligent Controller (BELBIC) is a neuroemotional computational model that aims to trigger a system response based on artificial emotions. This controller is based on the limbic system of mammalian brain composed by a set of interconnected brain structures involving the amygdala, orbitofrontal cortex, sensory cortex and thalamus [1]. The emotional learning process modeled by the BELBIC algorithm has been successfully implemented on a wide-range of control engineering applications [2, 3, 4]. The BEL controller presents interesting features that can be exploited to design structural control systems for civil engineering applications. Thus, the presented semi-active control system was developed based on this bio-inspired controller.

In this paper, a numerical simulation of a semi-active neuroemotional based control system for vibration reduction of a 3-story framed building structure under seismic excitation is presented. Previously to this work, this controller was applied to control a single degree of freedom system, being the obtained results presented in [5]. The BELBIC is used to design a closed-loop control system that determines the required control action (emotional response) based on the desired and actual system response (sensory input). In this case, the control signal is used to adjust in real time the damping force of a MagnetoRheological (MR) damper to reduce the system response.

2 BELBIC Controller

The Brain Emotional Learning (BEL) controller is a novel bio-inspired control model based on the emotional learning mechanism of the brain limbic system which has been employed to develop feedback controllers for complex control problems [6, 7, 8].

Essentially the BEL controller comprises four main components, i.e., the amygdala (Am), the orbitofrontal cortex (OC), the sensory cortex (SC) and the thalamus (Th). The amygdala and the orbitofrontal cortex are used to process the emotional signal (SE) while the sensory cortex and the thalamus receive and processes sensory inputs (SI). The Simulink model of the BEL controller is shown in Figure [1].

![Figure 1: Simulink model of the BEL controller for the three DOFs system.](image)

Sensory inputs (SI) are processed in the thalamus initiating the process of response to stimuli and passing those signals to the amygdala and the sensory cortex. Then, the sensory cortex operates by distributing the incoming signals properly between the amygdala and the orbitofrontal cortex. In this controller, the learning procedure is mainly processed in the orbitofrontal cortex and is based on the difference between an expected punishment or reward and the received punishment or reward (Rew). The perceived punishment/reward (ES) is processed in the brain using learning mechanisms while the received punishment/reward represents an external input.
If these signals are not identical, the orbitofrontal cortex inhibits and restrains the emotional response for further learning. Otherwise, the controller generates an output response [1, 7].

In this case the sensory input (SI) and the emotional signal (ES) can be related with the system response \( y_d \) (inter-story drifts in this case) and the BEL model output \( u \), which are determined using the following equations:

\[
SI = \omega_1 y_d + \omega_2 u \tag{1}
\]

\[
ES = \omega_3 y_d + \omega_4 \int u \, dt \tag{2}
\]

where \( \omega_i \) are weight factors that define the relative importance given to the drift response \((z_1 = y_d)\) and the output of the BEL controller \((f = u)\). The sensory and emotional outputs are forwarded as the stimuli and the reward/punishment for the BEL controller, respectively. Finally, the BEL control block uses this information to construct a response (model output) that represents the control action.

The BEL algorithm can be also combined with a PID controller to improve the performance of the control system. The PID controller is integrated in the BEL controller as part of the emotional signal, i.e.,

\[
ES = K_P y_d + K_I \int y_d \, dt + K_D \frac{d}{dt} y_d + \omega_4 \int u \, dt \tag{3}
\]

where \( K_P, K_I, \) and \( K_D \) are weight factors of the PID controller that must be carefully selected to obtain the desired performance [8]. The learning system of both amygdala (AM) and orbitofrontal cortex (OC) are based on internal weight adjusting rules defined by

\[
\frac{dG_{Am,i}}{dt} = \alpha SI_i \max(0, ES - \sum A_{m,i}) \tag{4}
\]

\[
\frac{dG_{OC,i}}{dt} = \beta SI_i \max(MO - ES) \tag{5}
\]

where \( \alpha \) is the learning rate of the amygdala, \( \beta \) is the learning rate of the orbitofrontal cortex, \( ES \) and \( MO \) are the emotional signal and the model output, respectively. These learning rates represent model parameters that must be adjusted in accordance with the input variables (i.e., structural responses) to achieve the required control action.

3 Numerical simulation

In this section, a numerical simulation of a semi-active neuroemotional based control system for vibration reduction of a 3-story framed building structure under seismic excitation is presented. The Brain Emotional Learning Based Intelligent Controller is used to design a closed-loop control system that determines the required control action (emotional response) based on the desired and actual system response (sensory input). In this case study, the MR damper (reference RD-1005-3) is located between the ground and the first floor as shown in Figure 2.

The equation of motion describing the response of the system is defined as:

\[
M \ddot{X}(t) + C \dot{X}(t) + KX(t) = \Gamma f_{c1}(t) - M\lambda \ddot{x}_g(t) \tag{6}
\]

where \( X(t) \) defines the displacement response, \( f_{c1}(t) \) the control force, \( \ddot{x}_g(t) \) the ground acceleration, \( M_{3x3}, C_{3x3} \) and \( K_{3x3} \) are the mass, damping and stiffness matrices, respectively, given by:
and finally $\Gamma$ and $\lambda$ are location vectors of the control forces and the earthquake excitation, respectively and given by:

\[
\Gamma = [-1, 0, 0]^T \quad ; \quad \lambda = [1, 1, 1]^T
\] (10)

The mass, damping and stiffness are defined as: $m_1 = m_2 = m_3 = 100$ kg; $c_1 = 175$ Ns/m, $c_2 = c_3 = 50$ Ns/m and $k_1 = k_2 = k_3 = 6 \times 10^5$ N/m.

The seismic acceleration time history of the 1940 NS component of the El-Centro earthquake is used as the ground motion (see Figure 3). A time-scaled seismic record (1:5) was used to ensure a ground motion compatible with the scale of the structural model.

The Simulink model of the semi-active control system based on the BEL controller is displayed in Figure 4.

In this type of smart damping devices, the viscosity of the MR fluid within the damper can be controlled depending on a prescribed input voltage/current. There are several numerical models to represent the hysteretic behavior of MR dampers. A common approach is to use the modified Bouc-Wen model shown in Figure 5 [9].

The numerical formulation of this parametric and the corresponding model parameters are described by the following equations:

\[
F(t) = c \ddot{y} + K_1(x - x_0)
\] (11)

\[
\ddot{y} = \frac{1}{c_0 + c_1}(\alpha z + c_0 \dot{x} + K_0(x - y))
\] (12)
Figure 3: N-S El-Centro earthquake ground motion (time scale 0.2t).

Figure 4: Simulink model of the BEL control system.
\[ \dot{z}(t) = -\beta |\dot{x}(t)|z(t)|z(t)|^{n-1} - \gamma \dot{x}(t)|z(t)|^n + A \dot{x}(t) \]  

The model parameters are defined based on experimental tests and some parameters are current (or voltage) independent, i.e., their values are not significantly affected by the magnetic field applied to the MR fluid. A commercial MR damper (RD-1005-3 by Lord Corp., USA) was experientially tested to obtain the model parameters \[ 10 \]. In this case, the current/voltage independent parameters are \( A = 10.013, \beta = 3.044 \text{ mm}^{-1}, \gamma = 0.10 \text{ mm}^{-1}, K_0 = 1.121 \text{ N/m}, f_0 = 40 \text{ N} \text{ and } n = 2 \). The remaining parameters are current dependent and can be defined by the following polynomial expressions:

\[ \alpha(I) = -826.67 I^3 + 905.14 I^2 + 412.53 I + 38.24 \] 

\[ c_0(I) = -11.73 I^3 + 10.51 I^2 + 11.02 I + 0.59 \] 

\[ c_1(I) = -54.40 I^3 + 57.03 I^2 + 64.57 I - 4.73 \] 

A first-order time lag involved in the current driver/electromagnet during a step command signal is also included in the numerical model of the device, which in this case is defined by a first order filter \( (\eta = 130 \text{ sec}^{-1}) \).

In this case, the inter-story drifts are used to determine the control action. The learning rates for the amygdala and orbitofrontal cortex were defined after a trial-and-error procedure and are computed to be \( \alpha = 0.8 \) and \( \beta = 0.5 \). Likewise, the sensory and the emotional outputs are determined by applying weight factors \( \omega_1 = 2, \omega_2 = 0.56, \omega_3 = 2 \text{ and } \omega_4 = 0.85 \), which provide the best structural performance.

The structural responses obtained with the BEL control system along with the uncontrolled response of the third floor are displayed in Figure 6. The results demonstrate the effectiveness of the proposed controller in reducing the response of the three DOFs structure.

As can be seen, the BEL controller achieves a good performance in reducing the structural responses using only floor displacements. Thus, the main advantage of the BEL based control system is that only inter-story drift responses of the structure are required to determine the control action. This means that the damping force generated during the control process does not need to be monitored, as happens in other controllers. Obviously, the main drawback...
regarding the implementation of the BEL based control system is related with the optimization of the controller parameters. It should be also noted that the combination of a BEL controller with other control techniques (e.g. PID control) has shown to be able to improve the overall performance of the resultant control system.

The damper force and the corresponding control signal are presented in Figure 7. As can be observed in the Simulink model depicted in Figure 4, the control system uses an inverse Bingham model to adjust the controller output to a continuous control signal (command current) compatible with the semi-active control operation of the MR damper. The resulting control action is similar to that obtained with other continuous controllers. The hysteretic behavior of the MR damper during the numerical simulation is portrayed in the force-displacement and force-velocity plots presented in Figure 8. The proposed control system is capable to explore the dissipative nature of this type of actuators. The shape and range of values of the hysteretic loops are comparable with those achieved with other semi-active controllers, whether they are based on a bi-state or a continuous operation of the actuator.

4 Conclusions and Future Work

Generally, the proposed BEL controller presents a remarkable overall performance being effective in reducing the peak responses and the control force compared with the passive modes, in particular the high damping state (passive on). This controller allows a consistent reduction in almost all structural responses (between 4 % and 13 % ), but was unable to decrease the peak acceleration of the first floor that remained nearly the same as that of the best passive control mode. The maximum damping force generated during the simulation is also reduced which indicates that using large control input may not lead to a better control outcome. The main drawback of the BEL controller is related essentially with the appropriate definition of
Figure 7: Damper force and corresponding operating current.

Figure 8: Three DOFs system - MR damper control force.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>$\chi$ (cm)</th>
<th>$\dot{\chi}$ (cm/s)</th>
<th>$\ddot{\chi}$ (cm/s²)</th>
<th>$\Delta \beta$ (cm)</th>
<th>$f$ (N)</th>
</tr>
</thead>
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<td>1305</td>
<td>0.695</td>
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<td></td>
<td>1.251</td>
<td>45.78</td>
<td>1736</td>
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</tr>
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<td>20.02</td>
<td>999</td>
<td>0.518</td>
<td>166.4</td>
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<tr>
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<td>34.51</td>
<td>1358</td>
<td>0.443</td>
<td>166.4</td>
</tr>
<tr>
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<td>42.79</td>
<td>1791</td>
<td>0.292</td>
<td></td>
</tr>
<tr>
<td>Passive ON</td>
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<td>7.77</td>
<td>613</td>
<td>0.171</td>
<td>1048.9</td>
</tr>
<tr>
<td>Modified Bouc-</td>
<td>0.423</td>
<td>19.36</td>
<td>1066</td>
<td>0.253</td>
<td>1048.9</td>
</tr>
<tr>
<td>Wen</td>
<td>0.560</td>
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<td>964</td>
<td>0.239</td>
<td></td>
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<tr>
<td>(-5%)</td>
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<td>1252</td>
<td>0.190</td>
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<tr>
<td>(-6%)</td>
<td></td>
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</table>

Figure 9: Peak responses (time-scaled el-centro earthquake).
emotional and sensory signals that are able to represent with sufficient precision the system state and the control objective in order to maximize the performance of the control system.

There are numerous optimization methods available for tuning these parameters (e.g. genetic algorithms) although a common approach is to use a trial-and-error procedure. As future work the authors intend to use this controller applied to complexer systems and to optimize its performance applying soft computing techniques.

REFERENCES


ONE DIMENSIONAL CONSOLIDATION AND DIRECT SHEAR TESTS: EXPERIMENTAL SETUP BASED ON A LABVIEW APPROACH

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Abstract. This work describes an experimental setup that was developed in order to automate the one-dimensional consolidation and the direct shear Tests. This experimental setup assures repeatability in the data acquisition, avoiding human errors, mainly when the tests data vary with a high dynamic. The described setup is based on LabVIEW, LVDT sensors and a 16 Bit Data Acquisition Board. For the one-dimensional consolidation test it was used a Load device and a consolidometer, being the experimental setup developed according to the standard ASTM D2435 / D2435M - 11. For the direct shear Test it was used an apparatus, covered in ASTM standard D-3080 / D3080M - 11, "Standard Method for direct shear test on soils under consolidated drained conditions".
1 INTRODUCTION

In this work, it is described an experimental setup, existent in the Geotechnical Laboratory of the Polytechnic Institute of Bragança, shown in Figure 1, that was developed in order to automate the one-dimensional consolidation properties of soil and the direct shear tests. A preliminary version of this acquisition system was presented in [1], being improved to allow the user to perform the experiment working with three consolidation cells, instead of one, and also to perform the direct shear test simultaneously with the consolidation tests, sharing the same data acquisition board. This approach has as goal speeding up the consolidation tests and to also perform the direct shear test, allowing a better rentabilization of the data acquisition board, which is an expensive resource. This experimental setup assures repeatability in the data acquisition test, avoiding human errors. Human errors occur mainly when the tests data vary with a high dynamic, being impossible to humans, that use an analog sensor with a display, to register the correct values.

The system, developed for acquisition and data register for both tests, is based on LVDT (Linear Variable Differential Transformer) sensors, a data acquisition board, a PC software LabVIEW application and specific dedicated hardware.

For the one one-dimensional consolidation test it was used a Load device and a consolidometer, being the experimental setup developed according to the ASTM D2435 / D2435M - 11 standard [2]. Consolidation is the process of time-dependent settlement of saturated clayey soil when subjected to an increased loading. In this test a soil specimen is restrained laterally and loaded axially with total stress increments. Each stress increment is maintained until excess pore water pressures are completely dissipated. During the consolidation process, measurements are made of change in the specimen height and these data are used to determine the relationship between the effective stress and void ratio or strain, and the rate at which consolidation can occur by evaluating the coefficient of consolidation.

For the direct shear test it was used an apparatus, covered in ASTM D-3080 / D3080M - 11 standard [3], "Standard Method for direct shear test on soils under consolidated drained conditions". The direct shear device is used to determine failure envelopes for soils. This device is not suitable for the determination of stress-strain properties of soils. In many engineering
problems such as design of foundation, retaining walls, slab bridges, pipes, sheet piling, the
value of the angle of internal friction and cohesion of the soil involved are required for the
design. Direct shear test is used to predict these parameters quickly. Based on the analysis
of the acquired data, a laboratory report is produced, covering the laboratory procedures for
determining these values, for cohesionless soils.

The paper is structured as follows, initially the one one-dimensional onsolidation test, its
setup and an example of a test data analysis are presented. Then the shear test, which uses the
Data acquisition Board as a shared resource is also described, being shown the setup and an
eexample of a test data analysis. Finally some conclusions and future work are presented.

2 CONSOLIDATION SETTLEMENT

2.1 CONCEPT

Consolidation settlement occurs in clays where the value of permeability prevents the ini-
tial excess pore water pressures from draining away immediately. The design loading used to
calculate consolidation settlement must be consistent with this effect.

The term consolidation is used to describe the pressing of soil particles into a tighter packing
in response to an increase in effective stress as shown in Figure 2. The volume of solids remains
constant (i.e., the compression of individual particles is negligible), only the volume of the voids
changes. The resulting settlements is known as consolidation settlement, $\delta_c$. This is the most
important source of settlement in soils, and its analysis is one of the cornerstones of geotechnical
engineering [4].

![Figure 2: Consolidation of solid particles under the influence of an increasing vertical effective stress](image)

Consolidation analysis usually focus on saturated soils ($S = 100\%$), which means the voids
are completely filled with water. Both the water and the solids are virtually incompressible, so
consolidation can occur only as some of the water is squeezed out of the voids.

2.2 CONSOLIDATION TEST

To predict consolidation settlement in a soil, it is necessary to know its stress-strain properties
(i.e, the relationship between $\delta_z$ and $\epsilon_z$). This normally involves bringing a soil sample to the
laboratory, subjecting it to a series of loads, and measuring the corresponding settlements.

The apparatus generally used in the laboratory to determine the primary compression characteristics of soil is known as the consolidation test apparatus (or oedometer), being illustrated in Figure 3. The soil sample is encased in a steel cutting ring. Porous discs, saturated with air-free water, are placed on top of and below the sample which is then inserted in the oedometer. A vertical load is then applied and the resulting compression measured by means of a dial gauge at interval time, readings being taken until the sample has achieved full consolidation (usually for a period of 24 hours). Further load increments are then applied and the procedure repeated, until the full stress range expected in situ has been covered by the test (Figure 4).

![Dial gauge and soil sample](image1.png)

**Figure 3: Tensile test results: consolidation apparatus [5]**

![Sample compression over time](image2.png)

**Figure 4: Tensile test typical results [5]**

The vertical pressure depends mainly on the expected site pressure, including overburden pressure. A load increment ratio of unity is used in conventional testing. Load increment ratio (LIR) of unity means that the load is doubled each time. As referred to each pressure increments are maintained for a period of 24 hours. The specimen consolidates with free drainage occurring from top and bottom faces. Dial gauge readings are noted at 6, 15, 30 secs, 1, 2, 4, 8, 15, 30 min, 1 hr, 2, 4, 8 and 24 hrs.
When consolidation under the final pressure is complete, the specimen is unloaded and allowed to swell. In this way an expansion to time curve can also be obtained. After the loading has been completely removed the final thickness of the sample can be obtained, from which it is possible to calculate the void ratio of soil for each stage of consolidation under the load increments. The graph of void ratio to consolidation pressure can then be drawn, such a curve generally being referred to as an e-p curve (Fig 5).

Figure 5: Typical e-p curve Void ratio to effective pressure [5]

The results can also be presented in a semi log graph paper with applied pressure on log scale at abscissa and corresponding void ratio as ordinate on linear scale. The void ratio corresponding to each applied LIR is defined as the pressure can be calculated from the dial gauge reading and dry weight of specimen is taken at the end of the test.

2.3 DATA ACQUISITION SYSTEM

The system, developed for acquisition and data register for soil consolidation tests, is based on LVDT (Linear Variable Differential Transformer) sensors, a data acquisition board and a PC software LabVIEW application, being possible to run in parallel three independent soil consolidation tests as well as the direct shear test, using the data acquisition board as a shared resource. The new proposed system is intended to replace an old analog measurement instrument, in which all data was registered manually, as shown in Figure 6. This experimental setup assures repeatability in the data acquisition test, avoiding human errors.

The applied LVDT is appropriate to be used in the existing mechanical system in order to execute the necessary tests. The used sensor has built in the necessary signal conditioning in order to obtain an output that is directly proportional to the linear displacement. The used sensor is DC powered and has an input range from ± 5mm. Its characteristic was obtained experimentally being powered with a source voltage of ± 5V. In this conditions the sensor sensitivity is nearly 600mV/mm, having an output voltage of ± 3,5V and an input current of 12 mA. The used data acquisition board was the NI PCI-6202, which is a generic multifunctional board. Due to its characteristics it was considered adequate for this application, mainly due to its high resolution. The data acquisition board features are PCI Bus, 8 differential analog inputs, 16 single ended analog inputs, 16 bit analog to Digital converter, sampling rate of 250kS/s, 24 digital I/O. Figure 7 shows the main window of the developed LabVIEW software interface for the soil consolidation test. This environment provides a friendly user interface for the final user.
Figure 6: Current and old systems setups

Figure 7: User interface for three independent consolidation tests
The user has the possibility to select the logging time, being the available 5, 10, 15, 20 and 30 seconds. In the developed tests, the most commonly used logging time, was of 5 seconds. The data is registered in a CSV file (comma separated values file), being very common in data acquisition systems. Each register adds a line to the stored file, being its limit restriction imposed by the Excel Software. The data is acquired with sampling rate of 5000 Hz. Before the data register the user must introduce some necessary information, such as soil description and the applied load, being the file name assigned automatically. The applied load corresponds to weight of the load that can have valued from 0,25 kg to 256 kg. Each time that the load has to be changed, the data acquisition is stopped, this parameter has to be changed in the software interface, being also possible, at this moment, to change the sample time. In the main software window, there is some information available, namely a table with vertical scroll that shows the stored files, a horizontal slide that shows the LVDT output voltage and it is also shown the linear displacement in $10^{-2}$ mm scale. In the user interface it are also available Boolean controls that allows the user to start and stop registering data and also to set a new zero for the sensor.

### 2.4 RESULTS OF A REAL TEST

This section presents the results of a real test. The results obtained in the software are presented in Figure 8, where the layout file is presented (in a CSV file). Briefly, this file presents the time vs the vertical displacement reading. With these results, it is possible to make the graph presented in Figure 9 (representation of the vertical displacement of a sample with the square root of time). This type of graph is used to determine the rate of soil consolidation, being very important to determine the time needed to process the consolidation.

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</tr>
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</tr>
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</table>

Figure 8: Example of csv file layout of the data acquisition program, time vs vertical displacement reading.
3 Direct shear test

3.1 CONCEPT

A shear test is designed to apply stress to a test sample so that it experiences a sliding failure along a plane that is parallel to the force applied. Generally, shear force cause one surface of material to move in one direction and the other surface to move in the opposite direction so that the material is stressed in a sliding motion. Shear tests differ from tension and compression tests in that the forces applied are parallel to the two-contact surface, whereas, in tension and compression they are perpendicular to the contact surface. The direct shear test as we now know it [3] was perfected by several individuals during the first half of the twentieth century [4].

3.2 DIRECT SHEAR TEST PROCEDURE

The test apparatus, shown in Figure 10, typically accepts a 60-75 mm (2. 5-3. 0 in) rectangular sample and subjects it to a vertical load, \( P \). The vertical total stress, \( \sigma'z \), is thus equal to \( \frac{P}{\text{Area}} \). The sample is contained inside a water bath to keep it saturated, but the hydrostatic pore water pressure is very small, so it can be assumed that the vertical effective stress, \( \sigma'z \), also equals \( \frac{P}{\text{Area}} \). It is important to select \( P \) values such that \( \sigma'z \) is close to the field stresses. The sample is allowed to consolidate under this load.

Once the soil has fully consolidated, a shear force, \( V \), is gradually applied. This shear force induces a shear stress \( \tau = \frac{V}{\text{Area}} \). Usually \( V \) is applied slowly enough to maintain drained conditions. In sands, the required rate of loading is such that failure occurs in a couple of minutes. However, clays must be loaded much more slowly, possibly requiring a time to failure of several hours.

The shear stresses are then plotted against shear displacement, as shown in Figure [11]. This procedure is then repeated two more times on “identical” new samples using different magni-
Figure 10: Cross-section through sample holder showing the sample and shearing action [7].

The shear stress vs. shear displacement curves in Figure 11 continue until the direct shear machine reaches its displacement capacity. Unlike stress-strain curves in steel or other familiar materials, there is no rupture point. Some shear resistance always remains, no matter how much displacement occurs.

Figure 11: Shear stress vs. shear displacement curve from three direct shear test.

The peak shear strength, $s$, for each test is the highest shear stress obtained. These values are then plotted on a Mohr-Coulomb diagram as shown in Figures 12 and 13 where the best-fit line is shown. The $\tau$-intercept is the effective cohesion, $c'$, and the slope of the line is the effective friction angle, $\phi'$. Sometimes direct shear tests are performed more quickly, thus simulating partially drained or undrained conditions. In this case the test results are expressed in terms of the total stress parameters $c_T$ and $\phi_T$.

The direct shear test has the advantage of being simple and inexpensive. It is especially useful for obtaining the drained strength of sandy soils. It also can be used with clays, but produces less reliable results because it is difficult to fully saturate the sample and because we have no way of controlling the drainage conditions other than varying the speed of the test. The direct shear test also has the disadvantages of forcing the shear to occur along a specific plane instead of allowing the soil to fail through the weakest zone, and it produces non-uniform strains in the sample, which can produce erroneous results in strain softening soils [4] [7].
Figure 12: Mohr-Coulomb diagram (vertical effective stress).

Figure 13: Mohr-Coulomb diagram (vertical total stress).
3.3 DATA ACQUISITION SYSTEM

The system, developed for acquisition and data register, described in subsection 2.3, is based on LVDT (Linear Variable Differential Transformer) sensors, a data acquisition board and a PC software LabVIEW application, being possible to run in parallel three independent soil consolidation tests, as well as the direct shear Test, using the data acquisition board as a shared resource.

Figure 14 shows the main window of the developed LabVIEW software interface for the shear test. This environment provides a friendly user interface for the final user.

Figure 14: Shear test user interface

The sensors that instrument the direct shear apparatus can be seen in Figure 15, where it can be observed that for the force measurement a dynamometer ring was used [6] [8].

Figure 15: Direct shear apparatus instrumentation.
3.4 RESULTS OF A REAL TEST

In this section the results of a real test are presented. The results obtained in the software are presented in Figure 16, where the layout file is presented (in CSV file). Briefly, this file presents the time vs displacements reading. LVDT1 is the deformation of the dynamometer ring (Force V), LVDT2 is the displacement of the lower box and LVDT3 is the displacement of the upper rigid plate (where the normal load P is applied). With these results, it is possible to obtain the graph presented in Figure 17 (representation of the shear displacement (LVDT2) of a sample with the shear stress, which is a function of the deformation of the dynamometer ring (LVDT1)). This type of graph is used to determine the maximum value of the shear force and thus determine the mechanical parameters of the soil.

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Figure 16: Example of csv file layout of the data acquisition program, time vs displacements reading.

4 CONCLUSIONS AND FUTURE WORK

This paper describes an experimental setup that was developed in order to automate the One-dimensional consolidation and the direct shear test. This experimental setup has the goal to assure robustness and repeatability in the data acquisition test, avoiding human errors. The described setup is based on LabVIEW, LVDT sensors, a 16 Bit Data Acquisition Board, a Load device, a consolidometer and a direct shear apparatus. The experimental setup was developed according to the ASTM D2435 / D2435M 11 and the D-3080 / D3080M - 11 standards. The
Figure 17: Example of shear stress vs. shear displacement curve

system, developed for acquisition and data register, is based on LVDT (Linear Variable Differential Transformer) sensors, a data acquisition board and a PC software LabVIEW application, being possible to run in parallel three independent soil consolidation tests, as well as the direct shear Test, using the data acquisition board as a shared resource. As future work the authors intend to develop studies based on the analysis of data acquired with the described acquisition system.

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CONTROL PROBLEM IN PASSIVE TRACER ADVOTION BY POINT VORTEX FLOW: A CASE STUDY

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Keywords: Point Vortex Flow, Passive Tracer, Control Problem, Numerical Optimization.

Abstract. Vortex dynamics and passive tracers in vortex-dominated flows form a vast area of research that continues to attract the attention of numerous studies. Among these studies, it has emerged in recent times a special interest in the use of control theory applied to vortex dynamics. Point vortices are singular solutions of the two-dimensional incompressible Euler equations. These solutions correspond to the limiting case where the vorticity is completely concentrated on a finite number of spatial points, each with a prescribed strength/circulation. By definition, a passive tracer is a point vortex with zero circulation. We are concerned with the dynamics of a passive tracer advected by two-dimensional point vortex flow. More precisely, we want to drive a passive particle from an initial starting point to a final terminal point, both given a priori, in a given finite time. The flow is originated by the displacement of \( N \) viscous point vortices. More precisely, we look for the optimal trajectories that minimize the objective function that correspond to the energy expended in the control of the trajectories. The restrictions are essentially due to the ordinary differential equations that govern the displacement of the passive particle around the viscous point vortices.
1 INTRODUCTION

Vortex dynamics and passive tracers in vortex-dominated flows form a vast area of research that continues to attract the attention of numerous studies. Point vortex are mathematical models used to describe the dynamic of vortex-dominated flows. These models are based on a low dimensional description of the flow features [1]. Vortex dynamics, based on point vortex models, have been employed in many science and engineering areas like geophysics, turbulence, superfluids or hydrodynamic [2, 3, 4, 5, 6]. The solution of these vortex dynamics models is usually obtained with low computational costs. Whereas, the solution of partial differential equations in realistic problems requires high CPU time and large memory storage. This makes these models very attractive, especially in flow control problems [7].

It has emerged in recent years a special interest in the use of control theory applied to vortex dynamics. In most of the control problems, concerning realistic flows, the solution is achieved by means of simplified models such as point vortex [1]. In consequence, there is a special interest in the use of control methods applied to vortex dynamics, namely in the fields of geophysical fluid dynamics, aeronautics and hydrodynamics [7].

In the context of hydrodynamics, the fish-like locomotion is an application of point vortex that have received some attention in the last years. This is mostly due to the development of autonomous underwater vehicles for the collection of data concerning the multiple oceanic phenomena [8]. In some approaches the robotic fish locomotion is modelled through a point mass in vector fields defined by the vortex dynamics. In [9], the displacement is achieved by the constant generation of periodic and predefined vortices. In [10], the control action is exercised by the generation of one vortex.

Point vortex dynamics is an area of mathematical physics that has served as a classical mathematics playground [11]. Over the times, many different methods from pure and applied mathematics have been used in this area. From the control point of view, several techniques have been also applied. Some of there had-oc and other based on solid mathematical foundations [7].

There is two main class of control methods: direct and indirect. Generally speaking, the direct approach consist to firstly discretize the problem and after that optimize, while the indirect approach, firstly optimize and after discretize. Direct methods discretize the problem relatively to the time in order to get a Non-Linear Programming (NLP) that can be solved by an optimization method like Interior Point. These methods are generally handy for singular or constrained arcs of the trajectory, but their accuracy can be affected by the discretization [12, 13]. Indirect methods use Pontryagin’s Maximum Principle to derive optimal conditions, where it is necessary to maximization the Hamiltonian, which can be achieved by collocation or shooting methods [14]. Indirect methods are fast and accurate, but they are also sensitive to the starting guess of the adjoint problem.

This work is concerned with the dynamics of a passive particles advected by a two-dimensional point vortex flow. A passive particle is small enough not to perturb the velocity field, but also large enough not to perform a Brownian motion. Particles of this type are the tracers used for flow visualization in fluid mechanics experiments [15]. We consider also that the passive particle have the same density of the fluid in which it is embedded.

We want to drive a passive particle from an initial starting point to a final terminal point, both given a priori, in a given finite time. The flow is originated by the displacement of a certain number, say $N$, of point vortices. This problem as some similarities with the fish-like locomotion problem. Here the vortex dynamics is governed by $N$ point vortices and the control
is due to the possibility of impulsion in any direction of the two dimensional plane. Of course we want minimize the total amount of energy spent in the impulsion. This issue can also be seen as part of the set of general open control problems proposed by Protas [7].

The displacement of the passive particle is then transformed in a control problem that we solve by a direct approach. The time disposable to preform the displacement is divided in a fixed number $n$ of subinterval, where the control variables are constant. The discretized problem is solved numerically by mean of a single shooting method. In each subinterval the vortex dynamics is integrated by the fourth order Runge-Kutta method.

Our approach is different from the usual ones, that are based on the indirect approach. Normally the Pontryagin’s Maximum Principle is used to derive the necessary conditions of optimality of the problem. This results in a problem with hight complexity, hose resolution needs some simplifying assumptions. On of these simplifications is to consider that the vortex dynamic is induced just by two point vortices [8].

In these paper we consider that the passive particle moves in a two dimensional flow whose dynamic is given, at any time interval, by $N$ point vortices. We consider four different problems, each one corresponding to a different value of $N$, ranged from one to four. We formulate the non linear programme (NLP) corresponding to each case and we show numerically that there is an optimal control for each one. The solution of the NLP is achieved by mean of the Matlab Optimization Toolbox™.

In Section 2 we introduce the equations that give the motion of passive particles advected by $N$ point vortices in the infinite real plane. After that, in Section 3, we formulate the control problems. Section 4 is devoted to the numerical solution of these control problems. We start by a passive particle moved by a single point vortex (Subsection 4.1.1) and terminate with a movement induced by four point vortex (Subsection 4.2.3). This paper is closed in Section 5 with some conclusions.

2 TWO-DIMENSIONAL EULER EQUATION AND POINT VORTEX

The two-dimensional incompressible Euler equations are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p,$$  \hspace{1cm} (1)

and

$$\nabla \cdot \mathbf{u} = 0,$$ \hspace{1cm} (2)

plus initial and boundary conditions, wherein $\mathbf{u} = (u_1, u_2)$, $u_i = u_i(X, t)$ ($i = 1, 2$) is the two-dimensional incompressible velocity field, $X = (x, y) \in \mathbb{R}^2$ the space coordinates, $t$ is the time variable, $\nabla = (\partial_x, \partial_y)$ is the gradient, and $p$ is the pressure.

Due to the incompressible condition in two spatial dimensions, i.e., $\nabla \cdot \mathbf{u} = 0$, one can express the velocity field $\mathbf{u}$ in term of the so-called stream-function $\Psi$:

$$\mathbf{u} = (u_1, u_2) = (\partial_y \Psi, -\partial_x \Psi).$$ \hspace{1cm} (3)

Introducing the vorticity vector (consider $\mathbf{u}$ in $\mathbb{R}^3$, where the 3rd component is zero):

$$\mathbf{w} = \nabla \times \mathbf{u} = (0, 0, \partial_x u_2 - \partial_y u_1)$$

the 2D scalar vorticity, given by

$$\omega = (\mathbf{w})_3 = \partial_x u_2 - \partial_y u_1,$$ \hspace{1cm} (4)
is linked with the stream-function through the Poisson equation
\[ \nabla^2 \psi = -\omega. \]  
(5)

Taking the curl in both sides of 2D Euler equation leads us to the vorticity formalism of the Euler equations [16]

\[ \partial_t \omega + (u \cdot \nabla) \omega \equiv \frac{d\omega}{dt} = 0. \]  
(6)

The equation (6) enables us to get the time evolution of the vorticity. Once we have the vorticity, the stream-function is computed through the solution of the Poisson equation (5). In turn, the velocity field is obtained by equation (3). This is the typical procedure used to solve the Euler equation.

Effectively, it can be shown (see [17]) that the solution of the Poisson equation (5), in the whole plane, is given by

\[ \psi = \frac{1}{2\pi} \int \int_{\mathbb{R}^2} \ln |X - X'| \omega(X') \, dX', \]  
(7)

and that \( u \) is equal to the convolution between the kernel \( K(\cdot, \cdot) \) and the given vorticity:

\[ u = (\partial_y \psi, -\partial_x \psi) = K * \omega, \]  
(8)

with

\[ K(x, y) = \frac{1}{2\pi} \left( \frac{y}{x^2 + y^2}, -\frac{x}{x^2 + y^2} \right). \]  
(9)

The point vortices correspond to the particular case where the vorticity is given by the weighted sum

\[ \omega = \omega(t, X) = \sum_{\alpha=1}^{N} k_{\alpha} \delta(X - X_{\alpha}(t)), \]  
(10)

where \( N \) is equal to the number of vortices and \( \delta(\cdot) \) is the \( \delta \)–Dirac function. The quantity \( k_{\alpha} \) is the circulation of the vortex \( \alpha \) \( (\alpha = 1, 2, \cdots , N) \) that, at time \( t \), is located in \( X_{\alpha}(t) = (x_{\alpha}(t), y_{\alpha}(t)) \).

The vortex equations are

\[ \frac{dx_{\alpha}}{dt} = -\sum_{\beta=1}^{N} \frac{k_{\beta} y_{\alpha} - y_{\beta}}{2\pi r_{\alpha\beta}^2}, \]
\[ \frac{dy_{\alpha}}{dt} = \sum_{\beta=1}^{N} \frac{k_{\beta} x_{\alpha} - x_{\beta}}{2\pi r_{\alpha\beta}^2}, \]  
(11)

supplemented by appropriate initial conditions. Here, \( r_{\alpha\beta}^2 = (x_{\alpha} - x_{\beta})^2 + (y_{\alpha} - y_{\beta})^2 \).

The equations (11) can be written using the complex variables

\[ \frac{dz_{\alpha}^*}{dt} = \frac{1}{2\pi i} \sum_{\beta=1, \beta \neq \alpha}^{N} \frac{k_{\beta}}{z_{\alpha} - z_{\beta}} \quad (\alpha = 1, 2, \ldots, N), \]  
(12)

whereas \( z_{\alpha} = x_{\alpha} + y_{\alpha} i \) \( (i^2 = -1) \) and \( z_{\alpha}^* \) its complex conjugate.
A passive particle is by definition a point vortex with circulation $k = 0$. Thus, the dynamics of a system with $P$ passive particles advected by $N$ point vortices is given by \((12)\) together with the equations for the passive particles

\[
\frac{dz^*_\alpha}{dt} = \frac{1}{2\pi i} \sum_{\beta=1}^{N} \frac{k_\beta}{z_\alpha - z_\beta} \quad (\alpha = N + 1, N + 2, \ldots, N + P),
\]

with the respective initial conditions.

### 3 STATEMENT OF THE CONTROL PROBLEM

The abstract formulation of the problem we want to address can be stated as follows. Consider the state vector \((M = N + P)\)

\[
X(t) = [x_1(t) \ y_1(t) \ x_2(t) \ y_2(t) \ \cdots \ x_M(t) \ y_M(t)]^T \in \mathbb{R}^{2M},
\]

and the evolution systems \((12)-(13)\) concisely written as

\[
\frac{dX}{dt} = f(X), \quad X(0) = X_0,
\]

where \(f : \mathbb{R}^{2M} \to \mathbb{R}^{2M}\) is the function describing the advection velocities of the vortices and particles due to the induction of all the vortices.

If we denote \(U : [0, +\infty[ \to \mathbb{R}^m\) a time dependent input control with \(m\) degree of freedom, the system \((15)\) can be reconfigured to

\[
\frac{dX}{dt} = f(X) + b(X)U(t), \quad X(0) = X_0.
\]

where \(b(X) : \mathbb{R}^m \to \mathbb{R}^{2M}\) is the control operator that describes the control action, \(U(\cdot)\), in the system dynamics.

The control problem can then be stated as follows: given the initial state \(X_0\) of the system and the prescribed \(X_T = X(T)\) terminal state, determine \(U(\cdot)\) that drives \(X_0\) to \(X_T\) during the time interval \([0, T]\) and minimizing, for instance, \(T\), or some cost function, etc.

### 4 SOLVING THE CONTROL PROBLEM BY DIRECT APPROACH

In this Section, we present the numerical approach used to solve the control problem presented above. Our direct approach consists in discretizing the problem and solving it using an optimization method. In this way, the control function \(U(\cdot)\) is replaced by \(n\) control variables \(u_0, u_1, \ldots, u_{n-1}\). The calculations were performed in Matlab\textsuperscript{®}, with the nonlinear programming solver \texttt{fmincon}. This solver provides some constrained optimization algorithms, such as the Interior Point or the Active-Set (see \[18\] and \[19\], for instance). We begin, in Section 4.1, by solving this problem for the case of a single passive particle in a single vortex flow and, after that, in Section 4.2, we address the cases involving up to four vortices. We would like to point out that the cases \(N = 2\) and \(N = 3\) correspond to integrable point vortex dynamics, whereas \(N = 4\) (or higher) corresponds to chaotic point vortex dynamics \[15, 20\].

#### 4.1 Flow created by one single vortex \((N = 1)\) and one particle \((P = 1)\).

In this Section, we address the case of a flow created by one single vortex. In subsection 4.1.1, we introduce the corresponding discretized optimization problem. For the case of a particular
complex control variable, we show that the problem formulated in Section 3 has an analytical solution. For the general case we show the numerical solutions corresponding to a different number of control variables \( n = 1, 2, 3 \) and 4.

4.1.1 Optimization Problem.

From a practical point of view, the control problem introduced in Section 3 can be illustrated for the case of one single passive particle \((P = 1)\) which moves thanks to the presence a single vortex \((N = 1)\) as follow:

\[
\begin{align*}
\text{(P)} & \quad \text{Minimize} & & \int_{0}^{T} |u(t)|^2 \, dt \\
& \quad \text{subject to} & & \dot{z}^* = \frac{1}{2\pi i} \times \frac{k}{z} + u \\
& & & z(0) = z_0 \\
& & & z(T) = z_f \\
& & & |u| \leq u_{\text{max}}
\end{align*}
\]

with \( u \in \mathbb{C} \), and \( z_0, z_f \in \mathbb{C}, \ T > 0 \) and \( u_{\text{max}} > 0 \) given.

In this optimization problem, the objective (cost) function represents, for instance, the energy spent by the control made by the passive particle along the displacement from \( z_0 \) to \( z_f \). The first restriction corresponds to the state equation that governs the position \( z \) of the particle as a function of the time. The control function \( u \) is introduced in this equation in order to move the particle from the starting point \( z_0 \) to the end point \( z_f \) in a fixed value of time \( T \). Theses two points are previously defined, as well as the time \( T \) available for travel from \( z_0 \) to \( z_f \). Additionally, in the fourth restriction, we impose that the control module is not greater than a prescribed value \( u_{\text{max}} \).

To address this problem, we proceed to the discretization of the control function. We replace \( u(\cdot) \) by \( n \) (discrete) variables defined as \((t_0 = 0, \ t_n = T)\)

\[
\begin{align*}
& & & u(t) = u_0 \quad \text{if} \quad t_0 \leq t < t_1, \\
& & & u(t) = u_1 \quad \text{if} \quad t_1 \leq t < t_2, \\
& & & u(t) = u_2 \quad \text{if} \quad t_2 \leq t < t_3, \\
& & & \vdots \\
& & & u(t) = u_{n-1} \quad \text{if} \quad t_{n-1} \leq t < t_n.
\end{align*}
\]

Thus, each variable \( u_i \) \((i = 1, 2, \cdots, n)\) corresponds to a constant value of the control exercised in the sub-interval \([t_{i-1}, t_i)\). Each of these subintervals has the constant length \( \Delta t = (t_n - t_0) / n \).

The discretization of the objective function by the rectangle method lead to the approximation

\[
\int_{0}^{T} |u(t)|^2 \, dt \approx \Delta t \left( |u_0|^2 + |u_1|^2 + |u_2|^2 + \cdots + |u_{n-1}|^2 \right). \tag{17}
\]

The control problem \((P)\) is then replaced by its discretized version:

\[
\begin{align*}
\text{(DP}_n) & \quad \text{Minimize} & & \Delta t \sum_{k=0}^{n-1} |u_k|^2 \\
& \quad \text{subject to} & & \dot{z}^* = \frac{1}{2\pi i} \times \frac{k}{z} + u_0, \ z(0) = z_0, \ |u_0| \leq u_{\text{max}}, \ t_0 \leq t < t_1 \\
& & & \dot{z}^* = \frac{1}{2\pi i} \times \frac{k}{z} + u_1, \ z(t_1) = z_{t_1}, \ |u_2| \leq u_{\text{max}}, \ t_1 \leq t < t_2 \\
& & & \vdots \\
& & & \dot{z}^* = \frac{1}{2\pi i} \times \frac{k}{z} + u_{n-1}, \ z(t_{n-1}) = z_{t_{n-1}}, \ |u_{n-1}| \leq u_{\text{max}}, \ t_{n-1} \leq t < t_n \\
& & & z(t_n) = z_f
\end{align*}
\]
The case \( n = 1 \) with \( u_0 \in \mathbb{C} \) and \( b(z) = \alpha / z \). We begin by studying the case of a single vertex, located at the origin, for the particular situation, where the operator control \( b(\cdot) \) in (16) is of the form \( b(z) = \alpha / z \) (\( \alpha \in \mathbb{R} \setminus \{0\} \)) there is an analytical solution of the optima control.

Effectively, the state equation becomes

\[
\dot{z}^* = \frac{k}{2\pi i} \frac{1}{z} + \frac{\alpha}{z} u_0 ,
\]

with \( k \in \mathbb{R} \), \( u_0 = u + iv \in \mathbb{C} \). We want to find \( u \) and \( v \) in order to drive \( z(0) = \rho_0 e^{i\theta_0} \) to \( z(T) = \rho_T e^{i\theta_T} \), in time \( T \). For this, we start by writing \( z(\cdot) \) in polar coordinates, that is, \( z(t) = \rho(t) e^{i\theta(t)} \). Substituting this transformation into (18) and matching the real and imaginary parts to the resulting equation, we get

\[
\begin{align*}
\dot{\rho} &= \frac{\alpha}{\rho} u , \\
\dot{\theta} &= \frac{k}{2\pi \rho^2} - \frac{\alpha}{\rho^2} v .
\end{align*}
\]

A straightforward integration allows us to obtain \( \rho(0) = \rho_0 \) and \( \theta(0) = \theta_0 \)

\[
\begin{align*}
\rho(t) &= \sqrt{2\alpha u t + \rho_0^2} , \\
\theta(t) &= \theta_0 + \frac{k-2\pi\rho}{4\pi\alpha} \ln \left( 1 + \frac{2\alpha u t}{\rho_0^2} \right) .
\end{align*}
\]

Finally, the determination of the control \( u_0 = u + iv \) is made thanks to terminal conditions \( \rho(T) = \rho_T \) and \( \theta(T) = \theta_T \), getting then

\[
\begin{align*}
u &= \frac{\rho_T^2 - \rho_0^2}{2\pi \alpha} , \\
u &= \frac{k}{2\pi i} - \frac{1}{2\pi} \left( \rho_T^2 - \rho_0^2 \right) (\theta_T - \theta_0) \frac{1}{\ln(\rho_T) - \ln(\rho_0)} .
\end{align*}
\]

In this particular case there is an analytical solution. Next, we present the numerical resolution for the single vortex problem, located at the origin.

The case \( n = 1 \) with \( u_0 \in \mathbb{C} \). The general case of a single vertex, located at the origin, is solved numerically. We seek for a number \( u_0 \in \mathbb{C} \) that drives the passive particle from \( z_0 \) to \( z_f \) in exactly \( T \) units of time and minimizes the energy. This problem is formulated as follows:

\[
(\mathcal{DP}_1) \quad \text{Minimize} \quad \Delta t |u_0|^2 \\
\text{subject to} \quad \dot{z}^* = \frac{k}{2\pi i} \frac{1}{z} + u_0 , \quad z(0) = z_0 , \quad z(T) = z_f , \quad |u_0| \leq u_{\text{max}}
\]

This optimization problem is solved numerically with the Interior Points optimization algorithm [18], included in the \texttt{fmincon} solver of Matlab.

We want find \( u_0 \in \mathbb{C} \) that moves the particle from \( z_0 = -1 - i \) and the final point \( z_f = 2 + 2i \). We consider also a time of displacement \( T = 10 \) and a circulation of the vortex \( k = 10 \).

The control value obtained with the optimization solver \texttt{fmincon} is \( u_0 = 0.189 - 0.179 i \), that corresponds to a value of the objective function equal to 0.677979. In Figure 1 we can observe the trajectory of the particle obtained with this control.
The case \( n = 2 \) with \( u_0, u_1 \in \mathbb{C} \). We now consider the case of two complex controls. We seek for a number \( u_0, u_1 \in \mathbb{C} \) that drives the passive particle from \( z_0 \) to \( z_f \) in exactly \( T \) units of time and minimizes the energy. This problem is formulated as follows:

\[
\text{Minimize } \sum_{k=0}^{1} |u_k|^2 \\
\text{subject to } \dot{z}^* = \frac{k}{2\pi i} + u_0, \quad z(0) = z_0, \quad |u_0| \leq u_{\text{max}}, \quad t_0 \leq t < t_1 \\
\dot{z}^* = \frac{k}{2\pi i} + u_1, \quad z(t_1) = z_{t_1}, \quad z(t_2) = z_f, \quad |u_1| \leq u_{\text{max}}, \quad t_1 \leq t < t_f
\]

This optimization problem is also solved numerically by means of the \textit{fmincon} solver of Matlab. We want find \( u_0 \in \mathbb{C} \) that moves the particle from \( z_0 = -1 - i \) and the final point \( z_f = 2 + 2i \). We consider also a time of displacement \( T = 10 \) and a circulation of the vortex \( k = 10 \).

The value obtained with the optimization solver \textit{fmincon} is \( u_0 = 0.0123 - 0.2491 i \) and \( u_1 = 0.1089 - 0.0151 i \) and the value of the objective function is 0.371478. In Figure 2 we can observe the corresponding trajectory.

The case \( n = 3 \) with \( u_0, u_1, u_2 \in \mathbb{C} \). We now consider the case of two complex controls. We seek for a number \( u_0, u_1, u_2 \in \mathbb{C} \) that drives the passive particle from \( z_0 \) to \( z_f \) in exactly \( T \)
units of time and minimizes the energy. This problem is formulated as follows:

\[(D\mathcal{P}_3)\quad \text{Minimize} \quad \Delta t \sum_{k=0}^{2} |u_k|^2 \]

subject to

\[\dot{z}^* = \frac{1}{k} \frac{1}{z} + u_0, \quad z(0) = z_0, \quad |u_0| \leq u_{\text{max}}, \quad t_0 \leq t < t_1\]
\[\dot{z}^* = \frac{1}{k} \frac{1}{z} + u_1, \quad z(t_1) = z_{t_1}, \quad |u_1| \leq u_{\text{max}}, \quad t_1 \leq t < t_2\]
\[\dot{z}^* = \frac{1}{k} \frac{1}{z} + u_2, \quad z(t_2) = z_{t_2}, \quad z(t_3) = z_f, \quad |u_2| \leq u_{\text{max}}, \quad t_2 \leq t < t_f\]

This optimization problem is also solved numerically by means of the \textit{fmincon} solver of Matlab. We want find \(u_0 \in \mathbb{C}\) that moves the particle from \(z_0 = -1 - i\) and the final point \(z_f = 2 + 2i\). We consider also a time of displacement \(T = 10\) and a circulation of the vortex \(k = 10\).

The value obtained with the optimization solver \textit{fmincon} is \(u_0 = -0.1477 - 0.2297i, u_1 = 0.1171 - 0.0687i\) and \(u_2 = 0.0951 + 0.0268i\) and the value of the objective function is 0.342648. In Figure 3 we can observe the corresponding trajectory.

**The case \(n = 4\) with \(u_0, u_1, u_2, u_3 \in \mathbb{C}\).** We now consider the case of two complex controls. We seek for a number \(u_0, u_1, u_2, u_3 \in \mathbb{C}\) that drives the passive particle from \(z_0\) to \(z_f\) in exactly \(T\) units of time and minimizes the energy. This problem is formulated as follows:

\[(D\mathcal{P}_4)\quad \text{Minimize} \quad \Delta t \sum_{k=0}^{3} |u_k|^2 \]

subject to

\[\dot{z}^* = \frac{1}{k} \frac{1}{z} + u_0, \quad z(0) = z_0, \quad |u_0| \leq u_{\text{max}}, \quad t_0 \leq t < t_1\]
\[\dot{z}^* = \frac{1}{k} \frac{1}{z} + u_1, \quad z(t_1) = z_{t_1}, \quad |u_1| \leq u_{\text{max}}, \quad t_1 \leq t < t_2\]
\[\dot{z}^* = \frac{1}{k} \frac{1}{z} + u_2, \quad z(t_2) = z_{t_2}, \quad |u_2| \leq u_{\text{max}}, \quad t_2 \leq t < t_3\]
\[\dot{z}^* = \frac{1}{k} \frac{1}{z} + u_3, \quad z(t_3) = z_{t_3}, \quad z(t_4) = z_f, \quad |u_3| \leq u_{\text{max}}, \quad t_3 \leq t < t_f\]

We want find \(u_0 \in \mathbb{C}\) that moves the particle from \(z_0 = -1 - i\) and the final point \(z_f = 2 + 2i\). We consider also a time of displacement \(T = 10\) and a circulation of the vortex \(k = 10\). This optimization problem is also solved numerically by means of the \textit{fmincon} solver of Matlab.

The value obtained with the optimization solver \textit{fmincon} is \(u_0 = -0.0806 - 0.2483i, u_1 = 0.1064 - 0.1301i, u_2 = 0.1103 - 0.0474i\) and \(u_3 = 0.0769 + 0.0017i\) and the value of the objective function is 0.291819. In Figure 4 we can observe the corresponding trajectory.
4.2 Flow created by several vortices

In this Section we address the problem of a single passive particle \((P = 1)\) moved by multiple vortices \((N)\). We consider \(N = 2\) in Subsection 4.2.1, \(N = 3\) in Subsection 4.2.2 and finally, \(N = 4\) in Subsection 4.2.3. We solve these problems numerically by means of the Matlab solver \texttt{fmincon}.

4.2.1 Two vortices \((N = 2)\) and one particle \((P = 1)\).

In the two vortices and one particle problem, the vortices positions are given by [21]:

\[
\begin{align*}
    z_1(t) &= \frac{1}{k_1 + k_2} \left[ (k_1 z_1(0) + k_2 z_2(0)) + (z_1(0) - z_2(0)) k_2 e^{i\Omega t} \right] \\
    z_2(t) &= \frac{1}{k_1 + k_2} \left[ (k_1 z_1(0) + k_2 z_2(0)) + (z_2(0) - z_1(0)) k_1 e^{i\Omega t} \right]
\end{align*}
\]

(19)

where \(\Omega = \frac{k_1 + k_2}{2\pi D^2}\), \(D = |z_2(0) - z_1(0)|\) and \(z_1(0)\) and \(z_2(0)\) are the initial position.

The passive particle position is given by the equation

\[
\dot{z}^* = \frac{1}{2\pi i} \left( \frac{k_1}{z - z_1(t)} + \frac{k_2}{z - z_2(t)} \right) + u
\]

(20)

with the given initial condition \(z(0) = z_0\).

The optimization problem is similar to the one presented in Section 4.1.1 excepting the restriction given by the equation that describes the position of the particle that is replaced here by equations (19) and (20). As before we want find \(u = [u_0, u_1, \ldots, u_{n-1}] \in \mathbb{C}^n\) that moves the particle from \(z_0 = -1 - i\) and the final point \(z_f = 2 + 2i\). We consider also a time of displacement \(T = 10\) and the same circulation for the two vortices \(k_1 = k_2 = 1\). The initial vortices positions are: \(z_{10} = 0.5 + 0.5i\) and \(z_{20} = 1.5 - 0.5i\). This problem is solved numerically with the Interior Points optimization algorithm [18], included in the \texttt{fmincon} solver.

Figure 5 presents two possible solution obtained with one \((n = 1)\) and with a two control variable \((n = 2)\). In both cases the targeted displacement corresponds to the solution given by the interior-point method. In the first case, Figure 5(a), the optimal solution is not find because \(\min |z_f - z(T)| = 0.059844\). In the second case, Figure 5(b), the optimal solution is obtained and corresponds to \(u_0 = 0.3185 - 0.1744i\) and \(u_1 = 0.1808 - 0.3364i\) and the objective function is equal to 1.38819. These results show that there is an optimal control in the case of passive particle moved by a two vortices flow.
4.2.2 Three vortices \((N = 3)\) and one particle \((P = 1)\).

In the problem with three vortices \((N = 3)\) and one particle \((P = 1)\), the vortices equations are

\[
\begin{align*}
\dot{z}_1^* &= \frac{1}{2\pi i} \left( \frac{k_2}{z_1 - z_2} + \frac{k_3}{z_1 - z_3} \right) \\
\dot{z}_2^* &= \frac{1}{2\pi i} \left( \frac{k_1}{z_2 - z_1} + \frac{k_3}{z_2 - z_3} \right) \\
\dot{z}_3^* &= \frac{1}{2\pi i} \left( \frac{k_1}{z_3 - z_1} + \frac{k_2}{z_3 - z_2} \right)
\end{align*}
\]

(21)

with the given initial conditions \(z_1(0) = z_{10}, z_2(0) = z_{20}, \) and \(z_3(0) = z_{30}\).

The passive particle equation is

\[
\dot{z}^* = \frac{1}{2\pi i} \left( \frac{k_1}{z - z_1} + \frac{k_2}{z - z_2} + \frac{k_3}{z - z_3} \right) + u
\]

(22)

with the given initial condition \(z(0) = z_0\).

The optimization problem is similar to the one presented in previous Section 4.1.1. The restriction due to the position of the particle is now given by the initial value problem that includes equations (21) and (22) and respective initial conditions. As before we want find \(u = [u_0, u_1, \ldots, u_{n-1}] \in \mathbb{C}^n\) that moves the particle from \(z_0 = -1 - i\) to the final point \(z_f = 2 + 2i\) in exactly \(T = 10\) units of time. We consider also the same circulation for the three vortices \(k_1 = k_2 = k_3 = 1\). The initial vortices positions are: \(z_{10} = 0.5 + 0.5i,\) \(z_{20} = 1.5 - 0.5i\) and \(z_{30} = 1 + i\). This problem is solved numerically with the Active-Set optimization algorithm [19, 22], included in the \texttt{fmincon} solver.

Figure 6 presents the solution obtained with \(n = 1, 2, 3\) and 4. In all the cases the optimal solution is obtained. The objective function values equal to a) 137.526, b) 89.329, c) 83.497 and d) 84.245. These results show us that there is optimal control for this problem and that the value of the objective function tends to decrease as the number of control variables increase.

4.2.3 Four vortices \((N = 4)\) and one particle \((P = 1)\).

We address now the case of four vortices \((N = 4)\) and one particle \((P = 1)\). This is a very interesting case because it is considered as a chaotic advection in point vortex models and two-dimensional turbulence [15].

Figure 5: Two vortices problem: numerical solution. a) trajectory obtained with \(n = 1\) control variable. b) trajectory obtained with \(n = 2\) control variable.
The vortices equations are

\[
\begin{align*}
\dot{z}_1^* &= \frac{1}{2\pi i} \left( \frac{k_2}{z_1 - z_2} + \frac{k_3}{z_1 - z_3} + \frac{k_4}{z_1 - z_4} \right) \\
\dot{z}_2^* &= \frac{1}{2\pi i} \left( \frac{k_1}{z_2 - z_1} + \frac{k_3}{z_2 - z_3} + \frac{k_4}{z_2 - z_4} \right) \\
\dot{z}_3^* &= \frac{1}{2\pi i} \left( \frac{k_1}{z_3 - z_1} + \frac{k_2}{z_3 - z_2} + \frac{k_4}{z_3 - z_4} \right) \\
\dot{z}_4^* &= \frac{1}{2\pi i} \left( \frac{k_1}{z_4 - z_1} + \frac{k_2}{z_4 - z_2} + \frac{k_3}{z_4 - z_3} \right)
\end{align*}
\] (23)

with the given initial positions \( z_1(0) = z_{10}, z_2(0) = z_{20}, z_3(0) = z_{30} \) and \( z_4(0) = z_{40} \). The passive particle equation is

\[
\dot{z}^* = \frac{1}{2\pi i} \left( \frac{k_1}{z - z_1} + \frac{k_2}{z - z_2} + \frac{k_3}{z - z_3} + \frac{k_4}{z - z_4} \right) + u
\] (24)

with the given initial condition \( z(0) = z_0 \). As in the two previous problems, equations (23) and (24) and respective initial conditions gives rise to the initial value problem that it is included in the optimization problem as restriction.

We want find \( u = [u_0, u_1, \ldots, u_{n-1}] \in \mathbb{C}^n \) that moves the particle from \( z_0 = -1 - i \) to the final point \( z_f = 2 + 2i \) in exactly \( T = 10 \)unities of time. We consider as before the same values for the circulation for the four vortices \( k_1 = k_2 = k_3 = k_4 = 1 \). The initial vortices positions are: \( z_{10} = 0.5 + 0.5i, z_{20} = 1.5 - 0.5i, z_{30} = 1 + i \) and \( z_{40} = -1 - 2i \). This problem is solved numerically with the Interior Points optimization algorithm [18], included in the fmincon solver.

Figure 7 presents the solution obtained in the case of four vortices with \( n = 1, 2, 3 \) and 4 control variables. In all the cases the optimal solution is obtained because \( \min |z_f - z(T)| \) is...
close to zero. The objective function values equal to a) 112.278, b) 66.014, c) 61.174 and d) 61.937. These results show us that there is an optimal control for this problem and that the value of the objective function tends to decrease until the number of control variables increase to 3.

5 CONCLUSIONS

The singular solutions of the two-dimensional incompressible Euler equations are known as point vortices. Point vortices are used to describe the dynamic of vortex-dominated flows. Because they are based on a low dimensional description of the flow features and, consequentially, enables the solution of fluid dynamic problems with low computational costs.

By definition, a passive tracer is a point vortex with zero circulation. In our case, we consider the advection of one passive tracer by point vortices in the unbounded plane and we have presented the formulation of corresponding control problems. These control problems results from the necessity of displacing the particle between two point in a fixed interval of time.

We use a control strategy, based on a direct approach, that is unusual in these kind of problems. This approach enables to work with vortex dynamics resulting from the interaction of several point vortices. In our case we consider dynamics induced by \( N = 1 \), \( N = 2 \), \( N = 3 \) and \( N = 4 \) vortices.

We have discretize the main time interval in a certain number of subintervals, wherein the control variable is constant, and we solve numerically the resulting non-linear programming. The results show the existence of optimal controls for the cases of \( N = 1 \), \( N = 2 \), \( N = 3 \) and \( N = 4 \) vortices. The number of suitable control variables varies from problem to problem. However, there is a tendency for this number to increase with the number of point vortices.
REFERENCES


NON-LINEAR HYSERETIC BEHAVIOR OF AN SDOF FRAME CONTROLLED BY A TUNED MASS DAMPER

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Abstract

The presence of dynamic loads, such as seismic and wind excitations in framed building structures requires the structural control of the consequent lateral displacements in the structure. This control can be made by the implementation of a vibration control system. This paper presents an investigation about the influence that the presence of infill walls in a system with one degree of freedom (SDOF), has on the control action of a passive vibration control system, a Tuned Mass Damper, and vice-versa. A Macro-Simulink model is implemented with the goal of simulate the hysteretic behavior of the infill wall subjected to cyclic loading, contemplating the stiffness and strength degradation and the pinching effect. Numerical results will be presented and discussed, regarding the assessment of the influence that the hysteretic behavior of the infill wall has on the structure.

Keywords: Non-Linear, hysteretic behavior, structural control, passive systems, tuned mass dampers (TMD).
1 INTRODUCTION

Vibration control systems in civil engineering applications, has been growing significantly in recent years due to safety requirements in the protection of critical facilities or buildings under natural hazard events such as winds or severe earthquakes.

Considering the passive vibration control systems, and given its remarkably acceptance by engineers and constructors, there are nowadays several civil structures equipped with base isolation, viscous dampers and tuned mass dampers (TMDs).

In this paper the passive vibration control system, tuned mass damper, will be used in the control action of a single degree of freedom frame structure that may or may not have infill walls.

The presence of infill walls can significantly influence the behavior and, therefore the response of the structure in which they are inserted in. Thus, they should be considered in structural design of civil engineering constructions that foresees the use of these components. Although, the highly non-linear behavior with a large initial stiffness of these non-structural elements, usually leads to a structural design in which the presence of these elements is neglected.

This simplification should be assessed in whether is appropriate to design passive control systems for building structures. TMDs can be seen as secondary oscillators or harmonic absorbers designed to reduce the amplitude of structural or mechanical vibrations. The performance of these devices is strictly related with the accurate definition of the dynamic properties of the main structure. Therefore, the presence of infill walls may influence the structural behavior, and consequently, the performance of the control system.

A two degree-of-freedom (2-DOFs) system representing a single-story framed structure equipped with a TMD will be used in the assessment of the control system performance and so its effectiveness, when the structure has infill walls. Stiffness and strength degradation, and the Pinching effect models will be considered for the representation of the hysteretic response of the infill wall.

2 NUMERICAL MODEL

The schematic representation of the SDOF structure, $m_1$ is presented in Figure 1, as well as the 2DOF system, representing the main structure with infill walls, $m_1$, equipped with the TMD, $m_2$. The structure is connected to the exterior by a spring of stiffness $k_1$, and by a damping constant $c_1$. The TMD, in turn, is connected to the main structure by a spring of stiffness $k_2$, and by a damping constant $c_2$ (Folhento, 2017 [1]).

In the present study the following parameters were considered: mass of the structure $m_1=5000$kg; the period $T=1.0s$, the structural damping coefficient $\xi =0.05$ and the mass ratio between the TMD and the structure $\mu=0.15$.

The Macro-Simulink numerical model used in this investigation was adapted from Mousavi, et al, 2015 [2], and is based on a smooth hysteretic model that was originally suggested by Bouc, 1967 [3] (and developed by many others: Wen, 1976 [4], Baber & Noori, 1985 [5], Casciati, 1989 [6], Reinhorn et al. [7], 1995, Sivaselvan & Reinhorn, 2000 [8]).

The influence of the infill wall in the control action of the TMD was verified by considering four cases of hysteretic behavior of the non-structural element: plain hysteretic behavior, considering no degradation; only stiffness degradation, stiffness and strength degradation; and finally, the consideration of the pinching effect along with the stiffness and strength degradation.

Table 1 shows the parameters and their respective values for the different cases of hysteretic behavior of the infill wall considered in the present study.
Figure 1: Schematic representation of the SDOF structural system (on the left) and 2DOF structural system (on the right).

Table 1: Hysteretic parameters and their values, considered to simulate the different frame behaviors (in all cases: $N=3; \alpha=0.03; \eta=5$).

<table>
<thead>
<tr>
<th>Case</th>
<th>Hysteretic behavior</th>
<th>$\alpha$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\sigma$</th>
<th>$R_{sp}$</th>
<th>$\lambda_c$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Plain hysteretic behavior</td>
<td>50</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>Stiffness degradation</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Stiffness and strength degradation</td>
<td>1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Stiffness and strength degradation with pinching effect</td>
<td>1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Two different acceleration signals will be used in this investigation. The first one, shown in Figure 2 and represented by the function in Equation 1, is a harmonic generic signal composed by five sections with different acceleration values.

Figure 2: Acceleration of a growing harmonic generic signal.
The second acceleration signal, shown in Figure 3, is the ground acceleration of the well-known El Centro earthquake, occurred on May 18, 1940 in southeastern California.

3 CASE 0: PLAIN HYSTERETIC BEHAVIOR

The following equations represent the simple case of hysteretic behavior that has no stiffness and strength degradation, appropriate for well-detailed steel structures, e.g., special moment resisting frames:

\[ P_f = k_f x = (ak_0 + k_{hyst}) x \]

\[ k_{hyst} = (1-a)k_0 \left(1 - \frac{P_f}{P_y} \right)^N \left[ \eta \text{sgn}\left((1-a)P_f\right) \right] \]

Table 2: Parameters description of Equation 2.

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_f )</td>
<td>Nonlinear total lateral stiffness of the frame</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>Initial lateral stiffness of the frame</td>
</tr>
<tr>
<td>( a )</td>
<td>Post-yield stiffness ratio</td>
</tr>
<tr>
<td>( N )</td>
<td>Controls the transition smoothness from pre-yield to post-yield</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Controls the shape of the unloading path</td>
</tr>
<tr>
<td>( P_f )</td>
<td>Current frame shear</td>
</tr>
<tr>
<td>( P_y )</td>
<td>Yield value of the current frame shear</td>
</tr>
<tr>
<td>( \text{sgn} )</td>
<td>Signum function</td>
</tr>
</tbody>
</table>
The graph of Figure 4 presents the uncontrolled and controlled with TMD structural responses of the frame without infill wall and with infill wall exhibiting the plain hysteretic behavior, in terms of displacements with respect to time and under the acceleration signal depicted in Figure 2. These responses were obtained using the aforementioned Macro/Simulink numerical model with the parameters of the Case 0 referred in Table 1.

The same numerical model and parameters were used to compute the uncontrolled and controlled with TMD structural responses of the frame with and without infill wall, in terms of displacements with respect to time, considering the same case of hysteretic behavior (Case 0), but now subjected to the El Centro’s seismic acceleration signal (Figure 3). These responses are represented in Figure 5.

Figure 4: Structural responses of the frame in terms of displacements with respect to time, under the generic signal acceleration and considering Case 0 of hysteretic behavior for the infill wall.

Figure 5: Structural responses of the frame in terms of displacements with respect to time, under the El Centro’s signal acceleration and considering Case 0 of hysteretic behavior for the infill wall.
4 CASE 1: STIFFNESS DEGRADATION

Geometric effects cause stiffness degradation. Elastic stiffness decreases with increased ductility. The stiffness degradation is reflected in the Macro-Simulink model through the so-called pivot rule (Park et al., 1987 [9]). Most reinforced concrete undergoes stiffness degradation that should be considered in a nonlinear dynamic analysis. To address this case, \( k_{\text{hyst}} \) should be modified as follows:

\[
k_{\text{hyst}} = (R_k - a) k_0 \left\{ 1 - \left| \frac{P_t}{P_y} \right| ^N \right\} (1-a) \eta \text{sgn}\left((1-a)P_t\right) \cdot \eta P
\]

in which

\[
P_t = k_\tau x = (ak_0 + k_{\text{hyst}}) x
\]

\[
R_k = \frac{P_t + \alpha P_y}{k_0 x + \alpha P_y}
\]

Table 3: New Parameters description in Equation 3 and 4 with respect to Equation 2.

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Stiffness regulator parameter</td>
</tr>
<tr>
<td>( R_k )</td>
<td>Function that controls the stiffness – in function of ( \alpha )</td>
</tr>
</tbody>
</table>

Parameter \( \alpha \) can control the stiffness degradation, i.e., the higher the \( \alpha \), the lower the stiffness degradation. Parameter \( R_k \) is always a positive value and the unit is its maximum possible value. However, \( R_k \) is also a decreasing function of time, since the stiffness of the structure would not increase after deterioration, regardless of the current displacement.

Figure 6 shows the uncontrolled and controlled with TMD structural responses of the frame without infill wall and with infill wall considering the stiffness degradation, in terms of displacements with respect to time and under the acceleration signal depicted in Figure 2. The aforementioned Macro/Simulink numerical model was used to compute these responses considering the parameters of the Case 1 referred in Table 1.

Figure 6: Structural responses of the frame in terms of displacements with respect to time, under the generic signal acceleration and considering Case 1 of hysteretic behavior for the infill wall.
In the same way the above-mentioned numerical model and parameters were used to compute the uncontrolled and controlled with TMD structural responses of the frame with and without infill wall in terms of displacements with respect to time, considering the case of hysteretic behavior of stiffness degradation (Case 1), but now subjected to the El Centro’s seismic acceleration signal (Figure 3). These responses are represented in Figure 7.

![Figure 7: Structural responses of the frame in terms of displacements with respect to time, under the El Centro’s signal acceleration and considering Case 1 of hysteretic behavior for the infill wall.](image)

5 CASE 2: STIFFNESS AND STRENGTH DEGRADATION

To prevent P-Δ effects and strength degradation during cyclic loading, a deterioration of resistance based on energy and ductility is applied in the Macro-Simulink model. This is attained by the following modification on the yield strength:

\[
P_{fy} = P_{fy0} \left[ 1 - \left( \frac{x_{\text{max}}}{x_{\text{ult}}} \right)^\frac{1}{\beta_1} \right] \left[ 1 - \frac{\beta_2 H}{(1-\beta_2)H_{\text{ult}}} \right]
\]

(5)

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{fy} )</td>
<td>Degraded yielding strength of the frame</td>
</tr>
<tr>
<td>( P_{fy0} )</td>
<td>Initial yielding strength of the frame</td>
</tr>
<tr>
<td>( u_{\text{max}} )</td>
<td>Maximum displacement in the current load inversion</td>
</tr>
<tr>
<td>( u_{\text{ult}} )</td>
<td>Ultimate displacement capacity of the frame</td>
</tr>
<tr>
<td>( H )</td>
<td>Dissipated energy accumulated at the current displacement</td>
</tr>
<tr>
<td>( H_{\text{ult}} )</td>
<td>Ultimate dissipated energy under monotonic (non-cyclic) load</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>Degradation parameter based on ductility demand</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>Degradation parameter based on energy dissipation demand</td>
</tr>
</tbody>
</table>

Table 4: Parameters description of Equation 5.
Strength degradation should be considered for ordinary or intermediate moment resisting frames under great ductility demands. Most reinforced concrete frames and shear walls would also experience strength deterioration.

The structural responses in terms of displacements with respect to time, of the frame without infill wall and with infill wall considering the stiffness and strength degradation, uncontrolled or controlled with the TMD, under the generic acceleration signal, are represented in Figure 8. The referred Macro/Simulink numerical model was used to compute these responses considering the parameters of the Case 2 in Table 1.

![Figure 8: Structural responses of the frame in terms of displacements with respect to time, under the generic signal acceleration and considering Case 2 of hysteretic behavior for the infill wall.](image)

The structural responses now considering the El Centro’s earthquake signal acceleration (Figure 3) were obtained in the same way as the previous situation with the generic signal acceleration, using the Macro-Simulink model and the parameters from Table 1 regarding the stiffness and strength degradation case (Case 2), their representation can be seen in Figure 9.

![Figure 9: Structural responses of the frame in terms of displacements with respect to time, under the El Centro’s signal acceleration and considering Case 2 of hysteretic behavior for the infill wall.](image)
6 CASE 3: STIFFNESS AND STRENGTH DEGRADATION WITH THE PINCHING EFFECT

Most of reinforced concrete shear walls and structures with masonry infill walls revealed compressed (or pinched) hysteretic cycles in early tests (Mousavi et al., 2015 [2]). Thus, pinching can be considered in the above-mentioned model by the following modification on the lateral stiffness of the frame, given by the following expression:

\[ k_f = a k_0 + \frac{k_{\text{hyst}} k_{\text{slip-lock}}}{k_{\text{hyst}} + k_{\text{slip-lock}}} \]  

where

\[ k_{\text{slip-lock}} = \left\{ \sqrt{2} \frac{R_s}{\pi} \left( \mu_d - 1 \right) \left( x_{\text{max}}^+ - x_{\text{max}}^- \right) \exp\left[ -\left( \frac{P_f - \lambda_c P_{\text{dy}}}{\sigma P_{\text{dy}}} \right) \right] \right\}^{-1} \quad 0 \leq R_s, \sigma, \lambda_c \leq 1 \]

Table 5: Parameters description of Equation 6 and 7.

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_s )</td>
<td>Parameter that defines the slip length of the behavior</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Fraction of frame yield shear beyond which slip would not occur</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Parameter that accounts for variable slip length depending on the developed ductility demand.</td>
</tr>
<tr>
<td>( \lambda_c )</td>
<td>Fraction of frame yield shear around which slip occurs (in symmetric frames, ( \lambda = 0 ))</td>
</tr>
<tr>
<td>( \mu_d )</td>
<td>Ductility demand at the current displacement</td>
</tr>
<tr>
<td>( x_{\text{max}}^+ )</td>
<td>Maximum displacements of the current loop at the positive direction</td>
</tr>
<tr>
<td>( x_{\text{max}}^- )</td>
<td>Maximum displacements of the current loop at the negative direction</td>
</tr>
</tbody>
</table>

Figure 10: Structural responses of the frame in terms of displacements with respect to time, under the generic signal acceleration and considering Case 3 of hysteretic behavior for the infill wall.
As can be seen from Table 5, parameters $R_{sp}$, $\sigma$, $\kappa$ and $\lambda_c$ present in Equation 8, are parameters related to the pinching effect. The pinching effect will be more pronounced for higher values of $R_{sp}$ and lower values of $\sigma$.

The graph of Figure 10 shows the uncontrolled and controlled with TMD structural responses in terms of displacements with respect to time, of the frame without infill wall and with infill wall considering the stiffness and strength degradation with the pinching effect, and under the acceleration signal depicted in Figure 2. These responses were obtained using the aforementioned Macro/Simulink numerical model with the parameters of the Case 3 referred in Table 1.

The structural responses now considering the El Centro’s earthquake signal acceleration (Figure 3) were obtained in the same way as the previous situation with the generic signal acceleration, using the Macro-Simulink model and the parameters from Table 1 regarding the stiffness and strength degradation with the pinching effect (Case 3), and their representation can be seen in Figure 11.

7 RESPONSE OF THE TMD IN THE CONTROL ACTION OF THE FRAME WITH AND WITHOUT INFILL WALL

The graphs of Figures 12 to 15 present the response in terms of displacements with respect to time of the passive control system, the TMD, under the generic signal acceleration, being applied to the frame with or without infill wall, for the four cases of hysteretic behavior contemplated in the present study.

In the same way, Figures 16 to 19 present the response in terms of displacements in relation to time of the passive control system, the TMD, now under the seismic signal acceleration of the El Centro’s earthquake, being applied to the frame with or without infill wall, for the four cases of hysteretic behavior contemplated in the present study.
Figure 12: Displacement responses of the TMD under the generic signal acceleration, considering Case 0 of hysteretic behavior.

Figure 13: Displacement responses of the TMD under the generic signal acceleration, considering Case 1 of hysteretic behavior.

Figure 14: Displacement responses of the TMD under the generic signal acceleration, considering Case 2 of hysteretic behavior.
Figure 15: Displacement responses of the TMD under the generic signal acceleration, considering Case 3 of hysteretic behavior.

Figure 16: Displacement responses of the TMD under the seismic acceleration, considering Case 0 of hysteretic behavior.

Figure 17: Displacement responses of the TMD under the seismic acceleration, considering Case 1 of hysteretic behavior.
HYSTERETIC BEHAVIOR OF THE NON-STRUCTURAL WALLS

In this section the results concerning the hysteretic behavior of the frame with infill walls for the different cases previously contemplated are presented, using the referred numerical model.

This behavior is translated by force-displacement relations, and these representations are shown in the following graphs of Figures 20 to 23 for the system in study subjected to the generic acceleration signal (Figure 2), for the different cases of hysteretic behavior considered, and for both the situations of uncontrolled and controlled with the TMD.

Figures 24 to 27 present the structural responses of the system in study with and without infill wall, submitted to the seismic acceleration signal of the El Centro’s earthquake, considering the different hysteretic behaviors investigated, and for both the uncontrolled and controlled with the TMD cases.
Figure 20: Hysteretic cycles of the infill wall structure under the generic signal acceleration, considering the plain hysteretic behavior (Case 0) for the uncontrolled response (on the left) and the controlled response with TMD (on the right).

Figure 21: Hysteretic cycles of the infill wall structure under the generic signal acceleration, considering the stiffness degradation (Case 1) for the uncontrolled response (on the left) and the controlled response with TMD (on the right).

Figure 22: Hysteretic cycles of the infill wall structure under the generic signal acceleration, considering the stiffness and strength degradation (Case 2) for the uncontrolled response (on the left) and the controlled response with TMD (on the right).
Figure 23: Hysteretic cycles of the infill wall structure under the generic signal acceleration, considering the stiffness and strength degradation with the pinching effect (Case 3) for the uncontrolled response (on the left) and the controlled response with TMD (on the right).

Figure 24: Hysteretic cycles of the infill wall structure under the seismic acceleration, considering the plain hysteretic behavior (Case 0) for the uncontrolled response (on the left) and the controlled response with TMD (on the right).

Figure 25: Hysteretic cycles of the infill wall structure under the seismic acceleration, considering the stiffness degradation (Case 1) for the uncontrolled response (on the left) and the controlled response with TMD (on the right).
DISCUSSION OF RESULTS AND CONCLUSIONS

The results of the peak responses (displacements, velocities, accelerations and displacements between the structure and TMD) of the system under study when subjected to the generic and seismic acceleration are presented in Table 6 and Table 7, respectively.

Observing Tables 6 and 7, it is concluded that the presence of the TMD positively influences the behavior of the structure in all cases and types of responses, with and without infill wall, as it significantly reduces the dynamic responses of the system in study.

From the analysis of Table 6, it can be seen in all the peak responses with infill wall that, as one advances to a more realistic case of hysteretic behavior (Case 3 – stiffness and strength degradation with the pinching effect), the percentage of reduction in relation to the respective uncontrolled case increases in its absolute value.

Observing sequentially the graphs of Figures 4, 6, 8 and 10, it can be seen that the uncontrolled and controlled response of the infill wall frame increases from the simple hysteretic behavior to the case where the stiffness and strength degradation with the pinching effect are considered (green and blue lines).
The same conclusions drawn above regarding the generic acceleration signal are applied in the results presented in Table 7, concerning the system submitted to the seismic acceleration, although not so obvious due to the particular irregularity of the seismic acceleration.

Table 6: Peak responses of the frame under the generic signal acceleration.

| Hysteretic behavior case | Peak responses |  |  |  |
|-------------------------|----------------|-------------------------|-------------------------|-------------------------|-------------------------|
|                         | \(x\) (m)      | \(\dot{x}\) (m/s)      | \(\ddot{x}\) (m/s\(^2\)) | Drift (m)               |
| Without infill wall     | 0.673          | 4.84                   | 26.565                  | 0.673                   |
| Case 0                  | 0.145          | 0.860                  | 6.357                   | 0.145                   |
| Case 1                  | 0.186          | 1.134                  | 7.722                   | 0.186                   |
| Case 2                  | 0.433          | 2.929                  | 24.342                  | 0.433                   |
| Case 3                  | 0.545          | 3.372                  | 27.055                  | 0.545                   |
| Controlled with TMD     |  |  |  |  |
| Case 0                  | 0.233 (-189%)  | 1.443 (-190%)          | 9.007 (-195%)           | 0.233 (-189%)           |
| Case 1                  | 0.512          | 2.947                  | 18.267                  | 0.484                   |
| Case 2                  | 0.311          | 1.918                  | 11.997                  | 0.285                   |
| Case 3                  | 0.493          | 2.725                  | 16.796                  | 0.378                   |

The first and second lines represent the peak responses for the first and second floors, respectively, the main structure and the TMD. The percentage on the right stands for the percentage of increase (positive) of the peak responses with respect to the corresponding uncontrolled response.

Table 7: Peak responses of the frame under the seismic acceleration of El Centro’s earthquake.

<table>
<thead>
<tr>
<th>Hysteretic behavior case</th>
<th>Peak responses</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(x) (m)</td>
<td>(\dot{x}) (m/s)</td>
<td>(\ddot{x}) (m/s(^2))</td>
<td>Drift (m)</td>
</tr>
<tr>
<td>Without infill wall</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 0</td>
<td>0.128</td>
<td>0.906</td>
<td>7.025</td>
<td>0.128</td>
</tr>
<tr>
<td>Case 1</td>
<td>0.064</td>
<td>0.627</td>
<td>6.391</td>
<td>0.064</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.066</td>
<td>0.636</td>
<td>6.381</td>
<td>0.066</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.066</td>
<td>0.635</td>
<td>6.353</td>
<td>0.066</td>
</tr>
<tr>
<td>Controlled with TMD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 0</td>
<td>0.081 (-57%)</td>
<td>0.577 (-57%)</td>
<td>4.854 (-45%)</td>
<td>0.081 (-57%)</td>
</tr>
<tr>
<td>Case 1</td>
<td>0.160</td>
<td>0.922</td>
<td>5.899</td>
<td>0.147</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.101</td>
<td>0.643</td>
<td>5.206</td>
<td>0.106</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.059 (-11%)</td>
<td>0.620 (-3%)</td>
<td>5.975 (-7%)</td>
<td>0.059 (-11%)</td>
</tr>
<tr>
<td>Controlled with TMD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 0</td>
<td>0.059 (-11%)</td>
<td>0.619 (-3%)</td>
<td>5.952 (-7%)</td>
<td>0.059 (-11%)</td>
</tr>
<tr>
<td>Case 1</td>
<td>0.109</td>
<td>0.669</td>
<td>5.196</td>
<td>0.113</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.059 (-11%)</td>
<td>0.618 (-3%)</td>
<td>5.951 (-7%)</td>
<td>0.059 (-11%)</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.114</td>
<td>0.684</td>
<td>5.199</td>
<td>0.118</td>
</tr>
</tbody>
</table>

The first and second lines represent the peak responses for the first and second floors, respectively, the main structure and the TMD. The percentage on the right stands for the percentage of increase (positive) of the peak responses with respect to the corresponding uncontrolled response.
By the analysis of Figures 20 to 23, focusing only on the uncontrolled cases, it is possible to verify that in the first case where the stiffness degradation is not considered (Figure 20), the loading and unloading curves remain approximately parallel, which means that the stiffness degradation is almost non-existent, since the stiffness is translated by the slope of the load and unloading paths. In addition to this, the fact that the system possesses great stiffness leads to greater displacements to higher values of the strength capacity of the structure. The evolution of hysteretic loops with the action of acceleration signals over time implies a greater dissipation of energy translated by the increasing area of the cycles.

The stiffness degradation is now evident in Figure 21, where the slope of the loading and unloading paths varies from loop to loop. In addition to this, a slight increase of the displacement can be seen to the same value of the strength capacity of the wall, when compared with the case of simple hysteretic behavior.

In the graph of Figure 22, showing the case of stiffness and strength degradation, it can be observed that besides the slope variation of the loading and unloading paths, reflecting the stiffness degradation, there is also a decrease of the strength capacity of the wall, resulting in larger displacements. This is evident, since the transformation of the shape of the hysteretic cycles passes from an approximately vertical shape to a horizontal one.

The graph of Figure 23, presents the most realistic case of hysteretic behavior of the frame structure filled by a non-structural wall, that considers the stiffness and strength degradation along with the pinching effect. The pinching effect is entirely related to the phenomenon of wall cracking, due to the effect of shear stresses on the behavior of the wall, resulting in a reduction of the hysteretic cycle to values of displacement near to the origin. This compression leads to less energy dissipation of the system.

The last two cases of hysteretic behavior resulted in a numerical instability. In reality, this translates into a structural instability of the wall, specifically in the rupture of the wall out of its plane. This fact can be easily verified by observing the graphs of Figures 8 and 10, where the "green line" has a permanent displacement of about 5 and 10cm, respectively, when compared to its initial position.

The same characteristics of the hysteretic loops can be verified when observing the corresponding controlled cases with the TMD in Figures 20 to 23. However, the presence of the TMD, provides a significant reduction of the displacements while preserving the same strength capacity of the infill wall. The presence of the TMD then provides a minor contribution of the infill wall in the energy dissipation of the frame. It should be noted that structural instability does not occur in the presence of the TMD, which can be demonstrated by the graphs of Figures 8 and 10 (blue line) and the Figures 21 and 22 (images on the right).

In Figures 24 to 27 the system is submitted to the seismic acceleration, where it is possible to verify the influence that the presence of TMD has in reducing the displacements of the system, although it is not very perceptive as the situation in which the system is subjected to the generic acceleration signal, this due to the inherent irregularity of the seismic acceleration.

A new set of evaluation criteria including normalized and RMS (Root Mean Square) responses and also control requirements was used to complete the performance assessment of each controlled case of hysteretic behavior.

The first three criteria \((J_1, J_2, J_3)\) are based on peak responses. The next four criteria \((J_4, J_5, J_6, J_7)\) are related with the RMS structural responses (see Table 8). In these equations, \(|\cdot|\) denotes the absolute value and \(\|\cdot\|\) is the L2 norm given by (Braz-César, 2015 [10]):

\[
\|\cdot\| = \sqrt{\int_{t_0}^{t_f} |\cdot|^2 \, dt}
\]
where \( t_f = t_{\text{max}} \) represent the total excitation duration and \( t_f \) represent a sufficient large time to allow the response to attenuate.

### Table 8: Evaluation criteria for the structural responses in terms of peak responses and RMS ratios.

<table>
<thead>
<tr>
<th>Evaluation criterion</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_1 = \max_{t_f} \left( \frac{| x_{c,t}(t) |}{| x_{\max,u}(t) |} \right) )</td>
<td>Maximum peak floor displacement ratio whereby the floor displacements over time are normalized by the maximum peak uncontrolled displacement.</td>
</tr>
<tr>
<td>( J_2 = \max_{t_f} \left( \frac{| \dot{x}<em>{c,t}(t) |}{| \dot{x}</em>{\max,u}(t) |} \right) )</td>
<td>Maximum peak floor velocity ratio whereby the floor velocities over time are normalized by the maximum peak uncontrolled velocity.</td>
</tr>
<tr>
<td>( J_3 = \max_{t_f} \left( \frac{| \ddot{x}<em>{c,t}(t) |}{| \ddot{x}</em>{\max,u}(t) |} \right) )</td>
<td>Maximum peak floor acceleration ratio whereby the floor accelerations over time are normalized by the maximum peak uncontrolled acceleration.</td>
</tr>
<tr>
<td>( J_4 = \max_{t_f} \left( \frac{| x_{c,t}(t) |}{| x_{\max,u}(t) |} \right) )</td>
<td>Maximum RMS floor displacement ratio, which is given in terms of the maximum RMS absolute displacement over time with respect to the uncontrolled case.</td>
</tr>
<tr>
<td>( J_5 = \max_{t_f} \left( \frac{| \dot{x}<em>{c,t}(t) |}{| \dot{x}</em>{\max,u}(t) |} \right) )</td>
<td>Maximum RMS floor velocity ratio, which is given in terms of the maximum RMS absolute velocity over time with respect to the uncontrolled case.</td>
</tr>
<tr>
<td>( J_6 = \max_{t_f} \left( \frac{| \ddot{x}<em>{c,t}(t) |}{| \ddot{x}</em>{\max,u}(t) |} \right) )</td>
<td>Maximum RMS floor acceleration ratio, which is given in terms of the maximum RMS absolute acceleration over time with respect to the uncontrolled case.</td>
</tr>
<tr>
<td>( J_7 = \max_{t_f} \left( \frac{| d_{c,t}(t) |}{| d_{\max,u}(t) |} \right) )</td>
<td>Maximum RMS inter-story drift ratio, which is given in terms of the maximum RMS absolute inter-story drift over time with respect to the uncontrolled case. Inter-story drift ( d_i = \delta_i / h_i ).</td>
</tr>
</tbody>
</table>

Subscript \( i = 1, 2 \) denotes the story index and subscripts \( c \) and \( u \) represent controlled and uncontrolled cases.

Tables 9 and 10 were carried out taking into account the performance evaluation criteria of Table 8, presenting the values of the seven criteria based on peak responses and RMS responses, when the system is subjected to the generic signal and seismic acceleration, respectively, and for the four controlled cases of hysteretic behavior considered in the present study, comparing them with the controlled case without infill wall.

### Table 9: Evaluation criteria values for the structural responses under the generic signal acceleration in terms of peak responses and RMS ratios.

<table>
<thead>
<tr>
<th>Hysteretic behavior case</th>
<th>( J_1 )</th>
<th>( J_2 )</th>
<th>( J_3 )</th>
<th>( J_4 )</th>
<th>( J_5 )</th>
<th>( J_6 )</th>
<th>( J_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without infill wall</td>
<td>0.761</td>
<td>0.345</td>
<td>0.688</td>
<td>0.714</td>
<td>0.693</td>
<td>0.679</td>
<td>0.714</td>
</tr>
<tr>
<td>Case 0</td>
<td>0.462</td>
<td>0.144</td>
<td>0.452</td>
<td>0.458</td>
<td>0.448</td>
<td>0.442</td>
<td>0.427</td>
</tr>
<tr>
<td></td>
<td>(-65%)</td>
<td>(-139%)</td>
<td>(-52%)</td>
<td>(-56%)</td>
<td>(-55%)</td>
<td>(-54%)</td>
<td>(-67%)</td>
</tr>
<tr>
<td>Case 1</td>
<td>0.525</td>
<td>0.178</td>
<td>0.511</td>
<td>0.492</td>
<td>0.480</td>
<td>0.473</td>
<td>0.463</td>
</tr>
<tr>
<td></td>
<td>(-45%)</td>
<td>(-94%)</td>
<td>(-35%)</td>
<td>(-45%)</td>
<td>(-44%)</td>
<td>(-43%)</td>
<td>(-54%)</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.732</td>
<td>0.299</td>
<td>0.632</td>
<td>0.596</td>
<td>0.568</td>
<td>0.547</td>
<td>0.556</td>
</tr>
<tr>
<td></td>
<td>(-20%)</td>
<td>(-22%)</td>
<td>(-24%)</td>
<td>(-20%)</td>
<td>(-22%)</td>
<td>(-24%)</td>
<td>(-28%)</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.737</td>
<td>0.298</td>
<td>0.650</td>
<td>0.618</td>
<td>0.590</td>
<td>0.569</td>
<td>0.581</td>
</tr>
<tr>
<td></td>
<td>(-3%)</td>
<td>(-16%)</td>
<td>(-6%)</td>
<td>(-15%)</td>
<td>(-17%)</td>
<td>(-19%)</td>
<td>(-23%)</td>
</tr>
</tbody>
</table>

The percentage below the values stands for the percentage of increase (positive) of the peak responses with respect to the corresponding response without infill wall.
The first conclusion to be drawn from the analysis of Tables 9 and 10 is that as we move on to a more realistic case (Case 0 to Case 3) of hysteretic behavior, the smaller the reductions (in absolute value), with some exceptions when considering the seismic acceleration. Thus, it can be seen from the analysis of Table 9 that the case in which we consider the stiffness and strength degradation with the pinching effect, being a more realistic case, presents the smallest reductions (in absolute value) with respect to the case without infill wall, which is verified in all performance evaluation criteria.

Analyzing Table 10 where the system is subject to the seismic acceleration, the reductions are not as evident as in the previous situation, although it shows significant reductions in certain responses, with the exception of the peak velocity and acceleration responses. The peak responses of velocity increase in relation to the case with no infill wall. In the case of the peak responses of acceleration they only suffer a slight increase.

By the observation of Figures 12 to 15, it can be verified that the displacement amplitude of the TMD increases as one moves on to a more realistic case of hysteretic behavior of the wall, meaning that there is a higher contribution of the TMD in the vibration control of the system structural response, i.e., dissipates more energy, leading to a minor energy dissipation of the infill wall, as can be proven by the hysteretic loops of the controlled cases. This conclusion can also be verified when the system is subjected to the seismic acceleration, although not as evident as for the generic signal acceleration.

## 10 CONCLUSIONS

- It has been found that the non-structural wall has a significant effect on the response of the system. The non-linear behavior of the wall allows a reduction of the peak displacements. This effect is most evident in the controlled case where the TMD is used in combination with the hysteretic models of the infill walls.
- The TMD allows the infill wall frame to have less contribution in the energy dissipation, avoiding a possible structural instability.
- It was also verified that the presence of the non-structural element influences the performance of the TMD, reducing its displacement amplitude in its control action. Although, when moving on to a more realistic case of hysteretic behavior, the displacement amplitude tends to approximate the same values of the case without infill wall.
REFERENCES


NONLINEAR BENDING OF ROUND THIN SD PLATES

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Abstract

The problem of elastoplastic bending of a freely supported, round thin plate uniformly loaded with transverse pressure is considered. The plate is made of thin sheet metal with transversal isotropy properties and the effect of different resistance to tension and compression (SD effect) during plastic deformation. Plasticity area in the compression zone of the plate is substantially smaller than those in the tension zone. We assume that the yield strength during compression is greater than that under tension.

To calculate the bending, the COMSOL software package is used. Depending on the pressure, we calculate the sizes of plasticity zones. The problem used the classical Mises – Hill approach. According to the results of the calculation, the magnitude of the plasticity "spot" and the depth of plasticity areas significantly depends on the condition of compression or tension.

Keywords: SD-effect; state of biaxial stress; yield criterion; plates; anisotropy; distributed load; elastic-plastic bending
1 INTRODUCTION

The classical theory of elastic and elastoplastic bending was developed in the scientific works of R. von Mises, R. Hill, L. H. Donnell. For thin isotropic plates S.P. Timoshenko was consider elasticity theory. Further development, with allowance for plasticity, belongs to V.V. Sokolovsky [1]. The material of the plate has different strength properties under tension and compression. It is called as materials with SD-effect [2]. Impact of the SD effect is significant for anisotropic metal alloys [3], modern structural materials and carbon plastics [4] at the biaxial stress state.

Numerous experimental studies [5, 6] show that the difference in yield stress in transversely isotropic materials reaches 40%, and the difference in yield stress in stretching and compression in SD materials reaches 25-30%.

The use of metal alloys with SD-effect in structures such as plates and shells operating under biaxial stress conditions, gives a significant increase in strength and load-bearing capacity [7,8]. An important problem is the estimation of the stressed state of a circular transversely isotropic plate made of a material with an SD effect [9]. In the present paper, a freely supported plate is investigated under the action of a constant distributed pressure.

To study the elastoplastic bending, various criteria of fluidity were used [10]. They introduced the parameters of transverse isotropy and plastic anisotropy. The influence of these parameters on the development of plastic regions and on deflection was considered in [11].

2 MATHEMATICAL MODEL

Let us consider the problem of elastic-plastic bending of a round freely supported SD-plate possessing the properties of transverse anisotropy and uniformly loaded with pressure \( p \) on the upper surface.

Figure 1 shows the central cross section of a curved circular plate and the following notation is introduced: \( h \) is the half thickness of the plate, \( x_1, x_2 \) is the radius of the plastic regions from below and from above respectively, \( a_1, a_2 \) are the depths of the plastic zones from below and from above, respectively. The plastic regions are shaded. The neutral surface, in the case under consideration, does not coincide with the geometrically average surface. A solid line — the neutral surface, a dashed line — the geometrically the middle surface.

The beginning of the coordinate system is in the center of the plate on the neutral surface (point \( O \) in Figure 1.). The development of plastic zones is disturbed.

At first, we consider the simple case of elastic-plastic bending of the transversal isotropic plate without SD-effect and the corresponding yield criterion [8].

![Figure 1. Elastic-plastic bending of a circular plate from SD material.](image_url)
Here $\sigma_r$, $\sigma_\theta$ are the stresses in the plane of the plate. In this criterion uses the transversal isotropy parameter $A$, which varies from 1 to 2 and calculates by the formula

$$A = 2 - \frac{\sigma_p^2}{\sigma_{pz}^2}. \hspace{1cm} (2)$$

Here $\sigma_p$ is the yield point for uniaxial tension in the plane of the plate and $\sigma_{pz}$ is the yield point for uniaxial tension in a direction perpendicular to the plane of the plate.

The relationships between the stresses and the curvature parameters for a transversal-isotropic material without an SD effect were obtained in [8].

In the articles [9], the mathematical model for the SD-plate was made more complicated and a new criterion of fluidity was proposed:

$$\kappa = \sqrt{\sigma_p^2 - A\sigma_r\sigma_\theta + \sigma_\phi^2 + \alpha \beta}. \hspace{1cm} (3)$$

in which a parameter $\beta$ introduces. The $\beta$ characterizes the SD effect.

For uniaxial stretching (the formula on the left) and uniaxial compression (the formula on the right), criterion (3) is equal to:

$$\kappa_p = \sigma_p + \frac{1}{3} \sigma_p \beta, \hspace{1cm} \kappa_c = \sigma_c + \frac{1}{3} \sigma_c \beta. \hspace{1cm} (4)$$

Here $\sigma_p$ is the yield point for uniaxial tension in the plane of the plate and $\sigma_c$ is the yield point by uniaxial compression in the plane of the plate. Then the relationship between $\beta$, $\sigma_p$ and $\sigma_c$ is:

$$\frac{\sigma_c}{\sigma_p} = \frac{3+\beta}{3-\beta}. \hspace{1cm} (5)$$

We substitute $\beta$ from (5) into (3). Then:

$$\kappa = \frac{2\sigma_p \sigma_c}{\sigma_p + \sigma_c} \hspace{1cm} (6)$$

In the case of a biaxial stress state, the criterions for stretching and compression can be written accordingly:

$$\kappa_p = \sigma_{pz} \sqrt{2 - A + \frac{2}{3} \sigma_{pz} \beta}, \hspace{1cm} \kappa_c = \sigma_{cz} \sqrt{2 - A - \frac{2}{3} \sigma_{cz} \beta}. \hspace{1cm} (7)$$

Then

$$\beta = \frac{3\sqrt{2-A} \left(\sigma_{cz} - \sigma_{pz}\right)}{2 \left(\sigma_{cz} + \sigma_{pz}\right)} \hspace{1cm} (8)$$

Substituting the resulting expression for $\beta$ in (8), we obtain the formula for $A$:

$$A = 2 - \frac{(\sigma_{pz} + \sigma_{cz})^2}{(\sigma_p + \sigma_c)^2} \frac{\sigma_p^2 \sigma_c^2}{\sigma_{pz}^2 \sigma_{cz}^2}. \hspace{1cm} (9)$$

The bending of the plate is considered according to the model of a plane stress state. The deformation of the transverse shear is ignored. The stress in the direction perpendicular to the plane of the plate is assumed to be zero, then the average stress is equal to:

$$\sigma = \frac{\sigma_r + \sigma_\theta}{3}. \hspace{1cm} (10)$$

Proceeding from formulas (7), (8) and (9) it is possible to establish a connection between yield strengths

$$\frac{\sigma_c}{2 \sigma_{pz}} = \left(\frac{\sigma_c}{\sigma_p} - 1\right) \left(\frac{\sigma_{pz}}{\sigma_p} - 1\right)^{-1}. \hspace{1cm} (11)$$

If the $\sigma_p$, $\sigma_c$, $\sigma_{pz}$, $\sigma_{cz}$, are known, then the values of $\beta$ and $A$ can be calculated.
To most metal alloys, $\sigma_p \leq \sigma_c$ [6], therefore it follows from formula (8) that $\beta \geq 0$, from formula (9) $A \leq 2$.
In the article [10] shows a more complex the relationship between the stress and the curvature parameters $\xi_r, \xi_\theta$ for a transversely isotropic SD-material:

$$
\sigma_r = \frac{E}{\sqrt{3(2-A)}} \left( \frac{2\xi_r + A\xi_\theta}{\text{sign}(x)\xi} \right) \frac{2\beta}{\sqrt{3}} \left( 1 - \frac{\beta(\xi_r + \xi_\theta)(2+A)}{\text{sign}(x)\xi} \right)
$$

$$
\sigma_\theta = \frac{E}{\sqrt{3(2-A)}} \left( \frac{2\xi_\theta + A\xi_r}{\text{sign}(x)\xi} \right) \frac{2\beta}{\sqrt{3}} \left( 1 - \frac{\beta(\xi_\theta + \xi_r)(2+A)}{\text{sign}(x)\xi} \right)
$$

$$
\xi = \frac{\text{sign}(x)\beta(2+A)(\xi_r + \xi_\theta)}{\sqrt{3(2-A)}} + \sqrt{(2+A)(\xi_r^2 + A\xi_r \xi_\theta + \xi_\theta^2)}/3(2-A).
$$

The most critical from the point of view of the evaluation of the stressed state of the plate is its center, therefore we will consider the stresses in the plastic regions near the centers of the upper and lower surfaces of the plate.
Suppose that $\beta \ll 1$. In the center of the plate $\xi_r = \xi_\theta$, therefore $\sigma_\theta = \sigma_r$ and formulas (12), (13) take the form:

$$
\sigma_\theta = \sigma_r = \frac{E}{\sqrt{3(2-A)}} \left( \frac{1}{\text{sign}(x)\xi} \right) \frac{2\beta}{3\alpha} \left( 1 + \frac{1}{F^2} \right) + \frac{4\beta^2}{\text{sign}(x)\xi 9\alpha^2 F}.
$$

Here

$$
\alpha = \sqrt{2 - A}, \quad F = 1 - \frac{\text{sign}(x)2\beta}{3\alpha}.
$$

Now we will consider an isotropic material with an SD effect. Then, in (16) $A = 1, \alpha = 1$, exact formula for the stress at the center of the lower surface of the plate (stretching zone) is

$$
\frac{\sigma_+}{\sigma_p} = \frac{3+\beta}{3} \left( 1 - \frac{2\beta}{3} \right)^2
$$

for the stress at the center of the upper surface of the plate (compression zone)

$$
\frac{\sigma_-}{\sigma_p} = \frac{3+\beta}{3} \left( 1 + \frac{2\beta}{3} \right)^2
$$

Expanding (17) and (18) in a Maclaurin series with respect to the small parameter $\beta$ and neglecting terms of the order of $\beta^3$ and above, we write out approximate formulas for the stresses in the stretching and compression zones for an isotropic material with an SD effect:

$$
\frac{\sigma_+}{\sigma_p} = \frac{3+\beta}{3} \left( 1 - \frac{2\beta}{3} \right), \quad \frac{\sigma_-}{\sigma_p} = -\frac{3+\beta}{3} \left( 1 + \frac{2\beta}{3} \right)
$$

We now turn to an analysis of the mutual influence of the transversal isotropy and the SD effect. In this case, $A > 1, \alpha \neq 1$ and the calculated formulas take the form:

$$
\frac{\sigma_+}{\sigma_p} = \frac{\beta+3}{3\sqrt{(2-A)}} \left( 1 - \frac{2\beta}{3\sqrt{(2-A)}} \right),
$$

$$
\frac{\sigma_-}{\sigma_p} = -\frac{\beta+3}{3\sqrt{(2-A)}} \left( 1 + \frac{2\beta}{3\sqrt{(2-A)}} \right).
$$

Thus, it becomes possible to estimate the influence of the parameters $A$ and $\beta$ on the stresses in the plate, not solving the large problem of elastoplastic equilibrium of the plate [11].
The results of calculations using formulas (20) and (21) are in Tables 1, 2.
Table 1: Dependence of stresses in the plate on the parameters $\beta$ and $A$.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$A$</th>
<th>$\sigma_{-}/\sigma_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>1.1</td>
<td>1.054</td>
</tr>
<tr>
<td>0.01</td>
<td>1.1</td>
<td>1.065</td>
</tr>
<tr>
<td>0.05</td>
<td>1.1</td>
<td>1.110</td>
</tr>
<tr>
<td>0.1</td>
<td>1.1</td>
<td>1.165</td>
</tr>
</tbody>
</table>

Table 2: Dependence of stresses in the plate on the parameters $\beta$ and $A$.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$A$</th>
<th>$\sigma_{-}/\sigma_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>1.3</td>
<td>1.195</td>
</tr>
<tr>
<td>0.01</td>
<td>1.3</td>
<td>1.205</td>
</tr>
<tr>
<td>0.05</td>
<td>1.3</td>
<td>1.245</td>
</tr>
<tr>
<td>0.1</td>
<td>1.3</td>
<td>1.333</td>
</tr>
</tbody>
</table>

As the parameter of transversal isotropy increases, the stresses increase. An increase in the parameter $A$ by 20% causes an increase in stresses at $\beta = 0$ by 14%, and at $\beta = 0.1$ by 17%, therefore, the rate of stress growth with increasing $\beta$ increases. With an increase in $\beta$ by 5% and a fixed $A$, the compressive stress increases by 5.5%. Analysis of the results of numerical simulation shows that for weak plastic anisotropy, the influence of the transversal isotropy parameter is greater than the effect of the SD, but with a strong plastic anisotropy, the effect of the SD increases substantially. This conclusion becomes even more obvious, if we plot the stress functions that depend on the parameters $A$ and $\beta$.

3 GRAPHICAL REPRESENTATION

Let us turn to a graphic illustration of the calculations performed. The asymptotic formulas (20) and (21) allow one to analyze the influence of the parameters of the transversal isotropy $A$ and the plastic anisotropy $\beta$ on the stresses in the plate. Formulas (20), (21) do not depend on the level of pressure on the plate, nor on its dimensions. Therefore, they are universal.

The calculations made in the software package “MATHEMATICS 5.0”.

![Figure 2. Graph of the function of compressive stresses in the center of the upper surface of the plate $\sigma_{-}/\sigma_p(\beta, A)$.](image)
The graphs of the functions $\frac{\sigma_+}{\sigma_\text{p}}(\beta, A)$ and $\frac{\sigma_-}{\sigma_\text{p}}(\beta, A)$ clearly show convex subsets in which we can determine the values of $\beta, A$ for which the stresses $\frac{\sigma_+}{\sigma_\text{p}}, \frac{\sigma_-}{\sigma_\text{p}}$ — locally are minimal.

As a result, we know that plasticity area in the compression zone of the plate is substantially smaller than those in the tension zone. We assume that the yield strength during compression is greater than that under tension. To calculate the bending, the COMSOL 5.4 software package is used. Depending on the pressure, we calculate the sizes of plasticity zones. According to the results of the calculation, the magnitude of the plasticity "spot" and the depth of plasticity areas significantly depends on the condition of compression or tension (see fig.4,5).

4 CONCLUSIONS

- Numerical modeling and graphical representation of the elastoplastic properties of circular transversely isotropic and plastic anisotropic plates showed that for surface stress functions a solution to the problem of optimizing the selection of the parameters of transversal isotropy and plastic anisotropy under the condition of minimum stresses is possible.
• The application of the yield criterion, taking into account the transversal isotropy and the SD effect for the elastoplastic bending of a circular plate, made it possible to construct asymptotic formulas for their calculation. The formulas obtained are universal and estimate the influence of the parameters of transversal isotropy and SD effect on the stress-strain state of any material satisfying the described conditions. Asymptotic formulas allow us to make a rapid evaluation of the stress state of a plate without cumbersome calculations, which is important in engineering practice.

• We can conclude that the capabilities of the COMSOL software package allow us to investigate many problems of nonlinear deformation of SD-materials.

5 ACKNOWLEDGEMENT

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6 REFERENCES

BUCKLING OF AN ANNULAR NANOPLANE UNDER TENSIL POINT LOADING

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Keywords: Surface stresses, Gurtin–Murdoch, Effective elastic moduli, Nanoplate, Buckling.

Abstract. The classical nonlinear von Kármán theory is adapted for modeling the behavior of nanoplates with surface stresses taken into account according to the strain-consistent Gurtin-Murdoch model of the surface elasticity using the effective elastic moduli. This allows us to apply the known solutions and methods for macroplates to nanoplates. In particular, the problem of buckling an thin elastic annular plate stretched by two point loads has been solved by the authors early. In this paper, the same problem are solving for an annular plate having nano-size. In contrast to the previous studies, where the size effect under buckling of nanoplates was considered mainly in the case of a homogeneous stress field, here, the size effect is considered when a nanoplate is buckling under an inhomogeneous stress field with a singularity.
The study of the mechanical properties of nano-scale structures is becoming increasingly important in connection with the rapid development of nanotechnology. At present, two-dimensional models of elastic ultrathin-walled structures are widely used that take into account surface stresses within the framework of the linearized Gurtin-Murdoch relations \[1, 2\]. Somewhat later, this continuum surface model (the GM model) was supported by an atomistic model \[3, 4\]. However, the original constitutive GM equations have a nonclassical structure due to the presence along with the strain terms as well of the displacement gradient. It strongly limits the practical applicability of these models. Therefore, along with the complete GM model of the surface elasticity \[5, 6\], the simplified strain-consistent form of the linearized GM relations is used \[7, 8, 9\]. It preserves its basic physical essence and, at the same time, allows to reduce the solving equations to the classical structure of the macromechanics equations. The surface stresses according to the strain-consistent GM model in the Kirsch problem have been taken into account in \[10, 11\] and in the problem of the compressive buckling of a rectangular nanoplate in \[12, 13\]. However, the validity of such simplification in each individual case deserves attention.

An essential element of these two-dimensional models taken into account surface stresses is the use of effective elastic moduli, including the mechanical characteristics of the bulk phase and the surface layers. These effective moduli, unlike macromechanics, are not material constants, but depend on the thickness. Usually, effective tangential and flexural stiffnesses introduced in \[7\] are considered. Here, within the framework of the strain-consistent constitutive GM equations and the gradient-consistent ones, two pairs of moduli for nanoplates are compared: effective tangential and flexural Young’s modulus and Poisson’s ratio. Using these effective elastic moduli introduced to describe the complex bending of nanoplates, it was shown, that the potential energy of a nanoplate retains the classical structure like a macroplate.

As numerical results, a comparative analysis of the buckling problem of an annular nanoplate under tensile point loading is attached. Factors influencing the deviation of the magnitude of the critical Euler load from the macroplate \[14\] are revealed. It is shown which of them are dominant.

1 MODEL OF BUCKLING AN ANNULAR NANOPLATE WITH SURFACE STRESSES

1.1 Constitutive relation and equilibrium equation

Let us consider a homogeneous linear elastic plate with the nano sized thickness \(h\), which occupies in the Cartesian coordinate system the area \(\{ (x_1, x_2, z) \in \Omega \times [h/2, +h/2] \}, \Omega \subset \mathbb{R}^2\). The axis \(x_1, x_2\) are in the midplane of the plate and the axis \(z\) is orthogonal to it.

On the facial surfaces of the plate \(z = \pm h/2\) according to the strain-consistent GM model of surface elasticity \[1, 2\], there are the surface stresses

\[
\tau_\pm = \tau_0 \mathbf{A} + 2 \mu_0 \varepsilon_\pm + \lambda_0 \mathbf{A} \text{tr} \varepsilon_\pm, \quad (\tau_{1z}, \tau_{2z})_\pm = \tau_0 \nabla w_\pm, \quad (\mu_0 = \mu^s - \tau_0, \quad \lambda_0 = \lambda^s + \tau_0) \quad (1)
\]

where \(\lambda^s\) and \(\mu^s\) are the elastic analogues of the surface Lamé constants, \(\tau_0\) is the residual surface stress (both triples of the surface elastic constants under \(z = \pm h/2\) are assumed to be equal); \(\mathbf{A}\) is the unit tensor in the midplane, and \(\nabla\) is the two-dimensional nabla operator in the midplane, \(\varepsilon\) is a displacement, and \(w\) is its \(z\)-component or the plate deflection.

Opposite to the macro plate theory, the constitutive stress-strain relation of the bulk phase \((-h/2 < z < +h/2)\)

\[
(A \cdot \sigma) = \frac{E}{1 - \nu^2} ((1 - \nu)(A \cdot \varepsilon) + \nu A \text{tr}(A \cdot \varepsilon)) + \frac{\nu \sigma_{zz}}{1 - \nu} \mathbf{A} \quad (2)
\]
has a non-zero \( \sigma_{zz} \), because of the surface equilibrium condition \( \sigma_{zz} = \nabla \cdot \tau \) \cite{1, 2}. Here \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio.

On the base of the kinematic Kirchhoff hypothesis and its analog for the stress component \( \sigma_{zz} \) \cite{6}, we can pass to the two-dimensional equations of a nanoplate theory with the effective membrane forces \cite{10, 11}

\[
\mathbf{T}^* = 2\tau_0 \mathbf{A} + C^* \left( (1 - \nu^*_t)\mathbf{e}^{(0)} + \nu^*_t \mathbf{A} \operatorname{tr} \mathbf{e}^{(0)} \right)
\]

where \( \nu^*_t \) is the effective tangential Poisson ratio; \( C^* \) and \( C \) is the effective and usual tangential stiffness) and the bendings moments

\[
M^* = -D^* \left( (1 - \nu^*_f)\nabla \nabla w + \nu^*_f \mathbf{A} \operatorname{tr}(\nabla \nabla w) \right)
\]

Using the effective elastic moduli, the equilibrium of a nanoplate element is expressed by the equations \cite{11} like as in a macro theory

\[
\nabla \cdot \mathbf{T}^* + \mathbf{A} \cdot \mathbf{q}^* = 0,
\]

\[
D^* \Delta \Delta w = q^*_w + \mathbf{T}^* \cdot \nabla \nabla w
\]

(10)

(7)

(8)

(9)

(11)

\( \Delta \Delta F^* = 0 \),

\( D^* \Delta \Delta w = \mathbf{T}^* \cdot \nabla \nabla w \)

\(\nabla \cdot \mathbf{T}^* + \mathbf{A} \cdot \mathbf{q}^* = 0, \)\n
\( D^* \Delta \Delta w = \mathbf{T}^* \cdot \nabla \nabla w \)

(11)

1.2 Eigenvalue problem for an annular nanoplate

Consider an annular nanoplate with an inner radius \( r_1 \) and outer \( r_2 \), stretched by a pair of the point loading \( P \), free supported on the outer edge and been free on the inner. Then, in the polar coordinate system \( r, \vartheta \) \((x_1 = r \cos \vartheta, x_2 = r \sin \vartheta)\) with the center in the middle of the inner hole, the buckling of the nanoring is described by the generalized Sturm-Liouville problem on eigenvalues \( P \) and eigenforms \( w(r, \vartheta) \)

\[
\begin{cases}
\Delta \Delta F^* = 0, \\
D^* \Delta \Delta w = T^* \cdot \nabla \nabla w
\end{cases}
\]

\( q^*_w = 0 \).

\[
\left\{ \begin{array}{l}
\Delta \Delta F^* = 0, \\
D^* \Delta \Delta w = T^* \cdot \nabla \nabla w
\end{array} \right.
\]

\[
\begin{cases}
\Delta \Delta F^* = 0, \\
D^* \Delta \Delta w = T_{rr} w_{rr} + T_{\vartheta\vartheta} \frac{w_{\vartheta\vartheta}}{r^2} + 2T_{r\vartheta} \frac{w_{r\vartheta}}{r}
\end{cases}
\]

\[
\begin{align*}
T_{rr} \big|_{r=r_1} &= 2\tau_0, & T_{r\vartheta} \big|_{r=r_1} &= 0
\end{align*}
\]
• in-plane – the outer edge is loaded by a pair of the point forces

\[ T_{rr} \big|_{r=r_2} = 2\tau_0 + \frac{P}{\pi r_2} \left( \delta(\vartheta + \pi/2) + \delta(\vartheta - \pi/2) \right), \quad T_{r\vartheta} \big|_{r=r_2} = 0 \quad (12) \]

• flexural – the inner edge is free

\[ M^*_r \big|_{r=r_1} = -D^* \left( w_{rr} + \nu f \frac{w_r}{r} \right) \big|_{r=r_1} = 0, \]

\[ (Q^*_r - M^*_{r\vartheta,\vartheta}) \big|_{r=r_2} = -D^* \left( (\Delta w)_r - (1 - \nu f) \frac{(\Delta_2 w)_{r\vartheta}}{r} \right) \big|_{r=r_1} = 0 \quad (13) \]

• flexural – the outer edge is simple supported

\[ w(r_2, \vartheta) = 0, \quad M^*_r \big|_{r=r_2} = -D^* \left( w_{rr} + \nu f \frac{w_r}{r} \right) \big|_{r=r_2} = 0 \quad (14) \]

Here \( \delta(\vartheta) \) is the Dirac delta function along the edge \( r = \text{const} \) and

\( \Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \vartheta^2}, \quad \Delta_1 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \frac{\partial^2}{\partial \vartheta^2}, \quad \Delta_2 = \frac{1}{r^2} \frac{\partial}{\partial \vartheta} - \frac{1}{r} \frac{\partial}{\partial r} \frac{\partial}{\partial \vartheta} \quad (15) \)

1.3 Potential energy of a nanoplate and Rayleigh-Ritz method for critical Euler force

The potential energy of a nanoplate consists of the potential energy of the bulk phase and facial surfaces [1, 2]

\[ W = \int_{\Omega} + h/2 \int_{\Omega} - h/2 \left( \frac{\lambda}{2} \, tr^2 \varepsilon + \mu \varepsilon : \varepsilon \right) \, dz \, d\Omega + \int_{\Omega} \left( \tau_0 \left( tr \varepsilon_+ + tr \varepsilon_- \right) + \frac{\lambda_0}{2} \left( tr^2 \varepsilon_+ + tr^2 \varepsilon_- \right) + \mu_0 \left( \varepsilon_+ \cdot \varepsilon_+ + \varepsilon_- \cdot \varepsilon_- \right) \right) \, d\Omega \quad (16) \]

As can be shown in [11, 12], the potential energy of a nanoplate (16) can be recombined into the well-known structure of the energy increment of the deformed midplane and of the bending, which has following view in the polar coordinates

\[ \Delta W = \Delta W^{(1)} + \Delta W^{(2)} = \frac{1}{2} \int_{\Omega} \left( T_{rr}^* w^2_{rr} + T_{r\vartheta}^* \frac{w^2_{r\vartheta}}{r^2} + 2T_{r\vartheta}^* \frac{w_r w_{r\vartheta} \vartheta}{r} \right) \, r \, dr \, d\vartheta + \frac{D^*}{4} \int_{\Omega} \left( (1 + \nu f^*) \left( (\Delta w)^2 + (1 - \nu f^*) \left( (\Delta_1 w)^2 + (\Delta_2 w)^2 \right) \right) \right) \, r \, dr \, d\vartheta \quad (17) \]

As can see from the boundary condition (11) and (12), the the membrane forces have two terms, proportional to other physical values \( 2\tau_0 \) and \( P \): \( T^* = 2\tau_0 A + \frac{P}{\pi r_2} \mathbf{T}^* \), where \( \mathbf{T}^* \) is...
dimensionless. Then taking \( r_2 \) for the characteristic linear dimension and believing \( w = r_2 \tilde{w} \), \( \Delta W_i = r_2^2 \Delta W_i \), the linear functional of the energy increment (17) can be recombined as

\[
\Delta W^{(1)}(T^*; w, w) + \Delta W^{(2)}(w, w) = \\
\frac{Pr_2}{\pi} \Delta \tilde{W}^{(1)}(\tilde{T}^*; \tilde{w}, \tilde{w}) + (2\tau_0 r_2^2 \Delta \tilde{W}^{(1)}(A; \tilde{w}, \tilde{w}) + D^* \Delta \tilde{W}^{(2)}(\tilde{w}, \tilde{w})) = \\
D^* \left( \Lambda^s \Delta \tilde{W}^{(1)}(\tilde{T}^*; \tilde{w}, \tilde{w}) + (\Lambda_0 \tilde{W}^{(1)}(A; \tilde{w}, \tilde{w}) + \Delta \tilde{W}^{(2)}(\tilde{w}, \tilde{w})) \right)
\]

(18)

where dimensionless \( \Lambda^s = \frac{Pr_2}{\pi D^*} \) is the sought-for eigenvalue and \( \Lambda_0 = \frac{2\tau_0 r_2^2}{D^*} \) is a constant.

Let the deflection is approximated by a series

\[
\tilde{w}(r, \vartheta) = \sum_i C_i w_i(r, \vartheta)
\]

(19)

each term of which satisfies the kinematic boundary conditions from (13)-(14). Then the potential energy increment (18) is a quadratic form of the coefficients \( C_i \) considering as the generalized Lagrange coordinates

\[
\Delta W = D^* \left( \Lambda^s \sum_i \sum_j \tilde{W}^{(1)}_{ij} C_i C_j + \sum_i \sum_j \tilde{W}^{(2)}_{ij} C_i C_j \right)
\]

(20)

where the coefficients \( \tilde{W}^{(1)}_{ij} \) and \( \tilde{W}^{(2)}_{ij} \) are calculated from (18) for the functionals \( \Delta \tilde{W}^{(1)}(\tilde{T}^*; \tilde{w}, \tilde{w}) \) and \( \Delta \tilde{W}^{(2)}(\tilde{w}, \tilde{w}) \) correspondently.

According to the Rayleigh-Ritz method, the corresponding generalized forces are equal to zero in the equilibrium position

\[
0 = \frac{\partial \Delta W}{\partial C_j} = 2D^* \left( \Lambda^s \sum_i \tilde{W}^{(1)}_{ij} C_i + \sum_i \tilde{W}^{(2)}_{ij} C_i \right) \text{ or } 0 = \Lambda^s \tilde{W}^{(1)}(C_i + \tilde{W}^{(2)} C)
\]

(21)

that is known as a generalized eigenvalue problem in the matrix algebra. The minimal positive eigenvalue \( \Lambda^s_{\min} \) is proportional to the critical Euler force

\[
P_{cr}^s = \frac{\pi D^* \Lambda^s_{\min}}{r_2}
\]

(22)

1.4 Solving the plane problem

A plane problem on an annular macroplate stretched by two point loads has been studied in [14]. We can use this solving for a nanoplate with a little addition. We represent the stresses in the plane as a sum \( \sigma = \sigma^I + \sigma^II \) (Fig. 1), where \( \sigma^I \) are the known solution of the problem about stretching a circle of radius \( r_2 \) by concentrated forces [15]

\[
\sigma^I_{rr}|_{r=r_2} = \frac{P}{8\pi r_2} \left( \delta(\theta - \frac{\pi}{2}) + \delta(\theta + \frac{\pi}{2}) \right), \quad \sigma^I_{r\theta}|_{r=r_2} = 0
\]

(23)

and \( \sigma^II \) are the unknown solution of the problem on loading a ring on the inner edge and free on the outer edge

\[
\sigma^II_{rr}|_{r=r_1} = -\sigma^I_{rr}|_{r=r_1}, \quad \sigma^II_{r\theta}|_{r=r_1} = -\sigma^I_{r\theta}|_{r=r_2}; \quad \sigma^II_{rr}|_{r=r_2} = 0, \quad \sigma^II_{r\theta}|_{r=r_2} = 0
\]

(24)
that have been obtained as the trigonometric series from the polar angle \( \theta \)

\[
\sigma_{rr}^{II} = \sum_{m=0}^{\infty} s_{rr}^{(m)}(r) \cos 2m\theta, \quad \sigma_{r\theta}^{II} = \sum_{m=0}^{\infty} s_{r\theta}^{(m)}(r) \cos 2m\theta, \quad \sigma_{rr}^{II} = \sum_{m=1}^{\infty} s_{rr}^{(m)}(r) \sin 2m\theta
\]

(25)

with the coefficients \( s_{mn}^{(ij)} \) depending on the polar radius \( r \) in the polynomial form

\[
s_{mn}^{(ij)}(r) = A_{mn}^{(ij)} r^{2m} + B_{mn}^{(ij)} r^{2m-2} + C_{mn}^{(ij)} r^{-2m} + D_{mn}^{(ij)} r^{-2m-2}
\]

(26)

Finally for the same annular nanoplate, the effective membrane forces are expressed as the sum

\[
T^* = 2\tau_0 A + h(\sigma^I + \sigma^{II}) = 2\tau_0 A + \frac{P}{\pi r_2} T^*
\]

(27)

2 NUMERIC RESULTS

The sought-for deflection function \( w(r, \theta) \) is approximated by the sum

\[
\tilde{w}(r, \theta) = \sum_{i=1}^{\infty} \left( \sum_{j} c_{ij} (r - r_2) \right)^i \begin{cases} \cos(j \theta), & j \geq 0 \\ \sin(j \theta), & j < 0 \end{cases}
\]

(28)
The each term of the series (28) satisfies the first condition (14), which is a only kinematic. The rest boundary conditions, both (13) and the second one in (14) are static and satisfied naturally in the potential energy.

Figure 3: Comparison of the behavior of relative minimal dimensionless $\lambda_{\text{min}}$ and end-critical force $P_{\text{cr}}$ for a nano- (marked by *) and macro- annular plate depending on the thickness $h$ for Al[111] and Al[100]

The relative values of the minimal dimensionless $\lambda_{\text{min}}$ and end-critical force $P_{\text{cr}}$ are shown in Fig. 3 for the aluminum with the elastic moduli for the bulk phase $\lambda = 58.2$, $\mu = 26.1$ GPa and on the facial surfaces:

- $\text{Al}[111]$: $\lambda_s = 6.85$, $\mu_s = -0.38$, $\tau_0 = 0.91$ N/m;
- $\text{Al}[100]$: $\lambda_s = 3.49$, $\mu_s = -5.43$, $\tau_0 = 0.57$ N/m.

Like the macro- ring, the nano- ring with the big inner radius is buckling in the symmetric form Fig. 2 (a) and with the small one – in the antisymmetric form Fig. 2 (b).
A size effect is revealed because of the several reasons. One of them is the redistribution of the bending energy (17), which is reflected in the behavior of the relative minimum eigenvalue under \( \tau_0 = 0 \) in Fig. 3 (a) and (b). Here, the size effect is manifested in a decrease in the stiffness of a nano-ring with a decrease in thickness.

The influence of the surface pretension is more strongly and shown also in the behavior of the relative minimum eigenvalue under \( \tau_0 \neq 0 \) in Fig. 3 (c) and (d). It can lead both to a decrease in the stiffness with a decrease in thickness of the nano-ring with the big inner opening and to its decrease for the nano-ring with the small inner opening in the case of Al[111].

Finally, the relative end critical force is shown in Fig. 3 (e) and (f). They are similar to Fig. 3 (c) and (d), but differ in the overlay on them of the behavior of the relative flexural stiffness \( D^*/D \). This leads to an even greater increase in the stiffness of the nano-ring with a decrease in thickness in the case of Al[100] or to its decrease in the case of Al[111].

3 CONCLUSIONS

- The considering plate bending theory taking into account the surface stresses reduces to redefining the elastic moduli through their effective analogs.

- This theory allows to study the elastic and flexural behaviors of nanoplates like as macroplates.

- On the example of buckling a stretched nano-ring, it was shown the size effect. It manifests itself in that the critical load is significantly affected by three factors: relative change in the magnitude of the dimensionless minimal eigenvalue, relative change in the magnitude of the flexural stiffness (both depend on the thickness), and surface pretension. The overlay of three factors determines the end-critical force.

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FREE VIBRATIONS OF ANNULAR CIRCULAR AND ELLIPTIC PLATES

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Abstract. The aim of this research is to study free transverse vibrations of an elliptic solid thin plate with a varying eccentricity and an elliptic plate with an elliptic cutout with the same eccentricity. Both analytical and numerical methods are used to examine the problem. For plates with small eccentricity and/or small central elliptic cutout asymptotic methods are used to describe the effect of the eccentricity and cutout on the vibrational spectra.

For a circular plate a vibrational mode can be expressed by one particular solution of the differential equation governing vibrations, for an elliptic plate all appropriate particular solutions should be summed up to express even one vibrational mode. That makes the problem much more difficult.

The main aim of the research is to follow what happens to the spectrum of transverse vibration frequencies of a solid circular plate with increasing of the central hole area and/or the plate eccentricity. Frequencies and modes were also found by finite element method using the package ANSYS 18.1. The results were compared with those obtained by other authors who used Rayleigh-Ritz and other numerical and analytical methods.
1 INTRODUCTION

Circular and elliptic solid and annular thin plates of constant thickness are widely used in actual engineering structures and devices. Such models are also may be applied to describe some biological structures, for example, tiny elements of a human eye. The study of vibrational properties of such structures is an important part of any engineering analysis. While the transverse vibrations of solid and annular circular thin plates are examined in details [7] and [4], the spectra of solid and annular elliptic plates were studied not so often. To examine vibrations of elliptic plates the analytical approach were used in [10], [8] and [9] when the solutions were found using the series in Mathieu functions. Numerical methods such as Raleigh-Ritz method and FEM were used in [5] and [3].

In our research we intend to answer the question: what happens to the vibration spectrum of a solid circular plate of constant thickness, if i) it is transformed to elliptic plate with saving its area (and, therefore, mass) and/or ii) the area of the similar (of same eccentricity) central hole increases. Due to Raleigh theorem the increasing of only the eccentricity must lead to the growth of the frequencies, since the system becomes more stiff, but it mass is constant. At the same time with the hole the stiffness of the system becomes smaller and simultaneously the mass goes down. The play of the above factors makes the behavior of the spectra of annular elliptic plates rather complex.

2 SPECTRA OF ANNULAR CIRCULAR PLATES

Consider free transverse vibrations of thin circular or annual plates of radius \( R \). The equation for the plate deflection \( W(\tilde{r}, \varphi) \) is [7]

\[
D \Delta \Delta W(\tilde{r}, \varphi) - \rho h \omega^2 W(\tilde{r}, \varphi) = 0, \quad D = \frac{Eh^3}{12(1-\nu^2)}
\] (1)

where \( D \) is the plate stiffness, \( E \) — Youngs modulus, \( h \) — plate thickness, \( \nu \) — Poissons ratio, \( \rho \) — plate density, \( \Delta \) — Laplace operator, \( \omega \) is the natural frequency and \( \tilde{r} \) and \( \varphi \) are the polar coordinates of the point on the neutral plate surface.

Considering the plate with constant thickness and density we separate variables in the form \( W = \sum_m w_m(\tilde{r}) \cos(m\varphi) \). Then introducing non-dimensional frequency \( \lambda = R(\omega^2 \rho h / D)^{1/4} \) and radius \( r = \tilde{r} / R \) we come to the equation

\[
\Delta\Delta w - \lambda^4 w = 0, \quad \Delta w = \frac{1}{r} \frac{d}{dr} r \left( \frac{d w}{d r} \right) - \frac{m^2}{r^2} w
\] (2)

where subscript \( m \) is omitted. Later we examine the plate with the clamped edge, i.e. with the boundary conditions

\[
w(R) = w'(R) = 0
\] (3)

The edge of the central circular hole is free, i.e.

\[
M(\varepsilon) = Q(\varepsilon) = 0, \quad M = w'' + \nu(w' / r - m^2 / r^2 w), \quad Q = m^3 / r^3 (3 - \nu) w - w'/r^2 + (\nu - 2) m^2 / r^2 w' + w'' / r + w'''
\] (4)

where \( Q \) is the transverse force, \( M \) — the bending moment and \( \varepsilon \) is the hole radius. For the solid plate the last boundary conditions are replaced with the boundedness conditions at the center.
The general solution of equation (1) is the linear combination of Bessel functions \(J_m(\lambda), Y_m(\lambda)\) and modified Bessel functions \(I_m(\lambda), K_m(\lambda)\) of the first and the second kind. Substituting solution into the boundary conditions (2–3) we come to the characteristic equation

\[ P(\lambda; m, \nu) = 0 \]

from which two parametric set of frequency parameters \(\lambda_{m,n}\) may be found. In the general case the above equations may be solved only numerically, the results are included, for example, in [7],[4]. Here is the plot of the first 10 functions \(\lambda_{m,n}(\varepsilon)\), where \(m\) and \(n\) are the wave numbers in the circumferential and radial directions respectively.

Consider the case, when the non-dimensional radius of the central hole is a small value \(\varepsilon \ll 1\). To use the perturbation method we represent the frequency parameter in the form \(\lambda = \lambda_0 + \delta(\varepsilon) + \cdots\), where \(\lambda_0\) is the frequency parameter for the solid plate and \(\delta \ll 1\). Substituting \(\lambda\) in equation (1) and using the asymptotic approximations for Bessel functions at zero [1] we come to the characteristic equation

\[ P(\varepsilon, \delta, \lambda_0, m) = 0. \]  

The main (in \(\varepsilon\)) term of (5) provides equation to find \(\lambda_0\)

\[ J_m(\lambda_0)I_{m+1}(\lambda_0) + I_m(\lambda_0)J_{m+1}(\lambda_0) = 0, \]

the roots of which converges to \(\pi(m/2 + n)\) as \(n \to \infty\). The next term gives expression for \(\delta\). The asymptotic expansion of \(\delta\) at zero contains power and logarithmic functions, such that

\[ \delta = \alpha\varepsilon^2 + \beta\varepsilon^4 + \gamma\varepsilon^4 \log \varepsilon + \cdots. \]

The formulas for \(\alpha, \beta, \gamma\) were obtained with Mathematica11.0, they are rather complex. Here we write only expression for \(\alpha\)

\[
\alpha = -\frac{A_1}{A_2}, \quad A_1 = \lambda_0(\lambda_0\nu(2J_1(\lambda_0)K_0(\lambda_0) - K_1(\lambda_0)J_0(\lambda_0)) + \pi(I_1(\lambda_0)Y_0(\lambda_0) - Y_1(\lambda_0)I_0(\lambda_0) - 4); \\
A_2 = 2(\nu - 1)(2I_0(\lambda_0)J_0(\lambda_0) + I_2(\lambda_0)J_0(\lambda_0) + I_0(\lambda_0)J_2(\lambda_0))
\]
In Fig. 2 and Fig. 3 the exact values of the function $\lambda(\varepsilon)$ are compared with the asymptotic values.

Figure 2: Function $\lambda(\varepsilon)$ for $m = 0$ and a) $n = 1$, b) $n = 2$. Exact values — red, $\lambda_0 + \delta$ — dark blue, 2 terms representation — green, 4 terms representation — blue.

Figure 3: Function $\lambda(\varepsilon)$ for $n = 1$ and a) $m = 1$, b) $m = 2$. Exact values — red, $\lambda_0 + \delta$ — dark blue, 2 terms representation — green, 4 terms representation — blue.

For all $m$ and $n \alpha < 0$ and therefore $\lambda'(0) < 0$. The four term approximation describes well the behavior of the function for small $\varepsilon$ particularly its initial fall and then fast rise, but for larger $\varepsilon$ the formulas become inapplicable. Representation (6) is valid for any $m$, but with some specific for $m = 1$, for which $\alpha = 0$. It’s explained by the different forms of asymptotic expansions of Bessel functions for different $m$, say, $Y_m(x) \sim 2/\pi \log(x/2)$ for $m = 0$ and $Y_m(x) \sim -\Gamma(m)/\pi(2/x)^m$ for $m > 0$ as $x \to 0$.

To answer the question, for which hole radius the natural frequency attains the minimum, one should solve the equation $d\lambda(m, n, r)/dr = 0$. For both exact characteristic equation and
its approximations their solutions may found only numerically. It appears that for the fixed value of the wave number in the circumferential direction $m$ the value of the hole radius, for which the frequency $\lambda$ is minimal, decreases with the increasing of the wave number $n$ in the radial direction (see Fig. 4a). While in the case of the fixed wave number in the radial direction the value of the hole radius, for which the frequency $\lambda$ is minimal, increases together with the wave number in the circumferential direction (see Fig. 4b).

![Figure 4: a) Hole radius, for which the free frequency is minimal, vs. wave number $n$ (for fixed $m = 0$), b) Hole radius, for which the free frequency is minimal, vs. wave number $m$ (for fixed $n = 1$),](image)

Before we come to analysis of annular elliptic plates it’s useful to remind that for $m = 0$ all solid and annular circular plate vibration frequencies are simple, but for $m > 0$ they are double with the corresponding modes $w(r) \sin(m\varphi)$ and $w(r) \cos(m\varphi)$.

3 SPECTRA OF ANNULAR ELLIPTIC PLATES

Consider now vibrations of the elliptic plate with the similar central cut-out, i.e. such the eccentricities ($e$) of the elliptic plate and elliptic cut-out are equal. The semi-axes of the plate and the cut-out are $(a, b)$ and $(a_1, b_1)$ respectively ($\kappa = a/b = a_1/b_1$). After introducing elliptic coordinates the governing equation was delivered in [7], [10], [8] and analytical solution was obtained in Mathieu functions. Here we apply FEM to study the effects of eccentricity and hole size on the vibration spectra of transverse vibrations of an elliptic plate.

We start with solid elliptic plate with the clamped edge. Consider elliptic plates of equal areas, but with different eccentricity. Using software package ANSYS the dependence of lower vibration frequencies on excentricity is found and plotted in Fig. 5 The results well agree with those obtained by other researchers by different methods: with FEM [5], with Rayleigh-Ritz method [6] and using Mathieu functions [10]

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Table 1: Fundamental frequency of the solid elliptic plate for different values of the ratio $a/b$.

The frequencies of the solid circular plate are plotted for $e = 0$ and marked with wave numbers $(m, n)$. For $e > 0$ the plate loses its rotations symmetry and frequencies those are double
for the circular plate (for $m > 0$) split into $\lambda_{m,n}^+$ and $\lambda_{m,n}^-$. Both frequencies increases with $e$ but with different rate. Two vibrational modes $w_{m,n}\sin(\varphi)$ and $w_{m,n}\cos(\varphi)$, which are orthogonal for the circular plate, transform to $w^+(r, \varphi)$ and $w^-(r, \varphi)$. One mode becomes stretched along the large semi-axis, the other — along small semi-axis. The frequencies corresponding to the second mode grows faster with $e$ (see Fig.6). Frequencies of axisymmetric vibrations stay unsplitted for $e > 0$.

For the radially symmetric plate (solid circular or annular circular) the variables are separable and any vibration mode is determined by 2 wave numbers. For the elliptic plate any vibration mode is the sum of the infinite series in $m$ and $n$, its number $(m, n)$ means that the main term
corresponds to the $w_{m,n}$-mode of the circular plate. The splitted modes $w^+(r, \varphi)$ and $w^-(r, \varphi)$ may be rather unsimilar (see Fig. 7)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{vibration_mode}
\caption{Vibration mode $w_{3,1}$ for circular plate and $w^+_{3,1}$ and $w^-_{3,1}$ for elliptic plate}
\end{figure}

It appears that the splitting coefficient for the frequencies practically linearly depends on the eccentricity for small $e$. Indeed, in Fig. 8 the dependence of frequencies on the ratio $\kappa = a/b$ is plotted with almost straight lines. For the frequency $(1,1)$ formulas $\lambda^+_{1,1} = 4.60514 + 3.62924 a/b$ and $\lambda^-_{1,1} = 4.60229 + 1.38493 a/b$ give good approximation for $a/b < 1.5$, i.e. for $e < 0.75$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{dependence}
\caption{Dependence of lower vibration frequencies for small values of the ratio $a/b$}
\end{figure}

Now consider annular elliptic plate. Since we wish to trace what happens to the frequencies of the solid circular plate when both area of the central hole and the eccentricity increase we do it in the following two ways. Firstly, fix some eccentricity, say, $e = 0.8$ and than increase the hole area. As a parameter we select $\sqrt{a_1 b_1}$, which equal to radius of the circular hole for $e = 0$. For $a_1 b_1 = 0$ we have the spectrum of the solid elliptic plate, for which the sequence of splitted frequencies

$$\{(0, 1), (1, 1)^-, (2, 1)^-, (1, 1)^+, (2, 1)^+, \ldots\}$$

differs from that of the circular plate

$$\{(0, 1), (1, 1), (2, 1), (0, 2), (3, 1), \ldots\}$$

The frequencies $\lambda_{0,n}$ and $\lambda_{m,n}^-$ increase monotonously with the hole area. The behavior of frequencies $\lambda_{m,n}^-$ is more difficult since they have minimum at some points $\sqrt{a_1^2 b_1^2}$ (see Fig. 9).
Finally, consider annular circular plate $(e = 0)$ with clamped hole area, say, $\sqrt{a_1b_1} = 0.6$ and start to increase the eccentricity. Most of the lower frequencies stay constant for some small values of $e$ then split. After that frequencies $\lambda_{m,n}^+$ go up monotonously. The behavior of the frequencies $\lambda_{m,n}^-$ is more sophisticated, some of them increase monotonously, some of them slightly decrease after splitting, attain the minimum and then start to grow. The frequency $\lambda_{1,1}$ appears to be the most sensitive to the value of the eccentricity.

4 CONCLUSIONS

For elliptic plates of constant thickness with the clamped edge the effect of the eccentricity is the following: i) all frequencies increase with the eccentricity, but with different rate, ii) double frequencies of non-axisymmetric vibrations of a circular plate split into two, the lower frequency corresponds to the mode stretched along the long semi-axis, the effect on the frequencies for small values of the axis ratio (up to 1:2) is linear with the high accuracy, iii) lower frequencies go up with the eccentricity faster, as a result for some values of the eccentricity the mode order changes.

For annular elliptic plate with the clamped edge the effect of the hole area is the following: i) the frequencies of axisymmetric vibrations and one of the splitted frequency corresponding to the mode stretched in the direction of the minor semi-axis monotonously increase with the hole area, ii) the behavior of the second splitted frequency as the hole area goes up is complex depending on the wave numbers and eccentricity.

All said above is valid for the specific boundary conditions, free hole edge and clamped outer plate edge. For example, for the clamped hole edge all frequencies increase monotonously with the hole area. So, the effect of the boundary conditions requires special consideration. Also the effect of a non-similar hole, when its eccentricity differ from the eccentricity of a plate, briefly discussed in [11] and [2] should be examined in details.

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UNSYMMETRICAL BUCKLING OF ORTHOTROPIC ANNULAR PLATES AND SPHERICAL CAPS UNDER INTERNAL PRESSURE

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Abstract. The work is concerned with asymmetric buckling of clamped nonuniform orthotropic shallow spherical shells under internal pressure. The effect of degree of orthotropy and material inhomogeneity, the curvature of a shell and the radius of the circular hole in the center of a shell on the buckling load is examined.

A mesh-like structure in a human eye called Lamina Cribrosa (LC) may be modeled with such shell. The LC is a part of sclera, where the optic nerve fibres pass through and where the layer of sclera becomes thinner and many little pores appear. The buckling of the LC in the non-axisymmetric state in the neighborhood of the edge could cause edemas and folds at the periphery of the LC, atrophy of the optic nerve fibres and the eventual loss of the sight. The non-symmetric part of the solution is sought in terms of multiples of harmonics in angular coordinate. The numerical method is employed to evaluate the lowest value of the load, which leads to the appearance of waves in the circumferential direction. It is shown that if the modulus of elasticity decreases away from the center of the shell, the critical internal pressure for non-symmetric buckling is significantly lower than for a shell with constant mechanical properties and the number of waves in the circumferential direction increases with the degree of non-uniformity.
1 INTRODUCTION

Panov and Feodos’ev were the first who formulated and studied the problem of unsymmetrical buckling of a thin circular isotropic plate under normal pressure [1]. They represented nonaxisymmetric displacement in the form 
\[ w = (1 - r^2)^2(A + Br^4 \cos n\theta) \]
and examined the bending problem by Galerkin procedure. Later, Cheo and Reiss analyzed axially unsymmetric equilibrium states of a clamped circular plate subjected to a normal surface load [2]. They emphasized that the approximation function with two unknown parameters was "too inaccurate to adequately describe the wrinkling of the plate" and assumed that Panov and Feodos’ev had found unstable unsymmetric state in [1].

The postbuckling asymmetrical behaviour of annular plates and the dependence of the critical load and number of waves on the boundary conditions, the ratio between the inner and outer radii were considered in [3]. Experiments on uniform heating of thin circular plate with fixed edges and formation of waves near plate’s edge are discussed in [4].

Coman analyzed the wrinkling of circular plates and shallow spherical caps in [5, 6]. Two-term asymptotic predictions for the buckling pressure and one-term approximations of the corresponding wave number were derived and comparisons of the asymptotic approximations with numerical calculations was reported.

Unsymmetric equilibrium states of inhomogeneous circular plates under normal pressure were discussed in [7]. Such a plate or a cap can be used as the simplest model of the Lamina Cribrosa (LC) in the human eye [8]. The LC is a part of sclera, where the optic nerve fibres pass through and where the layer of sclera becomes thinner and many little pores appear. The buckling of the LC in the non-axisymmetric state in the neighborhood of the edge could cause edemas and folds at the periphery of the LC, atrophy of the optic nerve fibres and the eventual loss of the sight.

In this study we discuss with buckling of an orthotropic annular plates and spherical caps under internal pressure.

2 PROBLEM STATEMENT

Let us consider a thin shallow spherical elastic shell of uniform thickness \( h > 0 \) with a small circular opening at the top. The shell mid-surface can be represented by \( \zeta = H(1 - r^2/a^2) \), where \( a \) is the base radius, \( H \) is the rise of the middle surface at the center. The ratio of the shell thickness to its radius of curvature \( R = a^2/(2H) \) is much less than unity \( (h/R \ll 1) \), and the apex rise is much less that the curvature radius \( (H \ll R) \). The shell is subjected to uniformly distributed inner pressure \( p \).

We introduce the orthogonal coordinate system \((r, \theta, z)\), in which \( r \) and \( \theta \) are polar coordinates in the base plane and \( z \) is the distance along the normal to the mid-surface.

The stress–strain relations for the orthotropic shell under the assumption of the classical theory are given by
\[
\begin{align*}
e_{rr} &= \frac{\sigma_r}{E_r} - \frac{\nu_r}{E_r} \sigma_\theta, \\
e_{\theta\theta} &= -\frac{\nu_\theta}{E_\theta} \sigma_r + \frac{\sigma_\theta}{E_\theta}, \\
e_{r\theta} &= \frac{\sigma_{r\theta}}{G_{r\theta}},
\end{align*}
\]
with Young’s moduli in the \( r \) and \( \theta \)-directions \( E_r, E_\theta \), respectively, the Poisson ratios \( \nu_r, \nu_\theta \), the shear modulus in \( r-\theta \) direction \( G_{r\theta} \). The equality \( E_r \nu_\theta = E_\theta \nu_r \) must be satisfied due to the symmetry.

The fundamental equations can be written in the form [9]
\[
D_r L_w(w) = P + L(w, F) - \Delta F/R,
\]
\[
L_F(F)/(hE_\theta) = -L(w, w)/2 + \Delta w/R,
\]

where \(w(r, \theta)\) is the displacement in the \(z\) direction, \(F(r, \theta)\) is the Airy stress function, \(D_r = E_r h^3/12(1-\nu_r\nu_\theta)\) is the bending stiffness. The Laplacian is defined as

\[
\Delta = \left(\frac{\partial}{\partial r}\right)^2 + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2},
\]

with the short-hand notations \(\frac{\partial}{\partial r} \equiv \partial( )/\partial r, \frac{\partial}{\partial \theta} \equiv \partial( )/\partial \theta\). The differential operators \(L_w, L_F, L\) are listed in Appendix.

We rescale Eqs. (2) by introducing dimensionless quantities as

\[
\begin{align*}
  r^* &= \frac{r}{a}, \quad w^* = \beta w/h, \quad P^* = \beta^3 P a^4/E_r h^4, \quad F^* = \beta^2 \frac{F}{E_r h^3}, \\
  A &= \beta \frac{a^2}{R h}, \quad \beta^2 = 12(1-\nu_r\nu_\theta).
\end{align*}
\]

Dropping asterisks for notational simplicity, we obtain the dimensionless forms of Eqs. (2)

\[
L_w(w) = P + L(w, F) - A\Delta F,
\]

\[
L_F(F)/\lambda = -L(w, w)/2 + A\Delta w.
\]

We assume that the outer edge of the shell is clamped but can move freely in the radial direction without rotation. The inner edge of the shell is assumed to be supported by a roller which can slide along a vertical wall. These constrains can be written as

\[
\begin{align*}
  w &= w' = N_r = N_r\theta = 0 \quad \text{for} \quad r = 1, \\
  w' &= 0, \quad N_r\theta = 0, \quad (rM_r)' - M_\theta + 2\dot{M}_r\theta = 0, \quad u = 0 \quad \text{for} \quad r = \delta.
\end{align*}
\]

Here \(r = \delta = a_{in}/a\) is a dimensionless radial coordinate of the inner edge, \(u\) denotes the horizontal radial components of displacement, \(M_r, M_\theta, M_r\theta\) are meridional, circumferential and twisting moments, respectively, and \(N_r, N_r\theta\) are stress resultants. In the terms of the displacement component \(w\) and stress function \(F\), the dimensionless boundary conditions (5) are equivalent to

\[
\begin{align*}
  \text{for} \quad r = 1 & \quad w = w' = F'/r + \dot{F}/r^2 = -\left(\frac{\dot{F}}{r}\right)' = 0, \\
  \text{for} \quad r = \delta & \quad w' = 0, \quad \dot{F}/r^2 - \ddot{F}/r = 0, \\
  & \quad w'''' + \frac{\dot{w}'''}{r} - \frac{\lambda}{r^2} \left( w''' + \frac{\ddot{w}}{r} \right) - \frac{2\lambda r_\theta - \nu_\theta}{r^2} \left( \frac{\ddot{w}}{r} - \frac{w'}{r^2} \right) = 0, \\
  & \quad F'''' + 2\frac{F'''}{r} - \frac{\lambda}{r^2} \left( F' + \frac{\ddot{F}}{r} \right) - \frac{\nu_\theta}{r^2} \left( F + \dot{F} \right)' = -A \frac{w}{r},
\end{align*}
\]

Following [2, 10], we look for solutions of problem (4, 5) of the form

\[
\begin{align*}
  w(r, \theta) &= w_s(r) + \varepsilon w_{ns} \cos(n\theta), \quad F(r, \theta) = F_s(r) + \varepsilon F_{ns} \cos(n\theta),
\end{align*}
\]

where \(w_s(r), F_s(r)\) describe prebuckling axisymmetric state, \(\varepsilon\) is infinitesimal parameter, \(n\) is a mode number and \(w_n(r), F_n(r)\) are the non-symmetrical components.
Substituting (7) in Eqs. (4) and (6), one can obtain the governing equations of the symmetrical problem

\[ w''_s + \frac{w'_s}{r} - \lambda \frac{w_s}{r^2} = \frac{Pr}{2} + \frac{w'_0}{r} + A\Phi_0 + \frac{C}{r}, \]

\[ \Phi''_0 + \frac{\Phi'_0}{r} - \lambda \frac{\Phi_0}{r^2} = \lambda \left( \frac{(w'_s)^2}{2r} + Aw'_s \right), \]

with boundary conditions

\[ w_s = w'_s = \Phi_0 = 0 \quad \text{for} \quad r = 1, \]

\[ w'_s = 0, \quad \Phi'_0 - \frac{\nu\theta}{r} \Phi_0 + \lambda Aw_s = 0 \quad \text{for} \quad r = \delta, \]

and

\[ C = -P\delta^2/2 + A\delta\Phi_0(\delta), \quad \Phi_0 = F'_s. \]

After substitution of (7) in (4), using Eqs. (8), and linearization with respect to \( \varepsilon \) we obtain

\[ L^n_w(w_n) = -A\Delta_n F_n + \frac{w''_n}{r} \Phi_0 + \frac{F''_n}{r} \Theta_0 + \Theta'_0 \left( \frac{F'_n}{r} - \frac{n^2}{r^2} F_n \right) - \Phi'_0 \left( \frac{w'_n}{r} - \frac{n^2}{r^2} w_n \right), \]

\[ L^n_F(F_n) = \lambda^2 \left( A\Delta_n w_n - \frac{w''_n}{r} \Theta_0 - \Theta'_0 \left( \frac{w'_n}{r} - \frac{n^2}{r^2} w_n \right) \right), \]

where \( \Theta_0 = w'_s \) and operators \( L^n_w, L^n_F \) are as defined in Appendix.

Boundary conditions (6) are reduced to

\[ \text{for} \ r = 0 \quad w_n = w'_n = F_n = F'_n = 0 \]

\[ \text{for} \ r = \delta \quad w'_n = 0, \quad F'_n - r F_n = 0, \]

\[ w''_n + \frac{w''_n}{r} - \frac{\lambda}{r^2} \left( w'_n - \frac{n^2}{r^2} w_n \right) + \left( 2\lambda r \theta - \nu \theta \right) \frac{n^2}{r^2} \left( \frac{w'_n}{r} - w'_n \right) = 0, \]

\[ F''_n + 2 \frac{F''_n}{r} - \frac{\lambda}{r^2} \left( F'_n - \frac{n^2}{r^2} F_n \right) + \nu \theta \left( n^2 - 1 \right) \frac{F'_n}{r^2} + \lambda A w_n = 0, \]

Equations (10) with boundary constrains (11) constitute an eigenvalue problem, in which the parameter \( p \) appears implicitly through the functions \( \Theta_0 \) and \( \Phi_0 \). To solve nonlinear axisymmetric problem (8)-(9) we used the shooting method via standard MATLAB functions. The finite difference method was employed to obtain the buckling value of \( P \), for which Eqs. (10) with (11) have nontrivial solution [2, 10].

3 NUMERICAL RESULTS

As described in [1, 2], a ring of large circumferential compressive stress, developed near the edge of the shell or the plate, indicates the possibility of buckling about the axisymmetric state into an unsymmetric equilibrium state. The elastic moduli ratio \( \lambda = E_\theta/E_r \) impacts on the intensity of the compressive stress and the width of the compressive ring (see Fig. 1).

Comparisons between the critical load for different values of the orthotropy rate \( \lambda \) are presented in Fig. 2 (left). The lines with square markers correspond to the isotropic annular plates \( (A = 0) \) and shallow truncated shells \( (A = 5) \). The lines with circular and asterisks markers
Figure 1: Dimensionless circumferential stress resultant $T_\theta$ for different values of $\lambda = E_\theta/E_r$; here $A = 5$, $\delta = 0.1$, $\nu_\theta = 0.4$, $\lambda_r = \lambda$. The value of $\lambda = 1$ corresponds to an isotropic cap.

illustrate results for the orthotropic shells. The calculated load $P_{cr}$ is normalized to buckling pressure for a uniform circular plate $P_{cr}^{pl} = 64453$.

The critical values of $P_{cr}$ increase by a factor of 4 when the orthotropy rate $\lambda$ increases from 1/2 to 2. The dependence of the $P_{cr}$ on the opening radius is relatively weak: the buckling load $P_{cr}$ increases by 30% while the opening radius $\delta$ varies from 0.05 to 0.25 (see Fig. 2 (left)).

Figure 2 (right) illustrates dependence of the normalized critical load $P_{cr}$ on the critical mode number $m$ for a truncated shallow spherical shell. The normalized buckling pressure $P_{cr}/P_{cr}^{pl}$ increases as the radius of the opening $\delta$ increases, while the buckling mode $m$ has opposite behavior: it decreases when $\delta$ increases.

4 CONCLUSION

The wrinkling of the annular plates and shallow spherical shells subjected to internal pressure has been studied in this work. Prebuckling stress-state in a narrow zone near the shells edge makes a major contribution to the unsymmetrical buckling mode and the value of the critical load. It is shown that the buckling load and corresponding mode number increase as the shallowness parameter grows. The critical load increases as the ratio of the circumferential to radial elastic modulus increases. For a truncated shallow shell the wrinkling pressure increases as the radius of the opening increases, while the buckling mode decreases.

5 ACKNOWLEDGMENTS

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Appendix

The differential operators that appear in (2) are defined by

\[ L_w(y) = y''' + 2 \frac{y''}{r} + 2 \frac{\lambda r \theta}{r^2} \left( y'' - \frac{y'}{r} + \frac{\ddot{y}}{r^2} \right) - \frac{\lambda}{r^2} \left( y'' - \frac{y'}{r} - \left( 2 \frac{y'}{r^2} + \frac{\ddot{y}}{r^2} \right) \right), \]

\[ L_F(y) = y''' + 2 \frac{y''}{r} + \frac{\kappa}{r^2} \left( y'' - \frac{y'}{r} + \frac{\ddot{y}}{r^2} \right) - \frac{\lambda}{r^2} \left( y'' - \frac{y'}{r} - \left( 2 \frac{y'}{r^2} + \frac{\ddot{y}}{r^2} \right) \right), \]

\[ L(x, y) = x'' \left( \frac{y'}{r} + \frac{\ddot{y}}{r^2} \right) + y'' \left( \frac{x'}{r} + \frac{\ddot{x}}{r^2} \right) - 2 \left( \frac{\dot{x}}{r} \right) \frac{x'}{r} \]

where

\[ \lambda = \frac{D_\theta}{D_r}, \quad \lambda \theta = \frac{D_{r \theta}}{D_r}, \quad \kappa = \frac{E_\theta}{G_{r \theta}} - 2 \nu_\theta = \frac{\lambda - \lambda_\theta \nu_\theta}{\lambda_\theta - \nu_\theta}, \]

\[ D_j = \frac{h^3 E_i}{12(1 - \nu_r \nu_\theta)} (i = r, \theta), \quad D_k = \frac{h^3}{12} G_{r \theta}, \quad D_{r \theta} = D_r \nu_\theta + 2 D_k. \]

For an isotropic shell we have \( E_r = E_\theta, \nu_r = \nu_\theta, G_{r \theta} = E_c/2(1 + \nu_\theta), \lambda = \lambda_{r \theta} = 1, \kappa = 2, \)

and the operators \( L_w, L_F \) are

\[ L_w(y) = L_F(y) = \Delta \Delta y. \]

After separation of variables the linear differential operators take the form

\[ L_w^n(y) = y''' + 2 \frac{y''}{r} - 2 \lambda \theta \frac{n^2}{r^2} \left( y'' - \frac{y'}{r} + \frac{y}{r^2} \right) - \frac{\lambda}{r^2} \left( y'' - \frac{y'}{r} - \frac{n^4 - 2n^2}{r^2} y \right), \]

\[ L_F^n(y) = y''' + 2 \frac{y''}{r} - \frac{\kappa n^2}{r^2} \left( y'' - \frac{y'}{r} + \frac{y}{r^2} \right) - \frac{\lambda}{r^2} \left( y'' - \frac{y'}{r} - \frac{n^4 - 2n^2}{r^2} y \right). \]
REFERENCES


CHANGES IN THE STRESS-STRAIN STATE OF THE CORNEASCLERAL SHELL UNDER APPLANATION BY A VACUUM RING

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Abstract

Research is conducted into variation in the stress-strain state of the corneoscleral shell of the human eye under loading by a vacuum compression ring with variation of vacuum level. A three-dimensional finite-element model of the contact problem of loading of the corneoscleral shell in the COMSOL program package is presented. Cornea and sclera are modeled as conjugated transversely isotropic spherical shells. The cornea is modeled as a multilayer shell with variable thickness in which all modeled layers have their own individual elastic properties. The research deals with the numerical calculation of variety of vacuum level up to 550 mm Hg.

Keywords: Intraocular Pressure, Corneoscleral Shell, Vacuum Compression Ring, LASIK, FE Modeling.
1 INTRODUCTION

The study of intraocular pressure (IOP) is important, as the level of IOP is a fundamental indicator in the diagnosis of various eye diseases and anomalies. It is well-known fact that short-term increase in IOP is a natural physiological phenomenon [1]. When a person squints the IOP increased to 50 mm Hg, or rubs eyes with hand IOP can rise to 150-250 mm Hg.

To determine intraocular pressure (IOP) special instruments – tonometers are used. Tonometry measures IOP by determining reaction of the cornea to the pressure applied. That is, IOP is judged by the reaction of the eye’s structures for application of the tonometer. Pressure, thus obtained, is called a tonometric pressure. It is clear that tonometers give only approximate value of the true pressure. In the early 21st century a new period of development of tonometry has appeared, which is associated with the rapid development of refractive surgery, which significantly changes the curvature and thickness of the cornea. The IOP readings before and after vision correction surgery are significantly different.

LASIK (Laser-Assisted in Situ Keratomileusis) is today one of the most effective methods of vision correction. The essence of the operation firstly is cut out a corneal flap sufficient for the operation of vision correction. Then the laser ablation is performed by narrowly directed pulses on the main layer of the cornea; this process is controlled by a computer in accordance with the required profile of the cornea [2]. Vacuum compression ring (vacuum ring) is used during the refractive surgery (LASIK) for vision correction in purpose of better formation of the corneal flap. Vacuum ring allows increasing the intraocular pressure to values above 65 mm Hg, by applying vacuum from 520 to 550 mm Hg [3]. The increase of IOP to these values makes flap creation easier and more accurate layer-by-layer corneal incision during refractive surgery and allows improving the accuracy of vision correction.

Because of this, problem concerning applanation of vacuum compression ring during refractive surgery and the creation of models enabling the influence of various parameters of the eye on IOP to be estimated have once again come to the fore.

2 MATHEMATICAL MODEL

A Finite element (FE) model for estimating changes in the stress-strain state of multilayer corneoscleral shell under pressure of vacuum compression ring is performed in the software package of finite-element modeling COMSOL.

The corneoscleral shell is modeled as two spherical segments with variable thickness corresponding to the sclera and cornea. The cornea is divided into four layers: epithelium, Bowman’s membrane (BM), stroma of the cornea and Descemet membrane (DM). The thickness of stroma – the core layer of the cornea – varies in its center. The outer radius of the sclera is 12 mm [4], and the thickness varies from 1 mm at the base of the cornea to 0.8 mm [5], and further to 0.6 mm in the equatorial zone of the scleral shell.

Vacuum compression ring has three main parameters $r_1$ (inner radius), $r_2$ (outer radius) and $h$ (height), which are shown on Fig. 1 (left).

According to [6] both the cornea and the sclera can be considered as a transversely isotropic shell. The elastic constants should satisfy the following system of inequality [7]:

\[
|v'_i| < \left(\frac{E'_i}{R_i}\right)^{1/2}, -1 < v_i < 1 - 2(v'_i)^2 \frac{E'_i}{E_i}, (E_i > 0, E'_i > 0), i = 1, ..., 4,
\]

where $E_i$ and $E'_i$ - are the stress-strain moduli of elasticity for the surface of isotropy and in the direction perpendicular to it; $v_i$ and $v'_i$ are the Poisson’s ratios. The shear modulus for the surface isotropy $G_i$ is given as:
\[ G_i = \frac{E_i}{2(1 + \nu_i)} \]  

Table 1 specifies the thickness of the layers of the corneal membrane, values of elastic moduli and Poisson’s ratios in the tangential and thickness directions, which are used to solve the problem [8]. It is assumed that the corneal tissue and scleral shell are close to incompressible, so \( \nu_i = 0.48 \) and \( \nu_i' = 0.02 \). FE model of vacuum compression ring is modeled using rigid domain.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Epithelium</th>
<th>BM</th>
<th>Cornea</th>
<th>DM</th>
<th>Sclera</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_i )</td>
<td>mm</td>
<td>0.043</td>
<td>0.012</td>
<td>0.4-1.1</td>
<td>0.01</td>
<td>0.6-1</td>
</tr>
<tr>
<td>( E_i )</td>
<td>MPa</td>
<td>0.06</td>
<td>0.6</td>
<td>0.3</td>
<td>0.9</td>
<td>5</td>
</tr>
<tr>
<td>( E_i' )</td>
<td>MPa</td>
<td>0.003</td>
<td>0.03</td>
<td>0.015</td>
<td>0.045</td>
<td>0.5</td>
</tr>
<tr>
<td>( G_i )</td>
<td>MPa</td>
<td>0.02</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>1.69</td>
</tr>
<tr>
<td>( G_i' )</td>
<td>MPa</td>
<td>0.001</td>
<td>0.01</td>
<td>0.007</td>
<td>0.02</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 1: Parameters of the simulated sclera and the corneal layers.

Curvilinear coordinate system is built to create base vector system of normalize vector field on each step of solution. Whereby moduli of elasticity \( E_i \) is always normal to a deformed corneoscleral shell. The initial curvilinear coordinate system is shown on Fig. 1 (right).

Corneoscleral shell is loaded by internal pressure (IOP), which is set in the direction perpendicular to the inner surface (fig. 2 on the left). Boundary load of vacuum compression ring is shown on fig. 2 (right).
The boundary conditions are chosen so as to eliminate the offset and rotation of the model. A rigid fixation of the outer surface of the sclera section in the equatorial zone of the eyeball in the orthogonal direction is applied, as well as a rigid fixation of the outer surface of the sclera sections by two orthogonal planes passing through the axis of rotation in the direction of the normal of each section.

FE model is built with free triangular elements with sizes of 0.03 and 0.05 mm on the sclera shell and vacuum compression ring correspondently (fig. 3).

To prevent the interference of contacting surfaces, the method of penalty is used, the essence of which is that in case of interference the penalty factor which is the actual stiffness of a spring inserted between the boundaries in the contact pair. So, the spring gives a resisting force until convergence to the state where there is no interference.
3 RESULTS

Figure 4 shows total displacement distribution of the corneoscleral shell under the load of vacuum compression ring in case of normal IOP (15 mm Hg). Stress and total displacement distributions under the load of vacuum compression ring are shown on Figure 5.

Mathematical modeling of the process of increasing the level of applied vacuum for different levels of IOP shows that this increase of IOP does not depend on the value of the elastic modules of corneoscleral shell. However, FE modeling shows significant dependence of IOP level from inner radius $r_1$, outer radius $r_2$ and height $h$ of vacuum compression ring and the size of the eye (the anterior-posterior axis of the eye and radiiuses of cornea and sclera).
4 CONCLUSIONS

• IOP does not depend on the value of the elastic modules of corneoscleral shell while loading with vacuum of 500 mm Hg.

• IOP level depends mainly from inner radius $r_1$, outer radius $r_2$ and height $h$ of vacuum compression ring and the length of anterior-posterior axis of the eye.

• The results of FE modeling of loading the corneoscleral shell with vacuum compression ring are well correlated with the results of the study [3, 9].

5 ACKNOWLEDGEMENT

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REFERENCES


LINEAR TWO-DIMENSIONAL MODELS OF ANISOTROPIC PLATES IN HIGH APPROXIMATIONS

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Keywords: anisotropic heterogeneous plate, two-dimensional model, second-order accuracy, bending, vibrations, waves propagation.

Abstract. Thin elastic anisotropic plates heterogeneous in the thickness direction are studied. Delivering of 2D approximate models of plates and shells is one of the classical problems of mechanics. In the present study, 2D models are delivered by using asymptotic expansions of solutions of 3D equations of the theory of elasticity in power series of a small thickness parameter. The zero asymptotic approximation gives equations that coincide with the Kirchhof–Love model if we use in-plane elasticity relations. If the elastic moduli are of identical order, then this model gives results acceptable for applications. In the opposite case, it is desirable to construct higher approximations. For particular types of anisotropy, 2D models of the second-order of accuracy were constructed earlier, and here we present a second-order model for a plate made of a heterogeneous material with general anisotropy described by 21 elastic moduli. The structure of the model and the differential orders of PDE is the same as for the Timoshenko–Reissner model, but equations are significantly more cumbersome. For a plate infinite in the tangential directions, there exists a simple harmonic solution. This solution allows us to estimate an error of 2D models compared with the exact solution of 3D equations of the theory of elasticity. The model is used to solve static and vibration problems. Some numerical examples are considered.
1 INTRODUCTION

Derivation of two-dimensional approximate models of thin plates and shells is one of classical problems of mechanics [1–3]. Numerous investigations were devoted to delivering of 2D approximate models of thin plates and shells made of anisotropic materials (we mention the books [4–6], containing an extensive bibliography). Here (as in the previous studies [5, 7–9], etc.) we use asymptotic expansions of solutions of 3D equations of the theory of elasticity in power series of a small thickness parameter. The zero approximation coincides with the classical Kirchhof–Love (KL) model based on the kinematic hypotheses [1, 2, 10]. For thin plates, this approximation gives sufficiently exact results for isotropic homogeneous plates and as well for anisotropic heterogeneous plate if the elastic moduli of the latter have identical orders in the thickness direction. The opposite case calls for consideration of higher approximations. For plates which are transversely isotropic and strongly heterogeneous in the thickness direction, the 2D equations of the second order accuracy (SOA) were derived and investigated in [11–16]. This model is useful for multi-layered plates with the alternating hard and soft layers, because the classical KL and Timoshenko–Reissner (TR) models lead to the large errors. For these multi-layered plates the generalized TR model with an equivalent transversal shear modulus is constructed [13, 14]. An analysis of multi-layered orthotropic plates with the arbitrary orientation of the main directions of orthotropy is reduced to investigation of plates heterogeneous in the thickness direction monoclinic. An asymptotic analysis of monoclinic plates was performed in [17], and in [18], 2D equations of SOA were derived.

In the case of general anisotropy (with 21 elastic moduli), the classical KL and TR models are also unacceptable. In [19], for anisotropic plates, and in [20] for shells, the generalized TR model was proposed. In [21], based on the asymptotic expansions (as elaborated in [12–14]), a 2D model for a multi-layered plate with the general anisotropy was proposed. This model leads to equations of 8th differential order and it is similar to the KL model with equivalent elastic moduli. This model is of zero order of accuracy and it is not acceptable for a multi-layered plate with alternating hard and soft layers. In the present stud, we consider an anisotropic heterogeneous plate of general anisotropy (with 21 elastic moduli) and construct higher asymptotic approximations. For general anisotropy, the asymptotic solution is essentially more difficult and bulky as compared with that for monoclinic material. To construct a model of SOA for a material with general anisotropy, it is necessary to built asymptotic solutions of the zero, the first, and the second approximations, while for a monoclinic material the first approximation is absent. As a result, a PDE system with constant coefficients is obtained. The differential order of this system is the same as the order of the TR model, but this system is essentially more cumbersome. For a plate heterogeneous in the thickness direction, to find the coefficients it is necessary to calculate the repeated integrals of elastic moduli. Closed solutions of boundary value problems for a finite plate (for example, for a rectangular plate) do not exist, because a separation of variables is impossible. A plate infinite in the tangential directions was considered. In this case, the 3D problem is reduced to a 1D boundary value problem (in the thickness direction) that admits a simple numerical solution, and the 2D problem leads to a linear algebraic system. A comparison of these 3D and 2D problems allows one to estimate an error of the approximate 2D model. Here, this comparison is performed for some problems of bending and vibrations of anisotropic multi-layered plates. It is shown that the SOA model is acceptable for a very large level of heterogeneity of a plate cross-section.
2 THE EQUILIBRIUM EQUATIONS AND ELASTICITY RELATIONS

Consider a thin elastic plate of constant thickness \( h \). In the Cartesian co-ordinate system \( x_1, x_2, x_3 \), the equilibrium equations read as

\[
\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0, \quad i = 1, 2, 3, \quad \frac{h}{2} \leq x_3 = z \leq \frac{h}{2},
\]

where \( \sigma_{ij} \) are the stresses and \( f_i \) are the intensities of external forces.

Following \[14\], we present the stresses \( \sigma_{ij} \) and the strains \( \varepsilon_{ij} \) as 6D vectors, and write the elasticity relations in the matrix form:

\[
\sigma = \mathbf{E} \cdot \varepsilon, \quad \mathbf{E} = (E_{ij})_{i,j=1,\ldots,6}, \\
\varepsilon = (\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{13}, \varepsilon_{23})^T, \\
\sigma_{ij} = \frac{\partial u_i}{\partial x_j}, \quad \varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, \quad i \neq j, \quad i, j = 1, 2, 3.
\]

Here and in what follows, \( ^T \) denotes transposition, while bold letters are used for vectors, for matrices and for operators, the product of vectors and matrices is denoted by the dot sign (\( \cdot \)).

The matrix \( \mathbf{E} \) of elastic moduli is symmetric and positively definite. In the considered case of a general anisotropy, it contains 21 independent elastic moduli. It is assumed that the elastic moduli \( E_{ij} \) are independent of the tangential co-ordinates \( x_1, x_2 \), but they may depend on \( x_3 = z \). A dependence on \( z \) takes place for functionally graded plates, as well as for multilayered plates for which the moduli \( E_{ij} \) are piecewise constant functions of \( z \).

Following \[14\], for an asymptotic analysis we divide the stresses \( \sigma_{ij} \) and the strains \( \varepsilon_{ij} \) in groups of tangential \( \sigma_t, \varepsilon_t \) and transversal \( \sigma_n, \varepsilon_n \) stresses and strains with various asymptotic behavior and put

\[
\sigma_t = (\sigma_{11}, \sigma_{22}, \sigma_{12})^T, \quad \sigma_n = (\sigma_{13}, \sigma_{23}, \sigma_{33})^T, \\
\varepsilon_t = (\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12})^T, \quad \varepsilon_n = (\varepsilon_{13}, \varepsilon_{23}, \varepsilon_{33})^T,
\]

where

\[
\mathbf{A} = \{A_{ij}\} = \begin{pmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{pmatrix}, \quad \mathbf{B} = \{B_{ij}\} = \begin{pmatrix} E_{15} & E_{25} & E_{56} \\ E_{14} & E_{24} & E_{46} \\ E_{13} & E_{23} & E_{36} \end{pmatrix}, \\
\mathbf{C} = \{C_{ij}\} = \begin{pmatrix} E_{55} & E_{45} & E_{35} \\ E_{45} & E_{44} & E_{34} \\ E_{35} & E_{34} & E_{33} \end{pmatrix}.
\]

Then the elasticity relations (2.2) assume the form

\[
\sigma_t = \mathbf{A} \cdot \varepsilon_t + \mathbf{B} \cdot \varepsilon_n, \quad \sigma_n = \mathbf{B}^T \cdot \varepsilon_t + \mathbf{C} \cdot \varepsilon_n,
\]

Excluding the small transversal strains \( \varepsilon_n \), we obtain

\[
\sigma_t = \mathbf{A}' \cdot \varepsilon_t + \mathbf{B} \cdot \mathbf{C}^{-1} \cdot \sigma_n, \quad \varepsilon_n = \mathbf{C}^{-1} \cdot \sigma_n - \mathbf{C}^{-1} \cdot \mathbf{B}^T \cdot \varepsilon_t
\]

where

\[
\mathbf{A}' = \mathbf{A} - \mathbf{B} \cdot \mathbf{C}^{-1} \cdot \mathbf{B}^T.
\]
Taking into account that $|\sigma_n| \ll |\sigma_t|$, we obtain the approximate elasticity relations

$$\sigma_t = A^* \cdot \varepsilon_t,$$  \hspace{1cm} (2.8)

that relate the tangential strains and stresses. The matrix $A^*$ of the in-plane elastic moduli plays an important role in the following approximations.

We next introduce dimensionless variables (with the sign $\hat{\cdot}$) as follows:

$$\{x_1, x_2, u_1, u_2, u_3\} = l\{\hat{x}_1, \hat{x}_2, \hat{u}_1, \hat{u}_2, \hat{w}\}, \quad z = l\hat{z}, \quad \mu = h/l,$$

$$\{A_{ij}, B_{ij}, C_{ij}, \sigma_{ij}\} = E\{\hat{A}_{ij}, \hat{B}_{ij}, \hat{C}_{ij}, \hat{\sigma}_{ij}\}, \quad \{f_i\} = (E/l)\{\hat{f}_i\}, \quad i, j = 1, 2, 3,$$

(2.9)

here $l$ is the characteristic length of waves in tangential directions, $E$ is the characteristic value of elastic moduli, $\mu$ is the small parameter. In what follows, the hat-sign $\hat{\cdot}$ will be omitted. As a result, we get a system of 6th order equations with small parameter $\mu$,

$$\frac{\partial w}{\partial z} = \mu \varepsilon_{33},$$

$$\frac{\partial u_i}{\partial z} = -\mu (p_i w - \varepsilon_{33}), \quad i = 1, 2,$$

$$\frac{\partial \sigma_{33}}{\partial z} = -\mu (p_1 \sigma_{13} + p_2 \sigma_{23} + f_i), \quad i = 1, 2,$$

$$\frac{\partial \sigma_{33}}{\partial z} = -\mu (p_1 \sigma_{13} + p_2 \sigma_{23} + f_3),$$

(2.10)

where $p_1 = \partial(\cdot)/\partial x_1$, $p_2 = \partial(\cdot)/\partial x_2$.

3 TRANSFORMATION OF SYSTEM (2.10)

To represent Eqs. (2.10) in the vector form, we introduce the vectors

$$\mathbf{u} = (u_1, u_2)^T, \quad \sigma_s = (\sigma_{13}, \sigma_{23})^T, \quad \varepsilon_s = (\varepsilon_{13}, \varepsilon_{23})^T, \quad \mathbf{f}_t = (f_1, f_2)^T,$$

(3.1)

the differential operators

$$\mathbf{p} = (p_1, p_2)^T, \quad \mathbf{P} = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix},$$

(3.2)

and the integral operators

$$\mathbf{I}_u(Z) \equiv \int_{-1/2}^{1/2} Zdz, \quad \mathbf{I}(Z) \equiv \int_{-1/2}^{Z} Z(z)dz, \quad \mathbf{I}_0(Z) \equiv \int_{0}^{Z} Z(z)dz.$$ 

(3.3)

Now Eqs. (2.10) in the vector form read as

$$\frac{\partial \mathbf{u}}{\partial z} = \mu \mathbf{\varepsilon}_{33}, \quad \frac{\partial \sigma_s}{\partial z} = -\mu (\mathbf{P}^T \cdot \sigma_t + \mathbf{f}_t),$$

$$\frac{\partial \mathbf{u}}{\partial z} = -\mu (\mathbf{p} \cdot w - \mathbf{\varepsilon}_s), \quad \frac{\partial \sigma_{33}}{\partial z} = -\mu (\mathbf{p}^T \cdot \sigma_s + \mathbf{f}_3),$$

(3.4)

Here the main unknowns are $\mathbf{w}, \mathbf{u}, b_s \sigma_s, \sigma_{33}$. The remaining unknowns can be expressed as

$$\varepsilon_n = (\varepsilon_s^T, \varepsilon_{33})^T = C^{-1}.(\sigma_n - \mathbf{B}^T \varepsilon_t), \quad \varepsilon_t = \mathbf{P} \cdot \mathbf{u},$$

$$\sigma_t = A^* \cdot \mathbf{P} \cdot \mathbf{u} + \mathbf{B} \cdot C^{-1} \cdot \sigma_n, \quad \sigma_n = (\sigma_s^T, \sigma_{33})^T.$$ 

(3.5)
The orders of the functions \( \sigma_s \) and \( \sigma_{33} \) are different, and so we rewrite Eqs. (3.5) with the help of block-structure of the matrices \( C^{-1} = \{G_{ij}\} \) and \( C^{-1}B^T = \{S_{ij}\} \) as follows:

\[
C^{-1} = \begin{pmatrix} G & g \\ g^T & c_3 \end{pmatrix}, \quad G = \begin{pmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{pmatrix}, \quad g = (G_{13}, G_{23})^T, \quad c_3 = G_{33},
\]

\[
C^{-1}B^T = \begin{pmatrix} S \\ s \end{pmatrix}, \quad S = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \end{pmatrix}, \quad s = (S_{31}, S_{32}, S_{33}).
\]

Now Eqs. (3.5) read as

\[
\begin{align*}
\sigma_t &= A^* \cdot P \cdot u + S^T \cdot \sigma_s + s^T \sigma_{33}, \\
\varepsilon_s &= G \cdot \sigma_s + g \sigma_{33} - S \cdot P \cdot u, \\
\varepsilon_{33} &= g^T \cdot \sigma_s + c_3 \sigma_{33} - s \cdot P \cdot u.
\end{align*}
\]

As a result, the right-hand sides of Eqs. (3.4) can be expressed in terms of the main unknowns.

The right-hand sides of Eqs. (3.4) are small, and so to solve this system we use the method of iterations. For majority of bending and tangential deformations the iterations are different, and we consider here the bending deformations. For convenience, we change the scale of unknown functions according with their orders and put

\[
u = \mu \hat{u}, \quad \sigma_s = \mu^2 \hat{\sigma}_s, \quad \sigma_{33} = \mu^3 \hat{\sigma}_{33}, \quad f_t = \mu \hat{f}_t, \quad f_3 = \mu^2 \hat{f}_3.
\]

Again the sign \(^*\) is omitted.

Supposing that the planes \( z = \pm 1/2 \) are free, we get the boundary conditions

\[
\sigma_s = \sigma_{33} = 0, \quad z = \pm 1/2.
\]

The external surface forces can be included in the body forces by using the Dirac delta-function.

Now in the notation of (3.1)–(3.3), (3.6), and (3.8), Eqs. (3.4) can be written as a system of integral equations

\[
\begin{align*}
w &= w_0 - \mu^2 I_0(s_P \cdot u) + \mu^3 I_0(g^T \cdot \sigma_s) + \mu^4 I_0(c_3 \sigma_{33}), \\
u &= u_0 - I_0(p \cdot w) - \mu I_0(S_P \cdot u) + \mu^2 I_0(G \cdot \sigma_s) + \mu^3 I_0(g \sigma_{33}), \\
\sigma_s &= -I(L_0 \cdot u) - \mu I(S_{P \cdot \sigma_s}) - \mu^2 I(S_{P \sigma_{33}}) - I(f_t), \\
\sigma_{33} &= -I(p^T \cdot \sigma_s) - I(f_3),
\end{align*}
\]

where we use the following notation

\[
L_0(z) = P^T \cdot A^*(z) \cdot P, \quad S_P(z) = S(z) \cdot P, \quad s_P(z) = s(z) \cdot P.
\]

In Eqs. (3.10), the functions \( w_0(x_1, x_2) \), \( u_0(x_1, x_2) \) are arbitrary functions. The functions \( \sigma_s, \sigma_{33} \) due Eqs. (3.3) satisfy the boundary conditions (3.9) at \( z = -1/2 \), and the functions \( w_0, u_0 \) are to be found from conditions (3.9) at \( z = 1/2 \).

All the unknowns in Eqs. (3.10) are of the order of unity. System (3.10) is exact and it is convenient to construct a solution by using the method of iterations. In the next sections, the solution of the SOA with respect to the small thickness parameter \( \mu \) will be constructed. Therefore, for simplicity the small summands of the orders of \( \mu^3 \) and of \( \mu^4 \) can be omitted in the first two Eqs. (3.10).
4 THE ZERO APPROXIMATION

All the unknowns in Eqs. (3.10) will be sought in the form

\[ Z(x_1, x_2, z, \mu) = Z^{(0)}(x_1, x_2, z) + \mu Z^{(1)}(x_1, x_2, z) + \mu Z^{(2)}(x_1, x_2, z), \quad Z = \{w, u, \sigma, \sigma_{33}\}. \]  

(4.1)

The zero approximation was obtained in [21], and here we give shortly the results in our notation

\[ w^{(0)}(x_1, x_2, z) = \omega_0(x_1, x_2), \]
\[ u^{(0)}(x_1, x_2, z) = u_0(x_1, x_2) - z \text{p} w_0(x_1, x_2), \]
\[ \sigma^{(0)}_x(x_1, x_2, z) = -I(L(z) \cdot u^{(0)}) - I(f_t), \]
\[ \sigma^{(0)}_{33}(x_1, x_2, z) = II(p^T L_0(z) \cdot u^{(0)}) + I(p^T f_t) - I(f_3). \]

(4.2)

The boundary conditions (3.3) at \( z = 1/2 \) lead to a system of equations for \( w_0 \) and \( u_0 \) in the zero approximation:

\[ L_0 \cdot u_0 - N_1 w_0 + F_t = 0, \]
\[ N_1^T \cdot u_0 - Q_2 w_0 + m + F_3 = 0, \]

(4.3)

where

\[ L_0 = P^T I_a(A^*(z)) \cdot P, \quad N_0 = P^T I_a(z A^*(z)) \cdot P \cdot \text{p}, \quad F_t = I_a(f_t), \]
\[ Q_0 = p^T P^T I_a(z^2 A^*(z)) \cdot P \cdot \text{p}, \quad F_3 = I_a(f_3), \quad m = I_a(p^T f_t). \]

(4.4)

The detailed expressions of operators in Eqs. (4.3) read as [21]

\[ L_0 = \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix}, \]
\[ L_{11} = a^{(0)}_{11} p_1^2 + 2a^{(0)}_{13} p_1 p_2 + a^{(0)}_{33} p_2^2, \]
\[ L_{12} = a^{(0)}_{13} p_1^2 + (a^{(0)}_{12} + a^{(0)}_{33}) p_1 p_2 + a^{(0)}_{23} p_2^2, \]
\[ L_{22} = a^{(0)}_{33} p_1^2 + 2a^{(0)}_{23} p_1 p_2 + a^{(0)}_{22} p_2^2, \]

\[ N_0 = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}, \]
\[ N_1 = a^{(1)}_{11} p_1^3 + 3a^{(1)}_{13} p_1 p_2^2 + (a^{(1)}_{12} + 2a^{(1)}_{33}) p_1 p_2^2 + a^{(1)}_{23} p_2^3, \]
\[ N_2 = a^{(1)}_{13} p_1^3 + (a^{(1)}_{12} + 2a^{(1)}_{33}) p_1 p_2^2 + 3a_{23} p_1 p_2^2 + a^{(1)}_{22} p_2^3, \]
\[ Q_0 = a^{(2)}_{11} p_1^4 + a^{(2)}_{13} p_1^3 p_2^2 + (2a^{(2)}_{12} + 2a^{(2)}_{33}) p_1 p_2^2 + 4a^{(2)}_{23} p_1 p_2^2 + a^{(2)}_{22} p_2^4, \]
\[ a^{(k)}_{ij} = I(z^k A^*_{ij}(z)) = \int_{-1/2}^{1/2} z^k A^*_{ij}(z) \, dz, \quad i, j = 1, 2, 3, \quad k = 0, 1, 2. \]

Equations (4.3) describes approximately the bending deformations of an anisotropic plate in the frames of the KL hypotheses. In [17] it was shown that in the case when all the elastic moduli are of the same order, the zero approximation gives an acceptable exactness for approximate calculations. If some elements of the matrix \( C \) are small, then according to Eqs. (3.6) the corresponding summands that were not included in the zero approximation are large, and the exactness of the zero approximation is not enough. The main effect that is not described by the zero approximation is the transversal shear, which is essential for multi-layered plates with hard and soft alternating layers.

5 THE FIRST APPROXIMATION

The first approximation takes into account the summands of the order of \( \mu \) in Eqs. (3.10). If \( S = 0 \), then the summands of the order of \( \mu \) in Eqs. (3.10) are absent. For monoclinic materials (in particular, for isotropic, for transversely isotropic, and for orthotropic materials),
S = 0. Materials with S ≠ 0 are used very rarely. We shall call such an anisotropy an inclined anisotropy, because it can be obtained with a composite plate consisting of an orthotropic matrix reinforced by a system of fibres inclined to a plane of plate [22].

For an inclined anisotropy, the first approximation reads as

\[ w^{(1)} = w_0, \]
\[ u^{(1)} = u_0 - I_0(p w^{(1)}) - \mu I_0(S_P \cdot u^{(0)}) = u^{(0)} - \mu I_0(S_P \cdot u^{(0)}), \]
\[ \sigma_s^{(1)} = -I(L_0 \cdot u^{(1)}) - \mu I(S_P^T \cdot \sigma_s^{(0)}) - I(f_t) = \]
\[ = -I(L_0 \cdot u^{(0)}) + \mu I(L_0 \cdot I_0(S_P \cdot u^{(0)})) + \mu I(S_P^T \cdot I(L_0 \cdot u^{(0)} + f_t)) - I(f_t), \]
\[ \sigma_{33}^{(1)} = -I(p^T \cdot \sigma_s^{(1)}) - I(f_3) = \]
\[ = \Pi(p^T \cdot L_0 \cdot u^{(0)}) - \mu \Pi \left( p \cdot L_0 \cdot I_0(S_P \cdot u^{(0)}) + p \cdot S_P^T \cdot I(L_0 \cdot u^{(0)}) \right) + \]
\[ + \Pi(p^T \cdot f_t) - I(f_3). \]

Here the normal deflection \( w^{(1)} \) is the same as in the zero approximation, and the tangential deflections \( u^{(1)} \) and the stresses \( \sigma_s^{(1)}, \sigma_{33}^{(1)} \) are changed.

The equations for \( w_0 \) and \( u_0 \) are not given here, because they follow from the equations (6.4) of second approximation if we omit the summands with \( \mu^2 \).

6 THE SECOND APPROXIMATION

The second approximation reads as

\[ w^{(2)} = w_0 + \mu^2 I_0(\sigma_s^{(0)}) = w_0 - \mu^2 I_0(S_P \cdot u^{(0)}), \]
\[ u^{(2)} = u_0 - I_0(p w^{(2)}) - \mu I_0(S_P \cdot u^{(1)}) + \mu^2 I_0(G \cdot \sigma_s^{(0)}) = \]
\[ = u^{(0)} - \mu I_0(S_P \cdot u^{(0)}) + \mu^2 I_0(p \cdot S_P \cdot u^{(0)}) + \]
\[ + \mu^2 I_0(S_P \cdot I_0(S_P \cdot u^{(0)})) - \mu^2 I_0(G \cdot I(f_t)), \]
\[ \sigma_s^{(2)} = -I(L_0 \cdot u^{(2)}) - \mu I(S_P^T \cdot \sigma_s^{(1)}) - \mu^2 I(s_3^2) - I(f_t), \]
\[ \sigma_{33}^{(2)} = -I(p^T \cdot \sigma_s^{(2)}) - I(f_3). \]

Here it is necessary to substitute the values \( u^{(1)} \) and \( \sigma_s^{(1)} \) from Eq. (5.1) and the values \( u^{(2)} \) and \( \sigma_s^{(2)} \) from Eqs. (6.1). As a result, we can write \( \sigma_s^{(2)} \) in the form

\[ \sigma_s^{(2)} = -I(L^* \cdot u^{(0)} + f^*), \]
\[ L^* = L_0 + \mu L_1 + \mu^2 L_2, \]
with
\[ L_0 = P^T \cdot A^*(z) \cdot P, \quad L_1 = -L_0 \cdot I_0(S_P) - S_P^T \cdot I(L_0), \]
\[ L_2 = L_0 \cdot L_0(p \cdot S_P) + L_0 \cdot I_0(S_P) - L_0 \cdot I_0(G \cdot I(L_0)) + \]
\[ + S_P^T \cdot I(L_0 \cdot I_0(S_P)) + S_P^T \cdot I(S_P^T \cdot I(L_0)) + s_p^T \cdot I(L_0 \cdot i(L_0) - s_p^T \cdot I(f_3)). \]

Using the forth equation in (6.1), we find that

\[ \sigma_{33}^{(2)} = \Pi(L^* \cdot u^{(0)} + f^*) - I(f_3). \]
The boundary conditions \( \sigma_{z}^{(2)} = \sigma_{33}^{(2)} = 0 \) at \( z = 1/2 \) lead to equations for \( u_{0} \) and \( w_{0} \),

\[
I_{a}(L^{*}\cdot u^{(0)} + f^{*}) = 0, \quad u^{(0)} = u_{0} - p \cdot z \cdot w_{0} \\
I_{a}I(p^{T} \cdot (L^{*}\cdot u^{(0)} + f^{*})) - I_{a}(f_{3}) = 0. \tag{6.5}
\]

Employing the equality \( I_{a}I(Z(z)) = (1/2)I_{a}(Z(z)) - I_{a}(z \cdot Z(z)) \) at \( Z = \sigma^{*}\cdot u^{(0)} + f^{*} \) with \( I_{a}(Z) = 0 \), we can write Eqs. (6.5) in the final form

\[
I_{a}(L^{*}) \cdot u_{0} - I_{a}(L^{*} \cdot p \cdot z) w_{0} + I_{a}(f^{*}) = 0, \quad I_{a}(Z(z)) = \int_{-1/2}^{1/2} Z(z)dz, \\
I_{a}(z p^{T} \cdot L^{*}) \cdot u_{0} - I_{a}(z p^{T} \cdot L^{*} \cdot p \cdot z) w_{0} + I_{a}(z p^{T} \cdot f^{*}) + F_{3} = 0. \tag{6.6}
\]

Therefore, the model of the SOA is built. As one can see, the coefficients of Eqs. (6.6) are expressed through a very bulky operator \( L^{*} \) depending on the integrals of elastic moduli. Equations(6.6) form a PDE system with constant coefficients with respect to the unknown functions \( u_{0} = (u_{10}, u_{20}), \ w_{0} \). The differential order of system (6.6) is the same as the order of the TR model, but the system (6.6) is essentially more complex.

Let us discuss possible simplifications of Eqs. (6.6).

For a monoclinic material, \( S = 0 \), and all summands of the order of \( \mu \) are equal to zero.

For a plate symmetric in the thickness direction (namely, for a plate with even or constant in \( z \) elastic moduli), in the zero approximations the tangential and the transversal deflections can be investigated separately. In higher approximations, these deflections can be tied by small terms. For an inclined anisotropy, this relation is of the order of \( \mu \) and for a monoclinic material, it is of the order of \( \mu^{2} \).

The second-order operator \( L_{2} \) in Eqs. (6.2) and (6.3) can be simplified if we approximately put \( L_{2} = -L_{0} \cdot I_{0}(G \cdot I(I_{0})) \), because this term is large for a plate with large transversal shear compliance. The rest 5 summands in \( L_{2} \) are not large as a rule.

7 HARMONIC SOLUTION

The main difficulty in obtaining a closed-form solution of Eqs. (6.6) for a finite plate (for example, for a rectangular plate) consists in satisfying the boundary conditions, which for the general anisotropy do not admit the separation of variables. Further we consider an infinite plate and investigate the harmonic solutions admitting closed-form solutions.

We consider a static harmonic problem. Let the external forces be

\[
\{f_{t}, f_{3}\}(x_{1}, x_{2}, z) = \{f_{t}(z), f_{3}(z)\}e^{iy}, \quad Y = x_{1}q_{1} + x_{2}q_{2}, \quad i = \sqrt{-1}, \tag{7.1}
\]

where \( q_{1}, q_{2} \) are the real-valued wave numbers. We seek the solution of the three-dimensional equations (3.4) in the same harmonic form,

\[
\{w, u, \sigma_{t}, \sigma_{33}\}(x_{1}, x_{2}, z) = \{w, u, \sigma_{t}, \sigma_{33}\}(z)e^{iy} \tag{7.2}
\]

and the solution of two-dimensional equations Eqs. (6.6) has the form:

\[
w(x_{1}, x_{2}) = We^{iy}, \quad u(x_{1}, x_{2}) = Ue^{iy}, \quad U = (U_{1}, U_{2})^{T}, \tag{7.3}
\]

where \( U, W \) are the unknown complex amplitudes of deflection of plane \( z = 0 \). We consider a bending deformation under the action of a periodic compression applied to a lower plane,

\[
f_{t} = 0, \quad f_{3}(x_{1}, x_{2}, z) = F_{3} \delta(z + 1/2)e^{i(x_{1}q_{1} + x_{2}q_{2})}. \tag{7.4}
\]
Inserting Eqs. (7.1) and (7.3) into Eqs. (6.6) yields the following linear algebraic system the for unknown variables \( U, W, \)

\[
(\mathbf{I}_0 + \mu \mathbf{I}_1 + \mu^2 \mathbf{I}_2) \cdot U - (\mathbf{N}_0 + \mu \mathbf{N}_1 + \mu^2 \mathbf{N}_2)W = 0,
\]

\[
(\mathbf{N}_0 + \mu \mathbf{N}_1 + \mu^2 \mathbf{N}_2) \cdot U - (\mathbf{Q}_0 + \mu \mathbf{Q}_1 + \mu^2 \mathbf{Q}_2)W + F_3 = 0,
\]

with

\[
\mathbf{I}_k = I_a(L_k), \quad \mathbf{N}_k = I_a(L_k \cdot p z), \quad \mathbf{N}_k = I_a(zp^T \cdot L_k), \quad \mathbf{Q}_k = I_a(zp^T \cdot L_k \cdot p z), \quad k = 1, 2, 3,
\]

and the differential operators \( p \) and \( \mathbf{P} \) are to be formally replaced by

\[
\tilde{p} = i \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \quad \tilde{\mathbf{P}} = i \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \\ q_1 \end{pmatrix}^T.
\]

We represent the solutions of Eqs. (7.5) as expansions in power series in \( \mu, \)

\[
W_s = W_0 + \mu W_1 + \mu^2 W_2, \quad U = U_0 + \mu U_1 + \mu^2 U_2.
\]

In the zero approximation, we have

\[
U_0 = \tilde{L}_0^{-1} \cdot \tilde{N}_0 W_0, \quad W_0 = -\frac{F_3}{Q_0 - Q_*}, \quad Q_* = \tilde{N}_0 \cdot \tilde{L}_0^{-1} \cdot \tilde{N}_0
\]

where the value \( Q_* \) takes into account the asymmetry of the cross-section.

We consider deformation of a plate under unit normal external compression applied to plane \( z = -1/2, \) that is, \( \sigma_{33}(-1/2) = -1. \) Next, we introduce the vector \( Y \) of unknown variables of Eq. (3.4),

\[
Y(z) = \{y_1, y_2, \ldots, y_6\} = (w, u_1, u_2, \sigma_{13}, \sigma_{23}, \sigma_{33})..\]

To find a solution of Eq. (3.4) satisfying the boundary conditions (3.9) we solve the four Cauchy problems with the boundary conditions

\[
Y^{(1)}(-1/2) = (1, 0, 0, 0, 0, 0), \quad Y^{(2)}(-1/2) = (0, 1, 0, 0, 0, 0),
\]

\[
Y^{(3)}(-1/2) = (0, 0, 1, 0, 0, 0), \quad Y^{(4)}(-1/2) = (0, 0, 0, 0, 0, -1).
\]

Now the sought-for solution reads as

\[
Y(z) = C_1 Y^{(1)}(z) + C_2 Y^{(2)}(z) + C_3 Y^{(3)}(z) + Y^{(4)}(z),
\]

where the constants \( C_1, C_2, C_3 \) should be found from the equations

\[
\sigma_{33}(1/2) = 0, \quad i = 1, 2, 3.
\]

Solving Eqs. (7.12), we have \( w(-1/2) = C_1, u_1(-1/2) = C_2, u_2(-1/2) = C_3. \)

Consider bending deformations of an infinite multi-layered plate consisting of 5 isotropic layers of equal thickness \( (h_k = 0.2) \) under the action of the harmonic compression \( F_3(x_1, x_2) = F_3 e^{i(q_1 x_1 + q_2 x_2)}, \) as applied to the lower plane \( z = -1/2. \) The Young moduli and the Poisson ratios of layers are as follows:

\[
E_k = 1, \quad k = 1, 3, 5, \quad E_k = \eta, \quad k = 2, 4, \quad \nu_k = 0.3, \quad k = 1, \ldots, 5.
\]

We take \( \mu = 0.1, \) \( q_1 = 0.6, \) \( q_2 = 0.8. \) The deflection amplitudes \( W, \) as found for various values \( \eta, \) are given in Table 1.
Table 1. Deflection amplitude versus η.

<table>
<thead>
<tr>
<th>η</th>
<th>δ_s</th>
<th>W_e</th>
<th>W_0</th>
<th>ε_0%</th>
<th>W_s</th>
<th>ε_s%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0029</td>
<td>1094</td>
<td>1092</td>
<td>−0.2</td>
<td>1095</td>
<td>0.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.012</td>
<td>1358</td>
<td>1344</td>
<td>−1.0</td>
<td>1360</td>
<td>0.1</td>
</tr>
<tr>
<td>0.01</td>
<td>0.11</td>
<td>1527</td>
<td>1375</td>
<td>−10.0</td>
<td>1529</td>
<td>0.1</td>
</tr>
<tr>
<td>0.001</td>
<td>1.1</td>
<td>2857</td>
<td>1378</td>
<td>−52.2</td>
<td>2908</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Here $W_e$ is the exact value found by numerical solution of Eqs. (3.4), $W_0$ is the first approximation (7.8) at $Q_s = 0$, $W_s$ is the second approximation (7.7), $ε_0$ and $ε_s$ are the corresponding relative errors of approximate values $W_0$ and $W_s$. The parameter $η^{-1}$ describes the level of a plate heterogeneity, and the parameter $δ_s = μ^2 Q_s/Q_0$, the influence of the transversal shear. From Table 1 it follows that the first approximation is acceptable for a comparatively small level of heterogeneity, while the second approximation gives exact enough results for wide limits of the ratio $E_1/E_2$.

8 HARMONIC VIBRATIONS

We consider free vibrations of an infinite anisotropic plate, and seek solutions of the 3D and of the 2D equations in the same forms (7.2) and (7.3), respectively, after changing the factor $e^{i(x_1q_1 + x_2q_2)}$ by $e^{i(x_1q_1 + x_2q_2 + ωt)}$, where $ω$ is the natural frequency. According to the notation of Eqs. (2.9) and (3.8), the external (inertia) forces in the dimensionless form read as

$$f_i(x_1, x_2, z, t) = λ \dot{ρ}(z) u \ e^{iY}, \ f_3(x_1, x_2, z, t) = \frac{λ}{μ^2} \dot{ρ}(z) w \ e^{iY}$$  (8.1)

with

$$Y = x_1q_1 + x_2q_2 + ωt, \quad λ = \frac{ρ_0 l^2 ω^2}{E}, \quad \dot{ρ}(z) = \frac{ρ(z)}{ρ_0}, \quad ρ_0 = I_a(ρ(z)),$$

where $λ$ is the unknown frequency parameter, and $ρ(z)$ is the density, $ρ_0$ is the average density. The sign $\dot{\ }$ is again omitted.

The inertia forces (8.1) are to be inserted into 3D exact Eqs. (3.10) or (3.4) and into 2D approximate Eqs. (6.6). These equations contain the unknown frequency parameter $λ$.

The system (6.6) is a linear algebraic one, and its determinant satisfies the equation

$$Δ(λ, q_1, q_2, μ) = 0.$$  (8.3)

This equation is cubic in $λ$. It is easy to prove that the roots of Eq. (8.3) are real positive numbers, $λ_j > 0$. The smallest root gives a natural frequency of bending vibrations, $ω = \sqrt{λ_1(q_1, q_2, μ)}$.

As in Section 7, to find the exact eigenvalue $λ$ we solve numerically the three Cauchy problems for Eqs. (3.4) with the initial conditions

$$Y^{(1)}(-1/2) = (1, 0, 0, 0, 0, 0), \ Y^{(2)}(-1/2) = (0, 1, 0, 0, 0, 0), \ Y^{(3)}(-1/2) = (0, 0, 1, 0, 0, 0)$$  (8.4)

Using the first root of the equation

$$Δ(λ) = \begin{vmatrix}
σ_{11}^{(1)}(1/2) & σ_{12}^{(1)}(1/2) & σ_{13}^{(1)}(1/2) \\
σ_{11}^{(2)}(1/2) & σ_{12}^{(2)}(1/2) & σ_{13}^{(2)}(1/2) \\
σ_{11}^{(3)}(1/2) & σ_{12}^{(3)}(1/2) & σ_{13}^{(3)}(1/2)
\end{vmatrix} = 0,$$  (8.5)
we get the exact value $\lambda = \lambda_e$.

Proceeding in this way it is possible to estimate the error of approximate equations (6.6).

If we put $Y = x_1q_1 + x_2q_2 - vt$, then Eqs. (7.2) and (7.3) describes the wave propagation in the direction $q = (q_1, q_2)$ with velocity $v$, and Eqs. (8.3) and (8.5) are the dispersion equations. The velocity $v = v(q_1, q_2)$ depends on the direction of wave propagation.

As a numerical example, we consider the bending vibrations of a symmetric plate consisting of 5 layers with equal thickness. The first, the third, and the fifth layers are orthotropic. The third (middle) layer is turned by an angle $\alpha = \pi/2$ with respect to the rest of the layers. The second and the forth layers are isotropic, and we will change the stiffness of these layers within the wide limits. The elastic moduli of the layers in Eqs. (3.4) are given in Table 2 in the dimensionless form. Here, $N$ is the number of layers.

### Table 2. Elastic moduli of layers.

<table>
<thead>
<tr>
<th>N</th>
<th>$E_{11}$</th>
<th>$E_{22}$</th>
<th>$E_{33}$</th>
<th>$E_{12} = E_{13} = E_{23}$</th>
<th>$E_{44} = E_{55} = E_{66}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>12.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.59</td>
<td>0.69</td>
</tr>
<tr>
<td>2.4</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>11.0</td>
<td>1.0</td>
<td>0.30</td>
<td>0.35</td>
</tr>
</tbody>
</table>

We take the mass density $\rho = 1$ for all layers, the wave numbers $q_1 = 0.6$, $q_2 = 0.4$, and the small thickness parameter $\mu = 0.1$.

For elastic moduli of the second and the fourth layers, we consider 4 variants. The moduli of the first variant are given in Table 1. The moduli of the remaining variants can be obtained by multiplying the moduli of the first variant by the values $\eta = 0.1$, $\eta = 0.01$, and $\eta = 0.001$, respectively. By using the algorithm and formulas of Section 6, we get the following results.

### Table 3. The frequency parameter versus the ratio $\eta$ of elastic moduli.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\lambda_e$</th>
<th>$\lambda_0$</th>
<th>$\varepsilon(%)$</th>
<th>$\lambda_e$</th>
<th>$\varepsilon(%)$</th>
<th>$\delta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.002211</td>
<td>0.002229</td>
<td>0.9</td>
<td>0.002212</td>
<td>0.0</td>
<td>0.0079</td>
</tr>
<tr>
<td>0.1</td>
<td>0.001985</td>
<td>0.002073</td>
<td>4.5</td>
<td>0.001986</td>
<td>0.1</td>
<td>0.044</td>
</tr>
<tr>
<td>0.01</td>
<td>0.001482</td>
<td>0.002058</td>
<td>38.5</td>
<td>0.001459</td>
<td>1.6</td>
<td>0.41</td>
</tr>
<tr>
<td>0.001</td>
<td>0.000511</td>
<td>0.002056</td>
<td>303.0</td>
<td>0.000405</td>
<td>20.9</td>
<td>4.07</td>
</tr>
</tbody>
</table>

In Table 3, for four values of the parameter $\eta$ (i.e., for the measure of small stiffness of the second and the fourth layers), the corresponding values of the frequency parameter $\lambda$ and the shear parameter $\delta_s$ are presented. Here $\lambda_e$ is the exact value, as found by numerical solution of Eqs. (8.5), $\lambda_0$ is the first approximation, and $\lambda_e$ is the second approximation, as obtained from Eqs. (6.6). In columns 4 and 6, the relative errors of the approximate values are given.

As for the problem of plate bending (Section 7), here the first approximation is acceptable for a comparatively small level of heterogeneity, while the second approximation gives exact enough results for wide limits of the parameter $\eta$.

### 9 CONCLUSIONS

The 2D linear model (7.5) of the SOA describing deformations of a heterogeneous plate in the case of general anisotropy (with 21 elastic moduli) is constructed. It is necessary to continue
investigations of the problem under consideration. It is desirable to investigate the peculiarities that appear in the case of plates with inclined anisotropy. Of great value is also to estimate the errors for test examples of the 2D models and compare them with the exact solutions of 3D equations of the theory of elasticity. The model presented here is bulky, and so it is desirable to propose, in particular cases, more simple but sufficiently precise models.

The next step is to solve some static, vibration, and buckling problems for particular kinds of anisotropy and heterogeneity.

The algorithm used here is based on the Cartesian coordinate system. It is interesting to apply the obtained results for heterogeneous anisotropic shells, in partial, to shallow shells for which the metric is close to the Cartesian metric.

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REFERENCES


BUCKLING OF THIN CYLINDRICAL SHELL STIFFENED BY RINGS OF T-SHAPED CROSS-SECTION

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Abstract. The buckling under the uniform external pressure of a thin elastic circular cylindrical shell stiffened by the rings of the T-shaped cross-sections is analyzed. The narrow rings are considered as circular beams. The wide rings are treated as annular plates stiffened at one edge by a circular beam. In the both cases problem is reduced to solution of the eigenvalue problem for systems of linear differential equations containing a small parameter. By means of asymptotic approaches approximate formulas for the critical pressure are obtained. The optimal parameters corresponding to the maximal critical pressure of the ring-stiffened shell with a given mass are evaluated.
1 INTRODUCTION

To improve the antibuckling capacity of a cylindrical shell under the external pressure one use the stiffening rings. Often these rings have a rectangular or a T-shape cross-sections ([1]–[4]). The buckling of the shell stiffened by the rings of rectangular cross-sections is sufficiently well studied.

Differential equations describing the buckling of a thin shells contain the dimensionless shell thickness as a small parameter. By means of asymptotic approaches ([5], [6], [7]) complex eigenvalue problems of the stiffened shells theory can be transformed into problems which have analytical solutions.

If the width of the rings is sufficiently small the typical buckling mode is similar to the buckling mode of the shell without rings, and the shell surface is covered by a series of pits stretched along the generatrix of the cylinder. In this case, considering the rings as circular beams, it is possible to seek solutions as a sum of slowly varying functions and boundary layers near shell edges and stiffened parallels. In the first approximation one get eigenvalue problem describing also vibrations of a beam stiffened by springs. If the rings are identical and uniformly arranged along the shell then a solution of the last problem can be obtained in a closed form by means of a homogenization procedure ([8]). This same asymptotic approach is applied to analyze the buckling of the shell stiffened by the narrow identical rungs of the T-shape cross-sections.

When the width of a ring grows, the critical pressure increases until the typical buckling mode will be replaced with the buckling mode localized on the surface of the ring while the cylindrical shell itself does not actually deform ([9]). The beam model can not be used for the studying the buckling of the wide ring.

The wide ring of the rectangular cross-sections can be considered as an annular plate. The wide ring of the T-shape cross-section may be treated as an annular plate stiffened by a circular beam of the rectangular cross-section. An asymptotic analyze shows that in both cases the couple bucking problem for the cylindrical shell joined with the annular plate in the first approximation is reduced to the eigenvalue problem describing the buckling of the annular plate.

2 BASIC EQUATIONS

Consider the buckling under the uniform external pressure of a thin circular cylindrical shell stiffened by \( n_r \) identical rings at the parallels \( s = s_i \), where \( s \in [0, l] \) is the dimensionless coordinate in the longitudinal directions, \( i = 1, 2, \ldots n_r \) (see Fig. 1). The radius \( R \) of the cylindrical shell is taken as the characteristic size.
After the separation of variables the non-dimensional equations describing the buckling of a cylindrical shell under the external pressure \( p \) can be written in the following form \([7]\)

\[
\begin{align*}
T_1' + mS &= 0, \\
Q_1' + mQ_2 - T_2 + \lambda m\vartheta_2 &= 0, \\
Q_1 &= M_1' + 2mH, \\
Q_2 &= -mM_2, \\
T_1 &= \varepsilon_1 + \nu\varepsilon_2, \\
T_2 &= \varepsilon_2 + \nu\varepsilon_1, \\
S &= (1 - \nu)(\nu' - mu)/2, \\
M_1 &= \mu^4(\vartheta_1' + m\vartheta_2), \\
M_2 &= \mu^4(m\vartheta_2 + \nu\vartheta_1'), \\
H &= \mu^4(1 - \nu)\vartheta_2', \\
\varepsilon_1 &= w', \\
\varepsilon_2 &= mw + w, \\
\vartheta_1 &= -w', \\
\vartheta_2 &= 2mw + v,
\end{align*}
\]

where (') denotes the derivative with respect to the longitudinal coordinate \( s \), \( m \) is the circumferential wave number, \( u, v, w \) are the components of the displacement, \( \lambda = \sigma p/(Eh) \) is the loading parameter, \( \sigma = 1 - \nu^2 \), \( \nu \) is Poisson’s ratio, \( E \) is Young’s modulus, \( h \) is the dimensionless shell thickness, \( T_1, T_2, S, Q_1, Q_2, M_1, M_2, H \) are the dimensionless stress-resultants and stress-couples, \( \vartheta_1 \) and \( \vartheta_2 \) are the angles of rotation of the normal, \( \mu \) is the small parameter, \( \mu^4 = h^2/12 \).

We denote as \( y^{(j)} \) the solutions of equations (1) in the intervals \( s \in [s_{j-1}, s_j], j = 1, 2, \ldots, n, \)

\( n = n_r + 1 \). These solutions satisfy 8 boundary conditions on the shell edges \( s = 0 \) and \( s = l \) and 8\( n_r \) continuity conditions on the parallels \( s = s_i, i = 1, 2, \ldots, n_r \).

## 3 FIRST APPROXIMATION FOR NARROW RINGS

Assume that the width of the rings is sufficiently small. Then the lowest eigenvalues \( \lambda \sim m^{-6} \) corresponds to the large circumferential wave number \( m \sim \mu^{-1/2} \) and we seek unknown functions in equations (1) as

\[
y^{(j)}(s) = y_0^{(j)}(s) + \sum_{k=1}^{4} D_k^{(j)} y_k^{(j)}(s), \quad s \in [s_{j-1}, s_j], \quad j = 1, 2, \ldots, n.
\]

Here \( y_0^{(j)} \) are slowly varying functions, \( D_k^{(j)} \) are arbitrary constants. The edge effect functions \( Y_k^{(j)} \), are localized in a small neighborhoods of the parallels \( s_0, s_1, \ldots, s_n \).

The equations of the first approximation are

\[
\frac{d^4w^{(j)}}{ds^4} - \alpha^4w^{(j)} = 0, \quad \sigma\alpha^4 = m^6\lambda - \mu^4m^8, \quad j = 1, 2, \ldots, n.
\]

System of equations (3) has the order \( 4n \) and its solutions can not satisfy all \( 8n \) boundary conditions of the initial eigenvalue problem. The problem of the extracting the boundary conditions for equations (3) out of the boundary conditions for equations (1) is discussed in \([7]\).

The solutions of equations (3) must satisfy \( 4n_r \) continuity conditions on the parallels \( s = s_i \):

\[
w^{(i)}_0 = w^{(i+1)}_0, \quad w^{(i)}_0 = w^{(i+1)}_0, \quad w^{(i)}_0 = w^{(i+1)}_n, \quad w^{(i)}_0 - w^{(i+1)}_n = -cw^{(i+1)}_0,
\]

\[
c = m^8I/h,
\]

where \( I \) is the dimensionless moment of inertia of a ring cross-section with respect to the generatrix of the cylinder. The dimensionless sizes of the T-shaped cross-sections are shown in Fig. 2. The moment of inertia of the T-shaped cross-sections is

\[
I = a^4k^3[1 - (1 - k_a)(1 - k_b)^3]/3.
\]

Here \( k = b/a, k_a = a_1/a, k_b = b_1/b, a_1 \) if \( k_a = 1 \) or \( k_b = 1 \) then the cross-section is the rectangle.
The boundary conditions for equations (3) at the freely supported shell edges have the form
\[ w_0^{(1)}(0) = w_0^{(1)\prime}(0) = w_0^{(n)}(l) = w_0^{(n)\prime}(l) = 0. \] (5)

The eigenvalue problem (3–5) also describe the flexural vibrations of the simply supported beam, stiffened by \( n_r \) identical springs of the stiffness \( c \) at the points \( s = s_i \).

From the second formula (3), it follows that the approximate values of the loading parameter \( \lambda \) are
\[ \lambda_k(m) = \sigma \alpha_k^4(m)/m^6 + \mu^4 m^2, \quad k = 1, 2, \ldots, \]
where \( 0 < \alpha_1 \leq \alpha_2 \leq \cdots \) are eigenvalues of the eigenvalue problem (3–5). The minimal loading parameter \( \lambda_c \), corresponding to the critical pressure \( p_c \), can be obtained by the formula
\[ \lambda_c = \min_m [\sigma \alpha_1^4(m)/m^6 + \mu^4 m^2]. \] (6)

Assume that rings are uniformly arranged, i. e. \( s_i = il/n \). In this case eigenvalue problem (3–5) has stiffness-independent solutions
\[ w_0 = \sin(n\pi s/l), \quad \alpha_0 = \pi n/l. \]
Corresponding to this solution the minimal stiffness-independent eigenvalue is
\[ \lambda_n = \min_m [\sigma \alpha_0^4(m)/m^6 + \mu^4 m^2] \simeq \frac{4\pi n^2 \sigma^{1/4} \mu^3}{3l^{3/4}}. \] (7)

If the number of rings \( n_r \) is large and the stiffness \( c \) of rings is small, one can use the homogenization method [8] for the approximate evaluation of the eigenvalues \( \alpha_k \). After the homogenization we get the approximate equation
\[ \frac{d^4w_0}{ds^4} - \left( \alpha^4 - \frac{c n}{l} \right) w_0 = 0. \] (8)

The boundary conditions for equation (8) are
\[ w_0 = \frac{d^2w_0}{ds^2} = 0, \quad s = 0, \quad s = l. \] (9)

Eigenvalue problem (8, 9) describes the vibrations of a simply supported beam on an elastic base and has the solutions \( w_0 = \sin(\pi x) \),
\[ \alpha_1^4(\eta) = \frac{\pi^4}{l^4} + \frac{\eta \mu^4 m^8}{\sigma}, \quad \eta = \frac{\sigma cn}{m^8 \mu^4 l} = \frac{\sigma n I}{\mu^3 h l} = \frac{n D_r}{l D}, \] (10)
where \( \eta \) is the relative ring stiffness, \( D_r = EI \) is the bending stiffness of the ring and \( D = Eh^3/(12\sigma) \) is the bending stiffness of the shell.
Substituting \((10)\) into \((6)\) we get the following approximate formula
\[\lambda_1(\eta) = \lambda_1(0)(1 + \eta)^{3/4},\] (11)
where \(\lambda_1(\eta)\) and
\[\lambda_1(0) = \frac{4\pi \sigma^{1/4} l^3}{3^{3/4} l}\]
are the approximate values of the loading parameters for the ring-stiffened and the non-stiffened cylindrical shells accordingly.

The relation \(\lambda_c \simeq \lambda_1(\eta)\) is valid only if the relative ring stiffness \(\eta\) is not too large, because \(\lambda_c \simeq \lambda_n = n\lambda_1(0)\) for the large values of \(\eta\). Therefore,
\[\lambda_c(\eta) \simeq \begin{cases} 
\lambda_1(0)(1 + \eta)^{3/4}, & 0 \leq \eta \leq \eta_*, \\
n\lambda_1(0), & \eta > \eta_* 
\end{cases}\] (12)
where \(\eta_* = n^{4/3} - 1\) is the root of the equation \(\lambda_1(\eta) = \lambda_n\).

4 OPTIMAL PARAMETERS

We suppose that the mass \(M_s\) of a ring-stiffened simply supported cylindrical shell is given and seek the optimal parameters for which the external critical pressure \(p_c\) has the largest value.

For the unstiffened simply supported cylindrical shell of the thickness \(h_0\) and length \(l\) the external critical pressure \(p_0\) can be found by the approximate formula \([7]\)
\[p_0 = \frac{Eh_0}{\sigma} \lambda_0, \quad \lambda_0 = \frac{4\pi \sigma^{1/4}}{l} \left(\frac{h_0}{6}\right)^{3/2}.\] (13)
The mass of the unstiffened shell \(M_0 = 2\pi \rho l h_0 R^3\). Assume that \(M_s = M_0\). The values of the ring-stiffened shell parameters for which the ratio \(f = p_c/p_0\) attains its maximum are called the optimal values.

The approximate values of optimal parameters for the cylindrical shell stiffened by the rings of rectangular cross-sections are obtained in \([8]\). Consider the cylindrical shell stiffened by the rings of the T-shaped cross-sections.

The mass of a cylindrical shell stiffened by the rings of the T-shaped cross-sections is \(M_s = 2\pi R^3 \rho (hl + n_r S)\), where \(S = a^2 k(k_a + k_b - k_a k_b)\) is the area of the ring cross-section. It follows from the condition \(M_s = M_0\) that
\[z = 1 - Aa^2, \quad A = \frac{n_r k(k_a + k_b - k_a k_b)}{h_0},\] (14)
where \(z = h/h_0\), \(h\) is the thickness of the ring-stiffened shell.

Using formula \((12)\) we obtain
\[f = \frac{p_c}{p_0} = \frac{z^{5/2} \lambda_c}{\lambda_1(0)} \simeq z^{5/2} \begin{cases} 
(1 + \eta)^{3/4}, & 0 \leq \eta \leq \eta_*, \\
n, & \eta > \eta_* 
\end{cases},\] (15)
Taking into account that
\[\eta = \frac{12\sigma n I}{h^3 l} = \frac{Ba^4}{z^3} = \frac{B(1 - z)^2}{A^2 z^3}, \quad B = \frac{4\pi k^3 [1 + (1 - k_a)(1 - k_b)^3]}{h_0^3 l},\] (16)
from relation (15) we obtain
\[
f(z) \simeq z^{5/2} \begin{cases} 
  n, & 0 < z \leq z_s, \\
  [1 + \gamma(z - 1)^2/z^3]^{3/4}, & z_s < z \leq 1,
\end{cases}
\]
where \( \gamma = B/A^2 \) and \( z_s \) is the root of the cubic equation
\[
g(z) = z^3 - q(z - 1)^2 = 0, \quad q = \gamma/\eta_s.
\]
As was to be proved in [8], for the small values of \( h_0 \)
\[
f_\ast = \max_{0 < z < 1} f(z) = f(z_\ast) = nz_s^{5/2},
\]
and the optimal values of the parameters \( z \) and \( a \) are \( z_\ast \) and \( a_\ast = \sqrt{(1 - z_\ast)/A} \).

Let \( l = 4, h_0 = 0.01, \nu = 0.3 \). The values of \( z_\ast, a_\ast \) and \( f_\ast \) for various numbers of rings \( n_r \)
are given in Table 1. In left and right parts of the table are listed the results for the rings of the square cross-section \((k = 1, k_a = 1)\) and the T-shaped cross-section \((k = 1, k_a = k_b = 0.2)\) correspondingly. From Table 1 it follows that the replacement of the rings of the square cross-
sections to the rings of the T-shaped cross-sections leads to an increase in the critical pressure of the stiffened shell without the increase in its mass. For ten rings this increase reaches 39%.

Using formulas (14), (16) and (17) we obtain
\[
A = A_1k, \quad A_1 > 0, \quad B = B_1k^3, \quad B_1 > 0, \quad q = B/\eta_s A^2 = B_1k/\eta_s A_1^2, \quad \frac{dq}{dk} = \frac{B_1}{\eta_s A_1^2} > 0,
\]
\[
\frac{dq}{dz_\ast} = \frac{z_\ast^2(3 - z_\ast)}{(1 - z_\ast)^3} > 0.
\]
Therefore,
\[
\frac{df_\ast}{dk} = \frac{5n}{2} z_\ast^{5/2} z_\ast dz_\ast = \frac{dz_\ast}{dk} \frac{dz_\ast}{dq} > 0,
\]
and the function \( f_\ast(k) \) increases with \( k \). However, for large values of \( k \) formula (18) is not valid because a wide ring must be considered as an annular plate.

5 WIDE RINGS

We consider the wide rings of the T-shaped cross-section as annular plates of the thickness \( a_1 \)
and the width \( \varepsilon = b - b_1 \) stiffened at one edge by a circular beam. The beam has the rectangular cross-section of the sizes \( a \) and \( b_1 \) (see Fig. 2).
The equations describing the transverse flexural deformation of the plate have the form [9]

\[
(rQ_{1p})' + mQ_{2p} = rT_{1p}w'' + T_{2p}(w'_p - r^{-1}m^2w_p),
\]

\[
rQ_{1p} = (rM_{1p})' - M_{2p}, \quad rQ_{2p} = -mM_{2p} + 2H_p,
\]

\[
rM_{1p} = \mu'_p[r\vartheta'_p + \nu(m\vartheta_{2p} + \vartheta_{1p})], \quad rM_{2p} = \mu'_p(m\vartheta_{2p} + \vartheta_{1p} + \nu r\vartheta'_p),
\]

\[
H_p = \mu'_p(1 - \nu)\vartheta'_p, \quad \vartheta_{1p} = -w'_p, \quad r\vartheta_{2p} = mw_p.
\]

Here (′) denotes the derivative with respect to the radial coordinate, \( r \in [1, r_1], r_1 = 1 + \varepsilon \) is the outer radius of the plate, \( w_p \) is the transverse deflection, \( Q_{1p}, Q_{2p}, M_{1p}, M_{2p}, H_p \) are the dimensionless stress-resultants and stress-couples, \( T_{1p} \) and \( T_{2p} \) are pre-buckling stress-resultants, \( \vartheta_{1p} \) and \( \vartheta_{2p} \) are the angles of rotation of the normal, \( \mu'_p = a_1^2/12 \) is a small parameter.

The tangential (in plane) deformation of the plate describe the following equations [9]:

\[
(rT_{1p})' - T_{2p} + S_pr = 0, \quad rS'_p + 2S_p - mT_{2p} = 0,
\]

\[
rT_{1p} = ru'_p + \nu(mv_p + u_p), \quad rT_{2p} = u_p + mv_p + \nu v'_p,
\]

\[
2rS_p = (1 - \nu)(ru'_p - mu_p - v_p),
\]

where \( u_p \) and \( v_p \) are the tangential components of the displacement, \( T_{1p}, T_{2p}, S_p \) are the dimensionless stress-resultants.

At the parallel \( s = s_i \) the following continuity conditions have to be satisfied

\[
w^{(i)} = w^{(i+1)} = u_p, \quad u^{(i)} = u^{(i+1)} = -w_p, \quad v^{(i)} = v^{(i+1)} = v_p, \quad \vartheta^{(i)} = \vartheta^{(i+1)} = \vartheta_{1p},
\]

\[
h(T^{(i+1)}_1 - T^{(i)}_1) = a_1Q_{1p}, \quad h(S^{(i+1)}_1 - S^{(i)}_1) = -a_1S_{1p},
\]

\[
h(M^{(i+1)}_1 - M^{(i)}_1) = -a_1M_{1p}, \quad h(Q^{(i+1)}_1 - Q^{(i)}_1) = -a_1T_{1p},
\]

where \( u_p, w_p, \vartheta_p, \ldots \) are the solutions of equations (19) and (20).

For the simply supported shell

\[
T_{11}^{(1)} = V^{(1)} = W^{(1)} = M_{11}^{(1)} = 0, \quad s = 0, \quad T_{11}^{(n)} = V^{(n)} = W^{(n)} = M_{11}^{(n)} = 0, \quad s = l.
\]

At the outer plate edge \( r = r_1 \), stiffened by circular beam, following conditions should be fulfilled

\[
T_{1p} + T + m^2M_x = 0, \quad S_p + m(T + M_x) = 0,
\]

\[
M_{1p} - M_y - mM_k = 0, \quad Q_{1p} - m(mM_y + M_k) + T_{1p}\vartheta_{1p} = 0,
\]

where

\[
T = \frac{\sigma}{a_1}S_b(mv_p + u_p), \quad M_x = \frac{\sigma}{a_1}I_xm(mu_p + v_p),
\]

\[
M_y = -\frac{\sigma}{a_1}I(m^2w_p + \vartheta_{1p}), \quad M_k = \frac{1 - \nu}{2a_1}I_km(w_p + \vartheta_{1p}),
\]

\[
S_b = ab_1, \quad I_x = \frac{ab_1^3}{12}, \quad I_y = \frac{a^3b_1}{12}, \quad I_k \simeq \frac{4I_xI_y}{I_x + I_y}.
\]

To solve the eigenvalue problem (1), (19)-(23), it is necessary to find the pre-buckling stress state.

### 6 PRE-BUCKLING STATE

Let us consider the axisymmetric deformation of a cylindrical shell stiffened by identical annular plates under the uniform external pressure \( p \). The non-dimensional equations describing the axisymmetric deformation of the cylindrical shell can be written as [9]

\[
T'_1 = 0, \quad Q'_1 + T_2 + \lambda = 0, \quad Q_1 = -M'_1,
\]

\[
M_1 = \mu^3w'', \quad T_1 = w' - \nu w, \quad T_2 = \nu w' - w.
\]
We use in equations (24) the same symbols as in equations (1), but replace $Q_1$ and $w$ on $-Q_1$ and $-w$.

Taking $v_p = S_p = m = 0$ in equations (20), we obtain the following non-dimensional equations for a tangential axisymmetric plate deformation

\[(rT_{1p})' - T_{2p} = 0, \quad rT_{1p} = ru'_p + \nu u_p, \quad rT_{2p} = u_p + \nu ru'_p.\]  \tag{25}

The transverse bending stiffness of a plate is not taken into account because it is much smaller than the tangential plate stiffness. In the case $w_\ell = \psi_1 = M_1 = Q_1 = m = 0$ conditions (21)-(23) take the form

\[w^{(k)} = w^{(k+1)} = -u_p, \quad \psi_1^{(k)} = \psi_1^{(k+1)}, \quad T_1^{(k+1)} = T_1^{(k)}, \quad M_1^{(k+1)} = M_1^{(k)},\]

\[h(Q_1^{(k+1)} - Q_1^{(k)}) = a_1 T_{1p}, \quad s = s_k,\]

\[T_1^{(1)} = w^{(1)} = M_1^{(1)} = 0, \quad s = 0, \quad T_1^{(n)} = w^{(n)} = M_1^{(n)} = 0, \quad s = l.\]  \tag{26}

Let us seek a solution to equations (24) as the sum of the membrane state and the edge effect. In particular,

\[w^{(k)} = w^{(k)}_a + w^{(k)}_b, \quad w^{(k)}_b = \sum_{j=1}^{4} D_j^{(k)} f_j^{(k)}(s).\]  \tag{28}

Here $w^{(k)}_a = \lambda/\sigma$ is a solution of the membrane equations, $D_j^{(k)}$ are the arbitrary constants,

\[f_j^{(k)} = e^{\alpha_j(s - s_k - 1)}, \quad j = 1, 2, \quad f_j^{(k)} = e^{\alpha_j(s - s_k)}, \quad j = 3, 4,\]

\[\alpha_{1,2} = -\frac{q}{\mu \sqrt{2}}(1 \pm i), \quad \alpha_{3,4} = -\alpha_{1,2}, \quad q = \sigma^{1/4}.\]

The equations for an axisymmetric membrane deformation hold, taking $M_1 = 0$ in equations (24). Its solution satisfy the boundary conditions $T_1^{(0)} = T_1^{(l)} = 0$. The edge effect functions $f_1, f_2$ and $f_3, f_4$ are localized in a neighborhood of the parallels $s_{k-1}$ and $s_k$ correspondingly.

The general solution of equations (25)

\[u = C_1 r + C_2 / r, \quad T_{1p,2p} = \gamma C_1 \mp \delta C_2 / r^2,\]  \tag{29}

where $\gamma = 1 + \nu, \delta = 1 - \nu$, depends on two arbitrary constants $C_1$ and $C_2$. It follows from condition (27) that

\[C_2 = C_1 r_1^2 K, \quad K = \frac{\gamma r_1 a_1 + \sigma S_b}{\delta r_1 a_1 - \sigma S_b},\]

Hence

\[u_p(1) = C_1 + C_2 = C_1(1 + r_1^2 K), \quad T_{1p}(1) = \gamma C_1 - \delta C_2 = C_1(\gamma - \delta r_1^2 K).\]  \tag{30}

In the first approximation

\[T_1^{(k)} = 0, \quad \theta^{(k)} = \theta^{(k)}_b = -\frac{dw^{(k)}_b}{ds}, \quad M^{(k)} = -M^{(k)}_b = \mu \frac{d\theta^{(k)}_b}{ds}, \quad Q^{(k)} = Q^{(k)}_b = -\frac{dM^{(k)}_b}{ds}.\]  \tag{31}
Substituting relations (31) into first five boundary conditions (26) we obtain

\[ w_b^{(k)} = w_b^{(k+1)}, \quad \theta_b^{(k)} = \theta_b^{(k+1)}, \quad M_b^{(k)} = M_b^{(k+1)}, \quad hQ_b^{(k)} = hQ_b^{(k+1)} - a T_{1p}(1), \quad w_a^{(k)} + w_b^{(k)} = -u_p(1), \quad s = s_k. \]  

(32)

Assume that \( s_k - s_{k-1} \gg \mu \). Then

\[
\begin{align*}
w_b^{(k)}(s_k) & \simeq D_3 + D_4, \quad w_b^{(k+1)}(s_k) \simeq D_1 + D_2, \\
\theta_b^{(k)}(s_k) & \simeq -\alpha_3 D_3 - \alpha_4 D_4, \quad \theta_b^{(k+1)}(s_k) \simeq -\alpha_1 D_1 - \alpha_2 D_2, \\
M_b^{(k)}(s_k) & \simeq \mu^4(\alpha_3^2 D_3 + \alpha_4^2 D_4), \quad M_b^{(k+1)}(s_k) \simeq \mu^4(\alpha_1^2 D_1 + \alpha_2^2 D_2), \\
Q_b^{(k)}(s_k) & \simeq -\mu^4(\alpha_3^2 D_3 + \alpha_4^2 D_4), \quad Q_b^{(k+1)}(s_k) \simeq -\mu^4(\alpha_1^2 D_1 + \alpha_2^2 D_2),
\end{align*}
\]

where \( D_1 = D_1^{(k+1)}, \quad D_2 = D_2^{(k+1)}, \quad D_3 = D_3^{(k)}, \quad D_4 = D_4^{(k)} \). After the substitution (33) into conditions (32), linear algebraic equations containing unknown constants \( D_1, D_2, D_3, D_4 \) and \( C_1 \) arise. Excluding \( D_1, D_2, D_3 \) and \( D_4 \) from these equations we get

\[ C_1 = -\frac{\lambda}{A_p}, \quad A_p = \sigma(1 + r_1^2 K) + \frac{qa_1(\delta r_1^2 K - \gamma)}{2\sqrt{2}\mu h}. \]

(34)

It follows from relations (29) and (34) that the approximate formulas for pre-buckling stress-resultants \( T_{1p} \) and \( T_{2p} \) take the form

\[ T_{1p,2p} = C_1 \left( \gamma \mp \frac{\delta r_1^2 K}{r^2} \right), \quad C_1 = -\frac{\lambda}{A_p}. \]

(35)

7 FIRST APPROXIMATION FOR WIDE RINGS

In order to analyze the buckling of a cylindrical shell stiffened by identical annular plates under the external pressure \( p \) the same asymptotic approach is applied as for the solution of equations (24). The solution to equations (1) is searched as a sum of a membrane state and edge effect. Assume that the thickness of the plate \( a_1 \) and the thickness of the shell \( h \) have the same orders, i.e. \( \mu_p \sim \mu \). Then in the first approximation we get the eigenvalue problem for equations (19) with the boundary conditions (23) and

\[ w_p(1) = \vartheta_{1p}(1) = 0. \]

(36)

The eigenvalue problem (19), (23) and (36) describes the buckling of the annular plate stiffened by a circular beam and do not have an analytical solution.

Buckling of an annular plate joint with a circular beam under radial uniform tensile stresses \( \sigma_0 \) at the inner plate edge was studied in [10]. In this case formulas for pre-buckling stress-resultants \( T_{1p} \) and \( T_{2p} \) take the form

\[ \hat{T}_{1p,2p} = \hat{C}_1 \left( \gamma \mp \frac{\delta r_1^2 K}{r^2} \right), \quad \hat{C}_1 = -\frac{\beta \mu_p^4}{\gamma(r_1^2 - 1)}, \quad \beta = \frac{(1 - \nu^2)\sigma_0}{\mu_p^4 E}. \]

(37)

If \( C_1 = \hat{C}_1 \) then \( T_{1p,2p} = \hat{T}_{1p,2p} \) and eigenvalue problem (19), (23), (36) and eigenvalue problem [10] coincide. It follows from the equality \( C_1 = \hat{C}_1 \) that

\[ \lambda = \frac{\mu_p^4 A_p}{\gamma(r_1^2 - 1)} \beta. \]

(38)
To find the approximate value of the buckling stress parameter $\beta_c$ the Rayleigh-Ritz method and the shooting procedure in [10] are used. In particular, for the plate with the parameters $\varepsilon = 0.1$, $\nu = 0.3$, $a_1 = 0.01$ stiffened by the beam of a square cross-section the critical load $\beta_c$ vs. the dimensionless sizes of the beam cross-section $a$ was found.

Using results of [10] and formula (38) the dimensionless critical pressure $\lambda_c$ for the cylindrical shell stiffened by rings of the T-shape cross-section can be obtained. In Fig. 3 for the shell and the ring with the parameters $h = 0.04$, $a_1 = 0.01$, $\nu = 0.3$, $\varepsilon = 0.1$, $b_1 = a$ the critical pressure $\lambda_c$ vs. the size of the beam cross-section $a$ is plotted.

\[ \lambda_c \]

Value $a = 0$ corresponds to the ring of the rectangular cross-section. The critical pressure $\lambda_c$ increases with the size of the beam cross-section $a$.

8 CONCLUSIONS

Simple approximate formulas for the critical pressure of the cylindrical shell stiffened by narrow rings of T-shape cross-section are obtained by means of asymptotic methods. It was shown that the replacement of the rings of the rectangular cross-section to the rings of the T-shaped cross-section can lead to increase in the critical pressure of the ring-stiffened shell without the increase in its mass.

For the cylindrical shell stiffened by wide rings of T-shape cross-section the buckling problem can be reduce to the eigenvalue problem describing buckling of the annular plate stiffened by a circular beam. The critical radial tensile stress $\sigma_c$ for this problem were found in [10]. The asymptotic analyses of axisymmetric deformation of a cylindrical shell stiffened by annular plates under the uniform external pressure allows to obtain the approximate formula (38), which connects the critical pressure $p_c$ with the critical stress $\sigma_c$.

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REFERENCES


ON AN ATTRACTION BASIN OF THE GENERALIZED KAPITSAS PROBLEM

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\textbf{Keywords:} Kapitsa’s pendulum, stability, attraction basin, two-scale asymptotic expansions, harmonic and random vibrations.

\textbf{Abstract.} The classic Kapitsa’s problem and its various generalizations about a stability of an inverted pendulum under action of vertical vibrations of support are investigated. By using asymptotic method of two-scale expansions the level of vibrations such, that the vertical position of rod is stable, are found. An attraction basin of a vertical position of rod in the case when this position is stable is found. The both harmonic and random stationary vibrations of support are considered.
1 INTRODUCTION

The stability of a vertical position of rod under vertical vibration of the support has been investigated in various statements. A simple problem of this sort is concerned with stability of the inverted pendulum under action of the support vibrations. Firstly this problem was solved by A. Stephenson [1], and the more detailed theoretical and experimental investigation was performed by P.L. Kapitsa [2]. The various generalizations of this problem and some approximate methods of the solution are contained in the book by I.I. Blekhman [3]. In our recent works [4, 5] a vertical deformable rod with a free upper end and with a clumped or a simply supported lower end is considered. A vertical position of a simply supported rod is always unstable as a position of a classic Kapitsas pendulum, and as it was established by L. Euler a vertical position clamped rod under its weight is unstable if the rod is long enough. The support is subjected to harmonic vibrations. The level of vibrations such, that the vertical position of rod is stable, are found. Both bending and longitudinal vibrations are taken into account [4, 5], but here we are bounded with the bending vibrations, assuming that the rod is inextensible. To describe the bending vibrations the model of Bernoulli-Euler beam is applied. In order to determine the critical value of the vibration the two-scaled asymptotic expansions are used [6].

In all works [1, 2, 4, 5] only the stability of a vertical position is investigated, and the problem is reduced to linear differential equations. Here we seek an attraction basin of the stable vertical position. We begin with the classic Kapitsas problem, apply an averaging procedure [6] to obtain a nonlinear ODE that solves this problem. Then we go to a flexible rod with a clamped lower end and consider the case when its vertical position is unstable under the weight. According to the Euler's investigations the rod axis takes some stable curvilinear form. We take this stable position as the initial one and try to answer the question: when the rod takes a vertical position under action of vertical harmonic support vibration. The problem is reduced to a nonlinear PDE system that solution is difficult. For this reason we model the rod as a system with several degree of freedom. As a first step we take one degree of freedom and consider the deformable rod as a rigid rod with an elastic lower support. If the support stiffness is small enough then the vertical position is unstable and this rigid rod can model the deformable rod.

The second problem under discussion is the following. In all previous works the support vibrations is assumed to be harmonic. Now we shortly consider the classic Kapitsas problem in the case in which the vertical support vibration is a stationary random process. Similar to the case of high level of vibrations the vertical rod position is stable, and we determine the corresponding attraction basin.

2 KAPITSA’S PENDULUM

We consider a pendulum as a thin homogeneous rigid rod of length $L$. Its motion in the movable co-ordinate system is described by the equation

$$J\ddot{\varphi} + n_0 \dot{\varphi} - (mL/2)(g - a\omega^2 \sin(\omega t_0 + \beta)) \sin \varphi = 0,$$  \hspace{1cm} (2.1)\

where $\varphi(t_0)$ is the angle between a rod and a vertical axis; $n_0$, $J = mL^2/3$, $m$, $g$ are the damping coefficient, the inertia moment of a rod, its mass, and the gravitational acceleration; respectively, $a$, $\omega$, $\beta$ are the amplitude, frequency and initial phase of the support vibration. A derivative with respect to time $t_0$ is denoted by a dot.

We present Eq.(2.1) in the dimensionless form that is convenient for the following analysis:

$$\dot{\varphi} + \varepsilon n_0 \dot{\varphi} - (\varepsilon^2 q - \varepsilon \sin(t + \beta)) \sin \varphi = 0,$$  \hspace{1cm} (2.2)
where
\[ t = \omega t_0, \quad n = \frac{2n_0}{mL\omega}, \quad q = \frac{2Lg}{3a^2\omega^2}, \quad \varepsilon = \frac{3\delta}{2}, \quad \delta = \frac{a}{L}. \] (2.3)

Here \( q \) is the loading parameter, \( \varepsilon \) is the small parameter. As above, the derivative with respect to \( t \) is denoted by a dot. We introduce the relative acceleration of the support vibration \( \zeta \) in the form
\[ \zeta = \frac{a\omega^2}{g} = \frac{2}{3\delta q}. \] (2.4)

For small \( \varphi \) (namely, for \( \sin \varphi \approx \varphi \)) Eq.(2.2) is the Mathieu equation. The solution \( \varphi \equiv 0 \) for small \( \varepsilon \) is stable if \[ q < \frac{1}{2}, \quad \zeta > \frac{4}{3\delta}. \] (2.5)

Now we proceed to the attraction basin of this solution and consider the Cauchy problem consisting of Eq.(2.2) and the initial conditions
\[ \varphi(0) = \varphi_0, \quad \dot{\varphi}(0) = 0. \] (2.6)

We seek an asymptotic solution of Eq.(2.2) as a two-scale expansion [6]:
\[ \varphi(t, \theta, \varepsilon) = \sum_{m=0}^{\infty} (U_m(\theta) + V_m(t, \theta))\varepsilon^m, \quad \int_0^{2\pi} V_m(t, \theta)dt = 0, \quad m = 0, 1, \ldots, \] (2.7)

where \( \theta = \varepsilon t \) is the slow time and
\[ \dot{\varphi} = \frac{\partial \varphi}{\partial t} + \varepsilon \frac{\partial \varphi}{\partial \theta}, \quad \ddot{\varphi} = \frac{\partial^2 \varphi}{\partial t^2} + 2\varepsilon \frac{\partial^2 \varphi}{\partial t \partial \theta} + \varepsilon^2 \frac{\partial^2 \varphi}{\partial \theta^2}. \] (2.8)

An expansion of Eq.(2.2) in powers of \( \varepsilon \) yields consecutively
\[ V_0(t, \theta) = 0, \quad V_1(t, \theta) = \sin U_0 \sin(t + \beta), \quad \frac{\partial^2 V_2}{\partial t^2} + H(\theta, t) = 0, \] (2.9)

with
\[ H = 2 \frac{\partial^2 V_1}{\partial t \partial \theta} + \frac{d^2 U_0}{d\theta^2} + n \frac{dU_0}{d\theta} - q \sin U_0 + (U_1 + V_1) \cos U_0 \sin(t + \beta). \] (2.10)

According to (2.7) the average value in \( t \) of function \( H(t, \theta) \) is to be equal zero that gives an equation for function \( U_0(\theta) \)
\[ \frac{d^2 U_0}{d\theta^2} + n \frac{dU_0}{d\theta} + F(U_0) = 0, \quad F(U_0) = ((1/2) \cos U_0 - q) \sin U_0. \] (2.11)

Due to relation \( \dot{\varphi} = \varepsilon (dU_0/d\theta + \partial V_1/\partial t) + O(\varepsilon^2) = 0 \), we solve Eq.(2.11) with initial conditions
\[ U_0 = \varphi_0, \quad U_0' = dU_0/d\theta = -\sin \varphi_0 \cos \beta \quad \text{for} \quad \theta = 0. \] (2.12)

The problem (2.11), (2.12) is the zero asymptotic approximation of the exact problem (2.2), (2.6).
For a definiteness we take $\alpha = 0.01, n = 0.1$ and for some values $\varphi_0$ and $\beta$ we find $q_*(\varphi_0, \beta)$ such that for $q < q_*(f_0, \beta)$ the limiting equality

$$\varphi(t) \to 0 \text{ at } t \to \infty,$$  \hspace{1cm} (2.13)

is valid, whereas in the opposite case $q > q_*(f_0, \beta)$ Eq.(2.13) is not fulfilled. The boundary $q_*(\varphi_0, \beta)$ depends on the initial phase $\beta$ which is unknown in the general case. That is why we introduce two attraction basins in the plane of parameters $(\varphi_0, q)$

$$G_a(\varphi_0) : \quad q < q_-(\varphi_0), \quad q_-(\varphi_0) = \min q_*(\varphi_0, \beta), \quad \text{at } \beta \in [0, 2\pi)$$

$$G_p(\varphi_0) : \quad q_-(\varphi_0) < q < q_+(\varphi_0), \quad q_+(\varphi_0) = \max q_*(\varphi_0, \beta), \quad \text{at } \beta \in [0, 2\pi).$$  \hspace{1cm} (2.14)

see Fig.1. Eq.(2.13) is fulfilled for all values $\beta$ in basin $G_a$; it is fulfilled only for some values $\beta$ in basin $G_p$, and it is newer fulfilled in part $G_0$ of plane $(\varphi_0, q)$ Eq.(2.13).

![Figure 1: The attraction basins.](image1)

The boundaries $q_-^-(\varphi_0)$ and $q_+^-(\varphi_0)$ are numerical solutions of the exact problem (2.2), (2.6). The approximate problem (2.11), (2.12) gives the close results (the corresponding curve $q_-^-(\varphi_0)$ in Fig.1 is shown as a dashed line, and the difference between the exact and approximate curves is so small that it is impossible to see it in figure $q_+^+(\varphi_0)$).

Eq.(2.11) is convenient for qualitative analysis in the phase plane $(U_0, U_0')$. The trajectories $U_0(\theta), U_0'(\theta)$ for $q = 0.3, n = 0$ are shown in Fig.2. A bold curve separates the attraction basin while the possible values $|U_0'| \leq |\sin U_0|$ are marked by dashed lines.

![Figure 2: The attraction basin in the phase plane $(U_0, U_0')$ at $q = 0.3$.](image2)
3 KAPITSA’S PROBLEM FOR THE FLEXIBLE ROD

We consider a vertical flexible inextensible rod with a clamped lower end and a free upper end. The long rod buckles under its weight $P$ at $P = P_\ast = \lambda_\ast D/L^2$, $\lambda_\ast = 7.8373$, where $D$ is the bending stiffness, and $L$ is length [4,5].

For $P > P_\ast$ the rod takes curvilinear forms that are shown in Fig.3 for some values of a parameter $\lambda = PL^2/D$. These forms are obtained from the equations [8]:

$$D \frac{d\phi}{ds} = M = \rho \int_s^L (x(s_1) - x(s)) \, ds_1, \quad \frac{dx}{ds} = \sin\phi, \quad \frac{dz}{ds} = \cos\phi, \quad (3.1)$$

where $\phi(s)$ is the angle between the rod tangent and the vertical; $s$, $M$, $\rho$ are the length of curve, the bending moment, and the weight of a rod init length, respectively. After simplification Eqs.(3.1) is reduced to the boundary-value problem

$$\frac{d^2\phi}{ds^2} + \lambda (1-s) \sin\phi = 0, \quad \phi(0) = 0, \quad \phi'(1) = 0, \quad \lambda = \frac{PL^2}{D} > \lambda_\ast. \quad (3.2)$$

It is established [4,5], that at intensive harmonic vertical support vibration the vertical rod position becomes stable. The following approximate condition for the dimensionless acceleration $\zeta$ ensuring stability is obtained from under condition

$$\zeta > \frac{1.26}{\delta} \left( 1 - \frac{\lambda_\ast}{\lambda} \right), \quad \zeta = \frac{a\omega^2}{g}, \quad \delta = \frac{aL}{D}. \quad (3.3)$$

We want to seek an attraction basin of a vertical position of rod in the case when this position is stable. More definitely, as an initial conditions we take one of the stable positions, shown in Fig.3, and seek the answer to the following question: is it possible to return this initial position to a vertical position by means of the support vibration.

A motion of an inextensible rod under action of vertical vibrations of support are described by equation:

$$D \frac{\partial \phi}{\partial s} = M(s,t) = \int_s^L (F_x(s_1)(z(s_1) - z(s)) - F_z(s_1)(x(s_1) - x(s))) \, ds_1, \quad (3.4)$$

Figure 3: Equilibrium forms of rod after a stability loss.
with unknown angle $\varphi(s, t)$, and
\[
x(s) = \int_0^s \sin \varphi \, ds, \quad z(s) = \int_0^s \cos \varphi \, ds, \quad F_x = -\rho \ddot{x}, \quad F_z = -\rho(g + a\omega^2 \sin \omega t + \ddot{z}).
\] (3.5)

In the static case (with $a = 0$) from Eq.(3.4) it follows Eq.(2.1).

To solve Eq.(3.4) the approximate method elaborated in [8] can be used. In the framework of this approach the flexible rod is considered as a system of rigid rods connected by elastic ties. However the complete solution of the corresponding system with several degrees of freedom is beyond the scope of this paper. For this reason in the first approximation we consider a model problem with a single degree of freedom.

4 A KAPITSA’S PENDULUM ON THE FLEXIBLE SUPPORT

Let us consider a rigid rod with an elastically supported lower end. In terms of the dimensionless variables (2.3) the motion of rod on vibrating support is described by the equation
\[
\ddot{\varphi} + n\varphi + \alpha^2(b\varphi - q \sin \varphi) + \alpha \sin \varphi \sin(t + \beta) = 0, \quad b = \frac{4b_0 L}{3ma^2\omega^2}.
\] (4.1)

In addition to Eq.(2.2) describing the classic Kapitsa’s pendulum, the bending support stiffness $b_0$ is introduced.

At $b < q$ and $a = 0$ the vertical rod position is unstable. The rod is stable at $\varphi = \varphi_0$, where $\varphi_0$ is the root of equation
\[
b\varphi_0 = q \sin \varphi_0, \quad \text{or} \quad b = kq, \quad k = \frac{\sin \varphi_0}{\varphi_0} < 1.
\] (4.2)

Now we seek the conditions ensuring stable vertical position in the presence of support vibration.

We seek a solution of Eq.(4.1) satisfying the initial conditions $\varphi(0) = \varphi_0$, $\dot{\varphi}(0) = 0$. We assume that the angle $\varphi_0 < \pi$ is a leading parameter of problem, and a stiffness parameter $b = kq$. As in Section 2, we use two-scale expansions, that in the first approximation for a slowly changing function $U_0(\theta)$ lead to Cauchy problem:
\[
U''_0 + nU'_0 + F(U_0) = 0, \quad U_0(0) = \varphi_0, \quad U'_0(0) = -\sin \varphi_0 \sin \beta,
\] (4.3)

with $F(U_0) = kqU_0 + ((1/2) \cos U_0 - q) \sin U_0$. At $n > 0$, $q < q^+_0 = 1/(2(1 - k))$ the solution $U_0(\theta) \equiv 0$ is asymptotically stable. As in Section 2, at $q < q^+_0$ we seek an attraction basin of this solution. A plane of parameters $(\varphi_0, q)$ consists of three parts $G_a, G_p, G_0$ and for $\varepsilon = 0.01$, $n = 0.1$, $q \leq 3$ it is shown in Fig. 4. At $q > 3$ the boundaries $q^-$ and $q^+$ coincide, and $q^- \approx q^+ \approx q^+_0$.

At $n = 0$ the trajectories
\[
\dot{U}^2_0 + 2 \int_0^{U_0} F(U) \, dU = C,
\] (4.4)

(with arbitrary constant $C$) in a phase plane $(U_0, U'_0)$ are symmetric with respect to axes $OU_0$ and $OU'_0$, and we consider a quarter part of plane $U_0, U'_0 \geq 0$. At $n > 0$ a point passes from one trajectory to another with the lower value of $C$. In Fig.5 the phase planes for $\varphi_0 = 1$ and for two values $q = 1$ and $q = 0.1$ are shown. The direction of decreasing values of $C$ is indicated by arrow. A set of possible values of $|U'_0|$ is marked by a vertical line. For $q = 1$ all $|U'_0|$ lie in the attraction basin of point $U_0 = U'_0 = 0$, therefore, for all values of the initial phase $\beta$ Eq.(2.13) is fulfilled, and $(\varphi_0, q) \in G_a$. For $q = 0.1$ only a part of values $|U'_0|$ lie in the attraction basin of point $U_0 = U'_0 = 0$ (that is separated by a bold line), and as a result we have $(\varphi_0, q) \in G_p$. 

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5 KAPITSA’S PROBLEM AT RANDOM EXCITATION

Let the vertical support vibration \( x_e(t_0) = \xi_0(t_0) \) be random, and \( \xi_0(t_0) \) be a stationary process with zero excitation and spectral density \( S_{\xi_0}(\lambda_0) \). We consider the problems of Section 2 for the case of random excitation. Eq.(2.1) reads as:

\[
J \frac{d^2\varphi}{dt_0^2} + n_0 \frac{d\varphi}{dt_0} - mL \left( g + \frac{d^2\xi_0}{dt_0^2} \right) \sin \varphi = 0.
\] (5.1)

We rewrite Eq.(5.1) in the dimensionless form, relating time \( t_0 \) to \( 1/\omega \) (\( \omega \) is the typical frequency of vibration of support), and relating excitation \( \xi_0(t_0) \) to the average amplitude of support vibration \( \sigma_{\xi_0} \):

\[
\ddot{\varphi} + \varepsilon n \dot{\varphi} - \left( \varepsilon^2 q + \varepsilon \dot{\xi} \right) \sin \varphi = 0,
\] (5.2)

where derivative with respect to \( t \) is denoted by a dot, and

\[
t = \omega t_0, \quad \xi_0(t_0) = \sigma_{\xi_0} \xi(t), \quad \sigma_{\xi_0}^2 = \int_{-\infty}^{\infty} S_{\xi_0}(\lambda_0) d\lambda_0, \quad \varepsilon = \frac{3\sigma_{\xi_0}}{2L}, \quad q = \frac{3Lg}{2\sigma_{\xi_0}^2 \omega^2}.
\] (5.3)

Here \( \varepsilon \) is a small parameter that is proportional to the average amplitude of the support vibration \( \sigma_{\xi_0} \) and \( \xi(t) \) is the normalized process with a unit dispersion. The spectral densities and the dispersions of \( \xi(t) \) and its derivatives are as follows:

\[
S_{\xi}(\lambda) = \frac{S_{\xi_0}(\lambda_0 \omega)}{\sigma_{\xi_0}^2}, \quad S_{\xi}^2(\lambda) = \lambda^2 S_{\xi}(\lambda), \quad S_{\xi}^4(\lambda) = \lambda^4 S_{\xi}(\lambda), \quad \sigma_{\xi}^2 = \int_{-\infty}^{\infty} S_{\xi}(\lambda) \lambda^2 d\lambda.
\] (5.4)
We solve Eq.(5.2) with the initial conditions \( \varphi(0) = \varphi_0, \; \dot{\varphi}(0) = 0 \), and use two ways for solving the problem.

One of them is a statistical simulation [10,11]. We model a random process \( \xi(t) \) as a sum of harmonic summands with random amplitudes and phases. For this aim we choose \( \Lambda \) so that the part of frequencies \( \lambda > \Lambda \) can be neglected and divide the interval \( 0 \leq \lambda \leq \Lambda \) by points \( \lambda_n, \; n = 1, \ldots, N \). Then the approximate realization of a random process \( \xi(t) \) read as:

\[
\xi(t) = \sum_{n=1}^{N} p_n (\eta_n \cos(\hat{\lambda}_n t) + \zeta_n \sin(\hat{\lambda}_n t)), \quad p_n = \sqrt{2S_\xi(\hat{\lambda}_n)(\lambda_n - \lambda_{n-1})}, \quad \hat{\lambda}_n = (\lambda_n + \lambda_{n-1})/2,
\]

where \( \eta_n \) and \( \zeta_n \) are the random independent standard Gaussian numbers (\( \mathbb{E}\eta_n = \mathbb{E}\zeta_n = 0, \mathbb{E}\eta_n^2 = \mathbb{E}\zeta_n^2 = 1 \), and \( \mathbb{E} \) denotes expectation). Then a numerical solution of Eq.(5.2) with the initial conditions (2.6) gives a realization of a random process \( \varphi(t) \).

As an example, we consider random process \( \xi(t_0) \) with the spectral density

\[
S_{\xi_0}(\lambda_0) = \frac{c_0}{(\lambda_0^4 + 2(\alpha_0^2 - \omega^2)\lambda_0^2 + (\alpha_0^2 + \omega^2)^2)(\lambda_0^2 + \lambda_2^2)},
\]

According to Eqs.(5.3),(5.4), for the dimensionless process \( \xi(t) \) the spectral density reads as:

\[
S_\xi(\lambda) = \frac{c}{(\lambda^4 + 2(\alpha^2 - 1)\lambda^2 + (\alpha^2 + 1)^2)(\lambda^2 + 1)}, \quad \lambda = \lambda_0/\omega, \; \alpha = \alpha_0/\omega,
\]

where constant \( c \) is to be found from the condition \( \sigma_\xi^2 = \int_{-\infty}^{\infty} S_\xi(\lambda) d\lambda = 1 \). We find \( \sigma_\xi^2 = (1 + \alpha^2)/(1 + 2\alpha) \). The constant \( c_0 \) is introduced so that the value of \( \sigma_{\xi_0}^2 \) and small parameter \( \varepsilon \) can be taken arbitrary.

We take the following values: \( \varepsilon = 0.01, \; n = 0.1, \; \alpha = 0.2, \; N = 200 \) and consider the case \( \varphi_0 > 0 \). The spectral density \( S_\xi(\lambda) \) of the normalized process \( \xi(t) \) is plotted in Fig.6. The maximum of \( S_\xi(\lambda) \) is close to \( \lambda = 1 \) and \( \sigma_\xi = 1 \). The attraction basins \( G_a \) and \( G_p \) obtained by a numerical solution of Eq.(5.2) are shown in Fig.7. In each numerical experiments we take 10 independent realizations of process \( \varphi(t) \). A point \( (\varphi_0, q) \) is included in \( G_p \) if at least one realization converges to zero at \( t \to \infty \), and at least one realization converges to \( \pm \pi \). Hence in the areas \( G_p \) and \( G_0 \) all (10) realizations tends to zero and to \( \pm \pi \) at \( t \to \infty \), respectively. The boundaries of \( G_p \) are denoted by \( \hat{q}^- \) and \( \hat{q}^+ \).

![Figure 6: Spectral density (left) and Attraction basins (right).](image-url)
The second way of analysis of Eq.(5.2) is applying the two-scale expansion (3.1):

\[ \varphi(t, \theta, \varepsilon) = U(\theta, \varepsilon) + V(t, \theta, \varepsilon), \quad U(\theta, \varepsilon) = \sum_{m=0}^{\infty} U_m(\theta)\varepsilon^m, \quad V(t, \theta, \varepsilon) = \sum_{m=0}^{\infty} V_m(t, \theta)\varepsilon^m \]  

(5.8)

where the average value \( V \) is equal to zero

\[ \langle V \rangle = \frac{1}{T} \int_0^T V(t, \theta, \varepsilon)dt = 0, \quad T = O(\varepsilon^{-1}). \]  

(5.9)

Repeating the calculations of Section 2, we successively obtain:

\[ V_0(t, \theta) = 0, \quad V_1(t, \theta) = \xi(t) \sin U_0, \quad \frac{\partial^2 V_2}{\partial t^2} + H(t, \theta) = 0, \]  

(5.10)

with

\[ H = 2\frac{\partial^2 V_1}{\partial t \partial \theta} + \frac{d^2 U_0}{\partial \theta^2} + n\frac{dU_0}{\partial \theta} - q \sin U_0 - (U_1 + V_1)\frac{d^2 \xi}{dt^2} \cos U_0. \]  

(5.11)

The condition \( \langle H \rangle = 0 \) leads to equation for function \( U_0(\theta) \)

\[ U''_0 + nU'_0 + (\chi \cos U_0 - q) \sin U_0 = 0, \quad \chi = -\langle \xi(t)\dot{\xi}(t) \rangle, \quad U_0(0) = \varphi_0. \]  

(5.12)

The second initial condition \( \varphi(0) = 0 \) due to Eq.(2.8) and Eq.(5.10) yields

\[ U'_0(0) = -\dot{\xi}(0) \sin \varphi_0. \]  

(5.13)

The problem (5.12), (5.13) contains two random values: \( \chi \) and \( \dot{\xi}(0) \), and we use this problem to estimate the attraction basins. To construct them we note that for Gaussian values with a probability 0.95 the following inequalities are valid:

\[ E(\chi) - 2\sigma_\chi \leq \chi \leq E(\chi) + 2\sigma_\chi, \quad -2(E(\chi) - 2\sigma_\chi)^{1/2} \leq \dot{\xi}(0) \leq 2(E(\chi) + 2\sigma_\chi)^{1/2}, \]  

(5.14)

where \( E(\chi) \) is the expectation of \( \chi \) and \( \sigma_\chi \) is the root-mean-square.

For the taken values of the random process (5.5) we obtain

\[ E(\chi) \approx -\frac{1}{T} \int_0^T \xi(t)\dot{\xi}(t)dt \approx \frac{1}{T} \int_0^T (\dot{\xi}(t))^2 dt \approx \sigma_{\dot{\xi}}^2 = 0.743. \]  

(5.15)

From Eq.(5.5) we have:

\[ \chi \approx \frac{1}{2} \sum_{n=1}^{N} \eta_n \dot{\xi}_n^2 + \zeta_n^2. \]  

(5.16)

Taking a large number (say, 10000) of random sets \( (\eta_n, \zeta_n, n = 1, \ldots, N) \) and using (5.16) we obtain the following value of root-mean-square \( \sigma_\chi = 0.157 \).

We put the upper and lower bounds of values of \( \chi \) and \( \dot{\xi}(0) \) in Eqs.(5.14) and obtain from Eq.(5.12) the boundaries \( q^-\varphi_0 \) and \( q^+\varphi_0 \) of attraction basins which are shown in Fig.7. For comparison, curve \( q^\pm\varphi_0 \) corresponding to the values \( \chi = \sigma_{\dot{\xi}}^2, \dot{\xi}(0) = 0 \) is also given there.

In particular, it follows from Eq.(5.12) that the vertical position (with a probability 0.95) is stable provided that

\[ q < \sigma_{\dot{\xi}}^2 - 2\sigma_\chi, \]  

(5.17)

and for taken values if \( q < 0.429 \).
6 CONCLUSIONS

A vertical position of the inverted pendulum can be stable not only under action of harmonic vertical support vibration but also under random vibrations. Attraction basins of a vertical position are found. Three problems are considered: (i) the classic Kapitsa’s pendulum, (ii) the pendulum with an elastic lower support, and (iii) the inverted pendulum under action of random support vibration. In all cases the attraction basin in the plane of parameters consists of two parts $G_a$ and $G_p$. In part $G_a$ the pendulum always approaches the vertical position. In part $G_p$ the limiting behavior of pendulum depends on the additional conditions, namely, in the problems (i) and (ii) the limiting behavior depends on the initial phase of harmonic support vibrations, while in the problem (iii) of random support vibration the limiting behavior depends on the realization of random excitation.

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REFERENCES


POST-BUCKLING DEFORMATION AND FRACTURE OF A STRETCHED PLATE WITH A CRACK

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Abstract. The tension of the thin plate with a rectilinear crack is considered. It is known that in the neighborhood of the crack appear compressive stresses, which at a certain level of tension could lead to local buckling of the plate. Of interest is the concentration of maximum tensile stresses in plates after buckling. The purpose of the study is to clarify the issue of whether buckling promotes crack growth or leads to the stabilization of deformations. For the analysis of the stress state of the plate in the initial post-critical stage the approximate analytical solution is suggested.

The post-buckling stretching the plate with central crack is analyzed by the finite element method using ANSYS code. The effect of the buckling under tension of a plate with a central crack on the level of the stress state is estimated. An analysis of the stress state in the vicinity of the tip of the central crack under uniaxial tension shows that with possible local stability loss near the crack an increase in tensile stresses in the vicinity of the crack tip is observed, and, as a result, load leading to destruction should be reduced.
1 INTRODUCTION

Thin-walled elements are widely used in various designs. When analyzing their bearing capacity, it is necessary to take into account not only the loads leading to their destruction, but also the loads under which occurs the loss of stability. It should be noted that the stability loss can occur both during compression of these elements, and during stretching in the presence of defects such as cuts and inclusions in them, since in the vicinity of these defects there are areas of compressive stresses, which can lead to local buckling.

The problem of the loss of the plane form of deformation of the plate, weakened by a crack, under uniaxial tension was studied in a number of papers ([1] - [9]). However, the question of post-buckling deformation and its effect on fracture did not receive a final answer.

In this regard, we can point to the work [8], in which the experimental results for stretching paper sheets with a central crack were presented and it was stated that after the plate buckling the stress intensity decreases in the vicinity of the crack tip. It means that a greater tensile loading is required than for the sheet that retains a plane shape of deformation. At the same time, experiments on stretching of metal sheets with a central crack show that local buckling in the vicinity of the crack leads to an increase in the stress concentration in the vicinity of the crack tips, i.e. to reduce the fracture load [7]. In the framework of this article the post-buckling deformation and fracture under tension of a plate with a crack are analyzed.

2 ANALYSIS OF THE STRESS-STRAIN STATE AND LOCAL LOSS OF STABILITY OF A PLATE WITH A CRACK

A crack of length $2a$ in an elastic isotropic homogeneous infinite plate with a thickness of $h \ll a$ is considered. The plate is at infinity evenly stretched by tension $T > 0$ in the direction perpendicular to the crack. The crack is stress free. The solution of this problem in the framework of the linear theory of elasticity is known [10]. We introduce a Cartesian coordinate system $xOy$ with origin located in the middle of the crack and the axis $Ox$ is directed along it.

The stress state is described by the formulas (2.1) for stresses $\sigma_{xx}$, $\sigma_{xy}$, $\sigma_{yy}$ [11]

\[
\frac{\sigma_{xx}}{T} = \frac{x\sqrt{r-b} + y\sqrt{r+b}}{\sqrt{2}r} - \frac{a^2y(r-2b)\sqrt{r+b}}{\sqrt{2}r^3} - 1,
\]

\[
\frac{\sigma_{yy}}{T} = \frac{x\sqrt{r-b} + y\sqrt{r+b}}{\sqrt{2}r} + \frac{a^2y(r-2b)\sqrt{r+b}}{\sqrt{2}r^3},
\]

\[
\frac{\sigma_{xy}}{T} = -\frac{a^2y(r+2b)\sqrt{r-b}}{\sqrt{2}r^3},
\]

where $b$ and $r$ are defined as follows

\[
b = a^2 - x^2 + y^2, \quad r = \sqrt{x^4 + 2x^2(y^2 - a^2) + (y^2 + a^2)^2},
\]

(2.2)

It follows from the formulas (2.1) that, in the vicinity of the crack, regions of compressive stresses arise, in which a local loss of stability can occur. Dimensionless stresses $s_{ij}^0 = s_{ij}/T$ at $a = 1$ in the vicinity of the crack $x = 1$, $y = 0$ (at $0 \leq x \leq 2$ and values $y = 0.05k$, $k = 1, \ldots 6$) are shown in Fig. 1. In the framework of the classical Kirchhoff-Love model, the stability of the plane shape of equilibrium is described by the equation

\[
D \Delta^2 w - T \left( \sigma_{xx}^0 \frac{\partial^2 w}{\partial x^2} + 2\sigma_{xy}^0 \frac{\partial^2 w}{\partial x \partial y} + \sigma_{yy}^0 \frac{\partial^2 w}{\partial y^2} \right) = 0, \quad D = \frac{Eh^3}{12(1-\nu^2)},
\]

(2.3)
where \( w(x, y) \) is the deflection, \( \Delta \) is the Laplace operator, \( \sigma_{ij} = T \delta_{ij} \).

On the crack \(-a < x < a, y = 0\), the following boundary conditions for \( w(x, y) \) are satisfied

\[
D \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^3 w}{\partial x^2 \partial y} \right) = 0, \quad D \left( \frac{\partial^3 w}{\partial y^3} + (2 - \nu) \frac{\partial^3 w}{\partial x^2 \partial y^2} \right) = 0, \tag{2.4}
\]

The critical value \( T^* \) of tension \( T \) can also be found from the variational problem \([12]\)

\[
T^* = \min_{w} \frac{\Pi_b}{\Pi_{uw}}, \tag{2.5}
\]

where \( \Pi_b \) is the bending energy at buckling, \( \Pi_{uw} \) is a complementary stretching energy

\[
\Pi_b = \frac{D}{2} \int \int \left( \kappa_1^2 + 2\nu\kappa_1\kappa_2 + \kappa_2^2 + 2(1 - \nu)\tau^2 \right) dx \, dy, \tag{2.6}
\]

\[
\Pi_{uw}^0 = \frac{1}{2} \int \int \left( \sigma_{xx}^0 w_x^2 + 2\sigma_{xy}^0 w_x w_y + \sigma_{yy}^0 w_y^2 \right) dx \, dy.
\]

Here \( \kappa_1 = w_{xx}, \kappa_2 = w_{yy}, \tau = w_{xy} \) are curvature and torsion of the median plane.

Here and further, integration is carried out over a quarter of the plane \( 0 \leq x, y < \infty \), because the form of stability loss is symmetric in \( x \) and in \( y \). The boundary conditions (2.4) are natural for the problem (2.5). Therefore, when choosing the coordinate functions of the Ritz method, they can be ignored. Boundary conditions on the \( x \)th axis are for symmetric shape

\[
\frac{\partial w}{\partial y} = \frac{\partial^3 w}{\partial y^3} = 0, \quad |x - a| > 0, \quad y = 0, \tag{2.7}
\]

and for asymmetrical shape

\[
w = \frac{\partial^2 w}{\partial y^2} = 0, \quad |x - a| > 0, \quad y = 0, \tag{2.8}
\]

In addition, the damping condition \( w \to 0 \) is satisfied for \( x^2 + y^2 \to \infty \). We consider the plate deflection in the linear approximation in the presence of small imperfections of the form \( w^0(x, y) \). The total deflection \( W = w_0 + w \) is the sum of the initial deflection \( w_0 \) and the additional deflection \( w \), generated by the action of tension \( T \), and the equilibrium equation has the form:

\[
D \Delta^2 w + T \left( \sigma_{xx}^0 \frac{\partial^2 W}{\partial x^2} + 2\sigma_{xy}^0 \frac{\partial^2 W}{\partial x \partial y} + \sigma_{yy}^0 \frac{\partial^2 W}{\partial y^2} \right) = 0. \tag{2.9}
\]

We consider the boundary-value problem of plane shape bifurcation

\[
D \Delta^2 w_k + T_k \left( \sigma_{xx}^0 \frac{\partial^2 w_k}{\partial x^2} + 2\sigma_{xy}^0 \frac{\partial^2 w_k}{\partial x \partial y} + \sigma_{yy}^0 \frac{\partial^2 w_k}{\partial y^2} \right) = 0, \tag{2.10}
\]
where \( T_k \) and \( w_k(x, y), \ k = 1, 2, \ldots \) are eigenvalues and eigenfunctions, moreover \( T_1 < T_2 \leq T_3 \leq \ldots \). We expand the initial and additional deflections into Fourier series in eigenfunctions of the problem (2.10)

\[
w^0(x, y) = \sum_{k=1}^{\infty} c_k^0 w_k(x, y), \quad w(x, y) = \sum_{k=1}^{\infty} c_k w_k(x, y).
\] (2.11)

After substituting the series (2.11) in the equation (2.9) and taking into account the orthogonality of the eigenfunctions, we can find the coefficients \( c_k = T c_k^0 / (T - T_k) \).

Then the additional deflection \( w(x, y) \) has the form:

\[
w(x, y) = \sum_{k=1}^{\infty} \frac{T c_k^0 w_k(x, y)}{T - T_k},
\] (2.12)

whence it means that with \( T \approx T_1 \) only the first term of the series (2.12) is essential and with the subsequent non-linear analysis it is possible to confine with the one-term approximation.

3 ASYMPTOTIC ANALYSIS OF THE INITIAL POST-CRITICAL DEFORMATION

From (2.12) it follows that for \( T = T_1 \) there will be \( w = \infty \). To obtain the finite values of the deflection, we consider the problem in a geometrically nonlinear formulation, in which the tangential deformations are as follows:

\[
\varepsilon_{11} = \varepsilon_{11}^0 + w_x^2/2, \quad \varepsilon_{22} = \varepsilon_{22}^0 + w_y^2/2, \quad \varepsilon_{12} = \varepsilon_{12}^0 + w_x w_y,
\] (3.1)

where \( \varepsilon_{ij} \) are the linear part of the deformations \( (\varepsilon_{11}^0 = u_x, \varepsilon_{22}^0 = v_y, \varepsilon_{12}^0 = u_y + v_x) \) and \( u(x, y), v(x, y), w(x, y) \) are the projections of the displacement of the median plane of the plate.

We find a refined expression of the potential deformation energy

\[
\Pi = \Pi_e + \Pi_b,
\] (3.2)

where \( \Pi_e \) and \( \Pi_b \) are the strain and bending strain energies

\[
\Pi_e = \frac{1}{2} \int \int (\sigma_{11} \varepsilon_{11} + 2 \sigma_{12} \varepsilon_{12} + \sigma_{22} \varepsilon_{22}) \, dx \, dy,
\]

\[
\sigma_{11} = \frac{E h (\varepsilon_{11} + \nu \varepsilon_{22})}{1 - \nu^2}, \quad \sigma_{22} = \frac{E h (\varepsilon_{22} + \nu \varepsilon_{11})}{1 - \nu^2}, \quad \sigma_{12} = \frac{E h \varepsilon_{12}}{2(1 + \nu)};
\] (3.3)

\[
\varepsilon_{11}^0 = \frac{T (\sigma_{11}^0 - \nu \sigma_{22}^0)}{E h}, \quad \varepsilon_{22}^0 = \frac{T (\sigma_{22}^0 - \nu \sigma_{11}^0)}{E h}, \quad \varepsilon_{12}^0 = \frac{2 T (1 + \nu) \sigma_{12}^0}{E h}.
\]

Here \( \sigma_{ij}^0 \) are stresses in the plane problem with tension \( T = 1 \) (see Fig. 1).

Confining one-mode approximation, we set \( W = (c^0 + c) w \), where \( w = w_1(x, y) \) is the eigenfunction of the problem (2.10) with \( T_k = T_1 \) (we assume \( w_1(0, 0) = 1 \), \( c^0 = c_1^0 \) is the specified small amplitude of the initial irregularity in the form \( w_1 \) and \( c = c_1 \) is the desired amplitude of the additional deflection.

We rewrite the energy (3.2) in the form:

\[
\Pi = \Pi_u - \Pi_{uw} + \Pi_w + \Pi_b = \frac{T^2}{2E h} \Pi_u^0 - \frac{T (c^0 + c)^2}{2} \Pi_{aw}^0 + \cdots
\]
$\frac{Eh c^2 (c^0 + c)^2}{8(1 - \nu^2)} \Pi_w^0 + \frac{D c^2}{2} \Pi_b^0,$  

(3.4)

where

$$
\Pi_w^0 = \int \int \left( (\sigma_{11}^0)^2 - \nu \sigma_{11}^0 \sigma_{22}^0 + (\sigma_{22}^0)^2 \right) dx dy,
$$

$$
\Pi_{uw}^0 = \int \int \left( (w_x^0)^2 + 2(1 - \nu) (w_{xy}^0)^2 \right) dx dy,
$$

$$
\Pi_w^0 = \int \int \left( w_x^0 w_y^0 \right) dx dy,
$$

$$
\Pi_b^0 = \int \int \left( \Delta w^0 \right) dx dy.
$$

(3.5)

The multiplier $c^2 (c + c_0)^2$ in the expression for $\Pi_w$ is connected with the fact that in the first factor of the product $\sigma_{ij} \varepsilon_{ij}$ the initial irregularities are not included, and in the second factor they are taken into account. The initial irregularities also do not enter the energy of a bending strain $\Pi_b$.

We set $T = T_1 (1 + \Delta)$, where $T_1 = D \Pi_b^0 / (c^2 \Pi_{uw}^0)$ is the critical load (2.5).

Minimizing energy (3.4) in amplitude $c$ leads to a cubic equation

$$(1 + \Delta)(c + c_0) = c + \frac{g}{h^2} (c + c_0)(2c + c_0), \quad g = \frac{3 \Pi_b^0}{\Pi_w^0}.$$  

(3.6)

or, calculating the deflections in fractions of the thickness ($c = h \hat{c}$, $c_0 = h \hat{c}_0$), we get

$$(1 + \Delta)(\hat{c} + \hat{c}_0) = \hat{c} + g \hat{c} (\hat{c} + \hat{c}_0)(2\hat{c} + \hat{c}_0).$$

(3.7)

In particular, in the absence of an imperfection ($c_0 = 0$), we have $c = h \sqrt{\Delta / (2g)}$.

In Fig. 2 at $g = 0.25$ a plot of $c(\Delta)$ at $c_0 = 0.1$ and in the absence of a imperfection $c_0 = 0$ is shown.

In virtue of formulas (3.1), (3.3), the stress $\sigma_{yy}$ at a nonlinear approach is equal to

$$
\sigma_{yy} = T c_{yy}^0 + \frac{Eh c^2}{2(1 - \nu^2)} (\nu w_x^2 + w_y^2).
$$

(3.8)

Due to the fact that the second term in formula (3.8) is positive, the tensile stress $\sigma_{yy}$ on the prolongation of the crack after buckling is greater than before. Therefore, buckling increases the level of tensile stresses at the tip of the crack and promotes crack growth.
4 FINITE ELEMENT MODELING OF POST-BUCKLING DEFORMATIONS.

Using the ANSYS code, a finite-element nonlinear analysis of the post-buckling deformation of a plate with a crack under uniaxial tension is performed. The following material properties of the plate were chosen for the simulation: the Young’s modulus $E = 70290 \text{ MPa}$ and the Poisson’s ratio $\nu = 0.345$. The plate dimensions are the height $2H = 8 \text{ mm}$, the width $2L = 8 \text{ mm}$, the thickness $h = 0.01 \text{ mm}$. The plate is weakened by a central rectilinear crack with a length of $2a = 0.5 \text{ mm}$ ($-a < x < a$, $y = 0$). To ensure the necessary accuracy of calculations in the vicinity of the tips of the crack, elements with a minimum linear size $0.005 \text{ mm}$ are selected. The plate is stretched by a uniformly distributed tension applied to the upper and lower sides of the plate ($-L < x < L$, $y = \pm H$). When the critical load is reached, a plate buckling occurs (loss of the plane shape of deformation).

<table>
<thead>
<tr>
<th>Form number</th>
<th>Load of bifurcation $N/\text{mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6707</td>
</tr>
<tr>
<td>2</td>
<td>2.3344</td>
</tr>
<tr>
<td>3</td>
<td>5.9132</td>
</tr>
</tbody>
</table>

Table 1: Loads of bifurcation for the first three forms of linear buckling.

Figure 3: Plate buckling with tensile force $2N/\text{mm}$.

A nonlinear buckling analysis was performed to evaluate the stress-strain state of a plate in a post-critical state.

Following the algorithm for constructing the solution of the nonlinear buckling problem, initial imperfections of the plate are set. These imperfections are proportional to the sum of displacements $w_k(x, y)$ of the first three forms of linear buckling and their amplitude does not exceed 5% of the plate thickness. The tensile load varies from 0 to $2N/\text{mm}$, which slightly exceeds the critical load ($1.6707 N/\text{mm}$) derived from the linear theory of stability. Fig.3 shows the plate buckling in the vicinity of the crack under a load of $2.0N/\text{mm}$, i.e. beyond the limit of
stability. The deflection $w(x, y)$ of the plate corresponds to the first form of bifurcation obtained in the eigenvalue (or linear) buckling analysis.

![Graph showing deflection $w$ of the center point crack faces $(x = 0, y = +0)$ by tensile load from 0 to 2 N/mm.](image)

In Fig.4 the displacement $w(x, y)$ of the center point of the crack $v = 0, y = +0$ depending on the applied load is presented, to illustrate the nonlinear deformation of the plate with initial imperfections.

As follows from Fig.4, in the presence of initial imperfections, a noticeable increase in deflections is observed at a load of $1.6 \, N/mm$ which is less than the critical load according to the linear buckling analysis. It is confirmed by an analytical solution (see Fig.2).

Analysis of the post-buckling state of the plate by a non-linear formulation makes it possible to estimate the level of stresses near the tip of the crack. Table 2 shows the maximum tensile stresses on the element adjacent to the crack tip with a tensile force of $2 \, N/mm$ at a distance of $0.00125 \, mm$ from the crack tip. For comparison, stresses are given in the case of linear deformation of the plate by a uniformly distributed force $2 \, N/mm$ without possible buckling, in the case of solving a problem by the nonlinearly elastic formulation without possible buckling, and also in the case of solving a problem by the nonlinearly elastic formulation when buckling.

<table>
<thead>
<tr>
<th>Stress $\sigma_{yy}(N/mm^2)$</th>
<th>Linear solution (flat shape)</th>
<th>Nonlinear solution (buckling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3278</td>
<td>3379</td>
<td>3619</td>
</tr>
</tbody>
</table>

Table 2: Maximum tensile stresses on an element adjacent to the crack tip $(N/mm^2)$ at a distance of $0.00125 \, mm$ from the tip.
5 CONCLUSION

The analysis of the stress state in the vicinity of the crack tip under uniaxial tension shows that an increase in tensile stresses occurs in the vicinity of the crack tip when buckling, and, as a result, the load leading to fracture decreases. This is confirmed by experimental results [7].

The discrepancy between the above analysis and the experimental results of [8] can be apparently explained by the fact that the material used in the experiments (paper) has a more complex rheology.

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REFERENCES


NONLINEAR DYNAMIC ANALYSIS OF THIN-WALLED STRUCTURES ADOPTING A MIXED BEAM FINITE ELEMENT MODEL WITH OUT-OF-PLANE CROSS-SECTION WARPING

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Keywords: Nonlinear dynamic analysis, Thin-walled beam, Warping, Mixed Formulation

Abstract. This paper focuses on the dynamic response of thin-walled structural elements. A mixed three-dimensional (3D) beam formulation is adopted, that includes the effect of inertia forces under dynamic loading conditions and accounts for out-of-plane cross-section warping. This is introduced by adding a specific displacement field to those due to rigid body motions, and is interpolated in the element volume with the definition of specific shape functions. The element governing equations are derived by expressing the Lagrangian functional in terms of four independent fields, i.e. the material rigid displacements, the strains and stresses and the additional warping displacement field. Four Lagrange’s equations of motion result, corresponding to the element compatibility condition enforced in weak form, the material constitutive law, and two sets of element equilibrium conditions associated to the rigid and warping displacements, respectively. The FE model has been implemented in a standard numerical code and used to investigate the effect of cross-section warping on the dynamic response of thin-walled structures. A T-shape beam is analyzed by performing modal decomposition and time-history analyses under linear elastic and nonlinear constitutive behavior.
1 INTRODUCTION

Thin-walled structural elements are widely spread in modern civil engineering constructions, especially in steel tall buildings and bridges. Their mechanical response is strongly affected by warping deformations and multi-axial stress interaction, which can be described by 3D Finite Element (FE) or shell formulations. However, to reduce computational effort beam FE models are usually preferred, although enhanced approaches have to be adopted. Indeed, beam FEs based on classic Euler-Bernoulli and Timoshenko beam theories that assume rigid cross-section hypothesis fail in reproducing the global response of thin-walled beams and the correct local distributions of strains and stresses that develop in the material. To overcome the mentioned drawbacks, many authors have proposed extended and enriched beam formulations that account for warping deformations of the cross-section and are capable of capturing interaction of axial force and bending moments with shear and torsional effects. Comprehensive reviews can be found in [1, 2]. Among them, Bairán [3] developed a generalized beam model that describes non-uniform shear strain/stress distributions; Saritas [4] presented a 2-node force-based Timoshenko FE that assumes an assigned distribution for shear strains over the cross-section due to shear forces. Ferradi et al. [5], Vieira et al. [6] and Dikaros et al. [7] introduced independent warping modes to describe cross-section out-of-plane displacements; Genoese et al. [8] formulated a mixed Hellinger-Reissner beam model describing the stress fields as the composition of the exact De Saint Venant contribution and some additional terms due to warping.

Extension of the models including cross-section warping effects to the dynamic field requires the definition of the consistent element mass matrix [9]. This is straightforward for classic displacement-based FEs, but results more involved in force-based and mixed FE formulations [10]. The procedure proposed in [10] was recently introduced in enhanced models by [9, 11]. This paper adopts the enhanced 3D beam finite element (FE) presented in [2] and extended to dynamic field in [12] to investigate the modal properties and the dynamic response of a simple thin-walled structural element. According to [1], the beam FE includes cross-section warping by introducing an independent displacement field in addition to the classic rigid body motions. This is interpolated on the basis of a variable number of degrees of freedom (DOF) added to those commonly used for beam FEs. In the formulation here presented, the additional warping DOFs are treated as global nodal DOFs and added at the end nodes. As such, warping displacement continuity is ensured and inertia forces associated to warping accelerations are correctly accounted for. The main issues of the adopted formulation are recalled and the element governing equations are derived by enforcing the stationarity of an extended Lagrangian functional expressed as function of four independent fields: rigid displacements, strains, stresses and the additional warping displacement. The presented beam FE formulation is adopted to study the dynamic linear and nonlinear response of a T-shape beam. Both modal decomposition and time-history analyses are performed, assuming both linear elastic and nonlinear constitutive behavior for the latter. In particular, the role of the warping deformation is put in evidence, as well as the influence of the cross-section thickness with respect to the element length.

2 MIXED BEAM FINITE ELEMENT WITH WARPING

The mixed beam FE including section out-of-plane warping presented in [2] and extended in [12] to account for the effects of inertia forces under dynamic loading conditions is adopted. Here, the main issues of the formulation are recalled, for the details referring to the cited papers [2, 12]. In the adopted two-node 3D beam finite element the warping DOFs collected in the vectors $u_{wI}$ and $u_{wJ}$ are added at the end nodes $I$ and $J$ to the standard kinematic translational,
\( u_I \) and \( u_J \), and rotational, \( \varphi_I \) and \( \varphi_J \), DOFs, as shown in Figure 1(a). Additional internal nodes could be introduced to obtain a higher-order interpolation for the warping displacement field \( u_w \). The element global displacement vector results as:

\[
\tilde{u} = \begin{bmatrix} u_I^T & \varphi_I^T & u_J^T & \varphi_J^T & u_{wI}^T & u_{wJ}^T \end{bmatrix}^T = \begin{bmatrix} u \end{bmatrix} \begin{bmatrix} u_w \end{bmatrix}
\]

where \( u_{wI} \) and \( u_{wJ} \) are vectors collecting the warping DOFs \( u_{wij} \) defined on the cross-section at node \( I \) and \( J \), respectively.

By eliminating the element rigid body motions, the six deformation displacements are obtained in the local reference system \((I; x, y, z)\) in Figure 1(b), resulting as:

\[
v = \begin{bmatrix} u_{xJ} & \theta_{zJ} & \theta_{yJ} & \theta_{xJ} & \theta_{yJ} & \theta_{zJ} \end{bmatrix}^T
\]

Vector \( v \) is derived by vector \( u \) by means of the element compatibility matrix \( a_g = a_v a_r \) composed by the rotation matrix \( a_r \) from the global to local reference system and the kinematic matrix \( a_v \) removing the element rigid body motions. Vector \( q \) contains the basic element forces work-conjugate to the displacements \( v \). According to the equilibrated beam formulation here adopted, the generalized section stress vector \( s(x) \) equilibrated with the basic forces \( q \) is introduced as:

\[
s(x) = b(x) q + s_q(x)
\]

where \( s_q(x) \) contains the generalized section stresses due to distributed loads \( q_n(x) \) and \( b(x) \) is the equilibrium matrix. Vector \( s(x) \) contains the axial stress \( N(x) \), bending moments \( M_z(x) \) and \( M_y(x) \), torsional moment \( M_x(x) \) and shear stresses \( T_y(x) \) and \( T_z(x) \). Then, the generalized section strain vector \( e(x) \), containing axial strain \( \varepsilon_N(x) \), flexural curvatures \( \chi_z(x) \) and \( \chi_y(x) \), torsional curvature \( \chi_x(x) \) and shear strains \( \gamma_y(x) \) and \( \gamma_z(x) \), can be defined by introducing the generalized section constitutive equation in incremental form as:

\[
\Delta e(x) = k_{ss}^{-1}(x) \Delta s(x)
\]

with \( \Delta s \) denoting the increment of variable \( s \) and \( k_{ss}(x) \) being the cross-section tangent stiffness matrix. Section compatibility conditions also permit to relate the generalized cross-section strains \( e(x) \) to the generalized cross-section displacements \( u_s(x) \), as follows:

\[
u_s(x) = \begin{bmatrix} u(x) & v(x) & w(x) & \theta_x(x) & \theta_y(x) & \theta_z(x) \end{bmatrix}^T
\]
where $u(x)$, $v(x)$ and $w(x)$ are the rigid translations and $\theta_x(x)$, $\theta_y(x)$ and $\theta_z(x)$ the rigid rotations. By applying the virtual work equivalence, the incremental element basic constitutive relationship is derived on the basis of Equations (3) and (4):

$$\Delta v = f \Delta q$$  \hspace{1cm} (6)$$
f being the element tangent flexibility matrix, given by:

$$f = \int_L b^T(x) k_{ss}^{-1}(x) b(x) \, dx$$  \hspace{1cm} (7)$$

The displacement $u_{m}(x, y, z)$ at the cross-section material point $m$ is expressed as the additive composition of the classical rigid part $u_{r}(x, y, z)$ and the additional out-of-plane warping displacement $u_{w}(x, y, z)$ (Figure 2), being these two enforced to be orthogonal. It results:

$$u_{m}(x, y, z) = u_{r}(x, y, z) + u_{w}(x, y, z)$$  \hspace{1cm} (8)$$

where the warping field contains only the out-of-plane component $u_{w}(x, y, z)$ directed along the beam axis $x$. This latter is interpolated at two levels, that is along the beam axis $x$ and over the cross-section $(y, z)$, by using 1D and 2D Lagrange polynomials, $N_i(x)$ and $M_j(y, z)$, respectively. The resulting interpolatory expression reads:

$$u_{w}(x, y, z) = \sum_{i=1}^{n} N_i(x) M(y, z) u_{wi}$$  \hspace{1cm} (9)$$

where vector $u_{wi}$ collects the $m_w$ warping DOFs $u_{w,ij}$ located on the $i$-th cross-section and $M(y, z)$ is a row vector containing the corresponding shape functions $M_j(y, z)$. Here, $n_w = 2$ is adopted, locating the warping interpolation cross-section at the beam end nodes $I$ and $J$, that is Equation (9) is written as:

$$u_{w}(x, y, z) = N_1(x) M(y, z) u_{w,I} + N_2(x) M(y, z) u_{w,J}$$  \hspace{1cm} (10)$$

where $N_1(x)$ and $N_2(x)$ are linear Lagrange polynomials.

To derive the element governing equations, the four-field extended Lagrangian functional $L$ is introduced as:

$$L(u_s, e, s, u_w, \dot{u}_s, \dot{u}_w) = T(\dot{u}_s, \dot{u}_w) - \Pi(u_s, e, s, u_w)$$  \hspace{1cm} (11)$$
where $T$ and $\Pi$ are the element kinetic and potential energy, respectively. These are written as function of the independent fields $u_*(x)$, $e(x)$, $s(x)$ and $u_w(x, y, z)$. By expressing the cross-section displacement $u_s(x)$ in terms of the nodal DOFs $\bar{u}$ as follows:

$$u_s(x) = N_s(x) \bar{u} + N_w(x) u_w$$  \hfill (12)

where $N_s(x)$ and $N_w(x)$ are the shape function matrices relating $u_*(x)$ to the standard and warping nodal DOFs, $\bar{u}$ and $u_w$, respectively, and by applying some manipulations as detailed in [12], the following expressions result:

$$T(\dot{\bar{u}}, \dot{\bar{u}}) = \frac{1}{2} \dot{\bar{u}}^T m_{rr} \dot{\bar{u}} + \dot{\bar{u}}^T m_{rw} \dot{u}_w + \dot{u}_w^T m_{wr} \dot{\bar{u}} + \frac{1}{2} \dot{u}_w^T (m_{ww}^r + m_{ww}) \dot{u}_w$$  \hfill (13)

$$\Pi(u, e, s, u_w) = \int_V \left[ e_m^T \{ e, u_w(u_w) \} \right] \dot{\bar{u}}^T dV + \int_L s^T \{ e[u_s(u, u_w)] - e \} dx +$$

$$\Pi_{ext}[u, u_w, u_s(u, u_w)]$$  \hfill (14)

The element mass matrices $m_{rr}$, $m_{rw}$, $m_{wr}$ and $m_{ww}^r$, associated to the generalized rigid cross-section velocity $\dot{u}_s(x)$, are expressed as:

$$m_{rr} = \int_L N_r^T(x) m_s(x) N_s(x) \, dx, \quad m_{rw} = \int_L N_r^T(x) m_s(x) N_w(x) \, dx = m_{wr}^T,$$

$$m_{ww}^r = \int_L N_w^T(x) m_s(x) N_w(x) \, dx$$  \hfill (16)

with $m_s(x)$ denoting the cross-section mass matrix. This relates the cross-section inertia forces to the generalized cross-section accelerations $\ddot{u}_s(x)$. Finally, $m_{ww}$ is the element mass matrix associated to the warping velocity $\ddot{u}_w(x, y, z)$ and is defined as:

$$m_{ww} = \int_L \begin{bmatrix} N_1(x) m_{sw}(x) N_1(x) & N_1(x) m_{sw}(x) N_2(x) \\ N_2(x) m_{sw}(x) N_1(x) & N_2(x) m_{sw}(x) N_2(x) \end{bmatrix} \, dx$$  \hfill (17)

being $m_{sw}(x)$ the cross-section mass matrix associate to $\dot{u}_w$, that is:

$$m_{sw}(x) = \int_{A(x)} \rho(x, y, z) M^T(y, z) M(y, z) \, dA$$  \hfill (18)

where $\rho(x, y, z)$ is the material mass density and $A(x)$ the cross-section area. By enforcing the stationarity of the Lagrange’s functional with respect to the independent fields $u$, $e(x)$, $s(x)$ and $u_w$, the four element governing equations are derived as:

$$\sigma_m(x, y, z) = \tilde{\sigma}_s \{ e_m(x, y, z) \}$$  \hfill (19)

$$v = \int_L b^T(x) e(x) \, dx$$  \hfill (20)

$$m_{rr} \ddot{\bar{u}} + m_{rw} \ddot{u}_w + a_s^T q - a_s^T p_{rq} = p$$  \hfill (21)

$$m_{wr} \ddot{\bar{u}} + (m_{ww}^r + m_{ww}) \ddot{u}_w + \int_L \begin{bmatrix} \frac{\partial N_1(x)}{\partial x} \\ \frac{\partial N_2(x)}{\partial x} \end{bmatrix} \dot{s}_w(x) \, dx + \int_L \begin{bmatrix} N_1(x) \\ N_2(x) \end{bmatrix} \dot{s}_w(x) \, dx = p_w$$  \hfill (22)
Equation (19) is the nonlinear material constitutive law, Equation (20) expresses the element compatibility in weak form, Equation (21) represents element equilibrium conditions in weak form, requiring the nodal internal forces $q$ to be in equilibrium with the nodal external and inertia forces, vector $p_{eq}$ collecting the element nodal forces in the local reference system due to loads distributed along the element axis. Finally, Equation (22) gives the section equilibrium conditions related to the warping, requiring $s_{w}^{x}(x)$ and $s_{w}^{y}(x)$ to be in equilibrium with forces $p_{w}$ and inertia forces $m_{w} u_{w} + (m_{w}^{ww} + m_{w}^{ww}) u_{w}$. Vector $p_{w}$ collects the warping forces $p_{wij}$ work-conjugate to the displacements $u_{wij}$.

3 NUMERICAL APPLICATIONS

Modal decomposition and time-history analyses of a Z- and C-shape beam are conducted in the previous study [12], investigating the influence of the cross-section deformations on the vibration shapes, with respect to the element length and thickness. Indeed, as cross-section in-plane deformations are neglected in the adopted model, this fails in correctly reproducing highest bending and torsional modes of squat beams with very thin cross-sections. To extend the study in [12] and better understand the range of applicability of the adopted model, the dynamic response of a T-shape beam is studied in the following.

For all computations, mass matrix is derived by neglecting inertia term $m_{ww} u_{ww}$ in Equation (22). Indeed spurious modes associated to the cross-section warping and characterized by low natural frequencies likely appear when this term is included. By contrast, as demonstrated in [12], neglecting $m_{w} u_{w}$ is beneficial for faster computation and leads to negligible errors.

3.1 Modal decomposition of a T-shape beam

The vibration circular frequencies of the T-shape beam are computed. Figure 3 describes the beam geometry, where $L$ indicates the beam length, $h$ the cross-section width and depth and $t$ the web and flange thickness (Figure 3(a)). Three boundary restraint configurations are considered, that is a cantilever (R1), a fixed-fixed beam (R2) and a fixed-supported beam (R3), as depicted in Figures 3(b), (c) and (d), respectively, with warping restrained only at the fixed ends.

Assuming linear elastic isotropic material with Young’s modulus $E = 210$ GPa, Poisson ratio $\nu = 0.33$ and mass density $\rho = 7850$ kg/m$^3$, for each restraint configuration R1, R2 and

![Figure 3: T-shape beam geometry.](image-url)
R3, eight beam geometries are studied, considering two values of length $L = 250$ cm (short beam) and 500 cm (long beam), two values of width/depth $h = 20$ cm (small cross-section) and 40 cm (wide cross-section), two values of thickness $t = 2$ cm (thin cross-section) and 4 cm (thick cross-section). Each geometry is labeled with a letter from A to H for easier reference, as reported in Table 1.

Table 1: Geometric data assumed for the T-shape beam.

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
<th>Length $L$ [cm]</th>
<th>Width/depth $h$ [cm]</th>
<th>Thickness $t$ [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>short beam with small thin cross-section</td>
<td>250</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>short beam with small thick cross-section</td>
<td>250</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>short beam with wide thin cross-section</td>
<td>250</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>short beam with wide thick cross-section</td>
<td>250</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>long beam with small thin cross-section</td>
<td>500</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>long beam with small thick cross-section</td>
<td>500</td>
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<td>4</td>
</tr>
<tr>
<td>G</td>
<td>long beam with wide thin cross-section</td>
<td>500</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>long beam with wide thick cross-section</td>
<td>500</td>
<td>40</td>
<td>4</td>
</tr>
</tbody>
</table>

The first 15 vibration modes are computed by means of two FE models, the first considering the mixed formulation beam formulation with cross-section warping and the second considering a standard beam formulation with rigid cross-sections. Both models adopts 20 FEs to discretize the T-shape beam, as the study in [12] demonstrates that these are sufficient to obtain accurate results. For the model with cross-section warping, the control point layout in Figure 4(a) is considered for the warping interpolation over the cross-section. This uses Lagrange polynomials $M_j(y,z)$ with parabolic variation over the flange, quartic over the web and linear across the thickness. The total number of warping DOFs over the cross-section is 20 and the corresponding interpolation is indicated as Scheme L20. For the standard model, shear areas along $y$ and $z$ directions are assumed equal to that of web and flange, respectively. Moreover, cross-section torsional inertia is assumed according to the thin-walled beam theory [13], that is $J = \sum \frac{1}{2} h t^3 = k_t I_{\rho}$, being $I_{\rho}$ the polar moment of inertia and $k_t$ the torsional stiffness correction factor. The reference solution is obtained from a 20-node brick FE model with a mesh of 2730 FEs. Figure 4(b) shows the brick discretization adopted over the beam cross-section with the blue squares indicating the FE nodes. In this model, to apply the boundary conditions, all displacement DOFs of all nodes belonging to fixed ends are restrained, while only displacements in the $y$-$z$ plane are restrained for nodes belonging to the torsional supported end.
The reference results are reported in terms of circular frequencies $\omega$ in Tables 8, 9 and 10 in Appendix A for the configuration R1, R2 and R3, respectively, while Figures 5, 6 and 7 show the percentage errors with respect to the reference solution for the mixed formulation (red bars) and standard (blue bars) beam models. Percentage errors are computed as $\text{Error [\%]} = \frac{(\omega_{\text{beam}} - \omega_{\text{brick}})}{\omega_{\text{brick}}} \times 100$. Tables 2, 3 and 4 report average and maximum value of the error.
The circular frequencies resulting from the standard model significantly differ from the reference values, although this model includes correction factors to account for correct shear and torsional inertia. In fact, Figures 5, 6 and 7 show that in many cases (e.g. geometries A, G and H) large errors occur even for the first vibration modes. By contrast, circular frequencies resulting
from the mixed formulation model are in perfect agreement with the reference solution. Large errors occur only for highest modes when geometries C, D and G are considered, as in these cases in-plane cross-section deformations, that are neglected in the mixed formulation, have more influence on the solution. As example, the vibration shape of modes 1, 7 and 9 are plotted in Figures 8, 9 and 10 for the restraint configuration R1 with geometry C, R2 with geometry
Table 2: Average and maximum errors for the circular frequencies $\omega$ of the T-shape cantilever (R1).

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Scheme L20</th>
<th>Rigid cross-section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average error</td>
<td>Maximum error</td>
</tr>
<tr>
<td>A</td>
<td>1.4%</td>
<td>4.3% (mode 14)</td>
</tr>
<tr>
<td>B</td>
<td>1.0%</td>
<td>3.5% (mode 15)</td>
</tr>
<tr>
<td>C</td>
<td>28.0%</td>
<td>84.9% (mode 14)</td>
</tr>
<tr>
<td>D</td>
<td>13.4%</td>
<td>69.5% (mode 15)</td>
</tr>
<tr>
<td>E</td>
<td>0.8%</td>
<td>1.8% (mode 11)</td>
</tr>
<tr>
<td>F</td>
<td>0.5%</td>
<td>1.2% (mode 5)</td>
</tr>
<tr>
<td>G</td>
<td>2.3%</td>
<td>5.3% (mode 15)</td>
</tr>
<tr>
<td>H</td>
<td>1.4%</td>
<td>4.3% (mode 14)</td>
</tr>
</tbody>
</table>

Table 3: Average and maximum errors for the circular frequencies $\omega$ of the T-shape fixed-fixed (R2).

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Scheme L20</th>
<th>Rigid cross-section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average error</td>
<td>Maximum error</td>
</tr>
<tr>
<td>A</td>
<td>2.4%</td>
<td>5.4% (mode 15)</td>
</tr>
<tr>
<td>B</td>
<td>1.3%</td>
<td>3.1% (mode 11)</td>
</tr>
<tr>
<td>C</td>
<td>44.8%</td>
<td>104.3% (mode 15)</td>
</tr>
<tr>
<td>D</td>
<td>22.3%</td>
<td>58.5% (mode 11)</td>
</tr>
<tr>
<td>E</td>
<td>1.1%</td>
<td>3.2% (mode 13)</td>
</tr>
<tr>
<td>F</td>
<td>0.8%</td>
<td>1.8% (mode 13)</td>
</tr>
<tr>
<td>G</td>
<td>6.9%</td>
<td>38.2% (mode 15)</td>
</tr>
<tr>
<td>H</td>
<td>2.4%</td>
<td>5.4% (mode 15)</td>
</tr>
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</table>

Table 4: Average and maximum errors for the circular frequencies $\omega$ of the T-shape fixed-supported beam (R3).

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Scheme L20</th>
<th>Rigid cross-section</th>
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<tbody>
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<td>Maximum error</td>
</tr>
<tr>
<td>A</td>
<td>1.9%</td>
<td>4.8% (mode 15)</td>
</tr>
<tr>
<td>B</td>
<td>1.3%</td>
<td>3.1% (mode 12)</td>
</tr>
<tr>
<td>C</td>
<td>34.2%</td>
<td>90.8% (mode 15)</td>
</tr>
<tr>
<td>D</td>
<td>19.3%</td>
<td>61.4% (mode 15)</td>
</tr>
<tr>
<td>E</td>
<td>1.0%</td>
<td>2.5% (mode 12)</td>
</tr>
<tr>
<td>F</td>
<td>0.6%</td>
<td>1.6% (mode 14)</td>
</tr>
<tr>
<td>G</td>
<td>4.3%</td>
<td>22.3% (mode 15)</td>
</tr>
<tr>
<td>H</td>
<td>1.9%</td>
<td>4.8% (mode 15)</td>
</tr>
</tbody>
</table>

D and R3 with geometry G, respectively. The results show that in-plane deformations need to be included in numerical simulations when specific geometric configurations of the modeled
beams are considered, that is when the beam cross-sections are wide compared to their thickness and length. In other words, for wide cross-sections, in-plane deformations are essential if beams are short (e.g. geometries C and D) or cross-sections are thin (e.g. geometry G). However, this also depend on the boundary conditions. At fixed ends, both in-plane and out-of-plane deformations are prevented and the beam cross-sections gradually deform moving from these zones to the unrestrained part of the element. Hence, in shorter beams greater part of the length is affected by the boundary restraints and the cross-sections tend to deform more in their plane, especially for torsional modes. The opposite occurs in longer beams. The same is true for fixed-fixed configurations, where in-plane deformations are more significant, as opposed to cantilever
Figure 11: Circular frequency values of the T-shape cantilever (R1) compared with the brick model with diaphragm constraints for in-plane rigid cross-sections.

Figure 12: Circular frequency values of the T-shape fixed-fixed beam (R2) compared with the brick model with diaphragm constraints for in-plane rigid cross-sections.
Figure 13: Circular frequency values of the T-shape fixed-supported beam (R3) compared with the brick model with diaphragm constraints for in-plane rigid cross-sections.

configurations. Indeed, for geometries C, D and G better results are obtained with the mixed formulation model for the restraint configuration R1 than configurations R2 and R3.

To confirm previous findings, the circular frequency values obtained for geometries with wide cross-section (geometries C, D, G and H) are compared with those resulting from a brick FE model where in-plane deformations are prevented. This is enforced by applying diaphragm constraints to all nodes belonging to the generic plane orthogonal to the beam axis. The reference results are reported in terms of the circular frequencies $\omega$ in Tables [11, 12 and 13] in Appendix A for restraint configuration R1, R2 and R3, respectively, while Figures [11, 12 and 13]

Table 5: Average and maximum errors for the circular frequencies $\omega$ of the T-shape cantilever (R1) compared with the brick model with diaphragm constraints for in-plane rigid cross-sections.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Scheme L20</th>
<th>Rigid cross-section</th>
<th>Scheme L20 FW</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Average error</td>
<td>Maximum error (%)</td>
<td>Average error</td>
</tr>
<tr>
<td>C</td>
<td>1.9%</td>
<td>-4.6% (mode 12)</td>
<td>15.3%</td>
</tr>
<tr>
<td>D</td>
<td>1.7%</td>
<td>-6.0% (mode 2)</td>
<td>11.3%</td>
</tr>
<tr>
<td>G</td>
<td>1.6%</td>
<td>-3.6% (mode 8)</td>
<td>8.9%</td>
</tr>
<tr>
<td>H</td>
<td>2.0%</td>
<td>-6.5% (mode 2)</td>
<td>6.6%</td>
</tr>
</tbody>
</table>
show the percentage errors with respect to the reference solution for the mixed formulation (red bars) and standard (blue bars) beam models. The same figures also plot results obtained with the mixed formulation beam, but neglecting the warping restraints at the beam ends (green bars). This model is indicated as Scheme L20 FW. Tables 5, 6 and 7 report average and maximum error values for each model.

When the beam cross-sections are forced to remain rigid in their plane, the mixed formulation model perfectly describes its natural modes, even for wide cross-section geometries. The difference with respect to the reference values is lower than 6.5%. By contrast, large errors occur for the standard beam. Finally, the model with free warping results more flexible than that with warping restrained. However, better solutions result for Scheme L20 FW when thick cross-sections are considered, i.e. for geometries D and H, as warping end restraints have lower influence on the beam stiffness in these cases.

Table 6: Average and maximum errors for the circular frequencies $\omega$ of the T-shape fixed-fixed (R2) compared with the brick model with diaphragm constraints for in-plane rigid cross-sections.

<table>
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<th>Geometry</th>
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<th>Rigid cross-section</th>
<th>Scheme L20 FW</th>
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</thead>
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<td>Average error</td>
<td>Maximum error</td>
<td>Average error</td>
</tr>
<tr>
<td>C</td>
<td>1.1%</td>
<td>-2.6% (mode 5)</td>
<td>22.0%</td>
</tr>
<tr>
<td>D</td>
<td>1.2%</td>
<td>-2.9% (mode 13)</td>
<td>13.9%</td>
</tr>
<tr>
<td>G</td>
<td>1.1%</td>
<td>-3.3% (mode 5)</td>
<td>8.3%</td>
</tr>
<tr>
<td>H</td>
<td>1.6%</td>
<td>-3.8% (mode 8)</td>
<td>8.8%</td>
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</table>

Table 7: Average and maximum errors for the circular frequencies $\omega$ of the T-shape fixed-supported beam (R3) compared with the brick model with diaphragm constraints for in-plane rigid cross-sections.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Scheme L20</th>
<th>Rigid cross-section</th>
<th>Scheme L20 FW</th>
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</thead>
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<td>Average error</td>
<td>Maximum error</td>
<td>Average error</td>
</tr>
<tr>
<td>C</td>
<td>1.6%</td>
<td>-4.6% (mode 10)</td>
<td>18.1%</td>
</tr>
<tr>
<td>D</td>
<td>1.5%</td>
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</tr>
<tr>
<td>G</td>
<td>1.4%</td>
<td>-4.6% (mode 14)</td>
<td>7.5%</td>
</tr>
<tr>
<td>H</td>
<td>1.9%</td>
<td>-4.6% (mode 10)</td>
<td>8.1%</td>
</tr>
</tbody>
</table>
3.2 Time-history analysis of a T-shape beam under dynamic loads

For the T-shape beam studied in Section 3.1 time history analyses under dynamic loads are also performed. Same restraint configurations R1, R2 and R3 are considered, with geometry E in Table 1 i.e. long beam with small thin cross-section ($L = 250$ cm, $h = 20$ cm, $t = 2$ cm). The beam configurations are redrawn in Figure 14 where the loading conditions are also illustrated. A time-varying concentrated transverse force $F$ is applied at the free end for the cantilever configuration (R1) and at the mid-span for the fixed-fixed (R2) and fixed-supported (R3) configurations. In all cases, the force is applied at point $O$ of the loaded cross-section, corresponding to the center of the flange upper edge, as indicated in Figure 15(a). This figure also shows the force components $F_y$ and $F_z$ that are parallel to the cross-section plane. Keeping constant ratio $F_y = 2F_z$, force components linearly increase from $F_z = 0.0$ kN to $F_z = 2.5$ kN and then remain constant, i.e. they assume the linear ramp time-variation in Figure 15(b).

Cantilever beam in Figure 14(a) has been studied by Le et al. [14] yet considering a higher maximum load ($F_z = 25.0$ kN) to analyze the response under large displacements and assuming linear elastic material response. Both linear elastic and nonlinear material responses are

![Figure 14: T-shape beam configurations and loading conditions.](image)

![Figure 15: T-shape beam: (a) load components and (b) loading history.](image)
considered in this study. For the latter case, the J2 plasticity model with linear kinematic hardening is adopted. Yielding stress is assumed equal to $\sigma_y = 110 \text{ MPa}$ for the configuration R1, $11 \text{ MPa}$ for the R2 and $15 \text{ MPa}$ for the R3. Hardening modulus is always assumed equal to $H_k = 0.001E$.

Figures 16 and 17 plot the time-evolution of the transverse displacements $v_G$ and $w_G$ of the loaded cross-section, under linear elastic and nonlinear material behavior, respectively. Displacements $v_G$ and $w_G$ are measured at the centroid $G$, along the $y$ and $z$ directions, respectively. The solid green curves represent the reference solutions obtained with a 20-node brick FEs, while the dashed red curves represent those obtained with the mixed formulation model.

![Graphs showing time-evolution of displacements](image)

Figure 16: T-shape linear elastic beam: time-evolution of the displacements along $y$ and $z$ for the centroid $G$ of the loaded cross-section.

After maximum load is reached at $t = 0.05 \text{ s}$, the beam starts oscillating about a new dynamic equilibrium position, as usual for dynamic systems under linear ramp loading conditions [15]. For instance, the dynamic equilibrium position of the linear elastic cantilever (R1) is defined by transverse displacements equal to $v_G = -3.5 \text{ cm}$ and $w_G = 3.7 \text{ cm}$, i.e. average
displacement values for $t > 0.05$ s in Figures 16(a) and 16(b). For the nonlinear material case, plastic strains arise before load reaches his maximum level and, thus, dynamic equilibrium is reached for higher levels of displacement. The dynamic equilibrium position of the nonlinear cantilever (R1) is defined by transverse displacements equal to $v_G = -6.1$ cm and $w_G = 8.0$ cm (Figures 17(a) and 17(b)).

Perfect match results between the beam FE accounting for cross-section warping and the reference solution for linear elastic material and a negligible discrepancy emerges for the elasto-plastic material. In the latter case, the dynamic equilibrium positions resulting from the mixed formulation model correspond to slightly higher levels of displacement than those resulting from the reference model, but the beam vibration frequencies are perfectly detected.

4 CONCLUSIONS

- An mixed 3D beam FE formulation, including effects of cross-section warping, has been adopted for the dynamic analysis of thin-walled beams. Additional warping DOFs have
been added at the element end nodes and used to interpolate the warping displacement filed in the element volume.

• The numerical study conducted on a T-shape beam with different boundary conditions show that in-plane section deformations are relevant for very thin geometries and/or squat members. Indeed, for such elements variation of the cross-section shape clearly occurs in modal shapes, and these in-plane deformation modes significantly influence natural frequencies. The beam FE here adopted neglects these deformations, assuming the cross-section as rigid in its plane.

• The results of the modal decomposition and time-history analyses of the thin-walled beam illustrated in the paper prove that the proposed beam formulation is accurate and computational saving with respect to more refined 3D FE formulations.

• Further developments of the FE model presented are ongoing to include the effects of cross-sections in-plane deformations.

A CIRCULAR FREQUENCY VALUES FOR THE STUDIED T-SHAPE BEAM

Table 8: Circular frequency $\omega$ [rad/s] for the T-shape cantilever (R1) obtained with the brick model.

<table>
<thead>
<tr>
<th>Geom.</th>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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Table 9: Circular frequency $\omega$ [rad/s] for the T-shape fixed-fixed beam (R2) obtained with the brick model.

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Table 10: Circular frequency $\omega$ [rad/s] for the T-shape fixed-supported beam (R3) obtained with the brick model.

<table>
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<th>Geom.</th>
<th>Mode</th>
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Table 11: Circular frequency $\omega$ [rad/s] for the T-shape cantilever (R1) obtained with the brick model with diaphragm constraints for in-plane rigid cross-sections.

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Table 12: Circular frequency $\omega$ [rad/s] for the T-shape fixed-fixed beam (R2) obtained with the brick model with diaphragm constraints for in-plane rigid cross-sections.

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Table 13: Circular frequency $\omega$ [rad/s] for the T-shape fixed-supported beam (R3) obtained with the brick model with diaphragm constraints for in-plane rigid cross-sections.

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REFERENCES


BENDING OF MULTILAYERED PLATES AND CYLINDRICAL SHELLS

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Keywords: plate, shell, generalized Timoshenko-Reissner model, heterogeneous material, isotropic multilayered shell

Abstract. In this paper, the bending of thin multilayered plates and cylindrical shells are considered. The generalized Timoshenko-Reissner (GTR) model, the bending equation of second-order accuracy (the SA model) and the equivalent single layer (ESL) model will be used here. According to the GTR model, a shell that is heterogeneous can be replaced by a homogeneous shell with the equivalent bending and transversal shear stiffness. In this paper, these models are used for the equations that take into account the transversal shear. The ESL model is developed by Grigoliuk and Kulikov and completely based on the generalized kinematic hypothesis of Timoshenko for the whole laminated packet.

As an example, thin multilayered plates and cylindrical shells consisting of isotropic layers with hinged edges are considered. It is assumed that each layer has a constant thickness. The exact numerical solution is compared with the results of the GTR model and the SA model and the ESL model.
1 INTRODUCTION

Numerous researches have been made on bending problems of plates and shells \[1\, 2\, 3\]. The 2D equations of the Kirchhoff-Love theory \[4\, 5\] are used in the case of vibrations and bending of an isotropic homogeneous plate. In the case of shells that are heterogeneous in a thickness direction (including a multilayer shell with the highly variable stiffness of layers), it is necessary to take into account the transversal shear stiffness \[6\].

The GTR model for plates that are heterogeneous in the thickness direction was introduced in the papers \[7\, 8\]. According to this model, a shell that is heterogeneous can be replaced by a homogeneous shell with the equivalent bending and transversal shear stiffness. In these papers, these models are used for the equations that take into account the transversal shear. In the paper \[9\] an equation for a heterogeneous transversely isotropic plate was obtained. Thus, the equivalent shear stiffness is determined. In the papers \[7\, 8\] the error of this approach is estimated by comparison with test examples showing the exact solution.

The variant of the equivalent single layer (ESL) models developed by Grigoliuk and Kulikov \[6\] is used for the equations that take into account the transversal shear. This model is completely based on the generalized kinematic hypothesis of Timoshenko for the whole laminated packet. The papers \[10\, 11\, 12\] are devoted to the ESL theories. The Grigoliuk and Kulikov \[6\] have derived a system of five differential equations for elastic laminated cylindrical shells with constant and elastic moduli.

As an example, the bending plates and free vibrations of laminated medium-length cylindrical shells consisting of isotropic layers with hinged edges are analyzed. It is assumed that each layer has a constant thickness. The results of the GTR model are compared with the KL model and the ESL model results and with the exact numerical solution. For comparatively small values of the transversal shear parameter the numerical results differ slightly from each other, and for larger values of the parameter this difference becomes essential. The effect of the shear parameter on the bending and free vibrations is studied.

2 EQUILIBRIUM EQUATIONS OF A PLATE

Bending of a thin plate made of a transversally isotropic heterogeneous material. The 3D equilibrium equations are taken in the form

\[
\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0, \quad i, j = 1, 2, 3, \quad 0 \leq x_3 = z \leq h, \tag{1}
\]

where \(x_j\) are the Cartesian coordinates, \(f_i\) are the projections of the external load intensity, \(h\) is the plate thickness. Eqs. (1) can be used to solve problems of a static equilibrium and free vibrations.

The the elasticity relations write as

\[
\begin{align*}
\sigma_{11} &= E_{11} \varepsilon_{11} + E_{12} \varepsilon_{22} + E_{13} \varepsilon_{33}, & \sigma_{12} &= G_{12} \varepsilon_{12}, \\
\sigma_{22} &= E_{12} \varepsilon_{11} + E_{11} \varepsilon_{22} + E_{13} \varepsilon_{33}, & \sigma_{13} &= G_{13} \varepsilon_{13}, \\
\sigma_{33} &= E_{13} \varepsilon_{11} + E_{13} \varepsilon_{22} + E_{33} \varepsilon_{33}, & \sigma_{23} &= G_{13} \varepsilon_{23},
\end{align*}
\tag{2}
\]

\[
\begin{align*}
\varepsilon_{11} &= \frac{\partial u_1}{\partial x_1}, & \varepsilon_{22} &= \frac{\partial u_2}{\partial x_2}, & \varepsilon_{33} &= \frac{\partial u_3}{\partial x_3}, \\
\varepsilon_{12} &= \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}, & \varepsilon_{13} &= \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}, & \varepsilon_{23} &= \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2},
\end{align*}
\tag{3}
\]
with \( E_{11} = E_{12} + 2G_{12} \). Here \( \sigma_{ij} \) are the stresses, \( \varepsilon_{ij} \) are the strains, \( u_1, u_2 \) and \( u_3 \) are the displacements in the corresponding directions. The elastic moduli \( E_{ij}, G_{ij} \) are independent of \( x_1 \) and \( x_2 \) but they depend on \( x_3 = z \). Therefore, Eqs. (1)-(3) describe functionally graded and multilayered plates. For functionally gradient materials the moduli are continuous functions and depend on \( z \), while for the multilayered plates they are piecewise continuous functions.

For the isotropic material the elastic moduli are given by

\[
E_{ii} = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}, \quad E_{ij} = \frac{E\nu}{(1 + \nu)(1 - 2\nu)}, \quad G_{ij} = G = \frac{E}{2(1 + \nu)},
\]

where \( E \) and \( \nu \) are the Young’s modulus and the Poisson ratio, respectively.

Set the boundary conditions at \( z = 0 \) and \( z = h \)

\[
\sigma_{i3} = 0, \quad i = 1, 2, 3.
\]

If the surface forces are given, then they are included in the body forces by using the Dirac’s delta-function.

Introduce the new unknown functions \( u, v, \sigma, \tau \) as

\[
u = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}, \quad v = \frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1},
\]

\[
\sigma = \frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2}, \quad \tau = \frac{\partial \sigma_{13}}{\partial x_2} - \frac{\partial \sigma_{23}}{\partial x_1},
\]

and rewrite the system (1) in the form [9]

\[
\frac{\partial w}{\partial z} = -\mu c_\nu u + c_3 \sigma_{33}, \quad \frac{\partial u}{\partial z} = -\mu \Delta w + c_\sigma \sigma,
\]

\[
\frac{\partial \sigma}{\partial z} = -\mu^2 E_0 \Delta u - \mu c_\nu \Delta \sigma_{33} - m, \quad \frac{\partial \sigma_{33}}{\partial z} = -\mu \sigma - f_3,
\]

with

\[
w = u_3, \quad m = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2}, \quad \Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2},
\]

\[
c_3 = \frac{E_\sigma}{E_{33}}, \quad c_\nu = \frac{E_{13}}{E_{33}} = \frac{\nu}{1 - \nu}, \quad E_0 = E_{11} - \frac{E_{13}^2}{E_{33}} = \frac{E}{1 - \nu^2}, \quad c_\gamma = \frac{E_\sigma}{G_{13}}.
\]

Here \( E_\sigma \) is the average value of modulus \( E_0 \)

\[
E_\sigma = \frac{1}{h} \int_0^h E_0(z) \, dz.
\]

System (7) of the fourth order describes the plate bending. The asymptotic solution of the second order of accuracy \( w^{(2)} \) was presented in [9]. This solution is based on the expansions in powers of the small parameter \( \mu = h/L \). Here \( L \) is the representative wave length in the in-plane directions.
3 APPROXIMATE MODELS FOR COMPUTING STATIC DEFLECTION

In the static problem the total deflection of the reference plane \( z = 0 \) can be expressed as \[9\]

\[ D \Delta^2 w^{(2)}(x_1, x_2, 0) = F_3 - A \Delta F_3, \tag{10} \]

where \( D \) is the bending stiffness of the plate with changing elastic moduli, \( F_3 \) is the overall transversal force. The equation

\[ D \Delta^2 w^{KL}(x_1, x_2, 0) = F_3 \tag{11} \]

corresponds to the classic KL model and the second term on the right side of the equation is the correction. The coefficients \( D \) and \( A \) depend on the elastic moduli distribution in the plate thickness. Different possible models to calculate the corrective parameter \( A \) are presented below.

In the papers \[7, 8\] the GTR model for plates that are heterogeneous in the thickness direction was introduced. According to this model, a shell that is heterogeneous can be replaced by a homogeneous shell with the equivalent bending and transversal shear stiffness. It is assumed that layers have full contact with each other. According to these papers, Eq. (10) takes the form

\[ D \Delta^2 w^{(2)} = F_3 - (A_g - A_\nu) \Delta F_3, \tag{12} \]

Here the coefficients \( A_g \) and \( A_\nu \) take into account the transversal shear and the Poisson ratio, respectively. For a transversely isotropic heterogeneous plate the elastic parameters are found in the form

\[ A_g = \frac{1}{D} \int_0^h b(z)^2 \frac{G(z)}{E(z)} \, dz, \quad b(z) = \int_0^z E_0(z_1 - a) \, dz_1, \]

\[ A_\nu = \frac{1}{D} \int_0^h (z - a) \frac{\int_0^z \int_0^{z_1} (c_\nu(z) E_0(z_2) + E_0(z) c_\nu(z_2)) (z_2 - a) \, dz_2 \, dz_1 \, dz}{\langle E_0 \rangle}, \tag{13} \]

\[ D = \langle (z - a)^2 E_0 \rangle, \quad a = \langle z E_0 \rangle \langle E_0 \rangle, \]

\[ E_0(z) = \frac{E(z)}{1 - \nu^2(z)}, \quad G(z) = \frac{E(z)}{2(1 + \nu(z))}. \]

For the multilayered plates the elastic moduli \( E(z) \) and \( \nu(z) \) are piecewise continuous functions and depend on the transversal coordinate \( z \) changing in thickness direction \((0 \leq z \leq h)\). Here \( \langle Z \rangle = \int_0^h Z(z) \, dz \) is the averaging operator, \( a \) is the coordinate of the neutral surface, \( D \) and \( G \) are bending and transversal shear stiffness, respectively.

If Young’s longitudinal moduli have order \( E \), then the orders of the correction terms \( A_g \) and \( A_\nu \) are

\[ \{A_g \Delta F_3, A_\nu \Delta F_3\} \sim \frac{h^2 F_3}{L^2} = \mu^2 F_3 \tag{14} \]

as \( \{A_g, A_\nu\} \sim h^2 \) and \( \Delta \sim 1/L^2 \). Therefore, the correction term on the right side of Eq. (12) has order \( \mu^2 \) in comparison with \( F_3 \).

If a multilayer plate has layers with a small Young’s modulus, the coefficients \( A_g \) ceases to be small while \( A_\nu \) remains small. Based on this, the approximate equation can be entered as

\[ D \Delta^2 w^{TR} = F_3 - A_g \Delta F_3. \tag{15} \]

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Eq. (15) takes into account the shear deformation and the term $A_{\nu}$ is dropped.

In the book [6] the ESL model developed by Grigoliuk and Kulikov was introduced. This model is completely based on the generalized kinematic hypothesis of Timoshenko for the whole laminated plate. This model takes into account only the transversal shear. According to this book, Eq. (10) takes the form

$$D \Delta^2 w^{\text{ESL}} = F_3 - A^{\text{ESL}}_g \Delta F_3,$$

where the parameters $A^{\text{ESL}}_g$ and $D$ of the ESL model can be written as

$$A^{\text{ESL}}_g = \frac{h^2}{\beta}, \quad D = \frac{E h^3}{12(1-\nu^2)} \eta_3,$$

where $h = \sum_{k=1}^{N} h_k$ is the total thickness of the laminate. Here parameters $\eta, \beta$ characterize the reduced shear stiffness of the multilayered plate, $\beta$ is the principle shear parameter taking into account the transverse shears integrally over the plate thickness [13]. These parameters are calculated as follows

$$\nu = \sum_{k=1}^{N} \frac{E_k h_k \nu_k}{1-\nu_k^2} \left( \sum_{k=1}^{N} \frac{E_k h_k}{1-\nu_k^2} \right)^{-1}, \quad E = \frac{1-\nu^2}{h} \sum_{k=1}^{N} \frac{E_k h_k}{1-\nu_k^2},$$

$$\beta = \frac{12(1-\nu^2)}{E h \eta_1} q_{44}, \quad q_{44} = \frac{\sum_{k=1}^{N} \left( \lambda_k - \frac{\lambda_{k0}^2}{\lambda_{kk}} \right)^2}{\sum_{k=1}^{N} \left( \lambda_k - \frac{\lambda_{k0}^2}{\lambda_{kk}} \right) G_k^{-1}} + \sum_{k=1}^{N} \frac{\lambda_{k0}^2}{\lambda_{kk}} G_k,$$

$$\eta_1 = \sum_{k=1}^{N} \xi_k^{-1} \pi_{1k} \gamma_k - 3c_{12}^2, \quad \eta_3 = 4 \sum_{k=1}^{N} (\xi_k + 3\zeta_{k-1}\zeta_k) \gamma_k - 3c_{13}^2,$$

where

$$\lambda_k = \int_{\delta_{k-1}}^{\delta_k} f_0(z) dz, \quad \lambda_{kn} = \int_{\delta_{k-1}}^{\delta_k} f_k(z)n(z) dz,$$

$$\gamma_k = \int_{\delta_{k-1}}^{\delta_k} \frac{E_k h_k}{1-\nu_k^2} \left( \sum_{k=1}^{N} \frac{E_k h_k}{1-\nu_k^2} \right)^{-1} dz, \quad h\xi_k = h_k, \quad h\zeta_k = h_n \quad (n = 0, k),$$

$$\frac{1}{12} h^3 \pi_{1k} = \int_{\delta_{k-1}}^{\delta_k} g^2(z) dz, \quad \frac{1}{2} h^2 \pi_{3k} = \int_{\delta_{k-1}}^{\delta_k} g(z) dz,$$

$$c_{12} = \sum_{k=1}^{N} \xi_k^{-1} \pi_{3k} \gamma_k, \quad c_{13} = \sum_{k=1}^{N} (\zeta_{k-1} + \zeta_k) \gamma_k,$$

$$g(z) = \int_{\delta_{0}}^{\delta_0} f_0(x) dx.$$

Here $\delta_k$ is the distance between the reference surface of the cross section and the upper bound of the $k$th layer, $\delta_0$ is the distance from the reference surface to the inner bound of the plate, $\gamma_k$ is the dimensionless stiffness, $f_0(z)$ and $f_k(z)$ are continuous functions introduced by

$$f_0(z) = \frac{1}{h^2}(z - \delta_0)(\delta_N - z) \quad z \in [\delta_0, \delta_N],$$

$$f_k(z) = \frac{1}{h_k^2}(z - \delta_{k-1})(\delta_k - z) \quad z \in [\delta_{k-1}, \delta_k], \quad f_k(z) = 0 \quad z \not\in [\delta_{k-1}, \delta_k].$$
Let the external loads be the harmonic functions of $x_1$ and $x_2$:

$$F_3(x_1, x_2) = F_3^0 \sin(r_1 x_1) \sin(r_2 x_2).$$  \hfill (21)

Then the internal solution $w(x_1, x_2, z) = w^0(z) \sin(r_1 x_1) \sin(r_2 x_2)$ and $\Delta = -(r_1^2 + r_2^2)$.

Substitute Eq. (21) into Eqs. (12), Eq. (11), Eq. (15) and Eq. (16) and obtain

$$w_{SA} = w_{KL}(1 + r^2 (A_g - A_\nu)), \quad w_{KL} = \frac{F_3^0}{D r^4}, \quad w_{TR} = w_{KL}(1 + r^2 A_g), \quad w_{ESL} = w_{KL}(1 + r^2 A_{gESL}),$$

$$\{r^2 A_g, r^2 A_{gESL}\} \sim \mu^2 \frac{E}{G_{13}}, \quad \mu = rh, \quad r^2 = r_1^2 + r_2^2.$$  \hfill (22)

**Numerical example 1.**

Let a thin multilayer plate consisting of $N$ homogeneous isotropic layers with parameters: thickness $h_k$, density $\rho_k$, Young’s modulus $E_k$, shear modulus $G_k$ and Poisson’s ratio $\nu_k$, where $k = 1, 2, ..., N$ is a number of layers. It is assumed that each layer has a constant thickness.

Find the exact value $w(0)^{EX}$ by solving numerically Eqs. (8), (5). The approximate values $w_{KL}$, $w_{TR}$, $w_{SA}$ and $w_{ESL}$ correspond to the KL model, the generalized TR model, the SA model and the ESL model, respectively.

Two multilayer plates with a thickness $h = 1$, a small thin-walled parameter $\mu = rh = 0.2$ and a Poisson’s ratio $\nu = 0.3$ are considered.

Parameters of the three-layer plate:

$$h_1 = 0.1 h, \quad h_2 = 0.8 h, \quad h_3 = 0.1 h, \quad E_1 = E_3 = 1, \quad E_2 = \eta E_1.$$  

Parameters of the seven-layer plate:

$$h_1 = h_3 = h_5 = h_7 = 0.1 h, \quad h_2 = h_4 = h_6 = 0.2 h, \quad E_1 = E_3 = E_5 = E_7 = 1, \quad E_2 = E_4 = E_6 = \eta E_1.$$  

Parameter $\eta$ will be changed.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\eta$</th>
<th>$w(0)^{EX}$</th>
<th>$w_{KL}$</th>
<th>$w_{TR}$</th>
<th>$w_{SA}$</th>
<th>$w_{ESL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>6879</td>
<td>6825</td>
<td>6903</td>
<td>6879.6</td>
<td>6904</td>
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<td>13278</td>
<td>12658</td>
<td>13316</td>
<td>13276</td>
<td>13262</td>
</tr>
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<td>13840</td>
<td>20231</td>
<td>20187</td>
<td>16907</td>
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<td>13971</td>
<td>77666</td>
<td>77623</td>
<td>19298</td>
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<tr>
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<td>6903</td>
<td>6879.6</td>
<td>6904</td>
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<td>12052</td>
<td>11576</td>
<td>12087</td>
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<tr>
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<td>17264</td>
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<td>17308</td>
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<tr>
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<td>59361</td>
<td>12535</td>
<td>60955</td>
<td>60914</td>
<td>12792</td>
</tr>
</tbody>
</table>

T able 1. Comparison of exact and approximate values of deflections.
The approximate dispersion relation can be written as
\[ \lambda = \rho \left( \kappa \frac{h^2}{E_s} \right), \]
where \( \rho_0(z) \) is the density distribution in the thickness direction and is given by
\[ \rho_0(z) = \frac{\rho(z)}{\rho_s}, \]
and the seven-layer plate is small when the parameter \( \eta = 1 \) and \( \eta = 0.1 \) (the small level of heterogeneity) and increases with decreasing parameter \( \eta \). For \( \eta = 0.01 \) and \( \eta = 0.001 \) (the very high level of heterogeneity) the TR model and the SA model are more accurate than the KL model and the ESL model.

4 APPROXIMATE MODELS FOR COMPUTING FREE VIBRATION

Free vibrations of a plate are considered in the form
\[ w(x_1, x_2, z) = u^0(z) \sin(r_1 x_1) \sin(r_2 x_2). \]
The load can be expressed as
\[ f_3 = \lambda w(x_1, x_2, z) \rho_0(z), \quad m = \lambda u(x_1, x_2, z) \rho_0(z), \]
where
\[ \lambda = \frac{\rho_s h^2 \omega^2}{E_s}, \quad \rho_0(z) = \frac{\rho(z)}{\rho_s}, \quad \rho_s = \frac{1}{h} \int_0^h \rho(z) \, dz. \]
Here \( \lambda \) is the frequency parameter depending on the average mass density \( \rho_s \) and elastic modulus \( E_s \). In the problem of free vibrations Eq. (10) takes the form
\[ D \mu^2 \Delta^2 w(x_1, x_2, 0) = \lambda \left( w(x_1, x_2, 0) + \mu^2 A \Delta w(x_1, x_2, 0) + O(\mu^4) \right), \]
The approximate dispersion relation can be written as
\[ \lambda^{TR} = \lambda^{KL} \left( 1 + \mu^2 A_g \right)^{-1}, \quad \lambda^{KL} = \frac{D \mu^4}{\langle \rho(z) \rangle}, \quad \mu = rh, \quad r^2 = r_1^2 + r_2^2, \]
\[ \lambda^{SA} = \lambda^{KL} \left( 1 + \mu^2 (A_g - A_\nu - A_\rho) \right)^{-1}, \quad \lambda^{ESL} = \lambda^{ESL} \left( 1 + \mu^2 A_g^{ESL} \right)^{-1}, \]
where \( \langle \rho(z) \rangle = \frac{1}{h} \sum_{k=1}^n h_k \rho_k. \)
The coefficients \( A_g \) and \( A_\nu \) are found from Eqs. (13) and the coefficient \( A_\rho \) depends on the distribution of mass density in the thickness direction and is given by
\[ A_\rho = \int_0^h \left( (z - a)^2 \rho_0(z) - \rho(z) \int_0^z c_\nu(z_1) (z_1 - a) \, dz_1 + c_\nu(z) (z - a) \int_0^z \rho_0(z_1) \, dz_1 \right) \, dz. \]

Numerical example 2.
Consider two multilayer plates with the same parameters as in the numerical example 1 and set additionally for the three-layer plate
\[ \rho_1 = \rho_3 = 1, \quad \rho_2 = 0.4, \]
and the seven-layer plate
\[ \rho_1 = \rho_7 = 1, \quad \rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = 0.4. \]
Then the exact value is \( \lambda^{EX} \) from Eqs. (8), (5) and approximate values are \( \lambda^{KL}, \lambda^{TR}, \lambda^{SA} \) and \( \lambda^{ESL} \) from Eqs. (27).
Table 2. Comparison of the exact and approximate values of the frequency parameter.

<table>
<thead>
<tr>
<th>N</th>
<th>η</th>
<th>$\lambda^{EX} \cdot 10^4$</th>
<th>$\lambda^{KL} \cdot 10^4$</th>
<th>$\lambda^{TR} \cdot 10^4$</th>
<th>$\lambda^{SA} \cdot 10^4$</th>
<th>$\lambda^{ESL} \cdot 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>2.7767</td>
<td>2.8177</td>
<td>2.7859</td>
<td>2.7792</td>
<td>2.7855</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>1.4395</td>
<td>1.5193</td>
<td>1.4442</td>
<td>1.4437</td>
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<td>3</td>
<td>0.01</td>
<td>0.9497</td>
<td>1.3895</td>
<td>0.9506</td>
<td>0.9502</td>
<td>1.1375</td>
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<td>3</td>
<td>0.001</td>
<td>0.2514</td>
<td>1.3765</td>
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<td>0.2476</td>
<td>0.9965</td>
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<tr>
<td>7</td>
<td>1</td>
<td>2.7767</td>
<td>2.8177</td>
<td>2.78586</td>
<td>2.7792</td>
<td>2.7855</td>
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<tr>
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<td>0.1</td>
<td>1.5859</td>
<td>1.6613</td>
<td>1.5910</td>
<td>1.5907</td>
<td>1.6327</td>
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<td>7</td>
<td>0.01</td>
<td>1.1104</td>
<td>1.5457</td>
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<td>7</td>
<td>0.001</td>
<td>0.3246</td>
<td>1.5341</td>
<td>0.3155</td>
<td>0.3155</td>
<td>1.5033</td>
</tr>
</tbody>
</table>

The qualitative conclusions derived from Table 2 are the same as from Table 1 in the static case. As $\eta$ decreases (the level of heterogeneity increases) the values of the frequency parameter decrease and the error increases. For small values of $\eta$ the TR model is preferable that the SA model because it does not require calculation of the small coefficients $A_\nu$ and $A_\rho$.

5 APPROXIMATE MODELS FOR COMPUTING FREE VIBRATION OF A SHELL

Free vibrations of a circle cylindrical shell of intermediate length with radius $R$ are considered. Let us take as the curvilinear coordinates on a cylindrical surface $s, \varphi, z$ which are the shell generatrix ($0 \leq s \leq l$, $l$ is the height of cylinder), the angle in circumferential direction ($0 \leq \varphi \leq 2\pi$) and the thickness coordinate ($0 \leq z \leq h$). It is assumed that the shell is heterogeneous in the thickness direction, in particular, it may be multilayered. The technical shell theory equations for the TR model read as

$$\frac{1}{K_0} \Delta \Delta \Phi + \Delta_R w = 0, \quad w = \Psi - A_g \Delta \Psi,$$

$$D\Delta \Psi - \Delta_R \Phi - F_3 = 0,$$

where

$$K_0 = \int_0^h E(z) dz. \quad (30)$$

Here $\Delta$ is the Laplace operator in the curvilinear coordinates, $\Delta_R$ is the second-order operator depending on the shell curvature

$$\Delta_R w = -\frac{1}{AB} \left( \frac{\partial}{\partial \alpha} \left( \frac{B}{AR_2} \frac{\partial w}{\partial \alpha} \right) + \frac{\partial}{\partial \beta} \left( \frac{A}{BR_1} \frac{\partial w}{\partial \beta} \right) \right) \quad (31)$$

The first Eq. (29) is the compatibility equation, $\Phi$ is the stress function involving the tangential stress resultants. The second Eq. (29) is the equilibrium equation in the normal direction, $w$ is the normal deflection. The functions $\Psi$ and $\Phi$ are introduced instead of the rotation angles of normal fibers $\varphi_1$ and $\varphi_2$ as

$$\varphi_1 = -\frac{1}{A} \frac{\partial \Psi}{\partial \alpha} + \frac{1}{B} \frac{\partial \Theta}{\partial \beta}, \quad \varphi_2 = -\frac{1}{B} \frac{\partial \Psi}{\partial \beta} - \frac{1}{A} \frac{\partial \Theta}{\partial \alpha}. \quad (32)$$

The elastic parameters $E$, $D$, $A_g$ and $A_g^{ESL}$ for the GTR model and the ESL model are given in Eqs. (13) and Eqs. (17). The same values of parameters can be used for shells. Suppose that the
shell edges \( s = 0 \) and \( s = l \) are simply supported. The transversely vibration modes are in the same form as Eq. (23)
\[
w(x_1, x_2, z) = w^0(z) \sin(r_1 s) \sin(r_2 \varphi).
\] (33)

Eqs. (29) give the approximate formulas for values of the bending frequency parameter
\[
\lambda_{TR} = \frac{D r^4}{\langle \rho(z) \rangle (1 + g_{TR})} + \frac{K_0 r_1^4}{\langle \rho(z) \rangle r^4}, \quad g_{TR} = \frac{r^2 A_g}{R^2},
\]
\[
\lambda_{KL} = \frac{D r^4}{\langle \rho(z) \rangle} + \frac{K_0 r_1^4}{\langle \rho(z) \rangle r^4},
\]
\[
\lambda_{ESL} = \frac{D r^4}{\langle \rho(z) \rangle (1 + g_{ESL})} + \frac{K_0 r_1^4}{\langle \rho(z) \rangle r^4}, \quad g_{ESL} = \frac{r^2 A_{g_{ESL}}}{R^2},
\]

where
\[
r^2 = r_1^2 + r_2^2, \quad r_1 = \frac{m\pi}{l}, \quad r_2 = n \quad m, n = 1, 2, 3...
\] (35)

The exact numerical solution for shell is not considered in this paper. It is found from a system of differential equations with variable coefficients because of the influence of the shell curvature.

**Numerical example 3.**

Consider a three-layer isotropic cylindrical shell with a thickness \( h = 0.05 \) and the same parameters as in previous numerical examples and set additionally
\[
h/R = 0.05, \quad l/R = 4, \quad \eta = 0.01.
\]

For this shell the approximate values of the frequency parameter for various values of wave numbers \( n \) and \( m \) are calculated and presented in Table 3.

**Table 3.** Comparison of the approximate values of the frequency parameter for shell.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( n )</th>
<th>( \mu )</th>
<th>( g_{TR} \cdot 10^3 )</th>
<th>( g_{ESL} \cdot 10^3 )</th>
<th>( \lambda_{KL} \cdot 10^3 )</th>
<th>( \lambda_{TR} \cdot 10^3 )</th>
<th>( \lambda_{ESL} \cdot 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.11</td>
<td>0.13</td>
<td>0.06</td>
<td>0.588</td>
<td>0.561</td>
<td>0.575</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.16</td>
<td>0.28</td>
<td>0.13</td>
<td>1.086</td>
<td>0.868</td>
<td>0.968</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0.20</td>
<td>0.48</td>
<td>0.23</td>
<td>3.025</td>
<td>2.053</td>
<td>2.464</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>0.25</td>
<td>0.74</td>
<td>0.35</td>
<td>7.135</td>
<td>4.107</td>
<td>5.270</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.28</td>
<td>0.88</td>
<td>0.42</td>
<td>10.793</td>
<td>6.045</td>
<td>7.781</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.32</td>
<td>1.17</td>
<td>0.56</td>
<td>20.647</td>
<td>11.098</td>
<td>14.282</td>
</tr>
<tr>
<td>1</td>
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<td>1.39</td>
<td>109.896</td>
<td>28.155</td>
<td>45.920</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
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<td>10.01</td>
<td>4.80</td>
<td>1315.230</td>
<td>128.714</td>
<td>235.097</td>
</tr>
</tbody>
</table>

Table 3 presents numerical results illustrating the degree of proximity the asymptotically values of frequency parameter. In this test case, the shear parameters \( g_{TR} \) and \( g_{ESL} \) for the TR model and the ESL model differ by approximately 50%. While these parameters are small their influence on the values of frequency parameter is insignificant. The wave parameters \( m = 1 \) and \( n = 2 \) (\( \mu = 0.09 \)) correspond to the lowest oscillation frequency. The values of frequency parameter grow with the value \( \mu \). For small values of \( \mu \) the numerical values of frequency parameter differ slightly from each other. As \( \mu \) grows this difference becomes more significant. However, the difference between the values of frequency parameter for the shells are smaller than those for plates.
6 CONCLUSIONS

The bending problem of plates and shells were considered. It was assumed that plates and shells are strongly heterogeneous in thickness direction, in particular, they may be multilayered. The generalized TR model, the asymptotic SA model and the ESL model were analysed. Test cases for multilayered plates and shells were investigated. The difference between the asymptotic and the numerical solution for plates and shells was discussed. It was found that the error of approximation depends on the two dimensionless parameters: the small wave-thickness parameter $\mu = h \nu$ and the parameter of heterogeneity for a multilayered plate $\eta (\eta = E_2 / E_1)$.

It was established that in the case of the small level of heterogeneity ($\eta \sim 1$) the SA model is essentially more exact. However, the generalized TR model, the ESL model and the KL model are also exact with little significant difference. In the case of the high level of heterogeneity sufficiently exact results were obtained using the generalized TR model, as the more complex SA model. The generalized TR model is preferable because it is less complicated.

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REFERENCES


SHAKING TABLE TESTS ON AN INTEGRAL ABUTMENT BRIDGE MODEL: PRELIMINARY RESULTS

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Abstract

Over the last decade, there was a renewed interest on Integral Abutment Bridges (IABs), characterized by the absence of bearing supports and expansion joints, leading to reduced construction and maintenance cost over ordinary bridges. Due to monolithic connections between abutments and deck, complex Soil-Structure Interaction (SSI) phenomena tend to develop between bridge and backfill in static and dynamic conditions, due to thermal expansion and earthquake action, respectively. An experimental campaign was conducted using the 3x3 m 6 DOF shaking table and the 5 m long shear stack of the University of Bristol, focusing on SSI effects between the IAB model and the backfill soil under earthquake loading. After a description of test set-up and of protocol, preliminary results will be illustrated and discussed.

Keywords: Integral Abutment Bridges, Shaking Table, Soil-Structure Interaction.
1 INTRODUCTION

It is well known that bridges are the most vulnerable elements in transport networks. Due to ageing and poor maintenance, most of these structures is subjected to a worsening of mechanical properties. Additionally, many bridges in earthquake-prone areas were realized without considering seismic actions. The dynamic response of bridges can be highly affected by Soil-Structure Interaction, Site effects and Spatial variability of ground motion [1, 2, 3]. Therefore, innovative solutions for the construction of new bridges both in seismic and non-seismic areas are required.

Integral Abutment Bridges (IABs), which are characterized by the absence of bearing supports and joints between the bridge girder and the abutments, have received an increasing interest in the last years due to their advantages with respect to Conventional Bridges [4]. IABs are now very common in many countries worldwide, for new realization as well as for retrofitting of Conventional Bridges [5]. United States (US) has the widest experience on IABs [6] and it is reported that more than 13,000 such bridges [7] were built by the mid 2000’s. Nevertheless, a meaningful number of IABs can be found in central Europe, mainly in Germany, Switzerland, UK, Austria, Luxembourg and France [8]. IABs have been considered and realized in many other countries, including Japan [9] and Australia [10]. In China, the first applications started at the end of the 1990s, but only in the recent years the jointless bridges are becoming more popular [11, 12].

IABs have improved durability and less need for maintenance with respect to conventional bridges, bringing to a dramatic cost reduction. Despite these direct advantages, due to the monolithic connection between the bridge superstructure and the abutments there is an increased structural redundancy which is beneficial on one side, but on the other side this brings to a more complex interaction between the superstructure, the abutments and the backfill soil. In other words, Soil-Structure Interaction (SSI) has a significant role in the response of IABs to horizontal loads, both in the static (thermal variations) and the dynamic case (earthquake loading). Horizontal loads can cause an abrupt increase in earth pressure on the abutments, which was the object of various studies in the last twenty years. The contribution of the embankments was discussed in different numerical studies highlighting the importance of the dynamic properties of the embankments [13, 14]. Due to this complex behavior, the maximum length of IABs is a debated issue [15].

Experimental investigations on SSI and the role of abutments in the case of seismic excitations have been object of less investigations. Recently, a comprehensive joint research effort by several US universities (University of Nevada–Reno, University of California – San Diego, University of California - Berkeley) focused on seismic response of bridges with seat type abutments. In particular, Saiidi et al. [16] performed shaking table tests on large scale models of two-span and four-span bridges at the University of Nevada, Reno; Wilson and Elgamal [17] did shaking table tests to investigate the abutment contribution to the structural response.

Some authors [18] introduced the technique of interposing layers of geosynthetic materials between the abutment and the backfill soil with the aim to mitigate soil pressures. A typical material used for these applications is Expanded Polystyrene (EPS) Geofoam, which can be posed in blocks behind the abutment, acting as a compressible inclusion, allowing for expansion of the structure due to thermal variations. Numerical studies on seismic response of IABs have shown the benefits of compressible inclusions leading to reduction of soil pressures and settlements in the backfill [19].

The experimental program of project SERENA (Seismic Response of Novel Integral Abutment Bridges), developed under the Horizon2020 SERA (Seismology and Earthquake Engineering Research Infrastructure Alliance for Europe) program is presented herein. The aim of
this project was to perform shaking table tests on a scaled model of Integral Bridge, placed in a large (5m long) shear stack filled with sand to reproduce natural soil conditions. Different configurations of compressible inclusion behind the abutments the influence on the dynamic response of the disconnection of the foundation piles from the foundation [20] were investigated as design solutions for IABs in earthquake. After a description of the test set-up and of the test protocol, preliminary results regarding the dynamic response of the model will be illustrated and discussed.

2 EXPERIMENTAL PROGRAM

2.1 Shaking Table and Equivalent Shear Beam Container

The EQUALS-BLADe Earthquake Simulator, shown in Figure 1, consists of a 3 x 3m cast aluminum platform weighing 3.8 tons and capable of shaking a maximum payload of 15 t at an operational frequency range of 0-100Hz.

![Shaking Table at EQUALS-BLADe, University of Bristol.](image)

The Equivalent Shear Beam soil container (“Shear Stack”), shown in Figure 2 and Figure 3, consists of eleven rectangular aluminium rings, which are stacked alternately with rubber sections to create a hollow yet flexible box of inner dimensions 4.80m long, 1m wide and 1.15m deep [21, 22].

![Equivalent shear beam container (“shear stack”), filled with sand and with the bridge model; (b) Section of one of the side walls of the shear beam container.](image)
2.2 Properties of the bridge model and foundation piles

Photos of the instrumented model structure tested are reported in Figure 3a. In the design and construction of the bridge model, the aim was to reproduce the monolithic connection between the bridge superstructure and the abutments. The abutment and footings of the bridge were realized using 32 mm thick aluminium sheets, connected by 8 M12 bolts. The deck was realized using four steel beams with a length, width, and depth of 1000, 100, and 30 mm, respectively. This solution was preferred to the use of a continuous slab in order to permit the accessibility and inspection of the space located under the bridge deck.

![Figure 3: (a) tested IAB model and (b) model piles.](image)

Each of the abutment wall footings was supported by eight model piles. A total of 16 model piles was supporting the abutments. Piles were made up of hollow aluminium tubes with a height of 400 mm and an outer diameter and vessel thickness of 22.24 and 1.3 mm, respectively, as displayed in Figure 3b. A nylon plug was inserted at the top of each pile to screw a steel riveted bar and obtain the “connected Piles” configuration (simulating a perfectly fixed constraint between the pile and the slab footing of the abutment.

2.3 Property of Soil layer materials

The soil used in the shaking table tests was dry Leighton Buzzard fraction B sand [23]. The free surface of the soil deposit was at 1000 mm from the soil container bottom. The soil was deposited in the shear stack at two increments. Initially, a 400 mm thick layer was deposited around the model piles in the shear stack. The 400 mm thick layer was then densified with vibrations applied with the shaking table (i.e., white noise input).

![Figure 4: Schematic diagram of the shear stack: (a) phase 1 – sand filling up to 400 mm, (b) phase 2 – sand filling up to 1000mm (600 mm additional fill) up to the level of the bridge deck.](image)
Following the placement of abutment walls and the bridge deck, an additional 600 mm thick sand layer was placed behind the abutment walls. Figure 4 depicts the two filling stages of the shear stack and the position of the bridge model.

2.4 Instrumentation

The instrumentation used in this experimental program consisted of accelerometers, strain gauges, and LVDTs (Linear Voltage Differential Transducer). The shaking table, interior base and exterior frames of the shear stack, the backfill sand soil, the bases of the abutment wall, and the model bridge deck were instrumented with accelerometers. The model piles, bottom and top portion of the abutment wall footing, and the abutment wall were instrumented with strain gauges. The horizontal and vertical movements of the abutment wall were tracked by LVDTs. The overall instrumentation scheme showing only accelerometers and LVDTs is illustrated in Figure 5. In this figure are reported the accelerometers on the shear stack (colored in magenta), the array of sensors installed inside the shear stack between the bridge model and the container wall, aimed at reproducing free-field conditions (in red), the backfill array (in blue), the sensors placed on the bridge model (in black) and the ones in the piles area (in black). Strain-gauges on the abutment and piles are not reported in this figure and their response is not yet considered in this preliminary analysis of the experimental results. The layout of instrumentation was designed taking into account also similar shaking table tests performed in the EQUALS-BLADE Laboratory of the University of Bristol [23].

3 TEST PROTOCOL AND SELECTION OF ACCELEROMGRAMS

A total of five test configurations were investigated. To simulate the effect of compressible inclusion on the model, PU foam was adopted choosing a value of stiffness relatively low to maximise the benefits and simulate the effect of EPS in a typical IAB prototype. In the experiment, Polyurethane foam was preferred to EPS Geofoam because is easily workable in the laboratory and is characterized by a lower shear modulus. Material tests are ongoing in the University of Bristol EQUALS-BLADE Laboratory to assess the mechanical characteristics of the material. The number of PU layers and pile head connection were varied to obtain the different configurations. Tested configurations together with the date of experiments are reported in Table 1.
Initially, for the first test configuration (i.e., test number 1), two layers of PU foam were introduced behind the abutment wall, before filling the shear stack with the backfill sand. Then, the backfill sand was removed using a vacuum pump in order to realize the configuration with one layer (test number 2) and no compressible inclusion (test number 3), respectively. The box was then fully emptied to modify the pile connection and simulate a hinged constraint between the base footing of the abutments and the piles (i.e., noCP configurations). First, noCP configuration was run without any compressible inclusion (test number 4) and one layer of EPS was added for the final configuration (test number 5).

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Date</th>
<th>Designation</th>
<th>EPS Layers Installed</th>
<th>Pile Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>02.07.2018</td>
<td>EPS2&amp;CP</td>
<td>2</td>
<td>Fixed</td>
</tr>
<tr>
<td>2</td>
<td>04.07.2018</td>
<td>EPS1&amp;CP</td>
<td>1</td>
<td>Fixed</td>
</tr>
<tr>
<td>3</td>
<td>05.07.2018</td>
<td>noEPS&amp;CP</td>
<td>0</td>
<td>Fixed</td>
</tr>
<tr>
<td>4</td>
<td>09.07.2018</td>
<td>noEPS&amp;noCP</td>
<td>0</td>
<td>Hinged</td>
</tr>
<tr>
<td>5</td>
<td>10.07.2018</td>
<td>EPS1&amp;noCP</td>
<td>1</td>
<td>Hinged</td>
</tr>
</tbody>
</table>

Table 1: Test configurations 1 to 5

The signals used as input for the shaking table were selected among the seismic records of the Italian Accelerometric Network (RAN), which are openly accessible and can be downloaded by the website of the European Strong Motion Database (www.esm.mi.ingv.it). Only strong motion stations for which the shear wave velocity $V_S$ is known were taken into account.

Uniform Hazard Spectra (UHS) were defined for the site of L’Aquila, in Italy, which is characterized by a medium to strong seismicity, and using three different return periods ($T_R$), namely 50, 100 and 500/1000 years. For each $T_R$, depending on the disaggregation of hazard, different ranges of Magnitude $M$ and Source-to-site Distance $R$ (in this case epicentral distance was considered) were used to search the database. The complete list of selected accelerograms is reported in Table 2.

<table>
<thead>
<tr>
<th>N.</th>
<th>$T_R$</th>
<th>Station</th>
<th>Date</th>
<th>Comp.</th>
<th>$M$</th>
<th>$R$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>AMT</td>
<td>18/01/2017</td>
<td>NS</td>
<td>5.4</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>CSC</td>
<td>26/10/2016</td>
<td>NS</td>
<td>5.4</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>SELE</td>
<td>21/03/1998</td>
<td>EO</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>500-1000</td>
<td>CSC</td>
<td>30/10/2016</td>
<td>NS</td>
<td>6.5</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>500-1000</td>
<td>AMT</td>
<td>24/08/2016</td>
<td>NS</td>
<td>6.2</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>500-1000</td>
<td>AMT</td>
<td>30/10/2016</td>
<td>EO</td>
<td>6.5</td>
<td>26</td>
</tr>
<tr>
<td>1</td>
<td>VERT</td>
<td>AMT</td>
<td>18/01/2017</td>
<td>VERT</td>
<td>5.4</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>VERT</td>
<td>AMT</td>
<td>30/10/2016</td>
<td>VERT</td>
<td>6.5</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 2: Selected accelerograms.

The natural records (not scaled in amplitude) were compared with the UHS to check for a rough spectrum compatibility. In the selection of the signals, attention was paid to the frequency content, excluding records with frequencies exceeding 30 Hz to avoid a poor frequency content of the accelerogram after the necessary scaling in the experiment. Records were scaled in the frequency domain by a factor of 5 to adapt them to the scaling factor of the model (e.g., [24]). In Figure 6a are reported the time-scaled and unscaled versions of the horizontal component of Record 6 related to the 30 October 2016 earthquake recorded at the Amatrice station (AMT) in Italy (see Table 2). The effect of time scaling in time is evident in Figure 6b where the Fourier
amplitude spectra of the two waveforms are shown. The record is characterized by a peak positive acceleration of 0.44g. In the scaled case the frequency content is spread along a wider frequency range.

Figure 6: Comparison between the time-scaled and unscaled version of Record 6: (a) Acceleration time histories, (b) Fourier Amplitude spectra

4 PRELIMINARY RESULTS

In each test configuration, a sequence of shakings including the waveforms provided in Table 2 were employed. Hammer tests and white noises tests were performed between each shaking to assess the changes on the natural frequency of the system. In Table 3 is provided the initial frequency measure through white noise of each test configuration emphasizing that the initial frequency was ranging between 9 and 17 Hz for all models.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Test configuration</th>
<th>Frequency of model [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EPS2&amp;CP</td>
<td>16.7</td>
</tr>
<tr>
<td>2</td>
<td>EPS1&amp;CP</td>
<td>14.0</td>
</tr>
<tr>
<td>3</td>
<td>noEPS&amp;CP</td>
<td>13.8</td>
</tr>
<tr>
<td>4</td>
<td>noEPS&amp;noCP</td>
<td>14.3</td>
</tr>
<tr>
<td>5</td>
<td>EPS1&amp;noCP</td>
<td>9.9</td>
</tr>
</tbody>
</table>

Table 3: Initial frequency of the model measured through white noise in each initial test configuration

Figure 7 and 8 report some preliminary results in terms of acceleration responses of the accelerometers located in the soil and on the bridge model. In particular, the figures report the maximum acceleration response value corresponding to the time instant of peak acceleration towards West direction (corresponding to the positive peak in Figure 6a) for each horizontal accelerometer. Results in Figure 7 are related to the time-scaled Record 6, while in Figure 8 are depicted the results for unscaled Record 6.

As a general remark, it can be observed that the acceleration values in the unscaled case are higher of about 100% than those in the scaled case. This is expected, as in the unscaled case the same input acceleration is applied in a different frequency range. By observing the acceleration responses recorded by the Free Field Array (channels: ch3, ch4, ch5 and ch6), it can be noticed the amplification from the bottom to the top of the container. For example, in the configuration with no PU layer and connected piles, the acceleration responses are 0.2, 0.23, 0.26 and 0.36g.
(scaled case, Figure 7a) and 0.51, 0.54, 0.58 and 0.60g (unscaled case, Figure 8a) for channels 3, 4, 5 and 6, respectively. While in the scaled case the soil acceleration response close to the surface (ch6) is on average 40% higher than the one recorded about 0.4 m below the surface (ch5), in the unscaled case the two average values are very close (about 0.58g). The scaled case represents the reference for a prototype IAB as it takes into account the model scaling. The good response of the soil container can be double-checked through the acceleration response of ch22, which reported values of 0.24 and 0.56g for the same configurations described before, in the scaled and unscaled case that is an interpolated value between the response of ch4 and ch5 showing that the box was accommodating the soil deformation. The free-field array should be basically undisturbed by the change of configuration, so a similar trend of the array is expected for the five configurations. This a trend observed in the unscaled case for four channels of the array (i.e., Figure 8) but it shows some discrepancy for the surface channels of the scaled case (Figure 7). Difference between configurations in Figure 7, for ch5 and ch6 are not still not very high but they could indicate that the surface profile of the different configuration has affected the response recorded at ch6. This latter effect will be further investigated double-checking the settlement profile for each configuration. Comparisons among configurations in Figure 7 suggest that the mitigating solution of adding a compressible inclusion and disconnecting the pile tends to reduce the horizontal acceleration on the bridge and, in particular, the joint solution of one layer of compressible inclusion and disconnection of piles seems very efficient with respect to the reference case (i.e., compare Figure 7a and 7e). On the contrary, the mitigation strategies seem to increase the peak accelerations in the backfill array. This effect has to be investigated further considering also the other records considered in the experimental campaign to assess if it can be considered a systematic effect. Similar experimental campaigns are not available in literature and most of the experimental and numerical study on compressible inclusions tend to emphasize the decrease in displacement in the backfill and the reduction of settlements, e.g., [1, 14, 15]. The increasing trend of acceleration in the backfill caused by the compressible inclusion is still observable in the unscaled case (see Figure 8), but when the frequency of the input is not scaled it seems not to have a significant reduction effect on the acceleration of the bridge. The above observation can be explained by the fact that the frequency range in of the input, not being scaled, is not relevant for the excitation of the bridge. A number of white-noise and hammer tests have been performed on the bridge prototype and not shown herein emphasizing that the model bridge natural frequency varies in the range of 9 to 17Hz according to the configuration considered (i.e., being the configurations with compressible inclusion more deformable and so with lower characteristic frequencies) as shown in Table 3. Accelerations recorded within the piles in the lower stratum of sand (ch 7, ch8, ch14) seems to be undisturbed by the pile connection modifications (i.e., from fixed to hinged) and have acceleration ranges very similar to those shown in the bottom part of the free filed array for both the scaled and unscaled configuration.

5 CONCLUSIONS

A preliminary presentation of the experimental results of the SERENA experimental campaign and a general description of the full experimental program including set up and instrumentation layout was provided. Results are showing the importance of scaling relationships to be employed in soil-structure interaction problems beyond the conventional scaling approaches of purely structural and geotechnical problems focusing on relative stiffnesses for the scaling of each variable in the problem. A demonstration for this is provided though the comparison of scaled and unscaled time-history response on the integral abutment bridge model and its configuration in the shear stack.
A preliminary assessment of mitigation strategies such as addition of compressible inclusions and release of fixed connection between piles and abutment base have shown an indication of positive benefits such as a general reduction of the maximum acceleration on the abutment.

Figure 7: Map of horizontal acceleration responses to Record 6 (Time-scaled) input for all configurations (i.e., black time-history in Figure 6a): (a) No PU and connected piles, (b) 1 PU layer and connected piles, (c) 2 PU layers and disconnected piles, (d) No PU layers, (e) 1 PU layer and disconnected piles. All accelerations are expressed in g.
Figure 8: Map of horizontal acceleration responses to Record 6 (unscaled) input for all configurations (i.e., red time-history in Figure 6a): (a) No PU and connected piles, (b) 1 PU layer and connected piles, (c) 2 PU layers and disconnected piles, (d) No PU layers, (e) 1 PU layer and disconnected piles. All accelerations are expressed in g.
This was assessed considering only waveform for the five configurations investigated and further analyses on how systematic this trend is are needed. On the contrary, behavior of the backfill needs to be assessed including analysis of settlements as it seems that an increase of acceleration is recorded when the acceleration on the abutment decreases. Again, this needs further investigation considering the full sequence of waveforms considered in the experimental campaign. Analysis of responses though LVDT and strain gauges on the integral abutment will provide further insights on the benefits of the mitigation measures investigated.

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REFERENCES


A NONLINEAR MATERIAL MODEL OF CORRODED REBARS FOR SEISMIC RESPONSE OF BRIDGES

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Abstract

A generalized cyclic steel model characterized by isotropic and kinematic hardening, inelastic buckling in compression and corrosion for the rebar in reinforced concrete (RC) structures is presented. This model has been implemented in in-house fiber program CY.R.U.S.-M developed in MATLAB, to perform the seismic analysis of RC sections. The model is especially accurate, with respect to experimental cyclic behavior of rebars with buckling in compression, in case the strain in compression does not exceed 1.2 - 1.5 %. Four RC sections were selected as the case studies for a single concrete geometry and different steel configurations assumed representative of RC bridge piers (in a suitable scale) and subjected to a cyclic curvature history representative of a severe seismic load, not far from collapse. Different rebar characteristics (yielding stress, maximum stress, hardening ratio), axial loads, corrosion percentages have been selected to perform some cyclic parametric analyses. The numerical results have shown that the maximum strain of the rebar in compression is always smaller than 1.2 - 1.5 % and therefore the simple model for the steel is a valid tool for the structural assessment. Finally, corrosion of the rebars reduces the section capacity in term of strength and energy dissipation.

Keywords: RC bridge, corroded rebars, cyclic steel model, corrosion, buckling, seismic response
1 INTRODUCTION

In recent years, many reinforced concrete structures (RC) have reached their service life and therefore assessment of the structural capacity ([1]-[6]) has to be performed to evaluate the proper repair and/or retrofitting strategies ([7]-[15]). Corrosion of the steel reinforcement in RC structures reduces the structural capacity before the design prevision especially in case of improper maintenance interventions, design errors or construction defects. For that reason, structural assessment should be performed including corrosion effects. This is more critical in seismic areas because corrosion reduces the area of the steel rebars, in a not uniform way when inside concrete sections (pitting corrosion), resulting in smaller cyclic capacity (strength and ductility) of rebars ([16]-[24]) and therefore of the RC members.

The not uniform reduction of the rebar area not only reduces the rebar strength but also introduces an eccentricity of the resulting force that can be taken into account reducing the steel stress-strain behavior in tension/compression. Not uniform corrosion along the rebar focuses large local strain demand resulting in the decrease of the rebar ductility.

The reduction of the stirrups confinement due to corrosion reduces the member ductility and increases the buckling attitude of the longitudinal rebars. The reductions of the stirrup area and ductility reduces the plastic distribution of the forces among the stirrups and therefore the shear strength mechanism can be less effective. Cracking of concrete due to rush expansion modifies the concrete mechanical properties and the steel-concrete bond. Spalling of concrete cover modifies the section shape.

Numerical models may be valid tools to evaluate the structural behavior of RC sections with corroded rebars, but the model uncertainty is large, due to pitting corrosion. The use of simple models allows setting some bounds in the structural assessment.

In this paper, the fundamental aspects of the cyclic axial-bending behavior of RC sections with corroded rebars are discussed.

Expression to calculate the steel properties (stress and strain) for a given corrosion percentage $\Psi$ are given in §2. Due to pitting corrosion bar diameters of longitudinal and transversal reinforcements reduce and therefore the free length of longitudinal rebars increases. A simple expression for this free length is suggested.

A generalized model to describe the cyclic behavior of rebars including corrosion is presented in §3. The model is a generalization of the cyclic one for steel rebars with inelastic buckling in compression, characterized by isotropic and kinematic hardening effects proposed in Zhou et al. 2014 [25] and Zhou et al. 2015 [26]. The generalized model is especially accurate, with respect to experimental cyclic behavior of rebars with buckling in compression, in case the strain in compression does not exceed $1.2 - 1.5\%$. Larger strains should require a nonlinear envelope in compression. In fact, other models which aim at representing very large strain in compression, use more complex formulations for that branch (Kashani et al. 2013 [22]).

Four RC sections are selected as the case studies in §4. These sections have single concrete geometry and different steel configurations assumed representative of RC bridge piers (in a suitable scale). Parametric cyclic analyses were carried out on these sections using in-house software (CY.R.U.S-M) based on fiber model, where the cyclic behavior of the corroded rebars is the proposed generalized steel model. The cyclic curvature history is representative a severe seismic load, not far from collapse.

Different rebar characteristics (yielding stress, maximum stress, hardening ratio), axial loads, corrosion percentages are considered in §4. The results are shown in §5 in term of the maximum strain of steel in compression/tension and the moment Vs curvature behavior of the RC sections.

Finally, the first conclusions are given in §6. For the cases analyzed, the maximum strain of the rebar in compression is always smaller than 1.2 - 1.5 % and therefore the simple model for
the steel is a valid tool for the structural assessment. Corrosion of the rebars reduces the section capacity in term of strength and energy dissipation.

2 CORRODED REBAR GEOMETRIES AND MECHANICAL PROPERTIES

Corrosion changes the not corroded steel rebar geometries and the mechanical properties. The diameter of the corroded rebar (D’) may be evaluated by equation (1):

\[
D' = D \cdot \sqrt{\frac{1 - \frac{m_0 - m}{m_0}}}
\]

in which D is the not corroded rebar diameter, m₀ is the rebar mass before corrosion and m is the rebar mass after corrosion. The ratio \((m_0 - m) / m_0\) measures the amount of corrosion.

The mechanical property \(X'_i\) (i = stress / force / strain) of the corroded rebar in case of pitting corrosion may be calculated by equation (2).

\[
X'_i = X_i \cdot (1 - \beta'_i \cdot \Psi)
\]

Where \(X_i\) (i = stress / force / strain) is the mechanical property of the not corroded rebar, \(\Psi = (m_0 - m) / m_0 / 100\) is the corrosion percentage and \(\beta'_i\) is the coefficient for the pitting corrosion (Du et al. 2005 [24]).

The rebar behavior in tension or in compression can be approximated by numerical curves. These curves are described by the parameters in Figure 1: the yield (\(f_y\)), the maximum (\(f_{max}\)) and the ultimate (\(f_u\)) stresses; the maximum (\(\varepsilon_{max}\)) and the ultimate (\(\varepsilon_u\)) strains; the hardening coefficient (b) and the initial elastic modulus (\(E_0\)).

![Figure 1 Load-strain curves for the rebar corroded in RC members for different corrosion percentages; Picture adapted from Meda et al. 2014 [19]](image)

The coefficient \(\beta'_i\) has been calibrated on the base of the experimental data from Meda et al. 2014 [19] for i = \(f_y\), \(f_{max}\), \(f_u\), \(\varepsilon_{max}\) and b. In Figure 2, the values of \(\beta'_i\) are showed in function of the corrosion percentage (\(\Psi\)) for each steel parameter.
Figure 2 Coefficient $\beta'_i$ for the yield ($f_y$), the maximum ($f_{\text{max}}$) and the ultimate ($f_u$) stresses; the maximum strain ($\varepsilon_{\text{max}}$); the hardening ratio (b); red crosses for the experimental data; black dot lines for the analytical interpolation curves.

Equation of the analytical curve, that interpolates better the experimental data, is given in Figure 2 for each parameter. These equations predict the $\beta'_i$ values for the corrosion percentage values not included in the experimental data.

The slenderness of the longitudinal rebar ($\lambda$) is the ratio between the free length of the longitudinal rebars (l) and the rebar diameter (D) [25]. Buckling of rebar in compression happens when the slenderness ($\lambda$) is greater than the critical slenderness ($\lambda_{cr}$) defined by equation (3).

$$\lambda_{cr} = \frac{5}{\sqrt{\frac{f_{yc}}{450}}}$$

in which $f_{yc}$ is the yield stress in compression.

Corrosion increases the slenderness of the longitudinal rebar because the free length increases whereas the rebar diameter reduces. The free length increases because a gap arises between the stirrups and the longitudinal rebar due to the rebar diameters reductions and the stirrups confinement efficiency reduces (Figure 3).
The free length for the corroded rebars (l’) can be calculated by equation (4):

\[ l' = l + 2 \cdot \mu \cdot G \]  

(4)

where l is the free length of the rebar before corrosion, G is the gap and \( \mu \) is a coefficient greater than 1. The gap (G) can be evaluated by equation (5):

\[ G = \left( \frac{D_{st} - D'_{st}}{2} \right) + \left( \frac{D - D'}{2} \right) \]  

(5)

in which \( D_{st} \) is the diameters of the not corroded stirrup and \( D'_{st} \) is the diameter of the corroded stirrup. The authors propose a value for \( \mu \) equal to 5 but it should be evaluated better on the base of the experimental data.

The slenderness of the corroded rebar (\( \lambda' \)) is the ratio between the free length of the longitudinal rebars (l’) and the rebar diameter (D’). The critical slenderness (\( \lambda'_{cr} \)) for the corroded rebar can be evaluated by equation (6):

\[ \lambda'_{cr} = \frac{5}{\sqrt{\frac{f'_{yc}}{450}}} \]  

(6)

in which \( f'_{yc} \) is the yield stress of the corroded rebar in compression.

Buckling of the corroded rebar in compression happen for \( \lambda' > \lambda'_{cr} \). This means that buckling of the corroded rebars may arise even if the slenderness of the not corroded rebar \( \lambda \) was smaller than the critical slenderness \( \lambda_{cr} \).

Indeed, the critical slenderness itself is a function of corrosion (\( \lambda'_{cr} \)) because yield stress in compression is function of corrosion (\( f'_{yc}(\Psi) \)).

3 CYCLIC MODEL FOR CORRODED REBAR IN RC SECTION

The cyclic model for the steel rebars with inelastic buckling in compression, characterized by isotropic and kinematic hardening effects proposed by the authors in Zhou et al. 2014 [25] and Zhou et al. 2015 [26] can be used to simulate the behavior of the corroded rebars. This is a generalized model that is especially accurate, with respect to experimental cyclic behavior of rebars with buckling in compression, in case the strain in compression does not exceed 1.2 –
1.5 % Larger strains should require a more complex curve in compression (Kashani et al. 2013 [22]).

The proposed model is based on the Menegotto-Pinto curve (Monti et al. 1992 [27]) in Figure 4a. The curve parameters are: the slope of the first asymptote of the curve, the yield point (εy, f_y) that is the intersection point of the two asymptotes of the curve, the curvature radius (R) and the hardening ratio (b) used to define the slope of the second asymptote of the curve. These parameters are updated for each branch n+1 in function of the strain history applied on the rebar. The buckling of the rebar in compression is simulated by the update of the model parameters using specific formulations in case of buckling (Figure 4b). The model describes buckling phenomena in compression (Figure 4b) in case of slenderness λ greater than the critical slenderness λ_{cr}.

![Figure 4 Proposed steel model: (a) curve without buckling; (b) curve with buckling](image)

Each branch (n) of the model is described by equations (7) and (8):

\[ \sigma^* = b^n \cdot \varepsilon^* + \frac{(1 - b^n) \cdot \varepsilon^*}{1 + \varepsilon^* R^n} \]  
\[ \sigma^* = \frac{(\sigma - \sigma_{f-1}^n)}{(f_y - \sigma_{f-1}^n)} ; \varepsilon^* = \frac{(\varepsilon - \varepsilon_{f-1}^n)}{(\varepsilon_y - \varepsilon_{f-1}^n)} \]  

The initial values of the model parameters f'_{y1} and b'_{01} for the corroded rebars can be calculated by equations (9) and (10) based on equation (2).

\[ f'_{y1} = f_y^1 \cdot (1 - \beta_{fy}(\Psi) \cdot \Psi) \]  
\[ b'_{01} = b_0^1 \cdot (1 - \beta_{b}(\Psi) \cdot \Psi) \]
In which \( f_y^1 \) and \( b_0^\prime \) are the yield stress and the hardening ratio for the not corroded rebars, \( \beta'_{\delta}(\Psi) \) and \( \beta'_{\delta}(\Psi) \) are the coefficients defined by equations in Figure 2 for a given corrosion percentage (\( \Psi \)).

The parameters \( f_y^1 \) and \( b_0^\prime \) may assume different values in tension or in compression because the pitting corrosion has different effects on rebar in tension or in compression. The effect of corrosion on the model parameter \( R \) was not investigated yet.

In absence of buckling, the parameter \( f_y \) is updated for each branch \( n+1 \) by equation (11):

\[
f_y^{n+1} = f_y^1 \cdot \text{sign} \left( -\xi_{p}^n \right) + P \cdot \Delta \sigma_k^n \left( b^n, E^n, \xi^n_p \right) + (1 - P) \cdot \Delta \sigma_i^n (b^n, E^n, \gamma^n_p)
\]

In which \( \text{sign}(x) \) is the sign function, \( P \) is the weight coefficient (\( P \leq 1 \)) for the two hardening contributions, \( \Delta \sigma_k^n \) is the kinematic hardening contribution and \( \Delta \sigma_i^n \) is the isotropic hardening contribution for the branch \( n \).

The hardening contributions are functions of the plastic excursion \( \xi^n_p \), the additional plastic excursion \( \gamma^n_p \) and the elastic modulus of steel \( (E^n) \) for the branch \( n \). The parameter \( b^n \) depends on \( b_0^\prime, \gamma^n_p, f_1^1, y_0 \). The parameter \( R^n \) depends on the maximum plastic excursion during the strain history and the initial value \( R_0 \). \( R^n \) has different formulations for unloading and reloading branches.

The critical slenderness \( \lambda'_{cr} \) and the slenderness \( \lambda' \) for the corroded rebar can be calculated by equation (6). Buckling of the rebar in compression arises for \( \lambda' > \lambda'_{cr} \).

In presence of buckling, the parameter \( f_y \) is updated for each branch \( n+1 \) by equation (12):

\[
f_y^{n+1} = f_y^1 \cdot \text{sign} \left( -\xi_{p}^n \right) + P \cdot \Delta \sigma_k^n \left( b^n, E^n, \xi^n_p \right) + (1 - P) \cdot \Delta \sigma_i^n (b^n, E^n, \gamma^n_p, \phi^n_p)
\]

The parameter \( b^n \) depends on \( b_0^\prime, \gamma^n_p, f_1^1, y_0 \) (steel initial elastic modulus), the plastic work \( \Phi^n_p \), \( \lambda' \) and \( \lambda'_{cr} \) for the branch \( n \). The parameter \( R^n \) depends on the maximum plastic excursion during the strain history and on the initial value of \( R^n \). \( b^n \) and \( R^n \) have different expressions for unloading and reloading branches. The complete formulations of the model are given in \textit{Zhuo et al. 2014} [25].

4 CYCLIC RESPONSE OF RC SECTION WITH CORRODED REBARS

The cyclic response of a RC section with corroded rebars subjected to severe seismic events was investigated.

The section with corroded rebars tested by \textit{Meda et al. 2014} [19] was selected as the case of study.

In-house software (CY.R.U.S-M) was implemented in MATLAB [28] to perform cyclic analyses of a section simulated by a fiber model. The results of this program were checked simulating the response of the base section of the column tested in \textit{Meda et al. 2014} [19].

Parametric analyses were carried out on four RC sections with different steel reinforcement configurations. Different steel characteristics (yield stress, maximum stress, hardening ratio), axial load and corrosion percentages values were considered.
The steel model presented in §3 was implemented in this program to describe the rebars behavior.

4.1 Case of study

The base section of the column with corroded rebars tested in Meda et al. 2014 [19] was selected as the case of study to test the in-house fiber program CY.R.U.S-M. The column geometries and the characteristics of the materials are representative of RC columns designed by previous design codes without seismic details. The reinforcement consisted in 4 Ø16 mm longitudinal rebars and Ø8 mm stirrups, 300 mm spaced. The concrete square cross section had dimensions 300 mm x 300 mm. The theoretical rebar corrosion percentage (Ψ) was 20 %. The materials characteristics obtained by the experimental tests are given in Table 1.

<table>
<thead>
<tr>
<th>Unconfined Concrete</th>
<th>Steel (not corroded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_c = 17.0</td>
<td>f_y = 520.0</td>
</tr>
<tr>
<td>ε_c = 2.0 %</td>
<td>f_max = 620.0</td>
</tr>
<tr>
<td>E_c = 30000</td>
<td>b = 0.0035</td>
</tr>
<tr>
<td></td>
<td>ε_{int} = 14 %</td>
</tr>
<tr>
<td></td>
<td>E_{so} = 210000</td>
</tr>
</tbody>
</table>

Table 1 Material properties for the cases of study [MPa]

Other four sections with the same square cross-section (300 mm x 300 mm) but different reinforcement configurations were selected as the case of studies to be representative of existing RC column or RC bridge pier in a suitable scale. Two longitudinal rebar percentages (ρ) and two different stirrups configurations were assumed (Figure 5a). ρ is the ratio between the total longitudinal rebar area A_s and the concrete section area A_c. The selected sections were labeled as:

- Section 1 with ρ = 0.9 % and Ø8 mm stirrups, 70 mm spaced;
- Section 2 with ρ = 2.7 % and Ø8 mm stirrups, 70 mm spaced;
- Section 3 with ρ = 0.9 % and Ø8 mm stirrups, 130 mm spaced;
- Section 4 with ρ = 2.7 % and Ø8 mm stirrups, 130 mm spaced.

The two stirrups configurations permitted to consider buckling of the longitudinal rebar for l/D = 8 or not buckling for l/D = 4.5 with l (stirrup space) and D evaluated for the not corroded rebar.

![Figure 5](image)

Figure 5 (a) RC sections selected as the case of studies; (b) curvature history applied on the sections for each parametric analysis.

The experimental curvature history measured on the corroded column in Meda et al. 2014 [19] when the drift was 2 % at the top of the column, was selected to perform each parametric
analysis. This curvature history is representative of a severe seismic load, not far from collapse (Figure 5b). The experimental axial load on the section was equal to 400 kN.

4.2 CY.R.U.S-M software

In-house software (Cyclic Response of Upgraded Section; CY.R.U.S-M) was implemented in MATLAB [28] to perform cyclic analysis of RC sections. This program divides a RC section in concrete and steel fibers, whose stress-strain behavior is described by uniaxial material models, applies the fixed vertical axial load on the section and imposes a cyclic curvature history on the section. The hypotheses assumed to evaluate the section behavior are: (i) the plane sections remain plain (hypothesis of Bernoulli); (ii) there is a perfect bond between concrete and steel.

The program determines the equilibrium of the forces on the base of the concrete and steel fibers responses and the corresponding moment on the section for each imposed curvature value. The program outputs are given in term of fiber stress-strain history and moment-curvature histories.

The model proposed in §3 describes the steel fibers behavior. The modified Kent and Park (Scott et al. 1982) [29] model simulates the concrete fiber behaviors: not confined concrete for the cover fibers and confined concrete for the core fibers.

The result of this software was checked in Figure 6a comparing the experimental section response from Meda et al. 2014 [19] with the numerical simulation obtained by CY.R.U.S-M.

The section divided into steel (green circles) and concrete fibers (red rhombuses for the confined concrete fibers; blue crosses for the unconfined concrete fibers) is shown in Figure 6b.

The concrete fibers at the section corners were removed to simulate the concrete cover spalling.

The steel model described in §3 was calibrated on the base of the properties given in Table 1 using equations in §3 for the corrosion percentage equal to 20 %.

The confined and unconfined concrete fibers behaviors were defined using the data in Table 1 and formulations in Mander et al. 1988 [30].

The experimental axial load equal to 400 kN and the curvature history given in Figure 5b were applied on the section.

The comparison has shown that the numerical simulation can reproduce well the RC section behavior in term of cyclic moment-curvature responses.

![Figure 6](image.png)

Figure 6 (a) Experimental response Vs numerical response by CY.R.U.S-M RC for the section from Meda et al 2014 [19]; (b) fiber section model implemented in CY.R.U.S-M
4.3 Parametric analyses

The parametric analyses were carried out considering different rebar characteristics (yield stress, maximum stress, hardening ratio), axial loads, corrosion percentages and steel configurations (longitudinal rebar percentage, stirrup space).

Four cases of analyses (case 0, 1, 2, 3) were carried out on the four sections (section 1, 2, 3, 4) described in §4.1.

The properties of the not corroded steel rebar and the unconfined concrete are shown in Table 1. The properties of the steel were evaluated including the effects of corrosion by equations given in §2 and §3. The properties of the confined concrete were calculated considering the confinement effect of the transverse stirrups by formulations given in Mander et al. 1988 [30].

Case 0 was performed to evaluate the effect of the variation of the axial load applied on the section on the fibers and section responses. Different values of the axial load were considered: 0, 100, 150, 200, 250, 300, 350, 400 KN. The steel corrosion percentage was $\Psi = 20 \%$.

Case 1 was carried out to evaluate the effect of the variation of the corrosion percentage ($\psi$) on the fibers and section responses. Different values of the corrosion percentage were selected: 0, 5, 10, 15, 20 %. The axial load on the section was equal to 400 kN.

Case 2 was carried out to evaluate the effect of the yield stress ($f_y$) of the not corroded rebar on the fibers and section responses. Different values of the yield stress of the not corroded rebar were considered: 400, 520, 600 MPa. These values represent different steel types used in construction up today. The steel corrosion percentage was $\Psi = 20 \%$. The axial load on the section was equal to 400 kN.

Case 3 was performed to evaluate the effect of the hardening ratio (b) of the not corroded rebar on the fibers and section responses. Different values of the hardening ratio of the not corroded rebar were considered: 0.001, 0.02, 0.05. The steel corrosion percentage was $\Psi = 20 \%$. The axial load on the section was equal to 400 kN.
5 ANALYSES RESULTS

The results of the parametric analyses are shown in term of the maximum strain of the steel in compression and in tension in Figure 7 for section 1 and 2 with longitudinal reinforcement percentages $\rho = 0.9\%$ or $\rho = 2.7\%$ and $l/D = 4.5$.

Figure 7 Results of the parametric analyses in term of the maximum strains of the steel fibers: axial load variation (a1, a2); Corrosion percentage variation (b1, b2); Yield stress variation (c1, c2); Hardening ratio variation (d1, d2) for section 1 with $\rho = 0.9\%$ and $l/D = 4.5$ and section 2 with $\rho = 2.7\%$ and $l/D = 4.5$; Corrosion percentage $\Psi = 20\%$ except for the data in figures b1 and b2.
The results of the parametric analyses are shown in Figure 8 in term of the maximum strain of the steel in compression and in tension for sections 3 and 4 with longitudinal reinforcement percentages $\rho = 0.9\%$ or $\rho = 2.7\%$ and $l/D = 8$.

Figure 8 Results of the parametric analyses in term of the maximum strains of the steel fibers: axial load variation (a1, a2); Corrosion percentage variation (b1, b2); Yield stress variation (c1, c2); Hardening ratio variation (d1, d2) for section 3 with $\rho = 0.9\%$ and $l/D = 8$ and section 4 with $\rho = 2.7\%$ and $l/D = 8$; Corrosion percentage $\Psi = 20\%$ except for the data in figures b1 and b2.
Similar considerations can be done for the four sections:

(i) different yield strength and hardening ratio of the original rebars, for a fixed corrosion level, have a negligible influence on the maximum strain of steel in compression, that resulted to be about 0.5 %;

(ii) The axial load has a strong effect on the maximum strain. However, the maximum strains in compression remain always well below 1.5 %.

The results of the parametric analyses are shown in term of moment - curvature behavior in Figure 9 for sections 3 with $\rho = 0.9 \%$ and $1/ D = 8$.

![Figure 9](image_url)

**Figure 9** Results of the parametric analyses in term of moment-curvature curves: axial load variation (a1 - a3); Corrosion percentage variation (b1 - b3); Yield stress variation (c1 - c3); Hardening ratio variation (d1 - d3) for section 3 with $\rho = 0.9 \%$ and $1/ D = 8$; Corrosion percentage $\Psi = 20 \%$ except for the data in figures b1 and b2.
In Figure 9 $v_c$ is the ratio between the axial load applied on the section and the axial strength of the section. The axial strength of the section was calculated considering the yield strength variation due to corrosion. It is evident that corrosion increase reduces the maximum moments and the energy dissipation capacity of the section. The results for section 3 are representative of the ones obtained for the other sections.

6 CONCLUSION

Corrosion reduces the area of the steel rebars, in not uniform way when inside concrete sections (pitting corrosion) resulting in smaller cyclic capacity (strength and ductility) of rebars and therefore of the RC members.

Numerical models may be valid tools to evaluate the structural behavior of RC section with corroded rebars, but the model uncertainty is large, due to pitting.

The use of simple models, as in this paper, allows setting some bounds in structural assessment.

A model to describe the cyclic behavior of rebars including corrosion is presented. The model is a generalization of the cyclic one for steel rebars with inelastic buckling in compression, characterized by isotropic and kinematic hardening effects Zhou et al. 2015 [25].

Expressions with whom the model modifies yield stress and hardening ratio for a given corrosion percentage $\Psi$ are given. Due to pitting corrosion rebar diameters of the longitudinal and transversal reinforcements reduce. Therefore, the free length of the longitudinal rebars increases. A simple expression for this free length is suggested and used in the model. Given the aforementioned aspects, it may happen that buckling arises when the initial (before corrosion) slenderness is smaller than the critical one.

The generalized model is especially accurate, with respect to experimental cyclic behavior of rebars with buckling in compression, in case the strain in compression does not exceed 1.2 -1.5 %. Larger strains should require a nonlinear envelope in compression. In fact, other models which aim at representing very large strain in compression use more complex formulation for that branch Kashani et. 2013 [21].

In this paper it is shown that such large strains in compression cannot be achieved, therefore more sophisticated branch description in compression, with respect to the linear one, is useless.

Four RC sections were selected as case studies for a single concrete geometry and different steel configurations assumed representative of RC bridge piers (in a suitable scale) and subjected to a cyclic curvature history representative a severe seismic load, not far from collapse.

Parametric cyclic analyses were carried out on these sections using in house software (CY.R.U.S-M) based on fiber model, where the cyclic behavior of the corroded rebars is the proposed generalized steel model.

Different rebar characteristics (yield stress, maximum stress, hardening), axial loads, corrosion percentages are considered.

The first results showed that:

(i) different yield strength and hardening ratio of the original rebars, for a fixed corrosion level (severe corrosion $\Psi = 20 \%$), have a negligible influence on the maximum strain of steel in compression, that resulted to be about 0.5 %;

(ii) The axial load has a strong effect on the maximum strain. However, the maximum strains in compression remain always well below 1.5 %;

(iii) Corrosion increase reduces the maximum moments and the energy dissipation capacity of the section.
The generalized model unified for the original as well as corroded rebars, showed to be sufficiently accurate and the linear envelope branch in compression is accurate as strains, in concrete section, always remain well below 1.5%.

In the present paper other features of the model like the possibility of checking strain reduction in steel as well as low cycles fatigue effect, strongly affected by corrosion, are not dealt with.

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REFERENCES


BRIDGE-ABUTMENT-BACKFILL INTERACTION: BENEFICIAL OR DETRIMENTAL FOR INTEGRAL ABUTMENT BRIDGES?

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Abstract

Integral Abutment Bridges (IAB) are becoming the structural system of choice for many bridge jurisdictions in North America and Europe, because of their superior durability over conventional bridges. IABs design out the movement joints and bearings, eliminating the need for costly maintenance throughout the life of the bridge and improving the durability of the bridge structure. IAB bridges may appear as simple structural systems, but the interaction of the many structural and non-structural elements comprising the bridge structure during its lifetime make this type of bridges unique in their structural analysis and behaviour prediction under the different loads imposed on the bridge during its service lifetime. Thermal effects and vehicular load effect are well-studied areas in bridge engineering, and their effects on integral abutment bridges have been the subject of many published literatures, but little research is available for the effect of earthquake loading on the behaviour of the IAB bridge systems and its interaction with the backfill soil. This paper aims at answering a critical question in IAB seismic design, i.e. whether the consideration of the backfill soil and the abutment is beneficial or detrimental in earthquake-resistant integral abutment bridges.

Keywords: integral abutment bridge, seismic response, design, interaction, SSI, beneficial, detrimental
1 INTRODUCTION

Conventional bridge systems have experienced maintenance challenges related to bridge deck joints and bearings. Joints and bearings are expensive to buy, install, maintain, repair and replace and maintenance activities can cause significant traffic disruptions. These challenges drove the engineering community into finding innovative solutions to come up with jointless bridge systems. Integral Abutment Bridges (IAB) have proven themselves as viable solution to these issues, and it has been used since in Canada, the US and Europe to develop a “get-in, get-out, stay-out” sustainable, low maintenance bridges. [1]

IABs are unique in terms of their structural analysis and behaviour under diverse actions due to the interaction between the structural and non-structural, e.g. backfill, elements comprising the bridge structure through the life of the bridge. Thermal movements of the bridge will result in movements in the backfill soil, which in-turn affects the soil pressure acting on the abutment walls. The interaction between the backfill soil and the abutment affects the design of the abutment foundation. The configuration of the abutment foundation, in turn, affects the stiffness of the whole system, and that influences the forces attracted to the abutments and piers, which requires many iterations from the designer to reach an optimal design.

This research paper aimed at answering the question: is the backfill soil of Integral Abutment Bridges (IABs) a factor that can reduce the seismic response of the bridge or is it a factor that can increase the seismic response of bridges? For example, if the backfill is a source of damping and acts as an additional external support to the bridge, then it is safer to consider this as a second line of defence and ignore it when analysing the bridge. Thus, we can use the backfill soil as a potential retrofitting measure for existing bridges and enhance its earthquake resistance. On the other hand, if the backfill is mainly a source of inertia mass, then the effect of this additional mass should be considered in the analysis and design of the bridge, as this might lead to larger displacements and bending moment in the structural components of the bridge. This question can be broken down to smaller questions, such as, what is the damping ratio of IABs [2, 3]; what is the stiffness of the backfill soil; what are the soil properties of the IAB; what is the geometry and the typology of the bridge and the type of the abutment and how the above factors may affect the response of the bridge. There are also other factors that may seem irrelevant, but can still influence IAB responses drastically, for example, the thermal expansion and contraction of the deck and the consequent movement of the abutment during the bridge service or prior to the seismic action, which imposes backfill deterioration, i.e. soil flow, compaction and ratcheting. Other factors might be the deterioration and/or fatigue of the abutment and the foundation of the abutment, for example, when abutments are supported on piles. There are also other factors that may seem irrelevant but can still influence IAB responses drastically, for example, the thermal expansion and contraction of the deck and the consequent movement of the abutment during the bridge service, i.e. before the seismic action, which imposes backfill deterioration, soil flow, compaction and ratcheting [4]. Other factors might be the deterioration/fatigue of the abutment or the foundation of the abutment (e.g. the piles if any). Currently, appears to be no agreement in the engineering community on the above question, and the following are few variables that can influence the bridge behaviour.

With regard to the length of IABs, they are usually short bridges, and that is what the codes prescribe, e.g. there are length limitations in each State in the USA. However, practical designs resulted in much longer bridges. The research by Zhang and Makris [2] suggests that the seismic movement of the bridge increases by a factor of two as the backfill almost drives the motion of the bridge, which refers to the case where the bridge is of relatively small length and moves together with the backfill soil. Potentially this is the source of the q=1 in Eurocode 8-2 [5,6]. However, for a longer bridge length, it is expected that a different bridge behaviours will be obtained. The key here is to identify the effective length of the backfill soil as this defines the amount of dissipation that occurs due to the plastification of the backfill soil, which is beneficial for seismic design, but causes settlements and the need for backfill replacement. On the other hand, the backfill is not only a source of dissipation and stiffness, but also a source of inertia.
mass, which potentially may increase the seismic response of the bridge, yet may as well increase the period of the bridge.

In respect of the type and height of the abutment, the taller the abutment, the more significant the interaction with the backfill soil, because a substantial part of the backfill soil is mobilised. However, this might mean considerable energy dissipation and larger backfill inertia mass. On the other hand, a short abutment has less interaction with the backfill, yet the abutment is seating on the backfill soil, as opposed to retaining it, in which case the backfill is the foundation soil for the abutment, i.e. the backfill soil may induce larger movements to the bridge as it is softer than original foundation soil, thus is expected to magnify the movements. In the latter case, the abutment movement is strongly dependent on the stem movement of the backfill soil.

The foundation type can as well affect the behaviour of the bridge, whether the bridge is supported on shallow foundation or piled foundation. For IAB supported on piles, the flexibility of the piles will play a role in determining the displacement capacity of the bridge during an earthquake. The interaction between the soil and the piles is a factor in the overall behaviour of the bridge. This foundation type was not considered in this research but is suggested for future work on the subject.

With regard to the condition of IAB components (structural and non-structural), concerns the properties of soil, abutment and bridge components and how they might have been deteriorated during the bridge service due to several factors, such as thermal movements, environmental conditions, corrosion, and fatigue. This has not been examined in this research but is suggested for future studies of the problem [4].
2 FE ANALYSIS MODEL

For the purpose of this study, the complex soil-structure interaction for Integral Abutment Bridges (IAB) under seismic load was analysed using the plain strain finite element code PLAXIS 2D (2017). Multiple models were developed to consider the effects of backfill on the dynamic response in different geometric scenarios such as length of bridge (spans) and height of abutments.

Dynamic absorbent boundaries were used to stimulate the far field behaviour of the medium, while at the base the boundary conditions were fully restrained in translation and rotation [7]. The width adopted for the model is sufficient to minimise the boundary effects, without significant increase in computational cost verified by preliminary sensitivity analysis.

Grade C30/37 concrete is used for the abutment, piers and foundation. Typically for this grade of concrete, the Young’s modulus can be taken as $E=33$GPa (Eurocode 2) but to account for creep, shrinkage and cracking in the concrete cross section it has been reduced to 10.5GPa.

Plate elements were used to model the concrete box deck to which axial and bending stiffness are directly applied. Rotational restraints, to account for high flexural stiffness of the deck, were applied at the top of pier and abutment cluster elements.

The IAB backfill soil was modelled with 14 layers behind the abutments at 500mm each. To better model the interaction between the abutment and backfill material, interface elements have been input. The bearing soil/ foundation consists of undrained very stiff clay which has a depth of 24m. This has been modelled as 10 layers to have a realistic idealisation of how the soil would be present in reality. However, for the multilayer model the layers of backfill have been specified differently which can be seen further on. The interaction between different structural elements and the soil can be detailed and captured by using the “interface elements” PLAXIS feature. This allows soil structure interaction idealisation internally within the software for the chosen elements. Interface elements are defined by a material type as well a virtual thickness. Earthquake induced ground motion in the analysis of earthquake effects is one of the many possible sources of uncertainty and previous research has shown this to have the highest effect on variability in the observed structural response [8]. Five dynamic motions were identified for the study. They were chosen to reach the most unfavourable dynamic response of the bridge system based on the expected natural period of the structure.

![Analysis Model](image-url)
3 PARAMETRIC STUDY

Six different variations of integral abutment bridge (IAB) geometries were chosen to represent the variation of the bridge length, the abutment height and the type of fill retained by the bridge abutment.

The analysis models represent IABs of a single-span, two-span and three-span bridges, representing bridges of 34m, 68m and 102m lengths respectively. The three-span structure considered three variations for the abutment height of 3.5m and 5.5m and 8.0m. The sixth model represent a three-span bridge with the full abutment height of 8.0m, but with varying shear wave velocity and other soil properties of the backfill layers (Table 2). Also, models without fill were analysed.

<table>
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<tr>
<th>Model Designation</th>
<th>Model Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>REF/WF</td>
<td>base reference model, 3-span bridge, 8m high abutment</td>
</tr>
<tr>
<td>REF/MLF</td>
<td>3-span bridge, 8m high abutment, backfill consists of multiple layers</td>
</tr>
<tr>
<td>REF/NF</td>
<td>3-span bridge, 8m high abutment, without backfill soil</td>
</tr>
<tr>
<td>REF/3.5m-WF</td>
<td>3-span bridge, 3.5m abutment</td>
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<td>3-span bridge, 5.5m abutment</td>
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<tr>
<td>1S/WF</td>
<td>single-span bridge, 8m high abutment</td>
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<tr>
<td>2S/WF</td>
<td>two-span bridge, 8m high abutment</td>
</tr>
</tbody>
</table>

Table 1 - Analysis Models Description

This enabled the study of the soil mass effect on the bridge response to determine whether it influences the overall bridge system behaviour, or the integral bridge structure is driving the behaviour of the system. Research showed that one of the many possible sources of uncertainty in the analysis for earthquake effects is the assumed earthquake-induced ground motions used in the analysis model, and research has shown this to have the highest effect on the variability observed in the structural response [8].
The models are subjected to five earthquake excitation records, which were carefully selected based on real earthquakes to obtain the bridges responses to these applied motion records, see Figure 6 below. These records were chosen based on the expected natural period of the structure under investigation, to reach the most unfavourable dynamic response of the bridge system.

The signals obtained from seismic motion databases available are all recorded at 0.15g, therefore, to investigate the structural system response in the range of moderate to high strains, these signals were scaled to 0.3g and 0.6g, with maintaining their frequency content. The selection of 0.3g represents the upper limit for Performance Zone 2 (AASHTO 2010), and it appears to be a typical range of PGA in European Earthquake prone areas [7].

Through time history analysis, the outputs from these models were used to determine the effect of earthquake loading on the bending moment and shear forces of the bridge deck as well as the variations in Pier drifts due to the earthquake loading on the structure. These models were analysed under earthquake motions scaled at 0.3g and 0.6g to compare the earthquake effects. The above models were re-analysed precluding the backfill soil (NF models) to determine the effect of the soil mass, stiffness and damping on the bridge system behaviour. The results of the models that include the fill and the models that do not include fill were compared to determine the contribution of the backfill soil on the overall behaviour of the bridge system. The study looked at bending moments and shear forces at deck-to-abutment and pier connections, bending moments at the deck mid-span, and compared relative displacements, i.e. drift values, at the abutments and piers.

3.1 **Backfill Soil Properties**

The soil properties for the backfill were chosen for a soil above the phreatic line, hence the drained soil assumption, the properties were assumed for sandy soil. The properties of the backfill soil are assumed to be constant along the depth of the abutment. This model will be considered as the benchmark for subsequent analyses.
The following properties are assumed for the backfill soil used for the different models:

<table>
<thead>
<tr>
<th>Layer Designation</th>
<th>Thickness (m)</th>
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<th>Shear Wave Velocity (m/s)</th>
<th>Effective Shear Modulus (kPa)</th>
<th>Poisson’s Ratio</th>
<th>Young’s Modulus (kPa)</th>
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<tr>
<td>MC</td>
<td>7.0</td>
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<td>271</td>
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<td>BF1</td>
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<td>0.01</td>
<td>243</td>
<td>79548</td>
<td>0.40</td>
<td>477,300</td>
</tr>
<tr>
<td>BF5</td>
<td>1.0</td>
<td>0.01</td>
<td>256</td>
<td>88541</td>
<td>0.40</td>
<td>531,200</td>
</tr>
<tr>
<td>BF6</td>
<td>1.0</td>
<td>0.01</td>
<td>268</td>
<td>96639</td>
<td>0.40</td>
<td>579,800</td>
</tr>
<tr>
<td>BF7</td>
<td>1.0</td>
<td>0.01</td>
<td>278</td>
<td>104061</td>
<td>0.40</td>
<td>624,400</td>
</tr>
</tbody>
</table>

The base soil is assumed to have average damping of 6%. In PLAXIS 2D this is calculated by the software by defining Rayleigh Damping Coefficients. To reach the target damping of 6%, the frequencies used are 1Hz and 3Hz.

4 RESULTS

The time history analysis results of the different motion records were compared to the values from running the static analysis model, representing the bridge self-weight. The self-weight bending moments and shear forces at the critical locations shown in Figure 8 were considered as the reference datum for the comparisons to follow. The ratio of bending moment and shear force resulting from each motion input scaled to the self-weight effects represent the change in the deck forces due to earthquake effect. This ratio indicates the amplification or reduction in the bending moment/shear forces due to the seismic loading above the bending moment or shear forces resulting from the dead loads of the structure. This is important from the design engineer’s perspective, since the bending moment and shear forces
ultimately affect the selection of the different structural element sizes, which subsequently affects the mass and stiffness of the bridge structure.

4.1 Bending Moments and Shear Forces

Results extracted from REF/WF and REF/NF analysis models showed that the backfill soil had a surcharge effect on the bridge behaviour. An average increase in shear forces at abutment face of 8.1% in the case of REF/WF compared to 5.6% increase in the shear forces for the case of REF/NF was noticed. The bending moment at the abutment face increased by an average of 9.2% for REF/WF compared to 6.9% in REF/NF case. This suggests that the backfill soil increased the earthquake actions on the structure and worsened the demand on the structural elements. Table 5 below shows the ratio of outputs from REF/WF to outputs from REF/NF.

Table 5 below shows the ratio of outputs from REF/WF to outputs from REF/NF.

<table>
<thead>
<tr>
<th>Location</th>
<th>M-support-1</th>
<th>Q-support-1</th>
<th>M-support-2/Left</th>
<th>Q-support-2/Left</th>
<th>M-support-2/Right</th>
<th>Q-support-2/Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kypseli/VL</td>
<td>7.40%</td>
<td>5.30%</td>
<td>10.00%</td>
<td>7.20%</td>
<td>5.60%</td>
<td>1.40%</td>
</tr>
<tr>
<td>Gebze/VL</td>
<td>8.30%</td>
<td>7.10%</td>
<td>10.00%</td>
<td>8.00%</td>
<td>7.30%</td>
<td>1.40%</td>
</tr>
<tr>
<td>Duzce/VL</td>
<td>9.70%</td>
<td>8.70%</td>
<td>12.10%</td>
<td>9.60%</td>
<td>8.30%</td>
<td>4.10%</td>
</tr>
<tr>
<td>Hector/VL</td>
<td>8.80%</td>
<td>7.40%</td>
<td>11.60%</td>
<td>8.60%</td>
<td>8.10%</td>
<td>2.20%</td>
</tr>
<tr>
<td>Umbria/VL</td>
<td>11.90%</td>
<td>11.90%</td>
<td>14.40%</td>
<td>11.90%</td>
<td>9.30%</td>
<td>2.80%</td>
</tr>
<tr>
<td>Average</td>
<td>9.22%</td>
<td>8.08%</td>
<td>11.62%</td>
<td>9.06%</td>
<td>7.72%</td>
<td>2.38%</td>
</tr>
</tbody>
</table>

Table 3 - Ratio of Bending and Shear from Dynamic analysis to the Dead load – Model REF/WF (Base Model REF/WF Flexure (kN.m) and Shear (kN) results shown per 1m width of bridge).

Table 4 - Ratio of Bending and Shear from Dynamic analysis to the Dead load – Model REF/NF (No Fill Model REF/NF Flexure (kN.m) and Shear (kN) results shown per 1m width of bridge).
Table 5 – Ratio of M and Q from No Fill Model (three span) to the Reference Model (REF/NF vs. REF/WF) (Base Model REF/WF Flexure (kN.m) and Shear (kN) - results shown per 1m width of bridge)

<table>
<thead>
<tr>
<th>Location</th>
<th>M&lt;sub&gt;W&lt;/sub&gt;/M&lt;sub&gt;NoFill&lt;/sub&gt;</th>
<th>Q&lt;sub&gt;W&lt;/sub&gt;/Q&lt;sub&gt;NoFill&lt;/sub&gt;</th>
<th>M&lt;sub&gt;W&lt;/sub&gt;/M&lt;sub&gt;NoFill&lt;/sub&gt;</th>
<th>Q&lt;sub&gt;W&lt;/sub&gt;/Q&lt;sub&gt;NoFill&lt;/sub&gt;</th>
<th>M&lt;sub&gt;W&lt;/sub&gt;/M&lt;sub&gt;NoFill&lt;/sub&gt;</th>
<th>Q&lt;sub&gt;W&lt;/sub&gt;/Q&lt;sub&gt;NoFill&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kypseli</td>
<td>-0.08%</td>
<td>-0.03%</td>
<td>0.00%</td>
<td>0.08%</td>
<td>0.03%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Gebze</td>
<td>1.44%</td>
<td>0.62%</td>
<td>2.66%</td>
<td>1.58%</td>
<td>1.17%</td>
<td>0.65%</td>
</tr>
<tr>
<td>Duzce</td>
<td>2.69%</td>
<td>2.55%</td>
<td>3.46%</td>
<td>2.79%</td>
<td>3.02%</td>
<td>0.85%</td>
</tr>
<tr>
<td>Hector</td>
<td>0.70%</td>
<td>1.04%</td>
<td>1.26%</td>
<td>1.24%</td>
<td>1.65%</td>
<td>1.38%</td>
</tr>
<tr>
<td>Umbria</td>
<td>2.06%</td>
<td>2.12%</td>
<td>2.84%</td>
<td>1.81%</td>
<td>2.45%</td>
<td>1.74%</td>
</tr>
<tr>
<td>Average</td>
<td>1.36%</td>
<td>1.26%</td>
<td>2.05%</td>
<td>1.50%</td>
<td>1.66%</td>
<td>0.92%</td>
</tr>
</tbody>
</table>

The effect of the backfill soil considering the varying properties of soil layers, i.e. the multi-layer fill (MLF) showed that the backfill properties had a burdensome effect on the bridge response as it increased the demand on the structural elements of the bridge. The reference model (REF/WF) utilised a constant set of soil properties for the backfill mass, and the shear velocity was fixed at 271 m/seconds for this model, as compared to the multi-layer model (REF/MLF) which utilised varying soil properties for each 1m thickness of the backfill soil; the lowest shear velocity was taken at 174 m/s for the top layer and increases to 278m/s for the lowermost layer, demonstrating that the softer the backfill soil the worse for the seismic behaviour of the IAB. Table 6 below shows the bending moment and shear from REF/MLF compared to the self-weight effects. An average increase in shear at abutment of 16%, and a 21% increase in bending moment at face of abutment was recorded, the mid-span moment showed and average increase of 28% to the datum self-weight effects.

Table 6 - Ratio of Bending and Shear from Dynamic analysis to the Self-weight effect - Model REF/MLF (Multi-Layer Model REF/MLF Flexure (kN.m) and Shear (kN) - results shown per 1m width of bridge)

<table>
<thead>
<tr>
<th>Location</th>
<th>M&lt;sub&gt;Support-1&lt;/sub&gt;</th>
<th>Q&lt;sub&gt;Support-1&lt;/sub&gt;</th>
<th>M&lt;sub&gt;Mid-span&lt;/sub&gt;</th>
<th>M&lt;sub&gt;Support-2/L&lt;/sub&gt;</th>
<th>Q&lt;sub&gt;Support-2/L&lt;/sub&gt;</th>
<th>M&lt;sub&gt;Support-2/R&lt;/sub&gt;</th>
<th>Q&lt;sub&gt;Support-2/R&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kypseli/</td>
<td>15.37%</td>
<td>12.02%</td>
<td>18.35%</td>
<td>14.38%</td>
<td>11.21%</td>
<td>1.92%</td>
<td>3.06%</td>
</tr>
<tr>
<td>Gebze/</td>
<td>16.03%</td>
<td>13.46%</td>
<td>17.66%</td>
<td>13.55%</td>
<td>11.23%</td>
<td>1.87%</td>
<td>2.51%</td>
</tr>
<tr>
<td>Duzce/</td>
<td>31.82%</td>
<td>22.40%</td>
<td>42.40%</td>
<td>35.69%</td>
<td>27.26%</td>
<td>6.83%</td>
<td>7.33%</td>
</tr>
<tr>
<td>Hector/</td>
<td>17.30%</td>
<td>14.34%</td>
<td>21.02%</td>
<td>16.79%</td>
<td>12.09%</td>
<td>3.39%</td>
<td>3.49%</td>
</tr>
<tr>
<td>Umbria/</td>
<td>22.10%</td>
<td>20.31%</td>
<td>40.87%</td>
<td>22.07%</td>
<td>18.69%</td>
<td>4.65%</td>
<td>5.65%</td>
</tr>
<tr>
<td>Average</td>
<td>20.53%</td>
<td>16.51%</td>
<td>28.06%</td>
<td>20.50%</td>
<td>16.10%</td>
<td>3.74%</td>
<td>4.41%</td>
</tr>
</tbody>
</table>
The results showed that the abutment height had a beneficial effect on the bending moments and shears on the bridge system, i.e. the higher the abutment the better for the earthquake resistance of the bridge. This is clear form Figure 10 above, which showed a reduction in bending moment as the abutment height increases. This may be attributed to the backfill acting as an energy dissipation medium.

Table 7 - Single Span Bridge (With Fill vs. No Fill)

<table>
<thead>
<tr>
<th></th>
<th>Single Span Bridge (1S/WF)</th>
<th></th>
<th>Single Span Bridge (1S/NF)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flexure (kN.m) and Shear (kN)</td>
<td></td>
<td>Flexure (kN.m) and Shear (kN)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(M_{\text{support}})</td>
<td>(M_{\text{mid-span}})</td>
<td>(Q_{\text{support}})</td>
<td>(M_{\text{support}})</td>
</tr>
<tr>
<td>Kypseli/VL</td>
<td>4.09%</td>
<td>3.12%</td>
<td>4.68%</td>
<td>Kypseli/VL</td>
</tr>
<tr>
<td>Gebze/VL</td>
<td>2.38%</td>
<td>2.37%</td>
<td>5.67%</td>
<td>Gebze/VL</td>
</tr>
<tr>
<td>Duzce/VL</td>
<td>5.70%</td>
<td>5.44%</td>
<td>5.84%</td>
<td>Duzce/VL</td>
</tr>
<tr>
<td>Hector/VL</td>
<td>1.64%</td>
<td>1.45%</td>
<td>5.15%</td>
<td>Hector/VL</td>
</tr>
<tr>
<td>Umbria/VL</td>
<td>5.08%</td>
<td>4.93%</td>
<td>6.07%</td>
<td>Umbria/VL</td>
</tr>
<tr>
<td>Average</td>
<td>3.78%</td>
<td>3.46%</td>
<td>5.48%</td>
<td>Average</td>
</tr>
</tbody>
</table>

Analysis showed that increasing the length of the bridge had worsened the behaviour, since the bending moments and shear forces increased with increasing the number of spans of the bridge. Figure 11 above shows that increasing the bridge length, i.e. the number of spans, resulted in an increase in the bending moments and shear force values at the abutment face. FEM results showed that for the single span bridge, the existence of backfill had a beneficial effect on the bending moments and shears at the abutment face and the mid-span. On average, there was 9\% increase in shear at abutment for the case of no fill (1S/NF) versus a 5.5\% increase in shear when the fill existed. This, however, was not the same
for bending moment at the abutment face, and the moment at the abutment showed an average increase of 3.78% for case with fill (WF) versus a negligible reduction in moment from the case of no fill (NF).

These results show that the existence of backfill tends to increase the bending moments and shears on longer IABs, which should be accounted for in the design of the superstructure elements. On the contrary, structures that are relatively short showed that the backfill is beneficial to its behaviour under seismic loading.

4.2 Pier and abutment drifts

The following results represent the recorded displacements at the top of the abutment or pier and the recorded displacements at the abutment or pier bottom. These results represent the maximum displacements due to the earthquake loading. The difference between the top displacement and the bottom displacement, occurring simultaneously, i.e. at the same time instance, represents the differential movement of the bridge abutment or pier during the earthquake excitation and hence it is a measure of the expected bending moment for given boundary conditions. Residual displacements, after earthquake loading, represents the displacements recorded at the last step of the time history analysis, i.e. after the completion of the seismic motion as per the equation below:

\[ \text{Drift} = \frac{U_{\text{top}} - U_{\text{bottom}}}{\text{Height}} \text{ (mm/m)} \]  

Figure 12 - Drift Calculation

The drift values were summarised for both models including backfill and models without backfill. The results show that the backfill has a significant effect on the displacement of the bridge as expected. Comparing the drift values of models with backfill to those without backfill, show that the backfill limited the displacements of the bridges significantly.

Comparing the outputs from the 3.5m and 5.5m models to the base model, the results showed that the backfill acted as a mass driving the motion of the bridge, this behaviour tends to be very obvious when comparing the base model to the 3.5m and 5.5m abutment height models. The lower mass of backfill, behind the abutments of the 3.5m and 5.5m models, driving the motion resulted in reduced displacements as compared to the base model, which had a larger mass of soil retained by the abutment. This can also be attributed to the change in the bridge stiffness, which in turn affects its response to the input seismic motion.
The following tables show a comparison between the drift values at the abutments of REF/WF and both REF/1S-WF and REF/2S-WF for input motions scaled at 0.3g and 0.6g. Single span and two span bridges showed low stiffness compared to the three span bridge, and this resulted in the backfill soil acting as the driving mass of the system, and therefore the drift values showed an increase in drift values of both 1S and 2S bridges compared to the base model.

<table>
<thead>
<tr>
<th>Location</th>
<th>REF/WF</th>
<th>REF/NF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kypseli</td>
<td>1.31</td>
<td>2.67</td>
</tr>
<tr>
<td>Gebze</td>
<td>1.53</td>
<td>2.87</td>
</tr>
<tr>
<td>Duzce</td>
<td>2.81</td>
<td>6.05</td>
</tr>
<tr>
<td>Hector mine</td>
<td>1.98</td>
<td>2.17</td>
</tr>
<tr>
<td>Umbria</td>
<td>2.56</td>
<td>3.38</td>
</tr>
</tbody>
</table>

Table 8 - Drift ‰ (mm/m) for 0.3g Input Motion – Bridge with fill (REF/WF) vs No fill (REF/NF)

<table>
<thead>
<tr>
<th>Location</th>
<th>REF/1S-WF</th>
<th>REF/2S-WF</th>
<th>REF/WF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kypseli</td>
<td>1.32</td>
<td>1.84</td>
<td>1.31</td>
</tr>
<tr>
<td>Gebze</td>
<td>1.57</td>
<td>2.28</td>
<td>1.53</td>
</tr>
<tr>
<td>Duzce</td>
<td>3.31</td>
<td>4.53</td>
<td>2.81</td>
</tr>
<tr>
<td>Hector mine</td>
<td>1.15</td>
<td>1.72</td>
<td>1.98</td>
</tr>
<tr>
<td>Umbria</td>
<td>1.89</td>
<td>2.54</td>
<td>2.56</td>
</tr>
</tbody>
</table>

Table 9 – Drift ‰ (mm/m) for 0.3g Input Motion – With Fill

<table>
<thead>
<tr>
<th>Location</th>
<th>REF/1S-WF</th>
<th>REF/2S-WF</th>
<th>REF/WF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kypseli</td>
<td>2.40</td>
<td>2.90</td>
<td>1.31</td>
</tr>
<tr>
<td>Gebze</td>
<td>4.60</td>
<td>5.80</td>
<td>1.53</td>
</tr>
<tr>
<td>Duzce</td>
<td>5.00</td>
<td>7.50</td>
<td>2.81</td>
</tr>
<tr>
<td>Hector mine</td>
<td>2.20</td>
<td>3.00</td>
<td>1.98</td>
</tr>
<tr>
<td>Umbria</td>
<td>4.50</td>
<td>5.90</td>
<td>2.55</td>
</tr>
</tbody>
</table>

Table 10 – Drift ‰ (mm/m) for 0.6g Input Motion - With Fill

The analysis results using 0.6g input motions confirm the behaviour and conclusions drawn from the 0.3g analysis results. The results show that the backfill acted as a mass driving the motion of the bridge, and this behaviour tends to be obvious when comparing the base model (REF/WF) to the 3.5m and 5.5m abutment height models. The shorter height of the abutment meant a lower mass of backfill which reduced the bridge displacements compared to the full height base model.
Table 11 - Drift % (mm/m) for 0.6g Input Motion - With Fill (Abutment Height Effect)

<table>
<thead>
<tr>
<th>Location</th>
<th>REF/3.5m-WF</th>
<th>REF/5.5m-WF</th>
<th>REF/WF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kypseli</td>
<td>1.00</td>
<td>1.90</td>
<td>1.30</td>
</tr>
<tr>
<td>Gebze</td>
<td>1.00</td>
<td>1.60</td>
<td>1.50</td>
</tr>
<tr>
<td>Duzce</td>
<td>1.20</td>
<td>2.60</td>
<td>2.80</td>
</tr>
<tr>
<td>Hector mine</td>
<td>1.20</td>
<td>1.70</td>
<td>2.00</td>
</tr>
<tr>
<td>Umbria</td>
<td>1.20</td>
<td>2.60</td>
<td>2.60</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS AND FURTHER RESEARCH

The present work aimed at establishing the effect of backfill soil on Integral Abutment Bridges (IAB) seismic response and to answer the question as to whether it is a governing factor in the design of these types of bridges. The effort comprised the development of finite element models using PLAXIS 2D for the study of soil-structure interaction effects on a number of variations of integral abutment bridges under seismic ground motion. The study covered in detail the seismic behaviour of six IAB bridge configurations that varies in length, abutment height and type of backfill soil.

The study showed that the presence of backfill soil had affected the dynamic behaviour of the bridges in multiple ways, and it also showed that the interaction between the backfill soil and the bridge structure is complex enough to warrant soil-structure interaction sophisticated modelling to obtain reliable responses with regard to the behaviour of the bridge under seismic loading. Although more research is required on the topic, the present study confirmed that the interaction between the bridge and the backfill is of importance to be scrutinized by the design engineers and to be considered using sophisticated analysis models, rather than using simplified design formulas. The results to date do not lead to clear conclusions, i.e. whether the backfill soil is beneficial or detrimental for the seismic response of IABs, however it is evident that this is a strongly case-dependent effect and this hence this is an ongoing research project.

The effects of thermal cycles, creep and shrinkage, and prestressing forces in concrete bridges and the long-term effects’ evolution over time is neglected in the present research, but such effects on the behaviour of the bridge is expected to have significant effects on the behaviour of the bridge and should be considered in any future studies.

Future studies should research into establishing design guidelines or design formulas to be used in the design office as access to an FE sophisticated software is not always available for design offices, in addition to the computational cost of running such complicated analysis software.

The study also considered the case of supporting the abutments and piers on shallow foundation. This system may be standard for European countries, but the more common system in North America is to use piled foundation. The piles have some flexibility and would allow movement of the abutment and piers which would affect the natural period of the structure, and this effect should be considered in further studies.

REFERENCES


TOWARDS A SIMPLIFIED AND RIGOROUS PERFORMANCE-BASED SEISMIC DESIGN OF ORDINARY STANDARD BRIDGES IN CALIFORNIA

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Abstract

This paper focuses on laying the groundwork for risk-targeted performance-based seismic design (PBSD) of Ordinary Standard Bridges (OSBs) in California. Rooted in this formulation is an improved seismic performance assessment methodology that integrates site-specific seismic hazard analysis, structural demand analysis, and damage analysis in a probabilistic framework. At the crux of structural design lies the selection of practicable values of critical design parameters such that predetermined target specifications of performance measures are met. To this end, full-fledged parametric performance assessments of four testbed OSBs are carried out to investigate the effects of varying primary structural design parameters on structural performance. The parametric study indicates erratic levels of conservativeness exhibited by the as-designed testbed OSBs, thus illustrating the need for a PBSD framework for OSBs such that explicitly stated performance objectives are consistently satisfied. Thus, a simplified, non-traditional, PBSD methodology is distilled out of this work that can be used to: (1) find a design point in a two-dimensional primary design parameter space of a bridge being designed for multiple risk-based performance objectives; and (2) delineate a feasible design domain containing other acceptable design options in the primary design space.

Keywords: Ordinary standard bridges, performance-based seismic design, feasible design domain; performance-based earthquake engineering.
1 INTRODUCTION

Probability-based design provides the most scientific and rational solution to an earthquake-resistant structural design problem wherein an inherently uncertain structural system needs to be designed such that its performance entails, not only resisting highly uncertain seismic demands, but also meeting reliably societal demands of life safety, economy and resiliency. The classification of structural performance should therefore be predicated on an acceptable risk, defined by the risk tolerance of society as a whole. Fueled by such needs, the structural engineering community, over the last few decades, has moved on towards implementing the philosophy of probabilistic performance-based earthquake engineering (PBEE) in the assessment and design of civil infrastructures. Consistently improving over time, such efforts have culminated into the fully probabilistic, rigorous and advanced PBEE assessment framework [1-3] developed under the auspices of the Pacific Earthquake Engineering Research (PEER) Center.

The PEER PBEE methodology has been mainly developed for analysis and assessment and not directly for design, except for some initial efforts [4, 5], but has recently been recommended as a future alternative for bridge seismic design [6]. This study is therefore intended to address, without any compromise in rigor, the somewhat hindered implementation of the rather esoteric PEER PBEE framework in seismic bridge design practice, a less trodden area in terms of PBEE applications as compared to building structures. To this end, a computationally cost-effective, simplified, non-traditional, risk-targeted PBSD methodology is formulated herein for conventional, multiple-span, skewed reinforced concrete (RC) bridges, referred to as Ordinary Standard Bridges (OSBs), in California.

2 TESTBED OSBs AND COMPUTATIONAL MODELS

Four existing California OSBs, namely Bridges A, B, C and MAOC (see Figure 1) that conform to the definition of OSBs as described in Caltrans Seismic Design Criteria (SDC) v1.7 [7] and cover a range of realistic design situations, are selected as testbeds for this study. These bridges have been previously studied in research projects funded by the California Department of Transportation (Caltrans) and PEER.

Three-dimensional nonlinear finite element (FE) models (consisting of nonlinear fiber-section beam-column elements and nonlinear springs) of these bridges are constructed in OpenSees [8], the open-source FE analysis software framework developed at PEER. Initially inherited Tcl input files of the OpenSees models of these bridges from previous Caltrans/PEER funded projects are revisited, parameterized, and improved [9]. A schematic representation of the computational model of one of the four testbed OSBs (Bridge B) is shown in Figure 2.
FORWARD PBEE ANALYSIS AND FULL-FLEDGED PARAMETRIC STUDY

Metrics of structural performance considered in this study are the mean annual rates (MARs), or equivalently the mean return periods (MRPs), of exceedances for a selected set of practical limit-states (LSs), which according to the PEER PBEE methodology are given by:

$$\nu_{LS} = \int_{IM} \int_{EDP} P[Z_k < 0 | EDP_k = \delta] \cdot f_{EDP,IM}(\delta | x) \cdot d\delta \cdot dV_{IM}(x)$$  \hspace{1cm} (1)$$

where $P[Z_k < 0 | EDP_k = \delta]$ is the fragility function expressing the conditional probability of exceedance of $LS_k$ (i.e., safety margin $Z_k = C_k - EDP_k < 0$ where $C_k$ is the structural capacity associated with $LS_k$ and $EDP_k$ is the associated engineering demand parameter) given a specific value $\delta$ of $EDP_k$, $f_{EDP,IM}(\delta | x)$ is the conditional probability distribution of $EDP_k$ given a specific value $x$ of the intensity measure ($IM$), and $V_{IM}(x)$ is the MAR of $IM$ exceeding the specific value $x$.

3.1 Improvement of PEER PBEE assessment framework

Improvements of several aspects in the various stages of the state-of-the-art PEER PBEE assessment methodology are incorporated in this study [9]. These include:

- Introduction of an improved $IM$, i.e., average spectral acceleration over a period range [10, 11].

- Conditional mean spectrum (CMS)-based site-specific risk-consistent ground motion selection [12, 13] (100 ground motions per hazard level for six seismic hazard levels corresponding to MRPs of $IM$ exceedances equal to 72, 224, 475, 975, 2475, and 4975 years, respectively) for ensemble nonlinear response-history analyses.

- Introduction of material strain-based EDPs (see Table 1) associated with practical material strain-based LSs related to the desirable (targeted) failure mode concerning RC bridge columns (i.e., flexural hinging of columns), namely (1) concrete cover crushing, (2) a precursor to rebar buckling, and (3) a precursor to rebar fracture.

- Development of strain-based normalized fragility functions for the considered LSs. Fragility functions for LSs 1 through 3, normalized using appropriate deterministic capacity prediction equations [14, 15] are shown in Figure 3.
### 3.2 Full-fledged parametric probabilistic seismic performance assessment

Using the improved version of the PEER PBEE assessment framework, full-fledged parametric probabilistic seismic performance assessments of the testbed OSBs are carried out to investigate the effects of varying key structural design parameters on structural performance. A two-dimensional design space is defined in terms of primary design variables, viz., column diameter ($D_{col}$) and column longitudinal reinforcement ratio ($\rho_{long}$), to which the exceedances of the selected set of LSs are most sensitive. All other bridge design parameters to be determined by meeting the requirements of capacity design, minimum ductility capacity, reinforcement ratio restrictions, etc., and/or restricted by the geometry of the bridge, available real estate, traffic requirements, etc. are referred to as secondary design variables and are taken as per the original designs of the as-designed OSBs in the parametric study. This includes the transverse reinforcement ratio, which for the parametric analyses, is taken to vary as a practical fraction of $\rho_{long}$, i.e., $0.5\rho_{long}$.

A fully automated workflow (see Figure 4) incorporating an efficient utilization of available desktop computing as well as supercomputing resources is developed for a smooth and seamless execution of the parametric assessment. The seismic performance of re-designs of each testbed OSB generated by varying the primary design parameters, subject to practical constraints, are evaluated using the improved PEER PBEE framework. For each re-design of a testbed OSB, and for each of the considered LSs, a piecewise linear surface is fitted in the primary design parameter space to the computed MRPs of LS exceedances. A feasible design domain (see Figure 5), i.e., collection of design points in the primary design space of an OSB with MRPs of LS exceedances higher than or equal to respectively specified target MRPs, is delineated and LSs controlling its boundary are identified.

<table>
<thead>
<tr>
<th>LS</th>
<th>Associated strain ($\varepsilon$)-based EDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\max_{col} \left( \max_{\varepsilon} \left( \max_{t} \left[ \max_{\varepsilon, comp_{\varepsilon}} (t) \right] \right) \right)$</td>
</tr>
<tr>
<td>2</td>
<td>$\max_{col} \left( \max_{\varepsilon} \left( \max_{t} \left[ \varepsilon_{ten} (t) \right] \right) \right)$</td>
</tr>
<tr>
<td>3</td>
<td>$\max_{col} \left( \max_{\varepsilon} \left( \max_{t} \left[ \varepsilon_{ten} (t) - \min_{\varepsilon, comp_{\varepsilon}} (t') \right] \right) \right)$</td>
</tr>
</tbody>
</table>

Table 1: LSs and associated strain-based EDPs

Figure 3: Normalized fragility curves (experimental/numerical data shown as crosses)
The seismic performance of the as-designed testbed OSBs is found to show considerable variability. These bridges originally designed following a more traditional (prescriptive) seismic design philosophy, rather than an explicitly performance-based one, are found to exhibit irregular levels of conservativeness. While some of the as-designed testbed OSBs are found to be conservative, sometimes too much, with respect to the selected LSs and corresponding target MRPs, others are found to lie near the borderline of safety, or clearly in the unsafe domain. Erratic levels of conservativeness exhibited by the as-designed testbed OSBs illustrates the need for a PBSD framework for OSBs such that explicitly stated risk-targeted performance objectives are consistently satisfied by the population of OSBs in California.

4 SIMPLIFIED RISK-TARGETED PBSD METHODOLOGY

The full-fledged parametric probabilistic seismic performance assessment framework can be readily used for the design of a new OSB unless its computational cost is prohibitive for the computational resources available. For reasons of practicability in current bridge design practice, a computationally more economical, simplified, non-traditional, risk-targeted PBSD procedure is distilled out of this study based on the findings of the full-fledged parametric study. The proposed simplified design methodology is intended to:

- find a design point in the primary design parameter space of a bridge being designed for multiple risk-based performance objectives; and
- delineate an approximate, yet sufficiently accurate, feasible design domain and identify the LSs controlling its boundary in the primary design parameter space of the bridge;

at a computational cost significantly lower than that of the full-fledged parametric method.

4.1 Finding a design point satisfying multiple risk-based performance objectives

It is noted from the results of the parametric study that along any line \( \overline{D_p} \) connecting \( p \) design points \( (D_1, \ldots, D_p) \) in the primary design parameter space with a positive slope, i.e., along any direction with increasing values of \( D_{col} \) and \( \rho_{long} \), the MRP surfaces corresponding to the LSs of interest can be well-approximated using a piecewise power law/function (e.g., Figure 6). The equation of such a line is given by:

\[
\rho_{long}[-] = m \left[ \text{ft}^{-1} \right] \cdot D_{col} \left[ \text{ft} \right] + a \left[ - \right]
\]  

(2)
where \( m \) denotes the slope of the line, and \( \alpha \) is the intercept of the line along the \( \rho_{\text{long}} \) axis. Having selected such a line with a positive slope equal to \( m \) \( \text{ft}^{-1} \), in the design space, a unitless quantity, \( X \), with the role of continuously increasing values of \( D_{\text{col}} \) and \( \rho_{\text{long}} \) along that line is defined as follows:

\[
X [-] = \rho_{\text{long}} [-] + \frac{1}{m} \left[ \text{ft}^{-1} \right] D_{\text{col}} \left[ \text{ft} \right]
\]

Mathematically, Eq. (3) represents a family of lines with slopes equal to \( -\frac{1}{m} \) \( \text{ft}^{-1} \) which are perpendicular to the original line \( \overrightarrow{DD'} \) (of slope equal to \( m \) \( \text{ft}^{-1} \)) with different values of \( X \) representing the intercepts of these lines along the \( \rho_{\text{long}} \) axis as shown in Figure 7.

Figure 6: Approximation of MRP surfaces for different \( LS \)s along a positive gradient line in the primary design parameter space of Bridge B: (a) \( LS_1 \), (b) \( LS_2 \) and (c) \( LS_3 \)

Figure 7: Graphical interpretation of the quantity \( X \) representative of different design points along a positive gradient line in the primary design space
Thus, with the performance assessment carried out for any three design points \( \mathbf{D}_1, \mathbf{D}_2, \) and \( \mathbf{D}_3 \) named in order of increasing \( D_{\text{col}} \) and \( \rho_{\text{long}} \), i.e., corresponding to increasingly stronger designs) along any positive gradient line \( \mathbf{D}_1 \mathbf{D}_2 \mathbf{D}_3 \) in the primary design space of an OSB, one can interpolate a design point, \( \mathbf{D}^* \), satisfying multiple risk-based performance objectives with reasonable accuracy using the piecewise power law assumed to hold well along \( \mathbf{D}_1 \mathbf{D}_2 \mathbf{D}_3 \). The choice of the three design points to be assessed along an arbitrary positive gradient line is, however, not completely arbitrary. The procedure for choosing these design points is illustrated in the flowchart in Figure 8. Design point \( \mathbf{D}^* \) obtained for Bridge B is shown in Figure 9 as an example.

![Flowchart](image)

**Figure 8:** Procedure for selection of the three design points to be assessed in the simplified PBSD methodology

![Diagram](image)

**Figure 9:** Design point for Bridge B satisfying multiple risk-based design objectives

It is to be noted that, in practice, values of \( D_{\text{col}} \) and \( \rho_{\text{long}} \) are constrained by various factors such as the availability of existing prefabricated formwork, restrictions on rebar sizes, etc. In case the exact values of \( D_{\text{col}}^* \) and \( \rho_{\text{long}}^* \) obtained are not practically realizable, a viable design point nearest to \( \mathbf{D}^* \) on the safer side is chosen as the final design. This along with the need to make risk-informed design adjustments in the context of a design process to be carried out in stages (i.e., determining secondary design variables upon selection of a first set of primary...
design variables) highlights the importance of knowing the feasible design domain in the design parameter space, an approximate delineation of which is discussed next.

4.2 Approximate delineation of a feasible design domain

As mentioned previously, following the parametric study, contour lines of the MRP surfaces fitted for the LSs of interest, corresponding to the respective target MRPs, were superimposed in the design space to delineate the feasible design domain of an OSB. The next step in the simplified methodology for obtaining a feasible design domain is, therefore, to approximate the contours of the MRP surfaces for each LS of interest. From observations of the topology of the fitted MRP surfaces for the selected set of LSs, contour lines of these surfaces can be reasonably assumed as bilinear about the chosen positive gradient line and also approximately parallel to each other over the primary design space. This implies that determination of the orientation of any arbitrary bilinear contour line for an MRP surface corresponding to a LS is sufficient to approximate the orientation of the target MRP contour line for that LS.

The procedure requires the performance assessment for two additional design points, namely \( \mathbf{D}_1^a \) and \( \mathbf{D}_2^a \), as shown in Figure 10 (a) (for Bridge B), forming a rectangle with diagonal \( \mathbf{D}_1 \mathbf{D}_3 \) in the design space. Bilinear contour lines are split into two segments with respect to \( \mathbf{D}_1 \mathbf{D}_3 \), one corresponding to the region in the design space containing \( \mathbf{D}_1^a \), and the other corresponding to that containing \( \mathbf{D}_2^a \). Figure 10 (b) shows the procedure to determine the orientation of the bilinear contours of an MRP surface (corresponding to LS 2 for Bridge B in the example). For LS 2, the design point \(( \mathbf{D}_1^a )^{LS_2} \) (yellow diamond) along \( \mathbf{D}_1 \mathbf{D}_3 \) having the same MRP of exceeding LS 2 as that of \( \mathbf{D}_1^a \) is found using the piecewise power function fitted to MRPs along \( \mathbf{D}_1 \mathbf{D}_3 \). This gives the orientation of the bilinear contours with respect to \( \mathbf{D}_1 \mathbf{D}_3 \) in the region containing \( \mathbf{D}_1^a \). A similar approach is taken to determine the orientation of the bilinear contours in the region containing \( \mathbf{D}_2^a \) by locating the design point \(( \mathbf{D}_2^a )^{LS_2} \) (yellow diamond) along \( \mathbf{D}_1 \mathbf{D}_3 \) having the same MRP of exceeding LS 2 as that of \( \mathbf{D}_2^a \). Finally, the target MRP contour corresponding to LS 2 can be approximated using the contour orientations determined and the fitted piecewise power function. Approximated target MRP contours of all LSs of interest can be determined similarly and subsequently superimposed in the design space to delineate an approximate feasible design domain for an OSB.

The simplified methodology, formulated thus far, significantly reduces the computational workload, as compared to the full-blown parametric method, in terms of the number of design points to be assessed towards achieving the goal of finding a design point satisfying multiple risk-based performance objectives and obtaining a feasible design domain in the design space of an OSB. While maintaining reasonable levels of accuracy of results, it is found that further reduction in computational cost is possible in terms of the number of seismic hazard levels considered (from 6 to 3) and the number of nonlinear time-history analyses performed per hazard level (from 100 to as low as 20) in the performance assessment for a single design point. The set of three seismic hazard levels to be considered for the performance evaluation of a design point are recommended to be well-spaced in terms of MRPs of IM exceedance. Approximate feasible design domains obtained for the considered testbed OSBs are shown in Figure 11 and are found to tally reasonably well with the feasible design domains previously obtained from the full-blown parametric study.
5 CONCLUSIONS

The simplified PBSD methodology for OSBs proposed in this study can be considered as a step forward in the direction of a rigorously implemented PBSD framework for bridges. Knowledge of the feasible design domain of an OSB in its design space emerges as an extremely valuable resource as it can be utilized to make risk-informed design decisions leading to safe and economic design of OSBs. In addition, the distinguishing features of the proposed PBSD methodology are:

- explicit account of pertinent sources of uncertainty
- comprehensive seismic damage hazard assessments of OSBs
- state-of-the-art criteria for damage assessment
- ability to accommodate any number of pertinent LSs
- reliance on rigorous probabilistic performance assessments of design iterations (i.e., design points) to arrive at a final design

It is believed that the adoption of the proposed PBSD methodology, although non-traditional in its format, will be extremely beneficial in the medium and long-term. An immediate implementation of the proposed PBSD methodology in a practical design environment might still seem computationally overpriced and analytically cumbersome. However, such an initial venture will prove crucial in supporting and fostering future research work and innovative technological developments in bridge infrastructure engineering.

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REFERENCES


HYBRID BEM-FEM ASSESSMENT ON THE DYNAMIC BEHAVIOUR OF AN INTEGRAL BRIDGE

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Keywords: integral bridge, dynamic soil-structure interaction, natural frequency, coupled BEM-FEM, ABAQUS

Abstract. One of the most important steps in the design of dynamic-resistant structure is the identification of the natural frequencies of the structure which are the representation of the dynamic behaviour. In integral bridges, the omittance of joints and bearings leads to substantial soil-structure interaction, thus increasing the sensitivity of the dynamic behaviour of the bridge to the underlying and backfill soil characteristic. As a result, accurate identification of the natural frequencies can be a profound problem for integral bridges when encastre grounding or frequency-independent spring-damper system is utilized in the numerical model. Considering the current trend of infrastructure development, this situation can lead to the increase of the economic cost of infrastructure. In this article, steady-state dynamic assessment of phenomenon at hand is presented by utilizing hybrid method of an in-house BEM for describing the semi-infinite part of the geological region and the commercial FEM software, ABAQUS for modelling the finite part of the soil-structure system. The retained flexibility and versatility of FEM and the capability of satisfying the Sommerfeld’s radiation condition of BEM make it possible to approach the problem in a more realistic and accurate manner without losing practicality. The influences of the bridge geometry and the soil condition on the natural frequencies of the structure are investigated and comparison to the conventional results are presented.
1 INTRODUCTION

It is well documented that the pounding between and unseating of structural members are among the common types of damages found in bridges due to earthquake. The integral bridge concept with its monolithic joint between the superstructure and the abutment overcomes these problems and combined with its long-term durability, simplicity and low maintenance, its popularity is increasing in the field of infrastructure development in many countries. With increased demand in traffic characteristics and technological advancement, there is large demand in the bridges with longer span and slender structures. These bridges, on the other hand, have to account for earthquakes or blast induced vibrations in many parts of the world. There have been numerous incidents of bridges failed due to earthquakes or other dynamic loads in the past. [11] outlines some significant damage to bridges following Canterbury earthquake, 2010-11. [7] and [9] studied damages found in rail and road bridges during different earthquakes.

In the design process of dynamic-resistant structures, one of the important steps is the characterization of the dynamic behaviour which is performed through the quantification of its natural frequencies. In non-monolithic systems, the supporting pier or column and the abutment can be analysed separately in a relatively simple manner since their behaviours tend to be dominated by the fundamental frequency. The same cannot be applied to the integral bridges or other monolithic system where interaction between structural members takes place. The interaction between the superstructure, the substructure, the underlying soil and the backfill means that the dynamic response of the integral bridges is more complex with tightly spaced natural frequencies and more sensitive to soil-structure interaction.

Over the time the bridge design is mainly focused on modelling with spring-dashpot system as it is computationally very efficient and simplified. The finite element method or similar other numerical modelling approaches are used. These models account for most of the engineering features in near field problems. But as the bridge structure is a combination of the structural and geotechnical components and the ground expands up to the infinite space, the dynamic wave propagation plays important role. This phenomenon occurs in a coupled way between the structure, soil-structure interface and the extent of the ground space, which has to be accounted in the numerical model.

To account for these phenomenon in far-field and near-field, a coupled numerical model between FEM and BEM is developed and used to assess the behaviour of a bridge in steady-state dynamic. The FEM is efficient with modelling nonlinear behaviour in the near field but lacks ability to account for the Sommerfeld’s radiation condition in the far-field. The BEM, on the other hand, lacks efficiency in solving most of the non-linear problem but can account for terrain effects and satisfy the Sommerfeld’s radiation condition in the far-field. A hybrid FEM-BEM model can account for these effects for a more realistic dynamic behaviour of the bridge. The wave propagation through poroelastic media in half-plane is studied in [5]. The same approach is used to study the impedance functions for rigid foundations and soil tunnel system in elastic and poroelastic half-plane [3, 4].

2 PROBLEM ILLUSTRATION

Consider an infinite geological region $\Omega_{BEM}$ in half-space which can be either elastic or poroelastic ($C_{P(BEM)}$, $C_{S(BEM)}$ or $C_{P(BEM)}^{*}$, $C_{S(BEM)}^{*}$) with the free-surface of $\Gamma_{BEM}$. The continuum $\Omega_{FEM}$ is resting on the half-space with the surface $\Gamma_{FEM}$. The intersection between the surfaces is the interface denoted as $\Gamma_{i}$. $\Omega_{FEM}$ represents the structure and can include the homogeneous soil back filling $\Omega_{bf}$. 
2.1 BEM formulation in semi-infinite domain $\Omega_{BEM}$

For the semi-infinite domain $\Omega_{BEM}$, bounded by the traction-free surface $\Gamma_f$ and the interface $\Gamma_i$, in homogeneous half-space, the formulation of seismic wave propagation can be represented by the following integral equation, as given in Dominguez [2].

$$c_{lk}u_k^{\Omega_{BEM}}(x, \omega) = \int_{\Gamma_{BEM}} U^{*}_{lk}\Omega_{BEM}(x, \omega) t_k^{\Omega_{BEM}}(x, \omega) - \int_{\Gamma_{BEM}} T^{*}_{lk}\Omega_{BEM}(x, \omega) u_k^{\Omega_{BEM}}(x, \omega)$$  \hspace{1cm} (1)

Here, $\Gamma_{BEM} = \Gamma_f \cup \Gamma_i$ and $c_{lk}$ is a constant, which depends upon the type and shape of the domain. Various methods can be used to compute the value of this constant matrix, for example by the rigid body motion principle or by solving the traction fundamental solution over augmented spherical boundary covering the calculated point [2]. In this study, the latter analytical approach is derived and implemented for smooth and non-smooth points in 3D space. The boundary conditions are given by Equations 2 to 4.

$$t^\Omega_{BEM} = 0$$  \hspace{1cm} (2)

$$t^\Omega_{FEM} = -t^\Omega_{BEM} = -(t^\Omega_{i,ff} + t^\Omega_{i,sc})$$  \hspace{1cm} (3)

$$u^\Omega_{i,BEM} = u^\Omega_{i,FEM}$$  \hspace{1cm} (4)

Here, $t_s$ and $t_i$ are the total traction along the free surface and the interface between the respective domains. $t_i,ff$ and $t_i,sc$ are the free field and scattered traction along the interface $\Gamma_i$ for the respective domains. $u_i$ is displacement along the interface $\Gamma_i$ for the respective domains. The computation of the free field motion can be found in [1].

2.2 Hybrid BEM-FEM formulation of the boundary value problem

For the domain $\Omega_{BEM}$, Equation 1 can be represented in a general matrix form of scattered displacement and traction:
\[ [H] \{ u_{sc} \} = [G] \{ t_{sc} \} \]  
(5)  
\[ [H] \{ u \} - [G] \{ t \} = [H] \{ u_{ff} \} - [G] \{ t_{ff} \} \]  
(6)

The expressions for \([H]\) and \([G]\) matrices in Equation 5 are taken from [2]. The subscripts ‘ff’ and ‘sc’ represent the free field and scattered motions, respectively. In Equation 5, the free field motion is calculated analytically as was done in [1] and the equations can be rearranged for traction. The traction vectors are now converted into equivalent force terms. In the case the solution for the domain \(\Omega_{BEM}\) is not necessary, the degrees of freedom along the boundary that is not in \(\Gamma_i\) can be condensed and the system can be reduced to the degrees of freedom along \(\Gamma_i\).

The procedure to condense the degree of freedom for BEM matrix and to convert the traction vector to force vector can be found in [5, 10]. Now the condensed BEM matrix equation in equivalent force vector term can be obtained in the form:

\[ \{F_{\Gamma, i}^{\Omega_{BEM}}\} + \{\Phi_{\Gamma, i}^{\Omega_{BEM}}\} = [K^{\Omega_{BEM}}] \{u_{\Gamma, i}^{\Omega_{BEM}}\} \]  
(7)

Here, \(F\) represents the external force matrix, \(\Phi\) represents the body force matrix, \(K\) represents the stiffness matrix, and \(u\) represents the displacement matrix for the respective domains and boundaries. The procedure to derive these matrices from the BEM form of Equation 5 and 6 are given in [5, 10].

A general format of finite element matrix for equation for the domain \(\Omega_{FEM}\) is obtained in the form similar to Equation 7 as:

\[ \{F_{\Gamma, i}^{\Omega_{FEM}}\} + \{\Phi_{\Gamma, i}^{\Omega_{FEM}}\} = [K^{\Omega_{BEM}}] \{u_{\Gamma, i}^{\Omega_{FEM}}\} \]  
(8)

Now, by applying the boundary condition, \(F_{\Gamma, i}^{\Omega_{FEM}} = -F_{\Gamma, i}^{\Omega_{BEM}}\) along the interface \(\Gamma_i\), the body force terms \(\{\Phi_{\Gamma, i}^{\Omega_{BEM}}\}\) and the stiffness terms \([K_b]\) from Equation 7 can be added to the respective degrees of freedom body force and stiffness in Equation 8. The similar coupling approach can be found in [5, 10].

### 2.3 Implementation of BEM as substructure element in ABAQUS

The coupling approach mentioned in Sections 2.1 and 2.2 can be applied in substructure element approach. The FEM pre-processing software (ABAQUS) is used to model the geometry and to mesh the whole domain including the BEM subdomain for which dummy elements are generated using 4- or 8-nodes shell element definition. These dummy elements are subsequently used as BEM elements by an in-house MATLAB code for the BEM computation and condensation of the matrices. The results are equivalent force and stiffness matrices computed by the authors’ codes which are integrated back to ABAQUS using user subroutine ‘UEL’ or keyword ‘matrix input’ [8]. The complex body force terms are included to respective real and imaginary forces. The real part of the stiffness matrix is implemented into the FEM stiffness matrix, whereas the imaginary part is implemented into the structural damping matrix [8]. The global equation is then solved in the ABAQUS steady-state dynamics. A similar implementation for poroelastic half-space can be found in [5].
3 VALIDATION

To validate our numerical model, consider the geometry as presented in Figure 1 in which \( \Omega_{FEM} \) is a rectangular rigid massless foundation with the dimension of 2\( B \) and 2\( L \) resting on a homogeneous half-space \( \Omega_{BEM} \). Steady state unit vertical force, horizontal force, and rotational moment about y-axis \( H, V, M_y \) are applied to the foundation at a point at the middle-bottom of it. The impedance function for external force at direction \( i \) and the response at direction \( j \), \( S_{ij} \), can be obtained as:

\[
\begin{bmatrix}
S_{xx} & S_{xz} & S_{xr_y} \\
S_{zx} & S_{zz} & S_{zr_y} \\
S_{ry_x} & S_{ry_z} & S_{ry} \\
\end{bmatrix}
= 
\begin{bmatrix}
H & 0 & 0 \\
0 & V & 0 \\
0 & 0 & M_y \\
\end{bmatrix}
\begin{bmatrix}
u_{xx} & u_{xz} & u_{r_yx} \\
u_{zx} & u_{zz} & u_{r_yz} \\
u_{ry_x} & u_{ry_z} & u_{ry} \\
\end{bmatrix}^{-1}
\]

(9)

Here, \( u_{ij} \) denotes displacement and \( S_{ij} = K_{ij} + i\omega C_{ij} \) can be decoupled into the dynamic stiffness, \( K_{ij} \), and the radiation damping, \( C_{ij} \). These values are subsequently normalized to their static stiffness and damping constants, respectively, to obtain the frequency-dependent factors, \( k_{ij} \) and \( c_{ij} \). The simulation results of the 3D hybrid BEM-FEM is in good agreement to the solution published by [6] as presented in Figure 2. The discrepancies in the result is due to the different foundation-soil interface definition in which the referenced study used relaxed boundary while the current study used the rough interface. The unitless frequency \( a_0 \) is defined by the angular frequency \( \omega \), width of foundation \( B \), and shear wave velocity of the half-space \( V_s \).

Figure 2: Frequency-dependent factors for impedance functions obtained by hybrid BEM-FEM and [6] for rectangular rigid massless foundation.
4 SIMULATION AND PARAMETRIC ANALYSIS

The qualitative assessment object is a typical overhead crossing, single span integral bridge made of concrete with monolithic superstructure-abutment joints resting on a half-space shown in Figure 3. The span and width are 35m and 12m, respectively. The abutment height is 7.6m with wingwalls extending to (1 : 1) slope. The superstructure consists of 30cm thick deck, main girders (5@2.5m) with 1.6m height and 1.0m high diaphragm which are modelled as integrated shell and beam elements. The abutments as well as the backfill are meshed using continuum elements. The superstructure and backfill here belong to FEM whereas the underlying soil is in BEM.

![Cross section](image1.png)  
(a) Cross section

![Isometric view](image2.png)  
(b) Isometric view

Figure 3: An integral bridge under consideration for parametric analysis.

The concrete properties of the bridge are simplified with Young’s modulus of 300 GPa, Poisson’s ratio of 0.2, and density of 2400 kg/m³. The underlying soil and the backfill materials are arranged in Table 1. The interface between the structure and backfill is of frictional type with friction angle of 20 degrees. The interface between the FEM and the BEM subdomains is of tied constraint. In the hybrid simulation, a gravity load action on the backfill is applied in the first step to better simulate the backfill resistance to the abutments and wingwalls movement. It is then followed by the steady-state simulations to assess how the dynamic behaviour of the bridge in comparison to the conventional approach.

The mode shapes that are in the interest of this study are shown in Figure 4. In the current steady-state simulations, these modes are generated by applying one- or multi-directional (coupled) distributed load on to the deck. Mode shapes 1 to 4 are dominated by the superstructure’s
Figure 4: The six mode shapes of the integral bridge considered in this study.

geometry and property whereas mode shape 5 is a transitional one.

The latter is activated by applying horizontal distributed load on the lateral direction. Under low frequency, the loading scheme tends to activate rotational mode found in mode shape 2 while under higher frequency, it tends to generate lateral translational mode. The last 2 mode shapes are dominated by the underlying soil. In each activated mode shape, displacement is measured at the degree of freedom and a point at the deck which represent the maximum response.

The first study is focused on the assessment of the influence of the underlying soil which is
performed by assigning properties no. 2 of Table 1 to the backfill and properties no. 1, 2, and 4 to the underlying soil. The influence of the backfill is studied in the second simulation set which is performed by setting the property no.2 to the underlying soil and properties no. 2, 3, and 4 to the backfill.

<table>
<thead>
<tr>
<th>No.</th>
<th>Vs (m/s)</th>
<th>(\rho) (kg/m³)</th>
<th>(\mu) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>1650</td>
<td>16.50</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>1800</td>
<td>112.50</td>
</tr>
<tr>
<td>3</td>
<td>375</td>
<td>1840</td>
<td>258.75</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>1890</td>
<td>472.50</td>
</tr>
</tbody>
</table>

Table 1: Material properties of the soil considered in simulation.

4.1 Results and discussion

The results of the hybrid simulation presented in Figures 5 to 7 show the influence of the underlying soil dynamic properties to the natural frequencies of the structure-soil system. These plots also show the advantage of the hybrid assessment method to the conventional method in identifying the natural frequencies by taking into account the dynamic properties instead of the static parameters. The comparison in Figure 8 shows the influence of the underlying soil to the magnitude of the response which is otherwise unknown. As expected, minor shifts in peak-frequency were found between the modes dominated by the superstructure while significant shifts can be observed in mode 5 and 6.

The results of the hybrid simulation on the influence of the backfill are presented in Figure 9. It can be observed that only a low shift in natural frequencies occurred due to the increase

Figure 5: Displacement response magnitude vs frequency for six mode shapes of the integral bridge with underlying soil: \(Vs = 100\text{m/s}, \rho = 1650\text{kg/m}^3, \nu = 1/3\).
Figure 6: Displacement response magnitude vs frequency for six mode shapes of the integral bridge with underlying soil: $V_s = 250 m/s$, $\rho = 1800 kg/m^3$, $\nu = 1/3$.

Figure 7: Displacement response magnitude vs frequency for six mode shapes of the integral bridge with underlying soil: $V_s = 500 m/s$, $\rho = 1890 kg/m^3$, $\nu = 1/3$.

in stiffness of the backfill. The results also show the influence of the backfill properties to the magnitude of the response.
Figure 8: Comparison of displacement response magnitude vs frequency for six mode shapes for three underlying soil conditions.

Figure 9: Comparison of displacement response magnitude vs frequency for six mode shapes for three backfill conditions.

5 CONCLUSIONS

A qualitative assessment of dynamic behaviour of a typical integral bridge is presented. The study takes advantage of 3D steady-state hybrid BEM-FEM formulation in solving dynamic soil-structure interaction in which BEM is used to solve the infinite far-field domain and FEM is
used to handle near-field finite soil domain as well as the superstructure system and the frictional interface. The results highlight the advantages of the method in which dynamic properties of the underlying and backfill soil can be taken into account to provide more accurate quantification of the dynamic behaviour of the system. With this, the study illustrates the strong influence of the aforementioned properties to the response of the bridge in terms of the natural frequency and the response magnitude.

REFERENCES


THE EFFECT OF JOINT GAP SIZE ON THE SEISMIC PERFORMANCE OF RAILWAY BRIDGES

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Abstract

Boundary conditions play an important role in the response of bridges to different actions, particularly dynamic loads. Gap closure at end joints, resulting in activation of the abutment-backfill-embankment system, considerably affects the dynamic characteristics and, hence, the seismic response of the bridge. In particular, consideration of abutment-backfill flexibility may often result in substantial reduction of the contribution of piers to the bridge response under earthquake loading. Moreover, gap size and closure are related to various damage stages including backfill deformation/damage and deck unseating, while the varying boundary conditions during an earthquake affect the inelastic behaviour of critical bridge components (i.e. piers, piles), as well as the damage sequence.

The use of gap sizes larger than the ones resulting from Eurocode 8 provisions is a common practice in seismic design of bridges in Greece; this is apparently considered to be on the safe side and conveniently relieves the designer from the difficulty of accounting for gap closure effects in the analysis. However, this means over-dimensioning of bridge joints, with subsequent increase in both initial and maintenance costs, while it is not certain that preventing activation of the abutment-backfill system under seismic actions exceeding the design ones increases the safety of the bridge.

In this context, the aim of this paper is to study the effect of gap size on the seismic performance of concrete bridges, providing insight into the distribution of seismic action effects among the key components of the bridge i.e. piers and abutments, accounting for varying gap sizes and hence boundary conditions. The behaviour of an existing railway bridge with a passive system is evaluated using inelastic response history analysis for a set of design spectrum compatible artificial accelerograms, considering different gap sizes for the end joints, i.e. those resulting from Eurocode 8, as well as higher and lower values. Analyses are carried out for different levels of seismic action, up to twice the design one.

Keywords: Bridges, Boundary conditions, Joint gap size, Seismic performance.
1 INTRODUCTION

Joints are required to accommodate movements of the deck due to thermal expansion and contraction, shrinkage, creep and prestressing of concrete, and earthquake-induced horizontal movements. The use of gap sizes larger than the ones prescribed by Eurocode 8 is a common practice (particularly in Greece) in order to ensure that the gap remains open during the design earthquake and avoid verification checks to account for gap closure effects. Dual analysis is recommended by Caltrans [1], i.e. the bridge is analysed assuming either free movement or full restraint at the “compression end” of the bridge. The most unfavourable response quantities from either set of analyses are taken into consideration, rendering this practice conservative and cost ineffective. Since the boundary conditions are different depending on whether joints are open or closed, the necessity for joint modelling during assessment of the seismic behaviour of bridges emerges. When either of the abutment joints closes during the earthquake, substantial forces are transferred to the abutment-backfill system.

Kappos & Sextos [2] have studied the importance of boundary conditions on the seismic response of bridges, with a view to highlighting the necessity of capturing the effect of gap closure on the seismic behaviour. In the studied bridge, failure was expected for a displacement twice the gap size and was attributed to unrecoverable damage to the backfill soil, while the piers remained well within their rotational capacity. However, a different failure mechanism, i.e. exceedance of available pier ductility, would have been predicted if the end supports were modelled as longitudinal restraint (as per the Caltrans simplified approach).

The main goal of this paper is to investigate the optimum design of critical bridge components, based on a proper gap size selection. In this context, the effect of joint gap size on the seismic performance of an existing bridge is investigated. The bridge studied is a seismically isolated railway bridge in Northern Greece. Different scenarios regarding the gap sizes for the end joints are studied, considering values calculated according to the provisions of Eurocode 8 [3], as well as half and twice the code values. Inelastic response history analysis is performed, using a set of 10 artificial accelerograms compatible with the design spectrum. Two different levels of seismic intensity are considered, corresponding to 1.0 and 2.0 times the design level. The results clearly show that the response of the bridge is significantly affected by the gap size. An important increase in the shear forces carried by the abutments is observed for smaller gap sizes, still well within the capacity of the abutments. Conversely, the relevant seismic forces in the bridge piers are reduced. Overall, it appears feasible to optimize the design by a proper selection of the gap size; this means that gap closure should be accounted for in the analysis, which is feasible when advanced software packages are used.

2 CASE STUDY RAILWAY BRIDGE

2.1 Overview of the bridge

The T4 railway bridge is part of the new High-Speed Railway Network of Northern Greece, connecting Polykastro and Eidomeni. The bridge has four spans (39m, 45m, 45m and 39m respectively), and the deck consists of a bearing-supported continuous box girder with constant height of 3.6m and width at the top 13.90m (Fig. 1b). The longitudinal slope is 2% and the maximum height above ground level, 24 m. The bridge piers have a hollow rectangular section with outside dimensions 3x5.5m and wall thickness 0.45m (Fig. 1d). The seat-type abutments have longitudinal joints but not transverse ones, as required in railway bridges.
The bridge was designed according to the Greek Standards for Seismic design of Bridges ([4], [5]) which are similar to Eurocode 8-2 [3], using the seismic isolation concept. Lead rubber bearings were used at both piers and abutments (with dimensions (mm) equal to 900×900/231-200 and 1200×1200/286-250, respectively), while shear keys and fluid viscous dampers were provided at each abutment. The shear keys prevent end displacement of the bridge in the transverse direction (to prevent derailment), hence the bridge is not fully isolated in that direction. Two fluid viscous dampers are provided at each abutment, in the longitudinal direction of the bridge, to achieve the required amount of damping. The selected damping coefficient $C$ of the nonlinear dampers ($F=Cv^\alpha$) is 2350 kN·s/m and the velocity exponent $\alpha=0.15$. Finally, pile foundation was adopted (Fig. 1a) for both piers and abutments.

2.2 Finite element modelling of the bridge

The finite element model of the bridge was set up in OpenSees v.2.5.0. [6] (Fig. 2). The focus of this study was on the effect of joints on the seismic response of the bridge; response history analyses were performed to this purpose for different ‘scenarios’ of joint gaps. It is noted that in the analyses reported herein expansion joints were assumed along both the longitudinal and the transverse directions, to fully study the effect of gap size on the seismic performance.
Parameter | F1SD | F2SD | F3SD | F4SD |
---|---|---|---|---|
$A$ ($m^2$) | 15.4 | 10.2 | 14.1 | 17.7 |
$J$ ($m^4$) | 23.9 | 16.5 | 22.3 | 26.9 |
$I_z$ ($m^4$) | 25.4 | 17.8 | 22.8 | 25.9 |
$I_y$ ($m^4$) | 122.1 | 106 | 120.9 | 131.8 |

Table 1: Geometric characteristics of the deck.

The bridge deck, which is intended to remain elastic during the design earthquake was modelled with elastic beam-column elements with gross section properties given in Table 1 and Figure 2. Bridge piers are expected to develop inelastic behaviour; hence, a lumped plasticity model was used, accounting for the plastic hinge length given in equation (1), where $L$ is the length of the contraflexure point under seismic action, $f_{yk}$ is the characteristic yield stress of steel reinforcement and $d_{bl}$ the bar diameter.

$$L_p = 0.1 \cdot L + 0.015 \cdot f_{yk} \cdot d_{bl}$$  \hspace{1cm} (1)

Bilinearised moment-curvature (M-φ) curves were obtained from section analyses for the axial load of the seismic combination $N_G + 0.3Q$, using the AnySection software [7]. Regarding confined concrete of hollow rectangular sections, the stress-strain model by Kappos [8] was used (Figure 3). Pier properties were based on secant values at yield.

Figure 3: (a) Pier reinforcement (b) Stress-strain curves for the confined and unconfined concrete of the pier

Regarding lead rubber bearings, the horizontal shear stiffness ($K_h$), as well as the flexural ($K_b$) and axial stiffness ($K_v$) were calculated according to [9]. The hysteretic model for lead rubber bearings is depicted in Figure 4(a), while the effective stiffness was calculated using an iterative procedure. Viscous dampers were modelled using a two-node link element (Figure 4(b)), while the stiffness and damping parameters were taken from manufacturer data.

Figure 4: (a) Hysteretic model for lead rubber bearings, (b) Viscous Damper

Detailed modelling of abutments should account for all resistance mechanisms and components, including an accurate estimation of mass, stiffness and nonlinear hysteretic behav-
The resistance of the abutment-embankment system was modelled through springs, based on Caltrans recommendations [1] and simplifying assumptions.

Regarding the case-study bridge, the detailed model of the abutments is depicted in Figure 5(a). The springs for lead rubber bearings and viscous dampers are shown, along with gap elements and abutment-embankment springs in both directions (Figure 5(b)). The detailed, trilinear, inelastic model proposed by Nielson [11] was adopted for the resistance of the abutment-backfill system subsequent to gap closure.

![Abutment modelling](image)

(a) Opensees model; (b) Constitutive laws

### 3 JOINT GAP SIZE ACCORDING TO EC8-2

According to Eurocode 8-Part 2 [3], adequate gap sizes according to equation (2) should be provided for protection of critical or major structural bridge members

\[
d_{Ed} = d_E + d_G + \psi_2 d_T
\]

where:
- \(d_E\) is the design seismic displacement
- \(d_G\) is the displacement due to the permanent and quasi-permanent actions measured in long term (e.g. post-tensioning, shrinkage and creep for concrete decks)
- \(d_T\) is the displacement due to thermal movements
- \(\psi_2\) is the combination factor for the quasi-permanent value of thermal action

The detailing of non-critical structural components, such as deck movement joints but also abutments with ‘sacrificial’ backwalls, should be related to controlled and repairable damage, additionally accounting for creep and shrinkage effects. According to EC8-2, clearances should accommodate appropriate fractions of the seismic design displacement and thermal movement (\(p_E=0.4\) and \(p_T=0.5\), respectively), allowing for any long-term creep and shrinkage effects, so that damage under frequent earthquakes is avoided.

To study the effect of joint gap in both directions, in addition to the existing longitudinal joints, a case with joints in the transverse direction was also studied. The required value of design displacement (joint gap) is determined for both bridge directions, according to Eurocode 8; the resulting values for each action are given in Table 2.
Sotiria P. Stefanidou, Anastasia E. Gektsi and Andreas J. Kappos

### Actions

<table>
<thead>
<tr>
<th>Abutment</th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ux (m)</td>
<td>Uy (m)</td>
</tr>
<tr>
<td>Thermal Movements:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.016</td>
<td>0.0172</td>
</tr>
<tr>
<td>Permanent and Quasi-Permanent Actions:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prestressing</td>
<td>0.011</td>
<td>0</td>
</tr>
<tr>
<td>Shrinkage and Creep</td>
<td>0.044</td>
<td>0</td>
</tr>
<tr>
<td>Seismic Displacements:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>±Ex</td>
<td>0.102</td>
<td>0</td>
</tr>
<tr>
<td>±Ey</td>
<td>0</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 2: Design displacements at abutments.

### COMBINATION

<table>
<thead>
<tr>
<th>Abutment</th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X-X (m)</td>
<td>Y-Y (m)</td>
</tr>
<tr>
<td>d_{Ed}=+0.4d_{E}+d_{G}+0.5d_{T}</td>
<td>0.104</td>
<td>0.121</td>
</tr>
<tr>
<td>d_{Ed}=+0.4d_{E}+d_{G}-0.5d_{T}</td>
<td>0.088</td>
<td>0.103</td>
</tr>
<tr>
<td>d_{Ed}=-0.4d_{E}+d_{G}+0.5d_{T}</td>
<td>0.006</td>
<td>-0.131</td>
</tr>
<tr>
<td>d_{Ed}=-0.4d_{E}+d_{G}-0.5d_{T}</td>
<td>0.006</td>
<td>-0.131</td>
</tr>
<tr>
<td>Critical Combination</td>
<td>0.104</td>
<td>0.131</td>
</tr>
<tr>
<td>Design Value of the Gap Size</td>
<td>X-X (m)</td>
<td>0.104</td>
</tr>
</tbody>
</table>

Table 3: Required joint gap size: Case of acceptable damage (d_{Ed}=±0.4d_{E}+d_{G}±0.5d_{T})

### COMBINATION

<table>
<thead>
<tr>
<th>Abutment</th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X-X (m)</td>
<td>Y-Y (m)</td>
</tr>
<tr>
<td>d_{Ed}=+d_{E}+d_{G}+0.5d_{T}</td>
<td>0.165</td>
<td>0.289</td>
</tr>
<tr>
<td>d_{Ed}=+d_{E}+d_{G}-0.5d_{T}</td>
<td>0.149</td>
<td>0.271</td>
</tr>
<tr>
<td>d_{Ed}=-d_{E}+d_{G}+0.5d_{T}</td>
<td>-0.055</td>
<td>-0.289</td>
</tr>
<tr>
<td>d_{Ed}=-d_{E}+d_{G}-0.5d_{T}</td>
<td>-0.055</td>
<td>-0.289</td>
</tr>
<tr>
<td>Critical Combination</td>
<td>0.165</td>
<td>0.289</td>
</tr>
<tr>
<td>Design Value of the Gap Size</td>
<td>X-X (m)</td>
<td>0.165</td>
</tr>
</tbody>
</table>

Table 4: Required joint gap size: Case that no damage is allowed (d_{Ed}=±d_{E}+d_{G}±0.5d_{T})

The required gap size according to Eurocode 8-2 provisions for the cases that acceptable or no damage is anticipated, are presented in Tables 3 and 4, respectively, for the longitudinal and transverse directions. For the case that damage (under the design earthquake) is acceptable, gap sizes equal to 105mm and 130mm (for the longitudinal and the transverse direction respectively) are calculated. When no damage is sought, the longitudinal joint gap increases to 165mm and the transverse one to 290mm. However, in the actual bridge, the gap size at end joints is equal to 500mm, much higher value than the required based on EC8-2 provisions. As discussed in the Introduction, the use of gap sizes larger than the ones calculated according to Eurocode 8 is common practice in modern bridges in Greece, resulting in over-dimensioning of bridge joints and increase in both initial and maintenance costs. Furthermore, critical bridge
components (i.e. bridge piers) attract significant earthquake loading and deformation during an earthquake, whereas the abutments are not contributing to the seismic energy dissipation.

4 NON-LINEAR RESPONSE-HISTORY ANALYSIS

The case study bridge described in section 2 was assessed considering various gap sizes, namely 0.5, 1.0 (related to acceptable damage) and 2.0 times the calculated EC8-2 design gap size ($d_E$) for each direction. Nonlinear dynamic response history analyses were performed in the longitudinal and transverse direction, using 10 spectrum compatible artificial accelerograms scaled to two different intensities that correspond to the design, and twice the design, earthquake intensity.

Non-linear response history analyses were performed using the OpenSees software [6]. The mean spectrum of the ten artificial, spectrum compatible, accelerograms used in the analyses is depicted in Figure 6; the average value of response quantities is considered in the following, since the number of accelerograms is greater than 7.

The different gap size scenarios for both directions and earthquake intensities are summarised in Table 5. The key results used to compare and assess the seismic performance for varying gap sizes are section forces of critical components, namely bridge piers and abutments, and the corresponding horizontal displacements.

<table>
<thead>
<tr>
<th>Earthquake Intensity</th>
<th>Longitudinal Direction</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scenario</td>
<td>$d_{ed}$</td>
</tr>
<tr>
<td>0.24g</td>
<td>1X</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>2X</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3X</td>
<td>2</td>
</tr>
<tr>
<td>0.48g</td>
<td>4X</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>5X</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>6X</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5: Gap size ‘scenarios’ for the longitudinal and transverse direction of the bridge.

5 ANALYSIS RESULTS

Nonlinear response history analyses were performed considering the 6 scenarios described in Table 5 for ten spectrum-compatible artificial accelerograms (scaled to two intensities), re-
sulting in a total of 60 analyses for each direction (longitudinal and transverse). Shear forces of piers and abutments were obtained and discussed with a view to highlighting the effect of gap size on bridge performance and the feasibility of design optimization by a proper selection of gap size.

5.1 Longitudinal direction of the bridge

Regarding the longitudinal direction of the bridge, the design seismic level (0.24g) and the gap size of 10 cm (corresponding to the design values), closure of joints at both abutments (A1 and A2) occurs (although for some of the accelerograms). When a gap size equal to 20cm (twice the design value for the case of acceptable damage) is considered, no gap closure at the end joints for the design earthquake is recorded; however, when twice the design earthquake is considered, gap closure is observed. Furthermore, for the case that an earthquake equal to twice the design level is considered, gap closure at both ends is observed for a gap size equal to 5 cm (half the design value).

The results considering shear forces are provided, for the case of excitation along the longitudinal direction, and various gap sizes at the end joints (Figures 7,8,10).

Tables 6 and 7 show the (percentage) changes in pier shears for reduction of the gap size from the design value (10 cm) to half the design value (5 cm) and from twice the design value (20 cm) to the design value (10 cm).

It can be observed that in the case of the reduced gap size, there is a significant reduction in the pier shear forces. In the case of twice the design earthquake intensity, this reduction is, as expected, more significant. Varying the gap size from 20 cm to 10 cm, there is no significant reduction in the key results, as the end joints do not close for the gap size of 10 cm.
Table 6: Changes in pier response for different gap sizes – 1× Design Earthquake

<table>
<thead>
<tr>
<th>PIER</th>
<th>Shear Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from 10 to 5 cm</td>
</tr>
<tr>
<td>P1</td>
<td>Base</td>
</tr>
<tr>
<td>P2</td>
<td>Base</td>
</tr>
<tr>
<td>P3</td>
<td>Base</td>
</tr>
</tbody>
</table>

Table 7: Changes in pier response for different gap sizes – 2× the Design Earthquake

<table>
<thead>
<tr>
<th>PIER</th>
<th>Shear Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from 10 to 5 cm</td>
</tr>
<tr>
<td>P1</td>
<td>Base</td>
</tr>
<tr>
<td>P2</td>
<td>Base</td>
</tr>
<tr>
<td>P3</td>
<td>Base</td>
</tr>
</tbody>
</table>

The forces resisted by the abutment – backfill system for the different gap size scenarios are shown in Figure 9, along with the capacity of the abutment – embankment system, estimated according to Nielson [11]. It can be concluded that even for twice the design earthquake the forces at the abutments are well within their capacity and, hence, no failure of the abutments is expected when they are activated subsequent to joint gap closure.

Figure 9: Abutment ultimate capacity and abutment forces in relation to the gap size for 1× the design earthquake (up) and 2× the design earthquake (down)
5.2 Transverse direction of the bridge

Regarding the transverse direction of the bridge and the 13 cm gap size, calculated according to Eurocode 8 provisions (case of acceptable damage, ‘sacificial’ abutment), abutment joints are expected to close at the design earthquake level. For twice the gap value (26 cm gap), there is no joint closure. However, for twice the design earthquake intensity (0.48g), end joints are expected to close, even for the largest gap size.

Shear forces at pier top and bottom, considering excitation along the transverse direction of the bridge are shown in Figures 11 and 12, while tables 8 and 9 summarise the changes in these quantities for various gap size scenarios. Reduction in shear forces is observed, as anticipated, for decreasing gap sizes. This reduction is more significant for the case of twice the design earthquake. It is clear from Fig. 12 that the forces developing at the abutment – embankment system (in the transverse direction) are much lower than the corresponding capacity in all cases studied.
Table 8: Changes in pier response for different gap sizes – 1× Design Earthquake (transverse)

<table>
<thead>
<tr>
<th>PIER</th>
<th>Shear Forces from 13 to 6.5 cm</th>
<th>Shear Forces from 26 to 13 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>Base  -5.31%</td>
<td>-1.11%</td>
</tr>
<tr>
<td>P2</td>
<td>Base  -1.59%</td>
<td>-0.16%</td>
</tr>
<tr>
<td>P3</td>
<td>Base  -5.68%</td>
<td>-0.90%</td>
</tr>
</tbody>
</table>

Table 9: Changes in pier response for different gap sizes – 2× the Design Earthquake (transverse)

<table>
<thead>
<tr>
<th>PIER</th>
<th>Shear Forces from 13 to 6.5 cm</th>
<th>Shear Forces from 26 to 13 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>Base  -4.23%</td>
<td>-9.48%</td>
</tr>
<tr>
<td>P2</td>
<td>Base  -6.07%</td>
<td>-3.10%</td>
</tr>
<tr>
<td>P3</td>
<td>Base  -3.73%</td>
<td>-8.95%</td>
</tr>
</tbody>
</table>

Figure 13: Abutment ultimate capacity and abutment forces in relation to the gap size for (a) 1× the design earthquake (up) and (b) 2× the design earthquake (down)

Finally, Figure 14 depicts the shear forces of the critical abutment and critical pier for the different gap size scenarios. Pier shear forces decrease as the gap size becomes smaller, while
shear forces at the abutments are increasing. In the case of twice the design earthquake this behaviour becomes more noticeable (note the different scale in each figure).

![Figure 14](image)

**Figure 14:** Shear forces of the critical abutment and critical pier in relation to the gap size for 1× the design earthquake (left) and 2× the design earthquake (right)

## 6 CONCLUSIONS

The effect of the joint gap size on the seismic performance of bridges was studied, focusing on an existing railway bridge that was assessed using inelastic response history analysis for a set of artificial accelerograms. Different gap size scenarios were considered for the end joints, i.e. gap sizes equal to the ones resulting from Eurocode 8, as well as twice and half these values. The analyses were conducted for the design and twice the design seismic level for the longitudinal and transverse direction of the bridge. From the results obtained for both directions of the bridge, several conclusions were drawn.

- Considering the response of the abutments for the different gap size scenarios, it was concluded that for open joints, the participation of the (seat-type) abutments in seismic energy dissipation is low.
- On the contrary, when the end joints close, further activation of the abutment – embankment system was obtained, particularly for small gap sizes. In general, the forces carried by the abutments do not exceed the capacity values and, thus, are considered to be acceptable.
- At joint closure, the shear forces in the piers decrease. This reduction was more noticeable for smaller gap sizes and was attributed to the consideration of the abutment – backfill flexibility in the seismic response of the bridge. For twice the design earthquake intensity, the reduction in shear values was even higher. Therefore, the distribution of seismic energy across the members of the bridge was found to be more uniform when gap closure was taken into account in the analysis.
- Finally, the aforementioned decrease in pier response quantities and increase in the forces carried by the abutments was found to be well within the capacity of the abutment-backfill system. Therefore, it seems feasible to optimize the design by a proper selection of the gap size.
7 ACKNOWLEDGEMENTS

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THE SIGNIFICANCE OF INNOVATIONS ON THE STRUCTURAL SYSTEM WHEN SELECTING THE CONSTRUCTION METHOD OF EARTHQUAKE-RESISTANT BRIDGES

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Abstract

The present paper attempts to demonstrate the great importance of innovative interventions on the structural system of earthquake-resistant bridges. The type of structural system affects also the construction method which will be chosen. The problem of selecting the most appropriate among the five current bridge construction methods (Cast-in-place, Precast I-Girder, Incremental Launching, Balanced Cantilever, Advanced Shoring method) is the most creative part of the decision-making process related to the construction of a bridge project. Seven compliance criteria are taken into account in this study, namely safety, economy, durability, serviceability, construction speed, aesthetics and environmental harmonization. The large number of criteria leads to the use of Multicriteria Analysis, and specifically of two Multicriteria methods: AHP and PROMETHEE. Certain innovative interventions in bridge design are selected and presented, all of them having features of “Egg of Columbus” solutions. Afterwards they are compared with some conventional construction methods, using a double Case study, from which interesting conclusions are drawn. The study does not result simply in the subjective predominance of one method, but in the hierarchy of all the respective alternative construction methods, followed by a specific score for each one of them. To this end, the opinions of distinguished experts on bridges are utilized through suitable questionnaires.

Keywords: Bridge construction method, Multicriteria analysis, Earthquake resistance, Innovative intervention, Construction cost, Non-conventional
1. INTRODUCTION

Construction projects can be categorized into Buildings, Bridges and Special structures. Bridges are ranked second in terms of investment volumes, but first in terms of difficulty, compared to the other two types of construction projects. In the context of the design phase of a bridge project, there is a critical question about the selection of the most appropriate construction method. The reason for the difficulty in this decision lies in the fact that there are many different factors which must be taken into account, such as safety (earthquake resistance), economy, serviceability, durability, aesthetics, etc. Consequently, the compliance of the selected method with certain criteria should be ensured. It is also worth noting that these compliance criteria are usually conflicting, so a compromise is needed between them.

The five modern bridge construction methods include: 1) Cast-in-place, 2) Precast I-Girder, 3) Incremental Launching, 4) Balanced Cantilever and 5) the Advanced Shoring method. As far as their general principles are concerned, Cast-in-place is a conventional construction method for bridges with total height less than 10m, by using scaffolding that is stationary on the ground. In Precast I-Girder method the deck slab is cast in place on precast slabs, where steel reinforcement or sheets are embedded. According to Incremental Launching, the deck is constructed in segments from the one abutment to the other and is jacked forward to the final position with jacks. As for the Balanced Cantilever, this is a construction method where a structure is built outward from a fixed point to form a cantilever structure, without temporary support, using staged cast-in-place construction. Finally, with respect to the Advanced Shoring method, a launching girder moves forward on the bridge piers, span-by-span to allow placing of the cast-in-situ concrete, while the method can be applied both underslung and overhead [1].

The Egnatia Motorway in Greece, part of European route E90, is a 670 km mega-project extending from the western port of Igoumenitsa to the eastern Greek-Turkish borders at Kipoi. The project is of great interest in the field of bridge design, as it contains 177 major bridges (longer than 50 m) and many smaller ones, with a total length of 42 km. The large number of bridges can be explained by the diverse terrain crossed by the Egnatia Motorway together with the special environmental conditions encountered. It is also remarkable that all five modern construction methods were applied in the project for the construction of such a variety of bridges. Thus, the Egnatia Motorway presents the advantage of being a homogeneous sample of bridges which were constructed almost in the same period and with the same design principles.

The application of each one of the bridge construction methods uniquely results in a particular structural system, fact that also appears in the bridges of Egnatia Motorway. In particular, the Cast-in-place method results generally in a monolithic structural system, as the best option. The Precast I-Girder method results generally in a “floating” structural system on elastomeric bearings, or very rarely in a ductile system with stoppers that are activated during the earthquake. The Incremental Launching results always in a “floating” system; the Balanced Cantilever results in a monolithic system, while the Advanced Shoring method is generally associated with a ductile system.

As regards bridge construction, bearings and joints were unknown to bridge builders until well into the 20th century. The use of reinforced concrete and the resulting increases in span lengths necessitated these developments in order to reduce restraints and facilitate construction. Thus, it has been common practice for many years to use movable bearings and joints in bridges to respond to the displacements resulting from temperature, settlement, shrinkage, etc. Although bridges with bearings and joints may satisfy all design rules and
applicable codes, they are assured of subsequent deterioration, especially in marine climates or regions where they will be exposed to chloride attacks from road salts [2].

It is worth mentioning that in modern bridge design, and as far as the Cast-in-place method is concerned, the monolithic structural systems have displaced the joints and bearings (with the exception of the positions of abutments), which result in premature aging of the structure, and therefore to a large increase in maintenance costs [3,4]. In general, with a design process focused on minimizing constraint stresses, monolithic bridges are feasible today. Such bridges offer increased robustness compared to conventional designs with bearings and joints, and offer a multitude of approaches for improved design and aesthetics [2].

2. INNOVATIONS ON STRUCTURAL SYSTEMS OF BRIDGES

A brief look at the recent past of Bridges shows that four of the bridge construction methods, namely the Precast I-Girder, the Balanced Cantilever, the Incremental Launching and the Advanced Shoring method, in their era were considered to be construction innovations. Thanks to them, problems that could not be addressed by the conventional method (Cast-in-place) have been overcome, such as the great height of a bridge. This demonstrates the great importance of innovations for Bridges, something that does not apply to the same extent in Buildings. An essential definition of innovation could be the departure from the usual approach in dealing with a problem. In conceiving an innovation, in addition to the deep knowledge of the subject, imagination plays a very important role. It could be said that it is the driving force behind every innovation. Besides, there is also the French slogan of the May ‘68: "L’Imagination au pouvoir", which means “Imagination in power”.

An examination of the bridges of the Egnatia Motorway by category, based on their construction method, reveals the absence of innovations in their resulting structural systems. Therefore, a non-utilization of the possibilities offered by the activation of imagination is observed. That activation sometimes can even offer solutions such as the so-called “Egg of Columbus” solutions, with more cost-effective and better quality results. The hesitation of bridge designers could possibly be explained by their concern that the Contracting authority would prefer more conservative choices, which are very close to the exact implementation (by the book) of the relevant regulations, as providing the most secure result. Moreover, it is known that when a change or an innovative proposal first appears, it generally goes through three stages: In the first stage it is disapproved or even mocked. In the second stage it is combated, and finally, in the third stage it is adopted as self-evident.

In order to give a clearer picture of the type of innovations related to structural systems (and thus to the bridge construction methods), some of them are mentioned on a sample basis. These proposals have been received from the recent literature and concern bridges in Greece. Concerning the conventional (cast-in-place) bridges, there is the example of an innovative design approach which provides the ability of spanning long distances with integral bridge structures. This consists of the option of a curved in plan deck, in order to effectively deal with the long span related constraint effects, which are not easy to be undertaken in the case of rectilinear decks. When a superstructure is curved in plan and it can “escape” transversely, the constraint stresses are considerably smaller than in a fixed straight bridge, however horizontal moments occur [2]. A well-known example of this case is Sarantaporos River Bridge, a double-curve in plan integral bridge (in the shape of the letter “S”) with 6 spans of 35 m, and a total length of 210 m. The bridge, which is located near the city of Konitsa, was designed by the insightful engineer V. Kollias in the 60s.

As a second example of possibility for innovation, the proposal of attaining a monolithic system in a Precast bridge is mentioned. In this case, the monolithic connection of the deck
with the piers is achieved, without abandoning the advantages of a prefabricated bridge [5]. A third example of applying an innovation is the case of short span bridges (from 150 to 200 m) constructed with the Balanced Cantilever method, in which the use of reinforcement steel bars, instead of prestressed tendons is applicable to the bottom layer. Through this possibility, the risk of rupture of the bottom slab of the deck box-section, which may occur in cases of poor construction execution (i.e. when the cantilevers are forced to join at midpoint), could be alleviated. Finally, one last example of innovations in structural systems is a case that can substitute for the Advanced Shoring method, providing a monolithic outcome in a much more economical way and with unquestionable aesthetic advantages. This case is the construction method of self-launching formwork, which is proposed by Tegos, Tegou, et al [6].

3. CAPABILITIES OF INNOVATIONS ON EARTHQUAKE-RESISTANT BRIDGES

Seismic stoppers and elastomeric bearings constitute already established former innovations of earthquake-resistant bridges. The stoppers are a means of converting the structural systems of precast bridges into ductility systems. On the other hand, the elastomeric bearings are a means of seismic isolation for the “floating” systems resulting from the methods of Precast I-Girder and Incremental Launching. A third innovation that is in the final phase of research in order to be established in Bridges, but also in Buildings, is the Rocking structure design, which aims to the restoring of the structural system to its default position and also to eliminating the development of cracking to the piers due to seismic action. Thus, the use of Rocking in its various versions can drastically reduce or even eliminate the necessary repair works of the bridge after an earthquake [7].

The proposed innovations are certainly associated with the gaining of some advantages with respect to the compliance criteria applicable to the design of earthquake resistant bridges, such as seismic safety, economy, serviceability, aesthetics, etc., for which a detailed reference will be made below. Particularly for the economic criterion, it should be noted that it is usually related to three cost shares; namely the construction cost, the maintenance cost and the cost of repairing the damage caused by earthquakes during the lifetime of the project. Regarding these costs, it is known that monolithic structural systems have the minimum maintenance cost and the maximum cost of repairing earthquake damage, while the opposite is true for the “floating” structural systems. Furthermore, monolithic systems require less cost to achieve the same level of seismic safety, compared to “floating” systems. This is because the conventional lifetime of bridges is much shorter than the return period of design seismic action, as determined by modern seismic codes [8]. In addition, monolithic structural systems certainly have the advantage of the response of statically indeterminate structures. However, they are dramatically inferior to “floating” systems in terms of the conventional criterion of serviceability, and also in terms of the seismic criterion of regularity, concerning which the codes impose a heavy penalty [9].

According to the current seismic codes, the structural elements of bridges can be grouped into two categories: The first category includes the deck and the abutments, which are both not affected by the earthquake; while the second one includes the piers and the bearings, which are both affected by the seismic actions. Moreover, it is worth mentioning that the height of piers affects the magnitude of seismic deformations and of P-Δ effect. The necessary limitation of these two effects of earthquake is achieved only through costly and sometimes non-aesthetic choices of cross-sections. Consequently, a drastic limitation of them through another, less costly way would be very desirable. After the above clarifications, it is easy to understand the need to search for innovative proposals, in order to achieve both the relief of the structural members affected by seismic actions (piers) and the drastic reduction of seismic
movements. Achieving this dual goal is expected to have also a positive impact on both the
economy and the aesthetics [5].

The answer to the question of whether such innovative solutions can exist is affirmative,
but these are not expected to be conventional. These solutions are associated, on a case-by-
case basis, either with the exploitation of the two abutments [10,11], or with the exploitation
of the stiffness of the approach embankments at the ends of the bridge [12,13], or by
anchoring the deck to the ends of adjacent tunnels [14]. This exploitation is possible through
the extension of the deck slab on either side of the bridge, and with its connection to one of
the aforementioned members, depending on the case [15,16].

4. CRITERIA FOR THE SELECTION OF CONSTRUCTION METHOD

In the context of the search for the most suitable construction method in highway bridge
projects, which concerns the present study, it is possible that innovations on the structural
systems can play an important role. In fact, construction methods are also related to the
introduced innovations, although the latter ones concern the structural systems of bridges.
This is because these innovations can lead to the creation of new, independent (non-
conventional) construction solutions, which can be added to the existing alternatives of the
conventional construction methods and compete with them. Such an innovative solution,
inserted in the Multicriteria Analysis process (which will be explained in the next section),
could possibly affect directly the result of the selection of construction method.

As already mentioned in the Introduction, the selection of the most appropriate
construction method for a highway bridge project should be based on the evaluation of the
performance of the alternative solutions in terms of some certain compliance criteria that have
been set. In practice however, for years, almost exclusively the criterion of economy was
taken into account, with bridge designers considering the cost reduction as their top priority
for a bridge project. On the contrary, in the present study a more integrated approach to the
performance of construction methods is attempted. To that end, seven compliance criteria are
used, which are: safety, economy, durability, construction speed, serviceability, aesthetics and
environmental harmonization. A brief clarification on the concept and content of each
compliance criterion is provided below:

- **Safety**
  This criterion corresponds not only to the level of safety according to current codes (i.e.
  Eurocodes) for Bridges, but moreover to the additional level of safety resulting from the
  response of statically indeterminate structures. In countries with high seismicity, the
  term safety mainly corresponds to seismic safety.

- **Economy**
  The criterion is related to the intended reduction of the cost of the project, yet without
  devaluation of the rest of criteria. In the context of this study, the criterion of Economy
  is related only to the construction cost of the project, while the maintenance cost is
  taken into account in the criterion of Durability. It is noted that in the last years there
  have been continuous research efforts for the limitation of bridge construction cost
  [17,18,19,20,21].

- **Durability**
  The term Durability means the minimization of maintenance needs, during the lifetime
  of the project. In the case of bridges, maintenance needs are associated with the use of
  bearings, joints and seismic dampers. As mentioned above, the maintenance cost of a
bridge is reflected in the performance of the criterion of Durability, and not in the one of Economy. It is notable that sometimes maintenance cost could even reach the construction cost level [2,3,4].

- **Construction speed**
  The criterion of Construction speed is directly related to the expected completion time of the project, which is obviously intended to be minimized. The (average) construction speed is defined here as the ratio of the length of the bridge deck to the total construction time of the bridge (i.e. the deck, the abutments, the piers and their foundations). The aforementioned total construction time includes also the assembling, disassembling and transfer times of the necessary mechanical equipment used in the construction.

- **Serviceability**
  The usual concept of serviceability is linked to the quality of traffic service; however, in this study, the term “Serviceability” means the development of an adequate structural response to the imposed deformations (namely the expansion and contraction of the deck) during the operation phase of the bridge. It is noteworthy that the use of bearings is the best solution to the problem of serviceability.

- **Aesthetics**
  The concept of Aesthetics in the field of bridges includes certain established rules related to the choice of geometric dimensions, which attempt to reconcile safety and geometric proportions that contribute to an aesthetic effect. Some indicative rules are the following: All piers should have the same width, the variability of height of the deck cross sections (arc shape) is positively evaluated, etc.

- **Environmental harmonization**
  The criterion of Environmental harmonization is related to the existing architectural tradition of the area, as well as to the surrounding landscape of the bridge. These two factors should affect the selection of the deck type and the geometric dimensions, which must both be in harmony with them. Moreover, the concept of this criterion includes the desirable minimization of landscape alteration and environmental impact in the area caused by the bridge project.

5. **MULTICRITERIA ANALYSIS**

The tool of Multicriteria Analysis (MCA) is a particularly widespread application of Operations Research in decision-making. The objective of this method is the systematic and mathematically standardized effort to solve decision-making problems, where there is a number of alternative choices and many conflicting criteria are involved (such as those mentioned above, for the selection of bridge construction method). The aim is to achieve a rational compromise among these criteria, in order to make the optimal choice.

The Multicriteria analysis method used in this study is PROMETHEE. The acronym stands for “Preference Ranking Organization METHod for Enrichment Evaluations”. PROMETHEE is an outranking method that was developed by J.P. Brans in 1982 and further extended by Brans and Vincke [22], Brans et al. [23], while it is one of the most widely known and used MCA methods. Its characterization as an outranking method means that the method is based on pairwise comparisons of the alternatives. The ultimate goal of PROMETHEE is to provide the decision maker with a ranking of the existing alternatives.
Before applying the method, the decision maker needs at first to define the criteria taken into account for the decision. Then, all the alternatives to be ranked need to be evaluated according to those criteria [24]. Therefore, the evaluation table of the alternatives against the different criteria is the starting point of the PROMETHEE method. The implementation of PROMETHEE requires two additional types of information, which are: 1) the weights of the criteria under study, and 2) the preference function used by the decision maker when comparing the contribution of the alternatives in terms of each separate criterion. The preference function translates the difference between the evaluations (i.e. scores) obtained by two alternatives in terms of a particular criterion, into a preference degree ranging from 0 to 1 [25].

Consequently, the PROMETHEE method is based on the computation of preference degrees. A preference degree is a score which expresses how an alternative is preferred over another alternative, on the criterion considered, from the decision maker’s point of view. A preference degree of 1 thus means a total preference for one of the alternatives, a preference degree of 0 means that there is no preference at all, while if there is some preference but it is not total, then the intensity will be between 0 and 1 [24].

With regard to the mode of functioning of PROMETHEE, initially dual outranking relations are set up for the representation of the decision maker’s preferences, through pairwise comparisons of the alternatives against each criterion. These different relation-degrees are then used to set up a partial preorder (PROMETHEE I), or a complete preorder (PROMETHEE II) on a finite set of feasible solutions [26]. More specifically, according to Ishizaka and Nemery (2013), the method includes three main steps:

1. The computation of preference degrees for every ordered pair of actions (alternatives) on each criterion;
2. The computation of unicriterion flows (which are an aggregation of the criterion preference degrees, globally for an action);
3. The computation of global flows (which are an aggregation of the unicriterion flows, by taking into account the weights of the criteria).

In this way, the global positive, negative and net flows are computed. Eventually PROMETHEE II, which is used in this paper, provides a complete ranking of the alternatives from the best to the worst one, based on their global net flows [25].

6. CASE STUDY

A problem of selecting the construction method of a particular highway bridge in Greece is examined as a Case study, under two versions (solutions), the results of which are compared. The first version includes as alternatives the five currently existing construction methods, characterized as "conventional", while in the second version two more solutions, characterized as innovative, compete with the conventional ones. The alternative solutions in both versions are compared to each other by the Multicriteria analysis method of PROMETHEE. The Case Study aims to demonstrate the superiority of construction solutions which contain innovative interventions, even when multiple compliance criteria are used, and not just the economic one.

6.1 Conventional version

6.1.1. Data of the problem

The project that is examined in the Case study is the G4 Bridge of the Northern Road Axis of Crete, located in the Gournes - Hersonissos section of Heraklion Prefecture. The
construction of the bridge began in 2017 by the method of Balanced Cantilever and the project has the following data: It is a valley bridge; it is part of a National Road and has a total length of 440 m. This, according to the design of the bridge, is distributed in 5 spans as follows: $58.75 \, \text{m} + 3 \times 107.5 \, \text{m} + 58.75 \, \text{m}$. The bridge has a maximum height of 30 m. above ground, while its longitudinal slope is 4.272%. The deck of the bridge has a width of 13.25 m, which together with the pavements is 13.75 m. The bridge is in seismic zone II, while it is worth mentioning that there is the ability to place the piers densely. In addition, the corresponding equipment of all construction methods is available. The bridge is shown in Figure 1.

![Bridge of Case study, constructed by Balanced Cantilever method: (a) Plan, (b) Longitudinal section](image)

6.1.2. The feasible construction solutions

Based on the geometric data (total bridge length and maximum height of bridge above ground), it can be easily understood that, among the five existing construction methods, only the four of them are feasible, namely Precast I-Girder, Incremental Launching, Balanced Cantilever and the Advanced Shoring method. Apparently, the method that is rejected is the Cast-in-place, due to the large height of the bridge above ground.

6.1.3. Application of Multicriteria analysis

The first step of the process is the calculation of the weights of compliance criteria, which define their relative importance. These weights were calculated by utilizing another Multicriteria analysis method, the Analytic Hierarchy Process (AHP), which provides the relevant methodology. The required values for the calculation of the weights were obtained by a questionnaire survey addressed to 7 Greek experts in Bridge design, who were the same ones as in [1]. The resulted weights of the criteria are presented in Table 1.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Safety</th>
<th>Durability</th>
<th>Economy</th>
<th>Constr. speed</th>
<th>Aesthetics</th>
<th>Serviceability</th>
<th>Environ. harmoniz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0.220</td>
<td>0.179</td>
<td>0.172</td>
<td>0.147</td>
<td>0.124</td>
<td>0.081</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Table 1: Relative weights of the compliance criteria

Afterward, the PROMETHEE II method is applied, by using the relevant software of “Visual PROMETHEE”. In the context of this software, a scale of 1-9 was used for the
criteria, in order to provide the opportunity to reflect even the minor differences among the Alternatives’ performance against the criteria. All of the criteria need to be maximized, while the Preference function selected was the “Usual” because all criteria are qualitative.

The evaluation of the performance of each alternative construction method against each compliance criterion is presented in the following Table 2. It is noteworthy that the evaluations of the four Alternatives, with respect to each criterion, were derived from the above-mentioned questionnaire survey to the seven experts.

<table>
<thead>
<tr>
<th>Evaluations</th>
<th>Safety</th>
<th>Durability</th>
<th>Economy</th>
<th>Constr. speed</th>
<th>Aesthetics</th>
<th>Serviceability</th>
<th>Environ. harmoniz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precast I-Girder</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>9</td>
<td>4</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>Incremental Launching</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Balanced Cantilever</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Advanced Shoring</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2: Evaluation table of the four Alternatives against each criterion

The resulting positive, negative and net preference flows of the four alternative construction methods are presented in the following Table 3. The positive preference flow (Phi+) of an Alternative measures how much it is preferred to the other Alternatives, while its negative preference flow (Phi-) measures how much the other Alternatives are preferred to it. Finally, the net preference flow (Phi) of an Alternative is the balance between its positive and negative flows.

<table>
<thead>
<tr>
<th>Alternatives / Flows</th>
<th>Phi+</th>
<th>Phi-</th>
<th>Phi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precast I-Girder</td>
<td>0,3996</td>
<td>0,5744</td>
<td>-0,1748</td>
</tr>
<tr>
<td>Incremental Launching</td>
<td>0,4406</td>
<td>0,4602</td>
<td>-0,0196</td>
</tr>
<tr>
<td>Balanced Cantilever</td>
<td>0,6004</td>
<td>0,3726</td>
<td>0,2278</td>
</tr>
<tr>
<td>Advanced Shoring</td>
<td>0,4332</td>
<td>0,4665</td>
<td>-0,0333</td>
</tr>
</tbody>
</table>

Table 3: Preference flows of the four Alternatives

As a result, the Multicriteria method of PROMETHEE II indicated, according to the net preference flows, that the ranking of the four Alternatives, from best to worst is the following:

1. Balanced Cantilever
2. Incremental Launching
3. Advanced Shoring method
4. Precast I-Girder

Concluding, the method of Balanced Cantilever obtained the highest score among the 4 competing construction methods, with a fairly significant difference from the others, as it was the only one with a net preference flow that was positive. It is remarkable that the application of Multicriteria analysis confirmed the selection of bridge construction method that was made in practice for the project.
6.2 Non-conventional version

6.2.1 Introduction of two innovative solutions

In the second version of the Case study, two innovative construction solutions are selected, according to the specific data of the problem, and added to the existing four conventional alternatives. Thus, these two non-conventional solutions, which are both variants of the Precast construction method, are inserted in the Multicriteria Analysis process, which is performed again; this time having six alternatives. The two innovative solutions are presented below in general terms. For each one of them, the positive effects on the construction process and their advantages, in terms of the seven compliance criteria, are briefly described.

6.2.2 Precast construction method with a monolithic outcome

The first innovative proposal concerns a new type of prefabrication at concrete bridges. The goal of this proposal was to attain a monolithic system at the positions of piers. The result that emerged was better than the proposals so far and eliminated completely the bearings from the system. It solved also in a better way the problem of reinforcement at the support positions, by avoiding the “emerging” pretensioning tendons and using conventional reinforcement in these positions [27].

In fact, a new type of management of the problem of deck shaping was proposed, by using precast beams. Significant advantages are also achieved concerning the aesthetics and the economy. It is noted that the proposed utilization of the new type of prefabrication can substantially increase the length of the spans, which reaches up to 40m according to the existing practice. However, according to this proposal, this length can be significantly increased and therefore the number of the piers can be reduced. Moreover, there is a very important advantage in terms of earthquake resistance, which is the possibility of restraining the lateral seismic movements by utilizing the foundations (pile caps) of the abutments as seismic stoppers. This can drastically reduce the earthquake resistance cost of the resulting system [27]. The plan and the longitudinal section of the proposed bridge are presented in the following Figure 2, while Figure 3 provides an explanation of the design proposal.

![Figure 2: Bridge of Case study, according to the proposal of Precast Innovative Solution I: (a) Plan, (b) Longitudinal section](image)
The outcome of this innovative construction solution resembles the one of the method of Balanced Cantilever. In general, this solution appears to be significantly upgraded against almost all the compliance criteria. It should also be noted that the "disadvantage" of the large number of its piers (10 instead of the 4 of Balanced Cantilever) is greatly offset by its advantages over the entire construction process.

6.2.3. A solution of precast deck with slab connected to both abutments

The second innovative solution concerns the bridge decks of prefabricated girders and comprises the connection of the cast-in-place deck slab to both abutments, without the two end joints. The main objective of this proposal is the improvement of the earthquake resistance of the structure, mainly along the critical spanning direction of the bridge [28]. Nevertheless, while the connection of a deck slab to the abutments solves the seismic problem, it induces in severe way serviceability issues for the deck slab which will be integrally connected to the two abutments. Moreover, large deformations developed in the abutments, which increase their cost significantly, shall be handled [29]. Both consequences can be accommodated with an innovative altered precast solution as described in [28]. This proposal of the connection of the deck slab to both abutments avoids also the use of an expensive seismic isolation system (elastomeric bearings). A typical abutment of the innovative configuration of the precast bridge is depicted in Figure 4.

Although the design proposal seems to be the same as the conventional Precast construction method, nevertheless it is superior in many areas, such as: The complete absence of joints, the drastic reduction in the number of bearings and the almost complete elimination of the differential seismic movements. This elimination of the differential seismic movements allows the use of solid wall-section piers despite the large height, instead of using hollow-section piers, the construction of which is costly and time consuming. Through these solid wall-section piers, the disadvantage of the large number of 11 piers, instead of the 4 piers of Balanced Cantilever and the 10 of the first innovative solution, is offset. All of these
advantages of the second innovative proposal have a positive impact especially on earthquake resistance, economy and construction speed of the bridge.

6.2.4 Economic data of the two innovative solutions

Regarding the construction costs that determine the performance of the two innovative solutions in terms of the criterion of Economy, the diagrams of Figures 5 to 10 are provided. These diagrams show the quantities of used materials, the total construction costs, the construction costs per square meter, and also the construction costs per structural member. The estimated costs and quantities of the two innovative versions of the Precast construction method are presented in comparison with the actual ones of the bridge that was constructed by the method of Balanced Cantilever.

Figure 5: Volume of concrete (m$^3$) required for the three construction methods under consideration

Figure 6: Weight of loose reinforcement (kg) required for the three construction methods
Figure 7: Weight of prestressing steel (kg) required for the three construction methods

Figure 8: Total construction cost for the three construction methods

Figure 9: Total construction cost per square meter (€/m²) for the three construction methods
6.2.5 Second application of Multicriteria analysis

The MCA method of PROMETHEE is implemented for once again, this time containing six alternative solutions, which are the four conventional construction methods of the first version of Case study and the two aforementioned innovative solutions which have been added.

The weights of compliance criteria have already been calculated in the first version of the Case study. The PROMETHEE II method is applied in exactly the same way as before, using the software of “Visual PROMETHEE”. Consequently, the new evaluation table of the six alternative construction methods with respect to each compliance criterion was created, exploiting again the opinions of the seven experts in Bridge design (Table 4).

<table>
<thead>
<tr>
<th>Evaluations</th>
<th>Safety</th>
<th>Durability</th>
<th>Economy</th>
<th>Constr. speed</th>
<th>Aesthetics</th>
<th>Serviceability</th>
<th>Environ. harmoniz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precast I-Girder</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>9</td>
<td>4</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>Incremental Launching</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Balanced Cantilever</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Advanced Shoring</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Innovat. Precast I</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Innovat. Precast II</td>
<td>8</td>
<td>6</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4: Evaluation table of the six Alternatives against each criterion

Eventually, PROMETHEE provides the resulting positive, negative and net preference flows of the six construction methods, which are presented in Table 5.
Therefore, the ranking of the six Alternatives, from best to worst is the following:

1. Innovat. Precast II
2. Innovat. Precast I
3. Balanced Cantilever
4. Precast I-Girder
5. Incremental Launching
6. Advanced Shoring method

In conclusion, the “Innovative Precast II” (i.e. the one of precast deck with slab connected to both abutments) obtained the highest score among the others and is deemed thus selectable, with a very slight difference from the second one, which is “Innovative Precast I” (namely the precast construction method with a monolithic outcome). It is also remarkable that the implementation of Multicriteria analysis resulted in an indisputable superiority of the two innovative solutions considered in this study, over the conventional ones. In addition, in this second version on the Case study, the rest of the construction methods retained their original ranking, with the exception of Precast I-Girder, which went up to the 4th position (having the same score with Incremental Launching, and being above the Advanced Shoring method). This happened due to the phenomenon of rank reversal which sometimes occurs in the PROMETHEE, as well as in other MCA methods, when a new alternative or more are added in the original set of Alternatives. Nevertheless, in the present study, this minor change did not affect in fact the final result of selection.

7. CONCLUSIONS

The study aims to highlight the importance of innovative interventions in the structural systems of earthquake-resistant long concrete bridges. At the same time, it deals with the problem of prioritization of the possible construction methods in a bridge project, and eventually with the selection of the most appropriate, per case, among them. To that end, the use of seven compliance criteria is suggested, while the tool of Multicriteria analysis is utilized for the decision making. The study focuses on a specific Multicriteria analysis method (PROMETHEE), analyzing its main principles and its mode of functioning. The critical problem of the evaluation of the performances of the construction methods against each compliance criterion was addressed through the answers to appropriate questionnaires of a number of experts on Bridge design and construction. The main conclusions of the research are the following:

- There is a unique relation between a construction method and the resulting structural system.
Despite their simplicity (compared to the spatial complexity of the corresponding multi-storey building systems), and maybe exactly because of this simplicity, the structural systems of bridges can be subject to various innovative configurations. It is also remarkable that these configurations sometimes are related to a low, if not negligible additional cost, while they could upgrade not only the resulting earthquake resistance, but also the economy, the durability, even the aesthetics of the outcome too.

As demonstrated by the second version of the Case study, if a construction solution including innovative interventions is inserted in the Multicriteria analysis process, it could possibly obtain the highest score among the other competing alternatives. This can happen due to its upgraded qualitative characteristics, in comparison with the corresponding conventional construction method from which it originates. Thus, it could change dramatically the result that a usual implementation of Multicriteria analysis (only with conventional alternatives) would have. This would lead of course to a different hierarchy of the alternative construction methods and therefore to a different selection.

In the context of application of Multicriteria analysis, the calculation of the weights of criteria is a crucial issue, since it can affect decisively the process of the selection of construction method, and even define the final result to a certain extent.

The monolithic connection of the deck of a bridge with the piers or the abutments, combined with the complete, if possible, elimination of the bearings and the joints, constitutes the optimal solution for the structural system and therefore for the corresponding construction method, under the condition that the emerging problems with regard to serviceability are effectively addressed.

Concerning the bridge construction methods:

a) The Precast construction method and the Incremental Launching provide both a “floating” structural system and hence, the use of seismic isolation is advisable for the seismic protection of the resulting structural systems.

b) The Cast-in-place, the Balanced Cantilever and the Advanced Shoring method provide seismic safety based on ductility.

The total or significant reduction of the differential seismic movements of the bridge gives the desired possibility of an economical choice of the piers cross-section, due to relieving the piers from the seismic load effects and reducing the P-Δ effect. This unrestricted choice of the piers cross-section makes it possible not only to reduce their cost, but also to increase their construction speed and upgrade the aesthetic effect.

The possibility of innovative interventions on the piers cross-section gives the advantage of avoiding the heavy penalty that the seismic codes impose in cases where there is not regularity.

Concerning the first innovative proposal, it is remarkable that the option of reducing the seismic effects by using stoppers onto the abutment pile cap results to reducing the seismic loads undertaken by the piers, something which has a positive impact on safety, economy, construction speed and aesthetics.

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INTEGRAL BRIDGE DESIGN FROM THE UK HIGHWAYS PERSPECTIVE

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Abstract

This paper aims to illustrate the current design procedures adopted by Highways England for the design of integral bridges. This procedure is proposed within the Design Manual for Road and Bridges (DMRB). PD 6694-1:2011_Recommendations for the design of the structures subject to traffic loading to BS EN 1997-1:2004 [1] will be presented and discussed, with reference to the Guidance on Integral Bridge Design and Construction_Commentary on PD 6694-1 of 2011 [2].

A recent example of the application of the abovementioned procedures in a project located in England will be then presented. The design of a single span integral bridge with precast prestressed concrete deck and sheet pile walls will be used to illustrate some critical aspects of the procedure. The introduction of the Eurocode together with the adoption of integral bridge solutions is resulting in some impact on the precast prestressed beams industry practice. This will also be presented and discussed with reference to previous applications according to the British Standards now superseded.

Finally, some statistical data regarding integral bridges realisations within major motorway schemes currently under construction will be presented to highlight the significant impact that this typology currently has on the new UK Highways bridges stock.

Keywords: Integral Bridge, Highways England, design standards, design procedure, design guidance.
1 INTRODUCTION

Although the concept of integral bridges is currently well known, and its advantages widely recognized, the adoption of this typology is still very limited in the southern European countries. As emerged in many international conferences and research activities on the topic [3], the lack of specific standards is perceived as one of the key factors limiting a wider adoption of this bridge typology.

In the United Kingdom there is currently a different trend, especially within the Highway sector, that represents a highly regulated environment. For Highways England, currently the adoption of the Eurocode together with the UK National Annex represents only the starting point for the design on new structures. In fact, back in 1992, Highways England introduced and since then adopted the Design Manual for Road and Bridges (DMRB), a series of 15 volumes providing standards, advice notes and other documents which all the supply chain is required to comply with. DMRB covers much more than technical design standards, regulating topics like approval procedures, assessment of existing assets, inspection and maintenance, and much more from a highway perspective where structures like bridges represents only a marginal part of the whole asset.

BD 57/01, Design for Durability [4] indicates:

“bridges with lengths not exceeding 60m and skews not exceeding 30° shall be designed as an integral bridge”.

This recommendation has now become a sort of application rule in the UK, not only in the highway sector.

Highways England recognizes the value of constructing integral bridges, especially to minimize the long-term maintenance liability. Therefore, it aims to increase the percentage of integral bridges built and aims to ensure integral bridges are built in a consistent, safe, durable and cost-effective manner. As part of a general strategy, many research activities have been commissioned during the last two decades and the beneficial effects of those actions are tangible.

2 PD 6694-1:2011 PROCEDURE FOR INTEGRAL BRIDGE DESIGN

Before entering the details of the design procedure proposed by Highways England it is important to note one of the distinctive aspects of the design of integral bridges in UK: the recognition of a specific soil effect affecting granular soil (only) called “Strain Ratcheting". The standard defines this as follows:

“For integral bridges which are subject to many thermal cycles, the repeated backward and forward movement of the abutment generates pressures when the bridge is expanding which are significantly higher than those that would occur with a single thermal cycle. After many cycles, this pressure tends to a maximum value with a pressure coefficient of K*. K* is dependent on the total movement of the end of the deck from its maximum contraction position to its maximum expansion position”.

This effect results in a progressive year-on-year increase in soil pressure due to cyclic movements and has been recognized by different researchers in laboratory tests. The two procedures presented below both aim to include the soil ratcheting effects in the design.

PD 6694-1 Clause 9.2 identifies two possible methods of analysis for integral bridges:

• Limit Equilibrium methods, consisting basically of the application of pre-defined earth pressure distributions representing an upper bound of the actual allowable earth pressures
• Soil Structure Interaction, consisting of the explicit soil modelling to capture interaction with the structural elements.
The standard also defines limits of applications of both analysis methods, so limit equilibrium methods can only be adopted where:

a) the characteristic thermal movement of the end of the deck does not exceed 40 mm;

b) the skew does not exceed 30°; and

c) the depth of soil affected by the abutment movement can be identified without recourse to a soil–structure interaction analysis, for example, abutments founded on spread footings and end screen abutments.

Soil structure interaction is the identified suitable analysis method for structures where limit equilibrium methods are not appropriate. Those are:

1) abutments founded on a single row of piles;

2) embedded wall abutments;

3) over-consolidated backfill material;

4) cohesive soils; and

5) layered soils.

Moreover, in presence of granular soils, where the standard recommends the soil structure interaction analysis to be performed, this also requires account to be taken of:

- the non-linear response of the soil to deck expansion and contraction;
- the effect of strain ratcheting on soil properties, which may be based on 120 cycles with an amplitude of \(d_k\) (characteristic value of the movement);
- variations of soil properties at different depths;
- the degree of compaction of the soil;
- the rotational and axial stiffness of the deck;
- horizontal soil arching between the piles;
- the staged application of the thermal effects;
- the early and later life of the structure; and
- the envelope of possible combinations of minimum earth pressures with maximum expansion and maximum earth pressures with minimum expansion (this refers to the need of considering upper and lower bound for soil properties and favourable and unfavourable gamma factors for actions).

PD 6694-1 is the final result of a series of research commissions awarded by Highways England (Highways Agency at that time) to Ove Arup & Partners in conjunction with Parsons Brinkerhoff and Transport Research Laboratory (TRL). ARUP has defined a numerical soil model for integral bridges, and the software Oasys Frew released by the company does include that, together with the implementation of the method proposed for full soil structure interaction. The author is not aware of other software currently implementing the procedures required by PD 6694.

The Guidance on Integral Bridge Design and Construction_Commentary on PD 6694-1 of 2011 provides some background on the geotechnical numerical soil model developed by ARUP, a commentary section on Clause 9 of PD 6694-1, Integral Bridges, a section regarding the method for designing integral abutments using a soil-structure interaction analysis and two worked examples of soil structure interaction analysis developed with the software Frew. The use of this software represents an established and approved method, adopted by many of the major design firms in the UK market.

Regardless the method of analysis adopted, the last two bullet points above represent a considerable complication in terms of analytical effort for integral bridges design. In fact, the earth pressure factors \(K^*\) (between at rest \(K_0\) and passive \(K_p\) values, to include soil ratcheting effects) are dependent on the values of the design movements \(d_o\), which is defined as:

\[
d_o = \frac{1}{2} d_k (1 + \psi \gamma_Q) \tag{1}
\]
where, for the combination of actions under consideration:

\( \gamma_0 \) is the partial factor for thermal action

\( \psi \) is 1.00, \( \psi_0 \), \( \psi_1 \) or \( \psi_2 \) for thermal actions as appropriate (see BS EN 1990:2002+A1, 6.3.1), and

\[
d_k = \alpha L_x (T_{e;\text{max}} - T_{e;\text{min}})
\]  

(2)

where:

\( \alpha \) is the coefficient of thermal expansion of the deck

\( L_x \) is the expansion length measured from the end of the bridge to the position on the deck

that remains stationary when the bridge expands

\( T_{e;\text{max}} \) and \( T_{e;\text{min}} \) are the characteristic maximum and minimum uniform bridge temperature components for a 50-year return period given in the UK National Annex to BS EN 1991-1-5.

In fact, this results in a number of sets of earth pressures calculated for each design approach combination (Design approach 1 combination 1 and Design approach 1 combination 2 or Set B and Set C combinations in Eurocode terminology), those need to be calculated for combinations where temperature action is the leading action, and for combinations where temperature action is the accompanying action. Then, upper and lower bound of soil characteristics need to be considered, favourable and unfavourable partial factors need to be considered and the early and later life of the structure’s properties need to be considered as well (for example uncorroded and corroded sections for sheet pile walls). This results in several permutations that considerably enlarge the design effort required even for small structures, like the one illustrated in the following paragraph.

3 PROJECT EXAMPLE

This project includes 3 new bridge structures, of which 1 is an accommodation bridge and 2 are road bridges. Given the small spans (less than 30 m) all of them have been designed to be integral structures. Being bridges that are required to span over a new build canal, the project represented a perfect opportunity for a top down construction method. Sheet pile walls (steel) have been preferred to bored secant concrete piles for retaining walls, with no exception on the bridge locations, where sheet pile walls have been then integrated with a cast in situ external concrete layer and a capping beam on top to form the bridge abutment. The deck has been designed with precast prestressed concrete beams as a fast, cost effective and durable solution. A few details of one of the structures are shown in the figures below:
Soil structure interaction has been developed by the geotechnical team with software Frew, while the superstructure has been designed by the structures team with a 3D grillage model with the software MIDAS Civil. Given the high number of permutations required, and the continuous exchange of data between geotechnical team and structural team, the process resulted to be very time consuming and therefore not very efficient. In the author’s opinion, the design procedure should be tailored to the complexity of the structure, and given the nature of the structure presented, this approach was basically overcomplicated.

4 IMPACT OF EUROCODE ON PRECAST PRESTRESSED CONCRETE BEAMS MARKET

As shown, the internal beams are so called M4 precast prestressed beams, and the edge beams are so called UM4 precast prestressed beams. More precast concrete manufacturers (for example Shay Murtagh or Banagher) adopt these beams in the UK, and those have been used for long time within the bridge market.

After the adoption of the Eurocode some of those typologies (like the UM beams used in the Oxford project) have been slowly superseded given some additional difficulties in complying with the standard, especially when used for integral bridges.

The issue regards the introduction of the SLS check for decompression stated by the Eurocode and not previously included in the British Standards.

Table NA.2 of National Annex NA to BS EN 1992-2 [5, 6] says:

“The decompression limit requires that all concrete within a certain distance of bonded tendons or their ducts should remain in compression under the specified loading. The distance within which all concrete should remain in compression should be taken as the value of $c_{\text{min},\text{dur}}$. Where the most tensile face of the section is not subjected to XD or XS exposure but another face is, the decompression limit should require all tendons within 100mm of a surface subject to XD or XS exposure to have a depth $c_{\text{min},\text{dur}}$ of concrete in compression between them and surfaces subject to XD or XS exposure”

For integral bridges, the negative bending moments over the abutments are generally inducing tension in the upper part of the vertical webs of the UM beams. For bridges decks the exposure classes XD or XS are common, therefore the requirement should be satisfied. Given the small width of the web and the possible position of the strands, this requirement is generally very hard to be achieved, therefore there is currently an increasing tendency to adopt solid edge beams and UM beams are slowly disappearing.
5 SOME FIGURES REGARDING INTEGRAL BRIDGES IN CURRENT UK MAJOR SCHEMES

The research activity founded by Highways England included a desk study review of integral bridge usage in the UK market. The research shows some figures regarding integral bridge realizations since the introduction of BA 42/96 in UK [7]. The implementation of this code in 1996 lead to a dramatic increase in integral bridge construction, comprising 8% of bridges constructed in the first half of the 1990s, rising to 65% of all bridges constructed between 2000 and 2005.

The trend is still increasing as currently shown in some of the major work-in-progress highways schemes in the UK where the author is currently involved. A couple of examples are described below, without specific references for confidentiality reasons.

The first one, is a main strategic route between London, the West of England and Wales. It connects people, communities and businesses, carrying on average 130,000 vehicles per day and is prone to congestion. The project will upgrade the existing highway to a smart motorway which will provide much needed capacity and support the economy, facilitating economic growth within the region. An additional lane will increase capacity and reduce congestion, and more technology on the road will be provided to smooth flows and manage incidents. Within the scheme, two major overbridges are planned to be widened, and eleven bridges over the motorway need to be replaced. Of those, 7 overbridges are single span bridges and all of them are designed to be full integral abutment bridges, 3 of them are 3-span overbridges and all of them are designed to be semi integral abutment bridges, and the last one is a bridge-way bridge designed as a simply supported steel truss. So, for this scheme 90% of the new bridges will be integral bridges.

The second scheme is a dualling project to upgrade 20km of road in a very complex geomorphological environment. The contract to design, construct, finance, operate and maintain the road includes an upgrade from a three-lane single carriageway to a dual two-lane carriageway, and the creation of six new junctions. Major geotechnical works will be undertaken, mainly soil nailing, around 40 culverts will be built and kilometres of retaining walls will also be required. Regarding bridges, 6 new footbridges and 8 new road bridges will be built. Of those, all 6 steel footbridges (some straight and some curved) will be integral with the abutment, and 5 of 8 bridges will be fully integral with the abutment. The only exceptions are 2 composite girder bridges (for very unfavourable foundation conditions) and 1 major long span arch bridge. So, for this scheme 78% of the new bridges will be integral bridges.
6 CONCLUSIONS

- The paper presents the design procedure adopted by Highways England for the design of integral bridges. Two design approaches are presented: limit equilibrium methods and soil structure interaction. Limitations for the application of the first method are presented, while the more complex second approach covers the majority of structures and typologies.

- Soil structure interaction analysis imposes the adoption of advanced geotechnical finite element software. In the authors knowledge, only one software currently covers the British Standard requirements, and this represents a strong limitation.

- The necessity of combining more analysis software's represents a complication in the design process, as more professionals from different disciplines are generally involved and this creates unnecessary interfaces and a possible source of mistakes.

- The number of permutations required to design an integral structure according to the presented procedure is high and may result in excessive design effort especially when applied to the design of small span bridges.

- Highways England’s desire of design process simplification through the production of guidance still requires refinement.

- Some statistical data regarding integral bridge usage in the UK has also been presented, demonstrating the increased interest on this structural typology as a result of the introduction of specific standards within the Highways sector. Other countries may follow similar approaches for increasing the adoption of integral bridge solutions, to obtain more durable and less maintenance demanding bridge assets.

REFERENCES


TOWARDS ACCELERATED CONSTRUCTION AND COST REDUCTION OF MONOLITHICAL BRIDGES FACING EARTHQUAKE HAZARD

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Abstract

The aim of the present study is the reduction of construction time and cost of a monolithical bridge with tall piers. The optimum combination of the criteria of safety, economy, construction speed, serviceability, durability, aesthetics and environmental harmonization is difficult to be achieved in such bridges, since factors like local geomorphology and feasible construction methods should be considered. In areas of high seismicity, seismic hazard is also added to these factors, affecting especially the piers. The introduction of high pier flexibility can address seismic hazard, since it increases the structure’s fundamental period drastically and reduces the seismic forces on piers. High pier flexibility also addresses serviceability demand of the bridge. Thus, a novel method to increase pier flexibility is explored herein. The proposed modification is applied as an alternate solution in a R/C bridge of Egnatia Motorway, which is used as the case study. FE models of the benchmark and the modified bridge are developed in a robust analysis software and nonlinear time history analysis is performed to investigate their seismic performance. Experimental tests were conducted for the verification of the decrease in pier stiffness and the experimental results were used to increase the reliability of the proposed simulation. To arrive to an optimum solution with emphasis on the construction speed and cost, three alternatives, i.e. the modified bridge and two different conventional construction methods, are investigated regarding their compliance to the seven aforementioned criteria. The study is conducted using Multi-Criteria Analysis. Based on the analysis results, the differences between the alternatives are highlighted and the optimum construction solution is determined.

Keywords: Reinforced Concrete Bridges, Multicriteria Analysis, Flexible Piers, Seismic Resistance, Construction Speed, Construction Cost
1 INTRODUCTION

The complexity and the increasing demand on a wide range of projects have turned the need of managing construction time and cost to a first priority. For achieving the desired quality level in a specific time-frame all project participants from design phase to construction should cooperate efficiently. The term accelerating construction refers to the completion of a project to the required quality level by reducing the time needed. Increasing construction speed is a critical challenge and the reasons that create this need often lie mainly in the nature of the project [1]. Some first attempts to improve construction speed were made in the field of building structural systems [2-3] where the use of unbonded pre-stressing, the removal of slab beams and the use of high-performance concrete achieved a 4-day concreting cycle rate per floor in a 25-storey hotel in Stuttgart. Along with constructing buildings faster, bridges are also structures that the scientific community focuses on their fast construction since they are vital for day to day commuting and transfer of goods. The fast delivery of new bridge projects can boost economic growth and the demand for further aerial interconnections have increased the need for new bridge projects that can be delivered rapidly, safely and economically.

In the bridge sector when considering the conventional methods, e.g. cast in situ, precast I-girder placement, incremental launching, balanced cantilever [4]; the incremental launching can be considered as the fastest practice, with 4 workman-hours/m3. However, there is still plenty of room for improvement for the rest of the methods, especially for the most common methods the cast in situ and the precast I-girder bridges. Researchers and practitioners in bridge engineering seek also different technologies and innovative construction methods for fast and economic bridge construction, like Accelerated Bridge Construction (ABC) method [5], which involves the introduction of precast piers in the bridge structural system.

In the frame of the present study a novel method in bridge design that reduces construction time and the optimum selection of the best construction practice on a case study bridge is investigated. The method includes interventions to both the design of the deck and piers that transforms the construction procedure of the bridge. Regarding the formation of the piers, the concept originates from a previous research work by Tegos & Pilitsis [6] for the reduction of pier stiffness. The conventional pier cross section is replaced with a bundle of smaller columns, symmetrically arranged, statically independent, circular cross-sections that can be purged with paper cardboard for formwork. Since bridge flexibility is increased, the proposed method can contribute also to improving bridge regularity, as shown in the case study. Regarding deck, a slab type cross section is used to allow for easier installation of prestress tendons and reinforcement bars [7]. A six-span bridge is selected as the case study, the proposed methods are analytically explored and finally multi-criteria analysis is applied to decide among different construction methods for the optimum when considering speed and economy as high priority.

The desired feature of increased construction speed is associated with the importance of the road on which it is built. This means that in small-scale bridges such as those belonging to provincial roads, the problem is asymmetrical, with the criterion of increased project execution speed being upgraded to first priority, together with the other supercritical constructability, but this does not mean to neglect the remaining compliance criteria that are always in place, such as safety, durability, economy, serviceability, aesthetics and integration into the environment. On the contrary, on a bridge of that is characterized as significant belonging to a Highway, the problem is more symmetrical, with the construction speed criterion at the same level as the other above-mentioned compliance criteria. It is noted that the three seismic significance factors may be extended to the quality of the bridges, which is determined by the degree of fulfillment of these compliance criteria.
NOVEL APPROACH FOR ACCELERATED BRIDGE CONSTRUCTION

It should be noted that the present approach focuses on the possibility of reducing the construction time of bridge structures constructed with the conventional Cast in Situ Method. It is well known that the conventional Cast in Situ method is applied in the case of bridges of small height from the ground and is carried out on a scaffolding installation starting from the ground and reaching the top of the bridge. From the theoretical point of view, there are no issues related to the construction process when the intermediate work joints are placed in the proper locations, Fig.1.

![Critical joint location](image)

Figure 1. Critical joint location

It is noted that the formwork may not be used over the entire length of the deck but is usually interrupted in the non-critical locations. The pre-stressing installation is performed segmentally and not one-off from the edges of the deck (bi-lateral pretension). The first aim of this application is to reduce cost in scaffolding and the second is to avoid large losses in pre-stressing. The cost of the construction of the formwork is generally very important as it sometimes reaches 50% of the total cost of a concrete construction. Also, the cost of the formwork together with the corresponding installation and disassembling work, which is even larger than the cost of the formwork itself, may exceed the total cost of the concrete and reinforcement materials and their respective work. Sometimes, the cost of the materials of the formwork is only a small percentage of all the costs of the formwork. Therefore, it is self-evident that the prerequisite for reducing the cost of construction is the use of fast-installing and disassembling formwork systems. Reasonably, if we associate this time reduction with the intended construction speed of the project, an easy construction structure is required which reduces working time and leads to time and cost savings. The most appropriate approach to achieving this goal is the flat slab, without beams, and the avoidance of the box-like cross-section of the deck. Formwork on such cross-sections (flat) is much easier and faster than in the case of Fig. 2. since the segments do not have flatness interruptions (transverse interference) as no beams and ribs are present. They are thus assembled and removed very easily, accelerating the pace of process development and reducing labor costs together and improving manufacturing speed.
At the same time, recourse to a plate-type deck allows for the use of straight reinforcement bars and a more comfortable placement of pretension tension tendons. However, the requirements of large plate thicknesses to deal with the large spans result in large weights which neutralize the previously mentioned advantages. To modify the system, the variability of the cross-sections within the spans is proposed.

Figure 2. Formwork for box deck cross-section, source [8]

More specifically, for the usual cases of panel openings from 30 to 40 m, the central 1/3 has a fixed section thickness of from 0.75 m to 1 m, while the end portions are of a variable thickness with a maximum value in the supports from 1.5 to 2 m. In this way, it is thought that it is possible to deal with the problem of the large demand in concrete and with the high stresses of
the same weight. In addition, the areas of the large thickness of the slab are very close to the supports of the piers which contributes to addressing the resulting stresses.

The recurrent problem of reduced stiffness of the spans and the resulting increase in deformations is addressed with pre-stressing. Finally, it is pointed out that the selected variability of the cross sections in the spans transfers the large moments from the middle to the supports. The case study includes a more detailed presentation of the proposed method.

Herein, a way of improving the construction speed by varying the cross section of the piers is also presented. A novel formation of the piers is proposed. The concept originates from a previous research work by Tegos and Pilitsis (2014) [6] and Markogiannaki et al. (2018) [9] for the reduction of pier stiffness. Therefore, the proposed system aims also at altering the flexural stiffness of piers and improving bridge regularity when dealing with irregular bridges. The proposed column approach includes the following design aspects: Smaller circular cross-sections, equal in diameter, replace large solid cross sections either circular or rectangular. Herein a rectangular cross section of piers is replaced in the case study. The schematic representation of the proposed configuration is illustrated in Fig. 3. In this way the massive solid piers are segmented and have increased flexibility that improves serviceability respond and addresses seismic demand.

The proposed pier type is easy to construct, as it is possible to use paper disposable formworks (cardboard), Fig. 4 that are widely available and of low cost in current construction practice. Aesthetics are also taken into account, as the new column configuration has architectural characteristics that do not compromise aesthetics.
Regarding the structural response of the smaller circular columns buckling issues shall be taken into account. Therefore, a criterion is determined regarding the minimum column dimensions. The criterion relates the Euler buckling load to the upper bound of the normalized axial force due to ductility demand. Therefore, applying this rationale and taking into account a value of $\eta_k=0.30$ for the normalized axial force, in the case of the longitudinal direction, the critical cross section is calculated [6]. Furthermore, P-Delta effects have to be considered in the selection of the appropriate column cross sections and column reinforcement is calculated from critical seismic and non-seismic load combinations.

3 CASE STUDY

3.1 Description of the Case Study Bridge

The case study bridge is a concrete bridge representing irregular bridges with characteristics that are common in modern motorways in Europe. Specifically, it is the Arachthos-Peristeri Bridge, located in the western section of Egnatia Motorway in Greece. It has six spans (34m+4x43m+34m) and a total length of 240m (Fig. 5). The cross section of the deck is box-girder and has a total width of 12.90m (Fig. 5). The deck is rigidly connected with the five piers and is seated on both of the abutments on sliding bearings. Abutments’ bearings have a circular section with diameter equal to 600mm. There are also a shear keys for the transverse direction with a load capacity of 176ton. The piers have rectangular cross section with dimensions of 5.00mx1.50m. The varying pier heights are shown in Fig. 5. The bridge is founded on ground of type B according to Eurocode 8 and design ground acceleration used in the final design was equal to 0.16g [11]. The importance factor adopted was equal to 1.30 and the behaviour factor (q-factors) was equal to 3.50 for the longitudinal direction without taking into account the reduction due to irregularity. The bridge from a serviceability point of view has a significant advantage, since the piers at the end spans are tall, while in most cases these piers are the shorter ones. This configuration is beneficial to addressing serviceability demand. However, this advantage in serviceability does not reflect in seismic response. Therefore, it is necessary to find a way to improve the flexibility of the three internal shorter piers to improve the distribution of seismic forces to all piers. The proposed solution herein attempts to address this issue. A 3-D finite element bridge model, Fig. 6, was generated in the analysis software OpenSees, [12]. Bridge components were modeled as frame elements; the deck as elastic beam elements and the piers were modeled accounting for material nonlinearities. Section analysis was conducted for the assignment of concentrated plasticity (hinges) at the top and bottom of piers with the software AnySection v4.0.6, [13]. This model is called as the benchmark solution in the following sections.

![Figure 5: Monolithic irregular bridge under study](image_url)
3.2 Study on the Modified Bridge with the Novel Approach

Figure 7 gives the new deck characteristics for the same bridge based on the proposed innovative approach with flat slabs.

Figure 7. Novel deck approach (a) bridge deck elevation, (b) detail at a middle span (c) detail at an end span
The bridge was studied alternately and based on the data in Figure 7, and the comparison of the results of the two deck cases the following were derived:

- Due to the drastic reduction of the slab thicknesses which replaced the fixed height cross-section, concrete sizing appears to be close to each other, with a difference of 10% in favor of the box cross-section, which is however balanced by reducing the reinforcement shown by the other option.

- The above results arose with the assumption of the variation of limited pretension in both cases. However, as the finding (which was, moreover, expected) that comparatively the supports are more demanding than the middle of the deck spans, it is possible to manage the pre-stressing problem, under the condition of acceptable transverse deformations: Instead of the overall limited pre-stressing in the spans and supports, limited pre-stressing should be applied only to the spans, while in the supports partial pre-stressing can be performed. This would obviously serve both the economy and the construction process.

- To achieve the optimal result in terms of the intended construction speed, it is necessary to coordinate the three phases which contribute to the completion of the final result of the construction: the formwork assembling-disassembling, the reinforcement installation and the concreting. Firstly, comes the formwork assembling, then the reinforcement is placed, followed by the concreting. It is understood that on bridges of relatively short length the three construction phases will terminate at the end of the bridge end if not at the same time, at close times, whereupon the tendons will be tensioned after the necessary degree of concrete hardening (fast cure) concrete. In the case of relatively longer bridges, it is necessary to stop the concreting at an intermediate position of the length, preferably at the point of flexion of the wiring and the tendons which will then be continued by a plug connection. Then, the formwork disassembling of the already pre-stressed section is followed and then continues in the same manner. In the case of satisfactory co-ordination in the three phases, the progress of the construction of the deck is expected to exceed the corresponding case where the carrier was constructed by the precast or the conventional cast in situ method.

The second intervention in the benchmark bridge, to the piers, was also investigated. The pier in Figure 4 represents the cross section that replaces the rectangular cross section of the piers of the bridge. The pier is rectangular at the first 6m and the smaller cross sections continue at the rest of the height, Figure 8. The pier bridge model is replaced accordingly with 8 linear elements connected with a stiff element at their top and bottom. The new configuration is easy to construct and can be also “used” to improve the regularity of the bridge. Therefore, seismic analysis was conducted for the “benchmark” and the “modified” bridge. In Fig. 9 the fundamental periods (longitudinal direction) of the two finite element models developed are shown. It is evident that the reduction in the pier flexural stiffness elongates the fundamental period of the structure, giving seismic isolation characteristics to the system.
Figure 8. Modified bridge pier and FE Model (dimensions in m)

Figure 9. a) Fundamental periods (T) of benchmark and modified bridge b) T on mean and target spectra

For the response history analyses the bridge models are subjected to seven recorded accelerograms. The records were selected via Rexel [14] from the European strong motion database. As shown in Table 1, the records cover a range in magnitudes ($M_w=5.1$ to 6.9) and epicentral distances ($R=10$ to 24km). The selected earthquake records were chosen to have relatively low peak accelerations values varying from 0.47m/s$^2$ to 3.68m/s$^2$ and are of soil type B according to Eurocode 8 soil classification scheme [11]. The records were appropriate-
ly scaled so that their average elastic acceleration response spectrum is compatible with the elastic response spectrum proposed by Eurocode 8 Part 2 [15] for soil type B and PGA 0.24g.

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Station</th>
<th>Epic. Dist. (km)</th>
<th>Mw</th>
<th>Rec. PGA (m/s²)</th>
<th>Soil type (EC8)</th>
<th>Fault mech.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kalamata</td>
<td>ST163</td>
<td>11</td>
<td>5.9</td>
<td>2.35</td>
<td>B</td>
<td>normal</td>
</tr>
<tr>
<td>2</td>
<td>Umbria Marche</td>
<td>ST228</td>
<td>21</td>
<td>5.1</td>
<td>0.47</td>
<td>B</td>
<td>normal</td>
</tr>
<tr>
<td>3</td>
<td>Kalamata</td>
<td>ST164</td>
<td>10</td>
<td>5.9</td>
<td>2.11</td>
<td>B</td>
<td>normal</td>
</tr>
<tr>
<td>4</td>
<td>Erzincan</td>
<td>ST205</td>
<td>13</td>
<td>6.6</td>
<td>3.81</td>
<td>B</td>
<td>strike slip</td>
</tr>
<tr>
<td>5</td>
<td>Kalamata</td>
<td>ST164</td>
<td>10</td>
<td>5.9</td>
<td>2.11</td>
<td>B</td>
<td>normal</td>
</tr>
<tr>
<td>6</td>
<td>Gulf of Corinth</td>
<td>ST178</td>
<td>10</td>
<td>5.3</td>
<td>0.67</td>
<td>B</td>
<td>normal</td>
</tr>
<tr>
<td>7</td>
<td>Montenegro</td>
<td>ST67</td>
<td>16</td>
<td>6.9</td>
<td>3.68</td>
<td>B</td>
<td>thrust</td>
</tr>
</tbody>
</table>

Table 1. Earthquake records used in response history analyses

Table 2 presents the calculated values of the ratio r according to EC8–Part 2 [15], as described also in a previous paper of Markogiannaki & Tegos [16] for the two investigated cases. In the Greek National Annex, ρ₀ shall be less or equal to 2 to ensure bridge regularity and to avoid penalizing with reduced factors. It is clear in Table 1 that the “benchmark” solution does not fulfill this requirement and the bridge should have been designed as an irregular bridge, while the “modified” bridge reduces ρ₀ below the upper limit and transforms the bridge to a regular one.

<table>
<thead>
<tr>
<th>Bridge</th>
<th>rₘₐₓ</th>
<th>rₘᵢₙ</th>
<th>ρ₀ (&lt;=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>3.38 (P4)</td>
<td>1.19 (P5)</td>
<td>2.84</td>
</tr>
<tr>
<td>Modified</td>
<td>2.24 (P4)</td>
<td>1.35 (P5)</td>
<td>1.65</td>
</tr>
</tbody>
</table>

Table 2. Irregularity check according to EC8 provisions for benchmark and modified bridge

4 MULTICRITERIA ANALYSIS

4.1 The problem of selection

The Multicriteria analysis is performed on the case study bridge of Egnatia Motorway, containing three construction solutions as alternatives. The one of them corresponds to the proposed innovative approach of the Bridge which was described in the previous paragraph, namely the “Innovative Cast in Situ” solution. The other two alternatives are the conventional methods of Cast in Situ (which was constructed in practice) and Precast I-Girder. The aim of the process is to search for the superior among the three solutions, based on multiple compliance criteria which govern bridge design.

4.2 Compliance Criteria

The criterion of construction speed has already been mentioned. Additionally, six more compliance criteria are used in this study, namely: safety, durability, economy, serviceability, aesthetics and environmental harmonization. In the context of Multicriteria analysis, the performance of the construction solutions is evaluated in terms of each one of the seven above-mentioned compliance criteria. Thus, there can be a justified selection of the most appropriate construction solution in the specific problem.
4.3 The Multicriteria analysis method of PROMETHEE

To carry out the Multicriteria analysis, the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE), distinguished by the simplicity of the application and the clarity of its results, was selected among other methods. This is a widespread multicriterion method of outranking methods with various variants (PROMETHEE I, II, III et al.), while its basic principles are described by Brans and Vincke [17] and Brans et al. [18].

This Multicriteria analysis method requires two types of information from the user; the criteria weights and the preference function for each criterion [19]. In general, the method includes three phases: a) the construction of generalized criteria; b) the determination of the outranking relations between all the feasible alternatives and c) the evaluation of these relations in order to obtain a final solution [18]. PROMETHEE initially compares each pair of alternatives in terms of their performance on each criterion, and then builds binary outranking relations to represent the preferences of the decision maker. These relations are then used to create the partial ranking (PROMETHEE I) and the complete ranking (PROMETHEE II) of all alternatives, from best to worst [20].

4.4 Application of Multicriteria analysis

In applying the method of PROMETHEE, the calculation of the weights of compliance criteria was initially required. These weights were obtained by utilizing another Multicriteria analysis method, the Analytic Hierarchy Process (AHP) that provides the relevant methodology for this purpose. This methodology required some evaluation values for the calculation of the weights, which were obtained through a suitable questionnaire addressed to 7 Greek experts in Bridge design. The final weights of the criteria are presented in Table 3.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Safety</th>
<th>Durability</th>
<th>Economy</th>
<th>Constr. speed</th>
<th>Aesthetics</th>
<th>Serviceability</th>
<th>Environ. harmoniz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0.22</td>
<td>0.179</td>
<td>0.172</td>
<td>0.147</td>
<td>0.124</td>
<td>0.081</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Table 3. Estimated weights of the compliance criteria

Next, the PROMETHEE II method (one of the variants of the general method) was applied using the "Visual PROMETHEE" software. Initially, the properties of the criteria were selected. It was decided for them to be qualitative and to use the 1-9 scale in order to evaluate the alternatives. The Preference function selected was the “Usual” for all criteria without thresholds, while all of the criteria need to be maximized. The evaluation of the three construction methods against each individual compliance criterion was also based on the results of the above-mentioned questionnaire survey to the bridge experts, and is presented in Table 4.

<table>
<thead>
<tr>
<th>Evaluations</th>
<th>Safety</th>
<th>Durability</th>
<th>Economy</th>
<th>Constr. speed</th>
<th>Aesthetics</th>
<th>Serviceability</th>
<th>Environ. harmoniz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constructed Cast in Situ</td>
<td>9</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Precast I-Girder</td>
<td>5</td>
<td>3</td>
<td>9</td>
<td>8</td>
<td>4</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>Innovative Cast in Situ</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>
Consequently, the application of PROMETHEE II resulted in the positive, the negative and the net preference flow of each construction solution, which are presented in Table 5. Additionally, this Table provides the final ranking of each construction solution, which results from its net preference flow (Phi). The net preference flow of an alternative is the balance between its positive and negative flows, and needs to be maximized.

<table>
<thead>
<tr>
<th>Alternatives / Flows</th>
<th>Phi+</th>
<th>Phi-</th>
<th>Phi</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constructed Cast in Situ</td>
<td>0,3002</td>
<td>0,528</td>
<td>-0,2278</td>
<td>2</td>
</tr>
<tr>
<td>Precast I-Girder</td>
<td>0,3262</td>
<td>0,6004</td>
<td>-0,2742</td>
<td>3</td>
</tr>
<tr>
<td>Innovative Cast in Situ</td>
<td>0,6284</td>
<td>0,1264</td>
<td>0,502</td>
<td>1</td>
</tr>
</tbody>
</table>

As it can be observed from the results of the applied Multicriteria analysis method, the proposed Innovative Cast in Situ method presents an overwhelming superiority over the other two conventional construction methods, and thus is deemed selectable in the context of the present study. As far as the two conventional construction methods are concerned, they presented a very slight difference, according to their performance on the compliance criteria. More specifically, the conventional method of Cast in Situ that was used in practice was ranked second, while the conventional Precast I-Girder method was ranked third.

5 CONCLUSIONS

The present study focuses on the investigation of reducing the construction time of a medium length concrete bridge with the selection of an appropriate structural system and the corresponding construction method. The study considers the complete structural system of the bridge, namely the deck and the piers. For the evaluation of the proposed system seven compliance criteria were used and multi-criteria analysis was performed. In addition to the improvement in construction speed, the improvement to the regularity of the bridge was investigated. The main conclusions that are derived in the present study are shown in the following paragraphs.

- The construction methods that provide the possibility of significant reduction in construction time are the Precast and the conventional Cast in Situ, with close effectiveness for both. The first is offered for low and medium quality bridges, while the latter is proposed in an innovative version for bridges of superior quality, where the quality is determined by the degree of satisfaction of the established criteria of compliance with a relative low prioritization of the economy criterion.
- The proposed innovative construction process involves the construction of cast in place prestressed slab, which addresses all the compliance criteria to a high degree and without heavily affecting the criterion of economy. The proposed variation in the cross-sectional height also contributes considerably, apart from the aesthetics, to the reduction of concrete consumption without significant increase in the pre tensioning of the tendons. In addition, the economy criterion is improved by combin-
ing the application of limited pre-stress to the spans and partial pre-stress to the supports

- The speed in pier construction is achieved through the avoidance of the solid rectangular cross-section their replacement by a group of small circular cross-sections at the upper part of their height. In this way, besides the speed of construction, the problem of any irregularity of the structural system is also improved.
- To achieve the best possible effect in terms of shortening of the construction time, the optimal coordination of the assembling-disassembling of the formwork, reinforcement and prestress application and concreting systems clearly contributes is expected.
- The use of prestress to shorten the time of formwork disassembling in combination with the use of fast-curing concrete can improve the final result.

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REFERENCES


VIRTUAL SENSING TECHNIQUES FOR THE ESTIMATION OF JOINTS CONCEPT MODELS PARAMETERS

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Abstract

Within the current trend towards lightweight and multi-material structures, the design of structural joints has even more become a topic of particular interest. Including these joints in models that can be used in the design, monitoring and control of structures dynamics behavior is fundamental. In front of the multiple modelling strategies, the engineer always has compromise between accuracy and simplicity, according to the application requirements. On the one hand, high complexity models have the potential to be very accurate but they require long modelling and simulation times and an extensive set of to be identified parameters. On the other hand, simple concept models, that approximate the behavior on a system level, allows a still representative accuracy with a lower computational cost for a large set of applications. In this latter case, it is more challenging to identify the parameters and their corresponding application range. In this context the parameter estimation techniques that inherently account for the model inaccuracies gain importance. This work proposes a new methodology for the estimation of the parameters for an adhesive joint concept model. The novelty involves a virtual sensing strategy, where the model and the experiments are used in a coupled fashion, to obtain with a limited number of simple non-intrusive measurements, an extended set of virtual data. Particularly a stochastic estimator has been used: the Kalman Filter brings together measurements from accelerometers with a reduced order model. Sensitivity and observability analysis are performed to set the initial configuration of the algorithm. Finally the work shows, numerically and experimentally, that this methodology allows to estimate the parameters of an adhesive joint concept model, resulting in an accurate representation of the system.

Keywords: Joint modelling, parameter estimation, virtual sensing, Kalman filter, structural dynamics


1 INTRODUCTION

In lightweight systems, where different types of material are connected together, the joining technologies have a dominant effect on the overall behavior and they must be properly considered in the modelling phase. Mechanical joints often have an important non-linear influence on the transient behavior of the system, due to local changes in stiffness and damping. Hence in that respect having a non-negligible influence on the lifetime evaluation of the whole structure. It is evident that the analysis and monitoring of these connections is fundamental [1].

From the system level point of view, a detailed modelling for each of the joints present in a certain assembly is a cumbersome process as it includes many local changes in the mesh geometry and hence a significantly large model in terms of degrees of freedom. Then, the modelling strategy selection for the joints becomes really important; engineers have to find an optimum between complex joints models, requiring an extensive modelling effort and long simulation times, or simple concept models where it is unclear which approach yields at accurate results. For this latter case, the model parameters are usually fewer but they are difficult to identify a priori. For these reasons their estimation is crucial and particularly challenging. Often the parameters are not practically accessible in-situ, example given for the elastic module of a glue, or a bolt stiffness; in other cases several physical characteristics are condensed in few model parameters, that are not directly linked to the reality and thus, they are not directly measurable from experiments.

This article proposes to identify the parameters of these concept joint models by upgrading them to the Digital Twin level [2]: the purely mathematical representation, the model, is kept connected to the reality in a closed-loop fashion through a number measurements, resulting in a more reliable and accurate analysis tool. The Digital Twin can provide full-field information of the system and a selection of this information, later called virtual sensors, can be used for an accurate estimation of the model parameters. One of the main advantages of these novel virtual sensing methodologies is the feasibility of in-situ estimation, based on few non-intrusive measurements. Furthermore it only requires time-domain signals, opening the possibility of estimation in operational conditions. Finally there is the potential for improved accuracy with respect to the traditional identification methods. Essential readings about the traditional identification methods for parametric models are the surveys by Mottershead [3] and Åström [4].

The presented work consists of the introduction and experimental validation of a new virtual sensing technique for parameter estimation. It is based on a stochastic Kalman filter estimator [5] that elaborates a reduced order model and few dynamical measurements, together with their respective uncertainties, the process and measurements noises. The Kalman filter is fed with 4 real sensors signals attached to the test-rig and it is used to estimate the corresponding 4 virtual sensors, in this case accelerometers. The algorithm is repeated for different values of the parameter to be estimated, and the optimal parameter value is the one that minimize a cost function related to the error between the real and the virtual sensors. The selected validation case consists in two plates connected with an adhesive lap joint, where the elastic module of one of the two plates is the parameter to be estimated. It is relevant to notice that with the proposed methodology, the parameter can be positively estimated in a non-intrusive way, without sensors or excitation systems attached to the component to whom the parameter physically belongs.

The next 5 sections introduce the 5 fundamental steps of the proposed methodology: Model definition, Sensitivity analysis, Sensors configuration and observability, Experimental data, and Estimation. They are presented in the chronological and logical order that should be followed for the methodology implementation. For this reason this work could be also seen as a practical
guideline for this new parameter estimation technique.

2 MODEL DEFINITION

2.1 Finite element modelling

In the current engineering environment the most well-known modelling tool for structural analysis is the Finite Element (FE) method. For this reason the methodology reported in this article is based on a FE model, even though other modelling approaches could have been used with similar results. In a FE model the system is spatially discretized in a number of \( n_{\text{elem}} \) elements, defined by \( n_{\text{nodes}} \) nodes. Each node is represented by a number of degrees of freedom for a total of \( n_{\text{dofs}} \) degrees of freedom in the model, and for each degree of freedom there is an equation describing its interactions with the other ones. Of course these interactions are defined by the physical laws, considering the applied external forces, the elements topology and the elements material properties, mainly stiffness and mass. The model is finally a mathematical representation of the real system, consisting in a system of \( n_{\text{dofs}} \) equations to be solved. In this work the commercial FE framework SIMCENTER 3D – NX Nastran solver has been used. The system considered consists in two aluminum plates with dimensions 221mm x 132mm x 4mm, adhesively bonded together with commercial super-glue LOCTITE 41. The lap joint involves the whole width of the plates, and the overlap measures 20 mm in the length direction. Similar single lap joints are modelled also in the articles by He [6] and Van Belle [7]. In this work the system is clamped at one side, and the elastic module of the clamped plate is the parameter to be estimated.

The FE model includes 60260 hexagonal solid elements (8 nodes each), 73920 nodes and 3 translational degrees of freedom per node. At the adhesive location the substrate mesh is refined, resulting in cubic shaped elements with dimension 1 mm. The thickness of the plates is subdivided into 4 elements, whereas the adhesive part consists in one only layer of solid elements with thickness 0.5 mm. The material and geometric properties are set as shown in Table 1 and the meshed model can be seen in Figure 1. The coordinate system that will be used through all the article, is centered at the right corner opposite to the clamping, the x direction is along the width and the y is along the length of the system.

![Figure 1: meshed Finite Element model](image)

<table>
<thead>
<tr>
<th></th>
<th>Clamped Plate</th>
<th>Top Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td>0.221 m</td>
<td>0.221 m</td>
</tr>
<tr>
<td><strong>Width</strong></td>
<td>0.132 m</td>
<td>0.132 m</td>
</tr>
<tr>
<td><strong>Thickness</strong></td>
<td>0.004 m</td>
<td>0.004 m</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>2692 kg m(^{-3})</td>
<td>2692 kg m(^{-3})</td>
</tr>
<tr>
<td><strong>Elastic Module</strong></td>
<td>?</td>
<td>69 GPa</td>
</tr>
</tbody>
</table>

Table 1: System physical properties
### 2.2 Parametric model order reduction

The model is not directly solved within SIMCENTER 3D environment, but its input file is exported into Matlab, where the assembled stiffness and mass matrices are elaborated and the model is reduced, by solving the eigenvalues problem. However, in this mathematical operation the explicit influence of the parameter on the system equations would get lost and thus its estimation would not be possible. To overcome this issue, a parametric model order reduction (PMOR) scheme is adopted. Benner et al. [8] provide a wide survey on the different schemes available in literature. In this article, the used PMOR technique is similar to the one introduced by Naets [9], where it is used for a non-linear estimation within an Extended Kalman filter. It consists of the following steps: first, the parametric space has been sampled with a discrete number of values equally spaced around the expected one; second, the model has been locally reduced, meaning that the eigenvalues and the eigenvectors have been separately computed for each different parameter sample (Figure 2). The model can later be interpolated between those parametric samples, and then used in simulations or estimations.

![Figure 2: Parametric model order reduction scheme](image)

#### 2.3 State-space formulation

Given the equation of motion for the FE model:

$$ M\ddot{\chi} + C\dot{\chi} + K\chi = F $$ (1)

Where M, C and K are respectively the mass, damping and stiffness matrix, the vector $\chi$ contains the degrees of freedom and F is the input forces vector; the nodal coordinates can be projected on the modal space, and the original system of a number of $n_{dofs}$ equations is reduced to a system of $n_{modes}$ equations. This procedure has been repeated for several parameter values, as described in the previous section.

The reduced order model can then be written as:

$$ I\ddot{q} + 2\zeta\Lambda\dot{q} + \Lambda^2 q = \Psi^TF $$ (2)

Where I is the identity matrix, $\Lambda$ is the eigenvalues matrix, $\zeta$ is the modal damping factors matrix, $\Psi$ is the eigenvector matrix with dimensions $n_{dofs} \times n_{modes}$, and F is again the input forces vector. The state space formulation allows at this point to pass from a second order ODE system to a first order ODE:
$$\begin{bmatrix} \ddot{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \Lambda^2 & 2\zeta\Lambda \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \Psi^T \end{bmatrix} \begin{bmatrix} F \end{bmatrix}$$

Let the state vector be \( \mathbf{x} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \), the system can then be rewritten in the condensed form:

\[
\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + \mathbf{w} \tag{4}
\]
\[
y = C\mathbf{x} + D\mathbf{u} + \mathbf{v} \tag{5}
\]

The equations describe a continuous-time, linear, first order dynamical system where \( x \) is the vector of the states (in this work, the modal participation factors), \( \dot{x} \) is the derivative of the state vector, \( u \) is the input vector (in this work a concentrated force), \( A \) is the states matrix with dimensions \( 2n_{\text{modes}} \times 2n_{\text{modes}} \), \( B \) is the input matrix with dimensions \( 2n_{\text{modes}} \times 1 \), and \( w \) represents the process noise. The output equation (5) includes \( y \) that is the output vector (in this exercise, the accelerometers signals), \( C \) is the state-output matrix with dimensions \( n_{\text{meas}} \times 2n_{\text{modes}} \), \( D \) is the input-output matrix (zero in this case) and finally \( v \) is the measurement noise.

### 3 SENSITIVITY ANALYSIS

Having already computed the eigenvalues and eigenvectors of the problem for different values of the parameter to be estimated, a dynamical sensitivity analysis can be made. The purpose of this phase is to assess the influence of the parameter on the system dynamics, which is quantified by looking at how much the natural frequencies shift. The most sensitive modes are also the most important modes to be estimated for the next parameter estimation.

From Table 2 it can be observed that the high frequency modes are the most sensitive, when considering the magnitude of the frequency shift. A deeper evaluation should also consider the relative frequency shift, corresponding to a 5% perturbation of the parameter around its hypothetical value (69 GPa) and divided by the respective natural frequency.

<table>
<thead>
<tr>
<th>MODES</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) [Hz]</td>
<td>20.4</td>
<td>125.3</td>
<td>132.5</td>
<td>354.1</td>
<td>384.9</td>
<td>580.6</td>
<td>693.6</td>
<td>788.8</td>
<td>1083.5</td>
<td>1135.2</td>
<td>1326.4</td>
<td>1671.6</td>
<td>1677.5</td>
</tr>
<tr>
<td>( df ) [Hz]</td>
<td>0.5</td>
<td>1.5</td>
<td>2.5</td>
<td>4.3</td>
<td>2.8</td>
<td>11.4</td>
<td>7.1</td>
<td>10.7</td>
<td>10.8</td>
<td>13.7</td>
<td>1.7</td>
<td>16.7</td>
<td>29.0</td>
</tr>
<tr>
<td>( df ) [%]</td>
<td>2.3</td>
<td>1.2</td>
<td>1.9</td>
<td>1.2</td>
<td>0.7</td>
<td>2.0</td>
<td>1.0</td>
<td>1.4</td>
<td>1.0</td>
<td>1.2</td>
<td>0.1</td>
<td>1.0</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 2: Absolute and relative parameter sensitivities

The last row of Table 2 indicates that all the modes are relatively sensitive to the parameter, except the eleventh mode which is a local mode for the top plate. The first mode is the most sensitive, and the first three modes are chosen for the model order reduction (Figure 3).
The model is then reduced from 221760 \( n_{dofs} \) to 3 \( n_{modes} \), getting computationally lighter and observable with only few measurements as will be discussed in the next section. On the other hand the reduction limits the validity of the model to a frequency band included between 1 Hz (rigid body displacements excluded) and 150 Hz. These frequency limit is set in between the last included mode (132 Hz), and the first not included one (354 Hz), precisely at the frequency value where a reduced model FRF and the corresponding full model FRF amplitudes relatively differ by only the 5%.

The model is experimentally validated from the dynamical point of view: considering an hypothetical value for the parameter to be estimated (69 GPa), its first 3 simulated modal frequencies are well matching with the experimental ones. Particularly the relative error between simulated and experimental modal frequencies stays in a tolerance limit of ±4%. The experimental modal frequencies are retrieved by a simple hammer test elaborated with the Polimax algori

Other parameters have been initially investigated too: the adhesive elastic module and the adhesive length, that could be considered as a damage factor of the joint. However, their sensitivities are far lower, with a maximum 0.05% of frequency shift relatively to a 5% change in the parameter value, as can be seen in Figure 4. For this first validation case with the proposed methodology, a higher sensitivity parameter has been chosen, but future efforts towards low-sensitivity estimations will be considered. He [10] [11] and Vaziri [12] already provide experimental sensitivity analysis for a wide variety of parameters in adhesive lap joints, such as thickness, Poisson ratio and void size in the glue layer.

A final remark: from the practical point of view, the need of an accurate identification for low-sensitivity parameters should be properly justified. By definition these parameters have a low influence on the system, and thus an eventual estimation error will also have a low influence. We should anyhow consider that the same parameter could have a weak impact on one modelling domain, for instance the dynamical modelling, but at the same time a strong impact on another modelling domain, for instance the fatigue modelling. It is always recommended to contextualize the parameter to be estimated with the considered modelling domain: in this article all the considerations are related to the structural dynamics aspects.

![Figure 4: Relative sensitivity of several parameters](image-url)
4 SENSORS CONFIGURATION AND OBSERVABILITY

The sensors configuration is essential for the proposed methodology, not only because of the observability assessment, but also because the sensors are directly involved in the parameter identification procedure.

A system is observable when, simply put, it is possible to retrieve all its states given a certain set of measurements. For linear systems, as the one considered in this article, there are some well-established observability criteria [2]. The Kalman criterion states that a system is observable if the rank of the observability matrix given by

\[
OBS = \begin{bmatrix} 
C \\
CA \\
CA^2 \\
\vdots \\
CA^{n-1} 
\end{bmatrix}
\]  (6)

is equal to \( n \), the rank of the dynamical system, i.e. equal to the number of the states. This condition is fundamental for the effectiveness of the linear Kalman filter.

Another method is the Popov-Belevitch-Hautus (PBH) criterion [13], stating that a system is locally observable if the PBH matrix is of full rank for all eigenvalues \( \lambda \) of \( A \). The PBH matrix is defined as follows:

\[
PBH = \begin{bmatrix} 
A - \lambda I \\
C 
\end{bmatrix}
\]  (7)

Besides these yes-or-no criteria there are some quantitative assessments to measure the observability, based on cost functions related to the determinant, the condition number or the singular values of the matrixes \( OBS \) and \( PBH \). Several sensor placement algorithms reported in literature are actually structured on these quantitative assessments [14]. In this case we consider to have only accelerometers available as sensor type. The excitation characteristics are not a priori known, thus the sensor configuration should be generally robust. The method here proposed, aims to maximize the signal to noise ratios. Keeping into account the modal sensitivities with respect to the parameter to be estimated, the mode shapes displacements are at first weighted with the factors given in Table 2. Then a product-objective function is established (8), and the first sensor is placed at the \((\bar{x}, \bar{y})\) coordinates that maximize the objective function.

\[
\begin{array}{c|c|c}
\text{Accelerometer 1} & X [\text{mm}] & Y [\text{mm}] \\
\hline
\text{Accelerometer 2} & 0 & 0 \\
\text{Accelerometer 3} & 132 & 156 \\
\text{Accelerometer 4} & 132 & 156 \\
\end{array}
\]

Table 3: Sensors coordinates resulting from the optimal placement

Figure 5: Experimental setup
In equation (8), $\alpha_i$ is the $i$-th modal weight, proportional to the percentage sensitivity value of Table 2, and $\Psi_i(x, y)$ is the $i$-th displacement mode shape function.

$$OF_{sensor \ placement} = \prod_{i=1}^{n_{modes}} \alpha_i |\Psi_i(x, y)|$$

According to this objective function, that considers only the modes of the reduced model, the nodal positions are excluded from the candidate list since they would lead to a zero value for the displacement mode shape function, hence the whole objective function would be zero.

After the first sensor has been selected, the nodes within a certain distance from it are eliminated from the next candidates list, and the procedure continues iteratively, until the defined maximum number of sensors $n_{sens}$ is reached. As expected from the modes shapes depicted in Figure 3, the 4 accelerometers are located along the edges of the system (Figure 5), precisely at the coordinates reported in Table 3.

Considering the model (4) and measurement equations (5) the observability is positively assessed using the Kalman criteria. In this example, the system is actually observable just with one accelerometer, unless it is located at the clamping. This just means that it is ideally possible to estimate the full states of the system with one only sensor, but it is important to notice that adding more sensor makes the estimation more accurate and it allows to have more information for processing the parameter identification. For these reasons 4 sensors are used.

In order to further extend the methodology to the estimation of local low-sensitive parameters, more complex excitation design and sensor placement strategies should be considered. The location and frequency content of the excitation will play an important role to selectively excite the most sensitive modes.

5 EXPERIMENTAL DATA

The methodology proposed in this article is based on a Kalman filter state estimator. Such algorithm requires some measurements and the external disturbances as inputs: in this case the measurements are the 4 acceleration signals and the external disturbance is the force signal given by an hammer hit. Other disturbance types can be used, such as harmonic or random excitations, as long as they adequately excite the modes that are included in the model. The hammer hit is chosen because it requires simple equipment, it is less intrusive than using a shaker or an actuator, and it adequately excite a wide range of frequencies. Future extension of this methodology could include the excitation estimation as output of the Kalman filter, saving the effort of measuring it and possibly allowing the parameter estimation in operational conditions.

The results shown in this work are related to a hit located at the coordinates $x=20$ mm and $y=100$ mm, hence close to the 4th accelerometer, but other locations have been proved to allow positive estimation too, as long as the states are adequately excited. For instance the mode shapes nodes should be reasonably avoided as excitation location. The data acquisition system used is the portable LMS SCADAS III and the measurement software is Simcenter TestLab17.

Before using the acceleration signals in the estimator, they have been low-pass filtered with a cut-off frequency equal to 150 Hz, corresponding to the validity limit of the reduced model. This operation leads a twofold advantage: some measurement noise is filtered out, and the frequency content of accelerometers is now matching with the frequency content of the reduced order model, both including only the first three modes. In this way both the measurements and the model accuracies are improved. The original accelerometers signals are compared with the low-pass filtered ones in Figure 7. It can be noticed that in the original data all the modes are
initially excited, also the higher ones that are excluded in the reduced model. For this reason the maximum gap between the measured and the filtered data occurs at the beginning of the time signals, just after the hammer hit. Nevertheless the high modes are quickly damped out and just after one second of measurements, the gap between the measured and filtered signals becomes almost negligible. The excitation signal is time-domain filtered with an exponential window instead, to remove the sensor noise after the impact (Figure 6).

Figure 6: Excitation force signal

Figure 7: Accelerometers signals
6 ESTIMATION

6.1 Methodological scheme

The methodology is structured in a batch of Kalman filter state estimators, each one running for a different value of the parameter to be estimated. It is important to notice that in the current work the parameter is unknown but it is expected to be constant, or at least its dynamics are expected to be orders of magnitude slower than the ones of the states.

Considering that both the measurements and the model are subjected to errors, each Kalman filter estimates 4 virtual accelerometers, that are the virtual representation of the real accelerometers on the test rig. The virtual sensors are expected to give a more accurate and less noisy output w.r.t. to the measurements and the model based forward simulation when separately taken. Then the parameter estimation problem corresponds to a minimization problem with respect to the parameter, where the cost function to be minimized is related to the error between real and virtual sensors.

Specifically, after every run, the error between one real and the relative virtual sensor is integrated over time and normalized with respect to the time integral of the experimental signal. The 4 relative errors, one per sensor, are averaged to obtain the cost function. Then at each parameter value corresponds one Kalman filter run, and further one cost function value. Finally the estimated value for the parameter is the one that minimizes this cost function. The methodology is summarized in the scheme of Figure 8.

\[ CF_{estimation} = \frac{\int_0^T |y_{real}(t) - y_{virtual}(t,p)| \, dt}{\int_0^T |y_{real}(t)| \, dt} = \frac{\int_0^T |error(t,p)| \, dt}{\int_0^T |y_{real}(t)| \, dt} \] (9)

The cost function is described in equation (9), where \(y_{real}\) is the vector of time-domain signal extracted from the real sensors, \(y_{virtual}(t,p)\) is the vector of time-domain estimated signal, i.e. the virtual sensors signal. Summarizing the methodology proposes to identify system parameters by fitting the estimation to the measurement, rather then directly the forward simulation to the measurement. The estimation is more accurate and better synchronized to the measurements, and this allows the time-domain fitting.

6.2 Kalman filter

A Kalman filter is a stochastic estimator that can be used to estimate states, input and parameters, combining together a mathematical model and experimental measurements [5]. In this case the model is assumed to be linear, since only small displacements are expected, and there are not any strong geometric nor material nonlinearities. At each time-step, an a priori
estimation is elaborated based solely on the model; secondly the virtual measurements are computed based on the a priori estimation, the error between them and the real current measurements is calculated and weighted considering the known model and measurements uncertainties; with this information the a priori estimation is finally corrected with the a posteriori one. Before proceeding to the next time-step the relative process-measurements uncertainty is updated, taking into account the covariance of the estimation.

Before starting the Kalman filter time loop, the model is time-discretized with exponential integration, that leads to:

\[
F = e^{A\Delta t} \\
G = F[I - e^{-A\Delta t}]A^{-1}B \\
x_k = Fx_{k-1} + Gu_{k-1} \\
y_k = Cx_k + Du_k
\]

(10) (11) (12) (13)

After the time discretization, the Kalman filter procedures described above are applied time-step by time-step:

\[
P_k^- = FP_{k-1}^+F^T + Q \\
K_k = P_k^-C^T(CP_k^-C^T + R)^{-1} \\
\hat{x}_k^- = F\hat{x}_k^+ + Gu_{k-1} \\
\hat{x}_k^+ = \hat{x}_k^- + K_k[y_k - (C\hat{x}_k^- + Du_k)] \\
P_k^+ = (I - K_kC)P_k^- (I - K_kC)^T + K_k R K_k^T
\]

(14) (15) (16) (17) (18)

In these mathematical formulations \(\hat{x}_k^\) is the a priori estimation vector related to the k-th time-step, \(\hat{x}_k^+\) is the a posteriori one, \(P_k^-\) is the a priori covariance matrix, \(P_k^+\) is the a posteriori covariance matrix, \(Q\) is the process/model covariance matrix and \(R\) is the measurements covariance matrix. Finally \(K_k\) is the Kalman gain, that weights the relative reliability of the measurements with respect to the model.

6.3 Model and measurements uncertainties

The Kalman filter is defined stochastic estimator because it takes into account the expected uncertainties deriving from the measurements and the model. Measurements uncertainty is clearly due to the sensors noise, sensor nonlinearities, but also to eventual systematic errors such as imperfect calibration, fastening, external influences et cetera. On the other side, a model is by definition an approximation of the reality and as such, it always brings an error with itself. Some of the possible causes of this error are the neglected un-modelled behaviors, the parameters setting, the time and space discretization and last but not least the boundary conditions definition.

Within the Kalman filter algorithm the errors distribution is approximated to be Gaussian with mean equal to zero. The errors are then exclusively defined by their covariance values, indicated in the \(n_{sens} \times n_{sens}\) diagonal matrix \(R\) for the measurements, and in the \(2n_{modes} \times 2n_{modes}\) diagonal matrix \(Q\) for the states. The covariance setting is a crucial step for the estimation results, particularly in terms of relative weight between \(R\) and \(Q\). As mentioned above, this ratio will lead the filter to adjust the Kalman gain on every time-step.

If for the measurements it is reasonable to link their covariance to the sensor characteristics available on their data-sheets, it is less straightforward to set the model covariance, especially when the states are the modal contributions. In this work the derivatives covariance is set to zero. The non-derivatives standard deviation is initially set to a value two orders of magnitude
lower than the forward-simulated modal contributions (19).

\[
Q = \begin{bmatrix}
10^{-16} & 0 & 0 & 0 & 0 \\
0 & 10^{-16} & 0 & 0 & 0 \\
0 & 0 & 10^{-16} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

(19)

Setting \( Q \) as a diagonal matrix does not consider the eventual cross-coupled states errors. In order to extend the methodology to this case, the continuous-time \( Q \) is integrated with the procedure developed by Van Loan [15], and the discrete-time \( Q_D \) is obtained as a full matrix and used in the Kalman filter instead of \( Q \).

6.4 Results

The expected value for the aluminum elastic module is 69 GPa. Figure 9 and Figure 10 are showing the results of the Kalman filter when the parameter value is fixed at the expected one. At first, in Figure 9 the plot emphasizes the added value of using a closed-loop estimator rather than a forward simulation. In terms of amplitudes of the full time signal, the forward-simulation seems matching quite well the measurements, but when zooming in it is possible to quantify a significant gap between amplitudes and phases. The estimated acceleration is almost perfectly matching the measured one, instead.

![Figure 9: Full-time signal at accelerometer 1 (left), zoomed signal at accelerometer 1 (right)](image)

In Figure 10 the forward-simulation is not represented anymore, to give more visibility to the comparison between measurements and estimations.
The estimation at the accelerometers 3 and 4 seem to be more accurate, actually it is almost perfectly matching the measurements, probably because they are closer to the excitation source, then the model transfer path is shorter and less influent. Further investigation will be carried on this observation. A similar observation will hold for Figure 11 where the accelerometer 3 and 4 provide a more evident estimation convergence.

As mentioned earlier the Kalman filter is repeated for different values of the parameter, and for each run the cost function described in equation (9) is recorded.

These recorded values are reported in Figure 11, where the cost function in the right plot is
representing the average between the 4 curves in the left plot. It can be observed that the parameter estimation converges to the value of 66 GPa, slightly lower than the expected 69 GPa. This offset occurs because the clamping is modelled as ideally stiff, whereas in reality it always allows some flexibility. In this exercise, since the adherend (i.e. the clamped plate) elastic module is the only tunable parameter for a good match between experimental and model system flexibility, its estimation takes also into account the clamping flexibility, resulting in a lower than expected value. The estimated value, apparently off, anyway allows the updated model to provide results that are closer to the reality, thus more accurate. As a future step a multi-parameter estimation will be considered, in which the clamping stiffness will be included in the model and estimated alongside the elastic module of the clamped plate.

Taken the 4 accelerometers errors separately, they still converge to the same parameter, meaning that in future even less intrusive setups will be possible. Nevertheless it is important to notice that the accelerometer positions have an influence on the clearness of the convergence. As already observed before in Figure 9 and Figure 10, the sensors number 3 and 4 show a lower error with respect to the others, resulting in a more evident parameter estimation (Figure 11).

Another future improvement could involve the resolution of the parameter estimation: the parameter space could be further finer sampled in the surroundings of the first estimation and the same minimization procedure could be repeated until the desired precision.

Finally a consideration about the influence of the model uncertainty, i.e. the $Q$ matrix. As mentioned before, its setting has usually a big impact on the states estimation, and then also on the proposed parameter estimation methodology. For this final investigation, other values for the model standard deviations (square root of the $Q$ matrix diagonal) have been considered, linearly sampled around the initial value $10^{-8}$, within a range that begins from $10^{-20}$ and ends at $10^{0}$. Once more, the cost function has been recorded for each combination of parameters and model standard deviation values, and the results are represented in the 3D plot of Figure 12.

![Figure 12: Influence of the model uncertainty on the parameter estimation](image)

High values of $Q$ mean that the model uncertainty is high, then the Kalman filter gives high confidence to the measurements and the virtual sensors signals are perfectly matching the real ones. As a direct consequence, the error tends to a minimum when $Q$ tends to its maximum plotted value (Figure 12). On the other hand, low values of $Q$ mean that the model is almost perfect, then the algorithm ignores the measurements, and the estimation is nothing else than a forward-simulation. In this case the error is always high and the parameter estimation is not possible. In between these two sectors, the Figure 12 shows an intermediate region, where the influence of the parameter on the error is more evident. Then in conclusion, the model standard
deviation must be set to a value within this area to make the estimation possible. The transparent vertical plane shows that the initial setting of the model standard deviation, equal to $10^{-8}$ indeed enables the estimation. The intersection curve between the vertical plane and the 3D surface exactly corresponds to the right plot in Figure 12.

7 CONCLUSIONS

Within the modelling of lightweight assemblies, where different types of material are connected together with several structural joints, the parameters identification is challenging. Often these parameters are local and it is not easy to access them, often they do not even have a direct physical link. In this context, this work introduces and experimentally validates a novel virtual sensing technique for parameter estimation. The methodology is non-intrusive, as it does not require the sensors and the excitation sources to be located directly on the component which the parameter belongs to. The methodology can be applied in-situ, i.e. keeping the system in its operating conditions, without dismounting the component which the parameter belongs to. The estimation of the elastic module of a plate in a single lap adhesive joint has been chosen as experimental validation case. The technique consists in the minimization, with respect to the parameter, of the error between the real sensors placed on the test rig, and the virtual ones estimated through a Kalman filter. The same approach can be applied not only for joint substrates estimation, but also for the identification of the connecting elements themselves, such as adhesive layers, bolts, rivets.

The article proposes a modelling strategy for the adhesive connection, a parametric model order reduction, and it suggests how to properly select the reduced model modes according to the sensitivity and observability assessments. Some considerations about the Kalman filter configuration can be extended for other state/parameter/input estimation exercises in which the states are the modal contributions: the frequency content of the measurements should be adjusted to match the model one, the model covariance has a big impact on the estimation result and this work provides some guidelines on its setting. Another basic but significant observation about identification problems is that all the approximation errors of the model are condensed in the estimated parameter. Thus it should not be considered as the real parameter, but rather the parameter that allows the model to better match the reality.

Finally this work opens the way to some possible next steps. Multi-parameter estimations could limit the condensation of the approximation errors in one only parameter. In order to make this extension possible, different sampling techniques for the parameter space should be considered. Another challenge mentioned in this paper is the estimation of low-sensitivity parameters. If this is required, more complex and tailored strategies should be taken into account for the sensors placement and the excitation design.

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MODEL UPDATING OF A MASONRY HISTORICAL CHURCH BASED ON OPERATIONAL MODAL ANALYSIS: THE CASE STUDY OF SAN FILIPPO NERI IN MACERATA

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Abstract

The paper describes the approach followed in the characterization of the structural behavior of the historical masonry church of San Filippo Neri in Macerata, severely damaged and condemned after the Central Italy Earthquake occurred in October 2016. The case study of San Filippo Neri is particularly interesting: first for the historical and artistic importance of the church furthermore for the evidences of recurrent structural damage.

The laboratory of Proof and Research on Structures and Materials of Roma Tre University carried out an extensive onsite testing campaign – including geometric survey, flat-jack test and ambient vibration test – in order to investigate the state of the building. Operational modal analysis was used to assess the dynamic behavior of the church; the results of the testing campaign were interpreted and correlated with an accurate finite element model of the construction. The numerical model was finally tuned up based on the experimental results in order to match the dynamic behavior.

The aim of the research is to set the first steps for an integrated approach able to fit and combine the results from experimental onsite testing and numerical modelling.

Keywords: Historical Masonry, Operational Modal Analysis, Ambient Vibration Test, Structural identification
1 INTRODUCTION

During the last decades, conservation and structural safety assessment of historical buildings gained great importance especially in seismic countries with significant architectural heritage. Italy comprise a heterogeneous and extended cultural heritage, its proper conservation has become a relevant concern particularly after the latest seismic events [1].

The prediction of the mechanical response of historical masonry constructions is a complex task to accomplish where the expected behavior of the building is often misleading from the real one [2]; different strategies have been developed to approach masonry structural analysis: simplified kinematic methods, FEM, limit analysis and discrete element methods are the most common and available, however the application of these methods requires accurate geometry and material characterizations which strongly influence the results [3].

After the occurrence of severe seismic events [4], it has been developed in the Italian code an approach based on the level of knowledge where various information on history, geometry, materials and monitoring of existing constructions are integrated and correlated with safety coefficients [4].

In this scenario, the assessment of onsite testing plays a relevant role in the identification of the construction; however, the local characteristics of several tests (i.e. double flat-jack test) do not provide complete information for a satisfactory evaluation of the structural behavior. Structural health monitoring, in particular ambient vibration test (AVT), provide a global characterization of the dynamic characteristics by considering the response of the entire construction to random external forces.

This paper, through the case study of San Filippo Neri’s church, aims to propose an integrated approach in order to update and tune up numerical models of historical masonry buildings.

2 SAN FILIPPO NERI IN MACERATA

2.1 The construction of the building

Macerata is located in central Italy at 315 meters above sea level; the historical center is positioned on the top of a hill and it has maintained over the years its original structure such as the boundary wall defined in the Renaissance period and the widespread historical masonry buildings erected between the 16th and 19th centuries. The Church of San Filippo was commissioned by Cardinal Taverna in 1606 to complete the urban intervention which connected Piazza della Libertà to Piazza Vittorio Veneto [5]. The building is positioned on a slight slope between Corso della Repubblica and Via Santa Maria della Porta as it is showed in Figure 1.

In 1689, the Filippini’s congregation asked to architect Giambattista Contini, a project for the construction of a new church. In the same year, Contini proposed his project but his design was criticized as too expensive and innovative. On the 28th of November 1697, another project by architect Ludovico Gregoriani was accepted and the first stone was laid in the month of December. During the construction operations the two contracting supervisors (Father Catenacci and Father Canelli) came into conflict with the Gregoriani, therefore Contini was recalled for the execution of a definitive project that included the Church and the annex convent. The work was completed in 1732 under the guidance of Sebastiano Cipriani, a pupil and collaborator of Contini [6].
2.2 Historical evolution of the church

The church is formed by a rectangular narthex followed by the main oval aula, four radial chapels surround the aula which is surmounted by the elliptical dome with lantern. In 1697, during the construction of the building, the original foundations were enlarged due to the consistency of the soil [7].

On the 29th of January 1899, an inspection was carried out in the basement of the adjacent house where it was discovered a cave with presence of inconsistent friable sandstone that extended under the church [8].

In 1903, engineer Canaletti ascertained the connection between the crack on the dome and the lowering of one of the piers. Canaletti’s project considered the insertion of six iron stirrups on the large window over the entrance; furthermore, he exhorted the construction of a masonry vault inside the cave and its bricked up [9].

In 1929, two deep cracks protracted in the longitudinal direction; engineer Bonci’s intervention consisted in an iron reinforcement ring under the drum to compensate the thrust of the dome [10].

In July 2012, the last restoration work included the strengthening of the dome with composite materials, the repair of vertical and sub-vertical cracks with traditional procedures and the installation of a steel reinforcement ring in the intrados of the drum (in Figure 2, highlighted in light blue the external tie beam connected to the encircling ring and the reinforcement tie on the lantern).
2.3 Seismic history of the site

Since the 18th century, Macerata has been interested by several important seismic events (Figure 3).

On April 24, 1741; an earthquake of the VII grade involved the territory between Serrasanquirico and Fabriano, it had an extremely extensive damage area: from Pesaro and Urbino to Gubbio and Perugia, from Macerata to Fermo [11].

On the 1st of September 1951 an earthquake of 5.25 Moment Magnitude with epicenter in the Monti Sibillini struck the city of Macerata [12].

On September 4, 1997; a series of seismic events occurred in the area between Marche and Umbria. Three major shocks were recorded at 2:33 (Local Magnitude ML 5.5 and VII Mercalli-Cancani-Sieber MCS), at 11:40 (ML 5.8 and VIII-IX MCS) and at 11:46 (ML 4.7 and VII MCS), the serious and diffuse damaged area was identify with the Provinces of Perugia and Macerata [14].

On the 30th of October 2016 at 7:40, an earthquake of ML 6.5 interested the City of Macerata [15]; as reported by the technical office of the Diocese of Macerata-Tolentino-Recanati-Cingoli-Treia, in this occasion it was observed consistent damage to the dome of San Filippo Neri which caused the condemned of the building.
3 ONSITE TESTING

In April and July 2018, the Laboratory of Proof and Research on Structures and Materials (PRiSMa) of the Architecture Department of University of Roma Tre carried on an extensive experimental campaign on the church of San Filippo Neri in order to investigate the structural elements, characterize the mechanical property of the masonry and identify the dynamic behavior of the construction.

During the onsite experimental campaign different tests were performed including:

- n° 20 video endoscopy inspections of the masonry elements
- n° 1 sonic tomography of the south-eastern pillar
- n° 3 double flat jack tests of the masonry
- n° 3 dynamic penetrometer tests of the mortar joints
- ambient vibration test

In the present paper are reported only the most relevant for the purpose of this research.

3.1 Double flat jack test

Three double flat-jack tests were executed at the ground floor level of the building in order to estimate the elastic moduli of the brick masonry elements (Figure 4).

Figure 4 Double flat-jack test location

The double flat-jack test is a well-known slightly destructive technique; during the test two parallel slots are sawn into the mortar joints, then two flat-jacks are inserted into the slots finally the pressure is increased cyclically through a hydraulic pump. The elastic modulus of masonry is evaluated by measuring deformation due to the compressive stress.

The performed test is summarized in the following steps:
1) Set of the line cuts and holes to support the instrumentation;
2) Execution of the cuts with Husqvarna K960 Ring Saw;
3) Inserting of two flat-jack into the slots (dimensions 350 x 175 mm for the semi-circular shape plus 350 x 85 mm for the rectangular shape by 4 mm of thickness) and connection to pressure switch and 700 bar hydraulic pump;
4) Set of 3 displacement sensors Penny & Giles SLS130 in vertical direction and 1 in horizontal direction;
5) Connection of the equipment to data acquisition system National Instruments SCXI 1001 with SCXI 1314/1520 slot and NI SCXI-1600 analog to digital converter with 16-bit resolution;

The tests were executed in different loading cycles, the stress state was evaluated with (1) according to [16].

\[ \sigma = P \cdot K_m \cdot K_a \]  

\( P \) is the flat-jack pressure in MPa;
\( K_m \) is a constant dependent on geometry and stiffness of the flat-jack;
\( K_a \) is a constant dependent on the ratio between the areas of the flat-jack and the slot;

The results of the tests are reported in Table 1.

<table>
<thead>
<tr>
<th>Test</th>
<th>Elastic Modulus [MPa]</th>
<th>Poisson’s coefficient [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M01</td>
<td>4160,7</td>
<td>0,21</td>
</tr>
<tr>
<td>M02</td>
<td>7440,6</td>
<td>0,30</td>
</tr>
<tr>
<td>M03</td>
<td>2896,1</td>
<td>0,20</td>
</tr>
</tbody>
</table>

Table 1 Double flat-jack test results

3.2 Ambient vibration test

Ambient vibration measurements and output-only dynamic testing are consolidated techniques for dynamic characterization of historical masonry constructions [17]. The test was conducted on the church in July 2018, the response acceleration of the construction was recorded for about 1 hour by 16 accelerometers located in 7 points at two different heights (Figure 5, Figure 6).
The sensors were connected to two different 24-bit acquisition systems, each one equipped with 2 signal conditioners, 8 channels and 8 independent A/D converters. The two systems are provided with GPS antenna to record the data on absolute time and synchronize the acquisitions. The measures were recorded with 200 Hz sample frequency with dedicated anti-aliasing filter using 16 uniaxial ICP piezoelectric accelerometers (n° 9 PCB 393A03, 1 V/g sensitivity, range ± 5 g, 10 µg rms resolution and n° 7 PCB 393B12, 10 V/g sensitivity, range ± 0.5 g, 8 µg rms resolution).

4 OPERATIONAL MODAL ANALYSIS

Operational modal analysis is based on the assumption of white noise random vibrations; in this scenario, it is sufficient to analyze the response of the structure to unknown random vibration (output-only modal identification) to extract the modal parameters of the building (natural frequencies, modal shape vectors and modal damping).

The extraction of modal parameters from ambient vibration data was carried out by using the polyreference least-squares complex frequency-domain method [18], explained in 4.1.

4.1 PolyMAX method

The method is based on the processing of the Cross-power spectrum functions-$S_{xy}(j\omega)$ which are correlated with the frequency response function-FRF of the system and expressed as function of modal parameters.

The data recorded are represented in the $z$-domain (2):

$$z = e^{j\omega \Delta t} \quad (2)$$

$\Delta t$ is the sampling time.

$H(\omega)$ is the matrix containing the FRFs between all $m$ inputs and $l$ outputs (3).

$$[H(\omega)] = \sum_{j=0}^{p} z^{j} [\beta] \cdot \left( \sum_{j=0}^{p} z^{j} [\alpha] \right)^{-1}$$

$H(\omega), [\beta], [\alpha] \in \mathbb{C}^{l \times m}; [\beta]$, $[\alpha]$ are the numerator matrix polynomial coefficients; $[\alpha]$ are the denominator matrix polynomial coefficients; $p$ is the model order. Once the denominator coefficients $[\alpha]$ are determined, the poles and modal participation factors are retrieved as the eigenvalues and eigenvectors of their companion matrix.

Pole-residue model is considered in (4).

$$[H(\omega)] = \sum_{i=0}^{n} \frac{\{v_{i}\} \langle l_{i}^{T} \rangle}{j \omega - \lambda_i} + \frac{\{v_{i}^{*}\} \langle l_{i}^{H} \rangle}{j \omega - \lambda_i^{*}} - \frac{[LR]}{\omega^2} + [UR] \quad (4)$$

$n$ is the number of modes; $\{v_{i}\} \in \mathbb{C}^{l}$ are the mode shapes; $\ast$, $^{T}$, $^{H}$ respectively denote complex conjugate, transpose and complex conjugate transpose of a matrix.

$\langle l_{i}^{T} \rangle \in \mathbb{C}^{m}$ are the modal participation factors and the poles, which are occurring in complex-conjugated pairs, are related to eigenfrequencies $\omega_i$ and damping ratios $\xi_i$.

$$\lambda_i, \lambda_i^{*} = -\xi_i \omega_i \pm j \sqrt{1 - \xi_i^2 \omega_i} \quad (5)$$

$[LR], [UR] \in \mathbb{R}^{l \times m}$ are respectively the lower and upper residuals which model the influence of the out-of-band modes in the considered frequency band. The interpretation of the
stabilization diagram yields a set of poles $\lambda_i$ and corresponding participation factors $\langle l_i^T \rangle$. Since the mode shapes $\{v_i\}$ and the lower/upper residuals are the only unknowns, they are readily obtained by solving the equation in a linear least-square sense [20]. The PolyMAX method is implemented in the software Simcenter TestLab 18 [20] which was used in this work for the data processing.

4.2 Data processing and results

For the visualization of the results, it was considered a geometric model, defined in Figure 7.

![Geometric model](image-url)

**Figure 7 Geometric model**

In the pre-processing stage, there was applied a high-pass filters to the recorded accelerations in order to reduce the noise of the electronic devices. Thereafter, the Cross-power spectra were elaborated with 0,049 Hz frequency resolution and 5% windowing; for the identification of the global modes there were considered as reference the measurements recorded in node B and C while for the identification of the local modes of the dome the measurements recorded in node G.

The PolyMAX algorithm was run on different sets of Cross-power spectra to investigate the stable poles and estimate the modal characteristics of the construction.

In Table 2 are reported the experimental results of the operational modal analysis.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Damping</th>
<th>Mode</th>
<th>Frequency</th>
<th>Damping</th>
<th>Mode</th>
<th>Frequency</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,065 Hz</td>
<td>1,11%</td>
<td>2</td>
<td>2,972 Hz</td>
<td>2,84%</td>
<td>3</td>
<td>3,395 Hz</td>
<td>1,45%</td>
</tr>
</tbody>
</table>

![Mode shapes](image-url)
The results identify 1st and 2nd modes with the global translation respectively in Y and X direction, 3rd mode involves the global rotation of the church while 4th mode the rotation of the back of the church; 5th and 6th modes identify the global translational modes with in-plane deformation. 7th, 8th and 9th modes are local modes proper of the dome; 7th mode identify the translation of the dome in the Y direction, 8th mode has a prevalent deformation in the vertical direction while 9th mode identify the translation of the dome in the X direction.

5 GLOBAL FINITE ELEMENT MODEL

A numerical tridimensional model of the construction was built with the finite element software Midas Fea [21]. The model was based on accurate geometrical survey which allowed detailed representation of the volumes. There were considered 546049 3D-solid elements to model the masonry and 129 1D-elements for the steel reinforcements. The reinforced concrete floors of the annex building and the eastern roof of the church were modeled with relative constrains while the influence of the annexed building complex was taken into account with horizontal restraints. The model considers tridimensional translational restraints for the base nodes; lateral nodes on the ground-masonry interface were restrained only in the horizontal directions; in Figure 8 it is shown the FEM used for the analysis.
The materials were considered homogeneous and isotropic with elastic behavior. The FEM was divided in three main macro zones in order to assign different characteristics to the materials; for the mass density there were assumed the values suggested by the Italian code [22] as reported in Table 3.

<table>
<thead>
<tr>
<th>Material</th>
<th>Mass density [kN/m$^3$]</th>
<th>E [MPa]</th>
<th>$v$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern masonry – E1</td>
<td>18</td>
<td>4160,7</td>
<td>0,21</td>
</tr>
<tr>
<td>Northern masonry – E2</td>
<td>18</td>
<td>7440,6</td>
<td>0,30</td>
</tr>
<tr>
<td>Eastern masonry – E3</td>
<td>18</td>
<td>2896,1</td>
<td>0,20</td>
</tr>
<tr>
<td>Steel reinforcements</td>
<td>76,98</td>
<td>210000</td>
<td>0,3</td>
</tr>
</tbody>
</table>

Table 3 FEM materials

5.1 Model updating

On the basis of the experimental results, it was carried on sensitivity analysis in order to match numerical modal analysis with operational modal analysis. Considering the complexity of the building and the uncertainties related with the construction technology (especially in the dome), the analysis focused on the identified global modes of the church (first six modes of vibrations). The updating procedure of the model was based on the evaluation of two main quantities well known in the scientific literature [23]: The relative frequency error ($Df_{i,j}$), defined in (6).

$$Df_{i,j} = \frac{|f_{num,i} - f_{exp,j}|}{f_{exp,j}} \cdot 100$$  \hspace{1cm} (6)

$f_{num,i}$ is the numerical frequency and $f_{exp,i}$ the experimental frequency for the $i^{th}$ mode of vibration.

The modal assurance criterion (MAC) [24], defined in (7).
\[ MAC \left( \left\{ \phi_{num,i} \right\}, \left\{ \phi_{exp,j} \right\} \right) = \frac{\left| \left( \left\{ \phi_{num,i} \right\}^T \left\{ \phi_{num,i} \right\} \right) \left( \left\{ \phi_{exp,j} \right\}^T \left\{ \phi_{exp,j} \right\} \right) \right|^2}{\left( \left\{ \phi_{num,i} \right\}^T \left\{ \phi_{num,i} \right\} \right) \left( \left\{ \phi_{exp,j} \right\}^T \left\{ \phi_{exp,j} \right\} \right)} \]  

(7)

\( \phi_{num,i} \) is the numerical modal vector and \( \phi_{exp,i} \) is the experimental modal vector for the \( i \)th mode of vibration.

On the basis of \( D_f \) and MAC between each numerical and experimental mode, it was implemented a procedure in order to select the proper numerical mode.

The process is summarized in the following steps:
1. Identification of the numerical frequency \( (f_{num,i}) \) closest to experimental frequency \( (f_{exp,i}) \);
2. Selection of the interested range of numerical modes \([f_{num,i-4}, f_{num,i+4}]\);
3. Normalization of the relative frequency error with the maximum error in the interested range defined in (8);
\[
D_{nf,i,j} = \frac{Df_{i,j}}{\max[f_{num,i-4}, f_{num,i+4}]} 
\]  

(8)

4. Selection of the suitable \( i \)th numerical mode on the base of the minimum error defined in (9);
\[
error_{i,j} = 0.5 D_{nf,i,j} + \left( 1 - MAC_{i,j} \right) 
\]  

(9)

Model updating started with the identification of the proper boundary conditions. The updated models considered 3-D point spring elements according to Winkler’s theory of elastic foundations. The spring stiffness was modified within a range of reasonable values on the base of the moduli of subgrade reaction (ks) available in literature [25].

Three different sets of springs were considered in the global finite element model:
- 10 XY – Z (ks in horizontal direction 10 times than vertical direction)
- XYZ (equal ks in the three directions)
- 0,1 XY – Z (ks in horizontal direction 10% than vertical direction)

Sensitivity analysis of the three sets was carried on within a range of ks between 14400 and 36000 kN/m³; the results are reported in Figure 9, Figure 10 and Table 4.

![Graph 9](image1.png)  
Figure 9 Average relative frequency error

![Graph 10](image2.png)  
Figure 10 Average MAC

The average errors for the three sets of models were calculated according to (9);
Table 4 Average error for the three sets of models

<table>
<thead>
<tr>
<th>$k_s$ [kN/m³]</th>
<th>10XY-Z</th>
<th>XYZ</th>
<th>0,1XY-Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>14400</td>
<td>1,19</td>
<td>0,89</td>
<td>0,82</td>
</tr>
<tr>
<td>19200</td>
<td>1,14</td>
<td>0,88</td>
<td><strong>0,76</strong></td>
</tr>
<tr>
<td>24000</td>
<td>1,14</td>
<td>0,94</td>
<td>0,81</td>
</tr>
<tr>
<td>36000</td>
<td>1,15</td>
<td>0,96</td>
<td>0,83</td>
</tr>
</tbody>
</table>

Model 0,1 XY – Z with $k_s$ equal to 19200 kN/m³ corresponds to the minimum error thus it was selected as first updating step.

Material characterization was further investigated by carrying sensitivity analysis on the moduli of elasticity of the masonry; the reference values of the moduli were modified within reasonable range, the parameters were varied one at the time in order to highlight their influence on the modal behavior of the building.

The following graphs report frequency error and MAC of the first six modes of vibrations on the variation of modulus of elasticity.
Sensitivity analysis highlighted the influence of modulus of elasticity on certain modes. Variation of the elasticity of the southern masonry (E1) showed the main influence on the modal deformations of 5th and 6th mode (Figure 12) while frequency errors of 3rd, 4th and 5th mode decrease with the reduction of elasticity (Errore. L’origine riferimento non è stata trovata.). Northern masonry (E2) stiffness variation conditioned mainly the behavior of 5th mode of vibration (Figure 13 and Figure 14). Variation of the eastern masonry (E3) elasticity affected mainly modal deformation of 4th mode (Figure 16) and frequency error of the first two modes of vibration (Figure 15).

According to the previous considerations, the model updating suggests:
- strong reduction of elasticity in the southern masonry (E1) in order to decrease the frequency error;
- slight increasing of the northern masonry elasticity (E2) in order to raise MAC of the 5th mode
- small reduction of the eastern masonry elasticity (E3) to increase 5th mode MAC and decrease 2nd and 6th mode frequency error.

5.2 Discussion

According to the influence of elasticity on the modal parameters, three different tuned up models are proposed in agreement with the updating procedure.
- model 0 is the initial fixed base model
- model 1 is the elastic foundation model with masonry elasticity from double flat-jack results
- model 2 is the updated model with minimum average frequency error
- model 3 is the updated model with maximum average MAC

In Table 5 are reported the set of parameters of each model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Foundation [N/mm]</th>
<th>Modulus of elasticity [N/mm²]</th>
<th>Mass density [kN/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ks,xy</td>
<td>ks,z</td>
<td>E1</td>
</tr>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
<td>4160,7</td>
</tr>
<tr>
<td>1</td>
<td>333,7</td>
<td>3336,7</td>
<td>4160,7</td>
</tr>
<tr>
<td>2</td>
<td>333,7</td>
<td>3336,7</td>
<td>1664,3</td>
</tr>
<tr>
<td>3</td>
<td>333,7</td>
<td>3336,7</td>
<td>2704,4</td>
</tr>
</tbody>
</table>

Table 5 Model parameters
The updating procedure highlighted the necessity to reduce the stiffness of the southern masonry; this result is in accordance with the historical events which reported the settlement problems of the building. The result from the double flat jack offered reliable information on the materials, moreover the introduction of elastic foundation provided a better match between experimental and numerical behavior.

The model updating underlined the enhancing of the error quantities with the variation in the relationship between the modulus of elasticity in particular for the higher modes (4\textsuperscript{th}, 7\textsuperscript{th} and 8\textsuperscript{th} mode)

The final results from each model are presented in Table 6 and Table 7.

<table>
<thead>
<tr>
<th>Df [%]</th>
<th>Modes of vibration</th>
<th>Df</th>
<th>Df</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>110,82</td>
<td>79,93</td>
<td>104,08</td>
</tr>
<tr>
<td>1</td>
<td>21,40</td>
<td>12,93</td>
<td>16,27</td>
</tr>
<tr>
<td>2</td>
<td>24,55</td>
<td>9,53</td>
<td>13,89</td>
</tr>
<tr>
<td>3</td>
<td>25,16</td>
<td>12,58</td>
<td>16,11</td>
</tr>
</tbody>
</table>

Table 6 Relative frequency error of the models

<table>
<thead>
<tr>
<th>MAC</th>
<th>Modes of vibration</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0,89</td>
<td>0,48</td>
</tr>
<tr>
<td>1</td>
<td>0,77</td>
<td>0,44</td>
</tr>
<tr>
<td>2</td>
<td>0,76</td>
<td>0,45</td>
</tr>
<tr>
<td>3</td>
<td>0,74</td>
<td>0,52</td>
</tr>
</tbody>
</table>

Table 7 Model Assurance Criterion of the models

6 CONCLUSIONS AND FUTURE WORK

The results from the experimental campaign on the church of San Filippo Neri in Macerata were reported and discussed. The flat-jack test reported a significant diversity in the modulus of elasticity of the masonry: northern masonry was identified with the stiffer material followed by the southern masonry and then eastern masonry. The results from the static tests provided reliable information on the materials, moreover the relationship between elastic moduli was partially validated by the numerical modal analysis of the FEM.

Ambient vibration test and operational modal analysis were confirmed as appropriate techniques for the evaluation of the dynamic response of historical masonry construction; furthermore, the PolyMAX method allowed an accurate identification of the global dynamic behavior of the construction as well as the local behavior of large structural elements such as the dome. The updating procedure highlighted the complexity of the problem in the optimum selection of the parameters; the application of elastic foundations was confirmed to be the most relevant upgrade in order to match numerical and experimental modal behavior.

Model upgrading suggested the additional reduction of stiffness in the southern area of the church, the goal was pursued by decreasing the elasticity of the southern masonry; however, a more sophisticated path (such as the local variation of the spring elasticity) could have been followed in order to match the experimental dynamic behavior although the complexity of the problem returns the attempt to a future development. The upgrading procedure and the introduction of a criteria in the selection of numerical modes set the groundwork for a future automated model upgrading able to handle higher number of parameters.
ACKNOWLEDGEMENTS

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IN-PLANE BEHAVIOR OF CRACKED MASONRY WALLS REPAIRED WITH TITANIUM RODS

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Abstract

High-performance materials are being widely researched for repair and rehabilitation of historic masonry structures, especially in seismic prone areas. In such context the use of titanium bars, as a Near Surface Mounted (NSM) repair method of cracked brickwork wall panels, has been investigated in this paper. The potential benefits, liabilities, and architectural considerations regarding the use of titanium alloys for reinforcing and repairing historic masonry walls are discussed with an emphasis on in-plane behaviour, under lateral (shear) loading. Test programs are described and results from laboratory testing are included. It is demonstrated that it is possible to repair brickwork panels using titanium bars embedded into the panel’s cracks using an epoxy paste. Cracked brickwork masonry panels were stabilised by bonding titanium rods into the horizontal bed joints. On opposite, the use of a cement mortar for bar application resulted in unsatisfactory test results.

Keywords: Brickwork masonry, titanium, shear strength, earthquake engineering.
1 TITANIUM AS A STRUCTURAL MATERIAL

Titanium and its alloys have started their industrial history in aerospace applications. During the first 40 years from the beginning of their use, i.e. from the first half of the ‘50s to the first half of the ‘90s, other applications were rare. At the end of the Cold War, with the subsequent reduction of military expenditures in many countries, the cost of titanium decreased. This was compounded by the fact that former Soviet Union countries and the Popular Republic of China started to sell titanium on the global market. Nevertheless, about ¾ of Titanium production is still used for aerospace and military applications (Fig. 1).

The price of Titanium is often governed by speculation, and it is rather volatile and high. This is a reason for its limited use in other applications. Nevertheless, in non–aerospace applications, the technological benefits provided by Titanium could overcome its cost. Among the benefits, it is possible to locate: 1. the high corrosion resistance, resulting in lower maintenance, repair and part replacement costs; 2. the low weight density, which reduces, for example, inertial forces in structural parts that must move at high speed, resulting in higher speed and efficiency, lower vibration, noise and energy consumption; this could be also interesting for seismic reinforcement applications; 3. the high mechanical properties (also including here the low Young’s modulus), that allow the construction of smaller and flexible structural members.

The corrosion resistance is of interest of the chemical and petrochemical industries, in the desalinization plants, for the production of prosthetic and medical devices, of the industry of galvanic active and passive protection, and of heat exchangers for power plants. This is also interesting for outdoor, unprotected applications in earthquake engineering, or Civil engineering in general. Among the users that could benefit from its low weight density, we could list the automotive industry (cars, motorbikes and bicycles), the food and beverage industry and in general the industry of canning, and of sport goods. Furthermore, the industry of umbilical for oil platforms, of prosthetic and medical devices, and firearms could highly benefit from its high mechanical properties.

There is also a number of applications where titanium’s use is governed by aesthetical aspects, rather than by technical considerations. This is the case of artefacts, glass frames, jewelry items, decorative panels, etc. Even if these applications are minoritarian compared to aerospace ones, it demonstrates the growing interest for this material in society and industry.

2 MATERIAL PROPERTIES AND CIVIL APPLICATIONS

The use of titanium found applications in conservation engineering, especially of high heritage value buildings is gaining popularity. A large number of new titanium profiles now
coming onto the market are, unfortunately, rarely innovative (Fig. 2). In recent times, more attention was given to the analysis and consideration of the principles of building conservation philosophy, prior to making decisions relating to masonry repair and reinforcement, especially for listed buildings.

In seismic prone areas, structural repair and reinforcement are often required for historic masonry buildings. Several methods have been proposed and tested, both on site and in the laboratory. However, these repairs or reinforcements have varying degrees of defensibility, and will ultimately lead to good or bad conservation approaches.

In such context, the use of titanium alloys sparked the interest of structural engineers and conservators: titanium is among the most chemically stable materials in nature, with no risks of galvanic corrosion, its mechanical characteristics are typically much higher that standard carbon steel, making it suitable for light-weight intervention. Its intrinsic ductility can help to meet the deformation requirements for buildings in areas of high seismic risk: titanium structural members can resist to relatively large deformation demands. Typical mechanical properties of commercially available titanium alloys are summarized in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus (GPa)</th>
<th>Yield Strength YS0.2% (MPa)</th>
<th>Tensile Strength TS (MPa)</th>
<th>Elongation at Failure FE4D (%)</th>
<th>Weight Density (kg/m^3)</th>
<th>Type of Alloy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti Grade 1</td>
<td>102</td>
<td>138</td>
<td>240</td>
<td>24</td>
<td>4500</td>
<td>Pure α</td>
</tr>
<tr>
<td>Ti Grade 2</td>
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<td>275</td>
<td>345</td>
<td>20</td>
<td>4500</td>
<td>Pure α</td>
</tr>
<tr>
<td>Ti Grade 3</td>
<td>102</td>
<td>380</td>
<td>450</td>
<td>18</td>
<td>4500</td>
<td>Almost pure α</td>
</tr>
<tr>
<td>Ti Grade 4</td>
<td>104</td>
<td>483</td>
<td>550</td>
<td>15</td>
<td>4500</td>
<td>Almost pure α</td>
</tr>
<tr>
<td>Ti-6Al-4V ELI</td>
<td>95 - 105</td>
<td>759</td>
<td>828</td>
<td>10</td>
<td>4400</td>
<td>α + β</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>95 – 105</td>
<td>828</td>
<td>895</td>
<td>10</td>
<td>4400</td>
<td>α + β</td>
</tr>
</tbody>
</table>

Table 1: Typical mechanical properties of the most commonly used titanium alloys. Limits for YS0.2%, TS and FE4D from ASTM B348-13 [2].

Figure 2: Examples of titanium profiles: a) round bars, b) tubes, c) square-section bars, d) sheets.

Conservation bodies often restrict the use of modern techniques and materials for repair and reinforcement of historic masonry structures. For example, in the 1990s composite mate-
rials (FRPs) have been found to be a viable solution in order to reinforce shear walls. Many studies have demonstrated that it is possible to improve the seismic response of buildings with FRPs [3-7]. FRPs are typically applied using epoxy adhesives, but these resins are difficult to be removed and the so-called “reversibility” is compromised. One of the most important principles adopted by ICOMOS for interventions of monuments and sites is “Where possible, any measures adopted should be “reversible” so that they can be removed and replaced with more suitable measures when new knowledge is acquired. Where they are not completely reversible, interventions should not limit further interventions” [8].

Figure 3: Reinforcement of the Roman Bridge of Narni, Italy, with titanium rods. Intervention was aimed at containing the effects of the arch thrust.

Figure 4: Consolidation of the bell tower of the St. Mark church, Venice, Italy with titanium rods (design Mr G. Galeazzo)

In this situation, researchers reverted to more traditional metal reinforcements, also considering their positive ductile behavior, ease to application, reversibility and limited cost. In order to improve the effectiveness of the metal reinforcements, new materials and methods have been experimented. An example is the use of ultra-high strength steel fibres [9-13] and stainless steel profiles [14-18]. Composite materials continued to be used for historic masonry reinforcement, but their application was made not with epoxy adhesives, but with metal fasteners or inorganic (lime- or cement-based) mortars [19-23].

However, corrosion problems of steel reinforcements are difficult to solve. Corrosion not only reduces the resisting steel sections (and this could compromise the reinforcement action), but rusting could seriously damage historic masonry members. For these reasons, conservation bodies limit the use of steel reinforcement on listed buildings. Creep is another critical aspect to consider when steel reinforcements are applied.
Corrosion-resistant materials have been investigated: aluminum reinforcements could represent an interesting alternative, but the low-modulus and low-tensile strength of aluminum alloys are problematic.

The use of titanium for reinforcement of historic, both listed and non-listed, masonry structures is not new. However, interventions are still rare and limited. Figures 3 and 4 shows two recent applications. Titanium bars have been inserted inside the masonry of the Roman bridge of Narni, Italy to absorb the arch thrust (Fig. 3). By doing this, it was possible to prevent the overturning of the stone masonry pillar, and stabilize the monument (Fig. 4). Figure 4 shows the bell-tower of the Church of St. Mark in Venice, Italy. A confinement effect to the masonry foundations of the bell-tower was activated by adding a titanium belt, made of titanium rods.

3 EXPERIMENTAL WORK

3.1 The proposed repair method

In this experiment unreinforced brickwork wall (URM) panels were tested in shear and repaired using titanium rods. Crack stitching was carried out using titanium rods, inserted into the horizontal bed joints. To fix the rods to the masonry two types of bonding materials were used: a cement mortar or an epoxy paste. A total number of 5 wall panels were tested: 5 shear tests were carried out on control, unreinforced panels. Two additional tests were conducted on two cracked panels repaired with titanium rods and new repointing cement mortar. Finally, three additional shear tests were carried out on three cracked panels repaired with titanium rods, embedded into an epoxy paste. The total number of shear tests was 10.

<table>
<thead>
<tr>
<th></th>
<th>Mortar</th>
<th>Brick</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight density (kg/m³)</td>
<td>-</td>
<td>1613</td>
</tr>
<tr>
<td>Mix design (lime: sand: cement)</td>
<td>1:2:0.15</td>
<td>-</td>
</tr>
<tr>
<td>Sample dimensions (mm)</td>
<td>160x40x40*</td>
<td>250x120x55*</td>
</tr>
<tr>
<td>Compressive Strength / Standard Deviation (MPa)</td>
<td>6.61 / 1.53</td>
<td>25.3 / 3.77</td>
</tr>
<tr>
<td>Young’s modulus (MPa)</td>
<td>-</td>
<td>7565</td>
</tr>
<tr>
<td>Sample Size</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Bending Strength / Standard Deviation (MPa)</td>
<td>0.22 / 0.042</td>
<td>6.04 / 0.972</td>
</tr>
<tr>
<td>Sample Size</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2: Mechanical properties of the mortar and brick (*nominal dimensions).

Wall panels were assembled in the structures laboratory at the University of Perugia, Italy (Fig. 5). These had the dimensions of 880x880x250 mm. Solid fired clay bricks were used for panel’s construction (nominal dimensions 120x250x55 mm). A low cement mortar was employed for construction. The mortar mix was decided with the aim at reproducing a historic (low-strength) mortar. The mix proportions (by volume) of the mortar (aerial lime: sand: cement) were 1:2:0.15. A mechanical characterization was also performed for both the mortar and the bricks, and results in terms of compressive strength, tensile strength and Young’s modulus are given in Table 2. Mortar samples had dimensions of 40x40x160 mm and were first tested at 90 days after casting in bending and then in compression. The mechanical properties of the solid bricks used for the construction of the wall panels were obtained in the laboratory by performing compression and bending tests (Tab. 2). Results in terms of
compressive strength provided the values of 6.61 and 25.3 MPa, for the mortar and the clay bricks, respectively.

Following initial testing on URM panels, titanium repair was applied and panels re-tested. URM panels typically failed due to the formation of a crack along the panel’s diagonal in compression (see test result section for details). The titanium rods were alternatively inserted (every other bed joint) into the horizontal bed joints, on both panel’s sides, across the diagonal crack. Titanium rods had the nominal diameter of 7 mm, and a length of 250 mm. These were provided by Tifast ltd, Narni, Italy (Fig. 6). The installation phase consisted in the removal of pre-existing mortar along the bed joints of a cracked URM brickwork panel, insertion of the titanium rod and stripping the joint with the new cement mortar or the epoxy paste. All reinforcement materials (cement mortar, epoxy paste and titanium rods) were mechanically characterized before application. Heat elemental analysis and mechanical properties of the titanium rods are reported in Tables 3 and 4.

Before the application of the titanium reinforcement, bed joints were treated using compressed-air with the aim at removing any inconsistent material and improving the bonding between masonry substrate and new mortar or epoxy paste. After this preliminary operation, the new mortar or the epoxy paste was inserted into the cracked bed joints and the titanium rods were embedded (Fig. 7). After the curing period (72 hours and 21 days for the paste and the cement mortar, respectively), the repaired wall panels were re-tested in shear. The mechanical properties of the used cement mortar and epoxy paste are summarized in Table 5.

Figure 5: URM brickwork panels.
Figure 6: Microstructure of 7 mm diameter bars C020872 composed by a fine dispersion of the α and β phases resulting from processing within the α-β field. No continuous β network at prior grain boundaries. No coarse, elongated α platelets. Inclusion free. Etchant: Kroll’s Solution

<table>
<thead>
<tr>
<th>Sampling position</th>
<th>Al</th>
<th>C</th>
<th>Fe</th>
<th>H</th>
<th>N</th>
<th>O</th>
<th>V</th>
<th>Others, each</th>
<th>Others, total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>5.90</td>
<td>0.016</td>
<td>0.10</td>
<td>0.004</td>
<td>0.011</td>
<td>0.120</td>
<td>3.90</td>
<td>&lt; 0.10</td>
<td>&lt; 0.40</td>
</tr>
<tr>
<td>Bottom</td>
<td>5.70</td>
<td>0.016</td>
<td>0.11</td>
<td>0.004</td>
<td>0.009</td>
<td>0.114</td>
<td>3.85</td>
<td>&lt; 0.10</td>
<td>&lt; 0.40</td>
</tr>
</tbody>
</table>

Limits for Grade 23 (ASTM B348)
Min: 5.5 Max: 0.08 Min: 0.25 Max: 0.0125 Max: 0.03 Max: 0.13 Max: 3.5 Max: 0.10 Max: 4.5

Table 3: Heat elemental analysis [wt%] of 7 mm diameter bars C020872.
Table 4: Typical mechanical properties of used titanium rods (*0.2% offset strain method).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of tested samples</td>
<td>6</td>
</tr>
<tr>
<td>Alloy type</td>
<td>Ti-6Al-4V</td>
</tr>
<tr>
<td>Nominal diameter (mm)</td>
<td>7</td>
</tr>
<tr>
<td>Weight density (kg/m³)</td>
<td>4421</td>
</tr>
<tr>
<td>Yield Strength* / Standard Deviation (MPa)</td>
<td>977.5 / 118.3</td>
</tr>
<tr>
<td>Young’s Modulus / Standard Deviation (GPa)</td>
<td>112.9 / 7.06</td>
</tr>
<tr>
<td>Tensile Strength / Standard Deviation (MPa)</td>
<td>1100.6 / 110.0</td>
</tr>
</tbody>
</table>

Table 5: Mechanical properties of the bonding agents used for rod application (*nominal dimensions) [24-25].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mix design (lime: sand: cement)</td>
<td>1:2:1</td>
</tr>
<tr>
<td>Sample dimensions (mm)</td>
<td>160x40x40*</td>
</tr>
<tr>
<td>Compressive Strength / Standard Deviation (MPa)</td>
<td>22.17 / 3.59</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>18.75</td>
</tr>
<tr>
<td>Sample Size</td>
<td>6</td>
</tr>
<tr>
<td>Bending Strength / Standard Deviation (MPa)</td>
<td>0.22 / 0.042</td>
</tr>
<tr>
<td>Sample Size</td>
<td>3</td>
</tr>
</tbody>
</table>

The use of a cement mortar to embed the rods was aimed at achieving reversibility of the proposed retrofitting intervention, using materials with higher compatibility with the brickwork panel. On opposite, the use of the epoxy paste was decided in order to fix the rods with a bonding agent with higher mechanical properties.

3.2 Instrumentation and Test Layout

The diagonal compression test is frequently used to study the in-plane behaviour of masonry walls. In this experiment, a hydraulic cylinder was applied at one of the edges in order to stress the panel in compression along a diagonal until failure occurs. Loading steel shoes were placed at the panel’s corners in order to distribute the diagonal load. The test procedure consisted of loading and unloading cycles.

Four 28 mm-diameter steel rods were used to contrast the diagonal load and a digital pressure transducer was applied near the hydraulic jack to measure the pressure of the oil and con-
sequently the diagonal load. Four inductive linear transducers (LVDTs) were also positioned along the panel’s diagonals to measure its deformation (Fig. 8) under loading. The diagonal compression test is standardized by ASTM [26] and RILEM [27].

During the test, particular attention was paid to avoid local crushing of the masonry near the steel shoes. The shoes (one at each panel’s edge along the loaded diagonal) had the function of transferring and distributing the diagonal load to the wall panel and were purposely designed to avoid this problem, following the instructions of the ASTM standard.

Figure 8: Test layout.

Figure 9: Un-repaired brickwork panel: failure mode.

Failure always occurred starting from a point in the centre of the wall panel, where principal stresses were maximum. This type of test allows to derive the values of the masonry shear strength, shear modulus, diagonal strain in the elastic and post-elastic phases. Furthermore, as the diagonal load was applied cyclically, it was also possible to calculate the shear modulus $G$ for different load cycles. At the end of each diagonal test, the panel’s failure mode was analysed and recorded. Starting from the calculation of the deformations along the diagonals of the wall panel:

$$\varepsilon_c = \frac{\Delta D_1}{2D_1} + \frac{\Delta D_3}{2D_3}, \quad \varepsilon_t = \frac{\Delta D_2}{2D_2} + \frac{\Delta D_4}{2D_4}$$

(1)

where $D_1$ and $D_3$ are the gage lengths of the diagonals in compression, and $D_2$ and $D_4$ are the gage lengths of the ones in traction. $\Delta D_n$ ($n=1,2,3,4$) are the diagonal length elongations/shortenings, recorded using the LVDTs. The angular strain is given by:

$$\gamma = |\varepsilon_c| + |\varepsilon_t|$$

(2)

The shear stress, according to the ASTM instructions, is:

$$S_s = \frac{P}{\sqrt{2} \times A}$$

(3)

and, for the RILEM guidelines:

$$\tau_0 = \frac{(P_d)_{max}}{3 \times A}$$

(4)

where $A$ is the average area between the vertical and horizontal panel’s cross sections and $(P_d)_{max}$ is the maximum diagonal load applied. With regard to the shear modulus, this was evaluated using:
\[ G_{1/3} = 1.05 \frac{(P_d)_{\text{max}}}{3 \times A \times \gamma_{1/3} P_d} \] (5)

where \( \gamma_{1/3} P_d \) is the value of the angular strain in correspondence of 1/3 \((P_d)_{\text{max}}\).

4 TEST RESULTS

In this section, results of cyclic loading shear tests under load control are reported. Tests were carried out using a diagonal load gradient of approx. 0.5 kN/s. Diagonal compressive loads were applied by increasing each cycle of 10 kN until failure (Fig. 9). The diagonal load was applied using a manual pump. A typical load history is reported in Figure 10: the maximum load level for the cycle was also maintained for a duration of 30 seconds and then completely removed. The panels were then left unloaded for further 30 seconds before the application of the next loading cycle.

Each test is identified by an alpha-numeric code: BR-XXX00. The first two letters (BR) identify the construction material (solid bricks for all panels). The three-letter designation XXX (URM for unreinforced control panels, RCE for panels repaired with titanium rods embedded into a cement mortar, and REP for epoxy paste) was used to identify if the panels were unreinforced or repaired. Finally, the wall panels were identified with a number varying between 1 and 5.

4.1 Control panels

Shear failure produced a rapid loss in the lateral stiffness. Test results of all shear tests are summarized in Table 6. Control URM specimens failed due the development of diagonal cracks along the compressed diagonal. A diagonal crack (5-7 mm in width) typically opened in the horizontal and vertical mortar joints, following a zig-zag pattern (Fig. 9). Furthermore, micro-cracks also opened on the side of the diagonal crack. Solid bricks usually resulted undamaged and uncracked at the end of the shear tests.

During the shear test, the solid bricks did not suffer a damage and the crack only opened in the mortar joints or at interface brick-to-mortar. The mean lateral capacity of URM panels was 43.5 kN (corresponding to a shear strength \( \tau_0 \) of 0.0652 MPa, evaluated according to eq. (4)). The Coefficient of Variation (CoV) of the lateral load capacity was 10.1%: this can be considered a small value for masonry, highlighting a reliable result. On opposite, the scattering was higher for the shear modulus \( G_{1/3} \) (mean value 4307 MPa, CoV 27.5%).

![Figure 10: Typical load history.](image1)

![Figure 11: URM wall panel: shear stress vs. angular strain.](image2)

As for control URM panels, shear stress vs. angular strain curves show a quasi-elastic initial behaviour followed by a decrease in the lateral load sustained in the plastic region. The
reduction of the lateral capacity in the plastic region varied between 33 and 43% compared to maximum load (Fig. 11). The non-linear plastic behaviour was clearly produced by the progressive diagonal cracking of the masonry along the compressed diagonal. With increasing angular strains, the mechanical interlocking of the mortar along the cracks highly reduced and this was the main reason for the reduction of the residual lateral capacity.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Max Diagonal Load ( (P_{\text{d}})_{\text{max}} ) (kN)</th>
<th>( (P_{\text{d}})<em>{\text{max, repaired}} / (P</em>{\text{d}})_{\text{max, URM}} )</th>
<th>Shear Strength ( S_0 ) (MPa)</th>
<th>Shear Strength ( \tau_0 ) (MPa)</th>
<th>Angular Strain at 0.33 ( (P_{\text{d}})_{\text{max}} ) (%)</th>
<th>Shear Modulus ( G_{1/3} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR-UNR1</td>
<td>38.1</td>
<td>-</td>
<td>0.121</td>
<td>0.0571</td>
<td>0.01833</td>
<td>3271</td>
</tr>
<tr>
<td>BR-UNR2</td>
<td>45.0</td>
<td>-</td>
<td>0.143</td>
<td>0.0674</td>
<td>0.02252</td>
<td>3151</td>
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<tr>
<td>BR-UNR3</td>
<td>43.9</td>
<td>-</td>
<td>0.140</td>
<td>0.0658</td>
<td>0.01711</td>
<td>4041</td>
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<tr>
<td>BR-UNR4</td>
<td>50.6</td>
<td>-</td>
<td>0.161</td>
<td>0.0758</td>
<td>0.01495</td>
<td>5324</td>
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<tr>
<td>BR-URM5</td>
<td>43.5</td>
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<td>0.138</td>
<td>0.0652</td>
<td>0.01185</td>
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<tr>
<td>(mean)</td>
<td>(44.2)</td>
<td>(0.141)</td>
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<tr>
<td>(CoV)</td>
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<td>(10.1)</td>
<td>(10.1)</td>
<td>(23.4)</td>
<td>(27.5)</td>
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<tr>
<td>BR-RCE1</td>
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<tr>
<td>BR-RCE2</td>
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<td>(mean)</td>
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<td>(0.530)</td>
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<tr>
<td>(CoV)</td>
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<td>(12.5)</td>
<td>(12.5)</td>
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<td>BR-REP3</td>
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<td>0.103</td>
<td>0.0487</td>
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<td>BR-REP4</td>
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<tr>
<td>BR-REP5</td>
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<tr>
<td>(mean)</td>
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<td>(0.108)</td>
<td>(0.0508)</td>
<td>(0.293)</td>
<td>(206)</td>
</tr>
<tr>
<td>(CoV)</td>
<td>(12.0)</td>
<td>(12.0)</td>
<td>(12.0)</td>
<td>(42.0)</td>
<td>(44.7)</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Results of the shear tests (CoV=Coefficient of Variation).

4.2 Repaired panels using the cement mortar

Five repaired wall panels have been tested in this experiment. Titanium rods have been embedded into the cracked bed joints using a cement mortar (2 panels, test No. BR-RCE1 and BR-RCE2) or an epoxy paste (3 panels, test No. BR-REP3, BR-REP4 and BR-REP5). The response of the repaired was found to be very different in terms of lateral capacity and shear modulus, depending on the type of material (cement mortar or epoxy paste) used for rod embedding.

The use of a cement mortar was not able to stitch the crack and to guarantee a satisfactory stress-transfer, without slippage phenomena at interface rod-to-mortar, between the two wall parts of a cracked panel. Few seconds after the application of the diagonal load, for very small values of the diagonal loads (2-4 kN) the transducers (LVDTs) measuring the elongation of the panel’s diagonal in traction started to measure high diagonal strain values. This was the result of the debonding (from both the bricks and the titanium rods) of the cement mortar used to fix the rods inside the horizontal mortar joints. The diagonal crack, previously opened when the URM panel was initially tested, re-opened for a mean diagonal load of 20.3 kN (corresponding to a shear strength \( \tau_0 \) of 0.0304 MPa). It can be noted that this strength value is very similar to the residual load capacity of URM panels, after failure. This clearly demonstrates that the use of a cement mortar is not able to prevent the re-opening of the shear (diagonal) crack, limiting the repair action of the titanium rods. A possible reason for this
unsatisfactory structural behavior is the fact that titanium rods were smooth. In order to overcome this problem, different solutions are under consideration at the moment: the use of longer titanium rods, ribbed rods in order to activate mechanical interlocking, the use of stronger cement mortars. Since the deboning was also recorded at brick-to-mortar interface, the use of ribbed bars could result non-effective.

A similar unsatisfactory result can be noted in terms of shear modulus $G_{1/3}$ and corresponding angular strain: the high values of these mechanical parameters demonstrate again that the titanium rods did not contribute in the resisting action under panel’s diagonal loading, due to poor stress transfer. At the end of the tests, an inspection of the titanium rods highlighted their complete detachment from the cement mortar with no damage.

### 4.3 Repaired panels using the epoxy paste

The use of the epoxy paste to fix the rods into the horizontal mortar joints produced different test results. Three damaged (cracked) panels have been repaired according to this procedure. Figures 12 and 13 shows the shear stress – angular strain response, and load history of the repaired panel BR-REP5. This panel exhibited a linear elastic response up to a shear stress of approx. 0.12 MPa. The mean lateral capacity of the three repaired wall panels was 33.9 kN, corresponding to a shear strength of 0.0508 MPa. If we compare these results with the values recorded for URM panels (44.2 kN and 0.0662 MPa, respectively), it can be noted that the titanium rod repair was not able to restore the original lateral capacity of URM panels. However, the titanium rods were effective in crack stitching: the combined action of the titanium rod and the epoxy resin prevented the re-opening of the crack in the original position and no slippage phenomena were recorded at interface brick-to-resin or resin-to-rod.

In general, up to 88% of the original lateral capacity was exhibited by the repaired panels. The failure mode of the repaired panels consisted in the formation of a new diagonal crack (along the diagonal in compression), outside the area where the rods have been previously installed. This crack only opened in the mortar joints, without affecting the bricks and the bed joint repaired with the titanium rod.

It could be interesting to comment on the reduced lateral capacity of the repaired panels, compared to URM panels: firstly, it should be remarked here that the titanium rods were only located along the superficial area of bed joints (on both panel’s sides): the cracked vertical joints and the internal area of the cracked panels were not affected by the application of the rods. Furthermore, only every other bed joint was repaired with the rods: all this clearly partially weakened the effectiveness of the repair method, and it provides an explanation for the missing ability to restore the original lateral capacity after repair.

![Figure 12: Load history of Test No. BR-REP5.](image1)

![Figure 13: Test No. BR-REP5: shear stress vs. angular strain.](image2)
From the observation of the panels at the end of the shear tests, it can be observed that no-cracking occurred in the area repaired with the titanium rods and the epoxy paste. Perfect rod-to-brick bonding across the crack was guaranteed by the epoxy. The failure mode of the URM walls was not limited by the formation of a single, diagonal crack: micro-cracking and a diffuse damage occurred inside the panel. The localized application of the titanium rod-repair wasn’t able to repair the damage outside a limited panel’s area. It is evident that the effectiveness of the repair depends on the level of damage of the URM panels: when this is localized (i.e. a single shear crack) the repairing method is more effective. On opposite, when the damage is made of a large number of cracks, it is more difficult to restore the original lateral capacity after rod repair.

A similar comment could be made by looking at the shear moduli of both URM and repaired wall panels. The use of high Young’s modulus repair materials (52.1 and 112.9 GPa for the epoxy and the titanium rods, respectively, compared to the Young’s modulus of the brickwork) was not able to increase the panel’s lateral stiffness, given the diffuse damage and crack pattern. The non-linear (un-elastic) response of URM wall panels after cracking highly reduced the lateral stiffness of the repaired panels, and the repair was not able to restore the original pre-damage stiffness. Again, the titanium rods ability to restore the original shear stiffness depends on the level and extension of the damage.

5 CONCLUSIONS

Cyclic load shear tests were performed on two series of unreinforced masonry wall panels repaired with titanium rods, placed every other bed joint, using a cement mortar or an epoxy paste.

Based on this study, it was found the application of titanium rods with a cement mortar did not restore the original shear stiffness and lateral load capacity of URM brickwork wall panels. Comparison of destructive load test results both before and after repair shows that the use of titanium rods was also not effective in restoring the elastic angular strain experienced by the URM brickwork panels. Deboning of the titanium rods from the cement mortar and reopening of diagonal cracks compromised the effectiveness of the titanium rod repair.

The use of an epoxy paste for rod application partially overcome the above mentioned issues. From the results of this experimental study on the shear behavior of masonry wall panels with incorporated epoxy-bonded titanium rods subjected to static-cyclic shear loading the following conclusions can be drawn:

1. Repaired panels were able to exhibit a lateral capacity up to 88% of the original pre-damage one of URM panels. The effectiveness of the rod reinforcement depended on the level and extension of the damage (i.e. number, diffusion and position of diagonal (shear) cracks);
2. The ability of the rod repair to restore the shear stiffness of URM walls is very limited, given the localized application of the rod repair, leaving unrepaird the internal area of the wall panels and the vertical joints;
3. At 1/3 of the maximum diagonal compressive load, angular strains of repaired panels increased by 17.3 times. The shear tests confirmed that the titanium repair and the brickwork material are currently acting compositely (with the titanium bars absorbing tensile stresses and the brickwork material the compressive ones). No debonding phenomena were observed at interface brick-to-epoxy and epoxy-to-rod.

In conclusion, the shear strength was higher for the wall panels repaired with the epoxy paste, indicating that the stress transfer between the titanium rod and the masonry can be better achieved using when the epoxy paste is used for rod application. These findings contribute
to the ongoing research of the use of advanced materials for rehabilitation purposes of historic masonry structures. Testing on masonry walls using titanium alloys will be continued, as future results will be very useful in understanding the structural behavior of the masonry buildings repaired and reinforced with titanium elements.

ACKNOWLEDGEMENTS

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REFERENCES


Abstract

The recurrent earthquakes which recently stroked the center of Italy caused severe damages to the built heritage showing extensive disaggregation mainly due to poor masonry quality. In this phenomenon, the vertical seismic accelerations played a crucial role. The aim of this study is to define a methodology for considering the effects of the vertical seismic component in the seismic assessment of masonry buildings through kinematic and pushover analysis. In pushover analysis, inertial forces caused by the vertical accelerations are considered in the safety verifications of the elements resulting in a drop of the global resistance and displacement capacity of the structure. The proposed methodology is applied to the case study of a two-stories masonry building for several levels of seismic action and for different levels of masonry quality. The results of the analyses performed with and without the vertical seismic component are compared and discussed. Then, the methodology is applied to the seismic assessment of a real masonry building through pushover analysis of the current state and evaluation of possible strengthening measures. The effect of the vertical seismic component in both configuration of the building is discussed.

Keywords: Seismic assessment, Existing Masonry Buildings, Vertical Component, Pushover Analysis, Kinematic Analysis, Equivalent Frame Method
1 INTRODUCTION

The severe damages caused by recent earthquakes in the center of Italy confirmed the high vulnerability of existing masonry buildings (Figure 1). The damage investigation carried out in recent studies highlighted, among other aspects, the crucial role played by the vertical seismic component in the loss of ductility and consequent masonry disaggregation [1].

Figure 1. Extensive damages caused by earthquakes at Pescara del Tronto (top), Accumoli (bottom left) and Castelluccio di Norcia (bottom right)

Figures 2-4 show the accelerograms related to the earthquakes of L’Aquila 2009, Emilia 2012 and Norcia 2016. The graphs represent the horizontal (red) and vertical (blue) component of the seismic action in the first 5 second of the seismic excitation. It is evident that in all these earthquakes the vertical seismic component played an important role.

Figure 2. L’Aquila 2009. Horizontal and vertical component of seismic action
Among the damages caused by the earthquakes, several masonry buildings showed pure shear failures with clear separation of horizontal mortar beds and sliding for several centimeters (Figure 5). These types of failures typically occur where the walls feature variation of horizontal cross section due to the presence of openings or floors, or for discontinuity in elevation. The phenomenon can be explained considering the combined action of vertical acceleration that lightened the structure and horizontal acceleration that caused shear action in masonry piers.

Therefore, the effects of the vertical seismic component on the seismic capacity of existing masonry building should be taken into account. Previous studies already confirmed that in other structural typologies failure may ensue due to direct tension or compression as well as due to the effect of vertical motion on shear and flexural response [2].
The objective of this work is to define a methodology viable for professionals for considering the vertical seismic component in the seismic assessment of masonry building through kinematic and pushover analysis. In Pushover analysis the effects of the vertical seismic component are modelled through a field of vertical inertial forces resulting from modal response spectrum analysis. During the incremental analysis, safety verifications applied to the elements are carried out combining the internal actions induced by the vertical forces with those arising from static and incremental horizontal loading.

The proposed methodology is applied to the case study of a two-story masonry building considering different levels of masonry quality and several levels of seismic intensity.

Then, further aspects of the methodology are introduced in order to make it viable for professional application in the assessment of real masonry buildings: (a) effects of compression and decompression cycles due to vertical seismic excitation, (b) capacity of the structure in terms of PGA considering the variability of the vertical seismic effects. This defines an algorithm for considering the vertical seismic effects in any professional software able to perform kinematic and pushover analyses of masonry buildings.

The complete methodology is applied to the seismic assessment of a real masonry building evaluating the effects of possible strengthening interventions.

2 EFFECTS OF VERTICAL SEISMIC COMPONENT ON SEISMIC ASSESSMENT OF MASONRY BUILDINGS

The vulnerability of an existing masonry building is first and foremost conditioned by the quality of masonry itself. Under seismic loading, masonry of poor quality is very likely to develop phenomena of disaggregation which can hardly be assessed by any mechanical model. Therefore, the seismic assessment of masonry buildings cannot prescind from an accurate analysis of the masonry quality and only if disaggregation phenomena are prevented it makes sense to investigate other failure mechanisms.

The next source of vulnerability for masonry structures is associated to local mechanisms mainly due out-of-plane behavior of walls. The seismic response of the building is governed by such mechanisms when connections between orthogonal walls and between walls and floors are particularly poor. Only if connections are improved by proper devices (e.g. tie-rods), local mechanisms can be prevented, and a global behavior governed by the wall in-plane behavior can develop.

In summary, safety assessment of a masonry building could be performed in three consequential phases:
1. Analysis of masonry quality in order to prevent eventual phenomena of disaggregation and define adequate mechanical parameters
2. Analysis of local collapse mechanisms through kinematic analysis
3. Analysis of in-plane response of masonry elements through pushover analysis. The analysis may be applied to single walls or global structure depending on the quality of connections between the elements.

Seismic assessment of an existing masonry building highlights its vulnerability and allows to define an adequate plan of strengthening interventions. Therefore, in order to assess the real capacity of the structure it is important to consider all the effects of the seismic action.

The vertical component of the seismic action affects negatively all the phases of the seismic assessment: the vertical dynamic actions accentuate eventual masonry disaggregation and tend to worsen the capacity of the structural elements with respect to local and global failure mechanisms.
In the next paragraphs a methodology for considering the effects of the vertical seismic component in kinematic and pushover analysis is presented and applied to the case study of a two-story masonry building.

3 KINEMATIC ANALYSIS

According to current Standards, Linear Kinematic Analysis is performed with the following steps:
1. Definition of the collapse mechanism: axes of rotations, participating bodies, forces.
2. Calculation of the collapse multiplier of the seismic action and the correspondent spectral acceleration that activates the mechanism.
3. Calculation of the capacity in terms of PGA, that is the ground acceleration correspondent to the spectral acceleration activating the mechanism.

In this context, the effects of the vertical seismic component may be modelled as a field of inertial forces directed upwards which tend to anticipate the activation of the mechanism. As a result, the collapse multiplier and the capacity in terms of PGA decrease.

Consider the mechanism in Figure 6, simple overturning of a masonry wall. The forces involved in the mechanism are the self-weight of the rigid body P, the horizontal inertial force \( \alpha P \) and the vertical inertial force \( \alpha_v P \).

![Figure 6. Simple overturning of masonry wall with horizontal and vertical inertial forces](image)

Both inertial forces are proportional to the self-weight. The ratio between the multiplier of vertical inertial force \( \alpha_v \) and the multiplier of horizontal inertial force \( \alpha \) assumes different values depending on whether the element is considered isolated or resting on the ground (rigid system) or it is located at a certain elevation on the building (deformable system):

\[
\frac{\alpha_v}{\alpha} = \frac{k \cdot S_{ez}(0)}{S_{ex}(0)} \quad \text{in case of rigid system}
\]

\[
\frac{\alpha_v}{\alpha} = \frac{k \cdot S_{ez}(0)}{S_{ex}(T_1) \cdot \Psi(Z) \cdot \gamma} \quad \text{in case of deformable system}
\]

where: \( S_{ex}(T) \) and \( S_{ez}(T) \) are the values of the elastic response spectra of horizontal and vertical acceleration for period \( T \); \( T_1 \) is the fundamental period of vibration of the whole structure in the horizontal direction; \( \Psi(Z) \) is the correspondent mode of vibration normalized so to be 1 at the top of the building; \( \gamma \) is the correspondent modal participation factor; \( Z \) is the
elevation of the rotation axis; \( k \) is the combination factor of the vertical component of the seismic action. \( k \) may be taken equal to 0.3 considering the indication given in [8] (4.3.3.5.2), or equal to 1.0 considering that the maximum horizontal and vertical accelerations may occur simultaneously.

Note that in both expressions the multiplier of vertical inertial forces is considered proportional to \( S_{\text{ez}}(0) \), the elastic response spectrum of vertical acceleration for \( T = 0 \). This because in the proposed methodology the multiplier of vertical inertial forces is always considered equal to the one on rigid system.

The collapse multiplier of the horizontal inertial forces \( \alpha_0 \) can be calculated applying the principle of virtual works, with the following relations:

\[
\begin{align*}
LV1 + \alpha \cdot LV2 + \alpha_v \cdot LV3 &= 0 \\
LV1 + \alpha_0 \cdot LV2 + \alpha_0 \cdot \frac{\alpha_v}{\alpha} \cdot LV3 &= 0 \\
\alpha_0 &= \frac{-LV1}{LV2 + \frac{\alpha_v}{\alpha} \cdot LV3} 
\end{align*}
\]

where: \( LV1, LV2 \) and \( LV3 \) are respectively the virtual works of static forces, horizontal inertial forces and vertical inertial forces obtained considering \( \alpha = 1 \) and \( \alpha_v = 1 \).

As said, the ratio \( \alpha_v/\alpha \) varies depending on whether the system underneath is considered rigid or deformable, but it also varies depending on the Limit State and the level of seismic intensity. Therefore, in the iterative procedure for the calculation of the capacity in terms of PGA the collapse multiplier must be always recalculated based on the current ratio \( \alpha_v/\alpha \). However, given the fact that virtual works \( LV1, LV2, LV3 \) remain constant during the iterations, the calculation of the collapse multiplier is not demanding in terms of computational effort.

\section*{4\hspace{1em}PUSHOVER ANALYSIS}

In Pushover analysis the seismic capacity of the structure is described by its behavior under a system of incremental forces which should simulate in the best possible way the inertial forces resulting from the seismic action in the horizontal direction. The capacity of the structure is represented by the capacity curve which is a plot of the total base shear versus the displacement of the control point (normally taken as the center of mass of the roof). In order to assess the vulnerability of the structure several pushover curves must be elaborated for different directions of the seismic action, distributions of lateral forces, effects of accidental eccentricity (modelled as additional twisting moments) and different control points.

Each pushover curve is elaborated through an incremental non-linear procedure performing a series of linear static analyses and keeping the model constantly updated in order to account for the stiffness reduction of the elements which enter the plastic range or reach collapse. According to the current Standards [7], masonry elements are modelled with an elastic-plastic bilinear behavior where the end of the elastic branch is determined by the minimum resistance in terms of different failure mechanisms (bending or shear) and the ultimate displacement is defined through a limit drift (ultimate chord rotation at the two ends of the elements). At each step of the incremental procedure, safety verifications are applied to the elements and whenever the internal actions overcome the resistance, or the deformations overcome the limits, the model is updated accordingly.

Once the pushover curve has been elaborated, each limit state may be associated to a specific point of the curve finding the related capacity in terms of displacement. The target displacement is then defined based on the displacement demand of an equivalent single-degree-of-freedom system derived from the elastic response spectrum. Knowing both capacity and demand in terms of displacement the safety verification can be applied by comparing the two. Moreover,
considering that the elastic response spectrum varies with the peak ground acceleration, by iteration one can find which is the capacity of the structure in terms of PGA and calculate the seismic risk index $\zeta_E$ as the ratio between capacity and demand in terms of PGA.

The objective of this work is to define a methodology in agreement with current Standards and viable for professionals for considering the vertical component of the seismic action in Pushover analysis. Previous studies proposed an improvement of the traditional pushover analysis taking into account the inertial forces caused by the vertical earthquake. The method was validated through nonlinear time-history analysis [3].

The methodology proposed in this work models the vertical seismic component through a field of vertical spectral forces derived from modal response spectrum analysis considering CQC combination and the elastic response spectrum of vertical acceleration. At each step of the incremental analysis safety verifications are applied to each element combining the effects of static and incremental horizontal loading with those arising from the vertical spectral forces. The latter must be considered both upwards and downwards in order to simulate the effects of the vertical excitation, thus, the verifications are applied twice considering each time the most severe effect. The decompression induced by the forces directed upwards results in a reduction of shear and bending moment resistance while the overpressure induced by the forces directed downward may anticipate a compression failure [5].

According to current Standards [6,8] the action effects due to the combination of the seismic components may be computed combining 100% of the effects in one direction with 30% of the effects in the other directions. The application of this combination to the proposed methodology would imply that the effects of the vertical component should be reduced to 30% since the pushover analysis account for 100% of the effects of the horizontal component. However, for the purposes of this work, the effects of the vertical component will not be reduced since the analysis of the accelerograms of recent earthquakes showed numerous impulses where the maximum ground accelerations in the three directions occur simultaneously.

### 4.1 Ultimate drift of masonry piers

As stated previously, the shear-displacement behavior of masonry elements is considered bilinear elastic-plastic and the ultimate displacement is defined as a limit drift, that is a limit in terms of chord rotation at the ends of the element. According to current Standards [7] the limit drift for unreinforced masonry piers at the ultimate limit state may be taken as:

$$\delta_u = 0.0125 \cdot (1 - \nu) \geq 0.01, \text{ where } \nu = \frac{\sigma_0}{f_a} \quad \text{in case of bending failure}$$

$$\delta_u = 0.005 \quad \text{in case of shear failure}$$

Thus, the current Standards, in case of bending failure consider that the ultimate drift may decrease for high values of compression but in case of shear failure and in any case for low compression they provide a fixed value of the limit.

It is understood that the vertical seismic component leads to a drop of resistance with respect to bending and shear mechanisms [4], it shall though be investigated whether it affects also the displacement capacity (ultimate drift) of the elements. Works based on the observation of the damages caused by recent earthquakes showed that the vertical seismic excitation leads to loss of ductility in masonry elements [1]. Since literature does not provide specific experimental evidence, in the proposed methodology the provisions of the current Standards in terms of ultimate drift have been integrated assuming that, applying the vertical seismic component, ductility remains constant.
Figure 7 shows in dashed line the shear-drift behavior of a masonry pier according to current Standards. When the pier reaches the ultimate resistance \( F_u; \delta_e \) it continues to sustain the same action until it reaches the ultimate drift \( \delta_u \). The ductility of the pier is defined as the ratio between the ultimate drift and the drift at the end of the elastic branch:

\[
\mu = \frac{\delta_u}{\delta_e}
\]  

The solid line represents the behavior of the same piers under the effects of the vertical component according to the proposed methodology. This time the ultimate resistance drops to \( F_{uV} \) which correspond to the drift \( \delta_{eV} \). The ultimate drift \( \delta_{uV} \) is obtained assuming that the ductility of the elements remains constant. Therefore:

\[
\delta_{uV} = \delta_{eV} \cdot \mu
\]

In this way, every time a masonry pier reaches the ultimate resistance, the ultimate drift is set accordingly based on the value provided by the Standards and reduced in order to account for the effects of the vertical seismic component.

5 CASE STUDY

In order to evaluate the negative effects of the vertical seismic component, the proposed methodology was applied to the case study of a two-story rural building consisting of rubble stone masonry. Pushover analyses were performed with and without the vertical seismic component considering different levels of masonry quality and several levels of seismic action.

The building model shown in Figure 8 features a rectangular floor plan of 13.90\times7.40 \text{ m} with a constant wall thickness of 40 cm. The height of the ground floor is 3.60 m while the first floor with gable roof features eaves height of 3.10 m and ridge height of 3.60 m. Openings are not aligned among the two storeys apart from the ones on the west elevation of the building. Floor and roof consist of timber beams topped with 5 cm concrete slab well connected to the perimetral walls.
For the purposes of this work, modelling and analyses of the building were performed using the commercial software Aedes.PCM [9] which implements the proposed methodology. The building was modelled according to the equivalent frame method where the structure is discretized in a set of masonry panels (piers and spandrels) connected through rigid links. However, the proposed methodology applies to any modelling strategy and the analyses may be carried out with any software able to perform modal and pushover analysis of masonry buildings.

The main characteristics of the model considered in this work are as follows.
- Masonry piers are modelled with a trilinear in-plane behavior consisting of two elastic branches and one perfectly plastic. When tensile stresses appear in the cross-section shear and stiffness are reduced by 50% and the element enters the second elastic branch. Then, when the element reaches ultimate resistance in terms of shear or flexure mechanisms plastic hinges are introduced and the element enters the plastic branch. In this last branch of the shear-displacement behavior, shear and bending moment remain constant until the element reaches ultimate deformation.
- In order to simplify the analysis, masonry spandrels are considered able to couple masonry piers only with respect to horizontal translation, thus the rotations are released at both their ends.
- Restraints. Joints at the foundation of the building are assumed fully fixed.
- Vertical loading. The intermediate slabs carry a dead load of 2.45 kN/m² and a live load of 2.00 kN/m² (cat. A). The roof carries a dead load of 2.45 kN/m² and a live load of 1.20 kN/m² (snow). Since the timber beams span along the longitudinal direction of the building.
building, all the slabs distribute 90% of the loads to the transversal walls and 10% to the longitudinal ones.

- Verifications. The in-plane resistance of masonry piers is governed by the following mechanisms: flexure, diagonal shear. Considering the relevant thickness of the piers (40 cm) their transversal stiffness is also accounted in the analysis and out-of-plane flexure verification applied at both their ends.

The walls of the building consist of rubble stone masonry. In order to evaluate the effects of the vertical seismic component, six different levels of masonry quality were considered in the analysis. The mechanical properties associated to each level are based on the reference values for rubble stone masonry provided by the Italian Standards [7]. Assuming a knowledge level KL2, the reference values of resistance and moduli of elasticity were taken as the mean values of the provided range and a confidence factor CF = 1.2 was accounted in the analysis. Corrective coefficients related to specific characteristics or strengthening measures (also provided by the Standards) were applied to create a scale of quality levels as specified in Table 1.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
<th>E</th>
<th>G</th>
<th>f_m</th>
<th>(\tau_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>core of poor quality</td>
<td>783</td>
<td>261</td>
<td>1.35</td>
<td>0.0225</td>
</tr>
<tr>
<td>A2</td>
<td>standard condition</td>
<td>870</td>
<td>290</td>
<td>1.50</td>
<td>0.0250</td>
</tr>
<tr>
<td>A3</td>
<td>mortar of good quality</td>
<td>1305</td>
<td>435</td>
<td>2.25</td>
<td>0.0375</td>
</tr>
<tr>
<td>A4</td>
<td>mortar of good quality and lacing courses</td>
<td>1305</td>
<td>435</td>
<td>2.92</td>
<td>0.0487</td>
</tr>
<tr>
<td>A5</td>
<td>reinforced mortar coating</td>
<td>2175</td>
<td>725</td>
<td>3.75</td>
<td>0.0625</td>
</tr>
<tr>
<td>A6</td>
<td>best possible interventions</td>
<td>3045</td>
<td>1015</td>
<td>5.25</td>
<td>0.0875</td>
</tr>
</tbody>
</table>

Table 1. Mechanical properties of six different level of masonry quality: modulus of elasticity (E), shear modulus (G), mean compressive strength (f_m), initial shear strength under zero compression (\(\tau_0\)). Values in N/mm²

The building is assumed located in Perugia (Italy). The Italian Standards [6] provide, for each location of the Italian territory and for different return periods \(T_R\), the reference parameters of the seismic action and the methods to consider stratigraphic and topographic amplifications. The parameters of the elastic response spectra of horizontal and vertical accelerations with respect to the Ultimate State of Severe Damage (\(T_R = 475\)) are given in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground acceleration</td>
<td>(a_g)</td>
<td>0.186</td>
</tr>
<tr>
<td>Soil factor</td>
<td>S</td>
<td>1.200</td>
</tr>
<tr>
<td>Maximum amplification factor</td>
<td>F</td>
<td>2.425</td>
</tr>
<tr>
<td>Periods</td>
<td>(T_B) 0.103</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(T_C) 0.310</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>(T_D) 2.344</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 2. Seismic parameter for \(T_R = 475\) years
One of the aims of this work is to evaluate how the effects of the vertical seismic component vary considering increasing levels of seismic intensity. Therefore, the analyses were performed considering 13 different levels of seismic action. The parameters of the response spectra associated to each level were obtained from the reference values provided by the Standards considering increasing values of horizontal and vertical ground accelerations assuming that all the other parameters of the spectra remain constant.

Table 3 provides the values of ground acceleration for the 13 different levels of intensity considered in the analysis. The values are given with respect to a return period $T_R=475$ years (the parameters associated to other return periods necessary for the calculation of the seismic risk index are updated accordingly).

<table>
<thead>
<tr>
<th>Level</th>
<th>$a_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.186 g</td>
</tr>
<tr>
<td>2</td>
<td>0.250 g</td>
</tr>
<tr>
<td>3</td>
<td>0.300 g</td>
</tr>
<tr>
<td>4</td>
<td>0.350 g</td>
</tr>
<tr>
<td>5</td>
<td>0.400 g</td>
</tr>
<tr>
<td>6</td>
<td>0.450 g</td>
</tr>
<tr>
<td>7</td>
<td>0.500 g</td>
</tr>
<tr>
<td>8</td>
<td>0.550 g</td>
</tr>
<tr>
<td>9</td>
<td>0.600 g</td>
</tr>
<tr>
<td>10</td>
<td>0.650 g</td>
</tr>
<tr>
<td>11</td>
<td>0.700 g</td>
</tr>
<tr>
<td>12</td>
<td>0.750 g</td>
</tr>
<tr>
<td>13</td>
<td>0.800 g</td>
</tr>
</tbody>
</table>

Table 3. Horizontal and vertical ground acceleration for 13 different levels of seismic action with respect to $T_R = 475$ years.
5.1 Modal Analysis

Modal analysis was performed considering the diaphragm actions provided by the rigid levels. The results of the analysis are given in Table 4, while Figure 10-12 show the deformed shapes associated to the fundamental mode of vibration in X Y and Z direction.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue (rad/sec)$^2$</th>
<th>Period (sec)</th>
<th>Participating mass ratio (%)</th>
<th>Participating mass ratio (progressive total %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>4.13E+02</td>
<td>0.309</td>
<td>0.356</td>
<td>76.240</td>
</tr>
<tr>
<td>2</td>
<td>4.36E+02</td>
<td>0.301</td>
<td>92.813</td>
<td>0.016</td>
</tr>
<tr>
<td>3</td>
<td>7.64E+02</td>
<td>0.227</td>
<td>1.086</td>
<td>15.680</td>
</tr>
<tr>
<td>4</td>
<td>3.65E+03</td>
<td>0.104</td>
<td>0.000</td>
<td>7.619</td>
</tr>
<tr>
<td>5</td>
<td>4.43E+03</td>
<td>0.094</td>
<td>5.482</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>6.10E+03</td>
<td>0.080</td>
<td>0.083</td>
<td>0.066</td>
</tr>
<tr>
<td>7</td>
<td>1.12E+04</td>
<td>0.059</td>
<td>0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>8</td>
<td>1.14E+04</td>
<td>0.059</td>
<td>0.033</td>
<td>0.007</td>
</tr>
<tr>
<td>9</td>
<td>1.22E+04</td>
<td>0.057</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>1.23E+04</td>
<td>0.057</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>11</td>
<td>1.35E+04</td>
<td>0.054</td>
<td>0.000</td>
<td>0.008</td>
</tr>
<tr>
<td>12</td>
<td>1.44E+04</td>
<td>0.052</td>
<td>0.007</td>
<td>0.039</td>
</tr>
<tr>
<td>13</td>
<td>1.51E+04</td>
<td>0.051</td>
<td>0.000</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 4. Modal analysis results

Figure 10. Fundamental mode of vibration in X direction
It is interesting to observe the high frequencies associated to the vertical modes of vibration. In order to activate 85% of the total mass in each direction and consider all the modes with at least 5% participating mass it was necessary to use all the 13 modes listed in Table 4. For each level of seismic intensity considered in the analysis, the vertical dynamic forces associated to each mode of vibration were calculated using the elastic response spectrum of vertical acceleration and combined through CQC method. The resulting field of forces is represented in Figure 13. The internal actions arising from this field of forces were taken into account at each step of Pushover analysis in the safety verifications of masonry elements.
5.2 Pushover analysis

Pushover analyses were performed for 6 different levels of masonry quality and 13 different levels of seismic action, thus for a total of 78 cases.

For each case the analysis was performed through elaboration of 16 pushover curves differing for:

- distributions of horizontal forces: a linear distribution with forces proportional to masses and elevations and a uniform distribution with forces proportional only to masses
- directions: the positive X and Y direction, respectively the longitudinal and transversal direction of the building
- effects of accidental eccentricity: considering additional twisting moment applied clockwise and counterclockwise
- effects of the vertical seismic component: curves with and without the effects of the vertical forces

The results of the analyses will be compared with respect to the Seismic Risk Index $\zeta$, that is the ratio between capacity and demand in terms of PGA. For the purposes of this work the capacity in terms of PGA was calculated assuming that the effects of the vertical seismic component remain constant throughout the iterative procedure. For professional applications a method for considering the variability of the vertical seismic effects in the calculation of the capacity in terms of PGA is provided in §6.1.2.

Figure 14 shows two of the pushover curves elaborated for masonry A6 considering the first level of seismic intensity ($a_g = 0.186$ g). The curves refer to the analysis in +X direction with linear distribution of horizontal forces and twisting moments applied counterclockwise. The blue line represents the curve without the effects of the vertical seismic component while the red line is the curve obtained considering those effects. The point shown in both curves represents the capacity with respect to the Ultimate Limit State of Severe Damage while the dashed line represents the relevant displacement demand.
It is evident that the vertical seismic component produces a reduction of global resistance and displacement capacity. Specifically, the maximum base shear drops by 26% while the displacement capacity at the ULS drops by 11%. At the same time the displacement demand increases by 34%. Finally, the Seismic Risk Index $\zeta_E$, that is the ratio between capacity and demand in terms of PGA, drops from 1.381 to 1.081 with a reduction of 22%.

Figure 15 shows the configuration of the structure at the last step of the pushover curve with the effects of the vertical seismic component. The different colors represent the state of the piers: elastic (green), partially plastic (yellow), plastic (orange), collapsed (red).
The following considerations are based on the minimum values of the Seismic Risk Index arising from the elaboration of all the pushover curves for each of the 78 cases considered.

Figures 16-21 show for each level of masonry quality the variation of the Seismic Risk Index with the level of seismic action represented by the vertical ground acceleration $a_{gV}$. The diagrams show with blue bars the Seismic Risk Index obtained without the vertical seismic component and with red bars the value obtained considering those effects.
Figure 18. Variation of Seismic Risk Index for masonry A3

Figure 19. Variation of Seismic Risk Index for masonry A4

Figure 20. Variation of Seismic Risk Index for masonry A5
The Seismic Risk Indexes decrease as the ground acceleration increases and the application of the vertical seismic component always leads to lower indexes. Moreover, as the ground acceleration increases also the gap between the two cases increases.

This behavior is more evident in Figure 22 that shows for each levels of masonry quality the reduction of the Seismic Risk Index caused by the application of the vertical seismic component. As expected, the reduction increases with the value of the ground acceleration $a_g V$. However, we also notice that the reduction is higher for masonries of better quality, meaning that the vertical seismic component tends to reduce the beneficial effects of eventual strengthening interventions.

The diagrams in Figures 16-21 may be assembled in the following diagrams showing the variation of the Seismic Risk Index with the level of seismic intensity and the level of masonry quality in the cases with and without the vertical seismic component.
Figure 23. Variation of the Seismic Risk Index with levels of masonry quality and seismic action ignoring the vertical seismic effects.

Figure 24. Variation of the Seismic Risk Index with levels of masonry quality and seismic action considering the vertical seismic effects.
When the effects of the vertical seismic component are ignored (Figure 23) the Seismic Risk Index decrease as the ground acceleration increase while it increases with the level of masonry quality. Considering the effects of the vertical seismic component (Figure 24) the improvement obtained with masonries of better quality is still evident but not so sharp like in the other case.

Figure 25 shows the variation of the Seismic Risk Index with the quality of masonry for the first level of seismic action \( (a_{AV} = 0.186 \text{g}) \). In both cases the indexes tend to increase with the quality of masonry. However, considering the vertical seismic effects the increment is more moderate.

![Figure 25. Variation of the Seismic Risk Index with the level of masonry quality for the first level of seismic action \( (a_{AV} = 0.186 \text{g}) \).](image)

6 PROFESSIONAL APPLICATION

The proposed methodology for considering the vertical seismic component in the assessment of masonry buildings may be applied to real cases taking into accounts the aspects highlighted in §6.1. The safety assessment of a real masonry building with the application of the proposed methodology is presented in §6.2.

6.1 Specific aspects of the methodology

6.1.1 Influence of compression and decompression cycles on resistance of masonry piers

In pushover analysis the progressive deterioration caused by horizontal and vertical components of the seismic action must be addressed in order to adequately represent the real masonry behavior. During seismic excitation numerous cycles of compression and decompression induced by the vertical seismic component may determine irreversible deterioration and decohesion of mortar joints. The poorer the quality of mortar, the more important this phenomenon is.

The phenomenon could be represented through reduction of the compressed part of the cross section caused by decompression. This leads to reduction of bending and shear resistance provided that the sliding shear mechanism is considered in the model. In irregular masonries this is often not the case due to the absence of a clearly horizontal mortar bed that could activate
this failure mechanism. In the proposed methodology, in order to account for decohesion of the mortar joints caused by the vertical cycles, verification with respect to sliding shear mechanism is applied also for irregular masonries and the value of initial shear strength under zero compressive stress is assumed equal to minimum value provided by the current Standards, that is $f_{v0}=0.07 \text{ N/mm}^2$.

![Figure 26. Possible sliding shear failure in uncut stone masonry caused by compression and decompression cycles due to vertical seismic action](image)

### 6.1.2 Capacity of the structure in terms of PGA

Once pushover curve has been elaborated it allows to perform a safety verification of the structure comparing the capacity and the seismic demand in terms of displacement. Considering that the displacement demand varies with the value of PGA, the pushover curve allows to find the capacity of the structure in terms of PGA and calculate the Seismic Risk Index $\zeta_E$ as the ratio between capacity and demand.

In pushover curves that ignore the vertical seismic component the capacity in terms of PGA can easily be found through an iterative procedure applying the displacement verification for different values of the demand. During this iteration the pushover curve remains the same because it is an intrinsic property of the structure and does not depend on the seismic demand.

This is not the case in pushover curves elaborated considering the vertical seismic component, given that the vertical spectral forces are calculated based on the elastic response spectrum of vertical acceleration. Therefore, the pushover curve depends on the seismic demand, as a result the iterations to find the capacity in terms of PGA would require re-elaboration of the pushover curve at each step for different values of the seismic demand. This procedure would be very demanding in terms of computational effort and hardly feasible in professional applications. For this reason, this work proposes an alternative manner for individuating the capacity in terms of PGA based on a linear interpolation between the results obtained with and without the vertical seismic component.
Consider the graph in Figure 27, where the $a_{gV}$ axis represents vertical ground acceleration and the $a_g$ axis represents horizontal ground acceleration. Point 1 ($a_{gV1}$; $a_{g1}$) represents the capacity of the structure in terms of horizontal ground acceleration resulting from a curve without the vertical seismic component ($a_{gV1} = 0$). Point 2 ($a_{gV2}$; $a_{g2}$), instead, represents the capacity of the structure resulting from a curve where the applied vertical ground acceleration is equal to the seismic demand (in this case the capacity in terms of horizontal ground acceleration $a_{g2}$ has been calculated assuming that the capacity curve remains the same throughout the iterative procedure). The blue line passing by point 1 and point 2 represents an estimation of how the capacity in terms of horizontal acceleration varies with the vertical acceleration applied in the analysis. The red line represents the ratio between horizontal and vertical acceleration, which, according to current Standards, is assumed constant for each level of seismic intensity. The intersection between the blue line and red line individuates the real capacity of the structure in terms of horizontal ground acceleration when the effects of the vertical component are considered. In this way the Seismic Risk Index in terms of PGA can be calculated and the procedure can be applied for each limit state.

### 6.2 Seismic assessment of a real masonry building

The objective of this study is the seismic assessment of a real masonry building located near Macerata, Italy. The assessment will be carried out through pushover analysis applying the proposed methodology for taking into account the vertical seismic effects. First the building will be assessed in its current state, then the effects of possible strengthening interventions will be evaluated. The two-story building shown in Figure 28 features a hipped roof and rectangular floor plan of 26x16 m.

Walls are made of uncut stone masonry and are well connected with each other. Perimetral walls feature a thickness of 50 cm while internal walls are 30 cm thick. The mechanical parameters of masonry are as follows: $f_m=2.00$, $\tau_0=0.035$, $E=1230$, $G=410$ (N/mm²). The achieved knowledge level is KL1, thus a confidence factor $CF=1.35$ is accounted in the analysis.

Floors consist of a series of littles vaults supported by steel beams while roof is made of timber beams and timber planks. Both floors and roof may be considered non rigid in their plane. Vertical loading is given in Table 5.
<table>
<thead>
<tr>
<th></th>
<th>Dead load</th>
<th>Live load</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First floor</strong></td>
<td>3.10</td>
<td>2.00 (Cat.A)</td>
</tr>
<tr>
<td><strong>Second floor</strong></td>
<td>4.50</td>
<td>2.00 (Cat.A)</td>
</tr>
<tr>
<td><strong>Roof</strong></td>
<td>2.25</td>
<td>1.33 (Snow)</td>
</tr>
</tbody>
</table>

Table 5. Vertical loading on floors and roof. Values in kN/m²

The parameters of the seismic action with respect to the Ultimate State of Severe Damage ($T_R = 475$) are given Table 6.

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ground acceleration</strong></td>
<td>$a_g$</td>
<td>0.223</td>
</tr>
<tr>
<td><strong>Soil factor</strong></td>
<td>$S$</td>
<td>1.185</td>
</tr>
<tr>
<td><strong>Maximum amplification factor</strong></td>
<td>$F$</td>
<td>2.415</td>
</tr>
<tr>
<td></td>
<td>$T_B$</td>
<td>0.147</td>
</tr>
<tr>
<td><strong>Periods</strong></td>
<td>$T_C$</td>
<td>0.442</td>
</tr>
<tr>
<td></td>
<td>$T_D$</td>
<td>2.492</td>
</tr>
</tbody>
</table>

Table 6. Seismic parameter for TR = 475 years

Figure 28. Floor plans and axonometric views of the building
Modelling and analyses of the building were performed using the commercial software Aedes.PCM [9] which implements the proposed methodology. The building was modelled according to the equivalent frame method with the following characteristics:

- Masonry piers are modelled with a trilinear in-plane behavior consisting of two elastic branches and one perfectly plastic. In the second elastic branch stiffness are reduced by 50%.
- Masonry spandrels are considered able to couple masonry piers only with respect to horizontal translation, thus the rotations are released at both their ends.
- Restraints. Joints at the foundation of the building are assumed fully fixed.
- Verifications. The in-plane resistance of masonry piers is governed by the following mechanisms: flexure, diagonal shear, sliding shear in case of vertical seismic action. Considering the relevant thickness of the piers their transversal stiffness is also accounted in the analysis and out-of-plane flexure verification applied at both their ends.

The results of modal analysis with respect to the fundamental mode of vibration in X, Y and Z direction are given in Table 7. Figures 29-31 show the correspondent deformed shapes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>T (sec)</th>
<th>Part. mass ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.310</td>
<td>70.1</td>
</tr>
<tr>
<td>Y</td>
<td>0.503</td>
<td>86.3</td>
</tr>
<tr>
<td>Z</td>
<td>0.085</td>
<td>14.9</td>
</tr>
</tbody>
</table>

Table 7. Fundamental mode of vibrations

Figure 29. Fundamental mode of vibration in X direction
Pushover analysis was performed through elaboration of 48 capacity curves differing for:

- distributions of horizontal forces: a linear distribution with forces proportional to masses and elevations and a uniform distribution with forces proportional to masses
- directions: X and Y directions, respectively the longitudinal and transversal direction of the building
- effects of the vertical seismic component: curves with and without the effects of the vertical forces

Figure 32 shows the curves that yielded the minimum values of the seismic risk index in terms of PGA with and without the effects of the vertical seismic effects. They are the curves in the positive and negative Y direction obtained with uniform distribution of the horizontal forces. The arrows highlight the reductions of maximum base shear and displacement capacity at ULS due to the vertical seismic component given in more detail in Table 8.
Table 8. Reduction of maximum base shear, displacement capacity at ULS and seismic risk index due to vertical seismic component $E_Z$

<table>
<thead>
<tr>
<th></th>
<th>Without $E_Z$</th>
<th>With $E_Z$</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum base shear (kN)</td>
<td>1837</td>
<td>1496</td>
<td>-19%</td>
</tr>
<tr>
<td>Displacement capacity (mm)</td>
<td>24.80</td>
<td>18.88</td>
<td>-24%</td>
</tr>
<tr>
<td>Seismic Risk Index $\zeta_E$</td>
<td>0.700</td>
<td>0.609</td>
<td>-13%</td>
</tr>
</tbody>
</table>

In order to improve the seismic capacity of the structure the following strengthening interventions were considered: (a) application of reinforced mortar coating on the perimetral walls and on the internal walls in the transversal direction of the building as shown in Figure 33, (b) stiffening of the floors through lightweight concrete slabs well connected to perimetral walls.
The application of reinforced mortar coating was accounted in the model updating the mechanical properties of masonry with a correction factor equal to 2: $f_m=4.00$, $\tau_0 =0.035$, $E=1230$, $G=410$ (N/mm$^2$). Stiffening of the floors was modelled accounting for diaphragm action on the relevant nodes.

Table 9 provides the results of modal analysis before and after interventions with respect to the fundamental mode of vibration in X, Y and Z direction. The strengthening interventions yielded an overall stiffening of the structure resulting in reduction of periods of vibrations.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Before interventions</th>
<th>After interventions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T (sec)</td>
<td>Part. mass ratio (%)</td>
</tr>
<tr>
<td>X</td>
<td>0.310</td>
<td>70.1</td>
</tr>
<tr>
<td>Y</td>
<td>0.503</td>
<td>86.3</td>
</tr>
<tr>
<td>Z</td>
<td>0.085</td>
<td>14.9</td>
</tr>
</tbody>
</table>

Table 9. Modal analysis results before and after interventions

The Seismic Risk Indexes in terms of PGA arising from Pushover analysis performed before and after interventions are given in Table 10. The proposed strengthening measures increased the seismic capacity of the structure yielding higher values of the indexes. In particular, the analyses performed without the vertical seismic effects show an increment of the index equal to $+0.182$ ($+26\%$) while the analyses performed considering the vertical seismic effects show a lower increment of the index $+0.080$ ($+13\%$). Therefore, the analysis carried out ignoring the vertical component of the seismic action tends to overestimate the beneficial effects of the strengthening measures.

<table>
<thead>
<tr>
<th></th>
<th>Before interventions</th>
<th>After interventions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without Ez</strong></td>
<td>0.700</td>
<td>0.882</td>
</tr>
<tr>
<td><strong>With Ez</strong></td>
<td>0.609</td>
<td>0.689</td>
</tr>
</tbody>
</table>

Table 10. Pushover analysis results. Seismic risk indexes $\zeta_\xi$ before and after interventions
According to current Italian regulations [6] the interventions of seismic improvement on existing buildings are accepted if they yield an increment of the Seismic Risk Index equal to 0.100. In this case the proposed strengthening measures would be acceptable if the vertical seismic effects are ignored, but they would not be enough if we consider those effects.

7 CONCLUSIONS

This study highlighted the crucial role played by the vertical component of the seismic action in the damages caused by recent earthquakes.

A methodology for considering the effects of the vertical seismic component in the seismic assessment of existing masonry building was introduced. The methodology in agreement with the current Standards allows to consider the vertical seismic effects in the local and global behavior of the structure. The local behavior governed by the out-of-plane response of walls is assessed through kinematic analysis taking into account the inertial forces induced by the vertical seismic component. The global behavior governed by the in-plane response of walls is assessed through pushover analysis, where the effects of the vertical seismic component result in a field of vertical inertial forces calculated through modal response spectrum analysis. The internal actions induced by the vertical forces are combined with those of static and incremental horizontal loading and considered in the safety verification of the elements. This leads to a global loss of resistance and displacement capacity of the structure.

The pushover analysis methodology was applied to the case study of a two-story masonry building considering different levels of masonry quality and several levels of seismic intensity. The analyses highlighted the negative effects of the vertical seismic component even for low values of the seismic action. The effects become more important as the ground acceleration increases, while the improvement of masonry quality can contrast and sometimes compensate the effects.

Further aspects of the methodology were introduced in order to make it viable for professional application in the assessment of real masonry buildings: (a) effects of compression and decompression cycles due to vertical seismic excitation and (b) capacity of the structure in terms of PGA considering the variability of the vertical seismic effects. The complete methodology was applied to the seismic assessment of a real masonry building evaluating the effects of possible strengthening interventions. The study confirmed the importance of considering the vertical seismic effects since the analysis carried out ignoring the vertical seismic component tends to overestimate the benefic effects of the proposed strengthening measures.

REFERENCES


https://www.ingenio-web.it/sfogliabile/EffettiSismaVerticale/Index.html


APPLIED ELEMENT MODELLING AND PUSHOVER ANALYSIS OF UNREINFORCED MASONRY BUILDINGS WITH FLEXIBLE ROOF DIAPHRAGM

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Abstract

Old and poorly constructed unreinforced masonry buildings are inherently vulnerable to earthquakes. Seismic analysis and assessment of these buildings with flexible diaphragms i.e. without box behavior is challenging as the available assessment methods for reinforced concrete framed or masonry structures with rigid diaphragms are not directly applicable to these buildings. In such structures, the walls loaded in out-of-plane direction and those loaded in in-plane direction behave in significantly different ways during earthquakes, making a conventional global analysis ambiguous and unreliable. In this study, using the applied element method for numerical modelling and analysis, a simple and reliable methodology for seismic performance assessment of unreinforced masonry buildings with flexible roof diaphragm is presented which includes the out-of-plane behavior in the global analysis. The applicability of the methodology for subsequent steps in fragility and vulnerability assessment of such buildings using non-linear static procedures is also clarified.

Keywords: Unreinforced Masonry Building, School Building, Applied Element Method, Out-of-Plane Behavior, Pushover Analysis, Seismic Performance Assessment
1 INTRODUCTION

It is observed that old and poorly constructed unreinforced masonry (URM) structures are susceptible to partial damage or collapse even under light to moderate earthquake shaking [1-3]. In URM buildings, one of the key construction features in controlling the seismic performance is the level of connection between the URM walls and the horizontal structures [4]. When floors and roof don’t have enough in-plane rigidity or are not properly connected to the URM walls, then the floors or roof act as a flexible type diaphragm and the commonly held assumption that all vertical structures at a story level will displace by the same amount, does not hold true. This has implications for the validity of results obtained by performing a conventional pushover analysis of such structures, hence the main aim of this paper is to present a valid strategy for pushover analysis and seismic performance assessment of URM buildings with flexible roof diaphragm.

When considering individual URM walls, the out-of-plane (OOP) response is weaker compared to the in-plane (IP) response, mainly because only the tensile capacity is mobilized dependent on cohesion, instead of the shear capacity which benefits from both cohesion and friction. Moreover, the out-of-plane behavior is also usually affected by lower stiffness and, in slender walls, subjected to P-Delta effects. Conversely, in 3-D global building configurations, where cross-walls are connected to each other as well as to the horizontal structures, the walls acting in OOP direction have better seismic behavior than a single detached wall because of the frictional resistance at the cross-wall connection and the displacement constraint provided by the horizontal structures. A comprehensive overview of the different OOP failure mechanisms of URM walls in existing buildings is presented in D’Ayala and Speranza [5]. Observations from several post-earthquake damage surveys [2-3, 6] have shown that the OOP failure modes have caused significant damage in URM structures with flexible diaphragms. Several experimental studies on such buildings [7-8] have also proven that partial or full OOP collapse are most likely to occur when the diaphragms are flexible or if the diaphragm to wall connections are poor. There are several experimental [9-12] and analytical as well as numerical [5, 13-15] works investigating the lateral OOP behavior and failure mechanisms of URM walls. A good review of the assessment methods (force-based and displacement-based methods as well as recommendations given in codes and standards) for OOP behavior of URM walls is presented in Ferreira et al. [16] and Sorrentino et al. [17]. It is observed that so far developed force-based procedures are conservative in terms of displacement capacity of OOP walls and the displacement-based procedures are based on few and specific experimental scenarios [16]. There are some issues that are still challenging in the reliable seismic assessment of OOP behavior in 3-D URM buildings when using these simplified approaches. First is the simplification of the boundary conditions. In real structures, the boundary conditions are not always rigid or simply supported as assumed in simplified assessment approaches [15]. Further, the extent of boundary conditions can vary depending on the strength of connections to the cross-wall or the geometry (e.g. opening position) and the material quality in the cross-walls. Consequently, the OOP collapse mechanism can involve just the separation of inter-locking at the connection or it can include certain portion of cross-walls to form a combined failure mechanism [5, 14]. Secondly, the presumed crack patterns have limitations and can change depending on either the geometry (e.g. wall aspect ratio, brick laying pattern) or mechanical properties of masonry or loading conditions.

In the assessment of global seismic behavior of URM structures with flexible diaphragms, many researchers have neglected or given little importance to OOP behavior and failure modes [e.g. 18-21]. This is mainly because of two reasons: the lack of real-world experimental works to characterize and quantify the load-deformation behavior in the OOP behavior and the
difficulty of modelling the large non-linearity attained by the walls in the OOP deformation before failure using FEM based procedures.

Pushover analysis based seismic performance assessment methods such as capacity spectrum method [22] and N2 method [23] are computationally efficient alternatives to time history analysis. In reinforced concrete or masonry structures with rigid diaphragm, the global seismic analysis can easily be carried out as the building’s behavior can be idealized as that of a single degree of freedom (SdOF) system. However, in flexible diaphragm type URM structures, because of the substantial difference in stiffness and displacement capacity, walls acting in IP and OOP directions usually have significantly different natural frequency of vibration and mode shapes, and hence it becomes inaccurate to represent the whole building with one SdOF system. A logical simplified approach to conduct global analysis of such buildings is currently lacking in literature.

Thus, this paper is focused on the development of a methodology for conducting non-linear static procedure based global seismic performance assessment of URM structures with flexible roof diaphragm. The paper is structured as follows: Section 2 introduces the overall methodology for the pushover-based seismic performance assessment of URM structures with flexible roof diaphragm within which the application of applied element method for the numerical modelling and analysis of URM structures is also discussed. Section 3 of the paper introduces the construction characteristics of the case study URM building typology. Section 4 presents and discusses the results from this study. Finally, some conclusions and recommendations are presented in Section 5.

2 METHODOLOGY FOR SEISMIC PERFORMANCE ASSESSMENT OF URM BUILDINGS WITH FLEXIBLE ROOF DIAPHRAGM

The overall methodology for the non-linear seismic performance assessment of single storied URM buildings with flexible roof diaphragm is presented in Figure 1. The first step involves the collection of geometrical data and identification of reliable material properties either via experimental tests or from literature on similar construction types. In the second step, the 3-D numerical model of the building is created which includes all the load bearing structures, appropriate material constitutive laws and boundary conditions. Finite element based, discrete element based, combined finite/discrete element based or applied element-based modelling tools can be used. The modelling environment should be able to simulate both the IP and OOP behavior of URM walls and the interaction among these walls as well as with the horizontal structures. In the next step, a global pushover analysis is conducted on the numerical model in which the model is subjected to an increasing lateral force pattern in a direction until global collapse is reached. By employing a global pushover analysis on a 3-D model, effects of cross-wall and diaphragm inter-actions are included in the load-deformation behavior of individual walls. From the results of this analysis, one can generate the pushover curves for each single IP and OOP walls by recording and plotting the base shear and the top displacement in the individual walls at each step of loading. If the displacement history for all the walls acting in IP (or OOP) direction are similar, then global IP (or OOP) pushover curves can be generated by summing up the individual wall’s base shear and computing the average displacement of the walls at each particular instant of the analysis. Then, the seismic performance assessment using capacity spectrum approach or N2 method can be conducted at each wall level or at global IP or OOP level. Finally, these results can be interpreted at individual wall level, global IP or OOP level or even at global building level. These different steps are further discussed in the subsections below.
2.1 Numerical modelling of unreinforced masonry

Different modelling environments and strategies have been used for numerical modelling of masonry, e.g. limit analysis based methods [e.g. 5], finite element based methods [e.g. 24], discrete element based method [e.g. 25], applied element based methods [e.g. 26], equivalent frame based method [e.g. 27] etc. Each method has pros and cons and comparison of these different methods is beyond the scope of this paper. Applied element method is used in the present study and is briefly discussed below.

Applied element method (AEM) [28], an improved variation of distinct element method, allows us to model the blocky and discontinuous nature of masonry structures. In AEM, a structure is modelled as an assembly of rigid elements connected to each other by means of distributed deformable springs. Several past studies [26, 29-30] have proved the applicability of the applied element method in simulating the crack initiation, propagation and large displacement response of URM structures under static and dynamic loading. As the global 3-D behavior of URM structures including the interaction of cross-walls can be studied, from the crack initiation to complete collapse with reliable accuracy, Extreme Loading for Structures (ELS) software [31], based on AEM, is used in the present study for the numerical modelling and pushover analysis.

In AEM, masonry is usually modelled by adopting simplified micro-modelling technique (Figure 2). Two types of springs are used, viz. ‘unit’ or ‘element’ springs connecting the applied
elements of the units, and ‘joint’ springs connecting the individual applied elements to represent the equivalent properties of mortar and mortar-unit interface. A detailed overview of the formulation, constitutive laws, failure criteria etc. for masonry modelling in AEM can be found in Malomo et al. [26].

![Figure 2: Schematic of the simplified micro-modelling of masonry using AEM.](image)

2.2 Global non-linear pushover analysis

Conventional pushover analysis [e.g. 32] of masonry structures modelled using element-by-element modelling technique, with discontinuous joint represented by springs with finite strength and stiffness that can separate during loading, is complex as the application of pushover force or displacement imposed on the structure often causes stress concentration on a particular element or region thereby causing local failure without affecting the rest of the structure. Thus, in the present study, a different approach for applying pushover loading is proposed in which the numerical model is subjected to a linearly increasing ground acceleration, rather than a force pattern on the structure, until collapse. This works by applying an increasing ‘effective earthquake force’ on the structure. In masonry structures represented by element-to-element modelling, each of the elements with same mass will get same force exerted on it. Thus, if the mass of the structure is uniformly distributed along the height, the proposed pushover loading will cause a uniform force pattern applied along the height. Such analysis represents a force-based non-linear pushover analysis, as opposed to the displacement-based pushover analysis, usually implemented for framed structures.

![Figure 3: Example capacity curves for walls acting in IP and OOP directions for an URM building with flexible roof diaphragm.](image)
When a global pushover analysis is conducted for a 3-D building with flexible roof diaphragm, the force-deformation behavior for each wall, acting in IP and OOP directions, can be generated by recording the base shear resisted by the wall and the corresponding wall top displacement along the pushover loading direction, at each step of loading. Usually, the initial stiffness and hence the natural frequency of vibration and mode shapes, maximum capacity and the ultimate displacement of walls acting in IP direction and those acting in OOP direction are significantly different (Figure 3) and it will be meaningless to combine such curves to produce a global one as the deformation and base shear are attained for each wall following a different rate of load-deformation histories.

The damage states of individual walls within the global pushover analysis can also be observed to identify the thresholds of different damage states in terms of wall drift. With this information, procedures such as N2 method [23, 33] can be applied to each wall to determine its seismic performance with respect to response spectra and damage threshold, and thereby derive fragility and vulnerability functions at individual wall level. For simplicity and computational efficiency, if the walls acting in a particular (IP or in OOP) direction have similar geometry and mass distribution, and hence similar load-deformation behavior, then seismic performance at global IP or global OOP level can be conducted by generating the global pushover curves for IP behavior (or OOP behavior), separately, using equations (1) and (2). However, it should be noted here that the individual walls acting in a particular direction are not coupled.

The total base shear and corresponding average roof displacement for global IP (or OOP) behavior:

\[ V_{G,IP\text{ (or OOP)}} = \sum_{i=1}^{n} V_{i,IP\text{ (or OOP)}} \]

\[ d_{G,IP\text{ (or OOP)}} = \frac{\sum_{i=1}^{n} d_{i,IP\text{ (or OOP)}}}{n} \]

Where, \( V_{i,IP\text{ (or OOP)}} \) and \( d_{i,IP\text{ (or OOP)}} \) are the base shear and the roof displacement, respectively of the \( i^{th} \) wall acting in IP (or OOP) direction and \( n \) is the number of walls in a particular (IP or OOP) direction.

Finally, these individual wall level (or global IP or OOP level) results can be combined to understand the building level seismic vulnerability of URM structures with flexible roof diaphragm.

3 EXAMPLE CASE STUDY BUILDING TYPOLOGY

To show the application of the proposed methodology, a single-story index building of URM (brick in cement mortar) school typology from Nepalese school building portfolio is considered as the case study building. Figure 4 and Figure 5 show some representative photographs and a typical plan of these school buildings, respectively. These buildings are mostly rectangular in plan and single storied with a double pitched light roof structure. The story height is about 3 m. Such URM school buildings are widely present worldwide in many countries [34-35]. For example, in Nepal, more than 60% of the total school portfolio consists of URM construction [36] and a large share of these construction types are old and were designed and constructed before either development or implementation of seismic design codes in the country. Nonetheless, residential buildings of similar construction types also exist in many developing countries [30, 37].

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The walls of these buildings are usually constructed in English bond pattern (one brick thick) and the seismic design measures such as horizontal tying elements (e.g. seismic bands) or vertical elements (e.g. posts or corner reinforcements) are usually absent. The openings in long walls are usually large, many and are often placed near to the wall corners.

Present material quality of these buildings varies significantly from one building to another depending on various factors such as date of construction, state of deterioration, maintenance history etc. For the present study, the material properties obtained from in-situ tests on several similar construction types [30], as listed in Table 1, are used. The values of elastic material properties reported in Table 1 are largely dispersed and comparatively lower than those usually reported in literature or suggested in codes and standards [38]. It should be noted that the elastic properties listed here are calculated from the results of in-situ shear tests on existing old buildings.

<table>
<thead>
<tr>
<th>Masonry material properties</th>
<th>Average value</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>263 MPa</td>
<td>0.79</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>158 MPa</td>
<td>0.79</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>4.14 MPa</td>
<td>NA</td>
</tr>
<tr>
<td>Cohesion</td>
<td>0.17 MPa</td>
<td>0.67</td>
</tr>
<tr>
<td>Flexural tensile strength</td>
<td>0.069 MPa</td>
<td>NA</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.6</td>
<td>NA</td>
</tr>
</tbody>
</table>

Table 1: Elastic and non-linear material properties of brick in cement mortar masonry from Nepal.
4 ANALYSIS RESULTS AND DISCUSSION

4.1 Validation of AEM modelling for lateral behavior of URM walls

The reliability of the ELS software for the analysis of URM structures is first tested by validating the numerical analysis results with experimental results.

To validate the load-deformation behavior and the failure modes of masonry walls under IP and OOP loadings, two experimental tests are considered: a solid wall with a pre-compression of 1.21 MPa tested by [39] under IP loading and another wall tested by [40] under OOP loading. These are particularly selected for validation because both the inputs i.e. wall model and boundary conditions, material properties and the outputs i.e. load-deformation behavior and failure modes are clearly reported in the literature. Figure 6 and Figure 7 compare the crack patterns at ultimate stage and load-deformation behaviors obtained from the experiment and from numerical analysis in ELS, for IP loading and OOP loading, respectively. For both IP and OOP loading scenarios, the experimental crack patterns are satisfactorily reproduced by the numerical analyses. Also, the initial stiffness, peak strength as well as the ultimate displacement capacity obtained from the numerical analysis are all well correlated with the same obtained from the experiments.

Figure 6: IP behavior: a) Experimental damage pattern at failure (reproduced from [24]), b) Damage pattern (separation cracks) obtained from ELS analysis, and c) Comparison of load-deformation behavior.

Figure 7: OOP behavior: a) Experimental damage pattern at failure (reproduced from [40]), b) Damage pattern (separation cracks) obtained from ELS analysis, and c) Comparison of load-deformation behavior.
The validation study revealed that the numerical analyses conducted using the material properties reported in the experimental studies often produce erroneous results that are not in correlation with the experimental results. In general, the elastic material properties i.e. Young’s modulus and shear modulus need to be reduced considerably to get numerical results in agreement with the experimental results. The values of elastic material properties reported in the literature are usually obtained from uniaxial compression tests on the masonry prisms and it is reported in a recent study [41] that the elastic properties obtained from prism tests are about 34% higher than those obtained from wallette tests. Thus, the reported values need to be reduced accordingly in the numerical simulation. Hence, calibrated material properties are used in the numerical analysis in sub-section 4.2.

4.2 Numerical model of the case study building

![3-D numerical model of the case study building in ELS using AEM.](image)

In ELS, a 3-D numerical model (Figure 8) of the case study URM building is created using simplified micro-modelling technique. The masonry walls are modelled exactly in the English bond pattern to represent the actual construction. For computational efficiency, bricks are modelled as rigid elements i.e. the cracks are assumed to develop through the mortar joints only. The timber lintels above the openings are modelled as elastic elements. Since the roof in these buildings is light and the roof elements are poorly connected to the masonry walls, it is not modelled structurally, but its weight is directly applied to the top of the walls, as vertical pre-compression. For the joint springs, a failure criterion is defined by specifying a separation strain, 0.05 for this particular model, obtained from calibration and depending on the maximum crack width. Once this strain limit is exceeded in a spring, the spring is completely removed, after which contact can occur between the applied elements (rigid bodies) depending on the loading condition. The numerical model consists of 12,387 applied elements and 3 x 3 sets of springs are used at the interface between any two applied elements. The building is rigidly connected to the ground at its base. The 3-D numerical model is then subjected to a linearly increasing ground acceleration along the longer direction to perform a non-linear pushover analysis.

4.3 Non-linear pushover analysis results

In this section, the failure mechanisms and the load-deformation behavior of walls acting in IP and OOP directions are presented and discussed.
Damage pattern and progression

Figure 9: Global crack patterns in the building at different stages of pushover loading (blue color represents the major damage cracks, crack width more than 12.5 mm).

Figure 9 shows the damage patterns in the case study URM building at different stages of pushover analysis. The evolution of damage shows initially a gradual separation of the OOP walls from the IP walls starting at the top of cross-wall connection (Figure 9(a)). When separation of the connection gradually reaches the lower part, then a horizontal flexural crack starts to develop in the bottom layer of the OOP walls activating the overturning mechanism (Figure 9(c)). Similarly, in the IP walls, the damage starts at the spandrels (Figure 9(b)), then the end piers damage in shear and finally the middle piers start rocking in flexure at the ultimate stage of pushover analysis (Figure 9(c)).

Load-deformation behavior

Although all the walls have developed collapse mechanisms almost simultaneously for this particular URM building model (Figure 9(c)) which is coincidental, closer analysis of load-deformation behavior of individual walls reveals that the strength and deformation relationships in the IP and OOP walls are entirely different. The elastic period of vibration of the walls acting in IP direction is about 0.16 s while the same for the walls acting in OOP direction is as high as 0.3 s. Similarly, once the cracking starts, at any given instant of the loading sequence, the wall
top displacement in the OOP walls is substantially larger than that in the IP walls, see Figure 10. It is therefore not meaningful to consider the top displacement of any wall, acting either in OOP or IP behavior, as the control node displacement for pushover curve generation for the whole building. Even if one applies some form of simple or weighted average to the displacement, thus derived ‘average’ global displacement is not realistic as one cannot directly measure it in real buildings nor it is possible to back calculate the individual wall top displacements.

![Figure 10. Variation of top displacements in IP and OOP walls under increasing lateral load.](image1)

![Figure 11: Capacity curves for individual walls acting in IP and OOP direction, respectively (see Figure 8 for wall terminology).](image2)

![Figure 12: Global capacity curves for (a) IP behavior and (b) OOP behavior.](image3)

Figure 11 shows the capacity curves for individual walls acting in IP and OOP directions. Comparing the initial stiffness, capacity and the lateral displacement, it is clear that it will be entirely wrong to combine these IP and OOP walls having significantly different load-
deformation behavior to generate a global building level pushover curve and conduct subsequent seismic performance assessment.

However, as the elastic period and the displacement history of all the walls acting in IP behavior and OOP behavior are similar, the global pushover curves for IP and OOP behavior separately are generated as shown in Figure 12 using equations (1) and (2). For efficiency, these capacity curves can be used for the subsequent seismic performance assessments of global IP behavior and global OOP behavior of the building using non-linear static pushover-based procedures [33]. Using this approach, vulnerability functions for global IP and OOP behavior can be generated separately and then combined to obtain the building total vulnerability function.

If the individual walls acting in a particular direction (IP or OOP direction) show dissimilar pushover curves (e.g. because of different wall thickness, opening layouts, connections etc.), then each of the walls should be analyzed separately and the seismic performance assessment becomes similar to a component-based approach, as detailed in FEMA P-58 [42], for example, for reinforced concrete structures.

Figure 13: Comparison of OOP pushover curves from this study with the one obtained by using analytical procedure suggested in Casapulla and Argiento [43].

Figure 13 shows a comparison of capacity curves of the walls acting in OOP behavior from this study to those obtained from non-linear kinematic analysis considering frictional resistance at cross-walls as suggested in Casapulla and Argiento [43]. The peak lateral capacity as well as the ductility from both studies show a good match. It indicates that simplified analytical procedures including frictional resistance such as the one used here for comparison seem to be useful for lateral analysis of OOP loaded masonry walls including the effect of friction at cross-wall connections. If the role of friction is neglected, the rigid body analysis of the same wall would yield a lateral capacity of about 0.1 i.e. about 75% less than the lateral capacity shown in Figure 13. Thus, the role of friction at the connection with the orthogonal walls cannot be neglected.

Is global seismic performance assessment necessary?

From the literature review and results of this study, it becomes clear that the seismic analysis of URM buildings with flexible diaphragms cannot be reliable when only the IP behavior is considered in the analysis. Conversely, one can argue that in URM buildings with flexible roof diaphragms, the vulnerability analysis of OOP walls might be sufficient when assessing the collapse. It should be noted that the walls acting in OOP behavior have considerable deformation capacity, as seen in the results presented in this study (Figure 11, 13) and supported by other experimental studies [15, 40] compared to that of the walls acting in IP behavior. Coincidentally, for this particular case study building, both the IP and OOP walls develop the collapse mechanisms at about the same instant of pushover loading application (refer to Figure 9 (c)). If
the wall thickness is more, then the walls acting in IP direction might get damaged before the collapse of walls acting in OOP direction as the OOP walls can have stability up to a top displacement equal to the wall thickness for one-way bending [15, 44-45] or even larger in case of two-way bending [11]. Furthermore, when the damage is limited and repairable (i.e. when the building is in slight or moderate damage state), it becomes necessary to understand the damage level in all the walls i.e. global damage level for deciding the repair and retrofit methods, costs etc. Thus, the global analysis of building is important and necessary in post-earthquake damage and repairability assessment as well as in the seismic risk assessment studies of existing URM buildings.

5 CONCLUSIONS

- AEM can be used for the 3-D numerical modelling and lateral load analysis of URM structures with reliable accuracy. This has been proven by validations of the IP and OOP behavior of experimentally tested walls and the results of the analysis on a 3-D URM building.

- Application of linearly increasing ground acceleration seems to be an effective method of force-based pushover application in URM buildings with flexible diaphragms.

- In single storied URM buildings with flexible roof diaphragm, the individual walls in IP and OOP direction behave in significantly different manner, and a global pushover curve cannot be generated for reliable non-linear static procedures based seismic performance assessments. Instead, pushover curves for individual walls should be generated which includes the interaction effects with cross-walls; and each wall (acting in IP and OOP direction) needs to be analyzed individually. Global IP or OOP analyses can be conducted in case the pushover curves for all the walls in a particular (IP or OOP) direction are similar.

- The flexible diaphragm is not modelled in this study because of the poor quality of roof connections in the considered building typology from Nepal. However, flexible diaphragms with good connections can alter the failure modes and load-deformation behavior and should be modelled.

ACKNOWLEDGEMENT

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ASSESSMENT OF GEOTECHNICAL AND SEISMIC RISK FOR CULTURAL HERITAGE SITES – THE STABLE PROJECT

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Abstract

This paper presents the ongoing Research Project entitled: “STructural stABiLity and risk assEssment – STABLE” with the collaboration of four universities, three private companies and one research center located in Greece, Italy and Cyprus. The aim of the STABLE Research Program is to develop a combined seismic and geotechnical risk assessment platform. Two specific areas have been selected from Greece as Working Case Studies for the platform development. The first site is the Historic Center of Nafplion and the second site is the Technological Park of Lavrion in Attica. The reliability of the platform will be calibrated based on real earthquake results from Accumoli, a recently earthquake stricken area of central Italy (2016). The platform will be applicable in the future in other regions as well. The platform exploits the technological developments in the areas of structural health monitoring and remote sensing as well as new insights on the assessment of structural stability and vulnerability. It aims to be a useful tool for decision making on the protection of the built environment with an emphasis on cultural heritage.

Keywords: STABLE Research Project, Risk Assessment Platform, Earthquake Risk, Geotechnical Risk.
1 INTRODUCTION

Preserving the cultural heritage is an issue that has been a concern for many European countries, including Greece, Italy, Germany, France and England since the beginning of the 19th century. These countries developed and followed different approaches for the restoration of historic structures and monuments. An exemplary case is the rehabilitation program for the ancient monuments of Rome during the first three decades of the 19th century (1800-1830). The first effort to adopt a common European policy on the preservation of cultural heritage dates back to the Athens International Conference of October 1931, where the first European Charter for the Restoration of Monuments of Art and History was presented [1]. In the decades that followed a number of directives, e.g., [2, 3] provided the principles and the philosophy of interventions on historic buildings and monuments without, however, going into details on the suitable intervention methods.

In recent years it has become clear that collaboration of experts from different fields of science is required to achieve an efficient way of intervention, since the structural integrity of monuments may be threatened by different disasters, including the climate change, geotechnical risks or earthquakes. However, there are limited case studies that highlight this need for an interdisciplinary approach [4] involving architects, structural engineers, surveying engineers, chemical engineers, art historians, archaeologists, experts in the preservation of materials and conservators [4-6].

The assessment of the vulnerability of each monument presents significant challenges that may be related to the selection of proper modeling and analysis methods. Especially as regards existing structures made of unreinforced masonry, the current state of damage, the highly non-linear response, pounding phenomena and the development of local collapse mechanisms may complicate the computational effort [7-13]. On the other hand the relevant legislative documents remain particularly limited [14].

Additionally, recent disasters at historic centers have highlighted the need to assess the structural vulnerability of conventional and monumental constructions on a larger scale than that of a single structure in order [15-17]: (a) to assess the distribution of the damage during a potential future catastrophic event, like an earthquake or an earthquake-induced landslide; (b) to prioritize the need for interventions. Within this framework the vulnerability of structures is assessed with the use of probability-based approaches [18].

Within this context, this work presents an ongoing research project entitled: “STructural stABIliTy risk asEssment – STABLE”. The main goal of STABLE is the development of a platform that will address seismic and geotechnical risk for cultural heritage sites at a medium scale (block of buildings and large structures) in order to derive damage maps before the occurrence of a seismic or geotechnical event and define the areas of maximum probable loss. Two sites are selected in Greece for the application of the methodology and the development of the platform: (a) the Historic Center of Naobilion; (b) the Technological Park of Lavrion in Attica. Both sites include historic structures, representative of the architectural history of Greece from the early 19th until the mid 20th centuries. The first site includes mainly residential buildings while the second one includes mainly industrial buildings. The platform aspires to be a decision-making tool that will be useful for the preservation of cultural heritage assets.

2 PROJECT TEAM AND MANAGEMENT STRUCTURE

2.1 Work Packages

The conducted research is organized in ten Work Packages, WP, namely:

WP 1: Monitoring System.
The main objective of Work Package 1 is to perform the monitoring of cultural heritage sites and related surrounding areas, by integrating novel remote sensing technologies, such as satellite, terrestrial and autonomous airborne inspection systems in order to identify the critical areas and/or structures affected by deformation.

**WP2: Seismic movements.**

The main objective of WP2 is to provide revised estimations regarding earthquake hazard in order to arrive to the estimation of seismic risk maps in conjunction with the estimation of vulnerability within the context of WP3. In the context of WP2 a database including existing geological, geotechnical, geophysical and groundwater information will be created for each test sites.

**WP3: Structural Stability.**

The objective of this work package is the evaluation of the vulnerability of existing buildings with a focus on the residential and/or historic structures. The two key elements of a vulnerability analysis are the capacity (strength and deformation capacity) of a building and the demand either from earthquake loads or from geotechnical deformations.

**WP4: Thematic Platform.**

The objective is to design, develop, and verify a WebGIS based System (STABLE Web Portal) integrating EO methods and Risk Modelling systems, dedicated to the risk management of Cultural Heritage sites.

**WP5: Data Integration and fusion.**

The objective of this work package is to implement efficient techniques towards building detection from time series data (orthomosaics and DIM point clouds) for each study area. A 2D and 3D building change detection approach will be also carried out.

**WP6: Demonstration.**

The main objective of WP6 Demonstration is the validation of the STABLE methodologies against project’s test cases. The activities of the demonstration work package will address the risk modelling systems enabling the forecast of structural stability of CH in different scenarios of seismic movements.

Figure 1: Schematic interrelation of the work packages.
WP7: Transfer of Knowledge, training and networking.
This WP regards training and networking actions under an inter-disciplinary research and transfer umbrella. Workshops, conferences and summer schools will allow researchers to exchange their knowledge between the different fields of engineering.

WP8: Communication, Dissemination and Exploitation.
WP8 will provide communication and dissemination through conferences and outreach events.

WP9: Project Management.
The management of the project (WP9 Project Management) is based on shared responsibility, joint ownership and good communication.

WP10: Ethics requirements.
The interrelation of the different work packages is depicted in Figure 1.

2.2 Partners
The research team includes partners from five universities and research organizations and three companies, as listed in the following:
- ALMA Sistemi Srl, Guidonia (Rome), Italy
- National Technical University of Athens (NTUA), Athens, Greece
- Universita degli Studi di Roma «La Sapienza» (CERI-UNIRM), Roma, Italy
- Universita della «Tuscia» (UNITUS), Viterbo, Italy
- Frederick University (FU), Nicosia, Cyprus
- GeoSystem Hellas (GSH), Athens, Greece
- Space System Solutions Ltd (S3), Nicosia, Cyprus
- Foundation for Research and Technology Hellas (FORTH), Rethymno, Greece

Numerous young and senior researchers are participating from Greece, Italy and Cyprus.

3 RESEARCH TEAM FROM NTUA
The research team that participates in STABLE from the National Technical University of Athens includes researchers of two major Laboratories of the School of Civil Engineering:
- the Laboratory for Earthquake Engineering, LEE-NTUA (http://lee.civil.ntua.gr/index_en.html), and
- the Geotechnical Engineering Department, GED-NTUA (http://www2.civil.ntua.gr/geotech/index.php/en/).

The main research field of the NTUA team is related to the effects of earthquakes and vibrations on structures with emphasis on Cultural Heritage sites and lab-based testing of soils and rocks, monitoring of ground vibrations, engineering behaviour of anisotropic rocks and heterogeneous rock masses, risk assessment of geohazards (landslides and rockfalls) for the protection of Infrastructure and historical monuments.

3.1 LEE-NTUA Infrastructure and Experience
In LEE an Earthquake Simulator with six degrees-of-freedom has been installed and used since 1986. The shaking table consists of a rigid steel platform with dimensions 4x4x0.6 (m) and 100 kN weight. It is capable of simultaneous vibration in all six degrees-of-freedom (6
DOFs), with vertical load capacity of 640 kN and a horizontal reference at 320 kN. The maximum acceleration in both horizontal directions that can develop is 2g, while in the vertical direction is 4g. The operating frequency rate of the simulator ranges from 0.1 up to 50 Hz. The generation and processing of seismic excitations and analysis of the experimental results is performed by special signal processing programs available in the library of LEE.

An independent reaction wall is also available at the laboratory. Its capacity is 10 MNm, is 6 m in height and consists of two sections each 4 and 5 m long, 1.1 m width. Several jacks are installed and may function with independent control. The jacks have capacities of 500 kN, 300 kN, 200 kN and 100 kN. A mobile data multi-channels acquisition system is supporting this facility. An innovative testing apparatus is also functioning at the laboratory in order to test specimens under recyclic loads horizontally applied, such as walls with up to 1.5 m height, 1.0 m long and 0.5 m thickness. The vertically applied load may be up to 1000 kN and can be kept constant without any variation throughout the tests. Innovative testing apparatus are also functioning at the Laboratory in order to measure mechanical characteristics (compression strength, modulus of elasticity, Poisson’s ratio, stress-strain curve) of masonry specimens.

Diverse instrumentation is also available for in-situ testing such as flat jacks, ambient testing, displacement strain measurements, pull-out tests, etc.

In 2000 the LEE-NTUA was certified by TUV CERT (TUV Austria Hellas), for Dynamic and Seismic Tests using the earthquake simulator, according to ISO 9002/1994. Starting on 12/1/2007, it implemented a Quality Management System for DYNAMIC SEISMIC TESTS certified by TUV Austria (the Austrian Organization of Inspection and Certification). The current relevant certification of the LEE by this organization is according to standard EN ISO 9001:2008.

In addition, LEE-NTUA has a highly experienced staff in preparing, testing and analyzing the experimental results. The staff has over twenty five years of experience and participation in the successful completion of a multitude of national and international research programs. A representative number of international projects relevant to the proposed research are listed:

- Evaluation and Retrofit of the Church of the Holy Sepulcher (Patriarchate of Jerusalem, Israel, 2013-17)
- Seismic Protection of the Showcases of the Louvre-Abu-Dhabi Museum (Meyvaert Glass Engineering SA, United Arab Emirates, 2015-17)
- New Integrated Knowledge based approaches to the protection of cultural heritage from Earthquake-induced Risk – NIKER (DCT - Università degli Studi di Padova).
- Seismic Engineering Research Infrastructures for European Synergies – SERIES (University of Patras/ European Commission/ Framework Programme 7).
- Seismic Resistance of new Reinforced or Isolated Perforated- Brick Masonry Housing Construction in Low Seismic Region (ECOLEADER Access to Research Infrastructures).
- Seismic Behaviour of Capacity designed masonry Walls in low Seismicity regions (ECOLEADER Access to Research Infrastructures).
- Experimental evaluation of technical interventions for reduced seismic vulnerability of old existing buildings.
- Experimental evaluation of technical interventions to reduce seismic vulnerability of masonry buildings.

4 SELECTED SITES IN GREECE AND ASSESSMENT OF RISK

Five case-study sites have been selected in order to validate and demonstrate the platform that is developed within the STABLE Project. Two of them are located in Greece, namely, the historic center of the Nafplion town, in Peloponnese and the Lavrion Technological Park in Attica.

The town of Nafplion is a seaport town in the Peloponnese, Greece near the north end of the Argolic Gulf. The town was the capital of the First Hellenic Republic and of the Kingdom of Greece, from the start of the Greek Revolution in 1821 until 1834, while today is the capital of the regional unit of Argolis. Nafplion was originally founded on relatively stiff ground, but during the recent years expanded along the coastline on ground that presents a questionable safety factor against liquefaction risk [19]. The main threats that the site faces in addition to liquefaction are the subsidence and foundation problems because of changes in aquifer level and the vulnerability of historic buildings against earthquakes. An overall view of Nafplion is shown in Figure 2a. Currently the available data regarding the geological documentation and informations regarding the state of damage of the built environment are collected.

The city and ancient mines of Lavrion are located approximately 50 kilometers south of Athens, in Greece. Lavrion is geographically located in the area of the Geohazard Supersites and Natural Laboratories GEO initiative. The ancient mines were abandoned in the 1st century BC, "rediscovered" in the 19th century by French and Greek companies and worked until 1977. A number of remnants of the mines, including shafts, galleries and surface workshops are still present in the region. The Technological Park includes forty-one buildings of which approximately the 1/3 has been restored up today (Figure 2b). The age of construction ranges between 1875 and 1940. Currently all available data regarding the area are collected including, topography-surveying data, geotechnical data and structural details regarding the restored buildings. The site is submitted on the Tentative List of UNESCO World Heritage List. The main threat that the site faces is the subsidence by old mining works that may affect historical buildings. The site has not been investigated yet using satellite methods in regard to subsidence and structural integrity problems.

![Figure 2: Selected sites in Greece: (a) the historic center of Nafplion; (b) the Lavrion Technological Park.](image)

As a first step, the vulnerability of structures within the selected case-study sites should be assessed. Following a probabilistic approach the seismic vulnerability of a structure for earth-
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Quake loads can be represented in the form of a series of curves that illustrate the likelihood of exceeding a structural performance level, namely a Limit State, caused by different ground shaking intensities, the so-called fragility curves [20, 21]. Today there are numerous methods to assess structural vulnerability that may be divided to the following categories: direct, indirect, conventional and hybrid techniques [22]. Since the evaluation of risk should be made for extended areas, the application of a detailed mechanical assessment would be prohibitive, at least for all the buildings, located within the considered sites.

The estimation of the vulnerability at a site level will be performed by a simplified mechanical method, e.g., [23]. For limited structures more detailed vulnerability estimation will be performed, by developing thorough numerical models and performing suitable analysis to assess the exact response. In the first case, both the capacity curve, i.e., the correlation of lateral seismic force and equivalent deformation, and the variance of the damage at a specific level of the seismic intensity will be obtained by utilizing data from buildings of similar typology. The aim is to calculate the probability of exceeding the damage in an extended area in the city of Rhodes. The needed input parameters include geometrical data (number of floors, total height, floor plan, wall thickness), data related to the use of every structure, era and code of construction. These data will be derived from the existing urban model in three dimensions and the geospatial base, from collection of available information and from limited autopsy.

5 CONCLUDING REMARKS

This paper presents the main aims and aspects of the ongoing research program entitled: “STructural stABiLity and risk assEssment – STABLE”. The aim of the STABLE Research Program is to develop a seismic and geotechnical risk assessment platform.

Specific areas have been selected in Italy, Greece and Cyprus as case-studies for the platform development, including the Historic Center of the town of Nafplion and the Lavrion Technological Park in Greece. The reliability of the platform will be calibrated based on real earthquake results from past earthquakes in Accumoli that was recently stricken by earthquakes in central Italy in 2016. The platform will be applicable in the future for other sites as well. The platform exploits the technological developments in the areas of structural monitoring and remote sensing as well as new insights on the assessment of structural vulnerability under seismic loads and geotechnical parameters. It aims to serve as a useful tool for decision making on the protection of the built environment with an emphasis on the preservation of cultural heritage.

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